上海交通大学硕士学位论文

COMPETITION OF SOCIAL OPINIONS ON TWO LAYER NETWORKS

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COMPETITION OF SOCIAL OPINIONS ON TWO LAYER NETWORKS

摘 要

Social conflict can be explained with competition network of two layers. This paper is investigated for a model with the competition between two-layer opinions, where the first layer is opinion formation and the second layer is decision making, on interconnected networks. Networks show the two interacting social sectors, the civilians, and representatives. Layer A is civilian opinion layer consists of four states $(-2, -1, +1, +2)$. These states describe the level of influence of opinion dynamics with reinforcement parameter γ. The layer B is the decision making layer that consists of only two states $(+1, -1)$. This layer can influence the decision dynamics with the probability in which decision is proportional to the number of interaction with the opposite opinion population raised to the power of β. Starting with a polarized competition case, layer A is all positive and layer B is all negative. In this paper, we create new models by changing the network structure, and compare these models with the pre-existing model. Then conditions are investigated that have the influence to opposite side and that make consensus in the interconnected network. This study could help to analyze social networks, such as legalization of social issues and prediction of vote results. Further more, it could contribute to solving the social conflict. 关

键词：complex network, interconnected network, modeling and simulation, social network analysis, opinion dynamics, consensus, language competition dynamics

COMPETITION OF SOCIAL OPINIONS ON TWO LAYER NETWORKS

ABSTRACT

Social conflict can be explained with competition network of two layers. This paper is investigated for a model with the competition between two-layer opinions, where the first layer is opinion formation and the second layer is decision making, on interconnected networks. Networks show the two interacting social sectors, the civilians, and representatives. Layer A is civilian opinion layer consists of four states $(-2, -1, +1, +2)$. These states describe the level of influence of opinion dynamics with reinforcement parameter γ. The layer B is the decision making layer that consists of only two states $(+1, -1)$. This layer can influence the decision dynamics with the probability in which decision is proportional to the number of interaction with the opposite opinion population raised to the power of β. Starting with a polarized competition case, layer A is all positive and layer B is all negative. In this paper, we create new models by changing the network structure, and compare these models with the pre-existing model. Then conditions are investigated that have the influence to opposite side and that make consensus in the interconnected network. This study could help to analyze social networks, such as legalization of social issues and prediction of vote results. Further more, it could contribute to solving the social conflict.

KEY WORDS: complex network, interconnected network, modeling and simulation, social network analysis, opinion dynamics, consensus, language competition dynamics

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Nomenclature

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Chapter 1 Introduction

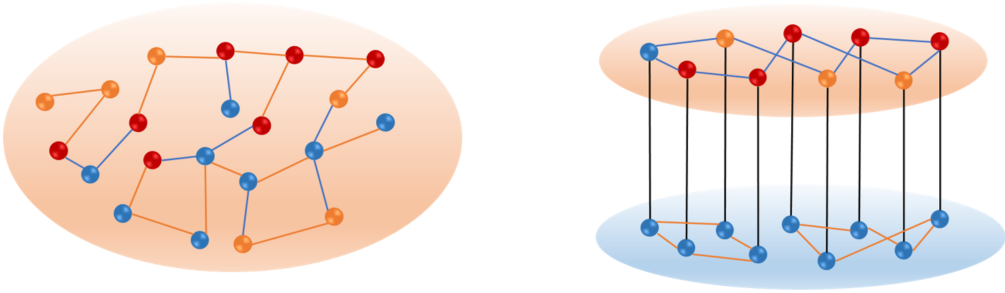
1.1 Introduction

People have their own opinions, and sometimes they change their opinions in response to others that hold views on given issues. Their opinions are reflected to the leader to make laws and vital decision. These phenomena can be found out in some cases, such as voting, legislation and adoption of new policies. It is widely recognized that opinion formation and decision making formation have mutual interaction as interconnected networks.[mikko2013, danziger2019, newman2010, boccaletti2014, domenico2013, tomasini2015, namkhanhvu2017]. And sometimes, opinion formation could be opposed to decision making formation. These situations often make social conflict and cause social confusion. To figure out these social conflicts, it is needed to understand and analyze the competition of interconnected networks. So far, physics and computer science have researched these social conflict by modeling and analyzing the complex systems[huberman2004, zuev2012, laguna2004, masuda2015]. The researches include opinion dynamics, voter model, game theory and many more[bianconi2018].

Competition of interconnected networks has been researched in many ways. These networks can be applied to the dissemination of computer viruses, messages, opinions, memes, diseases and rumors[hua2014, shenyu2018, zhou2018, alvarez2016, gomez2015, diep2017, rocca2014, velasquez2018]. Opinion dynamics on interconnected networks are investigated with various network models such as *Abrams-Strogatz(AS)* model[abrams2003, vazquez2010] and M model[rocca2014]. Based on the previous researches, we would study the main features of competing two-layer networks by changing network structures, changing the way to interact, and finding the key nodes on layers.

1.2 Related Work

In this research, we focus on social conflict and competition on multi layer network or interconnected network. Comparing with single layer, interconnected network has 2 dynamics, 2 parameters and include internal edge and external edge. Therefore, multi layer network would be more complex than single layer network. To make two layer networks under competition, each layer is made up with different dynamics network. Network dynamics are based on previous research such as[alvarez2016] One layer has the



Single layer	Multi layer(interconnected network)
1 dynamics	2 dynamics (each layer has it own dynamics)
1 parameter	2 parameter
Internal edge	Internal edge + external edge

Figure 1–1 Comparison between single layer and multi layer

function of social opinion and its own dynamics. Some opinion models provide social mechanism by means of a compromise process.[naim2003] Some opinion models represent persuasive process.[chau2014] In this paper, the social opinion layer is affected by the opinion dynamics which are also known as M-model[rocca2014], that includes compromise function and persuasion function. The other layer also has the function of decision-making and its own dynamics. The dynamics of the decision making layer is the language competition dynamics that are also called as Abrams-Strogatz model[abrams2003, vazquez2010, patriarca2012]. This model is useful to decide only one opinion from two opinions. For competition condition, the initial condition of the two layers is assumed to be in opposite states, social opinion layer has all positive states, decision making layer has all negative states.

So far, main researches have focused on what factors make a consensus or dissent, which have shown that the system can make positive consensus, negative consensus or coexistence under certain range of parameters, such as volatility, reinforcement, and prestige. Also, it is found out that the thresholds make the transition of states and they can explain and analyze the social phenomena in real world such as the legislation, election result, and social network[alvarez2016, amato2017, diep2017]. In[gomez2015], it is shown that the transition from localized to mixed status occurs through a cascade from poorly connected nodes in the layers to the highly connected ones. In addition, the main features, such as transition and cascade, found in Monte Carlo simulation are exactly characterized by the mean-field theory and magnetization[alvarez2016, diep2017, amato2017, gomez2015].

To change the structure of network, network structures would be investigated.

Network can be largely divided into regular network, random network[erdos1960], small world network[watts1998], scale free network[barabasi1999] and others. Regular network has lattice structure, and each node has exactly the same number of links. Random network is made up with edges that two node are connected with probability p in the systems with K nodes. Small world network is a network graph in which most nodes are not neighbors of one another, but most nodes can be reached from every other node by a small number of links. Small world network can be made by eliminating the edges with probability p and connecting two random nodes that are not connected in a regular network. Small world network has all characteristics of regular network and random network. Scale free network has the model that distribution of edges follows power function. Examples of scale free network are the World Wide Web (WWW), the Internet, movie star networks, protein interactions, metabolism, and so on. There are several ways to create a scale free network. Among them, the most typical way is Barabasi-Albert models.

The Barabasi-Albert model is growing networks in which nodes continue to be added, and connections between nodes has preferential attachment. The process of creating this model repeats the following two processes: First, add one node with a constant number of edges to the system every hour. Second, edges of the added nodes are connected in proportion to edges number of the pre-existing nodes In this paper, two type of general network would be applied such as Random-regular network and Barabasi-Albert network.

For further understanding the competition on two-layer network, it is very important to investigate the interaction between nodes or layers. Methods of interaction between nodes are very various.[sirbu2017] But, related to time, the types of interactions would be divided into two categories, simultaneous interaction and sequential interaction. In economics and social networks, it has been proven that there exists different results between simultaneous and sequential interactions.[hoffman2011, dietrich2004] In[hoffman2011], it was researched that how experimental subjects update exogenously induced prior information when receiving two information signals simultaneously versus receiving the same signals sequentially. As the experimental results, the simultaneous treatment is very different from sequential treatment, and under sequential information, subjects’ mean estimates of the two treatments(good news preceding bad news or vice versa) are also significantly different from each other. In conclusion, both sequencing and the order of information processing suggest which one arrives matters. And, in[dietrich2004], the usual random sequential updating rule is replaced by simultaneous updating on the Sznajd model. As the results, it is found out that this change makes a complete consensus much more difficult. The reason is analyzed as that for simultaneous updating some agents simultaneously receive conflicting messages from different neighbor pairs and thus refuse to change their opinion.

To find key nodes on two layers network, network centralities would be researched. Network centrality means the index to measure how close each node is to the center of the network. Than means answers to the question "What characterizes an important node?". The concept of network centrality was first introduced in the field of social network analysis.[freeman1979] After that, it has expanded to various areas where the concept of the network is related and has been used to identify which nodes are important in the network. So far, various criteria for assessing network centrality have been presented. Generally well-known network centralities include degree centrality, betweeness centrality, closeness centrality, eigenvector centrality and pagerank centrality.[koschutzki2008]

Degree centrality is the simplest but the most reliable concept. it is defined as the number of interacting neighbor nodes (or edges). Betweenness centrality is the concept of using the shortest path between two nodes on a network. It is explained as the concept to define two different node sets on the network (set 1, 2) and quantify how often each node appears on the shortest path for all combinations of nodes in set 1 and set 2. Closeness centrality is derived from that the shorter the path that one node reaches all the other nodes is, the more important the node is. Eigenvector centrality is the concept that the more a node is connected with critical nodes, the more important it is. Pagerank centrality measures the convergent value by repeating the process of propagating each node's influence to the other nodes.

In this paper, as the methods to find key nodes, network centralities are researched such as pagerank, degree centrality, eigenvector centrality, betweenness, and closeness.[francisco2019, bianconi2018] By using 5 centralities(pagerank, degree, eigenvector, closeness, betweenness), it would be found out that which property is the most influential. Finally, the best method would be provided to find key nodes on the competing two layers.

1.3 Thesis Objective

In this paper, opinion dynamics of a competing two-layer social network are investigated on the basis of the pre-existed research[alvarez2016, gomez2015, diep2017, rocca2014]. As the result of pre-existed research, interconnected competition of the social network have been researched by finding the threshold or critical point for consensus[alvarez2016, gomez2015, diep2017]. It has been proved that the system can make positive consensus, negative consensus or coexistence parts in interconnected competition of the social network[alvarez2016]. And it is shown that the number of external degree is very important to change the state of layers[gomez2015]. We develop the previous modeling and research to find out the characteristics of interconnected networks. By switching the network structure of each layer, such as changing the number of nodes or the number of edges, we can see how the consensus or coexistence states change and what conditions make the social consensus. This can help to explain social networks phenomena, such as social conflict between social opinion and the congress. Therefore, this research could be used as a tool for analyzing legislation problems, making efficient decision-making system and solving the social conflict.

Researching directions have 4 main topics. First, it would be provided how to make up competition models and how to measure the consensus for analysis. Second, it would be found out what factors make consensus by changing network structures. Second, it would be analyzed whether dynamics orders have an influence on status of two-layer. Third, it would be investigated which method is the best to identify key nodes based on node centralities.

This paper is organized as follows. In chapter 2, it is introduced that how competing two layers are made up and how the dynamics of each layer works. And some indexes are provided to measure and evaluate the simulation results. In chapter 3, with changing network structure, it would be found out that how the network structures have the influence on the consensus of two layers. In chapter 4, considering the dynamics orders and updating rules, simulation results would be compared and analyzed. In chapter 5, it would be researched that which nodes are important to affect the state of network by using node centralities. Finally, in chapter 6, all simulation results will be summarized and our findings are concluded. And it is considered that how the results are applied to the real world.

