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Deviating from full rationality but not from theoretical consistency: The behavior of inflation expectations in Brazil

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ABSTRACT

The aim of this paper is to investigate whether inflation expectations in Brazil have characteristics and statistical properties that can be correlated (possibly in a causal way) with observed variables of interest and expectations about them. Our analysis covers the period from December 2001 to August 2018. We test the hypothesis of rational expectations in the formation of inflation expectations by the respondents of the official survey conducted by the Central Bank of Brazil, examining the behavior of their forecast errors. As these errors are biased and can be predicted, we reject the hypothesis of full rationality. We also test models of noisy and sticky information, and we cannot conclude that the deviations from full rationality can be explained by information rigidity. Additionally, with a vector error correction model, we find evidence that the expectations about the related macroeconomic variables respond to each other as predicted by a theoretically-grounded macroeconomic model. Therefore, inflation expectations in Brazil are to an important extent consistent with more general expectations about the future performance of the economy.

1. Introduction

Since May 1999, during the transition to an inflation targeting regime, the Central Bank of Brazil (BCB) conducts the Focus Survey, collecting market expectations for the most relevant macroeconomic variables, which are used as an input for monetary policy decisions by the BCB.[1] In fact, the BCB attaches great importance to the Survey results in the minutes of the meetings of the Monetary Policy Committee (Copom) and in inflation reports, which are an important part of the communication and transparency strategy adopted by the Brazilian monetary authority. The expectations of private agents play a crucial role in the behavior of the economy, and they are important variables taken into account in economic policy decisions. Therefore, it is important (on both theoretical and empirical grounds) to understand the characteristics of such expectations, and how they are formed and revised.

Our aim in this paper is to investigate whether inflation expectations have characteristics that can be analytically associated with other observed variables and expectations about them. The assumption of full rationality, especially with respect to inflation expectations, has been assessed and challenged in the literature for quite a long time. Consequently, several models have been developed as an attempt to explain deviations from full rationality. Mankiw, Reis, and Wolfers (2004) show that a sticky information model is good for explaining U.S. survey data on inflation expectations by both professionals and households, although it is not enough to fully explain deviations from full rationality. Coibion and Gorodnichenko (2015) conclude that a model of information rigidity is adequate to explain the formation of inflation expectations by professional forecasters in the U.S. and other 11 industrialized countries. Meanwhile, Berge (2018), following the methodology used by Coibion and Gorodnichenko (2015), concludes that sticky and noisy information models do not explain all the deviations from full rationality in surveys conducted with U.S. professional forecasters and households.

Following Mankiw et al. (2004), Guillén (2008) finds that the inflation expectations included in the Focus Survey are neither

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[1] The Focus Survey compiles online-submitted forecasts of about 140 banks, asset managers and other institutions (real sector companies, brokers, consultancy firms, etc.). The provision of forecasts is not mandatory, but these institutions can submit forecasts as often as daily if they find it justified. The Survey daily monitors the market expectations for several inflation indices, the GDP and industrial production growth, the nominal exchange rate, the base interest rate, fiscal indicators and external sector variables. For more information about the Survey, see https://www.bcb.gov.br/en/monetarypolicy/marketexpectations.

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rational, even in the weak form of rationality, nor adaptive, for all forecast horizons between 2000 and 2007.[2] Guillén's (2008) results suggest that there is a cost associated with the collecting and processing of new information, which can be an indication that sticky information can provide a reasonable explanation. Meanwhile, Araujo and Gaglianone (2010), using several forecast horizons between 2002 and 2008, and Kohlscheen (2012), using one month ahead expectations between 2002 and 2010, do not reject a weak form of rationality in the formation of the inflation expectations included in the Focus Survey, as both studies were not able to reject the hypothesis of non-existence of a forecast bias. Both studies, however, reject the strong form of rationality, since forecast errors could be predicted. By analyzing the response of individual professional forecasters to new information between 2006 and 2013, Correa and Picchetti (2016) find support for sticky information and staggered updating for the formation of inflation expectations referring to the current month in Brazil.

We test the hypothesis of rational expectations about inflation by exploring the behavior of the forecast errors of the median of inflation expectations in the Focus Survey. As these errors are biased and can be predicted, we reject the hypothesis of full rationality. Furthermore, following Coibion and Gorodnichenko (2015) and Berge (2018), we test whether the process of inflation expectations formation of the respondents of the Focus Survey can be described by noisy and sticky information models. Although forecast revisions help to explain forecast errors, as expected by theory, the respective coefficient has an unexpected sign in the empirical analysis. Additionally, we are also able to use macroeconomic variables to predict forecast errors, even when we control for the revisions, which cannot occur in noisy and sticky information models. As a consequence, we cannot suggest that the deviations from rational expectations about inflation in the Focus Survey can be explained by information rigidity.

As the Focus Survey is not restricted to inflation expectations, we also investigate the consistency of a set of expectations with the established macroeconomic theory. In order to do so, we verify whether the median of expectations about inflation, the nominal exchange rate, the nominal interest rate, the level of economic activity, and the primary surplus behave as expected by the relationship between the respective observed variables. The theoretical consistency of expectations has been assessed mostly for individual series, and much less for a representative set of related expectations, as we carry out in this paper. Using a vector error correction (VEC) model, we look for evidence to confirm stylized facts emphasized in the established macroeconomic theory for the selected expectations. The confirmation of these stylized facts means that survey respondents inform expectations that are consistent with the operation of a macroeconomic model which is mostly theoretically grounded, although their inflation expectations cannot be described as fully rational.

Therefore, our paper is also related to the literature on anchoring of inflation expectations (see, e.g., Łyziak & Paloviita, 2017; Buono & Formai, 2018; Mehrotra & Yetman, 2018). In fact, for the most part, the issue of anchoring of inflation expectations has been investigated using deviations of short-run from long-run inflation forecasts, break-even inflation rates on financial assets with longer maturities and comparisons of survey data on inflation expectations with the behavior of current inflation. The focus of this paper is not on anchoring of inflation expectations *per se*, but on the extent of inflation forecast errors by survey respondents. Yet, our supplementary analysis of whether there is theoretical consistency among the median of expectations of a set of related macroeconomic variables sheds light on the issue of anchoring of more general expectations about the future performance of the economy.

As our sharp focus in this paper is on the aggregate behavior of expectations in Brazil, we inevitably and unavoidably eschew any inference about the behavior of individual respondents of the Focus Survey.[3] In this sense, our approach is closer to Guillén (2008), Araujo and Gaglianone (2010), and Kohlscheen (2012), which investigate the rationality of aggregate inflation expectations, than to Correa and Picchetti (2016), which use data from a panel of forecasters. We nonetheless extend the analysis carried out in these papers, by applying a different methodology for the rationality of inflation expectations, assessing the theoretical consistency of a set of related expectations. Given our main interest in the aggregate behavior of expectations, we use the median of the expectations informed by the Focus Survey respondents, which is the statistic used by the BCB in its communication strategy. While this kind of investigation for the U.S., for instance, typically covers several decades, as the Survey of Professional Forecasters began in 1968, our analysis covers only a more recent period, as determined by data availability. We use 12 months ahead expectations that were reported from December 2001, which is the first available observation to all series of interest, to August 2018.

The paper is organized as follows. After this introduction, Section 2 presents the theoretical framework regarding the expectations formation process and the relationship between the variables of interest. Data and the empirical setting for the analysis are presented in Section 3. The results are carefully discussed in Section 4, while Section 5 concludes.

2. Theoretical framework

In a setting of fully rational expectations, the difference between the observed value of a variable at a given moment in time, represented by x_{t+h}, and its forecast, given by $F_t x_{t+h}$, should be explained only by a random error term, such that:

$$x_{t+h} - F_t x_{t+h} = \epsilon_t. \tag{1}$$

Therefore, forecast errors cannot be either biased or predicted. Alternative models use information rigidity or noisy information to explain deviations from full rationality, such as the models in Eqs. (2) and (3), respectively, as represented in Coibion and Gorodnichenko (2015):

$$x_{t+h} - F_t x_{t+h} = \frac{\lambda}{1-\lambda}(F_t x_{t+h} - F_{t-1} x_{t+h}) + \nu_{t+h,t}, \tag{2}$$

$$x_{t+h} - F_t x_{t+h} = \frac{1-G}{G}(F_t x_{t+h} - F_{t-1} x_{t+h}) + \nu_{t+h,t}, \tag{3}$$

where λ is the probability that an agent does not update her or his information set at a given moment in time, G is the relative weight given to previous forecasts, and ν is the full-information rational expectations error. In a model of sticky information, as the model proposed by Mankiw and Reis (2002) represented by Eq. (2), there is a cost associated with information processing. As a result, agents do not update their information set every period, although their expectations are fully rational when updated. Meanwhile, in models of noisy information, like the model in Woodford (2003), represented in Eq. (3), agents not only update their information set periodically, but also know the true data generating process of the economy. Nonetheless, they do not perfectly observe the state of

[2] For the weak-form rationality, it suffices to have unbiased forecast errors – i.e., forecast errors should have a zero mean. For the strong-form rationality, on the other hand, in addition to being unbiased, forecast errors cannot be predicted by relevant data available in the information set of the forecaster (Grant & Thomas, 1999).

[3] Using the individual forecasts of the same database, Areosa (2016) found evidence of relevant strategic behavior.

the economy, and rely on the signal received by them, which affects the parameter G.

The relationship between the variables of interest can be drawn from established macroeconomic theory, as in the open-economy New Keynesian framework presented in Eqs. (4)–(9). As specified in the Phillips curve in Eq. (4), the current inflation, π_t, varies positively with the expected inflation, $E_t\pi_{t+1}$, the nominal exchange rate, e_t, as the price of one unit of the foreign currency in units of the domestic currency, and the output gap, y_t (or negatively with the unemployment rate). As specified in the IS curve in Eq. (5), an increase in the nominal interest rate, i_t, has a negative effect on the output gap, as it decreases investment and consumption, while an increase in actual (or expected) inflation, by lowering the real interest rate, and in government expenditures, g_t, have a positive effect on the output gap. According to a standard Taylor rule as in Eq. (6), the nominal interest rate should vary positively with the inflation rate and the output gap. In order to the uncovered interest parity (UIP) in Eq. (7) to hold, an increase in the domestic nominal interest rate, *ceteris paribus*, should be accompanied by an expected increase in the nominal exchange rate. The risk premium, ϱ_t, can be associated to a change in the UIP which is not explained by a change in the domestic interest rate; in the foreign interest rate, i_t^*; or in the exchange rate. According to equation (8), the terms of trade, s_t, defined as the price of foreign goods in terms of domestic goods, have a negative relationship with the nominal interest rate and a positive relationship with the expected inflation. The relationship of the terms of trade with net exports, x_t, given by Eq. (9), and, ultimately, with aggregate output, will depend on the parameters of the model (Galí, 2008). As a real depreciation in the value of the local currency typically causes (even if with some lag) an increase in net exports, aggregate output usually varies positively with the real exchange rate:

$$\pi_t = \beta E_t\pi_{t+1} + \kappa y_t + \gamma\Delta e_t + u_t, \tag{4}$$

$$y_t = -\frac{1}{\sigma}(i_t - E_t\pi_{t+1} - r_t^n) + E_t y_{t+1} + g_t, \tag{5}$$

$$i_t = \rho + \phi_\pi\pi_t + \phi_y y_t + v_t, \tag{6}$$

$$i_t = i_t^* + E_t\Delta e_{t+1} + \varrho_t, \tag{7}$$

$$s_t = (i_t^* - E_t\pi_{t+1}^*) - (i_t - E_t\pi_{t+1}) + E_t s_{t+1}, \tag{8}$$

$$nx_t = \alpha\left(\frac{\omega}{\sigma} - 1\right)s_t. \tag{9}$$

In order to account for several idiosyncrasies of the Brazilian economy, Castro, Gouvea, Minella, Santos, and Souza-Sobrinho (2015) develop and estimate the Stochastic Analytical Model with a Bayesian Approach (SAMBA), a dynamic stochastic general equilibrium model that yields theoretically consistent results and has been used by the BCB for carrying out monetary policy. According to the results of the SAMBA model, an increase in the nominal interest rate leads to a decrease in the real GDP and in inflation, to a real appreciation of the domestic currency, and to a procyclical behavior of the fiscal policy, or, in other words, an increase in the primary surplus. Meanwhile, a real depreciation of the local currency leads to an increase in the inflation rate, which is followed by an increase in the nominal interest rate by the monetary authority, which eventually causes a decrease in the real GDP and in the government expenditures. Lastly, a fiscal policy shock, represented by an increase in the government expenditures, leads to an increase in the level of economic activity, increasing the inflation rate as a consequence, with the monetary authority then raising the nominal interest rate in the sequence. A fiscal policy shock eventually leads to a real depreciation of the domestic currency.

Therefore, these stylized facts and model predictions should be confirmed by the estimated behavior of the aggregate expectations about the respective variables, if these expectations are to be seen as following a theoretically consistent pattern.

3. Data and empirical setting

We analyze the median of the 12 months ahead expectations for inflation, the nominal exchange rate, the nominal interest rate, the level of economic activity, and the primary surplus, calculated by the Market Expectations System, where professional forecasters inform their expectations to the BCB. Hence, when we refer to the expectations series, we mean the series of the median of the expectations informed by professional forecasters in the Market Expectations System. Although these series are daily, given that these professionals can post their forecasts in the system every day, the variables that are not market prices (inflation, economic activity, and primary surplus) are measured at least monthly. Therefore, it is necessary to choose a specific date in the month to be used. A possible choice would be the date of the public release of the Top 5 more accurate predictors.[4] However, the Top 5 ranking only takes into account predictions for inflation, the nominal exchange rate, and the nominal interest rate, with the further constraint that the respective reference dates varied across the expectations of interest (and sometimes also varied over time). In order to properly standardize the dates of the collected data, we reasonably use data for the first business day of the month for all variables. The first observation that is common to all series of interest is December 2001, which is a forecast for December 2002.

