Worksheet 01 - 01~10

In [1]:
```
from sympy import *
from sympy.plotting import plot, plot3d
import matplotlib.pyplot as plt
%matplotlib inline

plt.rcParams['figure.figsize'] = 10, 10
init_printing()
x, y, a, b, k, A, B, K, sh, r = symbols('x y a b k A B K sh r')
```

1. If $a(x+2)+b(x-1)=3$ for all x, then $a =$

```
(A) -1  (B) 0  (C) 1  (D) 2  (E) 3
```

Solution

$$\begin{gathered} a(x+2)+b(x-1)=3 \\ ax+2a+bx-b=3 \\ (a+b)x+(2a-b)=3 \end{gathered}$$

$$\swarrow \quad \searrow$$

$$a+b=0 \text{ or } 2a-b=3$$

In [2]:
```
solve((a*(x+2))+b*(x-1)-3, a, b)
```

Out[2]: $\{a : 1, \quad b : -1\}$

Answer: (C)

2. If $a+b=2$ and $ab=-1$, then $a^2+b^2 =$

```
(A) 4  (B) 5  (C) 6  (D) 8  (E) 10
```

Solution

- By my work

$$\begin{aligned} a^2+b^2 &= a^2+b^2+2ab-2ab \\ &= (a+b)^2-2ab \end{aligned}$$

- By SymPy

In [3]:
```
expr = (a+b)*(a+b)-2*a*b
expr.subs([(a+b, 2), (a*b, -1)])
```

Out[3]: 6

Answer: (C)

3. If the graphs of $3x + 4y = 5$ and $kx + 2y = 5$ are perpendicular, then $k =$

```
(A) -2  (B) -2.67  (C) 2.15  (D) 3.20  (E) 4
```

Solution

- From given

$$\begin{gathered}
3x + 4y = 5 \Rightarrow slop1 = -\frac{3}{4} \\
kx + 2y = 5 \Rightarrow slop2 = -\frac{k}{2} \\
slop1 * slop2 = -1 \\
(-\frac{3}{4}) * (-\frac{k}{2}) = -1 \\
3k = -8 \\
k \approx -2.67
\end{gathered}$$

In [4]:
```
solve((-3/4)*(-k/2)+1, k)
```

Out[4]: $[-2.66666666666667]$

Answer: (B)

4. If $K = \frac{AB}{A+B}$, then $B =$

$$(A)\ \frac{A}{1-A}$$
$$(B)\ \frac{AK}{A-K}$$
$$(C)\ \frac{AK}{K-A}$$
$$(D)\ \frac{A+K}{A}$$
$$(E)\ \frac{A-K}{AK}$$

Solution

- By my work

$$
\begin{aligned}
K &= \frac{AB}{A+B} \\
K(A+B) &= AB \\
KA + KB &= AB \\
KA &= (A-K)B \\
(A-K)B &= KA \\
B &= \frac{KA}{A-K}
\end{aligned}
$$

- By SymPy

In [5]:
```
solve(Eq(K, (A*B)/(A+B)), B)
```

Out[5]:
$$\left[\frac{AK}{A-K}\right]$$

Answer: (B)

5. If $\log 3 = a$, then $\log 90 =$

$(A)\ 1+2a$
$(B)\ 10a^2$
$(C)\ 10+2a$
$(D)\ 30a$
$(E)\ 10+3a$

Solution

- By my work

$$
\begin{aligned}
\log 90 &= \log(9 * 10) \\
&= \log(9) + \log(10) \\
&= \log 3^2 + 1 \\
&= 2\log 3 + 1 \\
&= 2a + 1
\end{aligned}
$$

- By SymPy

In [6]:
```
expr = 2*log(3)+1
expr.subs(log(3), a)
```

Out[6]: $2a + 1$

Answer: (A)

6. If $f(x) = 3 \ln x$ and $g(x) = e^x$, then $g(f(x)) =$

$(A)\ 3x$
$(B)\ e^x$
$(C)\ e^{2x}$
$(D)\ x^3$
$(E)\ x^2 + 1$

Solution

$$g(f(x)) = e^{f(x)} = e^{3 \ln x} = e^{\ln x^3} = x^3$$

In [7]:
```
F = logcombine(3*ln(x), force=True)
F
```

Out[7]: $\log\left(x^3\right)$

In [8]:
```
G = exp(x).subs(x, F)
G
```

Out[8]: x^3

Answer: (D)

7. The slant height of a regular circular cone is 20 cm and the radius of the base is 10 cm. Find the volume of the cone?

