Worksheet 001 (1~5)

%config IPCompleter.greedy=True

```python
In [1]: from sympy import *
        from sympy.geometry.line import Line
        from sympy.plotting import plot, plot3d
        import matplotlib.pyplot as plt
        %matplotlib inline

        plt.rcParams['figure.figsize'] = 10, 10
        init_printing(use_unicode=True)
        x, y, a, b, k, K, A, B = symbols('x y a b k K A B')
```

1. If $a(x + 2) + b(x - 1) = 3$ for all x, then $a =$

   (A) -1   (B) 0   (C) 1   (D) 2   (E) 3

Solution

My work

$$a(x + 2) + b(x - 1) = 3$$
$$ax + 2a + bx - b = 3$$
$$(a + b)x + (2a - b) = 3$$

$$\swarrow \qquad \searrow$$

$$a + b = 0 \text{ or } 2a - b = 3$$

$$+\begin{cases} a + b & = & 0 \\ 2a - b & = & 3 \end{cases}$$
$$\overline{\qquad 3a \quad = \quad 3 \qquad}$$

$a = \frac{3}{3} = 1$

$b = -a = -1$

$\begin{cases} a = 1 \\ b = -1 \end{cases}$

Using SymPy

Method 1

```
In [2]: eq = Eq((a*(x+2))+b*(x-1), 3)
        eq
```

Out[2]: $a(x+2) + b(x-1) = 3$

```
In [3]: solve(eq, a, b)
```

Out[3]: $\{a : 1, \quad b : -1\}$

Method 2

```
In [4]: solve((((a*(x+2))+b*(x-1)-3), a, b)
```

Out[4]: $\{a : 1, \quad b : -1\}$

Answer: (C)

2. If $a + b = 2$ and $ab = -1$, then $a^2 + b^2 =$

(A) 4   (B) 5   (C) 6   (D) 8   (E) 10

Solution

My Work

$$a^2 + b^2 = a^2 + b^2 + 2ab - 2ab$$
$$= (a+b)^2 - 2ab$$
$$= 2^2 - 2(-1)$$
$$= 6$$

Using SymPy

```
In [5]: expr = (a+b)*(a+b)-2*a*b
        expr.subs([(a+b, 2), (a*b, -1)])
```

Out[5]: 6

Answer: (C)

3. C. If the graphs of $3x + 4y = 5$ and $kx + 2y = 5$ are perpendicular, then $k =$

(A) -2　(B) -2.67　(C) 2.15　(D) 3.20　(E) 4

Solution

My Work

- Set $slop_1$ for $\ell_1 : 3x + 4y = 5$

$3x + 4y = 5$
$y = \dfrac{-3x + 5}{4} = -\dfrac{3}{4}x + \dfrac{5}{4}$
$slop_1 = -\dfrac{3}{4}$

- Set $Slop_2$ for $\ell_2 : kx + 2y = 5$

$kx + 2y = 5$
$y = \dfrac{-kx + 5}{2} = -\dfrac{k}{2}x + \dfrac{5}{2}$
$slop_2 = -\dfrac{k}{2}$

- ℓ_1 and ℓ_2 are perpendicular, it means $slop_1 * slop_2 = -1$

$(-\dfrac{3}{4}) * (-\dfrac{k}{2}) = -1$
$k = -\dfrac{2 * 4}{3} = -\dfrac{8}{3} \approx -2.67$

```
In [6]:  result = solve((-3/4)*(-k/2)+1, k)
         result[0].evalf(3)
```

Out[6]: -2.67

Answer: (B)

4. If $K = \frac{AB}{A+B}$, then $B =$

$(A)\ \dfrac{A}{1-A}$

$(B)\ \dfrac{AK}{A-K}$

$(C)\ \dfrac{AK}{K-A}$

$(D)\ \dfrac{A+K}{A}$

$(E)\ \dfrac{A-K}{AK}$

Solution

My Work

$$K = \dfrac{AB}{A+B}$$
$$K(A+B) = AB$$
$$KA + KB = AB$$
$$KA = (A-K)B$$
$$(A-K)B = KA$$
$$B = \dfrac{KA}{A-K}$$

By SymPy

```
In [7]:  solve(Eq(K, (A*B)/(A+B)), B)
```

$$\text{Out[7]:}\quad \left[\dfrac{AK}{A-K} \right]$$

Answer: (B)

5. If $\log 3 = a$, then $\log 90 =$

$(A)\ 1 + 2a$

$(B)\ 10a^2$

$(C)\ 10 + 2a$

$(D)\ 30a$

$(E)\ 10 + 3a$

Solution

My Work

$$\log 90 = \log (9 * 10)$$
$$= \log (9) + \log (10)$$
$$= \log 3^2 + 1$$
$$= 2 \log 3 + 1$$
$$= 2a + 1$$

By SymPy

In [8]:
```python
expr = 2*log(3)+1
expr.subs(log(3), a)
```

Out[8]: $2a + 1$

Answer: (A)

In [ ]: