AirGun1D User Guide

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I. Introduction

AirGun1D simulates a one-dimensional airgun coupled to an air bubble surrounded by water. The oscillations of the air bubble in the water generate acoustic waves that are used in marine seismic surveys. the air bubble is described with a lumped parameter model where the internal properties of the bubble are assumed to be spatially uniform. The airgun (or source) is described as one-dimensional such that properties can vary with space. **AirGun1D** is written in `MATLAB` and runs efficiently on a standard desktop/laptop computer. For more details and examples of the application of **AirGun1D** see:

- Watson, L. M., Werpers, J., and Dunham, E. M. (2019) What controls the initial peak of an air gun source signature? *Geophysics*, 84 (2), P27-P45, https://doi.org/10.1190/geo2018-0298.1.

AirGun1D is freely available online at https://github.com/leighton-watson/AirGun1D and is distributed under the MIT license (see `license.txt` for details).

This code extends **SeismicAirgun** (https://github.com/leighton-watson/SeismicAirgun), which describes the airgun (source) with a lumped parameter model. **AirGun1D** is more suitable for large sources where the spatial variations of properties inside the source is important.

II. Directory

- **SBPSAT** - source code for AirGun1D. Governing equations are solved using the summation-by-parts technique with simultaneous approximation term (SBP-SAT).
- **SeismicAirgunCode** - source code for lumped parameter model description of airgun.
- **doc** - documentation including user guide and license file.
- **MakeFigs** - script files that generate all of the figures of Watson et al. (2019).
- **Data** - data files that are needed to when generating the figures of of Watson et al. (2019).
- **BubbleCode** - bubble models that are compared in the appendix of Watson et al. (2019).

III. SBPLib

AirGun1D is a finite-difference code that uses summation-by-parts (SBP) operators Svärd and Nordström (2014) and relies on SBPLib, which is a library of primitives and help functions for working with summation-by-parts finite differences in Matlab developed by Ken Mattsson's group at Uppsala University (including Jonatan Werpers, Martin Almquist, Ylva Rydin and Vidar Stiernström). SBPLib is hosted through bitbucket and is available at https://bitbucket.org/sbpteam/sbplib/src/default/. Clone or download SBPLib and add it to your Matlab path:

```
addpath ../SBPLib/ % add path to SBPLib
```

IV. Model Description

Here, I describe the mathematical model of **AirGun1D**. The model can be divided into three components; i.) airgun, ii.) bubble, and iii.) acoustic radiation. The models for components ii) and iii) are the same as for **SeismicAirgun**. For more details see the **SeismicAirgun** user guide and Chelminski et al. (2019), Watson et al. (2016) and the references therein. Here, I provide a detailed discussion of the airgun model and a cursory description of the bubble and acoustic radiation models. For more details see Watson et al. (2019).

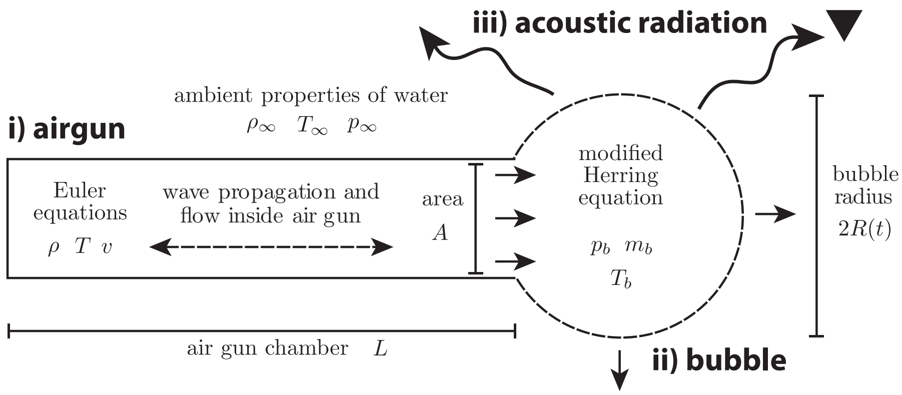

Figure 1: *Schematic diagram of AirGun1D model showing the three model components: i) airgun that is described by the Euler equation that allow for spatial variations of properties within the airgun, ii) bubble that is described with a lumped parameter model and the dynamics governing by the modified Herring equation, and iii) acoustic radiation that is modeled as a point source radiating in a half space.*

i. Airgun

The air gun firing chamber is idealized as a cylinder of length L and cross-sectional area A. The air inside is compressible and inviscid. We only consider waves and flow along the length of the firing chamber, which are governed by the one-dimensional Euler equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho v^2 + p)}{\partial x} = 0, \tag{2}$$

$$\frac{\partial e}{\partial t} + \frac{\partial[(e+p)v]}{\partial x} = 0, \tag{3}$$

where ρ, p, and v are the density, pressure, and axial velocity of the fluid inside the air gun, respectively, and e is the internal energy per unit volume. The system is closed with the ideal gas equation of state, written here as

$$p = (\gamma - 1)\left(e - \frac{1}{2}\rho v^2\right), \tag{4}$$

and the internal energy per unit volume is

$$e = c_v \rho T. \tag{5}$$

The model is initialized with spatially uniform pressure, density and temperature through the airgun and zero initial velocity. The initial air gun pressure is set equal to the operating pressure, p_0, and the initial temperature is assumed to have equilibrated with the ambient temperature T_∞ (since the air travels a long distance in a submerged umbilical hose from the compressor on the seismic vessel to the air gun).

