Worksheet 01~05

In [1]:
```
%config IPCompleter.greedy=True
```

In [2]:
```
from sympy import *
from sympy.geometry.line import Line
from sympy.plotting import plot, plot3d
import matplotlib.pyplot as plt
%matplotlib inline

plt.rcParams['figure.figsize'] = 10, 10
init_printing(use_unicode=True)
x, y, a, b, k, K, A, B = symbols('x y a b k K A B')
```

1. If $a(x+2)+b(x-1)=3$ for all x, then $a=$

   (A) -1 (B) 0 (C) 1 (D) 2 (E) 3

Solution

My work

$$a(x+2)+b(x-1)=3$$

$$ax+2a+bx-b=3$$

$$(a+b)x+(2a-b)=3$$

$$\swarrow \quad \searrow$$

$$a+b=0 \text{ or } 2a-b=3$$

$$+\begin{cases} a+b &= 0 \\ 2a-b &= 3 \end{cases}$$

$$3a = 3$$

$$a=\frac{3}{3}=1$$

$$b=-a=-1$$

$$\begin{cases} a=1 \\ b=-1 \end{cases}$$

Using SymPy

Method 1

In [3]:
```
eq = Eq((a*(x+2))+b*(x-1), 3)
eq
```

Out[3]: $a\,(x+2)+b\,(x-1)=3$

In [4]:
```
solve(eq, a, b)
```

Out[4]: $\{a:1,\quad b:-1\}$

Method 2

In [5]:
```
solve(((a*(x+2))+b*(x-1)-3), a, b)
```

Out[5]: $\{a:1,\quad b:-1\}$

Answer: (C)

2. If $a+b=2$ and $ab=-1$, then $a^2+b^2=$

   (A) 4 (B) 5 (C) 6 (D) 8 (E) 10

Solution

My Work

$$\begin{aligned} a^2+b^2 &= a^2+b^2+2ab-2ab \\ &= (a+b)^2-2ab \\ &= 2^2-2(-1) \\ &= 6 \end{aligned}$$

Using SymPy

In [6]:
```
expr = (a+b)*(a+b)-2*a*b
expr.subs([(a+b, 2), (a*b, -1)])
```

Out[6]: 6

Answer: (C)

3. C. If the graphs of $3x + 4y = 5$ and $kx + 2y = 5$ are perpendicular, then $k =$

(A) -2 (B) -2.67 (C) 2.15 (D) 3.20 (E) 4

Solution

My Work

- Set $slop_1$ for $\ell_1 : 3x + 4y = 5$

$$3x + 4y = 5$$
$$y = \frac{-3x + 5}{4} = -\frac{3}{4}x + \frac{5}{4}$$
$$slop_1 = -\frac{3}{4}$$

- Set $Slop_2$ for $\ell_2 : kx + 2y = 5$

$$kx + 2y = 5$$
$$y = \frac{-kx + 5}{2} = -\frac{k}{2}x + \frac{5}{2}$$
$$slop_2 = -\frac{k}{2}$$

- ℓ_1 and ℓ_2 are perpendicular, it means $slop_1 * slop_2 = -1$

$$(-\frac{3}{4}) * (-\frac{k}{2}) = -1$$
$$k = -\frac{2 * 4}{3} = -\frac{8}{3} \approx -2.67$$

In [7]:
```
result = solve((-3/4)*(-k/2)+1, k)
result[0].evalf(3)
```

Out[7]: -2.67

Answer: (B)

4. If $K = \frac{AB}{A+B}$, then $B =$

$(A)\ \frac{A}{1-A}$
$(B)\ \frac{AK}{A-K}$
$(C)\ \frac{AK}{K-A}$
$(D)\ \frac{A+K}{A}$
$(E)\ \frac{A-K}{AK}$

Solution

My Work

$$K = \frac{AB}{A+B}$$
$$K(A+B) = AB$$
$$KA + KB = AB$$
$$KA = (A-K)B$$
$$(A-K)B = KA$$
$$B = \frac{KA}{A-K}$$

By SymPy

In [8]:
```
solve(Eq(K, (A*B)/(A+B)), B)
```

Out[8]:
$$\left[\frac{AK}{A-K}\right]$$

Answer: (B)

5. If $\log 3 = a$, then $\log 90 =$

$(A)\ 1+2a$
$(B)\ 10a^2$
$(C)\ 10+2a$
$(D)\ 30a$
$(E)\ 10+3a$

Solution

My Work

$$\begin{aligned}\log 90 &= \log(9 * 10) \\ &= \log(9) + \log(10) \\ &= \log 3^2 + 1 \\ &= 2\log 3 + 1 \\ &= 2a + 1\end{aligned}$$

By SymPy

In [9]:
```
expr = 2*log(3)+1
expr.subs(log(3), a)
```

Out[9]: $2a + 1$

Answer: (A)

In [ ]:
```

```