Worksheet 06~10

In [1]:
```
%config IPCompleter.greedy=True
```

In [2]:
```
from sympy import *
from sympy.geometry.line import Line
from sympy.plotting import plot, plot3d
import matplotlib.pyplot as plt
%matplotlib inline

plt.rcParams['figure.figsize'] = 10, 10
init_printing(use_unicode=True)
x, y, a, b, r, h, sh, k = symbols('x y a b r h sh k')
```

6. If $f(x) = 3\ln x$ and $g(x) = e^x$, then $g(f(x)) =$

$(A)\ 3x$
$(B)\ e^x$
$(C)\ e^{2x}$
$(D)\ x^3$
$(E)\ x^2 + 1$

Solution

My Work

$$(g(x) \circ f(x)) = g(f(x)) = e^{f(x)} = e^{3\ln x} = e^{\ln x^3} = x^3$$

By SymPy

In [3]:
```
F = logcombine(3*ln(x), force=True)
F
```

Out[3]: $\log\left(x^3\right)$

In [4]:
```
G = exp(x).subs(x, F)
G
```

Out[4]: x^3

Answer: (D)

7. The slant height of a regular circular cone is 20 cm and the radius of the base is 10 cm. Find the volume of the cone?

$(A)\ 1813.8cm^3$
$(B)\ 3000.5cm^3$
$(C)\ 4120.4cm^3$
$(D)\ 7024.8cm^3$
$(E)\ 7046.6cm^3$

Solution

My Work

$V(r,h) = \frac{1}{3}\pi r^2 h$

$h(r,sh) = \sqrt{sh^2 - r^2}$

$$\begin{aligned} V(r,sh) &= \frac{1}{3}\pi r^2\sqrt{sh^2 - r^2} \\ &= \frac{1}{3}\pi 10^2\sqrt{20^2 - 10^2} \\ &= 1813.8 \end{aligned}$$

Using SymPy

In [5]:
```
V = Rational(1, 3)*pi*(r**2)*h
V
```

Out[5]:
$$\frac{\pi h r^2}{3}$$

In [6]:
```
V = V.subs(h, sqrt(sh**2-r**2))
V
```

Out[6]:
$$\frac{\pi r^2\sqrt{-r^2 + sh^2}}{3}$$

In [7]:
```
result = V.subs([(r, 10), (sh, 20)])
result
```

Out[7]:
$$\frac{1000\sqrt{3}\pi}{3}$$

In [8]:
```
result.evalf(6)
```

Out[8]: 1813.8

Answer: (A)

8. If $2 - i$ is one of the zeros of the polynomial $p(x)$, then a factor of $p(x)$ could be

$(A)\ x^2 - 2$
$(B)\ x^2 - 4$
$(C)\ x^2 - 4x + 4$
$(D)\ x^2 - 4x + 5$
$(E)\ x^2 + 4x + 3$

Solution

My Work

Method 1

$$(A)\ x^2 - 2 = (x - \sqrt{2})(x + \sqrt{2})$$
$$x \in \{-\sqrt{2}, \sqrt{2}\}$$

$$(B)\ x^2 - 4 = (x - 2)(x + 2)$$
$$x \in \{-2, 2\}$$

$$(C)\ x^2 - 4x + 4 = (x - 2)^2$$
$$x \in \{2\}$$

$$(D)\ x^2 - 4x + 5 = x^2 - 4x + 4 + 1 = (x - 2)^2 - i^2 = (x - 2 + i)(x - 2 - i)$$
$$x \in \{2 - i, 2 + i\}$$

$$(E)\ x^2 + 4x + 3 = (x + 3)(x + 1)$$
$$x \in \{-3, -1\}$$

Result: Answer should be (D)

Method 2

If one of the root is $2-i$, then $2+i$ should be another root. Set $x_1 = 2-i$ and $x_2 = 2+i$

$$\begin{aligned}(x-x_1)(x-x_2) &= 0\\ (x-(2-i))(x-(2+i)) &= 0\\ x^2-(2-i)x-(2+i)x+(2-i)(2+i) &= 0\\ x^2-2x+ix-2x-ix+(2^2-i^2) &= 0\\ x^2-4x+(4-(-1)) &= 0\\ x^2-4x+5 &= 0\end{aligned}$$

Using SymPy

In [9]:
```
x1 = 2 - I
x1
```

Out[9]: $2-i$

In [10]:
```
x2 = 2 + I
x2
```

Out[10]: $2+i$

In [11]:
```
eq = Eq((x-x1)*(x-x2), 0)
eq
```

Out[11]: $(x-2-i)(x-2+i) = 0$

In [12]:
```
simplify(eq)
```

Out[12]: $x^2-4x+5=0$

Answer: (D)

9. When a ploynomial function $f(x) = x^2+5x-k$ is divided by $(x-2)$, the remainder 5. What is the value of k?

(A) 19
(B) 18
(C) 16
(D) 10
(E) 9

Solution

My Work

$$f(x) = x^2 + 5x - k = (x-2)Q(x) + R$$

When $x = 2$, $R = 5$

$$\begin{aligned} f(2) = 2^2 + 5 * 2 - k = 4 + 10 - k &= 5 \\ 14 - k &= 5 \\ k = 14 - 5 &= 9 \end{aligned}$$

Using SymPy

Method 1

In [13]:
```
pdiv(x**2 + 5*x - k, x - 2)
```

Out[13]: $(x + 7, \quad -k + 14)$

In [14]:
```
eq = Eq(-k+14, 5)
eq
```

Out[14]: $-k + 14 = 5$

In [15]:
```
solve(eq, k)
```

Out[15]: $[9]$

Method 2

In [16]:
```
eq = Eq(prem(x**2 + 5*x - k, x - 2), 5)
eq
```

Out[16]: $-k + 14 = 5$

In [17]:
```
solve(eq, k)
```

Out[17]: $[9]$

Answer: (E)

In [ ]:
```

```