$$p_a(x,0) = p_0, \tag{6}$$

$$T_a(x,0) = T_\infty, \tag{7}$$

$$v_a(x,0) = 0. \tag{8}$$

Boundary conditions are also required for the 1D airgun model. At the back wall, $x = 0$, a solid wall boundary condition, $v(0) = 0$, is imposed. At the air gun port, $x = L$, the boundary condition depends upon the flow state. If flow out of the port is subsonic, $v(L) < c(L)$, continuity of pressure is required and the pressure in the air gun at the port, $p(L)$, is set equal to the pressure in the bubble, p_b. If the port is choked, $v(L) = c(L)$, waves cannot propagate upstream from the bubble into the air gun and pressure continuity is not required. The bubble pressure will be different than the air gun pressure. The latter is generally the case in our simulations.

ii. Bubble

The bubble is idealized as spherically symmetric with spatially uniform internal properties. The bubble wall dynamics are governed by the modified Herring equation (Herring, 1941; Cole, 1948; Vokurka, 1986; Appendix A):

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p_b - p_\infty}{\rho_\infty} + \frac{R}{\rho_\infty c_\infty}\dot{p_b} - \alpha\dot{R}, \tag{9}$$

where R and $\dot{R} = dR/dt$ are the radius and velocity of the bubble wall, respectively, and p_∞, ρ_∞ and c_∞ are the ambient pressure, density, and speed of sound, respectively, in the water infinitely far from the bubble. The ambient pressure is calculated by adding the hydrostatic pressure to the atmospheric pressure, $p_\infty = p_{\text{atm}} + \rho_\infty g D$, where p_{atm} is the atmospheric pressure, g is the acceleration from gravity and D is the depth of the air gun below the water. Numerical models of air gun signatures typically under predict the damping observed in the data. This is likely due to the models neglecting various dissipation mechanisms such as turbulence and energy loss to phase changes. Mechanical energy dissipation mechanisms, which are challenging to model directly, are parameterized into an extra damping term of the form of $-\alpha\dot{R}$ where α is an empirically determined constant (Langhammer and Landrø, 1996; **?**). This term dampens the bubble oscillations so that they are in agreement with the observations but has minimal impact on the initial bubble growth.

The pressure in the bubble is calculated from the ideal gas equation of state,

$$p_b = \frac{m_b Q T_b}{V_b}, \tag{10}$$

where m_b is the mass inside the bubble, T_b is the temperature inside the bubble in Kelvin, V_b is the volume of the spherical bubble and the internal energy is

$$E_b = c_v m_b T_b. \tag{11}$$

The internal energy of the bubble changes due to the advection of mass and associated transport of enthalpy into the bubble, work done by volume changes, and heat loss to the water:

$$\frac{dE_b}{dt} = c_p T_a \frac{dm_b}{dt} - 4\pi M\kappa R^2 (T_b - T_\infty) - p_b \frac{dV_b}{dt}, \tag{12}$$

where T_∞ is the ambient temperature in the water, κ is the heat transfer coefficient (Ni et al., 2011; de Graaf et al., 2014) and M is a dimensionless, empirically determined constant that accounts for the increased effective surface area over which heat transfer can occur as a result of turbulence in the bubble dynamics (Laws et al., 1990). Equation 12 is a statement of the first law of thermodynamics for an open system (Tolhoek and de Groot, 1952).

Mass flow into the bubble is determined by conservation of mass of the gas exiting the airgun:

$$\frac{dm_b}{dt} = v(L)\rho(L)A. \tag{13}$$

The cross-sectional area of the port is assumed to be the same as the cross-sectional area of the firing chamber (Figure 1).

iii. Acoustic Radiation

The pressure perturbation in the water is related to the bubble dynamics by (Keller and Kolodner, 1956)

$$\Delta p(r,t) = \rho_\infty \left[\frac{\ddot{V}(t - r/c_\infty)}{4\pi r} - \frac{\dot{V}(t - r/c_\infty)^2}{32\pi^2 r^4} \right], \tag{14}$$

where Δp is the pressure perturbation in the water, r is the distance from the center of the bubble to the receiver, and V is the volume of the bubble. The second term on the right side is a near-field term that decays rapidly with distance. In addition to the direct arrival there is a ghost arrival, which is the initially upgoing wave that is reflected with negative polarity from the sea surface. For a receiver directly below the source the observed pressure perturbation is

$$\Delta p_{\text{obs}}(r,t) = \Delta p_D(r,t) - \Delta p_G(r + 2D, t), \tag{15}$$

where Δp_D is the direct signal and Δp_G is the ghost arrival, which is assumed to have -1 reflection coefficient from the sea surface. D is the depth of the seismic airgun.

V. Numerical Implementation

The Euler equations inside the air gun are solved using finite difference summation-by-parts operators for spatial derivatives (Svärd and Nordström, 2014). The air gun and bubble governing equations are evolved in time using an explicit adaptive Runge-Kutta (4,5) method. The air gun and bubble models are coupled at the interface using the simultaneous-approximation-term framework (**??**Svärd and Nordström, 2014). The reader is referred to Watson et al. (2019) for more details.

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