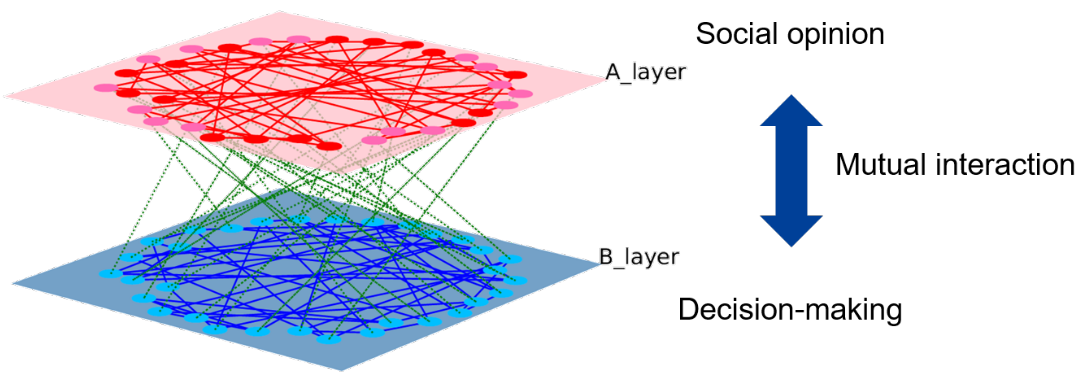


Figure 1–2 The example of competition on two-layer network

Chapter 2 Modeling and Analysis

In this chapter, a basic model would be introduced for competition on two-layer network. It would be also explained that how each layer is made up and what kind of function and dynamics it has. After modeling, many simulations would be fulfilled under the various conditions. Some indexes would be provided to analyze the interaction between two-layers. Simulation results would be analyzed with these indexes.

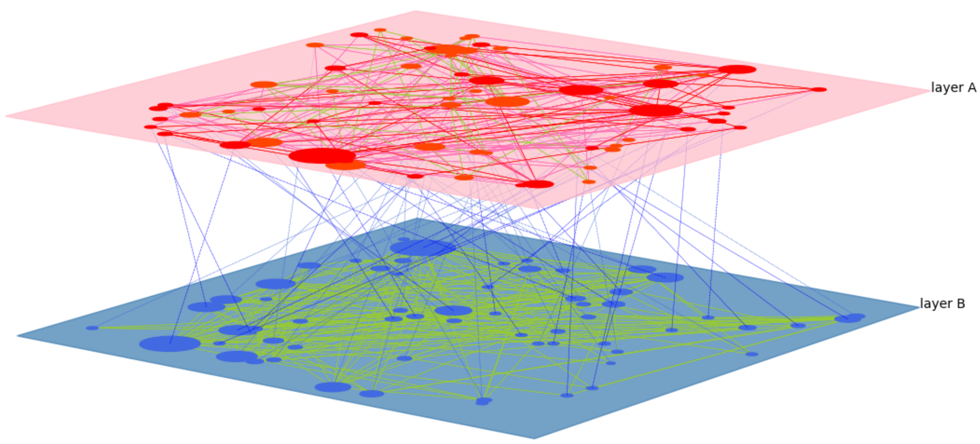


Figure 2–1 Competition of Interconnected Network

2.1 Modeling of two layer network

The model consists of two layers, and each layer has different dynamics. For layer A, the node change its states according to M model as introduced in[rocca2014]. Here, we choose $M = 2$, that each node has four states $(-2, -1, +1, +2)$. For each link (k, j) belong to layer A, the dynamics are designed as follows:

- Compromise : if two nodes connected with link(k, j) have opposite orientations, their states become more moderate with probability q :

$$\text{if } S_k < 0 \text{ and } S_j > 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^r, S_j^l) \text{ with } prob.q, \tag{2–1}$$

$$\text{if } S_k > 0 \text{ and } S_j < 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^l, S_j^r) \text{ with } prob.q. \tag{2–2}$$

If $S_k = \pm 1$ and $S_j = \mp 1$, one switches orientation at random:

$$(\pm 1, \mp 1) \rightarrow \begin{cases} (+1, +1) \text{ with } prob.q/2, \\ (-1, -1) \text{ with } prob.q/2. \end{cases} \tag{2–3}$$

- Persuasion : if two nodes connected with link(k, j) have the same orientation, their states become more extreme with probability p :

$$\text{if } S_k < 0 \text{ and } S_j < 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^l, S_j^l) \text{ with } prob.p, \tag{2–4}$$

$$\text{if } S_k > 0 \text{ and } S_j > 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^r, S_j^r) \text{ with } prob.p. \tag{2–5}$$

For each external link (k, j) with k belong to layer A, the state of node k is updated according to :

- $S_k \cdot S_j < 0$:

$$\text{if } S_k < 0 \text{ and } S_j > 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^r, S_j) \text{ with } prob.q, \tag{2–6}$$

$$\text{if } S_k > 0 \text{ and } S_j < 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^l, S_j) \text{ with } prob.q. \tag{2–7}$$

- $S_k \cdot S_j > 0$:

$$\text{if } S_k < 0 \text{ and } S_j < 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^l, S_j) \text{ with } prob.p, \tag{2–8}$$

$$\text{if } S_k > 0 \text{ and } S_j > 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^r, S_j) \text{ with } prob.p. \tag{2–9}$$

Here, S_k^r and S_k^l denote the right and left neighboring states of k, defined as

$$S_k^r = \begin{cases} +1, \text{ for } S_k = -1 \\ +2, \text{ for } S_k = +2 \\ S_k + 1, \text{ otherwise }, \end{cases} \qquad S_k^l = \begin{cases} -1, \text{ for } S_k = +1 \\ -2, \text{ for } S_k = -2 \\ S_k - 1, \text{ otherwise }. \end{cases} \tag{2–10}$$

The sign of S^A represents its opinion orientation and its absolute value $|S^A|$ measures the intensity of its opinion. So, $|S^A| = 2$ represents a positive or a negative extremist, while $|S^A| = 1$ correspond to a moderate opinion of each side. In case of internal link (k, j) belong to layer A, when the nodes have the same orientation($S_k S_j > 0$), if the states of nodes are moderate, then they become extreme($S_k = \pm1 \to \pm2, S_j = \pm1 \to \pm2$) with probability p. If they are already extreme, they remain extreme($S_k = \pm2 \to \pm2, S_j = \pm2 \to \pm2$). On the other hand, when the nodes have opposite orientations($S_k S_j < 0$), if they are extreme, the states of nodes become moderate($S_k = \pm2 \to \pm1, S_j = \pm2 \to \pm1$) with probability q. If they are already moderate, they switch orientations individually($S_k = \pm1 \to \mp1, S_j = \pm1 \to \mp1$). In case of interaction between node in layer A and node in layer B, node in layer A follows opinion dynamics formula, but the state of node in layer B does not change. In other words, the state of layer B affects layer A, but layer A dynamics does not affect the state of node in layer B. For example, one of the layer A node, $S_k = +2$ is connected with $S_j = -1$ node of layer B. Here, S_k will change into $S_k = +1$ with $prob.q$. But S_j will not change, which indicates that the states of layer B will influence the states of layer A.

The dynamics of layer B follows the decision-making dynamics as introduced in[abrams2003, vazquez2010]. The state of node i in layer B can be $+1$ and -1, and it updates according to

$$P_B(S_i \to -S_i) = \begin{cases} \left(\left(\dfrac{i_i + e_i}{n^{-S_i}} \right) \cdot \left(\dfrac{n^{-S_i}}{i_i + e_i} \right) \right)^{1/v}, \text{ if } v \neq 0 \\ 0, \text{ if } v = 0 \\ 0, \text{ if } n^{-S_i} = 0 \end{cases}, \tag{2–11}$$

where i_i is the number of internal edges and e_i is the number of external edges. n^{-S_i} is the number of neighbors of i with opposite state $-S_i$. v represents the volatility that measures how prone a node change its state. The scale of v is from 0 to 1. If $v \simeq 0$, a node is unlikely to change its state. On the other hand, if $v \simeq 1$, a node is very likely to change its state. Also, this formula shows that the more the number of nodes connected with the opposite state is, the easier the nodes are to change into the opposite state.

2.2 Simulations and Analysis

To start with a polarized competition, as the initial conditions, nodes in layer A are all positive, and nodes in layer B are all negative as shown in Fig. 2–1. For nodes in layer A, it begins with the status where half of nodes are +1 and the others are +2. The initial state of nodes in layer B have only -1.

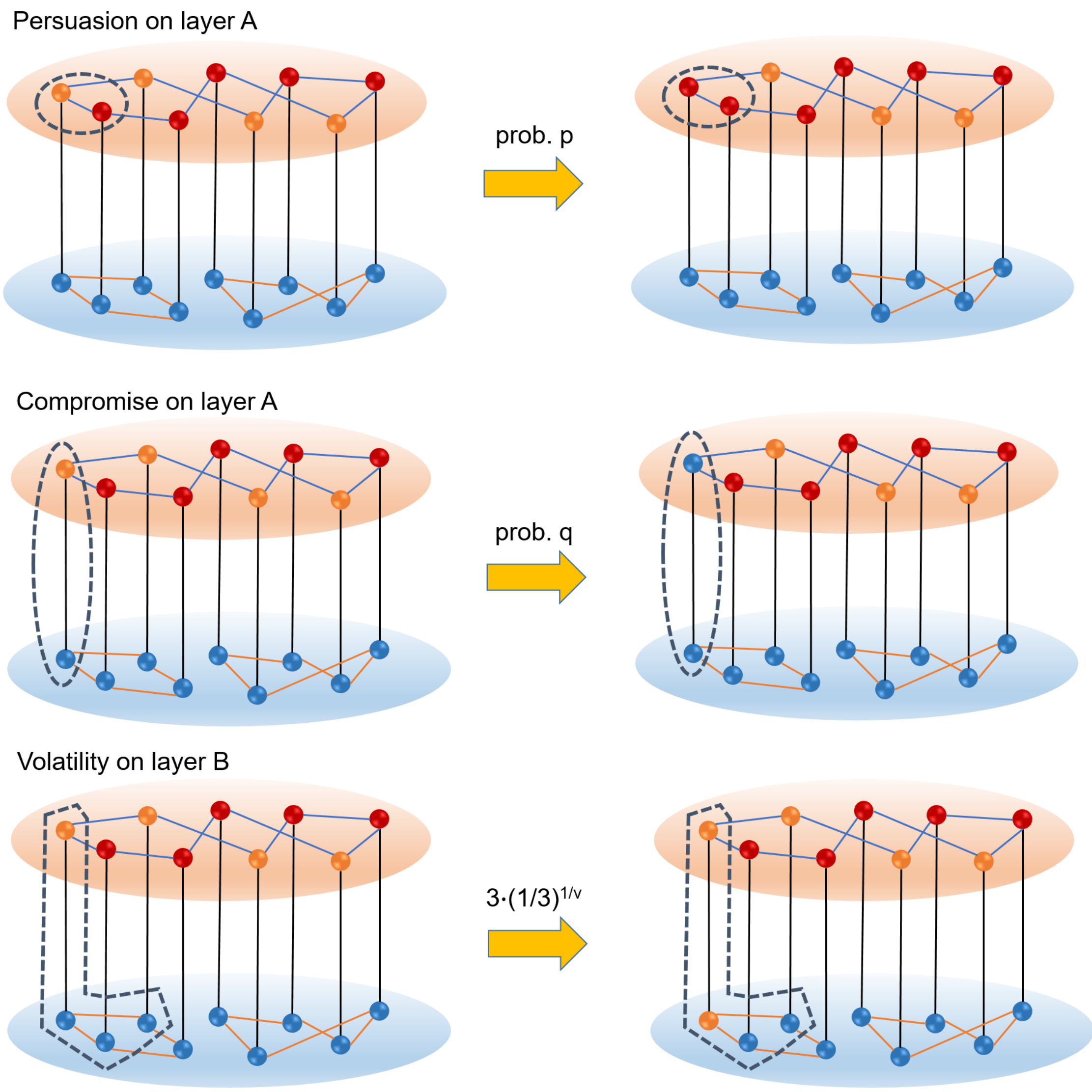


Figure 2–2 Dynamics on two layers

There are two parameters in the dynamics of layer A. To simply represent the probability p and probability q together, we set $p + q = 1$. So, p represents the tendency of opinion such as extreme or moderate, which is scaled to be 0 to 1. And, the scale of v, in the dynamics of layer B, is also 0 to 1.

To implement the interconnected dynamics, one step consists of two layers dynamics, where every node in layer A is checked with opinion dynamics, and every node in layer B updates its state according to the decision-making dynamics. Basically, the dynamics order follows updating state of layer B after updating state of layer A. The dynamics orders and updating rules would be discussed specifically in chapter 4.

Each simulation takes 100 steps, and 100 simulations are considered for average results. In the following simulations, we use *'Average State'(AS)* and *'Consensus Index'(CI)* to measure the competition result.

$$AS = avg\left(\sum_{i}^{K^A} S_i^A/4\right) + avg\left(\sum_{i}^{K^B} S_i^B/2\right). \tag{2–12}$$

$$CI = \frac{({K_+}^A \cdot {K_-}^B) + ({K_-}^A \cdot {K_+}^B)}{K^A \cdot K^B}. \tag{2–13}$$

In these formula, S_i^A means the state of node i in layer A, and K^A is the number of nodes in layer A. ${K_+}^A$ represents the number of nodes with positive state in layer A.

With AS, it could be verified whether the consensus happens in accordance with the change of p and v. If the positive consensus happens, it would be close to the value of $+1$ and if the negative consensus happens, it would be close to the value of -1. The values between $+1$ and -1 mean the states are belonging to the coexistence part. Figure. 2–3

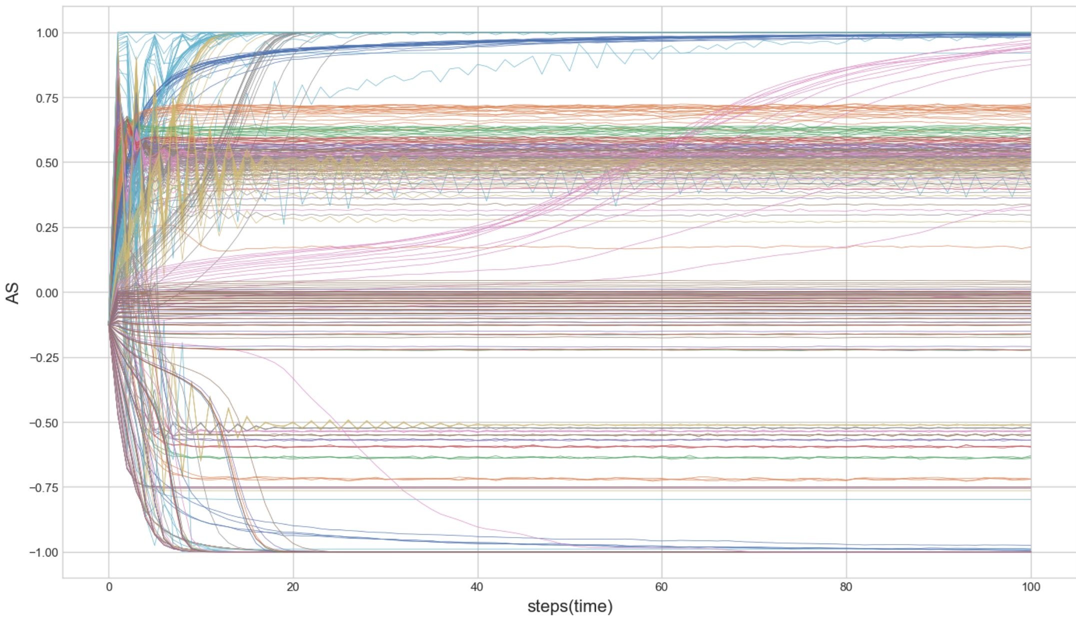


Figure 2–3 AS values per each step according to all parameters

shows that AS values are convergent to $+1$, -1 or other values as step(time) goes by. $+1$ means making positive consensus. -1 means making negative consensus. The other values mean coexistence state.

With *CI*, it could be measured how close the network state is to consensus. If the CI is close to 0, the state is close to positive or negative consensus. If the CI is close to 1, the state is separated coexistence where states of all nodes in layer A is opposed to states of all nodes in layer B. If the CI is close to 0.5, the state is mixed coexistence where each layer has both positive and negative states of nodes.

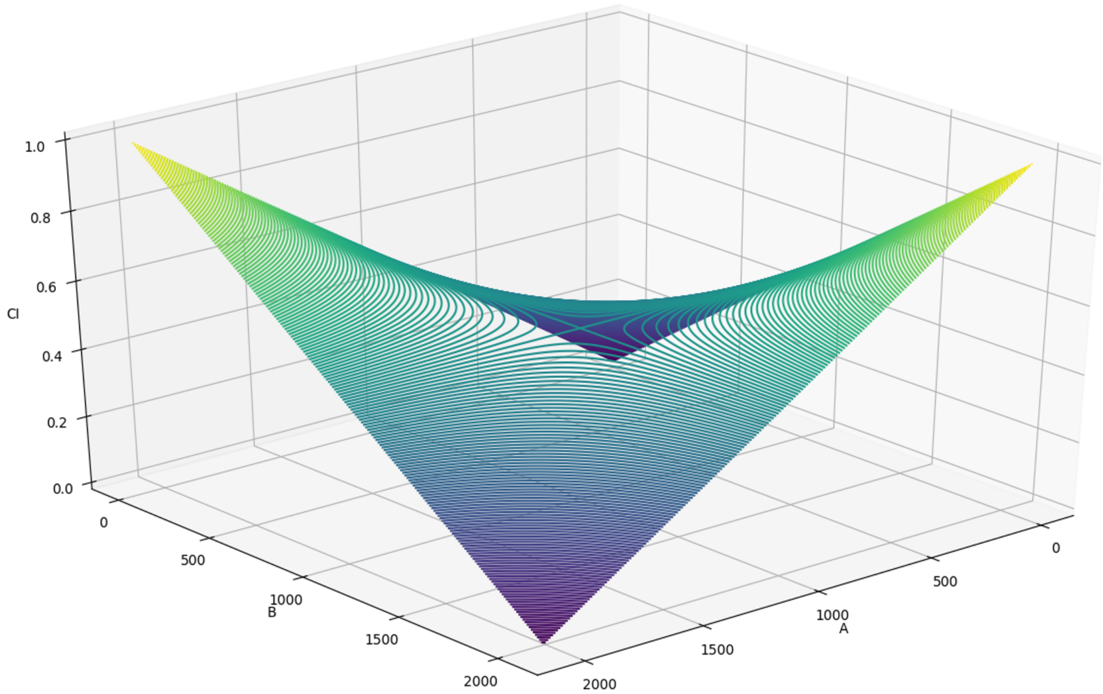


Figure 2–4 CI values according to all ${K_+}^A$ and ${K_+}^B$

Figure. 2–4 shows the characteristics of *CI*. Same orientation in two layers makes *CI* 0. Opposite orientation between two layers makes *CI* 1. And Mixed orientation in two layers makes *CI* close to 0.5.

As Figure. 2–5 shown, *CI* values are convergent to +1, 0, or other values as step(time) goes by. 0 means positive or negative consensus. +1 means opposite state between two layers. The other values means mixed state. By using *CI*, coexistence states can be divided into two categories, opposite state and mixed state.

To estimate and evaluate the consensus results regarding to different parameters p and v, we use four kinds of measures including *'AS total'*, *'Positive Consensus Ratio'(PCR)*, *'Negative Consensus ratio'(NCR)*, and *'Consensus Ratio'(CR)*. *AS total* means the summation of *AS* for all ps and all vs. *PCR* is the ratio of positive consensus over all simulations. Similarly, *NCR* is the ratio of experiments with negative consensus. *CR* is

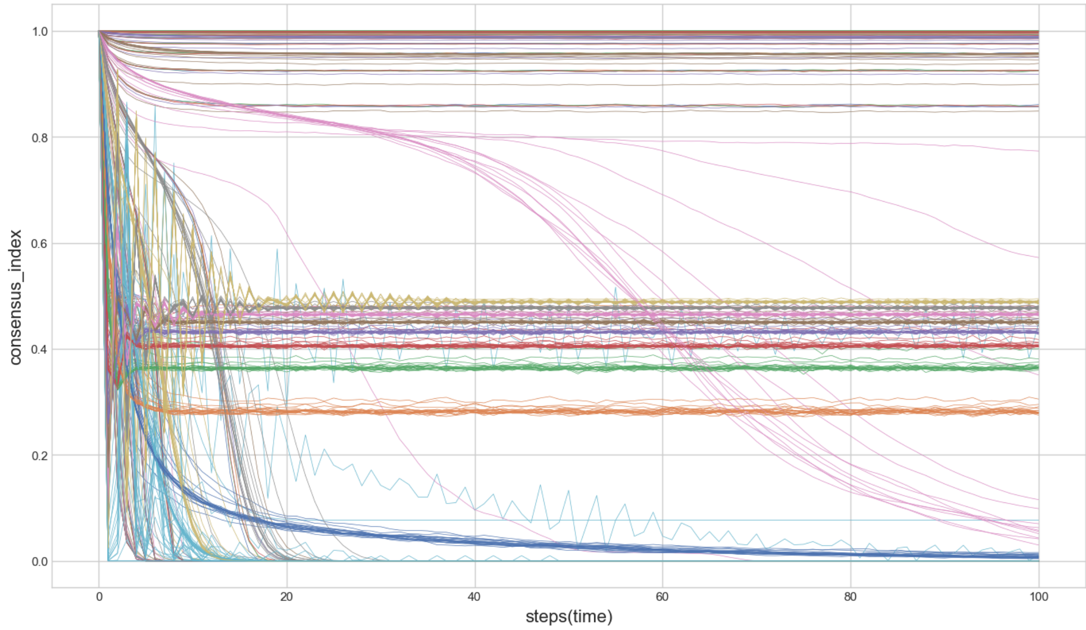


Figure 2–5 CI values according to all ${K_+}^A$ and ${K_+}^B$

the ratio of experiments reaching consensus, i.e. summation of PCR and NCR.

$$AS\ total = \frac{\sum_{j}^{m}\sum_{i}^{n} AS_{p_i,v_j}}{n \times m}, \quad \begin{array}{l} p = \{p_1, p_2, \cdots, p_n\} \\ v = \{v_1, v_2, \cdots, v_m\} \end{array}. \tag{2–14}$$

In Eq(2–14), AS_{p_i,v_j} means AS value with parameters p_i and v_j, which shows the total orientation and intensity of interconnected network.

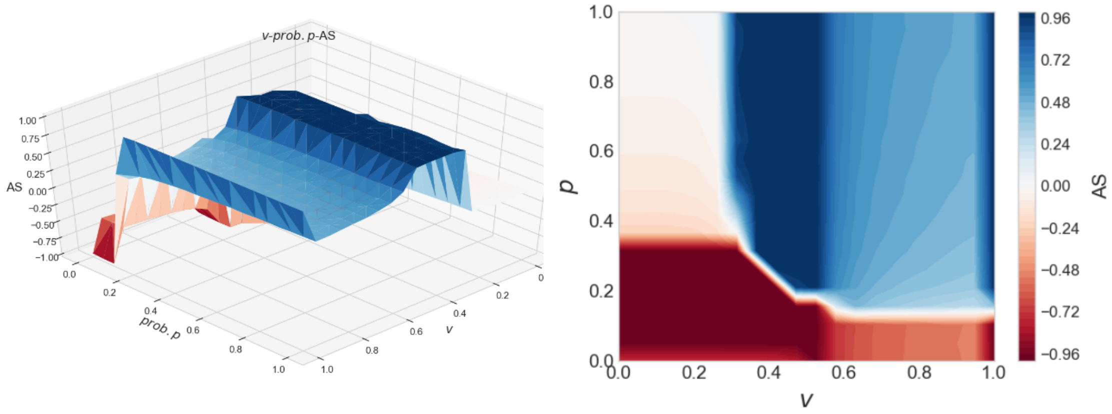
$$PCR = \frac{\sum_{j}^{m}\sum_{i}^{n} (AS_{p_i,v_j} \simeq 1)}{n \times m}. \tag{2–15}$$

In Eq(2–15), $AS_{p_i,v_j} \simeq 1$ means positive consensus.

$$NCR = \frac{\sum_{j}^{m}\sum_{i}^{n} (AS_{p_i,v_j} \simeq -1)}{n \times m}. \tag{2–16}$$

In Eq(2–16), $AS_{p_i,v_j} \simeq -1$ means negative consensus.