In the Market Expectations System, there are three alternatives for expectations about economic activity: monthly industrial production (12 months ahead), quarterly GDP growth rate (four quarters ahead), and annual GDP growth rate (one year ahead). We choose the annual GDP growth rate for this paper, since it is the longest series. Forecasts for the annual GDP and the primary surplus are made for the whole year, while we are working with 12 months ahead expectations. Thus, it is necessary to define some criterion for the selection of the date of reference. Inspired in Minella, De Freitas, Goldfajn, and Muinhos (2003), we use a weighted average.[5] Additionally, the series for expectations about the nominal interest rate covers the whole period, but there are no calculated expectations at some moments in time, which were approximated by linear interpolation.

The descriptive statistics of the expectations series are presented in Table 1. *INFLATION* is the median of expectations for the monthly percentage variation of the Extended National Consumer Price Index (IPCA); *INTEREST* is the median of expectations for the target Selic rate, which is the monetary policy instrument; *EXCHANGE* is the median of expectations for the nominal exchange rate as the price of one unit of U.S. dollar in units of the local currency, such that an increase in the expectations means an expected depreciation of the local currency; *GDP* is the median of expectations for the annual GDP growth rate; and *SURPLUS* is the median of expectations for the primary surplus as a proportion of GDP. The descriptive statistics show the high variability in data, which is a characteristic of emerging economies. This is also a consequence of the instability in the Brazilian economy, which has gone through quite different moments within the analyzed period. For instance, we can notice the effect on expectations of the fear gen-

[4] The Focus Survey Top 5 is a monthly and annual ranking of institutions, based on the accuracy of their forecasts, in order to encourage the improvement of predictions.

[5] The series are given by $S_t^* x_{j,t} = \frac{(12-t+1)}{12} S_t x_{j,t} + \frac{(t-1)}{12} S_t x_{j+1,t}$, such that $S_t x_{j,t}$ is the expectation of the variable x, which can be the GDP or the primary surplus, for the year j in the month t. We have modified the original weighted average in Minella et al. (2003), by adding one to $(12-t)$, and subtracting one to (t), since we use data for the first business day of the month.

Table 1
Descriptive statistics of the expectations series, December 2001–August 2018.[a]

	INFLATION	INTEREST	EXCHANGE	GDP	SURPLUS
Mean	0.406	12.099	2.589	2.622	2.300
Median	0.400	12.000	2.460	3.154	2.800
Std. dev.	0.069	3.013	0.703	1.781	2.119
Min.	0.267	6.500	1.630	−2.950	−2.292
Max.	0.661	20.000	4.300	5.845	4.350
ARIMA(1,1,0)[b]	0.139**	0.442***	0.640***	0.643***	0.494***
	(0.070)	(0.063)	(0.055)	(0.054)	(0.061)

[a] Values for the rates of inflation, nominal interest rate and GDP growth are in percentage, for the nominal exchange rate are the price of one unit of U.S. dollar in units of the local currency, and for the primary surplus are as a proportion of GDP.
[b] Value of the autoregressive coefficient of an ARIMA(1,1,0) estimated for each variable. Standard errors are in parentheses. Significance levels: * 10%, ** 5%, *** 1%.

Table 2
Expected relationships between pairs of variables.

	INFLATION	INTEREST	EXCHANGE	GDP	SURPLUS
RISK	+	+	+	?	?
INFLATION		+	?	?	?
INTEREST	–		+	–	+
EXCHANGE	+	+		+	?
GDP	+	+	?		?
SURPLUS	–	–	–	–	

This table shows the expected relationships between the variables, drawn from the review of theoretical and empirical results carried out in the preceding section. Positive relationships are represented by "+", and negative relationships by "–". When there is no defined prediction about the expected behavior of the variable, or the expected relation is ambiguous, it was used "?".

erated by the election of Luiz Inácio Lula da Silva for president. The expectations about inflation, the nominal interest rate, and the nominal exchange rate had a spike in November 2002, while the expectations about GDP decreased to their lowest value before the Great Recession. It is also noticeable the effect of the crisis related to the impeachment of President Dilma Rousseff, which formally happened between December 2015 and August 2016. The expectations about the nominal exchange rate reached their maximum in February and March 2016, while the expectations about GDP were in their minimum in January 2016, being negative between March 2015 and September 2016. The expectations about the primary surplus, which had already been worsening since 2009, rapidly decreased in 2015, becoming negative in October and reaching its minimum level one year later.

We explore the issue of whether inflation expectations are fully rational, by testing for the presence of a forecast error bias and the predictability of such an error. Furthermore, we run two regressions, as in Coibion and Gorodnichenko (2015). The first one, a Mincer and Zarnowitz (1969) regression, regresses the forecast error on the forecast itself, while the second one, a Nordhaus (1987) regression, regresses the error on forecast revisions. Following Berge (2018), we include the unemployment rate and a long-run interest rate in both regressions. Therefore, we estimate the following equations:

$$\pi_{t+12,t} - S_t\pi_{t+12,t} = \alpha + \varepsilon_t, \tag{10}$$

$$\pi_{t+12,t} - S_t\pi_{t+12,t} = \alpha + \beta_1 S_t\pi_{t+12,t} + \beta_2 u_{t-1} + \beta_3 i_{t-1} + \varepsilon_t, \tag{11}$$

$$\pi_{t+12,t} - S_t\pi_{t+12,t} = \alpha + \beta_1(S_t\pi_{t+12,t} - S_{t-1}\pi_{t+12,t}) + \beta_2 u_{t-1} + \beta_3 i_{t-1} + \varepsilon_t, \tag{12}$$

where $\pi_{t+12,t}$ is the observed inflation, $S_t\pi_{t+12,t}$ is the inflation expectation, $\pi_{t+12,t} - S_t\pi_{t+12,t}$ is the forecast error, u_t is the unemployment rate, and i_t is the interest rate. It should be noticed that the coefficient β_1 in Eq. (12) is equivalent to the coefficients $\frac{\lambda}{1-\lambda}$ and $\frac{1-G}{G}$ in Eqs. (2) and (3), respectively. According to Coibion and Gorodnichenko (2015), these coefficients should be positive in models with information rigidities. Additionally, when controlled for the forecast revision, as in Eq. (12), the forecast errors should not be predicted by related variables of interest.

Having computed such forecast errors, we are in position to test the hypothesis of rational expectations about inflation from December 2002, the first month for which we have a forecast, to August 2018. In 2016, the Brazilian Institute of Geography and Statistics (IBGE), the public institution responsible for producing statistical information, changed the methodology for the unemployment survey. Because none of the surveys cover the whole period, we do the test for three different periods. First, we test for the whole period and with the nominal interest rate only. The second period covers from December 2002 to February 2016, with unemployment data from the Monthly Employment Survey (PME), and the third period covers from March 2012 to August 2018, with data from the Continuous National Household Sample Survey (PNAD). Data for PME and PNAD are, respectively, series 2179 and 6381, from IBGE. For the observed inflation, we use the IPCA. The IPCA is also produced by IBGE, but we obtained the respective data from the BCB (series 433). Fig. 1 in Appendix shows both the actual and the expected monthly inflation in Brazil in the period covered in this paper. Lastly, the nominal interest rate is the swap reference rate, with preset DI rate, 360-term day (series 7827 from the BCB, provided by B3).

In order to investigate whether expectations about variables of interest interact as predicted by a theoretically robust macroeconomic model, we conduct a multivariate analysis within the period that is common to all variables, which is December 2001 to August 2018. It is important to consider that the nominal exchange rate has a strong contemporaneous relationship with the risk perception of the Brazilian economy abroad, the so-called country risk. Therefore, it is interesting to verify the relationship between risk and nominal exchange rate expectations, which can also affect inflation. In order to do so, we include the monthly EMBI+Br, calculated by J.P. Morgan, in the analysis. We use the end-of-period value of the series 40490, from the Institute of Applied Economic Research (IPEA). Given that expectations were collected in the first business day of the month, we use the values of EMBI+Br at the end of the month $t-1$ for expectations with respect to the month t. Additionally, the inflation expectations series is seasonally adjusted by the X13-ARIMA-SEATS.

In our framework, we can expect that any shock that causes an increase in inflation in the Phillips curve, in Eq. (4), will lead to an increase in the nominal interest rate, by the Taylor rule in Eq. (6). Therefore, we expect an increase in *INFLATION* with an increase in *GDP* and *EXCHANGE*, and, as a consequence, an increase in *INTEREST* with an increase in *INFLATION*, *GDP*, and *EXCHANGE*. An increase in *RISK*, by the UIP in Eq. (7), provided that both the domestic and the foreign nominal interest rates remain constant, increases *EXCHANGE*. Consequently, an increase in *RISK* should lead to an increase in *INFLATION* and *INTEREST* as well. With an increase in *INTEREST*, we expect a decrease in *GDP*, by the IS Curve in Eq. (5); a decrease in *INFLATION*, by the Phillips curve; an increase in *EXCHANGE*, by the UIP; and an increase in *SURPLUS*, by the results of Castro et al. (2015). Also from the results of Castro et al. (2015), we expect that *INFLATION*, *INTEREST*, *EXCHANGE*, and *GDP* decrease with an increase in *SURPLUS*. Lastly, we expect an increase in *GDP* with an increase in *EXCHANGE*. The expected signs of the relationships between the variables are summarized in Table 2.

According to the results of the Augmented Dickey–Fuller (ADF) test reported in Table 5 in Appendix, we fail to reject the null hypothesis of unit root for all series. Meanwhile, the series are stationary in first difference. Because all series are $I(1)$, it is necessary to test for the existence of cointegration relationships between them. We use the Johansen methodology and, as the results in

Table 3
Behavior of forecast errors in the formation of inflation expectations.

	Complete sample (12/2002–08/2018)	PME period (12/2002–02/2016)	PNAD period (03/2012–08/2018)
Mean and absolute forecast errors			
Mean error	0.082**	0.113***	0.059
	(0.038)	(0.038)	(0.057)
Mean absolute error	0.231	0.228	0.227
Persistence of forecast errors			
AR(1)	0.642	0.667	0.511
SARC[order]	0.595 [2]	0.605 [2]	0.511 [1]
Mincer-Zarnowitz regressions			
α	−0.085	0.126	0.300
	(0.162)	(0.167)	(0.338)
$S_t\pi_{t+12,t}$	−0.372	−0.090	−0.201
	(0.313)	(0.262)	(0.418)
i_{t-1}	0.024***	0.040***	0.026
	(0.009)	(0.010)	(0.016)
u_{t-1}	–	−0.118***	−0.045 **
		(0.031)	(0.019)
R^2	0.124	0.168	0.205
R^2 adj.	0.115	0.152	0.172
Nordhaus regressions			
α	−0.238**	0.090	0.243
	(0.102)	(0.096)	(0.220)
$S_t\pi_{t+12,t} - S_{t-1}\pi_{t+12,t}$	−0.633**	−0.681**	−0.759
	(0.293)	(0.284)	(0.498)
i_{t-1}	0.024***	0.041***	0.024
	(0.008)	(0.010)	(0.018)
u_{t-1}	–	−0.120***	−0.046**
		(0.031)	(0.019)
R^2	0.131	0.208	0.239
R^2 adj.	0.122	0.192	0.208
Observations	189	159	78

The first panel of this table shows the results for the mean and absolute forecast errors. For the mean error, it is presented the result for the following regression: $e_t = \alpha + \varepsilon_t$. The second panel shows the results for the coefficient of an AR(1) model and for the sum of the coefficients of an autoregressive model (SARC) with optimal lag chosen by the AIC criterion, which is indicated between brackets. The third panel shows the results for $e_t = \alpha + \beta_1 S_t\pi_{t+12,t} + \beta_2 i_{t-1} + \beta_3 u_{t-1} + \varepsilon_t$, and the fourth panel shows the results for $e_t = \alpha + \beta_1(S_t\pi_{t+12,t} - S_{t-1}\pi_{t+12,t}) + \beta_2 i_{t-1} + \beta_3 u_{t-1} + \varepsilon_t$. Newey-West standard errors are in parentheses. Significance levels: * 10%, ** 5%, *** 1%.

Table 6 in Appendix show, we reject the hypothesis of no cointegration relationship between the variables, and fail to reject the null hypothesis of one cointegration relationship. Therefore, we estimate the following VEC model, with one cointegration relationship and the country risk as an exogenous variable:

$$\Delta y_t = \alpha + \mathrm{ECT}_t + \Delta y_{t-1} + \Delta \mathrm{RISK}_{t-1} + \xi_t, \quad (13)$$

where ECT_t is the error correction term for the existing cointegration relationship and y_t is the vector of endogenous variables (*INFLATION*, *INTEREST*, *EXCHANGE*, *GDP*, and *SURPLUS*).

4. Results

The results for the forecast errors associated with the formation of inflation expectations are presented in Table 3, with ex-post forecast errors being given by $e_t = \pi_{t+12,t} - S_t\pi_{t+12,t}$. As it is possible to realize in the first panel of Table 3, the mean forecast errors are relatively low. These errors are even lower between March 2012 and August 2018. When the errors are regressed on a constant, as in Eq. (10), we reject the non-existence of a forecast bias and, therefore, the full rationality of inflation expectations in its weak form (as defined in Footnote [2]), except for the period between March 2012 and August 2018. Similarly, the persistence of forecast errors, which are shown in the second panel of Table 3, is lower for the second period, both for the coefficient of a first order autoregressive process and for the sum of the autoregressive coefficients of an autoregressive process with optimal lag chosen by the AIC criterion. Given that such forecast errors are persistent, they could be predicted and, therefore, inflation expectations cannot be seen as fully rational in the strong form as well (as defined in Footnote[2]), a result which is also obtained in Guillén (2008), Araujo and Gaglianone (2010), and Kohlscheen (2012).

Furthermore, the results for the Mincer-Zarnowitz and Nordhaus regressions, presented in the third and fourth panels of Table 3, show that the related macroeconomic variables of interest also help to predict forecast errors. These macroeconomic variables continue to be significant even when we control for the forecast revision, which suggests that models of information rigidities do not explain deviations from full rationality. Another argument against such models as a good explanation of the inflation expectations contained in the Focus Survey is the coefficient sign for the forecast revision, which is negative. This result differs from that obtained in Guillén (2008) and Correa and Picchetti (2016).

Table 4 presents the results for the estimation of the VEC model in Eq. (13), with the selected expectations as endogenous variables and the country risk as exogenous variable. The country risk has a negative significant relationship with the expectations about inflation and nominal exchange rate, different from what one would expect by the UIP. For the response of expectations about the relevant macroeconomic variables, one of the statistically significant results does not show the expected sign: the response of the expected economic activity to a change in the expected level of the nominal exchange rate, which is discussed in the next paragraph. The positive response of the expected inflation to a change in the expected nominal exchange rate is in line with the Phillips curve; the positive response of the expected nominal interest rate to a change in the expected nominal exchange rate is in line with the UIP and the Taylor rule; the positive response of the expected level of economic activity to a change in the expected inflation is in line with the IS curve; and the negative response of the expected primary surplus to a change in the expected inflation is in line with the results of Castro et al. (2015). The other significant results are the response of the expected nominal interest rate to a change in the expected nominal interest rate; the response of the expected nom-

Table 4
VEC estimation results, December 2001–August 2018.

	$\Delta INFLATION_t$	$\Delta INTEREST_t$	$\Delta EXCHANGE_t$	ΔGDP_t	$\Delta SURPLUS_t$
α	0.010	−0.142	0.003	0.190**	−0.256***
	(0.011)	(0.210)	(0.029)	(0.087)	(0.047)
ECT_t	−0.007	0.076	0.001	−0.144**	0.181***
	(0.008)	(0.154)	(0.022)	(0.063)	(0.035)
$\Delta INFLATION_{t-1}$	0.009	0.086	−0.171	1.330**	−0.653**
	(0.077)	(1.442)	(0.202)	(0.595)	(0.325)
$\Delta INTEREST_{t-1}$	0.001	0.385***	0.001	0.000	0.039**
	(0.004)	(0.071)	(0.010)	(0.029)	(0.016)
$\Delta EXCHANGE_{t-1}$	0.067***	1.222**	0.613***	−0.707***	−0.040
	(0.025)	(0.479)	(0.067)	(0.197)	(0.108)
ΔGDP_{t-1}	−0.002	0.218	−0.027	0.559***	0.014
	(0.008)	(0.148)	(0.021)	(0.061)	(0.033)
$\Delta SURPLUS_{t-1}$	0.018	−0.295	0.023	−0.035	0.267***
	(0.016)	(0.299)	(0.042)	(0.124)	(0.067)
$\Delta RISK_{t-1}$	−0.419***	−5.533*	0.594	−0.466	0.058
	(0.160)	(3.016)	(0.422)	(1.244)	(0.680)
R^2	0.093	0.249	0.425	0.472	0.396
R^2 adj.	0.055	0.217	0.401	0.450	0.370

This table shows the results for the VEC model $\Delta y_t = \alpha + ECT_t + \Delta y_{t-1} + \Delta RISK_{t-1} + \xi_t$ estimated for the expectations, so that ECT_t is the error correction term and y_t is the vector of endogenous variables (*INFLATION*, *INTEREST*, *EXCHANGE*, *GDP*, and *SURPLUS*). Standard errors are in parentheses. Significance levels: * 10%, ** 5%, *** 1%.

inal exchange rate to a change in the expected nominal exchange rate; the response of the expected level of economic activity to a change in the expected level of economic activity; and the response of the expected primary surplus to a change in the expected nominal interest rate and primary surplus.[6]

Figs. 2–6 in Appendix present the response of the several expectations to an impulse to each expectations series, which is calculated with the coefficient matrix of the moving average representation of the VAR form of the estimated VEC model. Fig. 3 shows that an expected increase in the nominal interest rate leads to an expected increase in the primary surplus, as in Castro et al. (2015). As can be seen in Fig. 4, an increase in the expected nominal exchange rate, that is, an expected nominal depreciation of the local currency leads to an expected increase in inflation, as one would expect according the Phillips curve in Eq. (4), and to an expected decrease in the level of economic activity. This latter result differs from what would typically be expected from the established macroeconomic theory, although it should be kept in mind that it does not refer to the expectation of a real depreciation of the local currency. However, an increase in the real exchange rate in one period leads to a decrease in the real GDP in later periods in Castro et al. (2015). Moreover, there are several studies about the contractionary effects of a real depreciation in the value of the local currency in emerging economies, e.g., Edwards (2003), Razmi (2007), and Ribeiro, McCombie, and Lima (2017). Galí (2008) also points out that the relationship of the terms of trade with net exports, and therefore the response of the economic activity to a change in the nominal exchange rate, depends on the parameters of the model. Lastly, we can note in Fig. 5 that an increase in the expected economic activity leads to an expected increase in the primary surplus. The impulse response functions are not statistically significant in the other cases.

[6] Since there is an incentive to make a forecast for the Top 5 variables, the Focus Survey usually has many more respondents informing forecasts for those variables than the others. In order to investigate if this difference might influence the results, we also estimate the system using only the variables that are part of the Top 5 (*INFLATION*, *INTEREST*, and *EXCHANGE*) as endogenous variables, with the country risk as exogenous variable. In this case, we no longer have a cointegration relationship, and estimate a VAR in first differences instead of the VEC. Yet the qualitative relationship between the variables does not change.

5. Conclusion

Our empirical exploration found that the median inflation expectations formed by the respondents of the Focus Survey conducted by the BCB cannot be considered as fully efficient information-wise. The reason is that the associated forecast errors can be predicted by the behavior of other related macroeconomic variables of interest, which violates the rational expectations hypothesis. Moreover, our results suggest that models with information rigidity do not satisfactorily explain such deviations from full rationality. As regards the pairwise relationship between the several expectations series of interest, the results for the VEC estimation suggest that to a great extent these expectations (especially inflation expectations) are formed in a way mostly consistent with a theoretically-grounded macroeconomic model. In fact, this theoretical consistency is likely to be a major reason why the forecast errors associated with the formation of inflation expectations are relatively low. This suggests, in turn, that the Brazilian monetary authority has somehow been successful in the management of inflation expectations, which is essential in its pursuit of a low and stable inflation rate.

As future research it will be worth investigating whether aggregate expectations about other measures of economic activity or forecast horizons also yield consistent results. Other issues for future research include how the relationship between inflation expectations and observed inflation impacts on the credibility of monetary policy; the potential asymmetry in the response of inflation expectations to increases and decreases in the observed inflation; and the relationship between inflation expectations compiled through the Focus Survey and those formed by the BCB itself, which are regularly published in the minutes of the Copom meetings and in official inflation reports.

Conflict of interest

None declared.

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Appendix.

Table 5
ADF test results, December 2001–August 2018.

Variable	Exogenous variables	Observations	Test statistic	Critical values		
				1%	5%	10%
INFLATION	Drift	201	−2.81	−3.46	−2.88	−2.57
ΔINFLATION	Drift	200	−9.77	−3.46	−2.88	−2.57
INTEREST	Drift and trend	201	−2.71	−3.99	−3.43	−3.13
ΔINTEREST	Drift	200	−6.43	−3.46	−2.88	−2.57
EXCHANGE	Drift	201	−1.24	−3.46	−2.88	−2.57
ΔEXCHANGE	Drift	200	−5.79	−3.46	−2.88	−2.57
GDP	Drift	201	−2.04	−3.46	−2.88	−2.57
ΔGDP	Drift	200	−4.66	−3.46	−2.88	−2.57
SURPLUS	Drift and trend	201	−2.03	−3.99	−3.43	−3.13
ΔSURPLUS	Drift	200	−6.39	−3.46	−2.88	−2.57
RISK	Drift	201	−2.14	−3.46	−2.88	−2.57
ΔRISK	Drift	200	−8.45	−3.46	−2.88	−2.57

This table shows the results of the Augmented Dickey–Fuller test, so that Δ*INFLATION*, Δ*INTEREST*, Δ*EXCHANGE*, Δ*GDP*, Δ*SURPLUS*, and Δ*RISK* are the first difference of *INFLATION*, *INTEREST*, *EXCHANGE*, *GDP*, *SURPLUS*, and *RISK*, respectively.

Table 6
Johansen cointegration test results, December 2001–August 2018.

	Trace statistic				Eigenvalue statistic			
	Test statistic	Critical values			Test statistic	Critical values		
		1%	5%	10%		1%	5%	10%
$r \leq 4$	1.78	11.65	8.18	6.50	1.78	11.65	8.18	6.50
$r \leq 3$	4.69	23.52	17.95	15.66	2.91	19.19	14.90	12.91
$r \leq 2$	12.21	37.22	31.52	28.71	7.52	25.75	21.07	18.90
$r \leq 1$	32.60	55.43	48.28	45.23	20.39	32.14	27.14	24.78
$r = 0$	71.51	78.87	70.60	66.49	38.92	38.78	33.32	30.84

This table shows the results of the Johansen cointegration test, for both the trace and the eigenvalue statistics, so that r is the number of cointegration relationships being tested.

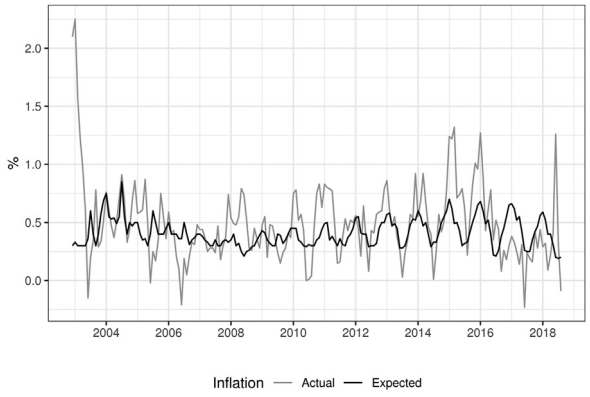

Fig. 1. Actual and expected monthly inflation in Brazil, December 2002–August 2018.
This figure shows the actual and expected monthly inflation in Brazil, between December 2002 and August 2018. Actual monthly inflation is the Extended National Consumer Price Index (IPCA), calculated by IBGE. Expected monthly inflation is the median of expectations for the IPCA in the Focus Survey, forecasted 12 months earlier.

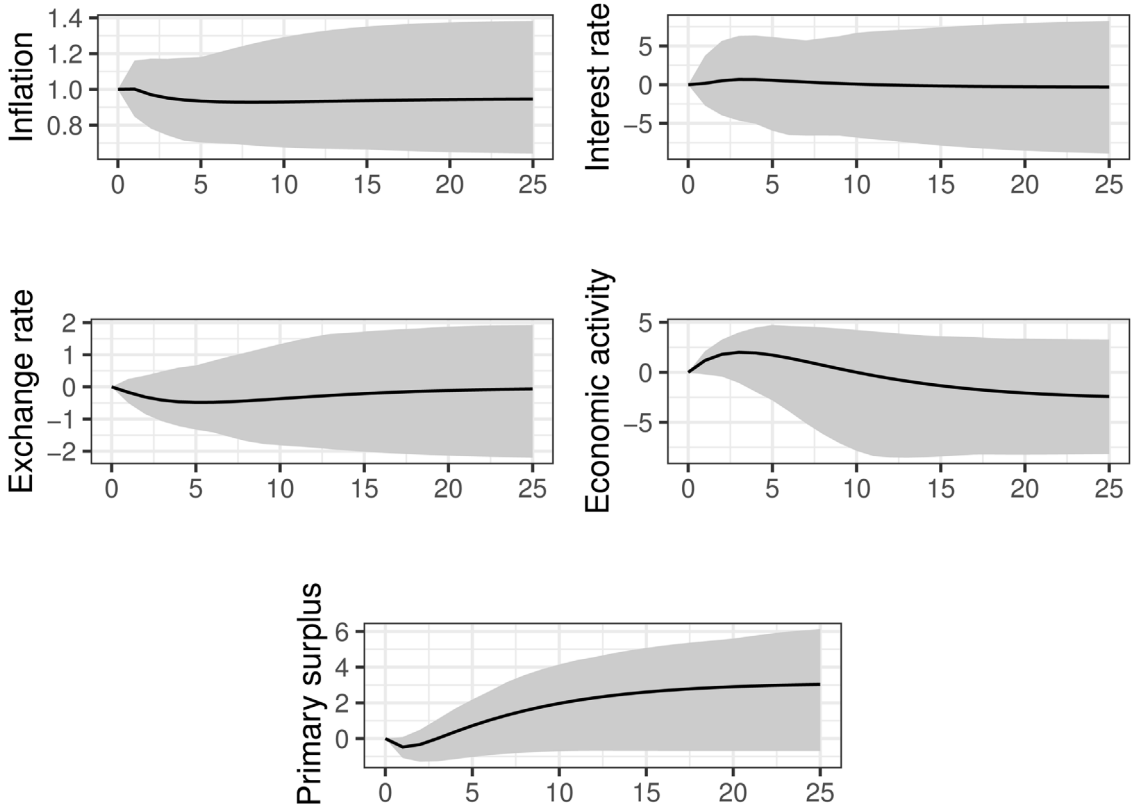

Fig. 2. Response of selected expectations to an impulse to inflation expectations.
This figure shows the response of the selected expectations, up to 25 periods ahead, to an impulse to inflation expectations, with the 95% confidence bands, estimated using bootstrap methods, represented by the gray area.

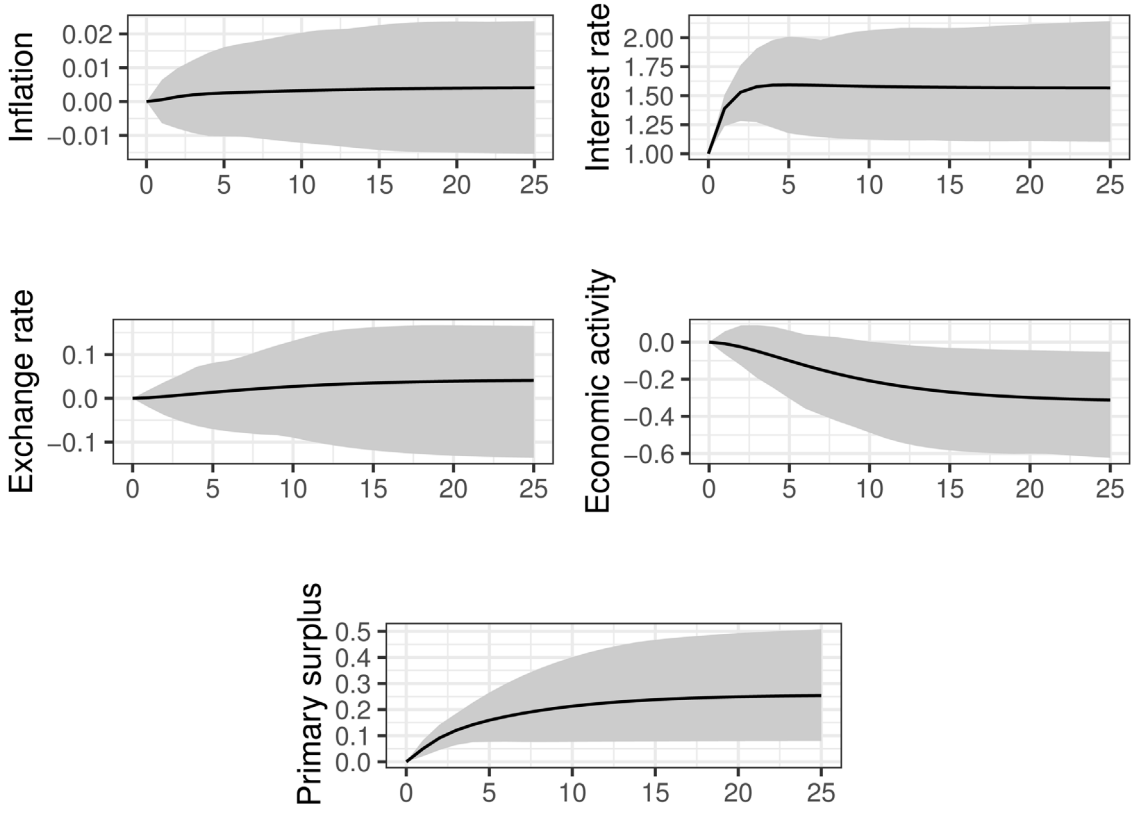

Fig. 3. Response of selected expectations to an impulse to nominal interest rate expectations.
This figure shows the response of the selected expectations, up to 25 periods ahead, to an impulse to nominal interest rate expectations, with the 95% confidence bands, estimated using bootstrap methods, represented by the gray area.

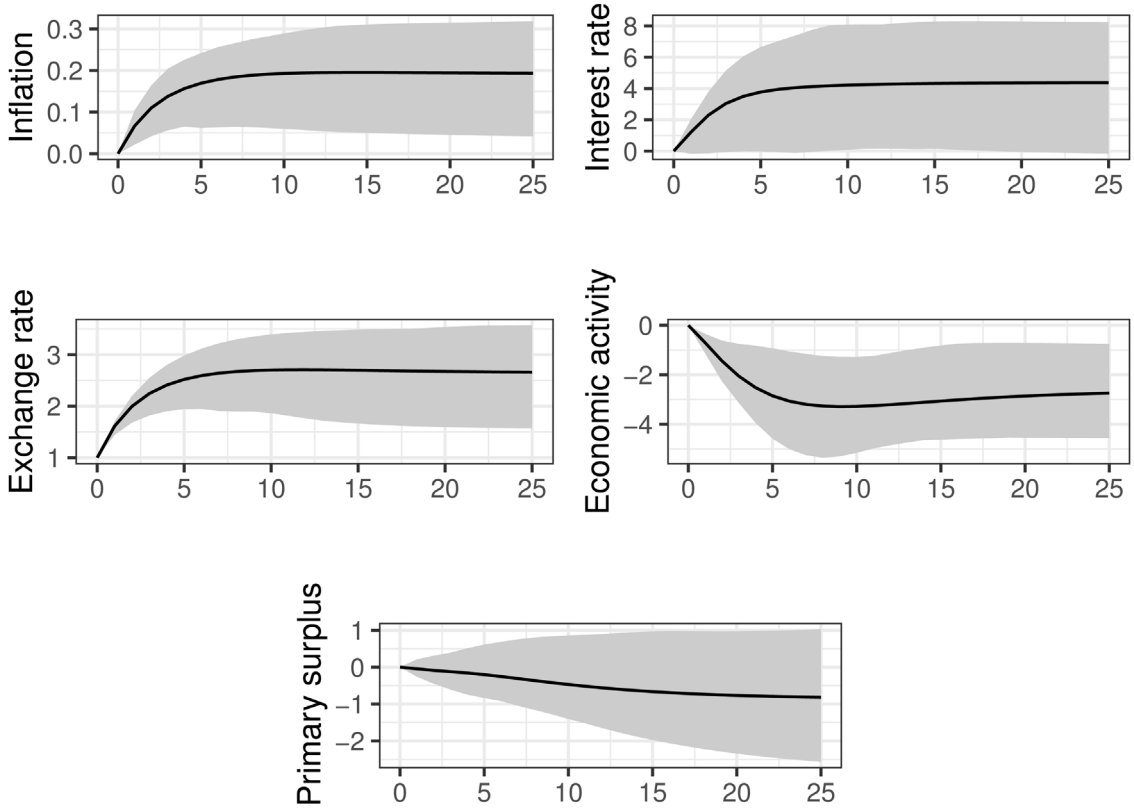

Fig. 4. Response of selected expectations to an impulse to nominal exchange rate expectations.
This figure shows the response of the selected expectations, up to 25 periods ahead, to an impulse to nominal exchange rate expectations, with the 95% confidence bands, estimated using bootstrap methods, represented by the gray area.

