上海交通大学硕士学位论文

COMPETITION OF SOCIAL OPINIONS ON TWO LAYER NETWORKS

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COMPETITION OF SOCIAL OPINIONS ON TWO LAYER NETWORKS

摘 要

Social conflict can be explained with competition network of two layers. This paper is investigated for a model with the competition between two-layer opinions, where the first layer is opinion formation and the second layer is decision making, on interconnected networks. Networks show the two interacting social sectors, the civilians, and representatives. Layer A is civilian opinion layer consists of four states $(-2, -1, +1, +2)$. These states describe the level of influence of opinion dynamics with reinforcement parameter γ. The layer B is the decision making layer that consists of only two states $(+1, -1)$. This layer can influence the decision dynamics with the probability in which decision is proportional to the number of interaction with the opposite opinion population raised to the power of β. Starting with a polarized competition case, layer A is all positive and layer B is all negative. In this paper, we create new models by changing the network structure, and compare these models with the pre-existing model. Then conditions are investigated that have the influence to opposite side and that make consensus in the interconnected network. This study could help to analyze social networks, such as legalization of social issues and prediction of vote results. Further more, it could contribute to solving the social conflict.

关键词：complex network, interconnected network, modeling and simulation, social network analysis, opinion dynamics, consensus, language competition dynamics

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ABSTRACT

Social conflict can be explained with competition network of two layers. This paper is investigated for a model with the competition between two-layer opinions, where the first layer is opinion formation and the second layer is decision making, on interconnected networks. Networks show the two interacting social sectors, the civilians, and representatives. Layer A is civilian opinion layer consists of four states $(-2, -1, +1, +2)$. These states describe the level of influence of opinion dynamics with reinforcement parameter γ. The layer B is the decision making layer that consists of only two states $(+1, -1)$. This layer can influence the decision dynamics with the probability in which decision is proportional to the number of interaction with the opposite opinion population raised to the power of β. Starting with a polarized competition case, layer A is all positive and layer B is all negative. In this paper, we create new models by changing the network structure, and compare these models with the pre-existing model. Then conditions are investigated that have the influence to opposite side and that make consensus in the interconnected network. This study could help to analyze social networks, such as legalization of social issues and prediction of vote results. Further more, it could contribute to solving the social conflict.

KEY WORDS: complex network, interconnected network, modeling and simulation, social network analysis, opinion dynamics, consensus, language competition dynamics

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Nomenclature

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Chapter 1 Introduction

1.1 Introduction

People have their own opinions, and sometimes they change their opinions in response to others that hold views on given issues. Their opinions are reflected to the leader to make laws and vital decision. These phenomena can be found out in some cases, such as voting, legislation and adoption of new policies. It is widely recognized that opinion formation and decision making formation have mutual interaction as interconnected networks.[mikko2013, danziger2019, newman2010, boccaletti2014, domenico2013, tomasini2015, namkhanhvu2017]. And sometimes, opinion formation could be opposed to decision making formation. These situations often make social conflict and cause social confusion. To figure out these social conflicts, it is needed to understand and analyze the competition of interconnected networks. So far, physics and computer science have researched these social conflict by modeling and analyzing the complex systems[huberman2004, zuev2012, laguna2004, masuda2015]. The researches include opinion dynamics, voter model, game theory and many more[bianconi2018].

Competition of interconnected networks has been researched in many ways. These networks can be applied to the dissemination of computer viruses, messages, opinions, memes, diseases and rumors[hua2014, shenyu2018, zhou2018, alvarez2016, gomez2015, diep2017, rocca2014, velasquez2018]. Opinion dynamics on interconnected networks are investigated with various network models such as *Abrams-Strogatz(AS)* model[abrams2003, vazquez2010] and M model[rocca2014]. Based on the previous researches, we would study the main features of competing two-layer networks by changing network structures, changing the way to interact, and finding the key nodes on layers.

1.2 Related Work

To make two layer networks under competition, each layer is made up with different dynamics network. Network dynamics are based on previous research such as[alvarez2016] One layer has the function of social opinion and its own dynamics. Some opinion models provide social mechanism by means of a compromise process.[naim2003] Some opinion models represent persuasive process.[chau2014] In this paper, the social opinion layer is affected by the opinion dynamics which are also known as M-model[rocca2014], that includes compromise function and persuasion function. The other layer also has the function of decision-making and its own dynamics. The dynamics of the decision making layer is the language competition dynamics that are also called as Abrams-Strogatz model[abrams2003, vazquez2010, patriarca2012]. This model is useful to decide only one opinion from two opinions. For competition condition, the initial condition of the two layers is assumed to be in opposite states, social opinion layer has all positive states, decision making layer has all negative states.

So far, main researches have focused on what factors make a consensus or dissent, which have shown that the system can make positive consensus, negative consensus or coexistence under certain range of parameters, such as volatility, reinforcement, and prestige. Also, it is found out that the thresholds make the transition of states and they can explain and analyze the social phenomena in real world such as the legislation, election result, and social network[alvarez2016, amato2017, diep2017]. In[gomez2015], it is shown that the transition from localized to mixed status occurs through a cascade from poorly connected nodes in the layers to the highly connected ones. In addition, the main features, such as transition and cascade, found in Monte Carlo simulation are exactly characterized by the mean-field theory and magnetization[alvarez2016, diep2017, amato2017, gomez2015].

To change the structure of network, network structures would be investigated.

Network can be largely divided into regular network, random network[erdos1960], small world network[watts1998], scale free network[barabasi1999] and others. Regular network has lattice structure, and each node has exactly the same number of links. Random network is made up with edges that two node are connected with probability p in the systems with K nodes. Small world network is a network graph in which most nodes are not neighbors of one another, but most nodes can be reached from every other node by a small number of links. Small world network can be made by eliminating the edges with probability p and connecting two random nodes that are not connected in a regular network. Small world network has all characteristics of regular network and random network. Scale free network has the model that distribution of edges follows power function. Examples of scale free network are the World Wide Web (WWW), the Internet, movie star networks, protein interactions, metabolism, and so on. There are several ways to create a scale free network. Among them, the most typical way is Barabasi-Albert models.

The Barabasi-Albert model is growing networks in which nodes continue to be added, and connections between nodes has preferential attachment. The process of creating this model repeats the following two processes: First, add one node with a constant number of edges to the system every hour. Second, edges of the added nodes are connected in proportion to edges number of the pre-existing nodes In this paper, two type of general network would be applied such as Random-regular network and Barabasi-Albert network.

To find key nodes on two layers network, network centralities would be researched. Network centrality means the index to measure how close each node is to the center of the network. Than means answers to the question "What characterizes an important node?". The concept of network centrality was first introduced in the field of social network analysis.[freeman1979] After that, it has expanded to various areas where the concept of the network is related and has been used to identify which nodes are important in the network. So far, various criteria for assessing network centrality have been presented. Generally well-known network centralities include degree centrality, betweeness centrality, closeness centrality, eigenvector centrality and pagerank centrality.[koschutzki2008]

Degree centrality is the simplest but the most reliable concept. it is defined as the number of interacting neighbor nodes (or edges). Betweenness centrality is the concept of using the shortest path between two nodes on a network. It is explained as the concept to define two different node sets on the network (set 1, 2) and quantify how often each node appears on the shortest path for all combinations of nodes in set 1 and set 2. Closeness centrality is derived from that the shorter the path that one node reaches all the other nodes is, the more important the node is. Eigenvector centrality is the concept that the more a node is connected with critical nodes, the more important it is. Pagerank centrality measures the convergent value by repeating the process of propagating each node's influence to the other nodes.

In this paper, as the methods to find key nodes, network centralities are researched such as pagerank, degree centrality, eigenvector centrality, betweenness, and closeness.[francisco2019, bianconi2018] By using 5 centralities(pagerank, degree, eigenvector, closeness, betweenness), it would be found out that which property is the most influential. Finally, the best method would be provided to find key nodes on the competing two layers.

1.3 Thesis Objective

In this paper, opinion dynamics of a competing two-layer social network are investigated on the basis of the pre-existed research[alvarez2016, gomez2015, diep2017, rocca2014]. As the result of pre-existed research, interconnected competition of the social network have been researched by finding the threshold or critical point for consensus[alvarez2016, gomez2015, diep2017]. It has been proved that the system can make positive consensus, negative consensus or coexistence parts in interconnected competition of the social network[alvarez2016]. And it is shown that the number of external degree is very important to change the state of layers[gomez2015]. We develop the previous modeling and research to find out the characteristics of interconnected networks. By switching the network structure of each layer, such as changing the number of nodes or the number of edges, we can see how the consensus or coexistence states change and what conditions make the social consensus. This can help to explain social networks phenomena, such as social conflict between social opinion and the congress. Therefore, this research could be used as a tool for analyzing legislation problems, making efficient decision-making system and solving the social conflict.

Researching directions have 4 main topics. First, it would be provided how to make up competition models and how to measure the consensus for analysis. Second, it would be found out what factors make consensus by changing network structures. Second, it would be analyzed whether dynamics orders have an influence on status of two-layer. Third, it would be investigated which method is the best to identify key nodes based on node centralities.

The paper is organized as follows. In section 2, the Basic Model is introduced and the dynamics, that is applied to each layer, are described. In section 3, the simulation results for the basic model and revised models are presented. In section 4 the characteristics of each model are described through the comparison and analysis. Finally, in section 5, the simulation results will be summarized and our findings are concluded.