$(A)\ 1813.8cm^3$
$(B)\ 3000.5cm^3$
$(C)\ 4120.4cm^3$
$(D)\ 7024.8cm^3$
$(E)\ 7046.6cm^3$

Solution

- By my work

$$V(r,h) = \frac{1}{3}\pi r^2 h$$
$$h(r,sh) = \sqrt{sh^2 - r^2}$$

$$\begin{aligned} V(r,sh) &= \frac{1}{3}\pi r^2 \sqrt{sh^2 - r^2} \\ &= \frac{1}{3}\pi 10^2 \sqrt{20^2 - 10^2} \\ &= 1813.8 \end{aligned}$$

- By SymPy

In [9]:
```
expr = (sympify(1)/sympify(3)) * pi * r**2 * sqrt(sh**2-r**2)
expr
```

Out[9]:
$$\frac{\pi r^2 \sqrt{-r^2 + sh^2}}{3}$$

In [10]:
```
result = expr.subs([(r, 10), (sh, 20)])
result
```

Out[10]:
$$\frac{1000\sqrt{3}\pi}{3}$$

In [11]:
```
result.evalf(6)
```

Out[11]: 1813.8

Answer: (A)

8. If $2 - i$ is one of the zeros of the polynomial $p(x)$, then a factor of $p(x)$ could be

$(A)\ x^2 - 2$
$(B)\ x^2 - 4$
$(C)\ x^2 - 4x + 4$
$(D)\ x^2 - 4x + 5$
$(E)\ x^2 + 4x + 3$

Solution

$$\begin{aligned} (A)\ x^2 - 2 &= (x - \sqrt{2})(x + \sqrt{2}) \\ x &\in \{-\sqrt{2}, \sqrt{2}\} \end{aligned}$$

$$(B)\ x^2 - 4 = (x-2)(x+2)$$
$$x \in \{-2, 2\}$$

$$(C)\ x^2 - 4x + 4 = (x-2)^2$$
$$x \in \{2\}$$

$$(D)\ x^2 - 4x + 5 = x^2 - 4x + 4 + 1 = (x-2)^2 - i^2 = (x-2+i)(x-2-i)$$
$$x \in \{2-i, 2+i\}$$

$$(E)\ x^2 + 4x + 3 = (x+3)(x+1)$$
$$x \in \{-3, -1\}$$

- For (A)

In [12]:
```
eq = Eq((x**2)-2)
eq
```

Out[12]: $x^2 - 2 = 0$

In [13]:
```
solve(eq, x)
```

Out[13]: $\left[-\sqrt{2}, \quad \sqrt{2}\right]$

- For (B)

In [14]:
```
eq = Eq((x**2)-4)
eq
```

Out[14]: $x^2 - 4 = 0$

In [15]:
```
solve(eq, x)
```

Out[15]: $[-2, \quad 2]$

- For (C)

In [16]:
```
eq = (x**2)-(4*x)+4
eq
```

Out[16]: $x^2 - 4x + 4$

In [17]:
```
solve(eq, x)
```

Out[17]: $[2]$

In [18]:
```
eq = x**2-4*x+5
eq
```

Out[18]: $x^2 - 4x + 5$

In [19]:
```
solve(eq, x)
```

Out[19]: $[2 - i, \quad 2 + i]$

In [20]:
```
eq = x**2+4*x+3
eq
```

Out[20]: $x^2 + 4x + 3$

In [21]:
```
solve(eq, x)
```

Out[21]: $[-3, \quad -1]$

Answer: (D)

9. When a ploynomial function $f(x) = x^2 + 5x - k$ is divided by $(x - 2)$, the remainder 5. What is the value of k?

(A) 19
(B) 18
(C) 16
(D) 10
(E) 9

Solution

- By my work

$$f(x) = x^2 + 5x - k = (x - 2)Q(x) + R$$

When $x = 2,\ R = 5$

$$\begin{aligned} f(2) = 2^2 + 5 * 2 - k = 4 + 10 - k &= 5 \\ 14 - k &= 5 \\ k = 14 - 5 &= 9 \end{aligned}$$

- By SymPy

Method 1

In [22]:
```
pdiv(x**2 + 5*x - k, x - 2)
```

Out[22]: $(x+7, \quad -k+14)$

In [23]:
```
eq = Eq(-k+14, 5)
eq
```

Out[23]: $-k+14=5$

In [24]:
```
solve(eq, k)
```

Out[24]: $[9]$

Method 2

In [25]:
```
eq = Eq(prem(x**2 + 5*x - k, x - 2), 5)
eq
```

Out[25]: $-k+14=5$

In [26]:
```
solve(eq, k)
```

Out[26]: $[9]$

Answer: (E)

10. A cube with edge of length 6, what is the length of diagonal $\overline{PQ}$?

$(A)18$
$(B)15$
$(C)6\sqrt{6}$
$(D)6\sqrt{3}$
$(E)6\sqrt{2}$

Solution

- My work

Set a is $\texttt{length}$ of cube, sd is $\texttt{surface diagonal}$, $\overline{PQ}$ is $\texttt{cube diagonal}$

$$sd = \sqrt{a^2 + a^2} = \sqrt{2}a$$

$$
\begin{aligned}
\overline{PQ} &= \sqrt{(a^2 + sd^2)} \\
&= \sqrt{(a^2 + (\sqrt{2}a)^2)} \\
&= \sqrt{3}a \\
&= 6\sqrt{3}
\end{aligned}
$$

- By SymPy

In [27]:
```
PQ = sqrt(a**2+(sqrt(a**2+a**2))**2)
PQ
```

Out[27]: $\sqrt{3}\sqrt{a^2}$

In [28]:
```
PQ.subs(a, 6)
```

Out[28]: $6\sqrt{3}$

Answer: (D)

In [ ]: