Simple Rules and Mandates for Foreign Exchange Intervention in a Small Open Economy

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Abstract

Foreign exchange intervention is an important additional instrument to conventional monetary policy in the form of nominal interest rate changes. An investigation of an optimal simple intervention rule is not complete without a joint optimisation of the monetary policy rule, which is something the literature has been neglecting. To fill this gap, the objective of this paper is to jointly optimise a simple interest rate rule and a simple foreign exchange intervention rule in a canonical New Keynesian small open economy model, taking into account constraints on both policies. I also investigate the use of mandates for intervention, motivated by the fact that a common reason given by central banks to intervene is to contain the volatility of the exchange rate. I find that, in an estimated model, the introduction of intervention leads to welfare gains, even if small, while the use of simple mandates for intervention has welfare costs that are associated with the mandate's design. Hence, this paper contributes to the literature by highlighting that foreign exchange intervention can be conducted together with monetary policy in a welfare-optimal way that is not only compatible with but also beneficial for inflation targeting.

Keywords: foreign exchange intervention, optimal simple rules, simple mandates, zero lower bound, exchange rate

JEL classification: F31, F41, E58

1 Introduction

In response to capital flows that appreciate or depreciate the domestic currency, concerned central banks regularly use foreign exchange (FX) intervention to influence the path of the exchange rate of the economy. In other words, central banks buy or sell foreign reserves in order to move the exchange rate in the desired direction. Not only is FX intervention compatible with inflation-targeting regimes, but it is also an instrument that can be useful for a price stability objective. As such, FX intervention is an important additional instrument alongside conventional monetary policy in the form of nominal interest rate changes. In fact, it can be shown that it is optimal to combine both policy types in response to a capital flow (Cavallino, 2019). Hence, an investigation of an optimal simple FX intervention rule is not complete without a joint optimisation of the monetary policy rule, which is something the literature has been neglecting.

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To fill this gap, the objective of this paper is to jointly optimise a simple interest rate rule and a simple FX intervention rule in a canonical New Keynesian small open economy (SOE) model based on Galí and Monacelli (2005). I also investigate the use of simple loss functions given to the central bank (mandates) for policymaking, motivated by the fact that a common reason given by central banks to intervene is to contain the volatility of the exchange rate.[1] To my knowledge, this is the first study to analyse optimised simple mandates for FX intervention policy. Previous studies used either a quadratic approximation of household welfare or an ad-hoc loss function.

This means that I contribute to the literature by analysing how FX intervention can be conducted in combination with monetary policy to increase welfare in a way that is beneficial to central banks' credibility. The use of simple rules has the advantage of increasing transparency in central bank policy-making. Transparency, in turn, helps with central bank credibility, which is crucial for the success of inflation targeting. Simple mandates in the form of objectives given to the central bank in a simple loss function reinforce the simple rules. Transparency increases with the adoption of simple mandates because the mandates make clearer to the other agents in the economy what is the central bank's policy. It also improves credibility, as the central bank becomes committed to its objectives.

The rules optimisation methodology requires an estimated model, such that I first estimate the SOE model with FX intervention using Brazilian data. After decades of fixed exchange rate regimes, Brazil adopted a floating exchange rate with inflation targeting in 1999. Since then, the Central Bank of Brazil (CBB) has been quite active in the foreign exchange market, conducting intervention in both the spot and the derivatives markets. In addition to actively using FX intervention, the CBB regularly publishes intervention data, which makes Brazil an interesting case study about FX intervention.[2]

In the model, the central bank of the SOE can use FX intervention in addition to monetary policy to stabilise inflation, output, and the exchange rate in response to a foreign capital outflow that depreciates the domestic currency. The FX intervention is sterilised, which means that it does not affect monetary policy.[3] Households face bond holding costs when trading foreign currency bonds, which increases imperfect substitutability between domestic and foreign currency bonds. This, in addition to being an important feature of emerging economies, makes sterilised FX intervention effective through the portfolio balance channel.

The SOE model is estimated using Bayesian methods with a mix of uninformative priors, for the intervention rule and bond holding costs, and priors that are commonly used in the literature for the interest rate rule. The empirical results suggest that, in addition to inflation-targeting, the Brazilian monetary authority indeed uses FX intervention in response to shocks. The results also confirm that FX intervention in Brazil

[1] Patel and Cavallino (2019) discuss the results of a survey of 23 central banks in emerging economies conducted by the Bank for International Settlements (BIS) regarding the practice of foreign exchange intervention. Financial stability is cited as an important goal.

[2] See, e.g., Kohlscheen and Andrade (2014) and Nedeljkovic and Saborowski (2019).

[3] An unsterilised FX intervention changes the central bank's balance sheet and is equivalent to an open market operation, i.e., it is equivalent to monetary policy.

leans against the wind. This means that the CBB sells (buys) foreign reserves following a currency depreciation (appreciation). The estimation of an FX intervention rule using FX intervention data in a structural macroeconomic model is also a contribution of this paper.

The simple rules and mandates are optimised in a two-stage delegation game solved by backward induction, following Deak et al. (2023) and adapting it when necessary. In the first stage, the mandate, which can be either a modified version of household welfare or a simple loss function, is designed. Then, in the second stage, the central bank receives the mandate and chooses the parameters of the simple rules such that household welfare is maximised, given a soft zero lower bound (ZLB) constraint on the nominal interest rate and a positive lower bound on foreign reserves. The lower bound constraints take the form of penalty functions in the mandate.

The results show that the introduction of FX intervention policy leads to welfare gains, compared to the case when only monetary policy is available to the central bank. The results also show that the adoption of simple mandates to conduct monetary policy and FX intervention is associated with welfare costs that depend on the mandate's design. The constraints on both policies are determined by how strongly the central bank desires to avoid hitting the lower bound in each instrument.

After this introduction, Section 2 reviews the related literature. Section 3 discusses the recent practice of FX intervention in Brazil. Section 4 presents the small open economy New Keynesian model with FX intervention. The details about the model estimation and its results are provided in Section 5. Sections 6 and 7 follow Deak et al. (2023). While Section 6 is dedicated to the optimised simple rules, Section 7 discusses simple mandates. Finally, Section 8 concludes.

2 Background and related literature

FX intervention can sometimes be substantial, even in advanced economies. While the Swiss National Bank (SNB) sold 24 billion dollars in foreign currencies in 2022, the Bank of Japan (BoJ) made a record intervention of 48 billion dollars in only one month of the same year.[4,5] During the global financial crisis (GFC), in 2008Q4, the Bank of Israel purchased 5 billion dollars (2.64% of GDP) in foreign reserves, while the Central Bank of Brazil and the Reserve Bank of India sold 23 billion dollars (1.65% of GDP) and 26 billion dollars (2.30% of GDP), respectively. Following the Taper Tantrum in 2013, the CBB sold 50 billion dollars in only four months.

Proxy data constructed by Adler et al. (2021) shows that FX intervention in the world was positive between January 2000 and December 2020, with countries using around 0.10% of GDP per month for intervention.[6] This means that, on average, countries purchased

[4] "SNB sold $24bln in foreign currencies during 2022 in intervention U-turn", https://www.reuters.com/markets/europe/snb-sold-24bln-foreign-currencies-during-2022-intervention-u-turn-2023-03-21/

[5] "Japan confirms record interventions to support yen", https://www.reuters.com/markets/currencies/japan-confirms-fx-interventions-twice-oct-support-yen-2023-02-07/

[6] A common challenge that studies about FX intervention face is data availability. Even though some results show that interventions work better when they are announced (Sarno and Taylor, 2001), many

around 0.10% of GDP per month of foreign reserves in the period, with the minimum and maximum intervention reaching -10.33 and 14.14% of GDP, respectively. Countries with fixed exchange rate regimes (i.e., hard peg) had the highest mean intervention (0.14% of GDP), while countries with completely free exchange rates had the lowest (0.05% of GPD).[7] On the other hand, countries with floating regimes, on average, intervened more than countries with soft peg regimes (when the exchange rate is kept within certain limits): the average intervention was 0.12% of GDP for the former and 0.09% of GDP for the latter. Overall, and as expected, the variance of intervention decreases as exchange rates become freer: from 1.25 in hard peg regimes to 0.69 in free-floating regimes. Emerging markets and developing countries intervened, on average, roughly the same as the whole sample.[8]

The increasing theoretical macroeconomic literature about FX intervention mostly investigates under which conditions the intervention is optimal.[9] For instance, it can be used to avoid sudden stops modelled as a tightening of borrowing constraints (Davis et al., 2020) and to lower welfare losses following a shock to risk appetite in international capital markets (Alla et al., 2020). However, sterilisation can be a costly policy, especially under capital controls and pegged exchange rates, which creates a trade-off between sterilisation costs and price stability (Chang et al., 2015).

Cavallino (2019) shows that the optimal policy in response to an exogenous shock to the foreign demand for home assets is to combine sterilised intervention and monetary policy. However, full stabilisation of the exchange rate is never optimal. Prasad (2018) shows that it is optimal to combine FX intervention and capital controls in response to a domestic productivity shock and to a foreign interest rate shock. However, Basu et al. (2020) argue that, when more instruments are available to the policymaker, it is not always optimal to use all of them at the same time, since they have non-trivial interactions. Davis et al. (2021) show that optimal FX intervention is equivalent to an optimal tax on foreign capital (capital controls).

A non-exhaustive list of examples that investigate FX intervention using simple rules is Devereux and Yetman (2014), Benes et al. (2015), Adler et al. (2019), Adrian et al. (2020), Agénor et al. (2020) and Lama and Medina (2020).

Benes et al. (2015) conclude that a fixed exchange rate regime using sterilised FX intervention leads to fewer welfare losses in response to a foreign interest rate shock, while a pure float inflation targeting regime is better for a terms of trade shock. Following a risk premium shock and a shock to an endogenous borrowing spread, Adrian et al. (2020) show

central banks around the world do not publish their intervention data, because they believe interventions should be secret in order to be effective.

[7] The Annual Report on Exchange Arrangements and Exchange Restrictions (AREAER), compiled by the International Monetary Fund, states that even free floating regimes are compatible with occasional interventions, "if intervention occurs only exceptionally and aims to address disorderly market conditions and if the authorities have provided information or data confirming that intervention has been limited to at most three instances in the previous six months, each lasting no more than three business days." (International Monetary Fund, 2022, p. 64)

[8] The descriptive statistics of the proxy FX intervention data by exchange rate regime type can be found in Appendix A.

[9] The empirical literature mainly focuses on the effectiveness of interventions. See Sarno and Taylor (2001), Neely (2005), Menkhoff (2010), and Chamon et al. (2019) for surveys about the empirical evidence. For a survey about the early theoretical literature, see Villamizar-Villegas and Perez-Reyna (2017).

that certain economies can benefit from FX intervention combined with capital controls. This happens mainly when inflation expectations are less well-anchored, when there are considerable currency mismatches, and when the economy is more vulnerable to shocks in global investor sentiment. However, both Benes et al. (2015) and Adrian et al. (2020) use only ad-hoc rules, without optimising them.

Adler et al. (2019) show that FX intervention works better to stabilise the economy after foreign interest rate shocks when a central bank has perfect credibility. Under imperfect credibility, the central bank faces a trade-off between output and inflation. Lama and Medina (2020) conclude that FX intervention and macroprudential policies complement monetary policy following capital outflows. However, FX intervention is better after a foreign interest rate shock, while macroprudential policy is better following domestic risk shocks. In both Adler et al. (2019) and Lama and Medina (2020) the policy parameters are chosen to minimise a given loss function based on output and inflation volatility.

Devereux and Yetman (2014) show that the use of FX intervention leads to welfare gains, which depend on the degree of financial internationalisation, goods market integration, and exchange rate pass-through. The welfare gains are greater when the intervention is sterilised than when it is not, but sterilised intervention cannot be used when the home economy is fully integrated into the international financial markets. Agénor et al. (2020) point out that fully sterilised FX intervention is not always optimal. In a framework in which there is a bank portfolio effect, even a fully sterilised intervention can be expansionary and magnify macroeconomic fluctuations. Devereux and Yetman (2014) and Agénor et al. (2020) optimise their FX intervention rules to maximise household welfare and perform welfare analysis. However, the imperfect international risk sharing in Devereux and Yetman (2014) is imposed rather than a feature of the model. Additionally, their analysis is only for productivity and cost-push shocks, not foreign shocks. While Agénor et al. (2020) perform their analysis in response to a world interest rate shock, their model is quite complex, with seven different types of agents.

The contribution of this paper is to jointly optimise a simple interest rate rule and a simple FX intervention rule in a canonical New Keynesian small open economy model, taking into account constraints on both types of policy. In this sense, the closest papers to this are Adler et al. (2019) and Agénor et al. (2020). The former is regarding the modelling of FX intervention, and the latter is because of the rules optimisation to maximise households welfare. When compared to the model in Adler et al. (2019), the model presented here is even simpler: the central bank has perfect credibility, there are no wage rigidities, and there is only one type of firm in the production sector, as in Galí and Monacelli (2005). Differently from Agénor et al. (2020), the optimised parameters of the FX intervention rules come from a mandate delegated to the central bank following Deak et al. (2023).

There are many possible reasons for the use of sterilised FX intervention by central banks, that could potentially motivate a mandate. Neely (2008), Patel and Cavallino (2019), and Hendrick et al. (2019) survey central banks about their practices. A common reason to intervene is to contain exchange rate movements or volatility. Investigating the motives for intervention, Adler and Tovar (2014) estimate a reaction function for a panel of 15 countries and find that central banks acted to mitigate currency appreciation pressures between 2004-2010 (2008-2009 crisis excluded) when many emerging economies were taking advantage of the favourable global conditions to accumulate reserves.

Because the FX intervention rule is optimised with a nominal interest rate rule, this paper also builds upon the literature on optimal monetary policy in open economies. Corsetti et al. (2010), Corsetti et al. (2018), Corsetti et al. (2020), and Senay and Sutherland (2019) focus on "targeting rules", with inflation rules that target welfare-relevant gaps created by the absence of risk-sharing. Devereux and Yetman (2014) conduct policy analysis in terms of "implementable" nominal interest rate rules. Here, I use implementable nominal interest rate and FX intervention rules and introduce soft ZLB constraints that avoid ZLB episodes on both instruments. While welfare-optimal simple rules, first introduced into the macroeconomic literature by Levine and Currie (1987), are inferior in welfare when compared to the Ramsey policy, the former closely mimics the welfare outcome of the latter (Schmitt-Grohe and Uribe, 2007).

Finally, this paper is also related to the literature on simple mandates in the form of loss functions. Debortoli et al. (2019) investigate the use of a dual mandate and Deak et al. (2023) develop a framework that formalises the ZLB constraint on the nominal interest rate. Both papers focus on monetary policy in a closed economy, while I investigate the use of mandates in a small open economy. Even though this paper closely follows the methodology proposed by Deak et al. (2023), it also builds on it, expanding the framework to be used with both a nominal interest rate rule and an FX intervention rule.

3 Foreign exchange intervention in Brazil

Figure 1 plots the quarterly change of the BRL/USD (Brazilian real/US dollar) nominal exchange rate, on the left-hand side, and the quarterly FX intervention between 2000Q1 and 2019Q4, according to FX intervention data published by the CBB, on the right-hand side. Table 1 reports the descriptive statistics of both variables. During those 20 years, the exchange rate varied considerably, having periods of significant depreciation. The currency depreciated 1.06% per quarter on average, with a standard deviation of 7.53%. The first large depreciation happened in 2001 during the country's energy crisis, which led to an energy rationing that lasted nine months. In 2002, during the presidential campaign, the market became increasingly nervous about the probable Lula's electoral victory. At that point in time, Lula was still seen as a far-left candidate, which caused fear about his future policies. After his election, this fear progressively dissipated, and the country enjoyed years of economic growth and appreciation of the domestic currency. The minimum depreciation of -15.66% (an appreciation) happened in 2003Q2 following the beginning of the first Lula government. The largest depreciation of 31.19% happened in 2008Q4, during the GFC. After that, the currency consistently depreciated during the political crisis that led to the impeachment of President Dilma Rousseff in 2016.

The average FX intervention in the period was positive. Loosely speaking, it means that the CBB increased its foreign reserves by 0.24% of GDP per quarter on average.

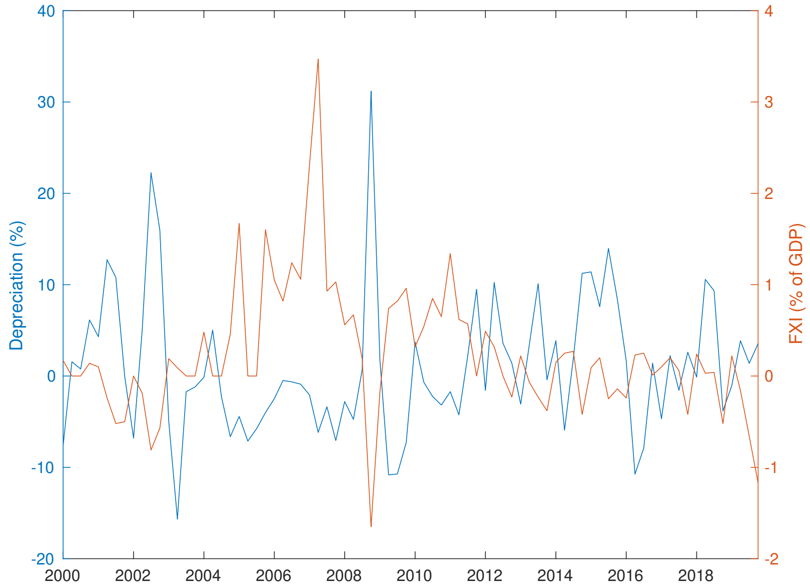

Figure 1: Quarterly depreciation and foreign exchange intervention, 2000Q1 to 2019Q4

Table 1: Descriptive statistics of the variables

Variable	Observations	Mean	Std. dev.	Minimum	Maximum
FX intervention	79	0.2443	0.7156	-1.6500	3.4700
Depreciation	79	1.0664	7.5333	-15.6618	31.1883

The largest positive intervention of 3.58% happened in 2007Q2, while the largest negative intervention of -1.65% of GDP happened during the GFC in 2008Q4. Between 2003 and 2012, and except during the worst moment of the GFC, Brazil accumulated reserves taking advantage of the favourable domestic and global conditions. The average intervention during the accumulation period, including the crisis, was 0.60% of GDP per quarter.