1.3.1 准备工作

要使用这个模板撰写学位论文，需要在 *TeX* 系统、*TeX* 技能上有所准备。

- TEX 系统：所使用的 TEX 系统要支持 XƎTEX 引擎，且带有 ctex 2.x 宏包，以 2017 年或更新版本的完整 TeXLive、MacTeX 发行版为佳。
- TeX 技能：尽管提供了对模板的必要说明，但这不是一份“LATEX 入门文档”。在使用前请先通读其他入门文档。
- 针对 Windows 用户的额外需求：学位论文模本分别使用 git 和 GNUMake 进行版本控制和构建，建议从 Cygwin[1]安装这两个工具。

[1]http://cygwin.com

1.3.2 模板选项

sjtuthesis 提供了一些常用选项，在 thesis.tex 在导入 sjtuthesis 模板类时，可以组合使用。这些选项包括：

- 学位类型：bachelor(学位)、master(硕士)、doctor(博士)，是必选项。
- 中文字体：fandol(Fandol 开源字体)、windows(Windows 系统下的中文字体)、mac(macOS 系统下的华文字体)、ubuntu(Ubuntu 系统下的文泉驿和文鼎字体)、adobe(Adobe 公司的中文字体)、founder(方正公司的中文字体)，默认根据操作系统自动配置。
- 英文模版：使用 english 选项启用英文模版。
- 盲审选项：使用 review 选项后，论文作者、学号、导师姓名、致谢、发表论文和参与项目将被隐去。

1.3.3 编译模板

模板默认使用 GNUMake 构建，GNUMake 将调用 latemk 工具自动完成模板多轮编译：

1.3.3.1 格式控制文件

格式控制文件控制着论文的表现形式，包括 sjtuthesis.cfg 和 sjtuthesis.cls。其中，“cls”控制论文主体格式，“cfg”为配置文件。

1.3.3.2 主控文件 thesis.tex

主控文件 thesis.tex 的作用就是将你分散在多个文件中的内容“整合”成一篇完整的论文。使用这个模板撰写学位论文时，你的学位论文内容和素材会被“拆散”到各个文件中：譬如各章正文、各个附录、各章参考文献等等。在 thesis.tex 中通过“include”命令将论文的各个部分包含进来，从而形成一篇结构完成的论文。对模板定制时引入的宏包，建议放在导言区。

1.3.3.3 各章源文件 tex

这一部分是论文的主体，是以“章”为单位划分的，包括：

- 中英文摘要 (abstract.tex)。前言 (frontmatter) 的其他部分，中英文封面、原创性声明、授权信息在 sjtuthesis.cls 中定义，不单独分离为 tex 文件。不单独弄成文件。

- 正文 (mainmatter)——学位论文正文的各章内容，源文件是 chapter*xxx*.tex。
- 附录 (app*xx*.tex)、致谢 (ack.tex)、攻读学位论文期间发表的学术论文目录 (pub.tex)、个人简历 (resume.tex) 组成正文后的部分 (backmatter)。参考文献列表由 bibtex 插入，不作为一个单独的文件。

1.3.3.4 图片文件夹 figure

figure 文件夹放置了需要插入文档中的图片文件 (支持 PNG/JPG/PDF/EPS 格式的图片)，可以在按照章节划分子目录。模板文件中使用`\graphicspath`命令定义了图片存储的顶层目录，在插入图片时，顶层目录名“figure”可省略。

1.3.3.5 参考文献数据库 bib

目前参考文件数据库目录只存放一个参考文件数据库 thesis.bib。关于参考文献引用，可参考第**??**章中的例子。

Chapter 2 Modeling and Analysis

In this chapter, basic model would introduced for competition on two-layer network and some indexes would be provided to analyze the interaction between two-layers.

2.1 Modeling of two layer network

The model consists of two layers, and each layer has different dynamics. For layer A, the node change its states according to M model as introduced in[rocca2014]. Here, we choose $M = 2$, that each node has four states $(-2, -1, +1, +2)$. For each link (k, j) belong to layer A, the dynamics are designed as follows:

- Compromise : if two nodes connected with link(k, j) have opposite orientations, their states become more moderate with probability q :

$$\text{if } S_k < 0 \text{ and } S_j > 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^r, S_j^l) \text{ with } prob.q, \tag{2–1}$$

$$\text{if } S_k > 0 \text{ and } S_j < 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^l, S_j^r) \text{ with } prob.q. \tag{2–2}$$

If $S_k = \pm 1$ and $S_j = \mp 1$, one switches orientation at random:

$$(\pm 1, \mp 1) \rightarrow \begin{cases} (+1, +1) \text{ with } prob.q/2, \\ (-1, -1) \text{ with } prob.q/2. \end{cases} \tag{2–3}$$

- Persuasion : if two nodes connected with link(k, j) have the same orientation, their states become more extreme with probability p :

$$\text{if } S_k < 0 \text{ and } S_j < 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^l, S_j^l) \text{ with } prob.p, \tag{2–4}$$

$$\text{if } S_k > 0 \text{ and } S_j > 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^r, S_j^r) \text{ with } prob.p. \tag{2–5}$$

For each external link (k, j) with k belong to layer A, the state of node k is updated according to :

- $S_k \cdot S_j < 0$:

$$\text{if } S_k < 0 \text{ and } S_j > 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^r, S_j) \text{ with } prob.q, \tag{2–6}$$

$$\text{if } S_k > 0 \text{ and } S_j < 0 \Rightarrow (S_k, S_j) \rightarrow (S_k^l, S_j) \text{ with } prob.q. \tag{2–7}$$

- $S_k \cdot S_j > 0$:

$$\text{if } S_k < 0 \text{ and } S_j < 0 \Rightarrow (S_k, S_j) \to (S_k^l, S_j) \text{ with } prob.p, \tag{2–8}$$

$$\text{if } S_k > 0 \text{ and } S_j > 0 \Rightarrow (S_k, S_j) \to (S_k^r, S_j) \text{ with } prob.p. \tag{2–9}$$

Here, S_k^r and S_k^l denote the right and left neighboring states of k, defined as

$$S_k^r = \begin{cases} +1, \text{ for } S_k = -1 \\ +2, \text{ for } S_k = +2 \\ S_k + 1, \text{ otherwise}, \end{cases} \qquad S_k^l = \begin{cases} -1, \text{ for } S_k = +1 \\ -2, \text{ for } S_k = -2 \\ S_k - 1, \text{ otherwise}. \end{cases} \tag{2–10}$$

The sign of S^A represents its opinion orientation and its absolute value $|S^A|$ measures the intensity of its opinion. So, $|S^A| = 2$ represents to a positive or a negative extremist, while $|S^A| = 1$ correspond to a moderate opinion of each side. In case of internal link (k, j) belong to layer A, when the nodes have the same orientation($S_k S_j > 0$), if the states of nodes are moderate, then they become extreme($S_k = \pm 1 \to \pm 2, S_j = \pm 1 \to \pm 2$) with probability p. If they are already extreme, they remain extreme($S_k = \pm 2 \to \pm 2, S_j = \pm 2 \to \pm 2$). On the other hand, when the nodes have opposite orientations($S_k S_j < 0$), if they are extreme, the states of nodes become moderate($S_k = \pm 2 \to \pm 1, S_j = \pm 2 \to \pm 1$) with probability q. If they are already moderate, they switch orientations individually($S_k = \pm 1 \to \mp 1, S_j = \pm 1 \to \mp 1$). In case of interaction between node in layer A and node in layer B, node in layer A follows opinion dynamics formula, but the state of node in layer B does not change. In other words, the state of layer B affects layer A, but layer A dynamics does not affect the state of node in layer B. For example, one of the layer A node, $S_k = +2$ is connected with $S_j = -1$ node of layer B. Here, S_k will change into $S_k = +1$ with $prob.q$. But S_j will not change, which indicates that the states of layer B will influence the states of layer A.

The dynamics of layer B follows the decision-making dynamics as introduced in[abrams2003, vazquez2010]. The state of node i in layer B can be $+1$ and -1, and it updates according to

$$P_B(S_i \to -S_i) = \begin{cases} \left(\dfrac{i_i + e_i}{n^{-S_i}}\right) \cdot \left(\dfrac{n^{-S_i}}{i_i + e_i}\right)^{1/v}, \text{ if } v \neq 0 \\ 0, \text{ if } v = 0 \\ 0, \text{ if } n^{-S_i} = 0 \end{cases}, \tag{2–11}$$

where i_i is the number of internal edges and e_i is the number of external edges. n^{-S_i} is the number of neighbors of i with opposite state $-S_i$. v represents the volatility that measures how prone a node change its state. The scale of v is from 0 to 1. If $v \simeq 0$, a node is unlikely to change its state. On the other hand, if $v \simeq 1$, a node is very likely to change its state. Also, this formula shows that the more the number of nodes connected with the opposite state is, the easier the nodes are to change into the opposite state.

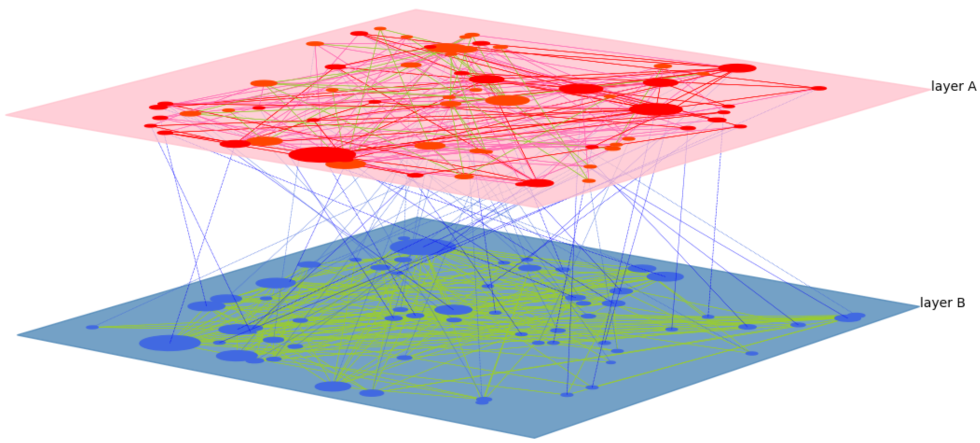

Figure 2–1 Competition of Interconnected Network

2.2 Simulations and Analysis

To start with a polarized competition, as the initial conditions, nodes in layer A are all positive, and nodes in layer B are all negative as shown in Fig. 2–1. For nodes in layer A, it begins with the status where half of nodes are +1 and the others are +2. The initial state of nodes in layer B have only −1.