Figure. 2–6 shows the states of two layers according to all ps and all vs. The X-axis is the p and the Y-axis is the v, and the Z-axis represents AS. The closer the color is to blue, the more it has positive consensus. And the closer the color is to red, the more it has negative consensus. A light and white areas have coexistence with positive states and



Div	AS_total(Average values)	PCR(Blue part)	NCR(Red part)	CR(Blue+Red part)
BA-BA network	0.1799	0.1750	0.1450	0.3200

Figure 2–6 The example of simulation：BA-BA network

negative states. Here, we can measure the consensus by using indexes, *‘AS total’*, *‘PCR’*, *‘NCR’*, and *‘CR’*. The average value of this chart means *‘AS total’*. The blue part area means *‘PCR’*, the red part area means *‘NCR’*, and the summation of those means *‘CR’*.

Chapter 3 Competition on two layer with various structural network

In this chapter, based on the competition model described in chapter. 2, simulation would be implemented with changing the network structures. As the basic model, interconnected layers with random regular network would be provided. And then, the interconnected network structure would be altered by changing the internal edges, external edges and network types. Finally, all simulations would be compared and analyzed with the indexes, *AS total*, *PCR*, *NCR* and *CR*

3.1 Competition on Random Regular Networks

In this section, simulation results on random regular network would be provided to comprehend the competition of two layers. Each layer consists of random regular network that has N nodes with k internal edges as introduced in[kimsangwoo2012, bela2001]. Each node of one layer is connected with a random node on the other layer. This means each node has only 1 external un-directed edge. Simulations are preformed on network with $N = 2048$, and $k = 5$.

The simulation results are shown in Fig. 3–2 and Fig. 3–3. Fig. 3–2 presents that how the Average State(*AS*) are changed according to each parameter, p and v. So we can know that how each parameter works on the network. Fig. 3–3 provides total results with all parameters. Through these figures, the characteristics of network would be analyzed.

Fig. 3–2(a) shows that when $p > 0.2$, $0.32 < v < 0.47$, it normally tends to positive consensus. But, if v is lower or larger than certain values, it doesn't make consensus. In Fig. 3–2(b), as v increases, it normally change from negative to positive consensus. But, when p is very low($p \leq 0.11$), it doesn't make positive consensus. On the other hand, when p is large enough, it makes positive consensus. But, when v is small enough, it is changed into negative consensus. When both of p and v are large enough, the state is in a coexistence part.

Fig. 3–3 shows the states of two layers according to all ps and all vs. As previously described in chapter. 2, blue area is for positive consensus, red area is for negative consensus, and light colored and white area is for coexistence. And indexes for consensus

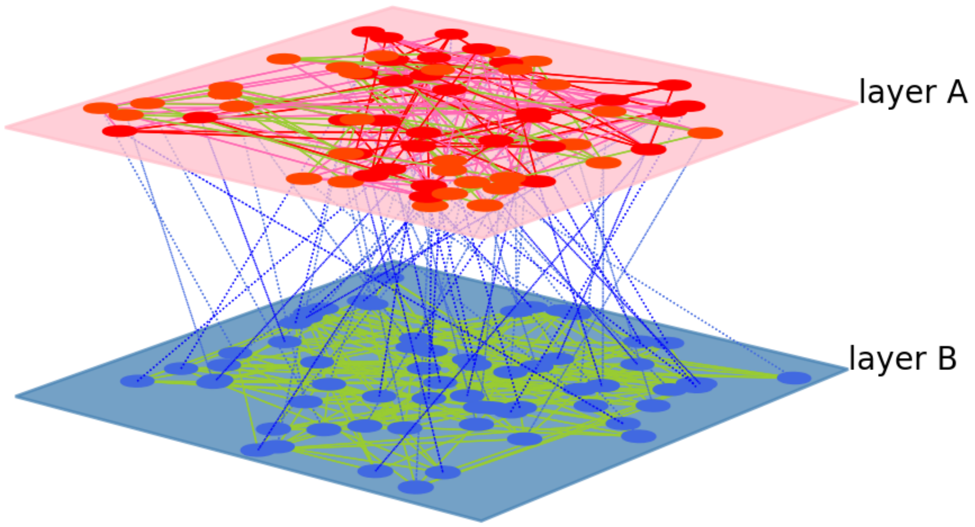


Figure 3–1 Competition on random regular network

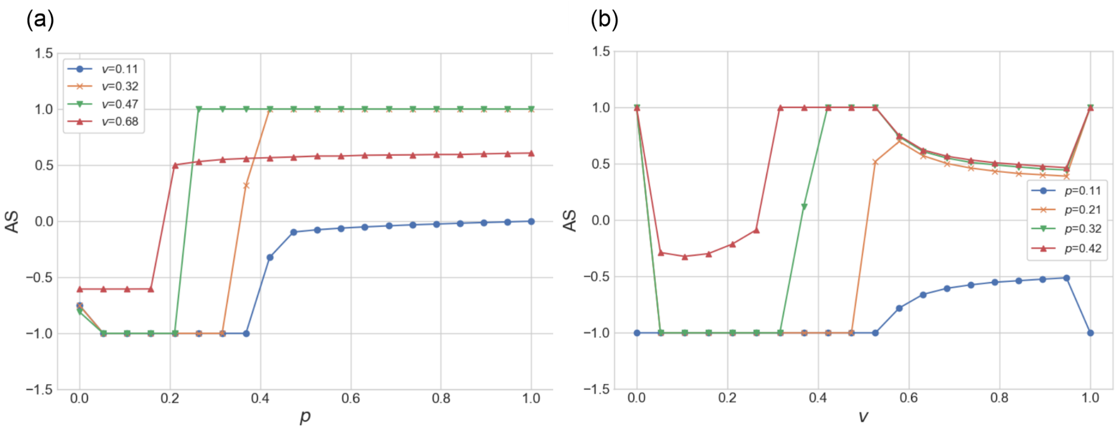
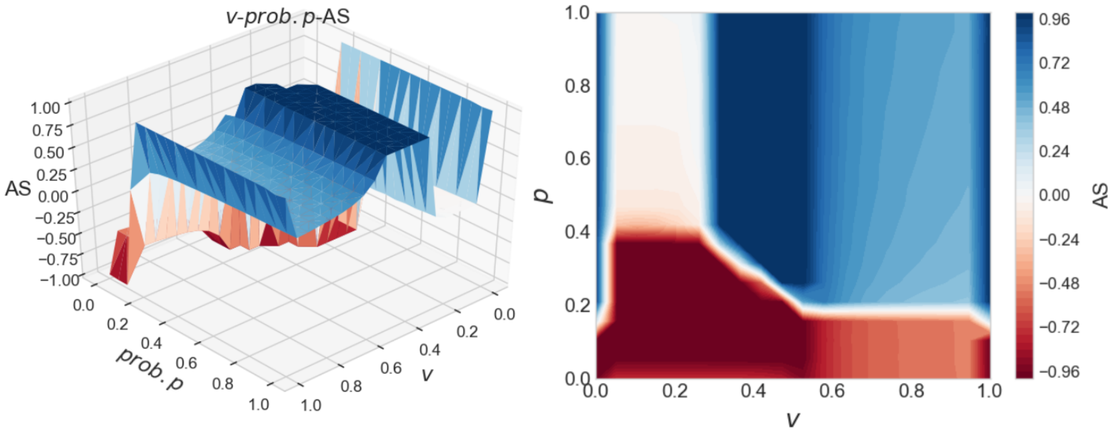


Figure 3–2 (a) p-AS chart according to certain v values. (b) v-AS chart according to certain p values.

are also measured.Positive consensus area is 0.2475, and negative consensus area is 0.1575. Coexistence area is $1 - CR = 0.5950$. By using these values and figures, this model would be compared with various structural networks in next section. Through these charts, several facts can be arranged. First, large p tends to make positive consensus and small p tends to make negative consensus. Second, small v tends to make negative consensus, and large v tends to make coexistence state.

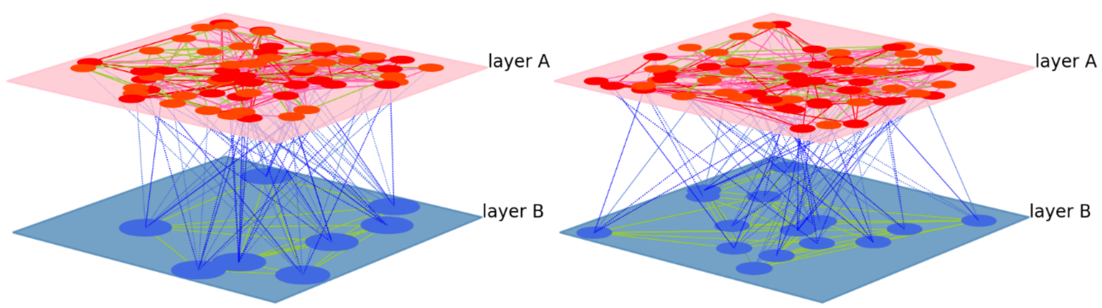


Div	AS_total (Average values)	PCR (Blue part)	NCR (Red part)	CR (Blue+Red part)
RR-RR network	0.2034	0.2475	0.1575	0.4050

Figure 3–3 simulation result : *AS* changing with all *p* and *v*

3.2 Competition on Networks with different number of external links

In this section, we consider the influence of external links. Based on the basic model in section 3.1, we reduce the number of nodes in layer B at a certain rate and increase the external links from nodes in layer B accordingly as shown in Fig. 3–4. We denote *HM(n)* as a hierarchical model with a level n, which means that the number of nodes in layer B is $1/n$ of the number of nodes in layer A, and the number of external links from node in layer B is n in view that the number of external links from node in layer A is 1. In other words, each node in layer A has one external edge, but each node in layer B has n external edges for *HM(n)*, which means one node in layer B can be influenced by n nodes in layer A.



(a) HM(8)　　(b) HM(4)

Figure 3–4 Competition on hierarchical model

To find out the significant influence of external edges, various *HM(n)s* were simulated. Totally, 8 *HM(n)*, *HM(2), HM(4), HM(8), HM(16), HM(32), HM(64), HM(128), HM(256)* were arranged as shown in Fig. 3–5. Fig. 3–5 shows that *HM(2)* has the most area for coexistence part(light and white area) and *HM(256)* has the most are for consensus part(blue and red area). As *n* in *HM(n)* is increased, coexistence area is decreased and consensus area is increased. Particularly, positive consensus area is significantly increased, negative consensus area is slightly decreased.

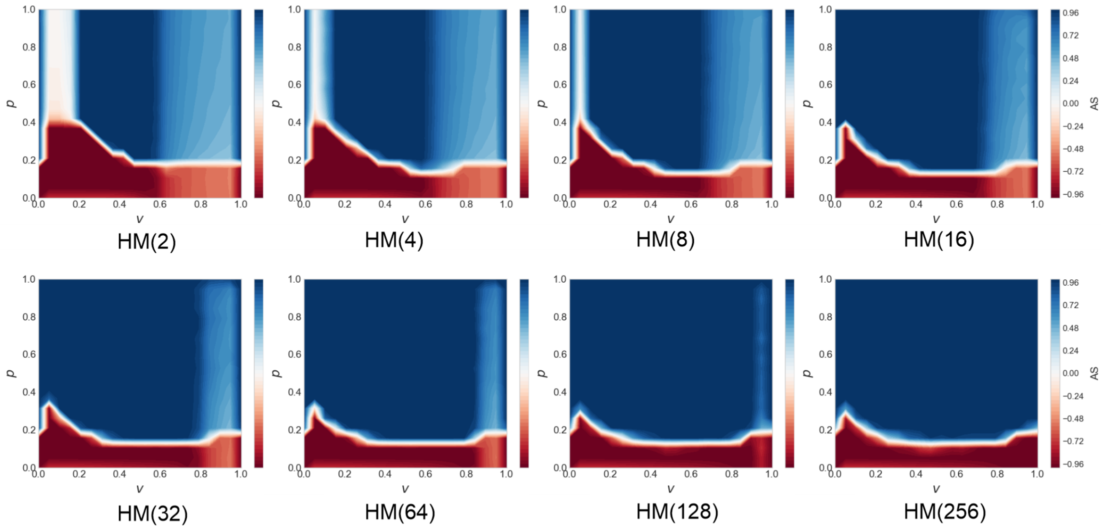


Figure 3–5 Competition on hierarchical model

To clearly find out the difference between models, we use the indexes, *PCR, NCR, AS total*. Fig. 3–6 shows the results to analyze *HM(n)* with indexes. Blue color bar is for PCR, red color bar is for NCR, and green color bar is for AS total. Comparing *HMs* with *Basic model(RR(5)-RR(5))*, *CR PCR* and *AS total* are all increased remarkably. *HMs* have more positive consensus part than *RR(5)-RR(5)*. But, *HMs* have less negative consensus part than *RR(5)-RR(5)*. It shows that as the number of B nodes are decreased, it is easy to make positive consensus(layer A opinion) and hard to make negative consensus(layer B opinion).

In summary, all the Hierarchical Models have more consensus ratio than Random Regular Networks Model. However, positive consensus ratio is increased, but negative consensus ratio is decreased. It is found out that as the number of B nodes are more decreased, it makes easier to make positive consensus and harder to make negative consensus. In real world, it would be analyzed that as the number of leaders is less,

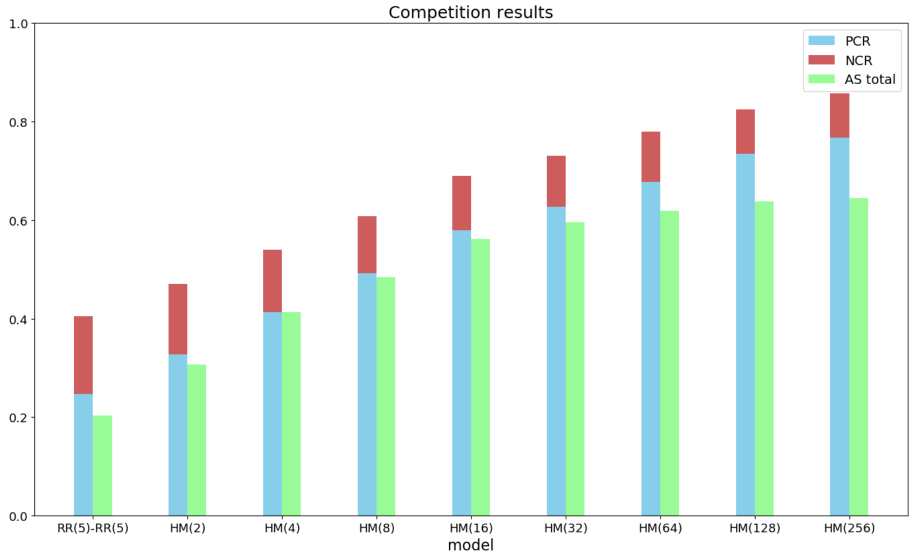


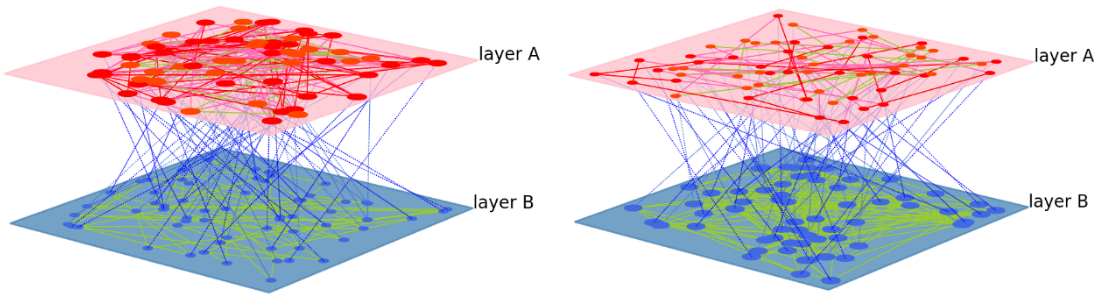
Figure 3–6 Simulation results of Hierarchical Models(*HM(n)*)

social conflict are decreased and the opinion is convergent to social opinion(layer A). But, sometimes there are some dangers to ignore the leader opinion(layer B), or to cause more social conflict when there are stubborn leaders.

3.3 Competition on Networks with different number of internal links

Next, the interconnected networks are simulated with different internal degrees in order to define and evaluate the influence of internal degrees. Random regular network would be applied. And the number of internal degrees on each node is switched to various numbers as shown in Fig. 3–7. But, there is no change on external degree, which would be fixed to only 1. Here, *RR(n)-RR(m)* represents layer A has random regular network with n internal edges, layer B has random regular network with m internal edges. Firstly, the internal degrees on layer A are changed. The internal degrees on layer B are fixed to $5,120$, which means each node has 5 internal degrees on layer B, and the internal degrees on layer A are switched into $2,048$, $3,072$, $4,096$, or $5,120$, which means each node has 2, 3, 4, or 5 internal degrees on layer A. Fig. 3–8 shows the simulation results for changing the internal degrees on layer A. As shows in Fig. **??** (a), as the number of internal degrees on layer A is increased, the red part is decreased and the blue part is increased.

To clearly compare and analyze the results, the results are presented with the indexes,



(a) RR(5)-RR(2) (b) RR(2)-RR(5)

Figure 3–7 Competition on interconnected networks with different internal edges

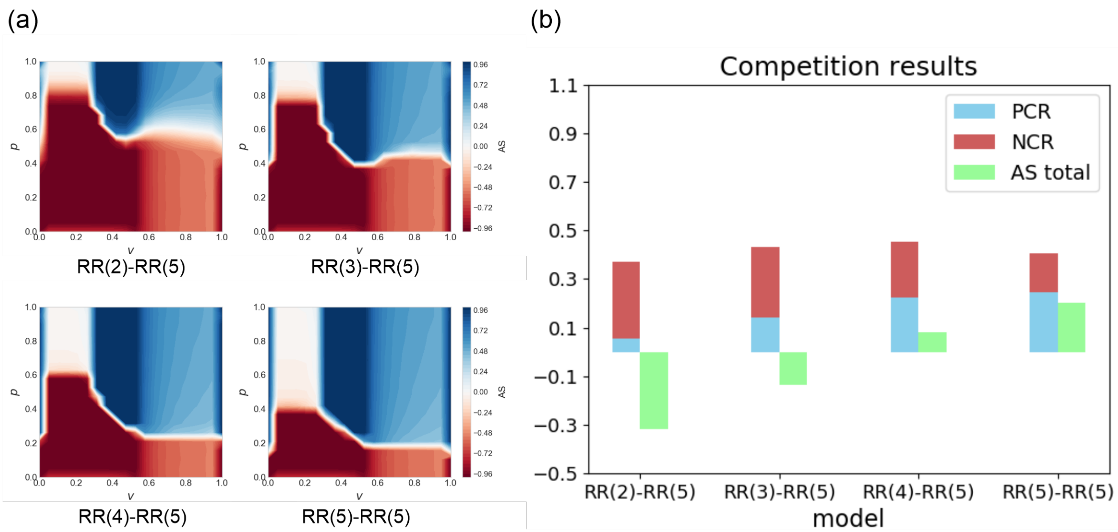


Figure 3–8 Simulation results with different internal degrees on layer A

PCR, NCR, AS total in Fig. 3–8 (b), which shows that as the number of internal degrees on layer A is increased, negative consensus is decreased and positive consensus is increased. As shown in Fig. 3–8, RR(5)-RR(5) has the most *PCR*, and RR(2)-RR(5) has the most NCR. However, *CR* is almost same with all models in Fig. **??**. It can be analyzed that the number of internal degrees on layer A has the tendency to keep positive state and to change negative state into positive state.

Next, the internal degrees on layer B are changed. The internal degrees on layer A are fixed to 5, 120, which means each node has 5 internal degrees on layer A, and the internal degrees on layer B are switched into 2, 048, 3, 072, 4, 096, or 5, 120, which means each node has 2, 3, 4, or 5 internal degrees on layer B.

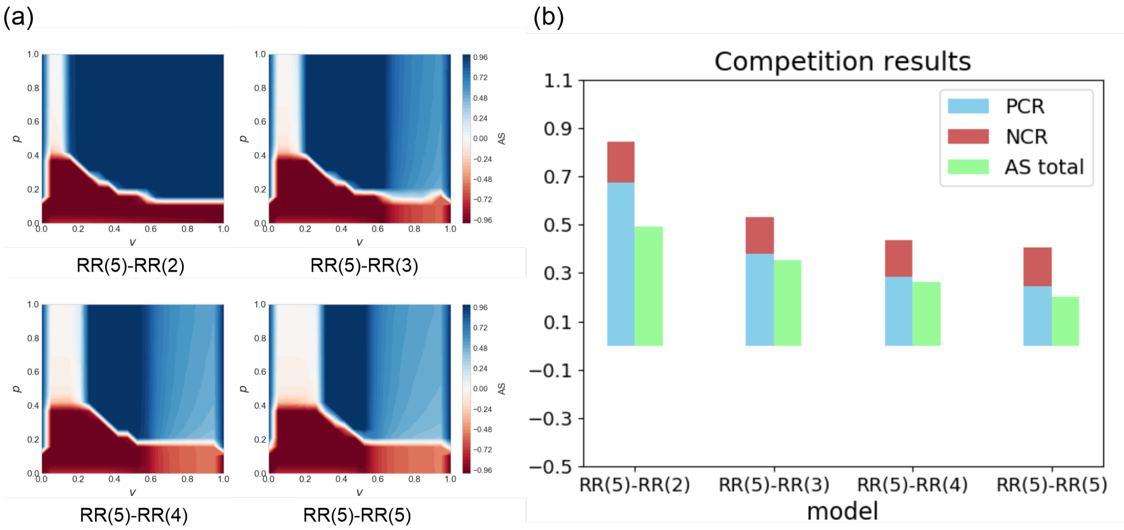


Figure 3–9 Simulation results with different internal degrees on layer B

Fig. 3–9 shows the simulation results with changing the number of internal degrees on layer B. As shows in Fig. **??** (a), as the number of internal degrees on layer B is increased, the blue part is decreased, the white and light color part is increased, and the red part is almost same, though the shape of red area is changed. As shown in Fig. 3–9 (b), RR(5)-RR(2) has the most *PCR* and *CR*, and RR(5)-RR(5) has the least PCR and *CR*. However, *NCR* is almost same with all models in Fig. **??**. It can be analyzed that the number of internal degrees on layer B has the tendency to hinder positive state and has the inverse relation with *CR*. As the number of internal degrees on layer B is increased, *PCR* and *CR* is inversely decreased. It is recognized that the role of internal degrees on layer A is different with internal degrees on layer B. The internal degrees on layer A has the function to keep the state of layer A, and the internal degrees on layer B has the function to restrain the state of layer A and make coexistence part.

Next, it is simulated that internal degrees are changed on both layer A and layer B, such as RR(2)-RR(2), RR(3)-RR(3), RR(4)-RR(4) and RR(5)-RR(5). Through these simulations, it would be found out that how total internal degrees affect the interconnected network.

Figure 3–10 Simulation results with different internal degrees on layer B

Figure 3–10

chap3$_internal_edge_two_total$

Fig. 3–10 presents that as the total number of internal degrees is increased, *CR* is inversely decreased.

In summary, *RR(5)-RR(2)* has the most *PCR*. *RR(2)-RR(5)* has the most *NCR*. When the number of internal edges in layer A are more than layer B, it has more positive consensus. On the other hand, when the number of internal edges in layer B are more than layer A, it has relatively more negative consensus. These results provide that the number of edges on layer A has the tendency to keep positive state, and the number of edges on layer B has the tendency to keep negative state. The number of internal edges have the influence on consensus result and a layer with more internal edges has the tendency to maintain its own state. In case of networks with same internal edges, *RR(2)-RR(2)* has more *PCR* and *AS total* than *RR(5)-RR(5)*. It can be analyzed that *RR(5)-RR(5)* is hard to make consensus, because it has more internal edges to cause inner conflict. Also, *RR(2)-RR(2)* has less *NCR* than *RR(5)-RR(5)*. It shows that the number of internal edges in layer B is more sensitive than layer A. As Eq(1) shows, layer B dynamics can have more various and extreme probabilities when it has more degrees. For example, in case of *RR(2)-RR(2)* with $\beta = 1$, the dynamics starts with $P_B = 1/3$ and in case of *RR(5)-RR(5)* with $\beta = 1$, the dynamics starts with $P_B = 1/6$.

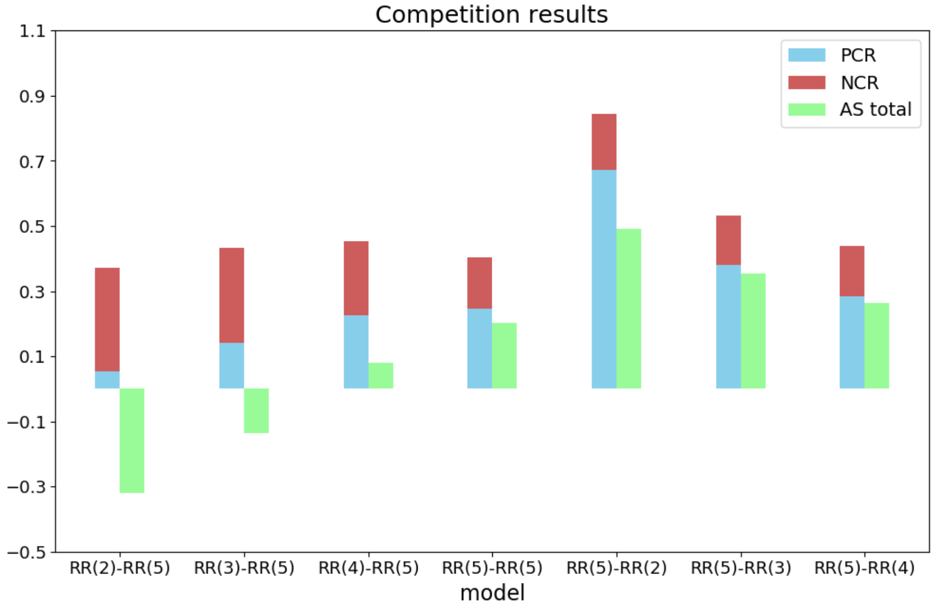


Figure 3–11 Simulation results with different internal degrees on two layers

3.4 Competition on Networks with different structures

So far, each layer of the interconnected network consisted of *RR(random regular networks)* that has the same number of edges for each node. Now, the simulation would be implemented on different network type. Here, we use *Barabasi-Albert network(BA)* structure as introduced in**barabasi1999**. *Barabasi-Albert(BA)* network has N nodes with attaching new nodes each with K edges that are preferentially attached to existing nodes with high degree. But, there is no change on external degree, which would be fixed to only 1.

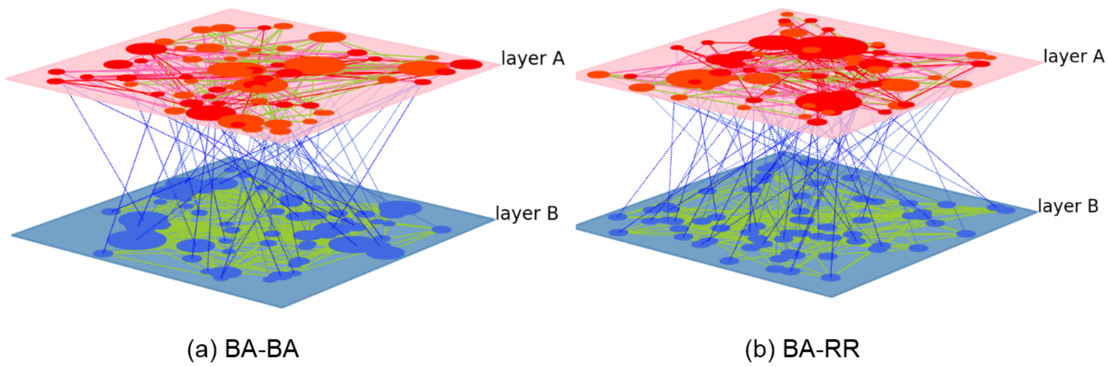


Figure 3–12 Competition on interconnected networks with different structures

To evaluate the influence of network structure, 4 simulations are implemented with switching network structures. The *BA* or *RR* network is applied for both layers or switched on each layer. To restrain the influence of internal degree number, the number of internal degrees in *BA* is set up to be similar with the number of internal degrees in *RR*. The number of internal degrees in *BA* is $6,135$, and the number of internal degrees in *RR* is $6,144$.

Figure 3–13 Simulation results with different network types

The simulation results are shown in Fig. **??**. The result of *BA-RR* and *RR(10)-RR(5)* have almost the same features. The gap of *CR* is almost same(less than 0.01). The structure of network make no obvious difference of consensus results. In case of *BA-BA*, the *CR* has the least ratio for consensus. *BA-BA* structure has lots of internal edges on each layer. Therefore, it is hard to make consensus due to inner conflict on each layer.

3.5 Conclusion

Table 3–1 Consensus properties of Simulation Models

Div	A nodes	B nodes	A edges	B edges	AS total	PCR	NCR	CR
RR(2)-RR(5)	2,048	2,048	2,048	5,120	-0.3186	0.0550	0.3175	0.3725
RR(3)-RR(5)	2,048	2,048	3,072	5,120	-0.1368	0.1400	0.2925	0.4325
RR(4)-RR(5)	2,048	2,048	4,096	5,120	0.0804	0.2250	0.2275	0.4525
RR(5)-RR(5)	2,048	2,048	5,120	5,120	0.2034	0.2475	0.1575	0.4050
* RR(6)-RR(5)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
* RR(7)-RR(5)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
RR(2)-RR(2)	2,048	2,048	2,048	2,048	-0.1412	0.1475	0.4050	0.5525
* RR(3)-RR(2)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
* RR(4)-RR(2)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
RR(5)-RR(2)	2,048	2,048	5,120	2,048	0.4927	0.6725	0.1725	0.8450
* RR(6)-RR(2)	2,048	2,048	5,120	4,096	0.2633	0.2850	0.1525	0.4375
* RR(7)-RR(2)	2,048	2,048	5,120	4,096	0.2633	0.2850	0.1525	0.4375
* RR(2)-RR(3)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
RR(3)-RR(3)	2,048	2,048	3,072	3,072	0.0084	0.2275	0.2825	0.5100
* RR(4)-RR(3)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
RR(5)-RR(3)	2,048	2,048	5,120	3,072	0.3555	0.3800	0.1525	0.5325
* RR(6)-RR(3)	2,048	2,048	5,120	4,096	0.2633	0.2850	0.1525	0.4375
* RR(7)-RR(3)	2,048	2,048	5,120	4,096	0.2633	0.2850	0.1525	0.4375
* RR(2)-RR(4)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
* RR(3)-RR(4)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
* RR(4)-RR(4)	2,048	2,048	4,096	4,096	0.5658	0.4828	0.0637	0.5466
RR(5)-RR(4)	2,048	2,048	5,120	4,096	0.2633	0.2850	0.1525	0.4375
* RR(6)-RR(4)	2,048	2,048	5,120	4,096	0.2633	0.2850	0.1525	0.4375
* RR(7)-RR(4)	2,048	2,048	5,120	4,096	0.2633	0.2850	0.1525	0.4375
* RR(5)-RR(6)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
* RR(5)-RR(7)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
RR(6)-RR(6)	2,048	2,048	6,144	6,144	0.2444	0.2350	0.1375	0.3725
RR(6)-BA(3)	2,048	2,048	6,144	6,135	0.2541	0.2275	0.1300	0.3575
BA(3)-RR(6)	2,048	2,048	6,135	6,144	0.2242	0.2300	0.1425	0.3725
* BA(2)-BA(2)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
* BA(3)-BA(3)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
* BA(4)-BA(4)	2,048	2,048	6,135	6,135	0.2197	0.1273	0.0190	0.1463
BA(5)-BA(5)	2,048	2,048	10,215	10,215	0.2496	0.1675	0.0675	0.2350
HM(2)	2,048	1,024	5,120	2,560	0.3073	0.3275	0.1425	0.4700
HM(4)	2,048	512	5,120	1,280	0.4128	0.4125	0.1275	0.5400
HM(8)	2,048	256	5,120	640	0.4846	0.4925	0.1150	0.6075
HM(16)	2,048	128	5,120	320	0.5610	0.5800	0.1100	0.6900
HM(32)	2,048	64	5,120	160	0.5959	0.6275	0.1025	0.7300
HM(64)	2,048	32	5,120	80	0.6185	0.6775	0.1025	0.7800
HM(128)	2,048	16	5,120	40	0.6379	0.7350	0.0900	0.8250
HM(256)	2,048	8	5,120	20	0.6454	0.7675	0.0900	0.8575

Especially, we provide three conclusions about the roles of edges. First, as hierarchical models show, when the number of external edges in decision making is more than opinion layer, it is easy to make consensus on both layers. Also, it is found out that there exists the efficient number of nodes in decision making layer for performing consensus. Second, a layer with more internal edges has more tendency to keep its own states. Third, too many internal edges on each layer can cause inner conflict, and that makes it hard to have consensus state.

Chapter 4　Competition on two layer with various updating rules

Here, we would control dynamics orders between layers and updating rules of nodes states. With changing dynamics orders and updating rules, it would be investigated how the state of network is changed. In this chapter, each layer consists of *Barabasi-Albert(BA)* network that has N nodes with attaching new nodes each with K edges that are preferentially attached to existing nodes with high degree as introduced in[barabasi1999]. Each node of one layer is connected with a random node on the other layer. This means each node has only 1 external un-directed edge. Simulations are preformed on network with $N = 2048$, and $K = 3$.

When considering dynamics order on two-layer networks, there are many ways to update the state of nodes. Dynamics order of two layer can be considered whether layer A works first or layer B. And, nodes can be thought about whether the states of nodes are changed simultaneously or sequentially or randomly. Links connected with a node also can be deliberated whether links are activated on a node sequentially or simultaneously or randomly. But, in layer B dynamics, order of edges in one node is always for simultaneous updating rule, because dynamics formula already considers states of all connected neighbor nodes simultaneously. To sum up, as shown in Table.4–1, 25 updating rules would be considered according to layers, nodes and edges.

In table remarks, 'O(o, o) → D(s)' means Opinion layer(node : sequential order updating, edges : sequential order updating) → Decision Making layer(node : simultaneous updating). And 'O(o, o) ⇔ D(o)' means that one node in Opinion layer is updated, and then one node in Decision Making layer is updated, this rule is repeated until all nodes are updated. Dynamics with 25 updating rules are simulated with parameter $p = 0.4$ and $v = 0.4$. Simulation results are divided by order of layers, nodes and edges.