FX intervention and depreciation have a moderate negative correlation of -0.53. In fact, when simply regressing intervention on its first lag and depreciation, I find a significant coefficient of -0.039 for the feedback on depreciation.[10,11] This means that a 10% depreciation of the domestic currency would be associated with a reserve sale of 0.39% of GDP. For an overview of the Brazilian experience in conducting FX intervention within the inflation targeting framework, see Barroso (2019).

Overall, FX intervention in Brazil is seen as effective and leaning against the wind, as estimated in the relationship between intervention and depreciation estimated in the previ-

[10]The regression model is $intervention_t = \gamma_0 + \gamma_1 intervention_{t-1} + \gamma_2 depreciation_t + v_t$, where γ_0 is a constant, γ_1 is a policy smoothing coefficient, γ_2 is the feedback on depreciation and v_t is the error term.

[11]The regression is estimated using OLS and it is worth mentioning that there is a possible endogeneity problem. Intervention happens due to the depreciation of the domestic currency at the same time they also have an effect on it. When this happens, OLS estimates are biased. However, Ghosh et al. (2016) and Chertman et al. (2020) argue that it is possible to use OLS in this case anyway and Carvalho et al. (2021) show that for an interest rate rule. Here, the bias works against the main hypothesis, such that, if the result is significant, it can be seen as a lower bound. Furthermore, they also argue that, because the exchange rate is difficult to forecast, it is difficult to find good instruments for it, such that it is not necessarily better to use instrumental variables.

ous paragraph. When directly estimating policy rules, Chertman et al. (2020) find that the dominant characteristic of interest rate policy in Brazil is inflation targeting and that the central bank used FX intervention in reaction to currency depreciation, such that 2.2 billion USD of reserves are sold in response to a 10% currency depreciation. Barroso (2014), Kohlscheen and Andrade (2014), Chamon et al. (2019), and Nedeljkovic and Saborowski (2019) conclude that FX intervention is effective in reducing the appreciation/depreciation of the Brazilian currency.

The structural macroeconomic literature that focuses on Brazil mainly analyses monetary policy, fiscal policy, and the interaction between them.[12] Studies that address a possible concern of the CBB with exchange rate movements have not come to the point of modelling foreign exchange intervention yet. Rather, the existing studies investigate changes in monetary policy that could be motivated by the depreciation of the domestic currency. Furlani et al. (2010), Palma and Portugal (2014), and Gómez et al. (2019) are examples of this. Furlani et al. (2010) stress that "there is qualitative evidence that the central bank seeks to reduce the exchange rate volatility, but that does not mean that the interest rate path is systematically changed due to such actions". This suggests that there is a role for FX intervention modelling.

4 A SOE model with foreign exchange intervention

This paper builds on a canonical small open economy model, which is a version of Galí and Monacelli (2005) where the small open economy is the home economy, denoted by H, and the rest of the world is the foreign economy, denoted by F. There are two one-period risk-free bonds that can be traded by home households: the domestic currency bond and the foreign currency bond. Both bonds cost one consumption unity in the respective currency at time t. However, households face bond holding costs, which are a function of (private) foreign bond holdings. The central bank can also hold foreign currency bonds (reserves) and, in addition to the policy interest rate, it can use FX intervention as a second instrument. Unlike households, the central bank does not face bond holding costs.

[12] Castro et al. (2015) develop a large DSGE model to account for many idiosyncrasies of the Brazilian economy, such as an explicit target for the primary surplus and the existence of administered or regulated prices in the economy, in order to be used by the Central Bank of Brazil as a part of their framework. Fasolo (2019) investigates the effect of monetary policy volatility shocks, Arruda et al. (2020) analyse monetary policy under credit constraints, and de Mendonça et al. (2022) study the effect of central bank credibility on the monetary transmission mechanism. Silva et al. (2019) analyse the impact of monetary policy on the housing market and Besarria et al. (2021) investigate the effect of fiscal policy and news shocks on housing prices. Costa Junior and García-Cintado (2021) investigate the rent-seeking behaviour in the Brazilian economy, while Costa Junior et al. (2021b) analyse the effects of labour informality on fiscal policy. Finally, de Jesus et al. (2020) analyse monetary policy with fiscal constraints, Costa Junior et al. (2021a) investigate political cycles, and Nobrega et al. (2022) study debt management. As Castro et al. (2015), none of these examples adds feedback on the foreign exchange rate in the interest rate policy.

4.1 Households

Households in the home economy choose consumption C_t, hours of labour N_t, and domestic and foreign currency bond holdings, B_t and B_t^* respectively, to maximise utility

$$\max_{\{C_t,N_t,B_t,B_t^*\}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right),$$

where

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

is a composite consumption index, $C_{H,t}$ is an index of consumption of domestic goods and $C_{F,t}$ is an index of imported goods, β is the discount factor, α is the trade openness (such that $1-\alpha$ is a measure of home bias), and η is the elasticity of substitution between domestic and foreign goods.

The maximisation is subject to the following nominal budget constraint:

$$P_t C_t + B_t + S_t B_t^* + P_t \Psi \left(\frac{S_t B_t^*}{P_t} \right) = R_{t-1} B_{t-1} + S_t R_{t-1}^* B_{t-1}^* + W_t N_t + P_t D_t + P_t T_t, \quad (1)$$

where R_t is the return on the domestic bond, R_t^* is the return on the foreign bond, W_t is the nominal wage, D_t is the profits received from firms, and T_t is the lump-sum transfer from the results of the sterilisation by the central bank. Bond holding costs are given by the convex bond holding costs function $\Psi\left(\frac{S_t B_t^*}{P_t}\right)$, such that the term $P_t \Psi\left(\frac{S_t B_t^*}{P_t}\right)$ can be seen as a transaction cost households face for trading foreign currency bonds.[13] The nominal exchange rate, S_t, is defined as the price in the domestic currency of one unit of the foreign currency, such that an increase in the exchange rate means a depreciation of the domestic currency. Finally, P_t is the consumer price index (CPI), given by

$$P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad (2)$$

where $P_{H,t}$ is the domestic price index and $P_{F,t}$ is the price index for imported goods expressed in domestic currency. Let $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ be the CPI inflation, then

$$\Pi_t = \left[(1-\alpha) \left(\Pi_t^H \frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\eta} + \alpha \left(\Pi_t^F \frac{P_{F,t-1}}{P_{t-1}} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

where $\Pi_t^H \equiv \frac{P_{H,t}}{P_{H,t-1}}$ and $\Pi_t^F \equiv \frac{P_{F,t}}{P_{F,t-1}}$ are, respectively, the inflation of domestically and foreign-produced goods.

The consumption demand for each type of good can be found in solving for the optimal allocation of expenditures between domestic and imported goods:

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t,$$

[13] The formulation with bond holding costs is equivalent to the commonly used formulation with a risk premium (Schmitt-Grohé and Uribe, 2003).

$$C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t.$$

The price ratio of home-produced goods $\frac{P_{H,t}}{P_t}$ given by

$$\frac{P_{H,t}}{P_t} = \left[1 - \alpha + \alpha \mathcal{T}_t^{1-\eta} \right]^{\frac{1}{\eta-1}},$$

and the price ratio of imported goods $\frac{P_{F,t}}{P_t}$ given by

$$\frac{P_{F,t}}{P_t} = \left[(1-\alpha) \mathcal{T}_t^{\eta-1} + \alpha \right]^{\frac{1}{\eta-1}},$$

where $\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ is the terms of trade, which are obtained by dividing equation (2) by $P_{H,t}$ and $P_{F,t}$, respectively. Adding terms of trade shock, we can write it as

$$\frac{\mathcal{T}_t tot_t}{\mathcal{T}_{t-1} tot_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}},$$

where the terms of trade shock tot_t follows the autoregressive process:

$$\log tot_t = \rho_{tot} \log tot_{t-1} + \epsilon_{tot,t}.$$

The bond holding costs function $\Psi(\cdot)$ is defined in terms of the private holding of foreign bonds in home consumption units, $\frac{S_t B_t^*}{P_t}$, and has the following functional form, as in Davis et al. (2021):

$$\Psi \left(\frac{S_t B_t^*}{P_t} \right) \equiv \frac{\kappa}{2} \left(\frac{S_t B_t^*}{P_t} - \frac{S B^*}{P} \right)^2 = \frac{\kappa}{2} \left(\frac{S_t B_t^*}{P_t} \right)^2,$$

where κ is the bond holding costs parameter and the last equality follows from the fact that foreign currency bond holdings in the steady state, B^*, are assumed to be equal to zero.

The first-order conditions of the households' problem give the labour supply

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t},$$

the Euler equation for the domestic currency bond

$$1 = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t,t+1}} \right\} R_t, \tag{3}$$

and the Euler equation for the foreign currency bond

$$1 = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\Pi_{t,t+1}^S}{\Pi_{t,t+1}} \right\} \frac{R_t^*}{1 + \kappa B_{F,t}}, \tag{4}$$

where $\Pi_{t,t+1} \equiv \frac{P_{t+1}}{P_t}$ is the (expected) inflation between periods t and $t+1$, $\Pi_{t,t+1}^S \equiv \frac{S_{t+1}}{S_t}$

is the (expected) nominal depreciation of the exchange rate, and $B_{F,t} \equiv \frac{S_t B_t^*}{P_t}$ is the private holdings of foreign assets in home consumption units. When κ is sufficiently small, the foreign currency bond becomes close to a perfect substitute for the domestic currency bond, such that FX intervention does not work. Equations (3) and (4) together give the uncovered interest rate parity (UIP) of this model, which includes a wedge between both bonds given by bond holding costs.

4.2 Firms

There is a continuum of firms that produce a differentiated good. Each period, only a measure of $1-\theta$ firms sets new prices, as in Calvo (1983). Firm j's production function is given by $Y_t(j) = A_t N_t(j)$, where A_t follows the AR(1) process

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t}.$$

The firm's problem is to choose the optimal price $\bar{P}_{H,t}$ that maximises real profits

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{H,t+k}} \left[Y_{t+k} \left(\bar{P}_{H,t} - MC_{t+k}^n MS_{t+k} \right) \right] \right\},$$

subject to the sequence of demand constraints

$$Y_{t+k}(j) \leq \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left(C_{H,t+k} + C_{H,t+k}^* \right) \equiv Y_{t+k}^d(\bar{P}_{H,t}),$$

where $\Lambda_{t,t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}$ is the stochastic discount factor, $MC_t^n = \frac{W_t}{A_t}$ is the nominal marginal cost, ε is the elasticity of substitution between varieties and MS_t introduces a mark-up shock:

$$\log MS_t = \rho_{MS} \log MS_{t-1} + \epsilon_{MS,t}.$$

The solution to the firm's problem results in the following price-setting dynamics:

$$\bar{\Pi}_t^H = \frac{\varepsilon}{\varepsilon - 1} \frac{X_{1,t}}{X_{2,t}},$$

where $\bar{\Pi}_t^H = \frac{\bar{P}_{H,t}}{P_{H,t}}$, and $X_{1,t}$ and $X_{2,t}$ are auxiliary variables given by

$$X_{1,t} = Y_t MC_t MS_t + \theta E_t \left\{ \Lambda_{t,t+1} (\Pi_{t,t+1}^H)^{\varepsilon} X_{1,t+1} \right\}$$

and

$$X_{2,t} = Y_t + \theta E_t \left\{ \Lambda_{t,t+1} (\Pi_{t,t+1}^H)^{\varepsilon - 1} X_{2,t+1} \right\},$$

where $MC_t = \frac{MC_t^n}{P_{H,t}}$ is the real marginal cost.

The price of the domestically produced good is given by the aggregation over both the firms that reset their price and the firms that do not:

$$P_{H,t} = \left[(1-\theta)(\bar{P}_{H,t})^{1-\varepsilon} + \theta (P_{H,t-1})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

Dividing both sides of the equation by $P_{H,t}$ results in:

$$1 = (1-\theta)(\bar{\Pi}_t^H)^{1-\varepsilon} + \theta(\Pi_{t-1,t}^H)^{\varepsilon-1}.$$

The price stickiness results in price dispersion, which lowers aggregate production. Then, the aggregate production becomes

$$Y_t = \frac{A_t N_t}{\Xi_t},$$

where $\Xi_t \equiv \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj$ is the price dispersion term.

Finally, the dynamics of price dispersion is given by

$$\Xi_t = (1-\theta)(\bar{\Pi}_t^H)^{-\varepsilon} + \theta(\Pi_{t-1,t}^H)^{\varepsilon}\Xi_{t-1}.$$

4.3 Central bank

The central bank sets the policy interest rate, R_t, and might use sterilised FX intervention in response to shocks that cause exchange rate movements. When setting the interest rate, the central bank follows a standard Taylor rule:

$$\begin{aligned} \log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\Bigg(\phi_\Pi \log\left(\frac{\Pi_{t-1,t}}{\Pi}\right) + \phi_Y \log\left(\frac{Y_t}{Y}\right) \\ + \phi_{dY} \log\left(\frac{Y_t}{Y_{t-1}}\right) + \phi_{\Pi^S} \log\left(\frac{\Pi_{t-1,t}^S}{\Pi^S}\right)\Bigg) + \epsilon_{M,t}, \end{aligned} \tag{5}$$

where variables without the time subscript denote steady-state values. Hence, the rule has interest rate smoothing $\left(\rho_R \log\left(\frac{R_{t-1}}{R}\right)\right)$ and feedback parameters on CPI inflation (ϕ_Π), output (ϕ_Y), change in output (ϕ_{dY}) and depreciation (ϕ_{Π^S}).

The central bank buys or sells foreign currency bonds to conduct FX intervention. For a sterilised intervention, the central bank must also trade domestic currency bonds in the opposite direction. The change in foreign assets in units of the domestic currency should be equal to the change in domestic liabilities for a fully sterilised intervention. Hence, the sterilisation condition is $\Delta B_t = S_t \Delta F_t^*$, where F_t^* denotes foreign reserves, i.e., foreign bonds held by the central bank to use foreign exchange intervention. Unlike households, the central bank does not face bond holding costs, such that the nominal results from the sterilisation operation are given by

$$P_t T_t = S_t F_{t-1}^*(R_{t-1}^* - 1) - B_{t-1}(R_{t-1} - 1), \tag{6}$$

which are transferred to households.

Let $F_t \equiv \frac{S_t F_t^*}{P_t}$ be the reserves in home consumption units, the FX intervention rule is

$$\log\left(\frac{F_t}{F}\right) = \rho_F \log\left(\frac{F_{t-1}}{F}\right) + (1-\rho_F)\left(\phi_{\Pi_f^S} \log\left(\frac{\Pi_{t-1,t}^S}{\Pi^S}\right)\right) + \epsilon_{F,t}, \tag{7}$$

where reserves in the steady state, F, are defined as a fraction δ of output in the steady state, i.e. $F = \delta Y$. As in the standard Taylor rule for the interest rate, the term $\rho_f \log\left(\frac{F_{t-1}}{F}\right)$ introduces policy smoothing. In addition to the persistence term, the rule in Equation (7) has feedback on depreciation. This means that the central bank uses FX intervention in response to any shock that causes exchange rate movements. While Adler et al. (2019) use a rule with feedback on the foreign interest rate, there is empirical evidence in favour of the rule with feedback on depreciation, which is more commonly found in the literature. Finally, when the central bank does not use the second instrument, reserves are held constant at their steady-state value, i.e. $F_t = F$.

4.4 External sector and the foreign economy

The nominal current account of the home economy is

$$CA_t = S_t(R^*_{t-1} - 1)(B^*_{t-1} + F^*_{t-1}) + P_t NX_t - P_t \frac{\kappa_t}{2}\left(\frac{S_t B^*_t}{P_t}\right)^2, \tag{8}$$

where $NX_t = \frac{P_{H,t}}{P_t} C^*_{H,t} - \frac{P_{F,t}}{P_t} C_{F,t}$ denotes net exports.

The nominal financial account is

$$FA_t = S_t(B^*_{t-1} + F^*_{t-1}) - S_t(B^*_t + F^*_t). \tag{9}$$

Then, Equations (8) and (9) combined give the balance of payments of the home economy ($CA_t + FA_t = 0$), or the evolution of foreign assets holdings in nominal terms:

$$S_t(B^*_t + F^*_t) = S_t R^*_{t-1}(B^*_{t-1} + F^*_{t-1}) + P_t NX_t - P_t \frac{\kappa_t}{2}\left(\frac{S_t B^*_t}{P_t}\right)^2,$$

which in real term becomes

$$B_{F,t} + F_t = \frac{\Pi^S_{t-1,t}}{\Pi_{t-1,t}} R^*_{t-1}(F_{t-1} + B_{F,t-1}) + NX_t - \frac{\kappa_t}{2} B^2_{F,t}. \tag{10}$$

The foreign economy is assumed to be exogenous to the model and is defined as in Adolfson et al. (2007), such that the vector of foreign variables X^*_t follow the multivariate process

$$\Phi_0 X^*_t = \Phi_1 X^*_{t-1} + \Phi_2 X^*_{t-2} + \Phi_3 X^*_{t-3} + \Phi_4 X^*_{t-4} + S_{X^*}\epsilon_{X^*,t}, \tag{11}$$

where $\epsilon_{X^*,t} \sim \mathcal{N}(0, I)$, S_{X^*} is a diagonal matrix with the standard deviations of the shocks in the main diagonal, and $\Phi_0^{-1} S_{X^*}\epsilon_{X^*,t} \sim \mathcal{N}(0, \Sigma_{X^*})$.

The foreign demand for home-produced goods, i.e., the export demand faced by the home country, denoted $C^*_{H,t}$, is given by

$$C^*_{H,t} = \alpha\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} Y^*_t = \alpha \mathcal{T}^{\eta}_t Y^*_t,$$

where $P^*_{H,t}$, $P^*_{F,t}$, and Y^*_t are the foreign counterparts of the same domestic variables. The last equality follows from the law of one price: $S_t P^*_{H,t} = P_{H,t}$, $P^*_t = P^*_{F,t}$, and $S_t P^*_t = P_{F,t}$.

The law of one price also gives a relationship between the imported goods price inflation, $\Pi^F_{t-1,t}$, and foreign inflation, $\Pi^*_{t-1,t}$:

$$\Pi^F_{t-1,t} = \Pi^*_{t-1,t}\Pi^S_{t-1,t}.$$

Finally, market clearing requires

$$Y_t = C_{H,t} + C^*_{H,t}.$$

The model is summarized in Appendix B.

5 Estimation

The model is estimated in two steps. First, because the foreign economy is defined to be completely exogenous, it is estimated separately. The VAR process in Equation (11) is estimated using quarterly US data, which is chosen to represent the rest of the world. Using these results to parameterise the foreign economy, then the home economy is estimated using quarterly data from Brazil.

5.1 Foreign economy

The data for foreign output (Y^*_t), foreign inflation (Π^*_t), and the foreign interest rate (R^*_t) for the estimation of Equation (11) is prepared as in Smets and Wouters (2007), except for when de-trending output (a one-sided HP filter is used instead of taking the first difference, which follows Adolfson et al. (2007)). The time series are obtained from the Federal Reserve Bank of St. Louis and span the period from 1953Q3 to 2019Q4. The structural shocks of the foreign economy are identified using short-run restrictions with the standard ordering found in the literature: inflation, output, and interest rate. Table 2 summarises the data and its transformations. The details are provided in Appendix D.

Table 2: Foreign economy data

Variable	Notation	Data
Foreign inflation	$\ln(\Pi^*_t)$	$\ln\left(\frac{\text{Implicit price deflator}_t}{\text{Implicit price deflator}_{t-1}}\right) \times 100$
Foreign output	$\ln(Y^*_t)$	$\ln(\text{Real GDP per capita}_t) \times 100$
Foreign interest rate	$\ln(R^*_t)$	$\frac{\text{Fed Funds effective rate}}{4}$

Finally, as in Adolfson et al. (2007), I assume and cannot reject that the coefficient matrix Φ_0 in Equation (11) has the following structure

$$\Phi_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \omega_{\Pi^*} & \omega_{Y^*} & 1 \end{bmatrix},$$

where ω_{Π^*} and ω_{Y^*} indicate coefficients to be estimated.

The VAR is estimated with four lags. Discarding the coefficients that were not significant at the 10% level, the resulting equations are:

$$
\begin{aligned}
\ln(\Pi_t^*) &= 0.4436\ln(\Pi_{t-1}^*) + 0.2409\ln(\Pi_{t-2}^*) + 0.2239\ln(\Pi_{t-4}^*) \\
&+ 0.3721\ln(R_{t-1}^*) - 0.3139\ln(R_{t-2}^*) + 0.2448\epsilon_{\Pi^*,t}, \\
\ln(Y_t^*) &= -0.3768\ln(\Pi_{t-3}^*) + 0.9965\ln(Y_{t-1}^*) - 0.1820\ln(Y_{t-3}^*) \\
&- 1.1716\ln(R_{t-2}^*) + 1.1541\ln(R_{t-3}^*) + 0.6464\epsilon_{Y^*,t}, \\
\ln(R_t^*) &= 0.0877\ln(\Pi_{t-1}^*) + 0.1420\ln(\Pi_{t-2}^*) + 0.0584\ln(Y_{t-1}^*) \\
&+ 1.1706\ln(R_{t-1}^*) - 0.5153\ln(R_{t-2}^*) + 0.4093\ln(R_{t-3}^*) \\
&- 0.1299\ln(R_{t-4}^*) + 0.0334\epsilon_{\Pi^*,t} + 0.0264\epsilon_{Y^*,t} + 0.1819\epsilon_{R^*,t},
\end{aligned}
$$

which are used to parameterise the foreign economy in the small open economy model.

5.2 Small open economy

The small open economy is estimated with Bayesian methods. I estimate five domestic shocks: technology (ϵ_A), mark-up (ϵ_{MS}), terms of trade (ϵ_{tot}), monetary policy (ϵ_M), and reserves (ϵ_F). For this, I use five observable variables: output (Y_t^{obs}), inflation (Π_t^{obs}), depreciation ($\Pi_t^{S,obs}$), interest rate (R_t^{obs}), and foreign exchange intervention (F_t^{obs}), which are constructed as follows:

$$
\begin{aligned}
Y_t^{obs} &= \left[\ln\left(GDP_t^{BR}\right) - \ln\left(GDP_{t-1}^{BR}\right)\right] \times 100; \\
\Pi_t^{obs} &= \left[\ln\left(CPI_t^{BR}\right) - \ln\left(CPI_{t-1}^{BR}\right)\right] \times 100; \\
\Pi_t^{S,obs} &= \left[\ln\left(ER_t^{BR}\right) - \ln\left(ER_{t-1}^{BR}\right)\right] \times 100; \\
R_t^{obs} &= \frac{R_t^{BR}}{4};
\end{aligned}
$$

where GDP_t^{BR}, CPI_t^{BR}, ER_t^{BR}, and R_t^{BR} denote the (transformed) times series of output, inflation, exchange rate, and the interest rate, respectively. F_t^{obs} is directly the FX intervention data published by the central bank as percent of GDP. The details about the data can be found in Appendix D.

The common start of all five time series is 1996Q1. However, as it was previously explained, Brazil only adopted a managed float exchange rate regime in 1999Q1. Therefore, I use the period from 1999Q1 to 2019Q4 for the estimation but keep the observations from 1996Q1 to 1998Q4 for initializing the Kalman filter.

5.2.1 Parameter values

For now, some parameter values are imposed rather than estimated. Table 3 summarises the chosen values for those parameters. Inflation in the steady state, $\Pi = \Pi^*$, is chosen to be equal to 1.0079, the average of the foreign inflation in the period. Following Galí and Monacelli (2005), the trade openness, α, is chosen to roughly correspond to the Brazilian imports/GDP ratio, which is equal to 0.09. I follow Castro et al. (2015) for the values of the labour supply elasticity ($\varphi = 1$), the elasticity of substitution between goods ($\varepsilon = 11$, to have a markup of 11%), and the risk aversion ($\sigma = 1.3$). For the Calvo parameter, I use $\theta = 0.74$, estimated by Castro et al. (2015). The elasticity of substitution between domestic and foreign goods, $\eta = 3.4$, comes from the estimation by Bajzik et al. (2020). Finally, reserves in the steady state as a fraction of GDP, δ, are chosen to correspond to the average in the period, which is 1.39.

Table 3: Parameter values

Parameter	Notation	Value
Inflation in the steady state	Π	1.0079
Trade openness	α	0.09
Labour supply elasticity	φ	1
Calvo parameter	θ	0.74
Discount factor	β	0.99
Risk aversion	σ	1.3
Elasticity of substitution between goods	ε	11
Elasticity of substitution between domestic and foreign goods	η	3.4
Reserves in the steady state as a fraction of GDP	$\frac{F}{Y} = \delta$	1.39

5.2.2 Bayesian estimation

I estimate the model using Bayesian methods. Bayesian estimation uses the well-known Bayes' rule

$$p(B|A) = \frac{p(A|B)p(B)}{p(A)},$$

where A and B are events, $p(A)$ is the probability of event A, and $p(A|B)$ is the probability of A given B, to estimate the parameters of a model, which are treated as random variables. For this, the rule is rewritten as

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)} \propto p(y|\theta)p(\theta),$$

where y is the data, θ is the model's parameters, $p(\theta)$ is the prior distribution, $p(y|\theta)$ is the likelihood distribution, and $p(\theta|y)$ is the posterior distribution, which is the object of interest. The likelihood density is computed by the Kalman Filter and the posterior distribution is updated using the rule $p(y|\theta)p(\theta)$. Ultimately, it is necessary to evaluate an integral, which cannot be done analytically, to compute the posterior distributions. Markov Chain Monte Carlo (MCMC) methods are used to draw from the posterior distribution and estimate it.

I estimate the autoregressive coefficients and the standard deviation of the shocks for the technology, mark-up, and terms of trade shock processes. I also estimate the interest rate smoothing coefficient (ρ_R), the feedback parameters (ϕ_Π, ϕ_Y, ϕ_{dY}, and ϕ_{Π^S}), and the

Table 4: Prior distributions

Parameter	Notation	Dist.	Mean, std. dev.
Standard deviation of the shock (Technology)	σ_A	IG	0.1, Inf.
Standard deviation of the shock (Mark-up)	σ_{MS}	IG	0.1, Inf.
Standard deviation of the shock (Terms of trade)	σ_{tot}	IG	0.1, Inf.
Autoregressive coefficient (Technology)	ρ_A	β	0.705, 0.2
Autoregressive coefficient (Mark-up)	ρ_{MS}	β	0.695, 0.2
Autoregressive coefficient (Terms of trade)	ρ_{tot}	β	0.685, 0.2
Standard deviation of monetary policy shock (Taylor rule)	σ_M	IG	0.1, Inf.
Interest rate smoothing (Taylor rule)	ρ_R	β	0.7, 0.1
Feedback on inflation (Taylor rule)	ϕ_Π	N	3, 0.25
Feedback on output (Taylor rule)	ϕ_Y	N	0.1, 0.005
Feedback on change in output (Taylor rule)	ϕ_{dY}	N	0.1, 0.05
Feedback on depreciation (Taylor rule)	ϕ_{Π^S}	N	0.1, 0.05
Standard deviation of reserves shock (FX intervention rule)	σ_F	IG	0.1, Inf.
FX intervention smoothing (FX intervention rule)	ρ_F	U	[-1,1]
Feedback on depreciation (FX intervention rule)	$\phi_{\Pi^S_f}$	N	0, 0.5
Bond holding costs	κ	IG	0.05, 0.05

standard deviation of the shock (σ_M) of the monetary policy rule:

$$\begin{aligned}\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left(\phi_\Pi \log\left(\frac{\Pi_{t-1,t}}{\Pi}\right) + \phi_Y \log\left(\frac{Y_t}{Y}\right)\right.\\ \left. + \phi_{dY}\log\left(\frac{Y_t}{Y_{t-1}}\right) + \phi_{\Pi^S}\log\left(\frac{\Pi^S_{t-1,t}}{\Pi^S}\right)\right) + \epsilon_{M,t},\end{aligned} \tag{12}$$

as well as the intervention smoothing coefficient (ρ_F), the feedback parameter ($\phi_{\Pi^S_f}$), and the standard deviation of the shock ($\sigma_{F,t}$) FX intervention rule:

$$\log\left(\frac{F_t}{F}\right) = \rho_F \log\left(\frac{F_{t-1}}{F}\right) + (1-\rho_F)\left(\phi_{\Pi^S_f}\log\left(\frac{\Pi^S_{t-1,t}}{\Pi^S}\right)\right) + \epsilon_{F,t}. \tag{13}$$

Finally, the bond holding costs parameter, κ, is also estimated.

Table 4 presents the prior distributions used in the estimation. For the shocks and the interest rate rule, I mainly follow the literature, slightly adapting the mean or standard deviation of some prior distributions when necessary to help with parameter identification.[14] I use an inverse gamma distribution for the standard deviation of the shocks (σ_A, σ_{MS}, σ_{tot}, σ_M, and σ_F), which should always be positive. The distribution is centred at 0.1 and has a degree of freedom equal to 2, which gives an infinite standard deviation. For the autoregressive coefficient of the shock processes (ρ_A, ρ_{MS}, and ρ_{tot}) and the interest rate smoothing (ρ_R), which should be bounded between 0 and 1, I use a beta distribution. The distribution is centred at 0.7 (but 0.705 for ρ_A, 0.695 for ρ_{MS}, and 0.685 for ρ_{tot}) with a standard deviation of 0.1 for ρ_R and 0.2 for ρ_A, ρ_{MS}, and ρ_{tot}. The feedback parameters on the policy rules follow normal distributions. The prior distribution of the feedback on inflation, ϕ_Π, is centred at 3 with a standard deviation of 0.25. For the feedback on output (ϕ_Y), change in output (ϕ_{dY}), and depreciation (ϕ_{Π^S}) the distribution is centred at 0.1. The standard deviation is 0.005 for the feedback on output and 0.05 for the feedback on

[14]The prior distributions affect the curvature of the likelihood function, which is key for parameter identification when using Bayesian methods.

the change in output and on depreciation. For the feedback parameter in the FX intervention rule ($\phi_{\Pi^S_f}$), I also use normal distributions to be in line with the usual practice for interest rate rules. However, in order to be as uninformative as possible, I centre the distributions at 0. The standard deviation is 0.5. The prior of the FX intervention smoothing coefficient, ρ_F, is a uniform distribution bounded between -1 and 1, also to be as uninformative as possible. Finally, because the bond holding costs should be positive to create a wedge in the right direction in the UIP, I choose an inverse gamma distribution for the bond holding costs parameter, κ. The distribution is centred at 0.05 and has a standard deviation of 0.05.

All of the 18 parameters to be estimated with the 5 observed variables are identified according to the tests of the Jacobian of the steady state and reduced-form matrices, the Jacobian of mean and spectrum as in Qu and Tkachenko (2012), and the Jacobian of the first two moments as in Iskrev (2010). The details of the identification tests plotted in Appendix E suggest that the FX intervention rule is weakly identified, which is at least partially given by construction, with the uninformative priors. However, estimation results and post-estimation diagnostics reported in Appendix F suggest that the rule is well identified. The technology and the mark-up shocks are highly colinear, such that their prior distributions were tweaked, as described in the previous paragraph. The likelihood seems to be flat for the feedback on output change in the interest rate rule (ϕ_{dY}) and the parameters of the terms of trade shock process in Appendix E, even though the information matrix plots suggest they are well identified. The post-estimation diagnostics in Appendix F, however, suggest that ϕ_{dY} is not identified, since its posterior distribution lies just on top of its prior. It might also be the case that data is not informative enough to update the prior. All other parameters, including the bond holding costs parameter, are well-identified.

5.3 Empirical results

Table 5 reports estimation results. The FX intervention rule has high policy smoothing, with ρ_F being higher than 0.9. The negative feedback on depreciation ($\phi_{\Pi^S_f} = -0.7966$), which is comparable to the one found in Section 3, suggests that FX intervention in Brazil leans against the wind.[15] This means that the central bank sells (buys) reserves in response to a currency depreciation (appreciation). Finally, the bond holding costs parameter, κ, is 0.0256.

Regarding monetary policy, the Central Bank of Brazil has mainly conducted inflation-targeting in the period, as the estimated interest rate rule displays a strong reaction to inflation (ϕ_Π is around 3) and small coefficients for the other feedback parameters. Both the feedback on output, ϕ_Y, and on output change, ϕ_{dY}, are around 0.10. The feedback on depreciation in the interest rate rule is positive ($\phi_{\Pi^S} = 0.03$) but it is not significant. This result is in line with the surveyed literature that suggests that the Brazilian monetary

[15] Before comparing values, however, it is necessary to consider that the coefficient estimated in Section 3 is not exactly the same as the one estimated here. In Section 3, the result is actually equivalent to $(1 - \rho_F)\phi_{\Pi^S_f}$, such that the value here to be compared is -0.0584.

Table 5: Bayesian estimation results, 1999Q1 to 2019Q4

Parameter	Notation	Posterior mean	HPD
Standard deviation of shock (Technology)	σ_A	6.2184	[4.9918,7.3382]
Standard deviation of shock (Mark-up)	σ_{MS}	5.0871	[3.9472,6.1758]
Standard deviation of shock (Terms of trade)	σ_{tot}	10.3146	[8.6622,11.9303]
Autoregressive coefficient (Technology)	ρ_A	0.6598	[0.5667,0.7506]
Autoregressive coefficient (Mark-up)	ρ_{MS}	0.9964	[0.9923,1.0000]
Autoregressive coefficient (Terms of trade)	ρ_{tot}	0.9945	[0.9890,1.0000]
Standard deviation of monetary policy shock (Taylor rule)	σ_M	0.2274	[0.1817,0.2730]
Interest rate smoothing (Taylor rule)	ρ_R	0.8686	[0.8446,0.8937]
Feedback on inflation (Taylor rule)	ϕ_Π	3.2399	[2.8936,3.5967]
Feedback on output (Taylor rule)	ϕ_Y	0.0962	[0.0879,0.1038]
Feedback on change in output (Taylor rule)	ϕ_{dY}	0.1135	[0.0372,0.1894]
Feedback on depreciation (Taylor rule)	ϕ_{Π^S}	0.0316	[-0.0114,0.0753]
Standard deviation of reserves shock (FX intervention rule)	σ_F	1.3665	[1.1665,1.5748]
FX intervention smoothing (FX intervention rule)	ρ_F	0.9267	[0.8986,0.9564]
Feedback on depreciation (FX intervention rule)	$\phi_{\Pi^S_f}$	-0.7966	[-1.0000,-0.5749]
Bond holding costs	κ	0.0256	[0.0157,0.0349]

authority does not change the interest rate in reaction to exchange rate movements.

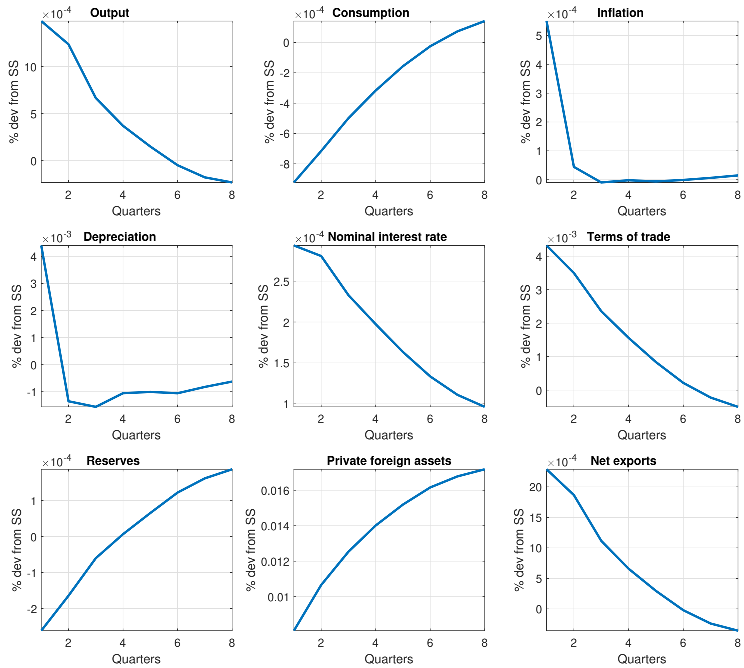

Figure 2: Impulse response to a foreign interest rate shock

Figure 2 presents the impulse response to a foreign interest rate shock. Following a positive foreign interest rate shock, there is an increase in capital outflow as shown by the response of private foreign assets. Such an outflow causes a depreciation of the domestic currency, which leads to an increase in inflation due to the pass-through. Hence, the central bank increases the nominal interest rate. With the depreciation of the domestic currency, home-produced goods become cheaper with respect to foreign-produced goods, such that, even though there is a decrease in aggregate consumption in the home economy, there is

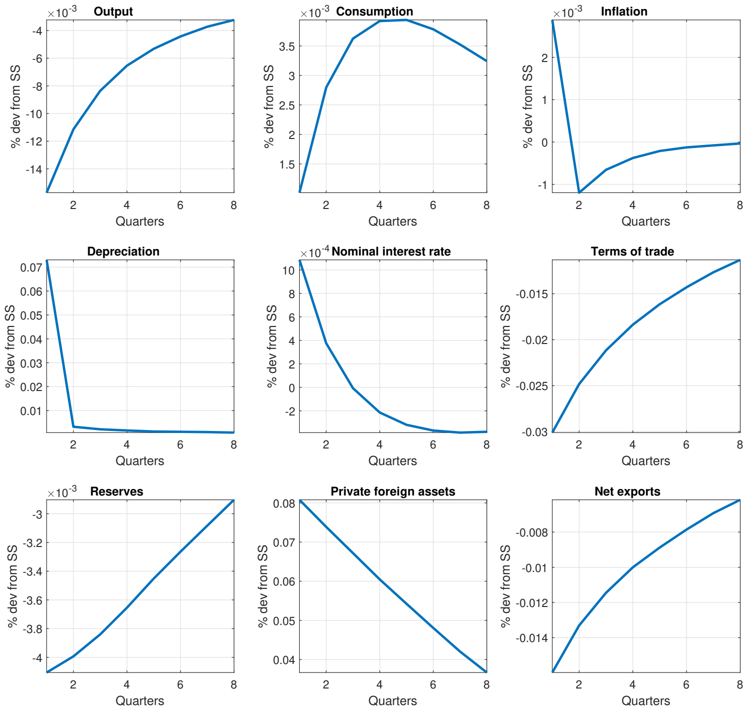

Figure 3: Impulse response to a terms of trade shock

an increase in output which is translated into an increase in net exports. Furthermore, reserves react following the estimated sign. Even though the model does not include feedback on the foreign interest rate, a change to it causes depreciation of the domestic currency, which triggers intervention.

Figure 3 presents the impulse response to terms of trade shock. The terms of trade are defined such that a positive shock leads to a decrease in them. Because now foreign-produced goods are cheaper with respect to home-produced goods, net exports and output decrease. Even though consumption does not react much on impact, it increases after a while. The decrease in net exports leads to an increase in private foreign assets and, consequently, the depreciation of the domestic currency. Hence, inflation increases due to the pass-through and the central bank reacts by increasing the nominal interest rate.

The variance decomposition in Table 6 reports the contribution of each shock to the variance of output, inflation, interest rate, depreciation, and reserves. The technology, markup, and terms of trade shocks are, in this order, the ones that contribute the most to the variance of output. The markup shock is the shock that mostly contributes to the variance of inflation and the interest rate, while the variance of depreciation is mainly explained by the terms of trade and foreign inflation shocks. All variables contribute somehow to explain the variance of FX intervention: 92% is explained by reserves shock, which might suggest that there are other factors that are more important when conducting FX intervention.

Figures 23 to 27 in Appendix G present the historical shock decomposition of each observable variable. Hence, in the figures, the deviation of each smoothed endogenous

Table 6: Variance decomposition of shocks in the estimated model

	ϵ_A	ϵ_{MS}	ϵ_{tot}	ϵ_M	ϵ_F	ϵ_{R^*}	ϵ_{Π^*}	ϵ_{Y^*}
Output	34.52	32.81	28.37	3.88	0.03	0.25	0.02	0.13
Inflation	24.45	61.78	6.54	2.01	0.01	0.97	2.79	1.45
Interest rate	18.63	73.18	1.53	1.00	0.02	1.02	2.93	1.69
Depreciation	0.25	1.17	38.13	0.27	0.03	10.31	33.60	16.24
Reserves	0.05	0.01	8.14	0.06	91.64	0.05	0.02	0.03

Table 7: Second moments of observed and simulated data

	Standard deviation		Crosscorr. with output	
Variable	Data	Model	Data	Model
Output	1.1926	3.5019	1.0000	1.0000
Inflation	0.8642	1.2898	-0.0379	-0.5473
Interest rate	1.8219	1.1860	-0.0868	-0.0848
Depreciation	8.0005	7.1678	-0.2835	-0.4741
Reserves	0.0973	1.3446	0.0073	0.1775

variable from its steady state is decomposed in the contribution of each smoothed shock in the period of estimation. Figure 23 shows that the deviation of output in the period was mostly due to technology and markup shocks, with a striking exception in the last quarter of 2008. The global financial crisis caused a big capital outflow represented by terms of trade shock, that contributed to a drop in output. The deviations of inflation and the interest rate respectively reported in Figures 24 and 25, follow the same pattern of output deviations, with technology and markup shock being responsible for most of the variation. Figure 26 shows that the deviation of depreciation is mainly given by the terms of trade shock, with the shocks to the foreign variables having a much smaller impact on the exchange rate. Finally, according to Figure 27 there is no clear pattern in the decomposition of FX intervention. However, it is possible to notice that the shock to reserves has a non-negligible contribution to the deviations, which might suggest that there are other factors related to the intervention that are not being captured by the model and its FX intervention rule.

Post-estimation diagnostics graphs are reported in Appendix F. It shows that the estimation converges with 200,000 draws. As mentioned previously, the prior-posterior plots show that the parameter ϕ_{dY} might not be identified or that data is not informative enough to update the prior. The others seem well identified.

After the estimation, the model is simulated using the posterior mean and Table 7 reports the second moments of simulated data. The model overpredicts the standard deviation of output, inflation, and reserves. On the other hand, it underpredicts the standard deviation of the interest rate and depreciation. Regarding the cross-correlation of variables with output, the model, with the exception of reserves, predicts the correct sign, getting really close to what is observed in the data for the interest rate.

Figure 4 displays the autocorrelations up to lag 10. For the first lag, the model correctly predicts the signs in most cases, but the results are not very close to the values observed in the data. Inflation and the nominal interest rate are the variables for which results are closest to the data in the first lag. However, the autocorrelation decay of these two

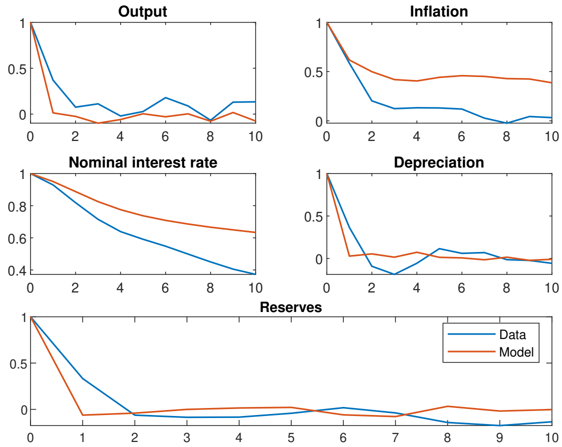

Figure 4: Autocorrelation of observed and simulated data up to lag 10

variables in the model is much slower than in the data, such that the difference between observed and simulated data becomes higher for higher-order lags.

Given the rather poor fit of some second moments, it is worth mentioning that the Bayesian estimation approach is based on the likelihood and not on moments, unlike the selected method of moments or the generalised method of moments. This could be seen as an argument for the use of endogenous priors, that use the data, or part of it, to compute priors to be used in the estimation, as developed by Christiano et al. (2011), which will be the subject of future work. However, in a previous estimation of this model using endogenous priors and the change in reserves as a proxy for FX intervention, the resulting second moments reported in Table 12 of Appendix H are not necessarily better. The choice of which type of priors to use might ultimately depend on what we are trying to achieve or predict.

6 Optimal simple rules

This section closely follows Deak et al. (2023), with some important differences that are discussed when relevant. As such, it focuses on the framework used to compute the optimal simple rules presented in Section 6.1. The optimised rules are chosen to maximise the welfare criterion discussed in Section 6.2. The delegation game, which introduces soft lower bound constraints on the nominal interest rate and foreign reserves is explained in Section 6.3.

6.1 Monetary policy and FX intervention rules

In what follows, the interest rate rule,

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left(\phi_\Pi \log\left(\frac{\Pi_{t-1,t}}{\Pi}\right) + \phi_Y \log\left(\frac{Y_t}{Y}\right)\right.$$
$$\left. + \phi_{dY} \log\left(\frac{Y_t}{Y_{t-1}}\right) + \phi_{\Pi^S} \log\left(\frac{\Pi^S_{t-1,t}}{\Pi^S}\right)\right) + \epsilon_{M,t},$$

and the FX intervention rule,

$$\log\left(\frac{F_t}{F}\right) = \rho_F \log\left(\frac{F_{t-1}}{F}\right) + (1-\rho_F)\phi_{\Pi_f^S}\log\left(\frac{\Pi_t^S}{\Pi^S}\right) + \epsilon_{F,t},$$

are optimised following the delegation game proposed by Deak et al. (2023). For this, it is first necessary to write the rules in the implementable form:

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + \gamma_\Pi \log\left(\frac{\Pi_{t-1,t}}{\Pi}\right) + \gamma_Y \log\left(\frac{Y_t}{Y}\right) + \gamma_{dY}\log\left(\frac{Y_t}{Y_{t-1}}\right) + \gamma_{\Pi^S}\log\left(\frac{\Pi^S_{t-1,t}}{\Pi^S}\right), \tag{14}$$

and

$$\log\left(\frac{F_t}{F}\right) = \rho_F \log\left(\frac{F_{t-1}}{F}\right) + \gamma_{\Pi_f^S}\log\left(\frac{\Pi_t^S}{\Pi^S}\right), \tag{15}$$

where $\gamma_i \equiv (1-\rho_j)\phi_i,\ i = \Pi, Y, dY, \Pi^S, R^*, \Pi_f^S$, such that $\rho_1 \equiv [\rho_R, \gamma_\Pi, \gamma_Y, \gamma_{dY}, \gamma_{\Pi^S}]$, and $\rho_2 \equiv [\rho_R, \gamma_\Pi, \gamma_Y, \gamma_{dY}, \gamma_{\Pi^S}, \rho_F, \gamma_{\Pi_f^S}]$ are the choices of feedback parameters that define the rules when the central bank is under Regime 1 (only monetary policy) and Regime 2 (monetary policy and FX intervention), respectively. This setup expands the one in Deak et al. (2023) by adding an FX intervention rule.

6.2 The welfare criterion

Following Deak et al. (2023), a welfare-based criterion based on the inter-temporal household expected utility

$$\Omega_t \equiv E_t \sum_{\tau=0}^{\infty} \beta^\tau U(C_{t+\tau}, N_{t+\tau}) \tag{16}$$

is used to rank alternative rules. In recursive form, the inter-temporal household expected utility at time t becomes:

$$\Omega_t = U_t + \beta E_t \Omega_{t+1}. \tag{17}$$

The welfare Ω_t is a function of policy, R_t, F_t, and all the estimated parameters in the model, including the shocks and the foreign economy modelled as a VAR.

Optimal monetary and FX intervention policies at time $t=0$, for given initial values for the predetermined variables Z_0, solve the maximisation problem:

$$\max_{\rho\in S} \Omega_0(\mathsf{Z}_0, \Pi, \rho), \tag{18}$$

which is a conditional and time-inconsistent criterion, as the optimised rule at time t becomes

$$\max_{\rho\in S} \Omega(\mathsf{Z}_t, \Pi, \rho) \Rightarrow \rho = \rho(\mathsf{Z}_t, \Pi). \tag{19}$$

This leads to an incentive to re-optimise, due to the dependence on Z_t.

The source of time inconsistency, however, is removed by choosing a welfare conditional on the economy being at the steady state $\mathsf{Z}_t = \mathsf{Z}$, which is exogenous and policy invariant.[16] The optimisation problem becomes

$$\max_{\rho \in S} \Omega(\mathsf{Z}, \Pi, \rho) \Rightarrow \rho = \rho(\mathsf{Z}, \Pi), \tag{20}$$

which now is time-invariant. Additionally, Z is policy invariant, such that we can write $\rho = \rho(\Pi)$. "Thus welfare at the steady state is maximised *on average* over all realizations of *future* shocks driving the exogenous stochastic processes given their deterministic steady states" (Deak et al., 2023). The optimal rule ρ^o is computed using second-order perturbation methods. The delegation game, described in the next section, introduces lower bound considerations for the nominal interest rate and reserves.

6.3 The delegation game

As in Deak et al. (2023), the actual household intertemporal welfare is given by

$$\Omega_t \equiv E_t \left[\sum_{t+\tau}^{\infty} \beta^{\tau} U_{t+\tau} \right], \tag{21}$$

where $U_t \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$ is the household utility. The modified welfare function is given by

$$\Omega_t^{mod} \equiv E_t \left[\sum_{t+\tau}^{\infty} \beta^{\tau} \left(U_{t+\tau} - w_r (R_{t+\tau} - R)^2 - w_f (F_{t+\tau} - F)^2 \right) \right], \tag{22}$$

where w_r and w_f are the weights that penalise the variance of the nominal interest rate and the variance of foreign reserves, respectively. Both the terms $-w_r(R_t - R)^2$ and $-w_f(F_t - F)^2$ modify the standard welfare function to include soft lower bound constraints in the problem. While the first one is associated with a standard ZLB constraint on the nominal interest rate, the second one creates a positive lower bound on reserves. Thus, in addition to hitting the zero lower bound in the interest rate with a very low probability, the central bank will also hit the lower bound in reserves with a very low probability. If $w_f = 0$, then the central bank is able to freely adjust reserves in response to shocks. The higher is w_f, the more constrained is the central bank to use FX intervention policy. Equation (22) can be seen as a mandate with a penalty function $P = w_r(R_t - R)^2 + w_f(F_t - F)^2$.

The delegation game has two stages. The mandate is designed in the first stage. This means that, for a chosen per-period probability $\bar{p}_R$ and $\bar{p}_F$ of the nominal interest rate and reserves, respectively, hitting the lower bounds, the central bank designs the optimal loss function in the mandate (i.e., the modified welfare function). In other words, the central bank chooses w_r and w_f in Equation (22) that maximise the actual household intertemporal welfare in Equation (21) for given $\bar{p}_R$ and $\bar{p}_F$.

In the second stage, the central bank receives the mandate and chooses optimised

[16] In the original framework presented by Deak et al. (2023), there is an additional source of time inconsistency, which is the choice of the steady-state inflation target. However, this is not a problem in the case of an SOE, which takes inflation in the steady state as given by the rest of the world, such that $\Pi^* = \Pi$.

rules to implement. Hence, the central bank chooses ρ for the rules in Equations (14) and (15), that maximises the modified welfare function in Equation (22). This results in the probabilities

$$p_R = p_R(\Pi, \rho^o(\Pi, w_r, w_f))$$

and

$$p_F = p_F(\Pi, \rho^o(\Pi, w_r, w_f))$$

of the nominal interest rate and reserves, respectively, hitting the lower bounds. The two-stage delegation game defines an equilibrium, which is solved by backward induction, in the choice variables w_r^o, w_f^o, and ρ^o that maximises the actual household welfare subject to the lower bound constraints:

$$R_t \geq 1 \text{ with high probability } 1 - \bar{p}_R$$

and

$$F_t \geq \bar{F} \text{ with high probability } 1 - \bar{p}_F,$$

where the first constraint is a ZLB constraint on the nominal interest rate and $\bar{F}$ is a positive lower bound on reserves. The most commonly used reserves adequacy measure around the world, according to a survey conducted by the World Bank, is import coverage (Alekasir et al., 2019), such that I assume $\bar{F} = C_F$.[17]

6.4 Results

The welfare outcome of each optimised rule is compared to the estimated rule using the consumption equivalent variation (CEV). For a given equilibrium in C_t and N_t and a single-period utility function, $U_t = U(C_t, N_t)$, the consumption equivalence (CE) is defined as the utility increase following a 1% increase in consumption. The dynamics of the CE follow:

$$CE_t \equiv 100\left(U_t(1.01C_t, N_t) - U_t(C_t, N_t)\right) + \beta E_t\left\{CE_{t+1}\right\}.$$

Then, the CEV is given by

$$CEV = \frac{\Omega_2 - \Omega_1}{CE},$$

[17] In the original framework proposed by Deak et al. (2023), the delegation game happens in three stages. In the first stage, as in this paper, the central bank designs the mandate. The choice of optimised rules to implement happens in the third stage. In the second stage, there is the choice of the steady state inflation rate Π that corresponds to the chosen probability of hitting the ZLB. This means that $\Pi = \Pi^o$ is chosen to satisfy

$$p(R_t \leq 1) \equiv p(\Pi^o, \rho^o(\Pi^o, w_r)) \leq \bar{p},$$

where $\bar{p}$ is the chosen probability of hitting the ZLB and $\rho^o(\Pi^o, w_r)$ is the optimised form of the rule given the steady-state target inflation Π^o, the weight on the interest rate volatility w_r, such that the ZLB constraint

$$R_t \geq 1 \text{ with high probability } 1 - \bar{p} \tag{23}$$

is successfully achieved. However, in the SOE and differently from the setup in Deak et al. (2023), the steady state inflation rate is always equal to the foreign steady state inflation rate, such that it is always fixed at $\Pi = \Pi^* = \Pi^o$ and there is no such a choice to be made in the SOE. Hence, the delegation game in the SOE has only two stages.

where Ω_1 and Ω_2 are actual welfare values and CE is the deterministic steady state of the consumption equivalence.

Table 8: Optimised simple rules

	Estimated	Regime 1	Regime 2
ρ_R	0.8669	0.9989	0.9998
γ_Π	0.4304	1.7894	1.8419
γ_Y	0.0128	0.0001	0.0000
γ_{dY}	0.0150	0.2222	0.2148
γ_{Π^S}	0.0044	0.0001	0.0002
ρ_F	0.9265	–	0.9902
$\gamma_{\Pi_f^S}$	-0.0582	–	-0.0457
p_R	0.0809	0.0095	0.0095
p_F	0.0000	–	0.0077
Act. welfare (Ω)	-51.1346	-47.7669	-44.1682
CEV (%)	0.0000	0.0337	0.0697
w_r	–	46	46
w_f	–	–	0.06

The CEV is calculated as $CEV = \frac{\Omega_2 - \Omega_1}{CE}$, where Ω_1 is the actual welfare of the estimated rule, Ω_2 is the actual welfare of the rule being compared, and CE is the deterministic steady state of the consumption equivalence, which is approximately 100.

Table 8 reports optimisation results for equilibrium optimal simple rules when both p_R and p_F are less than or equal to 0.01. The second column shows the estimated rule, which is the baseline for CEV comparisons. The third column shows the results for when the central bank has only one instrument, the nominal interest rate, available (Regime 1). The fourth column shows results for when the central bank can use both instruments (Regime 2).

The estimated monetary policy and FX intervention rules are associated with an actual welfare of -54.1346, such that every optimised rule in Table 8 represents a welfare improvement, even if small. The comparison between Regime 1 and Regime 2 illustrates the welfare benefits of the introduction of the second instrument. Regime 2 is associated with a welfare improvement of 0.0360% when compared to Regime 1.

With the estimated rules, there is a probability of 8.09% of the nominal interest rate hitting the ZLB, while reserves do not hit their lower bound. In order to decrease p_R to less than 1%, the weight on the interest rate volatility is chosen to be equal to 46 in both regimes. Because reserves naturally have a low probability of hitting the lower bound, a w_f of as low as 0.06 already guarantees $p_F < 0.01$. Generally, the lower the weight on the volatility of reserves, the more the central bank will use FX intervention in response to shocks and the more often reserves will hit the lower bound. In fact, when w_f is too low, the optimised FX intervention rule results in unreasonable high welfare and probability of hitting the lower bound, which illustrates the importance of modelling constraints and costs of FX intervention when analysing optimal policy and rules.

In terms of parameter values, the optimised interest rate rules have a high degree of interest rate smoothing, with $\rho_R > 0.99$, and a strong reaction to inflation, with $\gamma_\Pi > 1.78$. The feedback parameters on output and on depreciation in the interest rate rule are near zero in both cases. This shows that, even with the introduction of FX intervention policy, optimal monetary policy remains consistent with inflation targeting. The optimised FX intervention rule confirms that it is optimal to lean against the wind in response to currency depreciation, as the feedback parameter is negative ($\gamma_{\Pi^S_f} = -0.0457$). The rule also displays a high degree of policy smoothing, with $\rho_F = 0.9998$.

6.4.1 Impulse responses

Need to change the figures.

The rules optimisation process involves averaging across all realisations of future shocks. This means that the difference between Regime 1 and Regime 2 in the responses and the resulting welfare might be smaller for some shocks than for others. Figures 5 to 7 plot the impulse responses for the shocks with the highest difference in intertemporal welfare, defined by Equation (17), under both regimes. Figures 28 to 31 in Appendix I plot the other shocks. In every case, the difference in depreciation between Regimes 1 and 2 is tiny. This is because of the strength (or weakness) of the FX intervention rule combined with the size of the bond holding costs parameter κ. The higher is κ, the more efficient to stabilise the exchange rate the intervention will be. In other words, the same policy rule will have a higher impact on depreciation when κ is higher.

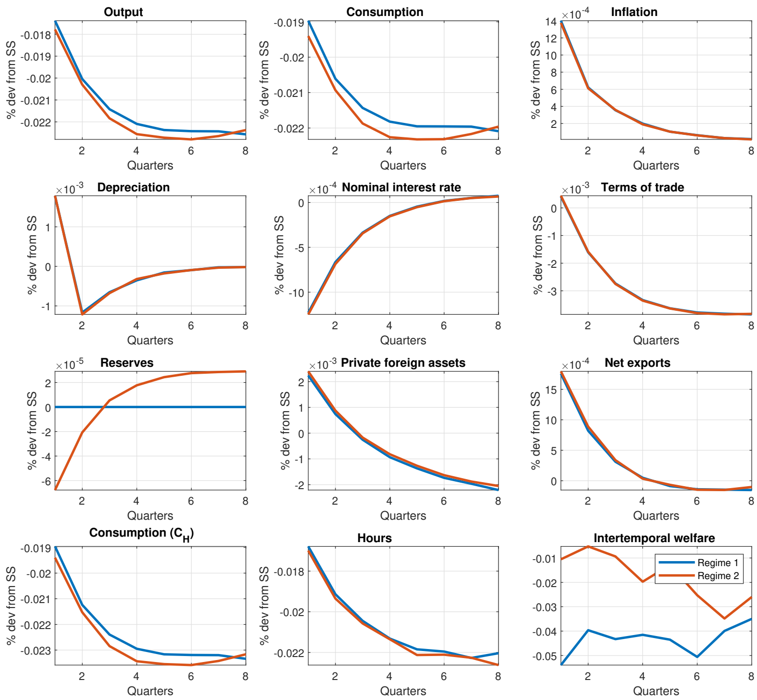

Figure 5: Impulse response to a mark-up shock

Figure 5 shows the response to a mark-up shock. Following the shock, which increases new prices, inflation increases, the currency depreciates, and output and consumption fall. The central bank decreases the nominal interest rate in response. In Regime 2, FX intervention is triggered, such that the central bank also sells reserves. Despite that, depreciation under Regime 2 is only slightly lower than under Regime 1. The same happens for the responses of inflation and the nominal interest rate. Nonetheless, it is possible to easily notice the difference in the intertemporal welfare between Regime 1 and Regime 2. Regime 2 yields higher welfare, even though the decrease in output and consumption is higher in Regime 2 than in Regime 1.

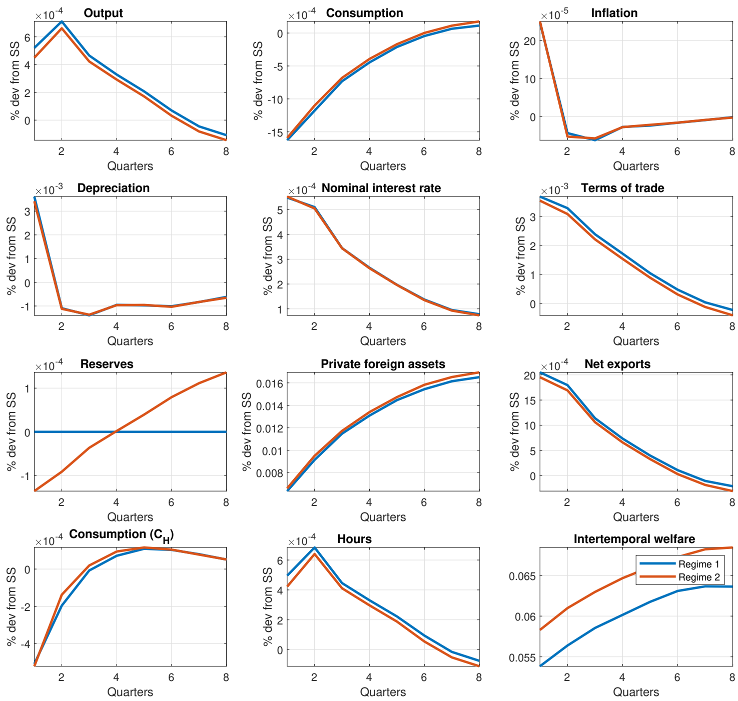

Figure 6: Impulse response to a foreign interest rate shock

In Figure 6, the dynamics of the responses to a foreign interest rate shock are the same as described in the estimated model. The foreign interest rate shock causes a depreciation of the domestic currency, which triggers FX intervention in Regime 2. As in the case of a mark-up shock, the differences in the responses of depreciation, inflation, and the nominal interest rate are quite small. Nonetheless, the intervention is enough to create a welfare gain when Regime 2 is compared to Regime 1.

Following a foreign productivity shock in Figure 7, net exports increase with the increase in foreign consumption (exports). The increase in the foreign demand is met by an increase in output and a decrease in the domestic consumption of domestically produced goods (C_H). With that, inflation and depreciation increase, triggering an increase in the nominal interest rate, which is lower under Regime 2, and FX intervention in the form of reserves sale. Like in the two previous cases, the difference between Regimes 1 and 2 in the resulting depreciation is small. However, the differences in responses between regimes are enough to result in higher welfare for Regime 2.

In conclusion, when the objective of the central bank is to simply maximise the household's welfare while avoiding hitting the ZLB, the optimal FX intervention does not act to contain the depreciation of the domestic currency, as seen in the impulse response

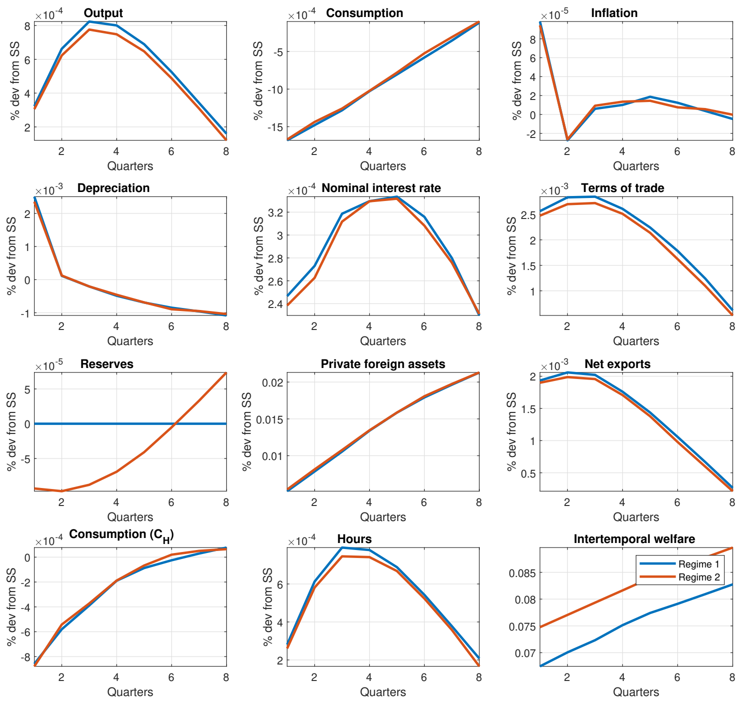

Figure 7: Impulse response to a foreign productivity shock

functions. On the contrary, the difference in depreciation between Regimes 1 and 2 is close to zero. This does not mean, however, that FX intervention is not meaningful. The intervention is enough to increase welfare.

7 Optimal simple mandates

Given that one of the main motives for conducting FX intervention is to contain the volatility of the exchange rate, suppose that the central bank receives a simple mandate, in the form of a simple loss function, for this. This means that instead of using the actual modified welfare function in Equation (22) to design the optimal simple rules, the central bank receives a function that actually accounts for the volatility of depreciation, and possibly other variables, instead of household welfare. Additionally, the central bank also receives a simple rule to match the mandate, meaning that it uses a rule that contains the same variables in the mandate.

7.1 Simple mandate 1

First, suppose that FX intervention is conducted independently from monetary policy (it might be the case that the policymaker responsible for each type of policy is not the same and they do not coordinate). Hence, the central bank, or the relevant policymaker, receives the function

$$\Omega_t^{mod} \equiv E_t \left[\sum_{t+\tau}^{\infty} \beta^\tau \left(-(\Pi^S_{t+\tau-1,t+\tau} - \Pi^S)^2 - w_f (F_{t+\tau} - F)^2 \right) \right],$$

instead of Equation (22), which is implemented using the FX intervention rule:

$$\log\left(\frac{F_t}{F}\right) = \rho_F \log\left(\frac{F_{t-1}}{F}\right) + \gamma_{\Pi^S_f} \log\left(\frac{\Pi^S_t}{\Pi^S}\right), \tag{24}$$

assuming that monetary policy is optimised independently using the previous framework (Regime 0). Then, the policymaker receives a mandate for FX intervention, for a given optimal monetary policy. This is Mandate 1. Because Mandate 1 takes the monetary policy of Regime 0 as given, however, it is not a Nash equilibrium in rules.

7.2 Simple mandate 2

Now, suppose that the central bank has a unique mandate to use both monetary policy and FX intervention to contain the volatility of inflation, output, and the depreciation of the domestic currency. This means that in this case (Mandate 2), instead of Equation (22), the central bank receives the function

$$\begin{aligned} \Omega_t^{mod} \equiv E_t \Bigg[\sum_{t+\tau}^{\infty} \beta^\tau \Big(&-(\Pi_{t+\tau-1,t+\tau} - \Pi)^2 - w_y (Y_{t+\tau} - Y)^2 - w_s (\Pi^S_{t+\tau-1,t+\tau} - \Pi^S)^2 \\ &-w_r (R_{t+\tau} - R)^2 - w_f (F_{t+\tau} - F)^2 \Big) \Bigg], \end{aligned} \tag{25}$$

where w_y and w_s are the relative weights on the volatility of output and of depreciation when the weight on the volatility of inflation is normalised to one. The mandate has the following policy rules:

$$\begin{aligned} \log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + \gamma_\Pi \log\left(\frac{\Pi_{t-1,t}}{\Pi}\right) + \gamma_Y \log\left(\frac{Y_t}{Y}\right) + \gamma_{dY} \log\left(\frac{Y_t}{Y_{t-1}}\right) \\ + \gamma_{\Pi^S} \log\left(\frac{\Pi^S_{t-1,t}}{\Pi^S}\right) \end{aligned} \tag{26}$$

and

$$\log\left(\frac{F_t}{F}\right) = \rho_F \log\left(\frac{F_{t-1}}{F}\right) + \gamma_{\Pi^S_f} \log\left(\frac{\Pi^S_t}{\Pi^S}\right). \tag{27}$$

In this case, there is the issue of the choice of the parameters w_y and w_s, in addition to w_r and w_f. For simplicity, I analyse two cases. First, I assume that the central bank puts on the volatility of output and of depreciation the same weight it puts on the volatility of inflation, i.e., $w_y = w_s = 1$. One could think about this as being a "triple mandate", in which the central bank has three goals (stabilisation of inflation, output, and depreciation) and two instruments, as opposed to the dual mandate in Debortoli et al. (2019). Second, I assume the weight on output volatility to be zero (i.e., $w_y = 0$), while the weight on depreciation volatility remains unchanged. This, then, would be a dual mandate, with two goals and two instruments.

7.3 Results

Table 9 reports optimisation results for the equilibrium optimal simple rules when both p_R and p_F are less than or equal to 0.01 under Mandates 1 and 2. The CEV is calculated as before, but now Regime 2, whose results are reported in the second column of the table, is used as the baseline for comparisons. The third column shows results for the independent mandate for FX intervention (Mandate 1), while the last two columns show results for the joint mandate for both monetary policy and FX intervention (Mandate 2). Column (A) reports the results for the case when $w_s = w_y = 1$. In column (B), $w_s = 1$ but $w_y = 0$. Need to produce results for $w_y = 0$!

Table 9: Optimised simple rules (Mandates)

	Regime 2	Mandate 1	Mandate 2 (A)	Mandate 2 (B)
ρ_R	0.9998	0.9989	0.9493	
γ_Π	1.8419	1.7894	1.5638	
γ_Y	0.0000	0.0001	0.0001	
γ_{dY}	0.2148	0.2222	1.8077	
γ_{Π^S}	0.0002	0.0001	0.0940	
ρ_F	0.9902	0.8935	0.7387	
$\gamma_{\Pi^S_f}$	-0.0457	-0.4990	-0.4503	
p_R	0.0095	0.0075	0.0090	
p_F	0.0077	0.0090	0.0000	
Act. welfare (Ω)	-44.1682	-44.2327	-49.6560	
CEV (%)	0.0000	-0.0006	-0.0549	
w_r	46	46	5	
w_f	0.06	0.0003	0	
w_s	–	1	1	
w_y	–	–	1	

The CEV is calculated as $CEV = \frac{\Omega_2 - \Omega_1}{CE}$, where Ω_1 is the actual welfare in Regime 2, Ω_2 is the actual welfare of the rule being compared, and CE is the deterministic steady state of the consumption equivalence, which is approximately 100.

The comparison of Mandates 1 and 2 with Regime 2 shows that simple mandates can be associated with welfare losses. While Mandate 1 results in a loss of 0.0006% in welfare, the welfare loss associated with Mandate 2 varies with the chosen weight on output volatility. The loss is of 0.0549% in column (A), when $w_y = 1$, and ??? in column (B), when $w_y = 0$. Hence, in this case, it is welfare-enhancing to remove output stabilisation from the central bank's objective.

The volatility of the nominal interest rate and the volatility of reserves are lower under the mandates than under Regime 2, such that the probability of each instrument hitting its lower bound is lower as well. Mandate 1 has a lower p_R than Regime 2 with the same weight on the interest rate constraint, $w_r = 46$. In Mandate 2, w_r is lowered to 5 in column (A) and ??? in column (B). In the mandates, w_f is lowered to 0.0003 in Mandate 1 and column (B), or even to zero in column (B), from 0.06 in Regime 2.

As before, every resulting rule has a high degree of interest rate smoothing and a strong reaction to inflation, with $\rho_R > 0.9$ and $\gamma_\Pi > 1.5$. The feedback on output is near or equal to zero in all cases. The feedback on depreciation in the interest rate rule is practically zero under Mandate 1, but it is higher in Mandate 2 (even though it is still small). Even though the depreciation volatility has the same weight in Mandates 1 and 2, the former is only used for FX intervention and not the interest rate rule. The FX intervention rules also have a high degree of policy smoothing, as $\rho_F > 0.73$, even though not as much as the rule in Regime 2, in which $\rho_F = 0.9902$. The reaction to currency depreciation is stronger in the mandates (around -0.5) than in Regime 2 (-0.04). This is because Regime 2 does not explicitly have a penalty for the depreciation volatility in the welfare function, as the mandates do. Need to check whether this is still valid in column (B) of Mandate 2.

8 Conclusion

FX intervention is widely used by central banks around the world, even by inflation targeters and particularly in response to changes in international financial conditions. Although there is an increasing literature that investigates under which conditions this type of intervention is optimal, there are not many known papers that study welfare maximising simple FX intervention rules. With the ultimate objective of contributing to filling this gap in the literature, this paper performs a welfare analysis of simple monetary policy and FX intervention rules in a canonical small open economy model. The paper also investigates the use of simple mandates in the form of objectives given to the central bank for conducting monetary policy and FX intervention, motivated by the fact that central banks usually use intervention to contain the volatility of the exchange rate.

Even though the analysis is dependent on the estimated model, the results shed light on the welfare benefits provided by the introduction of the second instrument/rule (FX intervention) when compared to the case in which the central bank can only use one instrument/rule (monetary policy). Regardless of the limited range of the tested parameters, the results also show that the adoption of simple mandates in the form of simple loss functions on the volatility of inflation, output, and the exchange rate is potentially associated with welfare costs that are dependent on the mandate's design. When using a simple mandate with lower bound constraints on the nominal interest rate and foreign reserves, however, the constraint on reserves (or the penalty weight in the mandate) matters less (i.e., reserves naturally hit the lower bound with a very low probability). Finally, the optimised rules show that it is optimal to lean against the wind when reacting to the depreciation of the domestic currency, but not to the point of completely stabilising it, as also concluded by Cavallino (2019). In conclusion, this paper highlights that FX intervention can be conducted together with monetary policy in a welfare-optimal way that is not only compatible with but also beneficial for inflation targeting.

The analysis of the optimised rules would be stronger if the results were compared to the Ramsey policy, although it has already been shown by Schmitt-Grohe and Uribe (2007) that simple rules can mimic the Ramsey welfare outcome. For the Ramsey policy, it is necessary to derive the linear-quadratic (LQ) approximation of the households' utility function, which can become challenging in models of imperfect international risk-sharing, such as the one in this paper. An LQ framework, which will be the subject of future work, would also allow for more general conclusions regarding the mandates and the optimal choices of the volatility weights. Finally, the analysis of optimal FX intervention policy in an LQ framework can be applied to compare commitment and discretion to study credibility, instead of the approach taken by Adler et al. (2019).

Future work will embed the sterilised FX intervention framework presented in this paper in the model developed by Mirfatah et al. (2022), in which the rest of the world is fully modelled. Future work can also include a framework for the delegation game in which the independent mandate is a Nash equilibrium in rules, which is not currently the case. It could be also interesting to allow for time-varying bond holding costs (i.e., a time-varying wedge in the UIP condition), which could be a more realistic modelling choice for an emerging economy such as Brazil.

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Appendices

A Foreign exchange intervention data

Table 10: Descriptive statistics of the foreign exchange intervention proxy by exchange rate regime type in the full sample

Regime type	Observations	Mean	Std. dev.	Minimum	Maximum
All	28,001	0.1008	1.0771	-10.33	14.12
Hard peg	2,460	0.1385	1.2464	-6.75	8.85
Soft peg	12,834	0.0917	1.2174	-10.33	14.12
Floating	8,791	0.1249	0.9401	-9.96	12.55
Free-floating	3,916	0.0532	0.6887	-6.51	12.74

"Hard peg" denotes fixed exchange rate regimes, "Soft peg" denotes regimes in which the exchange rate is fixed within limits, "Floating" denotes regimes in which the exchange rate floats freely and FX intervention does not aim a target rate, and "Free-floating" denotes freely floating exchange rates with only occasional interventions.

Table 11: Descriptive statistics of the foreign exchange intervention proxy by exchange rate regime type in emerging markets and developing countries

Regime type	Observations	Mean	Std. dev.	Minimum	Maximum
All	16,830	0.0984	1.1177	-10.33	14.12
Hard peg	1,608	0.1175	1.2775	-6.75	8.85
Soft peg	8,225	0.0906	1.3007	-10.33	14.12
Floating	5,304	0.1125	0.8577	-9.96	7.50
Free-floating	1,693	0.0740	0.5870	-5.48	7.19

"Hard peg" denotes fixed exchange rate regimes, "Soft peg" denotes regimes in which the exchange rate is fixed within limits, "Floating" denotes regimes in which the exchange rate floats freely and FX intervention does not aim a target rate, and "Free-floating" denotes freely floating exchange rates with only occasional interventions.

B Equilibrium conditions

Period utility

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi},$$

Stochastic discount factor

$$\Lambda_{t,t+1} = \beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma},$$

Labour supply

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t},$$

Euler equation – domestic currency bond

$$1 = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t,t+1}} \right\} R_t,$$

Euler equation – foreign currency bond

$$1 = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\Pi^S_{t,t+1}}{\Pi_{t,t+1}} \right\} \frac{R_t^*}{1+\kappa B_{F,t}},$$

Aggregate output

$$Y_t = \frac{A_t N_t}{\Xi_t},$$

Market clearing

$$Y_t = C_{H,t} + C^*_{H,t},$$

Marginal cost

$$MC_t = \frac{(1-\tau)\frac{W_t}{P_t}}{A_t \frac{P_{H,t}}{P_t}},$$

Prices

$$1 = (1-\theta)(\bar{\Pi}_t^H)^{1-\varepsilon} + \theta(\Pi^H_{t-1,t})^{\varepsilon-1},$$

Price dispersion

$$\Xi_t = (1-\theta)(\bar{\Pi}_t^H)^{-\varepsilon} + \theta(\Pi^H_{t-1,t})^{\varepsilon}\Xi_{t-1},$$

Optimal price setting

$$\bar{\Pi}_t^H = \frac{\varepsilon}{\varepsilon-1}\frac{X_{1,t}}{X_{2,t}},$$

Auxiliary price setting recursion 1

$$X_{1,t} = Y_t MC_t MS_t + \theta E_t \left\{ \Lambda_{t,t+1} (\Pi^H_{t,t+1})^{\varepsilon} X_{1,t+1} \right\},$$

Auxiliary price setting recursion 2

$$X_{2,t} = Y_t + \theta E_t \left\{ \Lambda_{t,t+1} (\Pi^H_{t,t+1})^{\varepsilon-1} X_{2,t+1} \right\},$$

Consumption demand – domestic production

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t,$$

Consumption demand – imports

$$C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t,$$

Consumption demand – exports

$$C^*_{H,t} = \mathcal{T}_t^{\eta} Y^*_t,$$

Net exports

$$NX_t = C^*_{H,t} - C_{F,t},$$

CPI inflation

$$\Pi_t = \left((1-\alpha) \left(\Pi^H_{t-1,t} \frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\eta} + \alpha \left(\Pi^F_{t-1,t} \frac{P_{F,t-1}}{P_{t-1}} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}},$$

Law of one price

$$\Pi^F_{t-1,t} = \Pi^*_{t-1,t} \Pi^S_{t-1,t},$$

Balance of payments

$$B_{F,t} + F_t = \frac{\Pi^S_{t-1,t}}{\Pi_{t-1,t}} R^*_{t-1} (F_{t-1} + B_{F,t-1}) + NX_t - \frac{\kappa}{2} B^2_{F,t},$$

Terms of trade

$$\log\left(\mathcal{T}_t tot_t\right) = \log\left(\mathcal{T}_{t-1} tot_{t-1}\right) + \log \Pi^F_t - \log \Pi^H_t$$

Price ratio – home produced goods

$$\frac{P_{H,t}}{P_t} = \left[1 - \alpha + \alpha \mathcal{T}_t^{1-\eta}\right]^{\frac{1}{\eta-1}},$$

Price ratio – foreign produced goods

$$\frac{P_{F,t}}{P_t} = \left[\alpha + (1-\alpha) \mathcal{T}_t^{\eta-1}\right]^{\frac{1}{\eta-1}},$$

Interest rate rule

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left(\phi_\Pi \log\left(\frac{\Pi_{t-1,t}}{\Pi}\right) + \phi_Y \log\left(\frac{Y_t}{Y}\right) \right.$$
$$\left. + \phi_{dY} \log\left(\frac{Y_t}{Y_{t-1}}\right) + \phi_{\Pi^S} \log\left(\frac{\Pi^S_{t-1,t}}{\Pi^S}\right)\right) + \epsilon_{M,t},$$

FX intervention rule

- No intervention:

$$\log(F_t) = \log(F)$$

- With intervention:

$$\log\left(\frac{F_t}{F}\right) = \rho_F \log\left(\frac{F_{t-1}}{F}\right) + (1-\rho_F)\left(\phi_{\Pi^S_f} \log\left(\frac{\Pi^S_{t-1,t}}{\Pi^S}\right)\right) + \epsilon_{F,t},$$

Technology shock

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t},$$

Mark-up shock

$$\log MS_t = \rho_{MS} \log MS_{t-1} + \epsilon_{MS,t},$$

Terms of trade shock

$$\log tot_t = \rho_{tot} \log tot_{t-1} + \epsilon_{tot,t}.$$

C Recursive steady state

$$\Lambda = \beta,$$

$$A = 1,$$

$$MS = 1,$$

$$\Pi^* = 1.0079,$$

$$\Pi = \Pi^*,$$

$$\Pi^F = \Pi,$$

$$\Pi^S = \frac{\Pi^F}{\Pi^*},$$

$$\Pi^H = \Pi,$$

$$\bar{\Pi}^H = \left(\frac{1-\theta(\Pi^H)^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}},$$

$$\frac{P_H}{P} = 1,$$

$$\frac{P_F}{P} = 1,$$

$$\mathcal{T} = 1,$$

$$\Xi = \frac{(1-\theta)\left(\bar{\Pi}^H\right)^{-\varepsilon}}{1-\theta(\Pi^H)^{\varepsilon}},$$

$$\frac{X_1}{X_2} = \frac{\varepsilon-1}{\varepsilon}\bar{\Pi}^H,$$

$$MC = \frac{X_1}{X_2}\frac{\left(1-\theta\Lambda(\Pi^H)^{\varepsilon}\right)}{\left(1-\theta\Lambda(\Pi^H)^{\varepsilon-1}\right)},$$

$$R = \frac{\Pi}{\beta},$$

$$R^* = \frac{\Pi}{\beta\Pi^S},$$

$$B_F = 0,$$

$$\frac{F}{Y} = \delta,$$

$$\frac{NX}{Y} = \left(1-\frac{\Pi^S}{\Pi}R^*\right)\frac{F}{Y},$$

$$\frac{C}{Y} = 1-\frac{NX}{Y},$$

$$\frac{W}{P} = \frac{MC\frac{P_H}{P}A}{(1-\tau)},$$

$$N = \left(\frac{W}{P} \Xi^{\sigma} \left(\frac{C}{Y} \right)^{-\sigma} \right)^{\frac{1}{\varphi+\sigma}},$$

$$Y^W = AN,$$

$$Y = \frac{Y^W}{\Xi},$$

$$F = \frac{F}{Y} Y,$$

$$NX = \frac{NX}{Y} Y,$$

$$C = \frac{C}{Y} Y,$$

$$C_H = (1 - \alpha) C,$$

$$C_F = \alpha C,$$

$$C_H^* = NX + C_F,$$

$$Y^* = C_H^*,$$

$$X_1 = \frac{YMCMS}{1 - \theta \Lambda (\Pi^H)^{\varepsilon}},$$

$$X_2 = \frac{Y}{1 - \theta \Lambda (\Pi^H)^{\varepsilon - 1}},$$

$$U = \frac{C^{1-\sigma}}{1 - \sigma} - \frac{N^{1+\varphi}}{1 + \varphi}.$$

D Data

D.1 Foreign economy data

The foreign economy VAR is estimated with quarterly US data for output, inflation, and interest rate for the period between 1954Q3 and 2019Q4. The time series obtained from the Federal Reserve Bank of St. Louis are:

- GDP^{US}: Real Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate, Series GDPC1;
- POP^{US}: Civilian Labor Force Level, Thousands of Persons, Monthly, Seasonally Adjusted, Series CLF16OV. It was transformed to quarterly and an index where $1992 = 1$;
- DEF^{US}: Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted, Series GDPDEF;
- R^{US}: Federal Funds Effective Rate, Percent, Monthly, Not Seasonally Adjusted, Series FEDFUNDS.

The variable foreign output (Y_t^*) is given by the one-sided-HP-filtered series of

$$\ln(GDP_t^{US}/POP_t^{US}) \times 100,$$

the variable foreign inflation (Π_t^*) is given by

$$\ln(DEF_t^{US}/DEF_{t-1}^{US}) \times 100,$$

and the variable foreign interest rate (R_t^*) is given by $R_t^{US}/4$.

D.2 Small open economy data

Data for output, interest rate, inflation, depreciation, and FX intervention of Brazil, for the period 1996Q1 to 2019Q4 comes from the following data:

- GDP^{BR}: Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for Brazil, Chained 2000 National Currency Units, Quarterly, Seasonally Adjusted. Series NAEXKP01BRQ652S from the Federal Reserve Bank of St. Louis;

- R^{BR}: Interest rate - Selic accumulated in the month in annual terms (basis 252), % p.y., Monthly. Series 4189 from the Banco Central do Brasil. It was transformed to quarterly using the period average;

- ER^{BR}: National Currency to US Dollar Exchange Rate: Average of Daily Rates for Brazil, National Currency Units per US Dollar, Quarterly, Not Seasonally Adjusted. Series
CCUSMA02BRM618N from the Federal Reserve Bank of St. Louis;

- CPI^{BR}: Consumer Price Index: All Items for Brazil, Index 2015 = 100, Quarterly, Not Seasonally Adjusted. Series BRACPIALLMINMEI from the Federal Reserve Bank of St. Louis. It was seasonally adjusted using X13-ARIMA;

- F^{BR}: Total Reserves excluding Gold for Brazil, Dollars, Not Seasonally Adjusted, Quarterly, Average. Series TRESEGBRM052N from the Federal Reserve Bank of St. Louis.

E Identification

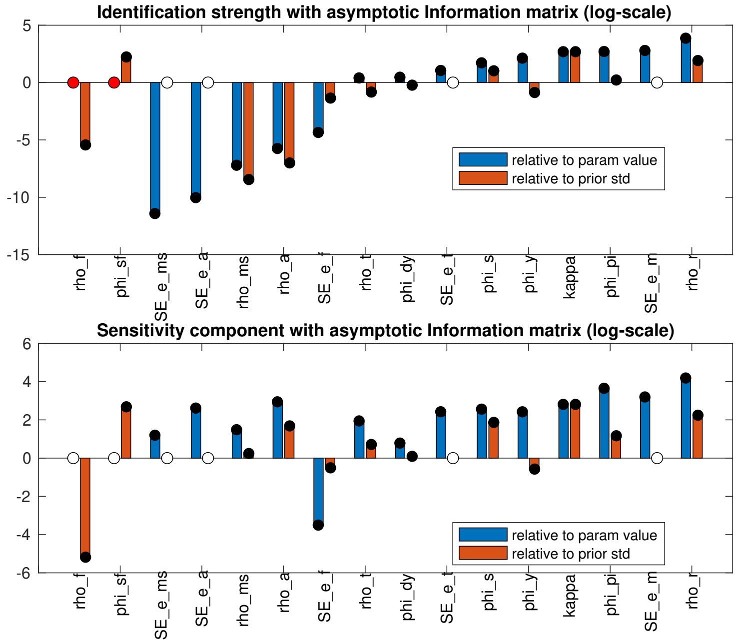

Figure 8: Identification strength based on the Fischer information matrix

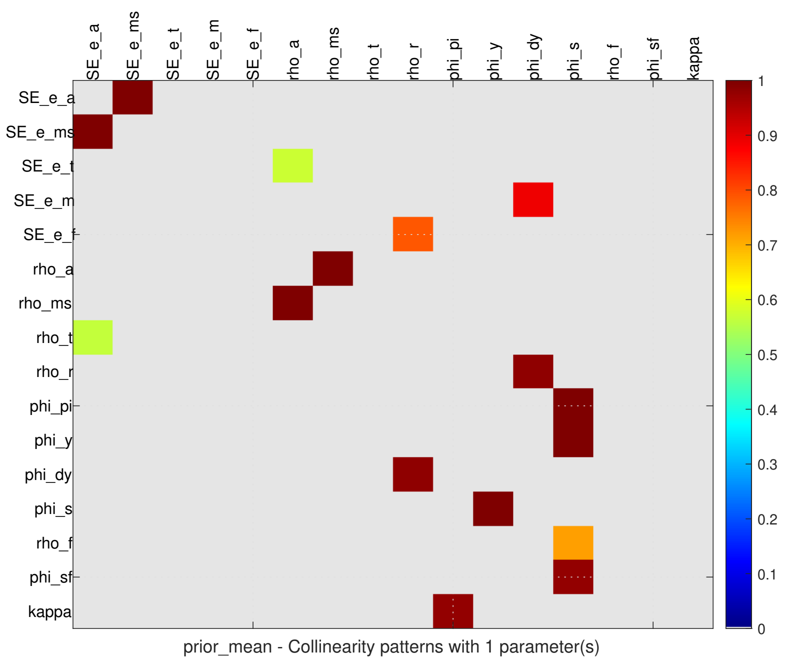

Figure 9: Collinearity pattern with 1 parameter

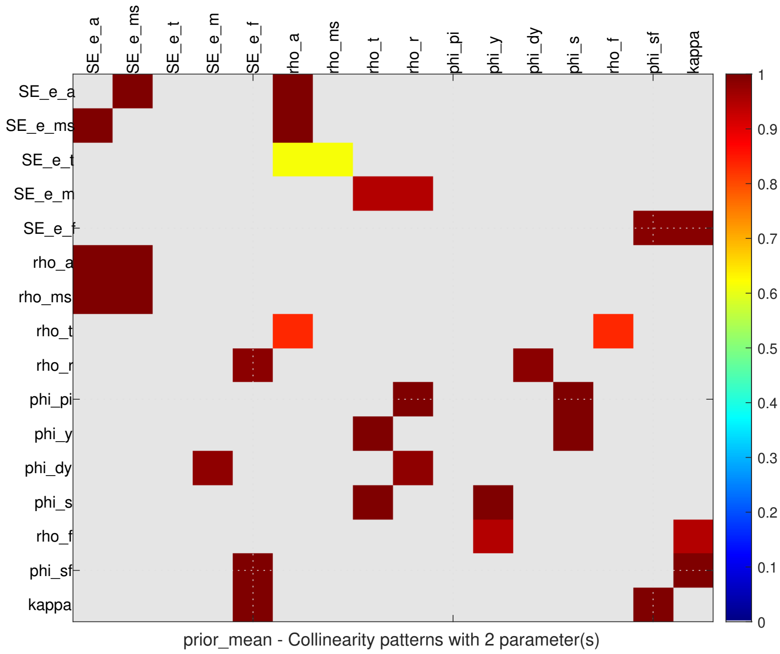

Figure 10: Collinearity pattern with 2 parameters

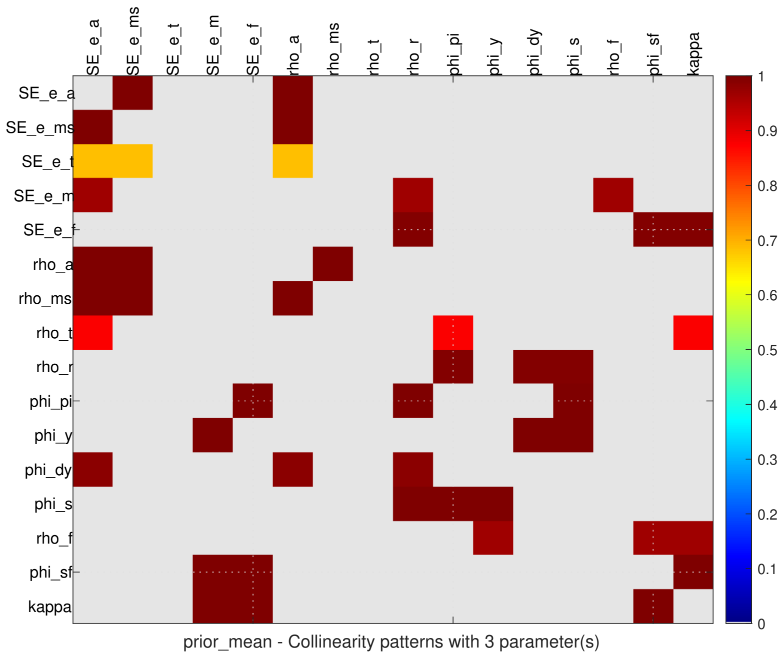

Figure 11: Collinearity pattern with 3 parameters

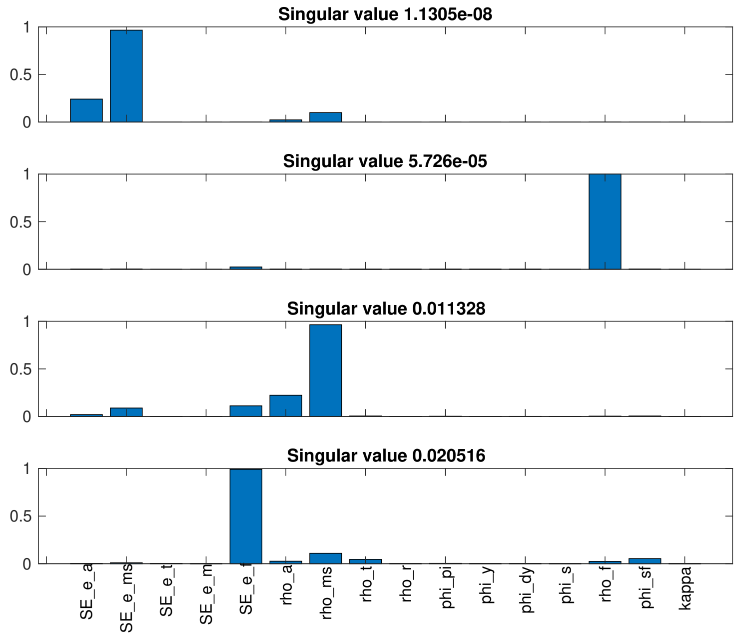

Figure 12: Smallest singular values in the information matrix

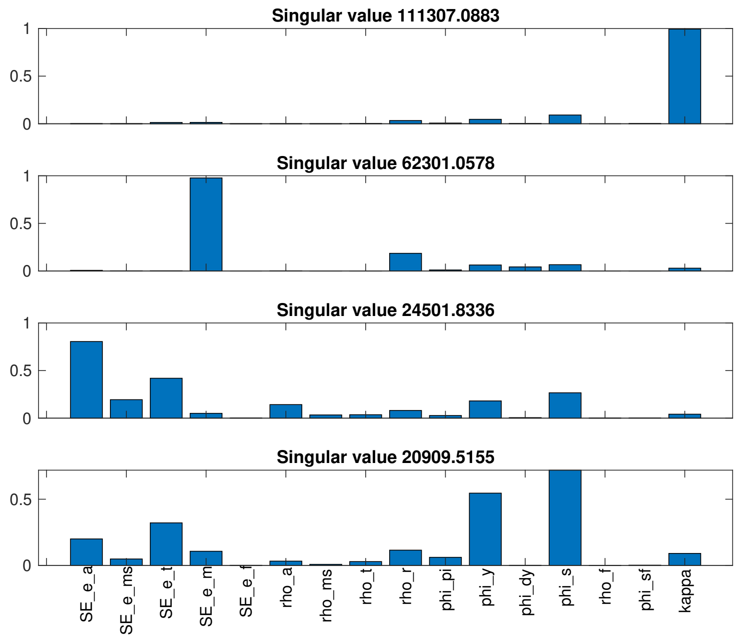

Figure 13: Highest singular values in the information matrix

F Post-estimation diagnostics

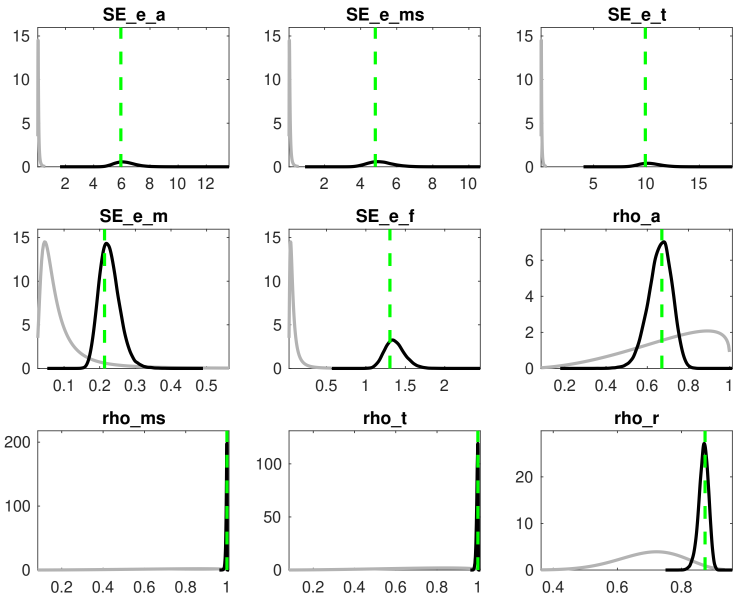

Figure 14: Priors and posteriors - part 1

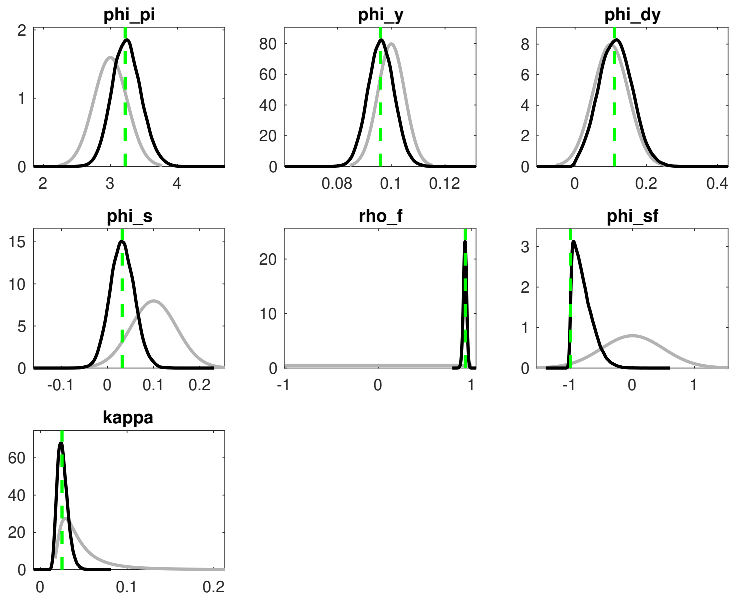

Figure 15: Priors and posteriors - part 2

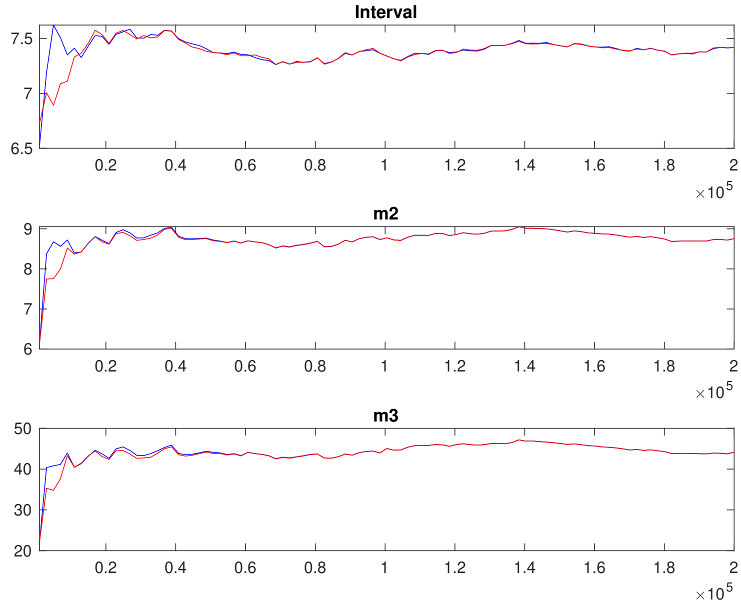

Figure 16: Multivariate convergence diagnostic

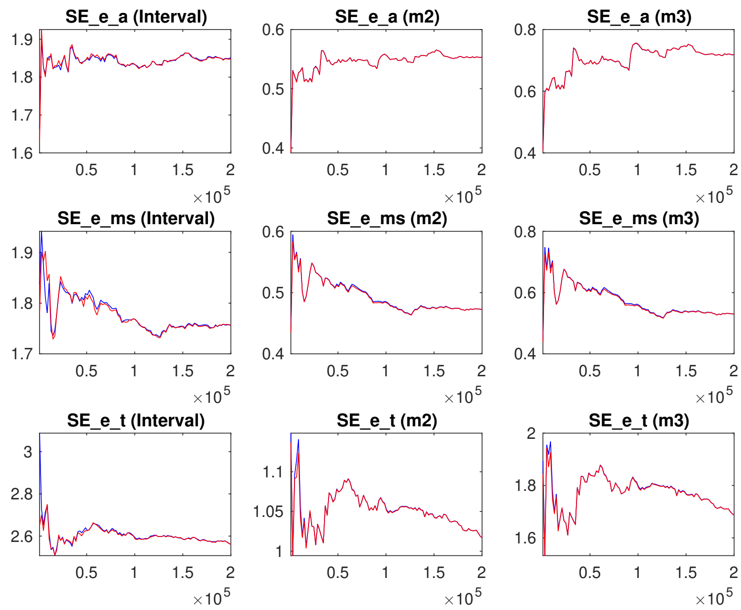

Figure 17: Monte Carlo Markov Chain univariate diagnostics - part 1

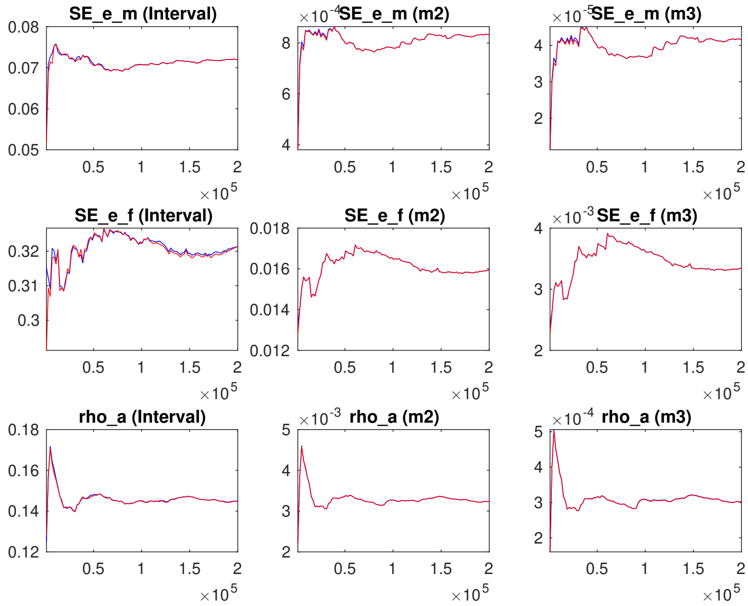

Figure 18: Monte Carlo Markov Chain univariate diagnostics - part 2

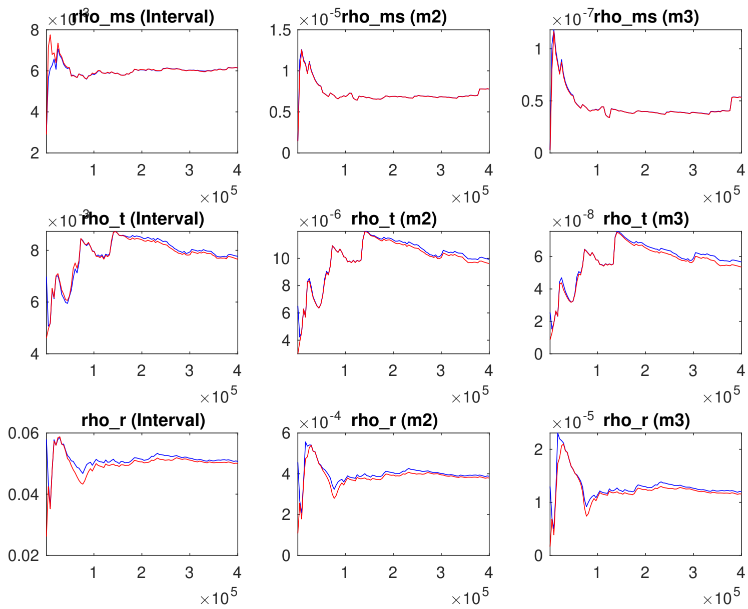

Figure 19: Monte Carlo Markov Chain univariate diagnostics - part 3

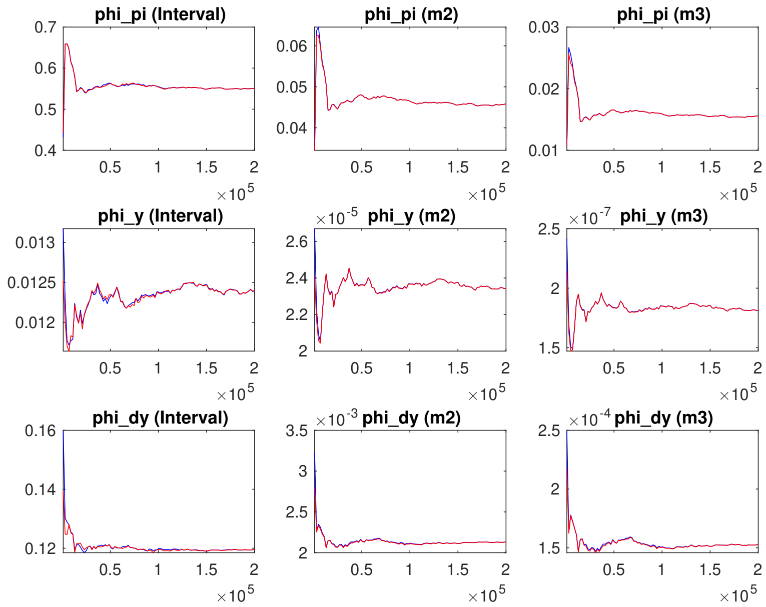

Figure 20: Monte Carlo Markov Chain univariate diagnostics - part 4

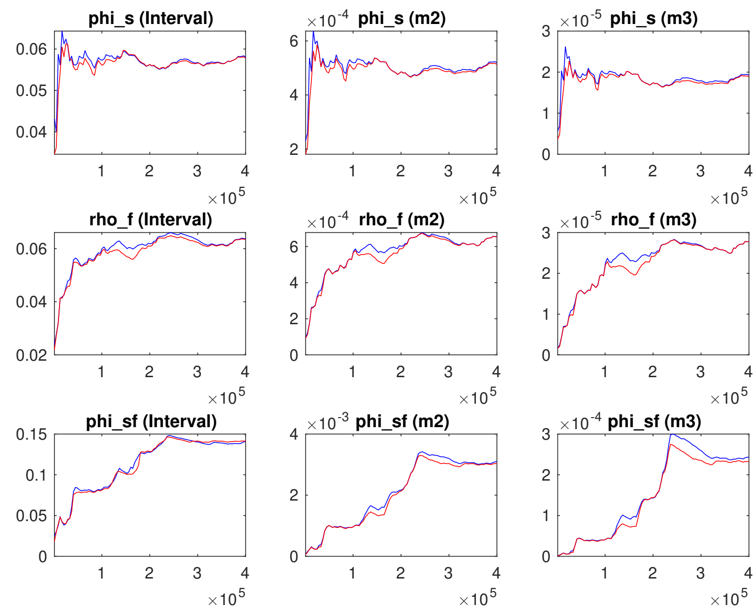

Figure 21: Monte Carlo Markov Chain univariate diagnostics - part 5

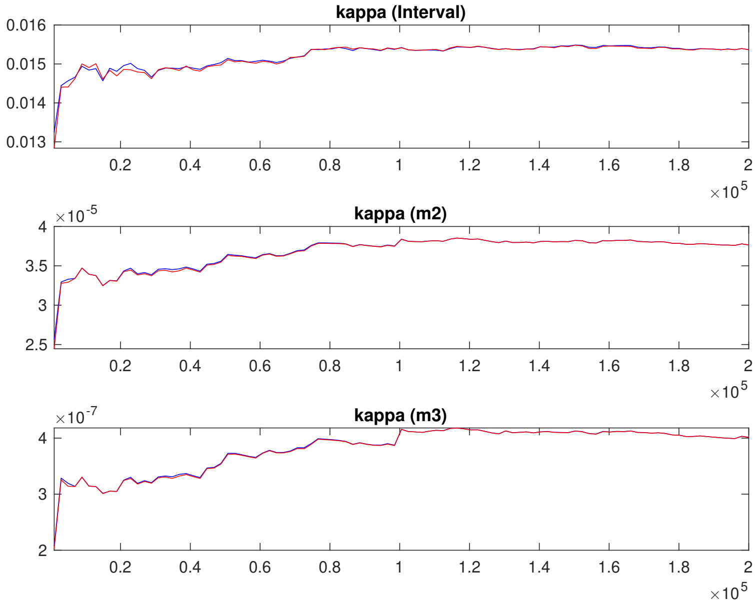

Figure 22: Monte Carlo Markov Chain univariate diagnostics - part 6

G Other estimation results

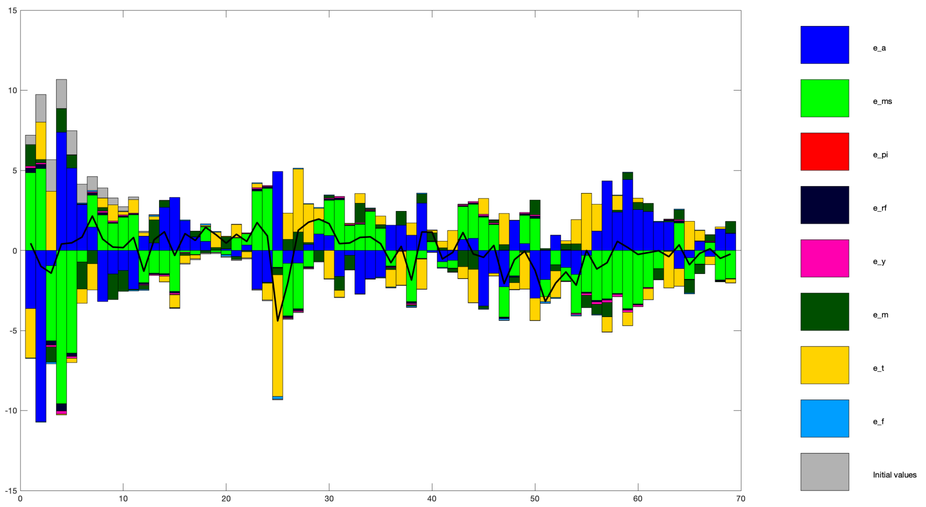

Figure 23: Historical shock decomposition of output

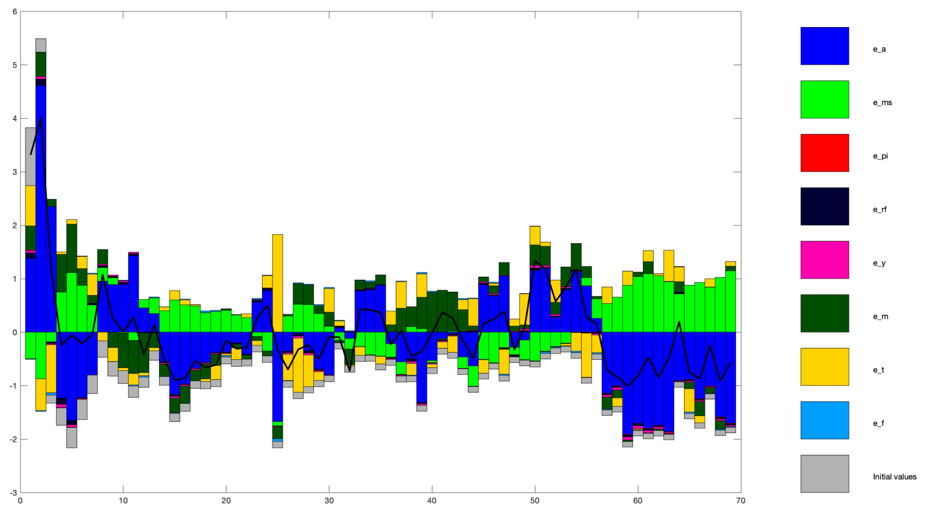

Figure 24: Historical shock decomposition of inflation

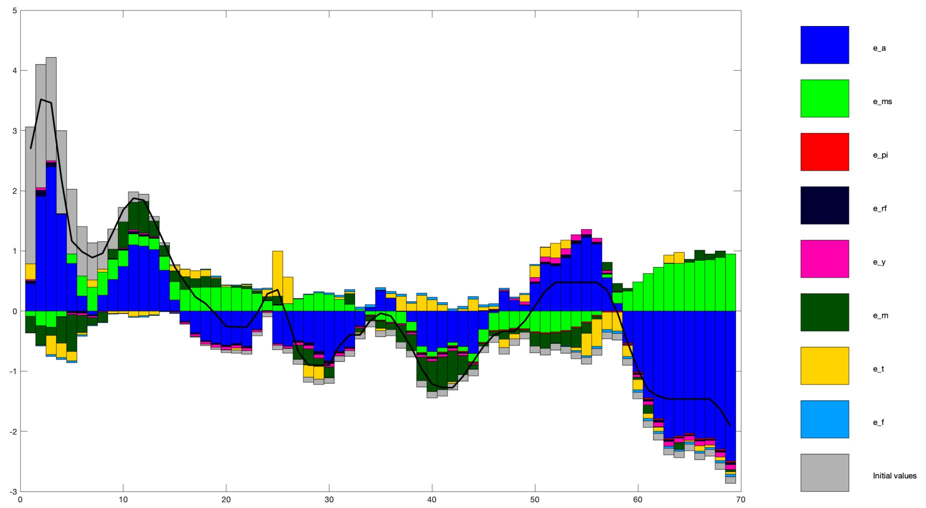

Figure 25: Historical shock decomposition of the interest rate

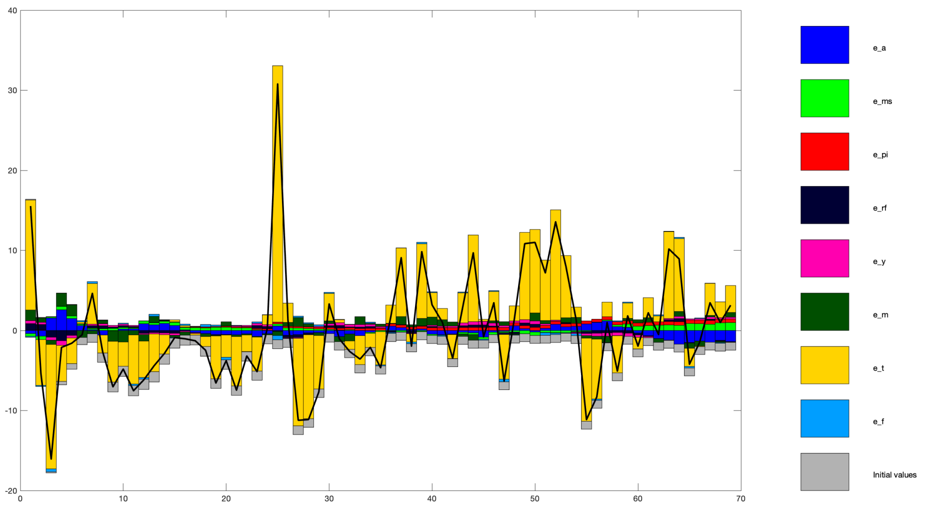

Figure 26: Historical shock decomposition of depreciation

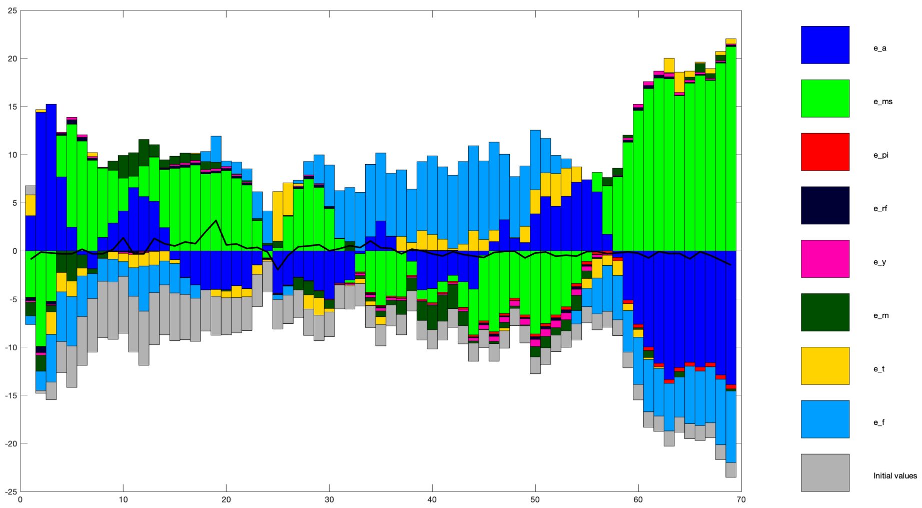

Figure 27: Historical shock decomposition of reserves

H Results with endogenous priors

Table 12: Second moments

Variable	Data	Estimation 1	Estimation 2	Estimation 3
Standard deviation				
Output	1.1926	4.2477	2.0016	3.4144
Inflation	0.8642	1.8173	0.9575	1.2933
Interest rate	1.8219	1.8201	0.8413	1.2376
Depreciation	8.0005	7.9227	6.5355	7.2049
Reserves	0.0973	0.0857	0.0809	1.4596
Crosscorrelation with output				
Output	1.0000	1.0000	1.0000	1.0000
Inflation	-0.0379	-0.5044	-0.5671	-0.5304
Interest rate	-0.0868	-0.1049	-0.2087	-0.0885
Depreciation	-0.2835	-0.2968	-0.3578	-0.4453
Reserves	0.0073	-0.2677	-0.2636	0.0914
Autocorrelation (order=1)				
Output	0.2485	0.0143	-0.0933	0.0096
Inflation	0.5713	0.6610	0.3761	0.6366
Interest rate	0.8121	0.9369	0.8508	0.9542
Depreciation	0.2450	0.0722	0.0604	0.0708
Reserves	0.2332	-0.0026	-0.0288	-0.0843

Estimation 1: reserves data and standard priors.

Estimation 2: reserves data and endogenous priors.

Estimation 3: FX intervention data and standard priors.

I Additional impulse response functions

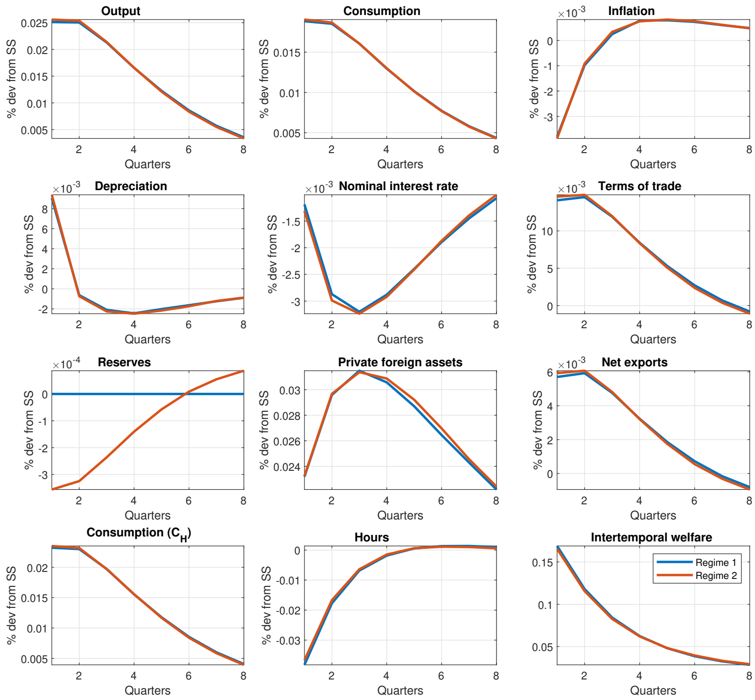

Figure 28: Impulse response to a domestic technology shock

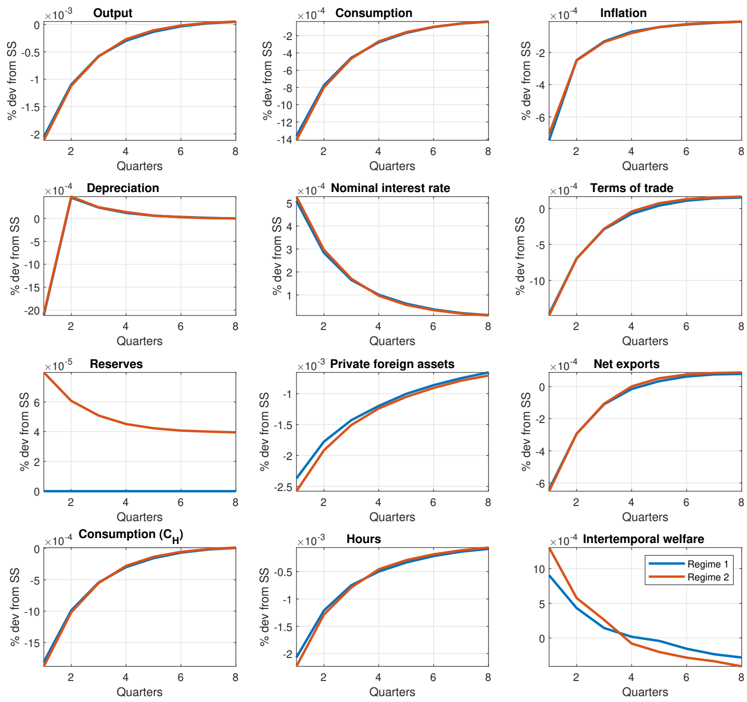

Figure 29: Impulse response to a monetary policy shock

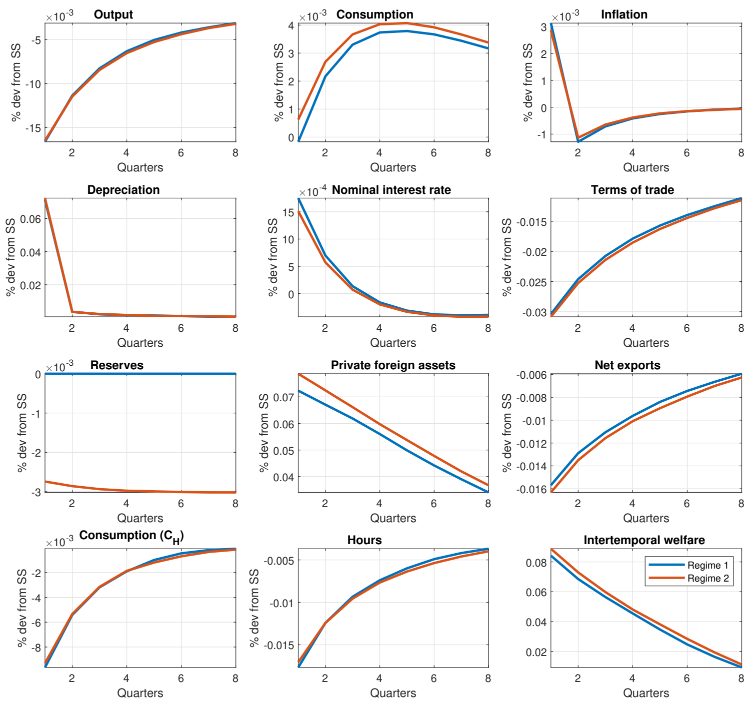

Figure 30: Impulse response to a terms of trade shock

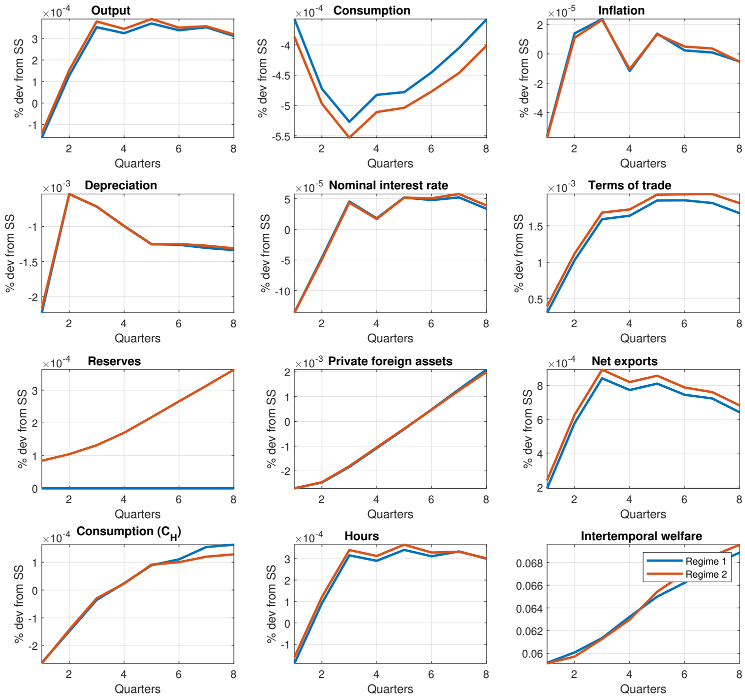

Figure 31: Impulse response to a foreign inflation shock