There are two parameters in the dynamics of layer A. To simply represent the probability p and probability q together, we set $p + q = 1$. So, p represents the tendency of opinion such as extreme or moderate, which is scaled to be 0 to 1. And, the scale of v, in the dynamics of layer B, is also 0 to 1.

To implement the interconnected dynamics, one step consists of two layers dynamics, where every node in layer A will be checked with opinion dynamics, and every node in layer B will updates its state according to the decision-making dynamics. Here, we would control dynamics orders between layers and updating rules of nodes states. With changing dynamics orders and updating rules, it would be investigated how state of network change.

Each simulation takes 100 steps, and 100 simulations are considered for average results. In the following simulations, we use *'Average State'(AS)* and *'Consensus Index'(CI)* to measure the competition result.

$$AS = avg\left(\sum_{i}^{K^A} S_i^A/4\right) + avg\left(\sum_{i}^{K^B} S_i^B/2\right). \tag{2–12}$$

$$CI = \frac{({K_+}^A \cdot {K_-}^B) + ({K_-}^A \cdot {K_+}^B)}{K^A \cdot K^B} \tag{2–13}$$

In these formula, S_i^A means the state of node i in layer A, and K^A is the number of nodes in layer A. ${K_+}^A$ represents the number of nodes with positive state in layer A.

With *AS*, it could be verified whether the consensus happens in accordance with the change of p and v. If the positive consensus happens, it would be close to the value of $+1$ and if the negative consensus happens, it would be close to the value of -1. The values between $+1$ and -1 mean the states are belonging to the coexistence part.

With *CI*, it could be measured how close the network state is to consensus. If the CI is close to 0, the state is close to positive or negative consensus. If the CI is close to 1, the state is separated coexistence where states of all nodes in layer A is opposed to states of all nodes in layer B. If the CI is close to 0.5, the state is mixed coexistence where each layer has both positive and negative states of nodes.

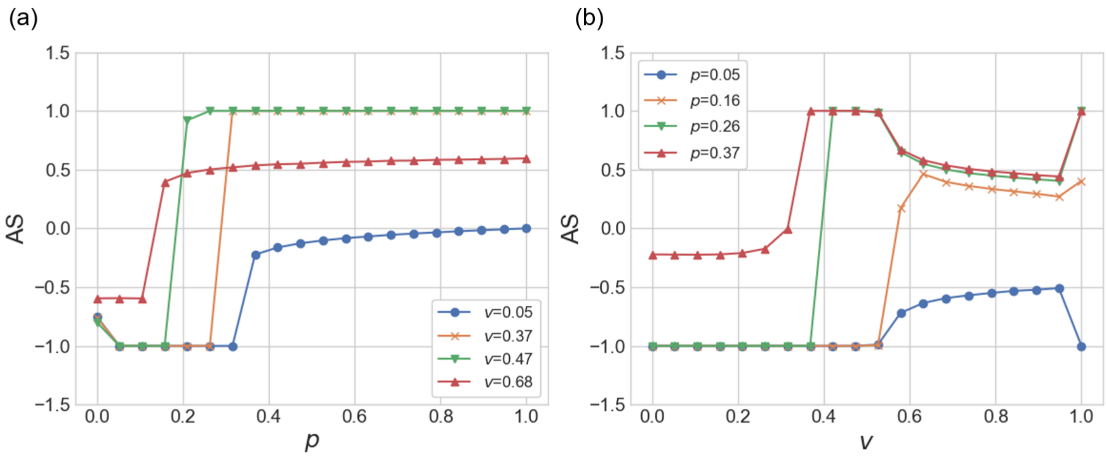

Figure 2–2 (a) *p-AS* chart according to certain *v* values. (b) *v-AS* chart according to certain *p* values.

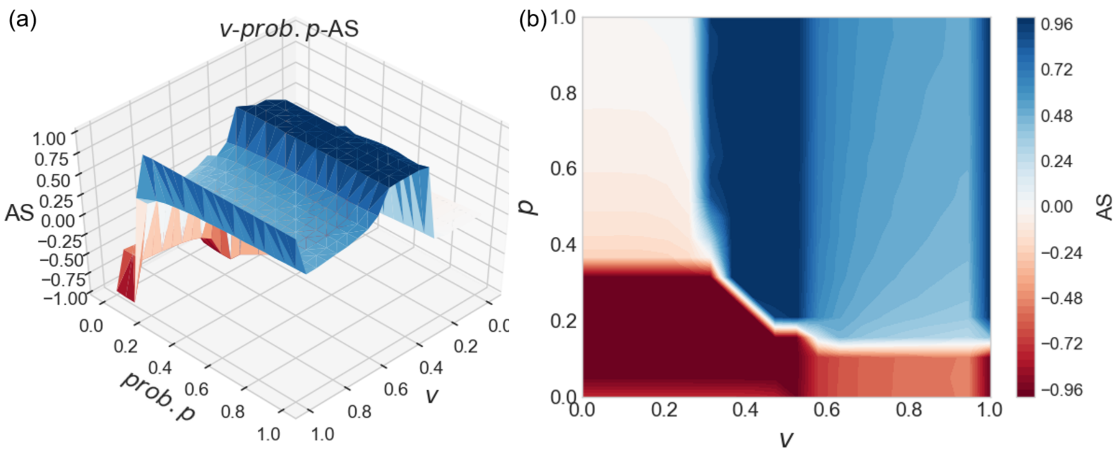

Figure 2–3 Two layer networks with sequential updating rule : *AS* changing with all *p* and *v*

Chapter 3 Competition on two layer with different structural network

3.1 Competition on Networks with different number of external links

In this subsection, we consider the influence of external links. Based on the basic model in Subsection 3.1, we reduce the number of nodes in layer B at a certain rate and increase the external links from nodes in layer B accordingly. We denote *HM(n)* as a hierarchical model with a level n, which means that the number of nodes in layer B is $1/n$ of the number of nodes in layer A, and the number of external links from node in layer B is n in view that the number of external links from node in layer A is 1. In other words, each node in layer A has one external edge, but each node in layer B has n external edges for *HM(n)*, which means one node in layer B can be influenced by n nodes in layer A. γ scale is same as the Random Regular Networks Model. But, β scale depends on the number of degrees. So the β scale is adjusted to have the same probability of volatility with Random Regular Networks Model(*RRM*) as following Equation.

$$\beta_{h,\max} = \beta_{rr,\max} \cdot \log\left(\frac{{n_{rr}}^{-S_i}}{i_{rr,i} + e_{rr,i}} \cdot \frac{i_{h,i} + e_{h,i}}{{n_h}^{-S_i}}\right). \tag{3–1}$$

Eq(6) is derived from Eq(1) at the initial states. $\beta_{h,\max}$ is the maximum value of β scale in *HM*, and $\beta_{rr,\max}$ is the maximum value of β scale in *RRM*. When *RRM* begins with initial state and the maximum of β scale, it has the lowest volatility except 0. In order to have the same probability in layer B dynamics for different network structures at the initial time, maximum value of β in *HM* is calculated based on Eq(6).

Fig. 4–1 shows the Hierarchical Model simulation results. Comparing *HMs* with *RRM*, *CR* and *PCR* are all increased remarkably. *HMs* have more positive consensus part than *RRM*. It shows that as the number of B nodes are decreased, it is easy to make positive consensus. Comparing *HM(16)* with other *HMs*, *HM(16)* has the most positive consensus part. In case of models where the number of nodes in layer B is less than *HM(16)*, *CR* and *PCR* of the models are decreased and *NCR* is increased slightly. Also, for models where the number of nodes in layer B is more than *HM(16)*, *CR* and *PCR* are also decreased. However, *HM(4)* has the most *AS total*. Although *HM(4)* doesn't have the most consensus part, it has more intensity for positive social opinion. It can be analyzed that strong social intensity usually can not make more consensus. These results indicate that network structure can contribute more for consensus.

Figure 3–1 Hierarchical Model(*HM(n)*)

In summary, all the Hierarchical Models have more consensus ratio than Random Regular Networks Model. Among *HMs*, *HM(16)* has the most positive consensus part. When the number of nodes in layer B is more or less than *HM(16)*, *CR* and *PCR* are decreased. This shows that there exists an efficient number for the decision making layer to perform positive consensus.

3.2 Competition on Networks with different number of internal links

Figure 3–2 Comparison of Networks with different internal degrees(*RR(n)-RR(m)*: layer A has random regular network with *n* internal edges, layer B has random regular network with *m* internal edges)

Next, the interconnected networks are simulated with different internal degrees in order to define and evaluate the influence of internal degrees. The number of internal degrees on each node is switched to 2 or 5.

Fig. 4–2 shows the simulation results with changing the number of internal edges. *RR(5)-RR(2)* has the most *PCR*. *RR(2)-RR(5)* has the most *NCR*. When the number of internal edges in layer A are more than layer B, it has more positive consensus. On the other hand, when the number of internal edges in layer B are more than layer A, it has relatively more negative consensus. These results provide that the number of edges on layer A has the tendency to keep positive state, and the number of edges on layer B has the tendency to keep negative state. The number of internal edges have the influence on consensus result and a layer with more internal edges has the tendency to maintain its own state. In case of networks with same internal edges, *RR(2)-RR(2)* has more *PCR* and *AS total* than *RR(5)-RR(5)*. It can be analyzed that *RR(5)-RR(5)* is hard to make consensus, because it has more internal edges to cause inner conflict. Also, *RR(2)-RR(2)* has less *NCR* than *RR(5)-RR(5)*. It shows that the number of internal edges in layer B is more sensitive than layer A. As Eq(1) shows, layer B dynamics can have more various and extreme probabilities when it has more degrees. For example, in case of *RR(2)-RR(2)* with $\beta = 1$, the dynamics starts with $P_B = 1/3$ and in case of *RR(5)-RR(5)* with $\beta = 1$, the dynamics starts with $P_B = 1/6$.

