

Worksheet 01~05

In [1]: `%config IPCompleter.greedy=True`

In [2]: `from sympy import *
from sympy.geometry.line import Line
from sympy.plotting import plot, plot3d
import matplotlib.pyplot as plt
%matplotlib inline

plt.rcParams['figure.figsize'] = 10, 10
init_printing(use_unicode=True)
x, y, a, b, k, K, A, B = symbols('x y a b k K A B')`

1. If $a(x + 2) + b(x - 1) = 3$ for all x , then $a =$

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

Solution

My work

$$a(x + 2) + b(x - 1) = 3$$

$$ax + 2a + bx - b = 3$$

$$(a + b)x + (2a - b) = 3$$

↙ ↘

$$a + b = 0 \text{ or } 2a - b = 3$$

$$+ \begin{cases} a + b = 0 \\ 2a - b = 3 \end{cases}$$

$$3a = 3$$

$$a = \frac{3}{3} = 1$$

$$b = -a = -1$$

$$\begin{cases} a = 1 \\ b = -1 \end{cases}$$

Using SymPy

Method 1

```
In [3]: eq = Eq((a*(x+2))+b*(x-1), 3)
eq
```

```
Out[3]: a(x + 2) + b(x - 1) = 3
```

```
In [4]: solve(eq, a, b)
```

```
Out[4]: {a : 1,  b : -1}
```

Method 2

```
In [5]: solve(((a*(x+2))+b*(x-1)-3), a, b)
```

```
Out[5]: {a : 1,  b : -1}
```

Answer: (C)

2. If $a + b = 2$ and $ab = -1$, then $a^2 + b^2 =$

(A) 4 (B) 5 (C) 6 (D) 8 (E) 10

Solution

My Work

$$\begin{aligned}
 a^2 + b^2 &= a^2 + b^2 + 2ab - 2ab \\
 &= (a + b)^2 - 2ab \\
 &= 2^2 - 2(-1) \\
 &= 6
 \end{aligned}$$

Using SymPy

```
In [6]: expr = (a+b)*(a+b)-2*a*b
expr.subs([(a+b, 2), (a*b, -1)])
```

```
Out[6]: 6
```

Answer: (C)

3. C. If the graphs of $3x + 4y = 5$ and $kx + 2y = 5$ are perpendicular, then $k =$
- (A) -2 (B) -2.67 (C) 2.15 (D) 3.20 (E) 4

Solution**My Work**

- Set $slop_1$ for $\ell_1: 3x + 4y = 5$

$$3x + 4y = 5$$

$$y = \frac{-3x + 5}{4} = -\frac{3}{4}x + \frac{5}{4}$$

$$slop_1 = -\frac{3}{4}$$

- Set $slop_2$ for $\ell_2: kx + 2y = 5$

$$kx + 2y = 5$$

$$y = \frac{-kx + 5}{2} = -\frac{k}{2}x + \frac{5}{2}$$

$$slop_2 = -\frac{k}{2}$$

- ℓ_1 and ℓ_2 are perpendicular, it means $slop_1 * slop_2 = -1$

$$\left(-\frac{3}{4}\right) * \left(-\frac{k}{2}\right) = -1$$

$$k = -\frac{2 * 4}{3} = -\frac{8}{3} \approx -2.67$$

```
In [7]: result = solve((-3/4)*(-k/2)+1, k)
result[0].evalf(3)
```

```
Out[7]: -2.67
```

Answer: (B)

4. If $K = \frac{AB}{A+B}$, then $B =$

- (A) $\frac{A}{1-A}$
 (B) $\frac{AK}{A-K}$
 (C) $\frac{AK}{K-A}$
 (D) $\frac{A+K}{A}$
 (E) $\frac{A-K}{AK}$

Solution

My Work

$$K = \frac{AB}{A+B}$$

$$K(A+B) = AB$$

$$KA + KB = AB$$

$$KA = (A-K)B$$

$$(A-K)B = KA$$

$$B = \frac{KA}{A-K}$$

By SymPy

In [8]: `solve(Eq(K, (A*B)/(A+B)), B)`

Out[8]: $\left[\frac{AK}{A-K} \right]$

Answer: (B)

5. If $\log 3 = a$, then $\log 90 =$

- (A) $1 + 2a$
 (B) $10a^2$
 (C) $10 + 2a$
 (D) $30a$
 (E) $10 + 3a$

Solution

My Work

$$\begin{aligned}\log 90 &= \log (9 * 10) \\ &= \log (9) + \log (10) \\ &= \log 3^2 + 1 \\ &= 2 \log 3 + 1 \\ &= 2a + 1\end{aligned}$$

By SymPy

```
In [9]: expr = 2*log(3)+1  
        expr.subs(log(3), a)
```

Out[9]: $2a + 1$

Answer: (A)

```
In [ ]:
```