4.1　Order of layers

There exist two layers on interconnected network. And each layer have its own dynamics, such as *M-Model* and *AS-Model*. Two dynamics can be operated simultaneously or sequentially. If they act sequentially, dynamics of layer A can act first or dynamics of

Order of layers	Layer A		Layer B	remarks
	Order of nodes	Order of edges	Order of nodes	
Layer A → Layer B	Sequential	Sequential	Sequential	O(o, o) → D(o)
			Simultaneous	O(o, o) → D(s)
		Simultaneous	Sequential	O(o, s) → D(o)
			Simultaneous	O(o, s) → D(s)
	Simultaneous	Sequential	Sequential	O(s, o) → D(o)
			Simultaneous	O(s, o) → D(s)
		Simultaneous	Sequential	O(s, s) → D(o)
			Simultaneous	O(s, s) → D(s)
	Random	Random	Sequential	O(r, r) → D(o)
			Simultaneous	O(r, r) → D(s)
Layer A ← Layer B	Sequential	Sequential	Sequential	O(o, o) ← D(o)
			Simultaneous	O(o, o) ← D(s)
		Simultaneous	Sequential	O(o, s) ← D(o)
			Simultaneous	O(o, s) ← D(s)
	Simultaneous	Sequential	Sequential	O(s, o) ← D(o)
			Simultaneous	O(s, o) ← D(s)
		Simultaneous	Sequential	O(s, s) ← D(o)
			Simultaneous	O(s, s) ← D(s)
	Random	Random	Sequential	O(r, r) ← D(o)
			Simultaneous	O(r, r) ← D(s)
Layer A ↔ Layer B	Simultaneous	Sequential	Simultaneous	O(s, o) ↔ D(s)
		Simultaneous	Simultaneous	O(s, s) ↔ D(s)
Layer A ⇔ Layer B	Sequential	Sequential	Sequential	O(o, o) ⇔ D(o)
		Simultaneous	Sequential	O(o, s) ⇔ D(o)
	Random	Random	Random	O(r, r) ⇔ D(r)

Table 4–1 25 updating rules according to order of layers, nodes, and edges

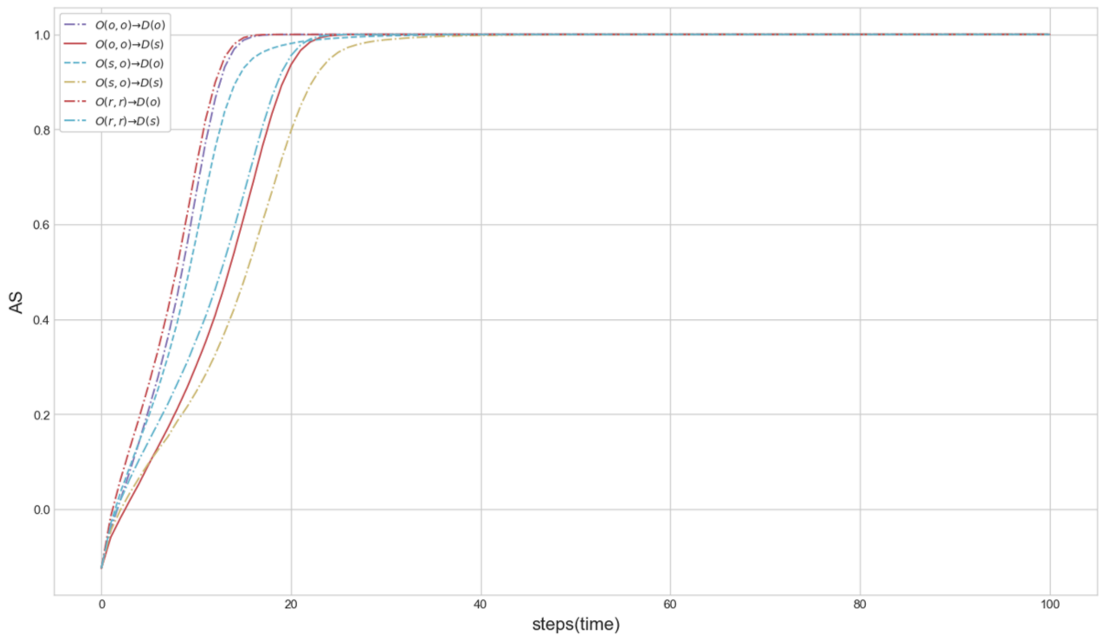
layer B can work previously. Otherwise, regardless of layers order, nodes of two layers can interact mutually, i.e. one node in layer A are updated and then one node in layer B are updated until all nodes are updated. Considering all situations, there are 4 ways in order of two layers, *Layer A → Layer B, Layer A ← Layer B, Layer A ↔ Layer B(simultaneous), Layer A ⇔ Layer B(interaction regardless of layers)*. Fig. 4–1 shows

Figure 4–1 Simulation results according to orders of layers

4 simulation results related to orders of layers. As seen in Fig. 4–1, it is shown that there is little difference between orders of layers. Consensus time and result are almost same, though dynamics order is different. Regardless of dynamics directions, when other conditions, such as order of nodes and edges are same, the dynamics results are also very similar. Dynamics order of layers does not have an significant influence on the network state.

4.2 Order of nodes

In the simulation model, each layer has 2048 nodes, and each node has interaction with other nodes. Now, interaction order of nodes would be considered. One node can be updated sequentially after neighbor nodes are updated. Otherwise, every node can be updated simultaneously. Simulation results would be different according to interaction order of nodes. In addition, random order between nodes is also simulated. In random order, one edge is selected randomly and updated until all edges are considered regardless of orders in nodes or links. Interaction order of nodes have meaning related to time. If networks have short time to change states, networks follow simultaneous updating rule. However, if networks have enough time to update states, networks follow sequential updating rules. In real world, discussion or conversation with enough time means sequential updating rule of nodes, and vote or election means simultaneous updating rule of nodes.



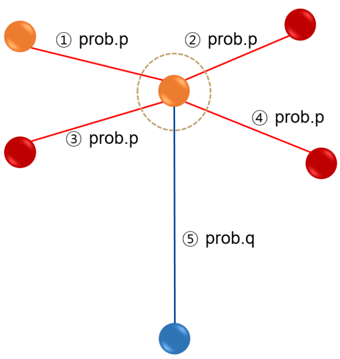
Div	Fast Consensus	Slow Consensus
Orders	① O(r, r) → D(o) ② O(o, o) → D(o) ③ O(s, o) → D(o)	④ O(r, r) → D(s) ⑤ O(o, o) → D(s) ⑥ O(s, o) → D(s)

Figure 4–2 Simulation results according to orders of nodes: comparison between order of nodes under same conditions such as order of layers and edges.

Fig. 4–2 shows simulation results. The results are classified to two categories, fast consensus and slow consensus. It is shown that simultaneous interaction between nodes makes slow consensus. Simultaneous order in layer A does not make large difference, but it make consensus slightly slow. Simultaneous interaction between nodes in layer B have more influence on consensus time than in layer A. Random order has similar results with sequential order and does not make different states. For quick social consensus, both opinion layer and decision making layer need sequential updating rule, such as conversation and discussion.

4.3 Order of edges

Each node has several edges connected with other nodes. Simulation results can be different according to that edges are activated sequentially or simultaneously. If edges of each node work sequentially, a state of node is changed whenever each edge is activated. However, If edges of a node are activated simultaneously, a state of node would be changed considering all connected nodes. In real world, order of edges in one node can be analyzed as characteristics of nodes. If order of edges is sequential, the node would be rash. If order of edges is simultaneous, the node would be considerate. For example,



Order of edges	Next state
Sequential	Hard to calculate (According to order of activated edges, final state is changed)
Simultaneous	Unchanged: prob.p(activated) = prob.q(activated) Persuasion : prob.p(activated) > prob.q(activated) Compromise: prob.p(activated) < prob.q(activated)

Figure 4–3 one node connected with other nodes changes its state with sequential or simultaneous order of edges

considering the case that one node is connected with other nodes as shown in Fig. 4–3, we can think how the state of node change. If the edges follow sequential updating rule, it is hard to calculate the probabilities, because the states can change according to sequential order of edges. Therefore, we can get next states of nodes by using computer simulation

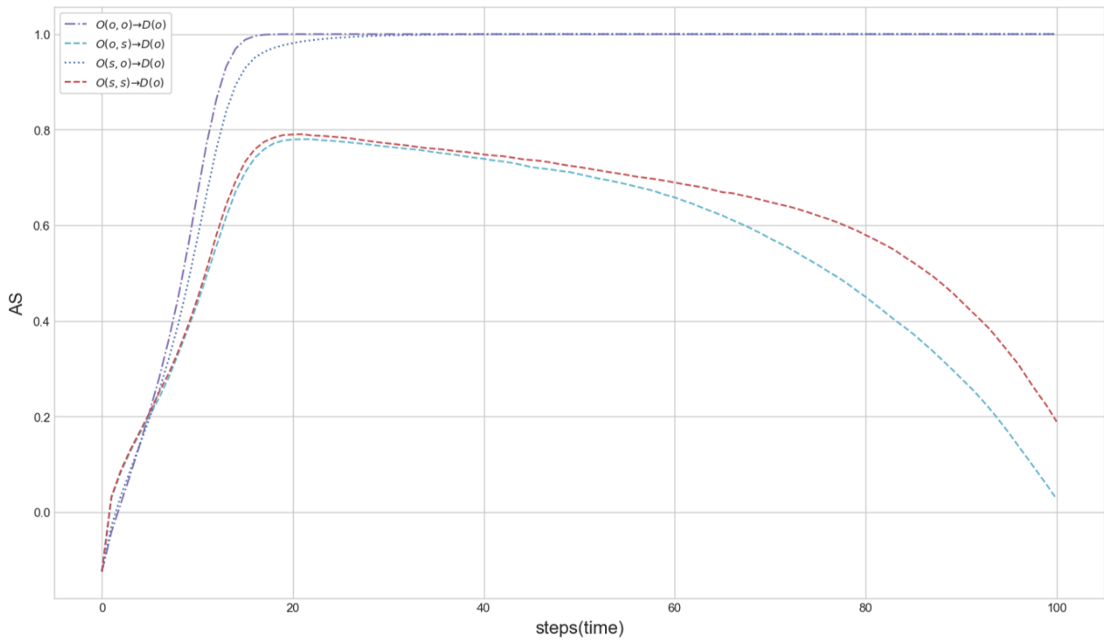
If the edges follow simultaneous updating rule, it needs some assumptions:

1. If the number of activated *prob.p* is more than the number of activated *prob.q*, persuasion dynamics would work.

2. If the number of activated *prob.p* is same with the number of activated *prob.q*, the state would be unchanged.
3. If the number of activated *prob.p* is less than the number of activated *prob.q*, compromise dynamics would work.

Through these assumptions, we can calculate probabilities of changing state in layer by considering all cases like these formula.

$$K = \{k \quad | \quad 0, \cdots, n^{-S_i}\}, \quad L = \{l \quad | \quad 0, \cdots, n^{S_i}\}, \quad M = \{m \quad | \quad k - l\},$$
$$P_A(S_i \mapsto S'_i) = \begin{cases} \text{unchanged}(k = l) : \sum p^{n^{-S_i}+m} \cdot (1-p)^{n^{S_i}-m} \cdot {}_{n^{S_i}}C_k \cdot {}_{n^{-S_i}}C_l \\ \text{persuasion}(k > l) : \sum p^{n^{-S_i}+m} \cdot (1-p)^{n^{S_i}-m} \cdot {}_{n^{S_i}}C_k \cdot {}_{n^{-S_i}}C_l \\ \text{compromise}(k < l) : \sum p^{n^{-S_i}+m} \cdot (1-p)^{n^{S_i}-m} \cdot {}_{n^{S_i}}C_k \cdot {}_{n^{-S_i}}C_l \end{cases} \tag{4–1}$$



Div	Consensus	Not reaching consensus
Orders	① O(o, o) → D(o) ② O(s, o) → D(o)	③ O(o, s) → D(o) ④ O(s, s) → D(o)

Figure 4–4 Simulation results according to orders of edges: comparison between order of edges under same conditions such as order of layers and nodes

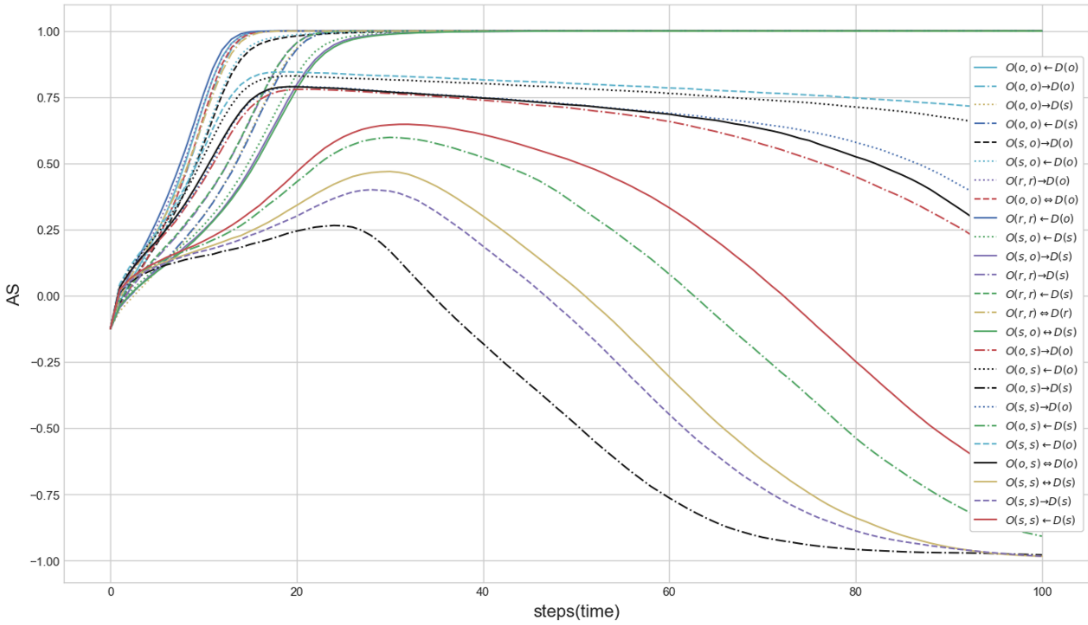
Fig. 4–4 shows the simulation result according to order of edges. The results are categorized to consensus and coexistence(not reaching consensus) due to order of edges. Sequential updating rule of edges makes consensus, i.e. rash nodes make consensus. But simultaneous updating rule of edges makes it hard to reach consensus, i.e. considerate nodes do not make consensus. It can be analyzed that rash node is easy to be extreme and make consensus, but considerate node is very moderate and hard to reach consensu

4.4 Comparison and Analysis

It is found out that there are different simulation results according to orders of layers, nodes, and edges. To sum up all updating rules, they can be categorized into 3 parts, positive consensus, coexistence, and negative consensus as shown in Fig. 4–5. To clearly classify the state of two-layers, the results can be analyzed by using *CI* as shown in Fig. 4–6. There are three branch points. In the first branch point, the results are divided according to whether order of nodes in layer B is sequential or simultaneous. In the second and third branch point, the results are divided according to whether order of edges in layer A is sequential or simultaneous. As the results, there are 4 categories such as fast positive consensus, slow positive consensus, coexistence and slow negative consensus.

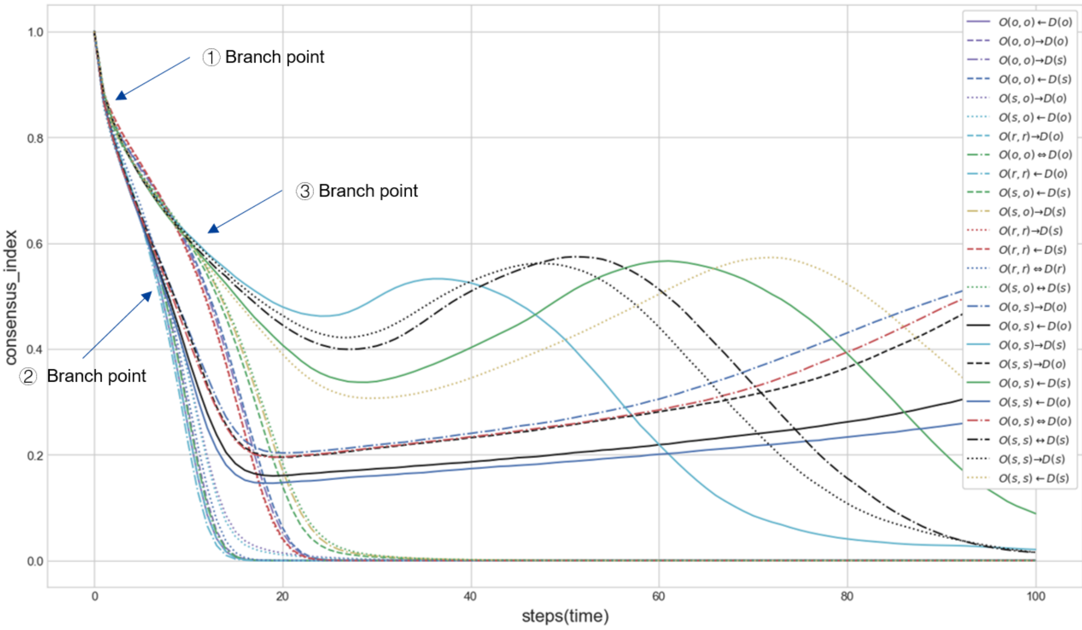
4.5 Conclusion

Through these results, several important facts can be arranged. First, networks with more simultaneous updating rules make slow consensus or coexistence, sometimes make transition to opposite orientation. On the other hands, networks with more sequential updating rules make fast consensus. In other words, if opinion layer has more rash nodes, more time to have some conversation and decision making layer has more time to discuss topics, the network have more probabilities to make consensus for opinion layer. Second, dynamics order between layers does not have an influence for network state, though there exists tiny consensus time gap. Third, order of nodes in layer B has more influence for network states than order of nodes in layer A. order of nodes in layer B makes the first branch point. But order of nodes in layer A does not make any branch point, though there exists tiny consensus time gap. Forth, order of edges in layer A is very influential so that it makes different network states. So to speak, characteristics of nodes in layer A, such as rash and considerate, affects consensus time and sometimes makes transition to coexistence or opposite orientation.



Div	Positive Consensus (close to positive)	Coexistence	Negative Consensus (close to negative)
Orders	O(r, r) ⟵ D(o) O(r, r) ⟶ D(o) O(o, o) ⟵ D(o) O(o, o) ⇔ D(o) O(o, o) ⟶ D(o) O(r, r) ⇔ D(r) O(r, r) ⟶ D(s) O(r, r) ⟵ D(s) O(o, o) ⟶ D(s) O(o, o) ⟵ D(s) O(s, o) ⟵ D(o) O(s, o) ⟶ D(o) O(s, o) ⟵ D(s) O(s, o) ⟶ D(s) O(s, o) ↔ D(s)	O(s, s) ⟵ D(o) O(o, s) ⟵ D(o) O(s, s) ⟶ D(o) O(o, s) ⇔ D(o) O(o, s) ⟶ D(o)	O(s, s) ⟵ D(s) O(o, s) ⟵ D(s) O(o, s) ⟶ D(s) O(s, s) ⟶ D(s) O(s, s) ↔ D(s)

Figure 4–5 Total results of 25 updating rules with *AS*



Div.		**States**	**Orders**
① Branch point	② Branch point : Sequential order of node in layer B	Fast positive consensus : Sequential order of edge	O(r, r) ← D(o) O(r, r) → D(o) O(o, o) ← D(o) O(o, o) ⇔ D(o) O(o, o) → D(o) O(r, r) ⇔ D(r) O(s, o) ← D(o) O(s, o) → D(o)
		Coexistence : Simultaneous order of edge	O(s, s) ← D(o) O(o, s) ← D(o) O(s, s) → D(o) O(o, s) ⇔ D(o) O(o, s) → D(o)
	③ Branch point : Simultaneous order of node in layer B	Slow positive consensus : Sequential order of edge	O(r, r) → D(s) O(r, r) ← D(s) O(o, o) → D(s) O(o, o) ← D(s) O(s, o) ← D(s) O(s, o) → D(s) O(s, o) ↔ D(s)
		Slow negative consensus : Simultaneous order of edge	O(s, s) ← D(s) O(o, s) ← D(s) O(o, s) → D(s) O(s, s) → D(s) O(s, s) ↔ D(s)

Figure 4–6 Total results of 25 updating rules with *CI*

Chapter 5 Finding key nodes on two layer networks

In this chapter, it would be investigated that what nodes are important to keep or change their orientation on two-layer networks. There exist many methods to find key nodes, such as pagerank, degree centrality, and eigenvector centrality. Based on these methods, it would be researched that which method is the most effective and the most influential for changing state on two layers.

5.1 Method for finding key nodes

We would find important nodes on two-layer networks by using node centrality. Here is the way to find key nodes.

1. All nodes are ranked by 6 node centralities(pagerank, degree, eigenvector, closeness, betweenness, random).
2. The nodes would be deactivated from high ranked order until the state of network has significant difference, i.e. the ratio of stubborn node would be increased according to high ranked order.
3. The results would be compared according to node centralities. If the least ratio of stubborn node makes the largest difference of network state, its node centrality is the most influential for competition of the interconnected network

As initial condition for finding key nodes, each layer is made of BA network with 2048 nodes and 1 external edge. Each simulation takes 100 steps, and 100 simulations are considered for average results. To demonstrate the difference of network state clearly, for finding key nodes on layer A, the parameters would be set to be negative state. Then, as the stubborn nodes on layer A are increased, the network state would be gradually changed into positive state. Inversely for finding key nodes on layer B, the parameters would be set to be positive. Then, as the stubborn nodes on layer B are increased, the network state would be gradually changed into negative state.

5.2 Key nodes on layer A

5.3 Key nodes on layer B

5.4 Key nodes on two layers with different structures

5.5 Conclusion

Chapter 6 Conclusion

6.1 Summary

We have researched the competition of two layer networks. To begin with, competing interconnected networks were introduced to have different dynamics on each layer. And some indexes were provided to measure how the network state is changed and evaluate the consensus on two layer. Based on this modeling, various simulations were implemented according to 3 main topics.

- Competition on two layer with different structural network
- Competition on two layer with different updating rules
- Finding key nodes on two layer networks

In chapter 3, we have investigated competition on two layer with different structural network. With changing network structure, it was measured that how the interconnected network change its state and make consensus. As the method to revise the network structure, 3 ways were provided. First, as the result of changing the internal edges, a layer with more internal edges has more tendency to keep its own states Second, as the result of changing the external edges, hierarchical model was provided.

Third, as the result of changing the network type,

In chapter 4, it has been researched that how the dynamics orders and updating rules have influence on the competition of two-layers network. Dynamics orders are divided into whether layer A first begin the dynamics or layer B first start the dynamics, or two layer begin together. Updating rules are divided into two categories. As one category, it could be considered that whether it is simultaneous updating rule or sequential updating rule. As the other category, it could be thought that how the updating rules are applied. When each node changes its state, it can be considered that all nodes are changed simultaneously or each node is changed sequentially. When a node change its state, it can be also thought that all connected edges are operated simultaneously or each edge is operated sequentially. According to dynamics orders and updating rules, 25 simulations were implemented.

Through simulation results, several conclusions can be derived. First, networks with more simultaneous updating rules make slow consensus or coexistence, sometimes make transition to opposite orientation. On the other hands, networks with more sequential updating rules make fast consensus. In other words, if opinion layer has more rash nodes, more time to have some conversation and decision making layer has more time to discuss topics, the network have more probabilities to make consensus for opinion layer. Second, dynamics order between layers does not have an influence for network state, though there exists tiny consensus time gap. Third, order of nodes in layer B has more influence for network states than order of nodes in layer A. order of nodes in layer B makes the first branch point. But order of nodes in layer A does not make any branch point, though there exists tiny consensus time gap. Forth, order of edges in layer A is very influential so that it makes different network states. So to speak, characteristics of nodes in layer A, such as rash and considerate, affects consensus time and sometimes makes transition to coexistence or opposite orientation.

In chapter 5, it has been studied that how the key nodes can be found out on the interconnected network. To find key nodes on the network,

6.2 Discussion

So far, we have researched and analyzed the competitions of two-layers network. It was found out that how network structures have the influence on the consensus of two-layers and what nodes have more influential to affect the network state. In real world, we can find out the phenomenon of these competitions, such as election, legislation, adoption of new policies and making decision on social conflict issues. These competitions of real world may have similar characteristics with our simulation results. Therefore, based on simulation results, these competitions can be applied to solve the social conflict.

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