3.3 Competition on Networks with different structures

So far, each layer of the interconnected network consisted of random regular networks that has the same number of edges for each node. Now, the simulation would be implemented on different network structures.

Figure 3–3 Comparison of Networks with different structures

Here, we use *Barabasi-Albert network(BA)* structure as introduced in[barabasi1999]. To evaluate the influence of network structure, 5 simulations are implemented with changing network structures. The *BA* network is applied for both layers or switched on each layer. And, because layer A with *BA* network structure has total 10, 215 internal edges, *RR(10)-RR(5)*, under the similar conditions such as the number of nodes and edges, is also simulated. The simulation results are shown in Fig. 4–3. The result of *BA*-*RR* and *RR(10)-RR(5)* have almost the same features. The gap of *CR* is almost same(less than 0.01). The structure of network make no obvious difference of consensus results. In case of *BA*-*BA*, the *CR* has the least ratio for consensus. *BA*-*BA* structure has lots of internal edges on each layer. Therefore, it is hard to make consensus due to inner conflict on each layer.

Chapter 4 Competition on two layer with different structural network

4.1 Competition on Networks with different number of external links

In this subsection, we consider the influence of external links. Based on the basic model in Subsection 3.1, we reduce the number of nodes in layer B at a certain rate and increase the external links from nodes in layer B accordingly. We denote *HM(n)* as a hierarchical model with a level n, which means that the number of nodes in layer B is $1/n$ of the number of nodes in layer A, and the number of external links from node in layer B is n in view that the number of external links from node in layer A is 1. In other words, each node in layer A has one external edge, but each node in layer B has n external edges for *HM(n)*, which means one node in layer B can be influenced by n nodes in layer A. γ scale is same as the Random Regular Networks Model. But, β scale depends on the number of degrees. So the β scale is adjusted to have the same probability of volatility with Random Regular Networks Model(*RRM*) as following Equation.

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In summary, all the Hierarchical Models have more consensus ratio than Random Regular Networks Model. Among *HMs*, *HM(16)* has the most positive consensus part. When the number of nodes in layer B is more or less than *HM(16)*, *CR* and *PCR* are decreased. This shows that there exists an efficient number for the decision making layer to perform positive consensus.

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Chapter 5 Competition on two layer with different updating rules

5.1 Competition on two-layer Networks with sequential updating rule

In this subsection, each layer consists of *Barabasi-Albert(BA)* network that has N nodes with attaching new nodes each with K edges that are preferentially attached to existing nodes with high degree as introduced in[barabasi1999]. Each node of one layer is connected with a random node on the other layer. This means each node has only 1 external un-directed edge. Simulations are preformed on network with $N = 2048$, and $K = 3$.

The simulation results are shown in Fig. 2–2 and Fig. 2–3. Fig. 2–2(a) shows that when $p > 0.2, 0.37 < v < 0.47$, it normally tends to positive consensus. But, if v is lower or larger than certain values, it doesn't make consensus. In Fig. 2–2(b), as v increases, the state of networks changes continuously. But, when p is very low($p \leq 0.05$), it doesn't make positive consensus. On the other hands, when p is large enough, it has the most positive consensus parts. When v are large enough(> 0.6) and less than 1, the state is in a coexistence part.

Fig. 2–3 shows the states of two layers according to all ps and all vs. The X-axis is the p and the Y-axis is the v, and the Z-axis represents AS. The closer the color is to blue, the more it has positive consensus. And the closer the color is to red, the more it has negative consensus. A light and white areas have coexistence with positive states and negative states. This chart has two areas for coexistence, when v is very low or very high. When v is in certain range, interconnected network can perform positive or negative consensus with different p values.

Fig. 5–1 shows AS value according to each step(time). As the steps are increased, the state of two-layers become stable. The closer AS value is to 1, the closer the state is to positive consensus. The closer AS value is to -1, the closer the state is to negative consensus. AS values between 1 and -1 represents coexistence states, but it cannot be classified whether they are mixed coexistence states or separated coexistence states. Fig. 5–2 shows CI value according to each step(time). With CI, the coexistence states of two-layer can be classified into mixed coexistence states and separated coexistence states.

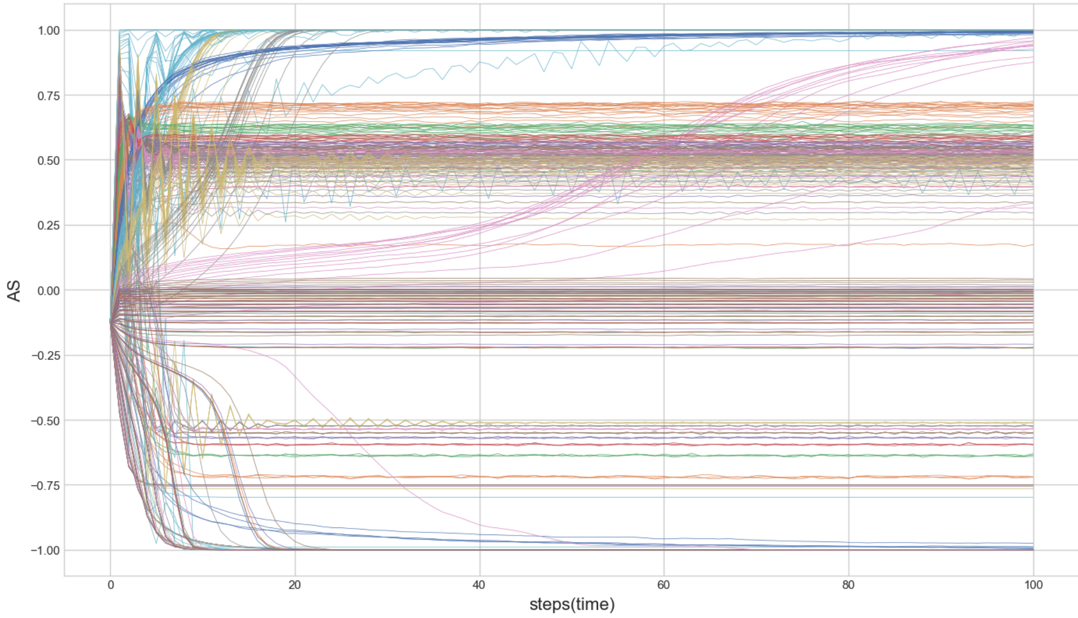


Figure 5–1 Two layer networks with sequential updating rule : *steps-AS* changing with all p and v

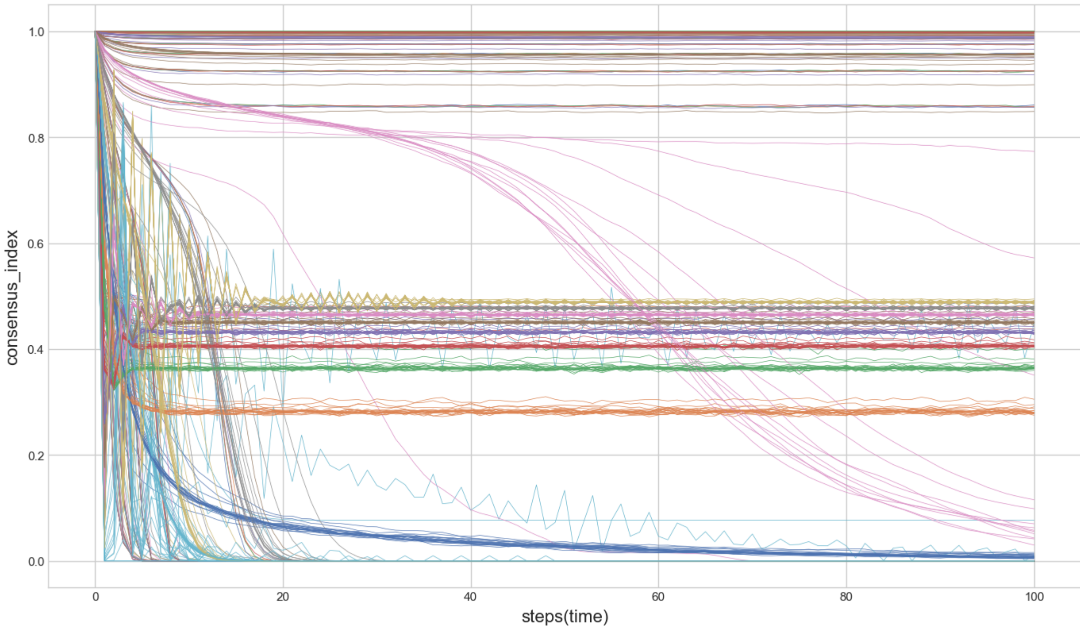


Figure 5–2 Two layer networks with sequential updating rule :*steps-CI* changing with all p and v

As the *CI* value is close to 0.5, the states are close to mixed coexistence states. And, as the *CI* value is close to 1, the states are close to separated coexistence states. If the values are close to 0, the states are close to consensus states.(Here, it is impossible to divide whether they are positive consensus or negative consensus.)

5.2 Competition on two-layer Networks with different updating rules

When considering dynamics order on two-layer networks, there are many ways to update the state of nodes. First, order of two layer dynamics can be considered. And then order of nodes in each layer can be investigated as updating rules. In addition, order of edges in one node also can be researched. But, in layer B dynamics, order of edges in one node is always for simultaneous updating rule, because dynamics formula already considers states of all connected neighbor nodes simultaneously. To sum up, as shown in Table.5–1, 25 updating rules would be considered according to layers, nodes, and edges.

Order of layers	Layer A		Layer B	remarks
	Order of nodes	Order of edges	Order of nodes	
Layer A → Layer B	Sequential	Sequential	Sequential	O(o, o) → D(o)
			Simultaneous	O(o, o) → D(s)
		Simultaneous	Sequential	O(o, s) → D(o)
			Simultaneous	O(o, s) → D(s)
	Simultaneous	Sequential	Sequential	O(s, o) → D(o)
			Simultaneous	O(s, o) → D(s)
		Simultaneous	Sequential	O(s, s) → D(o)
			Simultaneous	O(s, s) → D(s)
	Random	Random	Sequential	O(r, r) → D(o)
			Simultaneous	O(r, r) → D(s)
Layer A ← Layer B	Sequential	Sequential	Sequential	O(o, o) ← D(o)
			Simultaneous	O(o, o) ← D(s)
		Simultaneous	Sequential	O(o, s) ← D(o)
			Simultaneous	O(o, s) ← D(s)
	Simultaneous	Sequential	Sequential	O(s, o) ← D(o)
			Simultaneous	O(s, o) ← D(s)
		Simultaneous	Sequential	O(s, s) ← D(o)
			Simultaneous	O(s, s) ← D(s)
	Random	Random	Sequential	O(r, r) ← D(o)
			Simultaneous	O(r, r) ← D(s)
Layer A ↔ Layer B	Simultaneous	Sequential	Simultaneous	O(s, o) ↔ D(s)
		Simultaneous	Simultaneous	O(s, s) ↔ D(s)
Layer A ⇔ Layer B	Sequential	Sequential	Sequential	O(o, o) ⇔ D(o)
		Simultaneous	Sequential	O(o, s) ⇔ D(o)
	Random	Random	Random	O(r, r) ⇔ D(r)

Table 5–1 25 updating rules according to order of layers, nodes, and edges

In table remarks, 'O(o, o) → D(s)' means Opinion layer(node : sequential order updating, edges : sequential order updating) → Decision Making layer(node : simultaneous updating). And 'O(o, o) ⇔ D(o)' means that one node in Opinion layer is updated, and then one node in Decision Making layer is updated, this rule is repeated until all nodes are updated. Dynamics with 25 updating rules are simulated with parameter $p = 0.4$ and $v = 0.4$. Simulation results are divided by order of layers, nodes and edges.

5.2.1 Order of layers

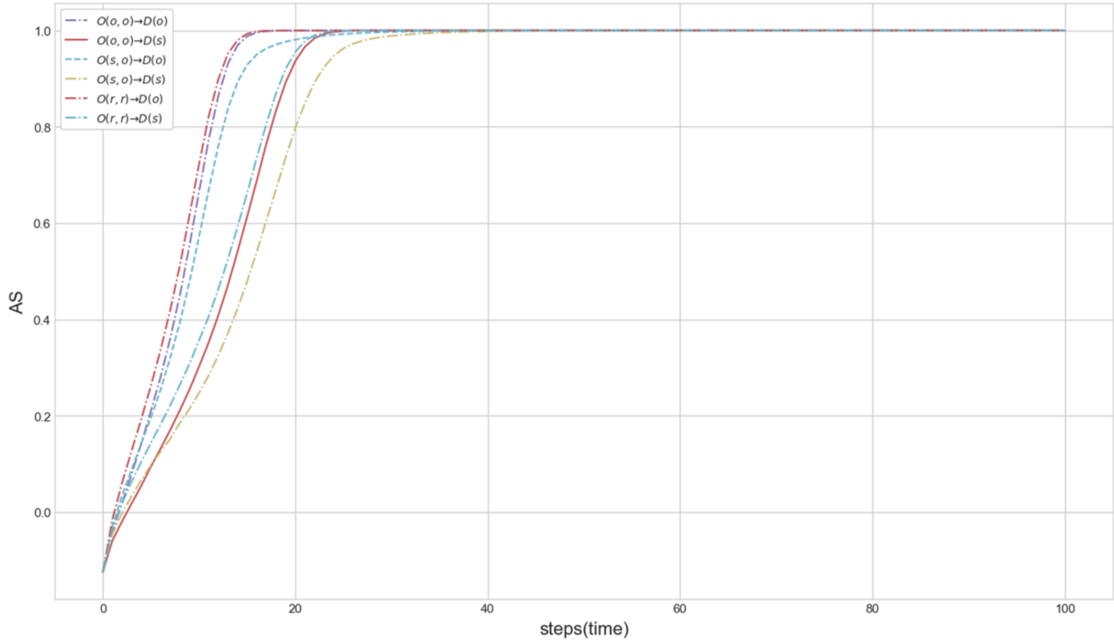
There exist two layers on interconnected network. And each layer have its own dynamics, such as *M-Model* and *AS-Model*. Two dynamics can be operated simultaneously or sequentially. If they act sequentially, dynamics of layer A can act first or dynamics of layer B can work previously. Otherwise, regardless of layers order, nodes of two layers can interact mutually. For example, one node in layer A are updated and then one node in layer B are updated until all nodes are updated. Considering all situations, there are 4 ways in order of two layers, *Layer A* $\rightarrow$ *Layer B, Layer A* $\leftarrow$ *Layer B, Layer A* $\leftrightarrow$ *Layer B(simultaneous), Layer A* $\Leftrightarrow$ *Layer B(interaction regardless of layers)*. As

Figure 5–3 Simulation results according to orders of layers

seen in Fig. 5–3, simulation results show that there is little difference between orders of layers. Consensus time and result are almost same, though dynamics order is different. Regardless of dynamics directions, when other conditions, such as order of nodes and edges are same, the dynamics results are also very similar.

5.2.2 Order of nodes

In the simulation model, each layer has 2048 nodes, and each node has interaction with other nodes. Now, interaction order of nodes would be considered. One node can be updated after other nodes are updated. Otherwise, every node can be updated simultaneously. Simulation results would be different according to interaction order of nodes. In addition, random order between nodes is also simulated. In random order, one edge is selected randomly and updated regardless of orders between nodes and edges until all edges are considered. Interaction order of nodes have meaning related to time. If networks have short time to change states, networks follow simultaneous updating rule. However, if networks have enough time to update states, networks follow sequential updating rules. For example, discussion or conversation with enough time means sequential updating rule of nodes, and election means simultaneous updating rule of nodes.



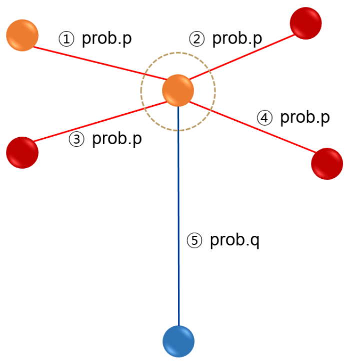
Div	Fast Consensus	Slow Consensus
Orders	① O(r, r) → D(o) ② O(o, o) → D(o) ③ O(s, o) → D(o)	④ O(r, r) → D(s) ⑤ O(o, o) → D(s) ⑥ O(s, o) → D(s)

Figure 5–4 Simulation results according to orders of nodes: comparison between order of nodes under same conditions such as order of layers and edges.

Simulation results shows that simultaneous interaction between nodes makes slow consensus. And, simultaneous interaction between nodes in layer B have more influence on consensus time than in layer A. Random order has similar results with sequential order and does not make different states. For quick social consensus, both opinion layer and decision making layer need sequential updating rule, such as conversation and discussion.

5.2.3 Order of edges

Each node has some edges connected with other nodes. Simulation results can be different according to that edges are operated sequentially or simultaneously. If edges of each node work sequentially, a state of node is changed whenever each edges works. However, If edges of a node work simultaneously, a state of node has to be changed considering all connected nodes. In real world, order of edges in one node can be analyzed as characteristics of nodes. If order of edges is sequential, the node would be rash. If order of edges is simultaneous, the node would be considerate. For example,



Order of edges	Next state
Sequential	Hard to calculate (According to order of activated edges, final state is changed)
Simultaneous	Unchanged: prob.p(activated) = prob.q(activated) Persuasion : prob.p(activated) > prob.q(activated) Compromise: prob.p(activated) < prob.q(activated)

Figure 5–5 one node connected with other nodes changes its state with sequential or simultaneous order of edges

considering the case that one node is connected with other nodes as shown in Fig. 5–5, we can think how the state of node change. If the edges follow sequential updating rule, it is hard to calculate the probabilities, because the states can change according to sequential order of edges. Therefore, we can get next states of nodes by using computer simulation

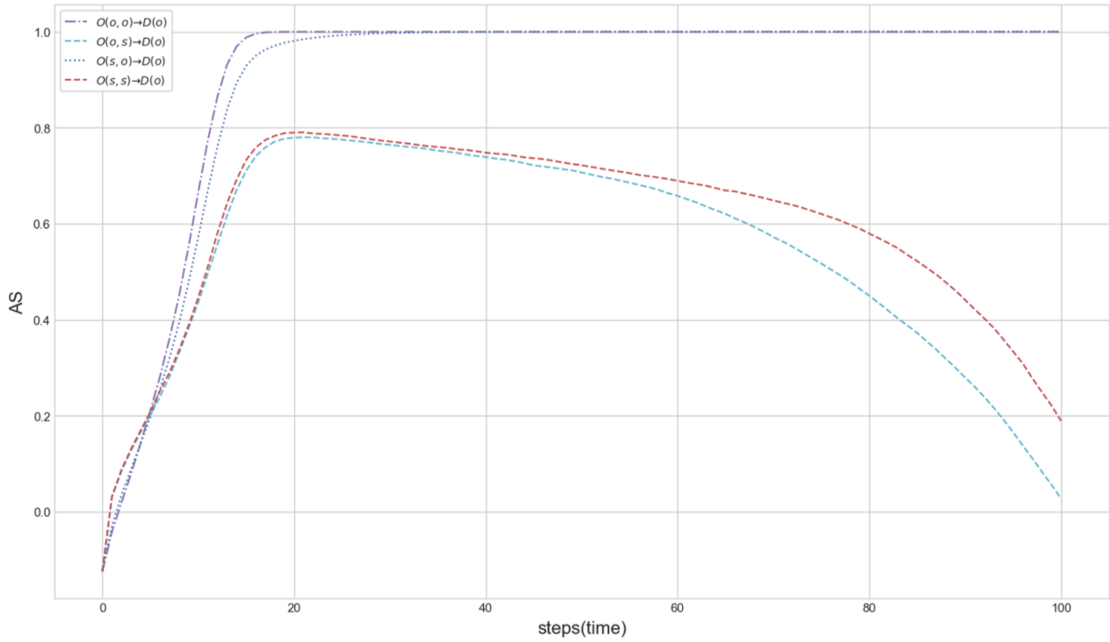
If the edges follow simultaneous updating rule, it needs some assumptions:

1. The number of activated *prob.p* is more than the number of activated *prob.q*, persuasion function work.
2. The number of activated *prob.p* is same with the number of activated *prob.q*, the state would be unchanged.
3. The number of activated *prob.p* is same with the number of activated *prob.q*, compromise function work.

Through these assumptions, we can calculate probabilities of changing state in layer by considering all cases like these formula.

$$
K = \{k \mid 0, \cdots, n^{-S_i}\}, \quad L = \{l \mid 0, \cdots, n^{S_i}\}, \quad M = \{m \mid k - l\},
$$
$$
P_A(S_i \mapsto S'_i) = \begin{cases} \text{unchanged}(k = l) : \sum p^{n^{-S_i}+m} \cdot (1-p)^{n^{S_i}-m} \cdot {}_{n^{S_i}}C_k \cdot {}_{n^{-S_i}}C_l \\ \text{persuasion}(k > l) : \sum p^{n^{-S_i}+m} \cdot (1-p)^{n^{S_i}-m} \cdot {}_{n^{S_i}}C_k \cdot {}_{n^{-S_i}}C_l \\ \text{compromise}(k < l) : \sum p^{n^{-S_i}+m} \cdot (1-p)^{n^{S_i}-m} \cdot {}_{n^{S_i}}C_k \cdot {}_{n^{-S_i}}C_l \end{cases} \tag{5–1}
$$

As shown in Fig. 5–6, sequential updating rule of edges(rash node) makes consensus. But simultaneous updating rule of edges(considerate node) makes it hard to reach



Div	Consensus	Not reaching consensus
Orders	① O(o, o) → D(o) ② O(s, o) → D(o)	③ O(o, s) → D(o) ④ O(s, s) → D(o)

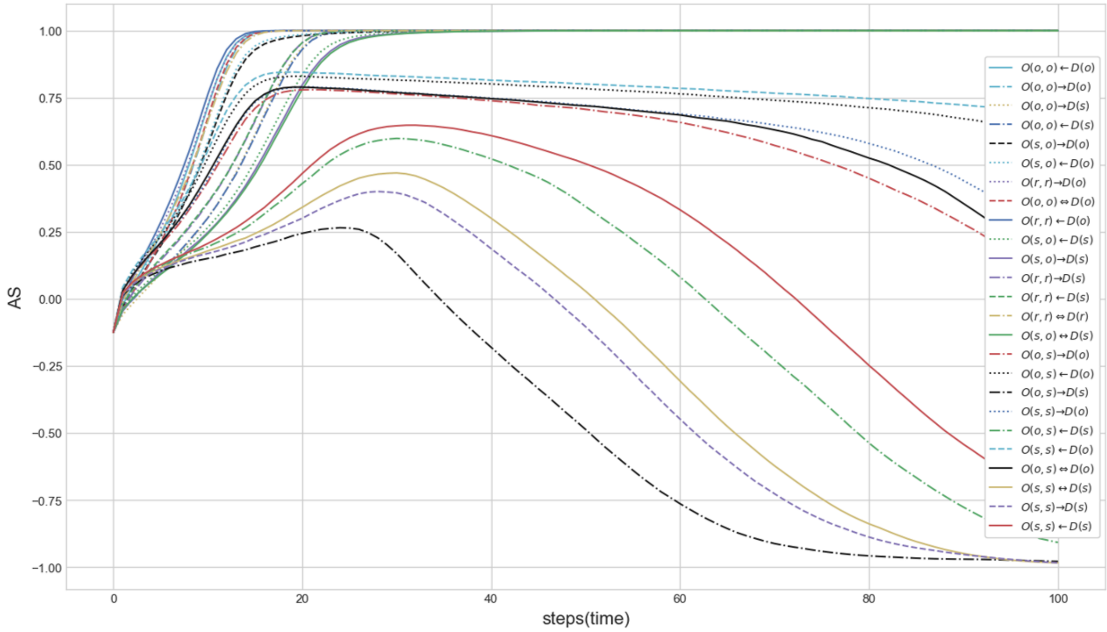
Figure 5–6 Simulation results according to orders of edges: comparison between order of edges under same conditions such as order of layers and nodes

consensus. It can be analyzed that rash node is easy to be extreme and make consensus, but considerate node is very moderate and hard to reach consensus.

5.2.4 Comparison and Analysis

It is found out that there are different simulation results according to orders of layers, nodes, and edges. To sum up all updating rules, they can be categorized into 3 parts, positive consensus, coexistence, and negative consensus as shown in Fig. 5–7.

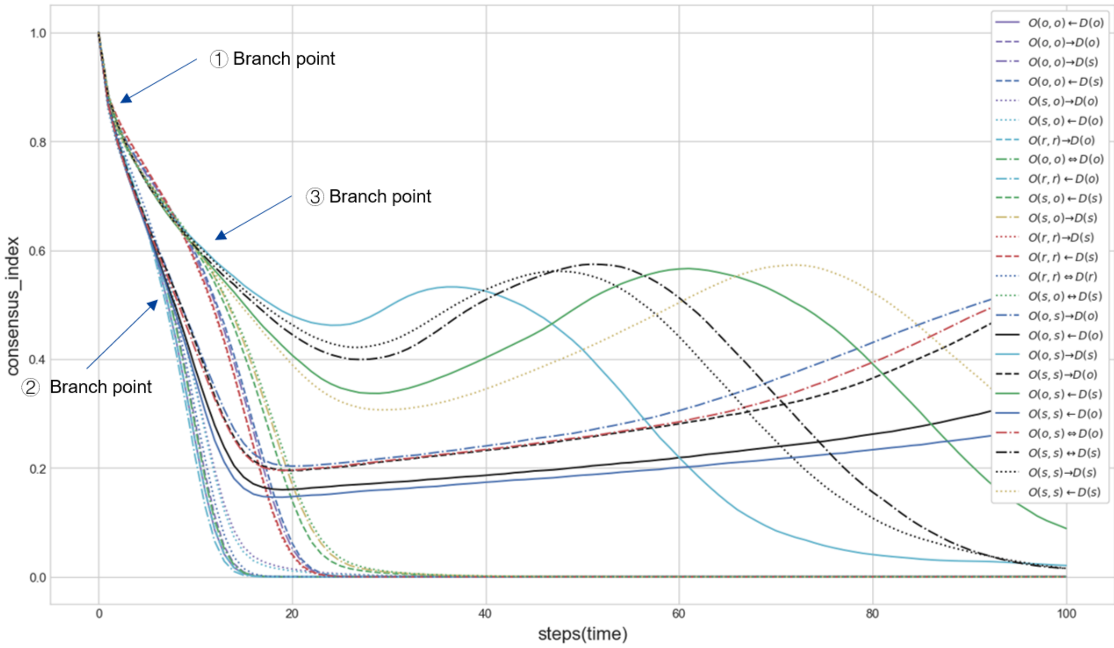
To clearly classify the state of two-layers, the results can be analyzed by using *CI* as shown in Fig. 5–8. There are three branch points. In the first branch point, the results are divided according to whether order of nodes in layer B is sequential or simultaneous. In the second and third branch point, the results are divided according to whether order of edges in layer A is sequential or simultaneous. As the results, there are 4 categories such as fast positive consensus, slow positive consensus, coexistence and slow



Div	Positive Consensus (close to positive)	Coexistence	Negative Consensus (close to negative)
Orders	O(r, r) ← D(o) O(r, r) → D(o) O(o, o) ← D(o) O(o, o) ⇔ D(o) O(o, o) → D(o) O(r, r) ⇔ D(r) O(r, r) → D(s) O(r, r) ← D(s) O(o, o) → D(s) O(o, o) ← D(s) O(s, o) ← D(o) O(s, o) → D(o) O(s, o) ← D(s) O(s, o) → D(s) O(s, o) ↔ D(s)	O(s, s) ← D(o) O(o, s) ← D(o) O(s, s) → D(o) O(o, s) ⇔ D(o) O(o, s) → D(o)	O(s, s) ← D(s) O(o, s) ← D(s) O(o, s) → D(s) O(s, s) → D(s) O(s, s) ↔ D(s)

Figure 5–7 Total results of 25 updating rules with *AS*

negative consensus. Through these results, several important facts can be arranged. First, networks with more simultaneous updating rules make slow consensus or coexistence, sometimes make transition to opposite orientation. On the other hands, networks with more sequential updating rules make fast consensus. In other words, if opinion layer has more rash nodes, more time to have some conversation and decision making layer has more time to discuss topics, the network have more probabilities to make consensus for opinion layer. Second, dynamics order between layers does not have an influence for



<table>
<tr><th colspan="2">Div.</th><th>States</th><th>Orders</th></tr>
<tr><td rowspan="4">① Branch point</td><td rowspan="2">② Branch point :
Sequential order of node in layer B</td><td>Fast positive consensus :
Sequential order of edge</td><td>O(r, r) ← D(o)
O(r, r) → D(o)
O(o, o) ← D(o)
O(o, o) ⇔ D(o)
O(o, o) → D(o)
O(r, r) ⇔ D(r)
O(s, o) ← D(o)
O(s, o) → D(o)</td></tr>
<tr><td>Coexistence :
Simultaneous order of edge</td><td>O(s, s) ← D(o)
O(o, s) ← D(o)
O(s, s) → D(o)
O(o, s) ⇔ D(o)
O(o, s) → D(o)</td></tr>
<tr><td rowspan="2">③ Branch point :
Simultaneous order of node in layer B</td><td>Slow positive consensus :
Sequential order of edge</td><td>O(r, r) → D(s)
O(r, r) ← D(s)
O(o, o) → D(s)
O(o, o) ← D(s)
O(s, o) ← D(s)
O(s, o) → D(s)
O(s, o) ↔ D(s)</td></tr>
<tr><td>Slow negative consensus :
Simultaneous order of edge</td><td>O(s, s) ← D(s)
O(o, s) ← D(s)
O(o, s) → D(s)
O(s, s) → D(s)
O(s, s) ↔ D(s)</td></tr>
</table>

Figure 5–8 Total results of 25 updating rules with *CI*

network state, though there exists tiny consensus time gap. Third, order of nodes in layer B has more influence for network states than order of nodes in layer A. order of nodes in layer B makes the first branch point. But order of nodes in layer A does not make any branch point, though there exists tiny consensus time gap. Forth, order of edges in layer A is very influential so that it makes different network states. So to speak, characteristics of nodes in layer A, such as rash and considerate, affects consensus time and sometimes makes transition to coexistence or opposite orientation.

Chapter 6 Finding key nodes on two layer networks

In this chapter, it would be investigated that what nodes are important for keep orientation on two-layer networks. There exist many methods to find key nodes, such as pagerank, degree centrality, and eigenvector centrality. Based on these methods, it would be researched that which method is the most effective and the most influential for changing state on two layers.

Here is the way to find key nodes on two-layer networks by using centrality. All nodes are ranked by node centrality, and the ratio of unchanged nodes are increased according to ranked order, until the average states of network have different states. When the ratio of unchanged nodes according to node centrality is the least, that centrality is the most influential property for interconnected network. As initial condition for finding key nodes, each layer is made of BA network with 2048 nodes and 1 external edge.

6.1 Key nodes on layer A

6.2 Key nodes on layer B

6.3 Key nodes on two layers with different structures

Chapter 7 Conclusion

Chapter 8 常见问题

Q: 我是否能够自由使用这份模板?

A: 这份模板以 Apache License 2.0 开源许可证发布，请遵循许可证规范。

Q: 我的论文是 Word 排版的，学校图书馆是不是只收 LaTeX 排版的论文?

A: 当然不是，Word 版论文肯定收。

Q: 我的论文是 LaTeX 排版的，学校图书馆是不是只收 Word 排版的论文?

A: 当然不是，PDF 版的电子论文是可以上交的。是否要交 Word 版就看你导师的喜好了。

Q: 为什么屏幕上显示的左右页边距不一样?

A: 模板默认是双面打印，迎面页和背面页的页边距是要交换的，多出来的那一部分是留作装订的。

Q: 为什么在参考文献中会有“//”符号?

A: 那就是国标 GBT7714 参考文献风格规定的。但可以使用 gbpunctin=false 选项将其还原成 in:,进一步可以在导言区加入`\DefineBibliographyStrings{english}{in=`将其去掉。

Q: 为什么参考文献中会有 [s.n.],[S.l], [EB/OL] 等符号?

A: 那也是国标 GBT7714 参考文献风格定义的。[s.n.] 表示出版者不祥，[S.l] 表示出版地不祥，[EB/OL] 表示引用的参考文献类型为在线电子文档。但可以使用 gbpub=false 选项将其缺省补充的出版项 [s.n.] 等去掉。也可以使用选项 gbtype=false 将参考文献类型标识去掉。

Q: 如何获得帮助和反馈意见?

A: 你可以通过在 github 上开 issue 、在水源 LaTeX 版发帖反映你使用过程中遇到的问题。

Q: 使用文本编辑器查看 tex 文件时遇到乱码?

A: 请确保你的文本编辑器使用 UTF-8 编码打开了 tex 源文件。

Q: 在 CTeX 编译模板遇到“rsfs10.tfm already exists”的错误提示?

A: 请删除`X:\CTEX\UserData\fonts\tfm\public\rsfs`下的文件再重新编译。问题讨论见水源 2023 号帖。

Q: 升级了 TeXLive 2012，编译后的文档出现“minus”等字样?

A: 这是 xltxtra 和 fontspec 宏包导致的问题。学位论文模板从 0.5 起使用 metatlog 宏包代替 xltxtra 生成 XeTeX 标志，解决了这个问题。

Q: 为什么在 bib 中加入的参考文献，没有在参考文献列表中出现?

A: bib 中的参考文献条目，常通过`\cite` 或`\parencite` 或`\supercite` 或`\textcite` 等命令在正文中引用进而加入到参考文献列表中。当需要将参考文献条目加入到文献表中但又不引用可以使用`\nocite` 命令，当 nocite 参数为 * 时则引入 bib 中的所有文献。

Q: 我可以使用 Sublime Text 编写学位论文吗？

A: 可以。首先下载并安装 Sublime Text，然后安装 Package Control，之后按`ctrl+shift+p` 或者`cmd+shift+p` 调出命令窗口，输入`install`，选择 *Package Control: Install Package*，按回车，稍等片刻，等待索引载入后会弹出选项框，输入`LaTeXTools` 并回车，即可成功安装插件。之后只需要打开`.tex` 文件，按`ctrl+b` 或者`cmd+b` 即可编译，如有错误，双击错误信息可以跳转到出错的行。

Q: 在 macTex 中，为什么 pdf 图片无法插入？

A: 如果报错是"pdf: image inclusion failed for "./figure/chap2/sjtulogo.pdf"."，则采取以下步骤

Code 8–1 编译模板

```
brew install xpdf
wget http://mirrors.ctan.org/support/epstopdf.zip
unzip epstopdf.zip
cp epstopdf/epstopdf.pl /usr/local/bin/
cd figure/chap2
pdftops sjtulogo.pdf
epstopdf sjtulogo.ps
pdfcrop sjtulogo.pdf
mv sjtulogo.pdf backup.pdf
mv sjtulogo-crop.pdf sjtulogo.pdf
```

Q: 为什么维普等查重系统无法识别此模板生成的 pdf 内所有的中文?

A: 中文无法识别的情况多半是由于使用了 ShareLaTeX 的原因，请尝试使用 TexStudio 等软件在本地进行编译。如果使用 TeXstudio 请在 Preferences-Build 中将 Default Compiler 和 Default Bibliography Tool 分别改为 XeLaTeX 和 Biber。

Q: 如何向你致谢?

A: 烦请在模板的github 主页点击"Star"，我想粗略统计一下使用学位论文模板的人数，谢谢大家。非常欢迎大家向项目贡献代码。

Summary

这里是全文总结内容。

2015 年 2 月 28 日，中央在北京召开全国精神文明建设工作表彰暨学雷锋志愿服务大会，公布全国文明城市（区）、文明村镇、文明单位名单。上海交通大学荣获全国文明单位称号。

全国文明单位这一荣誉是对交大人始终高度重视文明文化工作的肯定，是对交大长期以来文明创建工作成绩的褒奖。在学校党委、文明委的领导下，交大坚持将文明创建工作纳入学校建设世界一流大学的工作中，全体师生医护员工群策群力、积极开拓，落实国家和上海市有关文明创建的各项要求，以改革创新、科学发展为主线，以质量提升为目标，聚焦文明创建工作出现的重点和难点，优化文明创建工作机制，传播学校良好形象，提升社会美誉度，显著增强学校软实力。2007 至 2012 年间，上海交大连续三届荣获“上海市文明单位”称号，成为创建全国文明单位的新起点。

上海交大自启动争创全国文明单位工作以来，凝魂聚气、改革创新，积极培育和践行社会主义核心价值观。坚持统筹兼顾、多措并举，将争创全国文明单位与学校各项中心工作紧密结合，着力构建学校文明创建新格局，不断提升师生医护员工文明素养，以“冲击世界一流大学汇聚强大精神动力”为指导思想，以“聚焦改革、多元推进、以评促建、丰富内涵、彰显特色”为工作原则，并由全体校领导群策领衔“党的建设深化、思想教育深入、办学成绩显著、大学文化丰富、校园环境优化、社会责任担当”六大板块共 28 项重点突破工作，全面展现近年来交大文明创建工作的全貌和成就。

进入新阶段，学校将继续开拓文明创建工作新格局，不断深化工作理念和工作实践，创新工作载体、丰富活动内涵、凸显创建成效，积极服务于学校各项中心工作和改革发展的大局面，在上级党委、文明委的关心下，在学校党委的直接领导下，与时俱进、开拓创新，为深化内涵建设、加快建成世界一流大学、推动国家进步和社会发展而努力奋斗！

上海交通大学医学院附属仁济医院也获得全国文明单位称号。

Appendix A 搭建模板编译环境

A.1 安装 TeX 发行版

A.1.1 Mac OS X

Mac 用户可以从 MacTeX 主页[1]下载 MacTeX。也可以通过 brew 包管理器[2]安装 MacTeX。

```
brew cask install mactex
```

A.1.2 Linux

建议 Linux 用户使用 TeXLive 主页[3]的脚本来安装 TeXLive。以下命令将把 TeXLive 发行版安装到当前用户的家目录下。若计划安装一个供系统上所有用户使用的 TeXLive，请使用 root 账户操作。

```
wget http://mirror.ctan.org/systems/texlive/tlnet/
       install-tl-unx.tar.gz
tar xzvpf install-tl-unx.tar.gz
cd install-tl-20150411/
./install-tl
```

A.2 安装中文字体

A.2.1 Mac OS X、Deepin

Mac 和 Deepin 用户双击字体文件即可安装字体。

[1] https://tug.org/mactex/
[2] http://caskroom.io
[3] https://www.tug.org/texlive/

A.2.2 RedHat/CentOS 用户

RedHat/CentOS 用户请先将字体文件复制到字体目录下，调用 fc-cache 刷新缓存后即可在 TeXLive 中使用新字体。

```
mkdir ~/.fonts
cp *.ttf ~/.fonts                    # 当前用户可用新字体
cp *.ttf /usr/share/fonts/local/     # 所有用户可以使用新
       字体
fc-cache -f
```

Appendix B Maxwell Equations

选择二维情况，有如下的偏振矢量：

$$\mathbf{E} = E_z(r,\theta)\hat{\mathbf{z}} \tag{B–1a}$$

$$\mathbf{H} = H_r(r,\theta))\hat{\mathbf{r}} + H_\theta(r,\theta)\hat{\boldsymbol{\theta}} \tag{B–1b}$$

对上式求旋度：

$$\nabla\times\mathbf{E} = \frac{1}{r}\frac{\partial E_z}{\partial\theta}\hat{\mathbf{r}} - \frac{\partial E_z}{\partial r}\hat{\boldsymbol{\theta}} \tag{B–2a}$$

$$\nabla\times\mathbf{H} = \left[\frac{1}{r}\frac{\partial}{\partial r}(rH_\theta) - \frac{1}{r}\frac{\partial H_r}{\partial\theta}\right]\hat{\mathbf{z}} \tag{B–2b}$$

因为在柱坐标系下，$\overline{\overline{\mu}}$ 是对角的，所以 Maxwell 方程组中电场 **E** 的旋度：

$$\nabla\times\mathbf{E} = \mathbf{i}\omega\mathbf{B} \tag{B–3a}$$

$$\frac{1}{r}\frac{\partial E_z}{\partial\theta}\hat{\mathbf{r}} - \frac{\partial E_z}{\partial r}\hat{\boldsymbol{\theta}} = \mathbf{i}\omega\mu_r H_r\hat{\mathbf{r}} + \mathbf{i}\omega\mu_\theta H_\theta\hat{\boldsymbol{\theta}} \tag{B–3b}$$

所以 **H** 的各个分量可以写为：

$$H_r = \frac{1}{\mathbf{i}\omega\mu_r}\frac{1}{r}\frac{\partial E_z}{\partial\theta} \tag{B–4a}$$

$$H_\theta = -\frac{1}{\mathbf{i}\omega\mu_\theta}\frac{\partial E_z}{\partial r} \tag{B–4b}$$

同样地，在柱坐标系下，$\overline{\overline{\epsilon}}$ 是对角的，所以 Maxwell 方程组中磁场 **H** 的旋度：

$$\nabla\times\mathbf{H} = -\mathbf{i}\omega\mathbf{D} \tag{B–5a}$$

$$\left[\frac{1}{r}\frac{\partial}{\partial r}(rH_\theta) - \frac{1}{r}\frac{\partial H_r}{\partial\theta}\right]\hat{\mathbf{z}} = -\mathbf{i}\omega\overline{\overline{\epsilon}}\mathbf{E} = -\mathbf{i}\omega\epsilon_z E_z\hat{\mathbf{z}} \tag{B–5b}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH_\theta) - \frac{1}{r}\frac{\partial H_r}{\partial\theta} = -\mathbf{i}\omega\epsilon_z E_z \tag{B–5c}$$

由此我们可以得到关于 E_z 的波函数方程：

$$\frac{1}{\mu_\theta\epsilon_z}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_z}{\partial r}\right) + \frac{1}{\mu_r\epsilon_z}\frac{1}{r^2}\frac{\partial^2 E_z}{\partial\theta^2} + \omega^2 E_z = 0 \tag{B–6}$$

Appendix C 从 CJK–LaTeX 转向 XeTeX

我习惯把 v0.2a 使用 dvipdfmx 编译的硕士学位论文模板称为“CJK–LaTeX 模板”，而这个使用 XeTeX 引擎 (xelatex 程序) 处理的模板则被称为“XeTeX/LaTeX 模板”。从 CJK–LaTeX 模板迁移到 XeTeXLaTeX 模板的好处有下：

☺ 搭建 XeTeX 环境比搭建 CJK–LaTeX 环境更容易；

☺ 更简单的字体控制；

☺ 完美支持 PDF/EPS/PNG/JPG 图片，不需要“bound box(.bb)”文件；

☺ 支持 OpenType 字体的复杂字型变化功能；

当然，这也是有代价的。由于 XeTeX 比较新，在我看来，使用 XeTeX 模板所必须付出的代价是：

☹ 必须把你“古老的”TeX 系统更新为较新的版本。TeXLive 2012 和 CTeX 2.9.2 能够编译这份模板，而更早的版本则无能为力。

☹ 需要花一些时间把你在老模板上的工作迁移到新模板上。

第一条就看你如何取舍了，新系统通常意味着更好的兼容性，值得升级。而转换模板也不是什么特别困难的事情，可以这样完成：

1. 备份你要转换的源文件，以防你的工作成果丢失；
2. 将你原来的 tex 以及 bib 文件另存为 UTF-8 编码的文件。iconv、vim、emacs、UEdit 等等工具都可以完成。WinEdt 对文件编码识别功能很差 (到了 v6.0 还是如此)，不推荐作为字符编码转换工具；
3. 将 diss.tex 导言区中的内容替换为 XeTeX 模板 diss.tex 导言区的内容；
4. 将你对原先导言区的修改，小心翼翼地合并到新的导言区中；
5. 使用 XeTeX 模板中的 GBT7714-2005NLang.bst 替换原有的 bst 文件，新的 bst 文件只是将字符编码转换为 UTF-8；
6. 删除 bouding box 文件；
7. 使用本文1.3.3介绍的方法，重新编译文档；

Appendix D 模板更新记录

2018 年 1 月 v0.10 发布，项目转移至 SJTUG 名下，并增加了英文模版，修改了默认字体设置。

2016 年 12 月 v0.9.5 发布，改用 GB7714-2015 参考文献风格。

2016 年 11 月 v0.9.4 发布，增加算法和流程图。

2015 年 6 月 19 日 v0.9 发布，适配 ctex 2.x 宏包，需要使用 TeXLive 2015 编译。

2015 年 3 月 15 日 v0.8 发布，使用 biber/biblatex 组合替代 BibTeX，带来更强大稳定的参考文献处理能力；添加 enumitem 宏包增强列表环境控制能力；完善宏包文字描述。

2015 年 2 月 15 日 v0.7 发布，增加盲审选项，调用外部工具插入扫描件。

2015 年 2 月 14 日 v0.6.5 发布，修正一些小问题，缩减 git 仓库体积，仓库由 sjtu-thesis-template-latex 更名为 SJTUThesis。

2014 年 12 月 17 日 v0.6 发布，学士、硕士、博士学位论文模板合并在了一起。

2013 年 5 月 26 日 v0.5.3 发布，更正 subsubsection 格式错误，这个错误导致如"1.1 小结" 这样的标题没有被正确加粗。

2012 年 12 月 27 日 v0.5.2 发布，更正拼写错误。在 diss.tex 加入 ack.tex。

2012 年 12 月 21 日 v0.5.1 发布，在 LaTeX 命令和中文字符之间留了空格，在 Makefile 中增加 release 功能。

2012 年 12 月 5 日 v0.5 发布，修改说明文件的措辞，更正 Makefile 文件，使用 metalog 宏包替换 xltxtra 宏包，使用 mathtools 宏包替换 amsmath 宏包，移除了所有 CJKtilde(~) 符号。

2012 年 5 月 30 日 v0.4 发布，包含交大学士、硕士、博士学位论文模板。模板在github上管理和更新。

2010 年 12 月 5 日 v0.3a 发布，移植到 XeTeX/LaTeX 上。

2009 年 12 月 25 日 v0.2a 发布，模板由 CASthesis 改名为 sjtumaster。在 diss.tex 中可以方便地改变正文字号、切换但双面打印。增加了不编号的一章“全文总结”。添加了可伸缩符号 (等号、箭头) 的例子，增加了长标题换行的例子。

2009 年 11 月 20 日 v0.1c 发布，增加了 Linux 下使用 ctex 宏包的注意事项、.bib 条目的规范要求，修正了 ctexbook 与 listings 共同使用时的断页错误。

2009 年 11 月 13 日 v0.1b 发布，完善了模板使用说明，增加了定理环境、并列子图、三线表格的例子。

2009 年 11 月 12 日上海交通大学硕士学位论文 LaTeX 模板发布，版本 0.1a。

Acknowledgements

This is the most important stage of my life as I am graduating from the most prestigious institute ‘Shanghai Jiao Tong University (SJTU)’. Two and half years of studies at SJTU not only provided me with rich contents of research but also polished my professional skills. Armed with these tools and skills, I am ready to take on new challenges in life and pursue my dreams. First of all, I would like to thank my supervisor Professor Wang Lin for guiding me throughout my research and study. From discussing my research interests to this moment, even changing my research topic, she has been very helpful throughout this journey. I am very grateful to her for allowing me to explore my own capabilities and demonstrate my research attitude. I would also like to thank all my lab mates who accompanied me throughout this project: I want to thank them for the joy, laughter and occasional little surprises they brought to me, adding a lot of warmth to my dull postgraduate life. They inspired me in every communication with them and helped me make important decisions in life. Knowing each other is a precious memory in my life and I hope this friendship lasts forever. Finally, I would like to thank my parents and family who have always loved me and took care of me with their unconditional love, care, and support. I have no words that can express my gratitude for what they have done for me. Thank you!

Publications

[1] CHEN H, CHAN C T. Acoustic cloaking in three dimensions using acoustic metamaterials[J]. Applied Physics Letters, 2007, 91:183518.

[2] CHEN H, WU B I, ZHANG B, et al. Electromagnetic Wave Interactions with a Metamaterial Cloak[J]. Physical Review Letters, 2007, 99(6):63903.