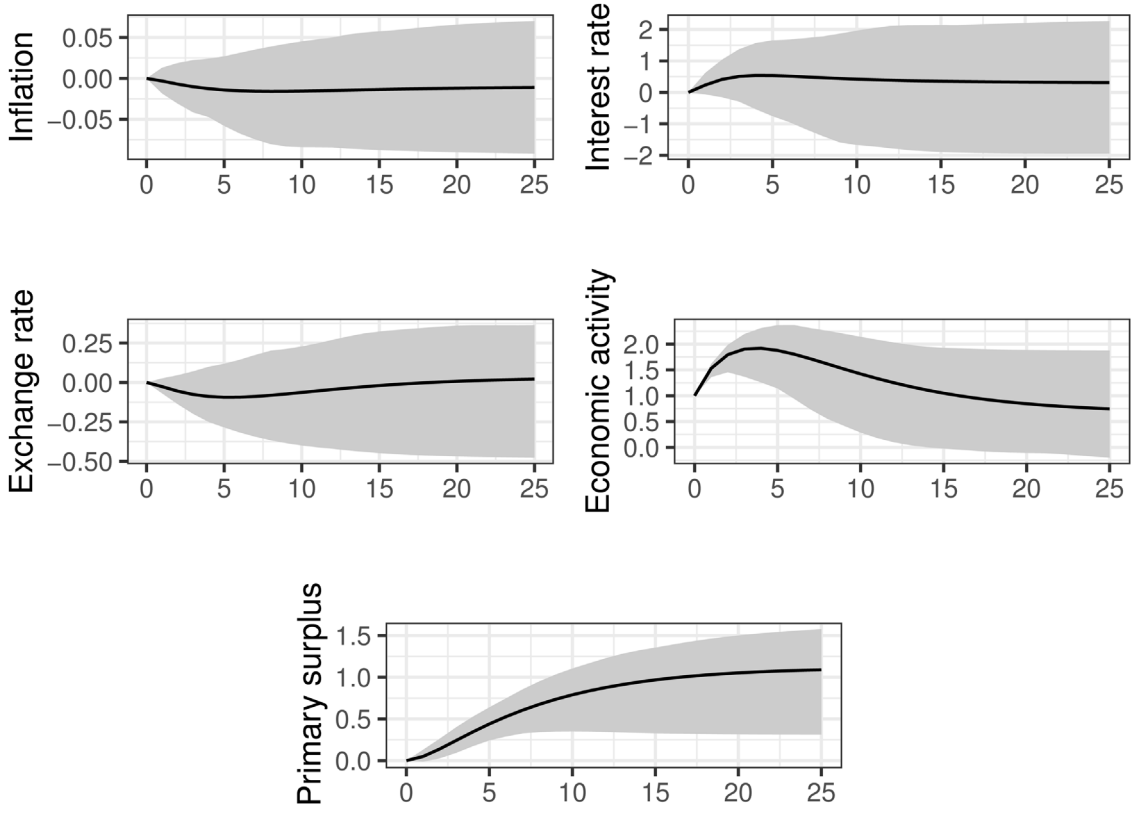

Fig. 5. Response of selected expectations to an impulse to economic activity expectations.
This figure shows the response of the selected expectations, up to 25 periods ahead, to an impulse to economic activity expectations, with the 95% confidence bands, estimated using bootstrap methods, represented by the gray area.

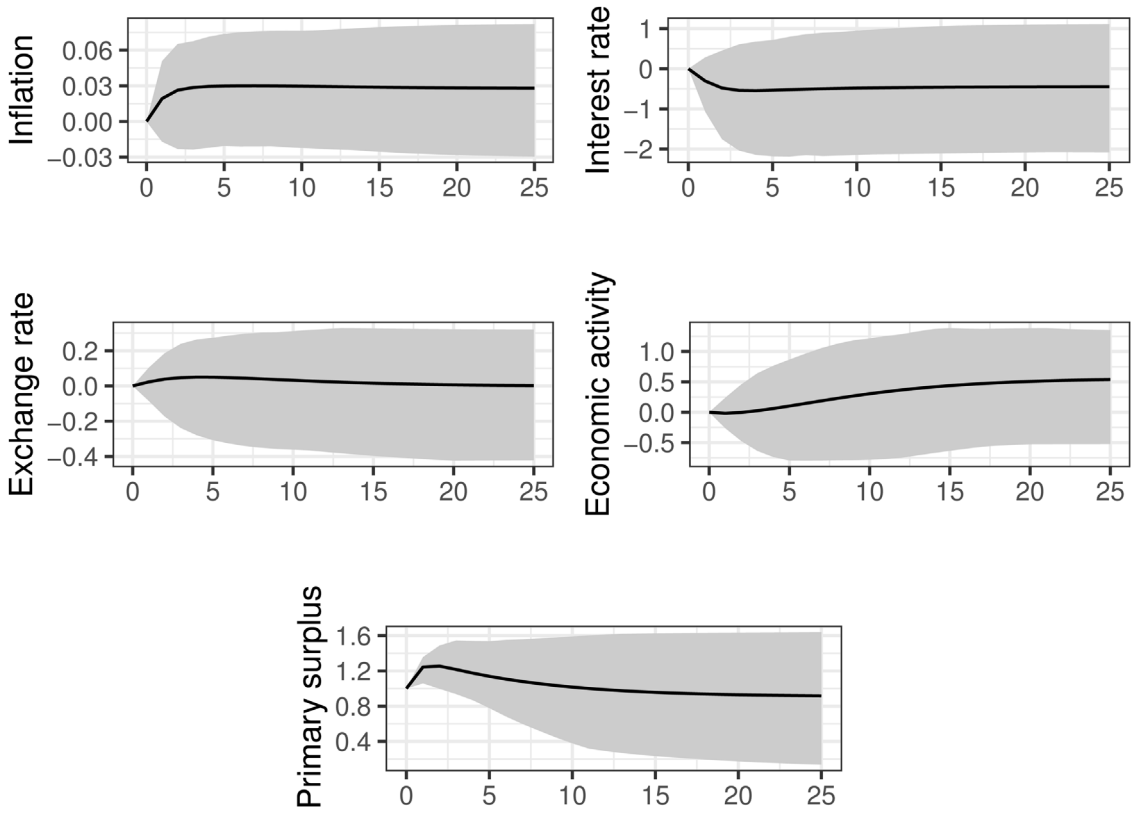

Fig. 6. Response of selected expectations to an impulse to primary surplus expectations.
This figure shows the response of the selected expectations, up to 25 periods ahead, to an impulse to primary surplus expectations, with the 95% confidence bands, estimated using bootstrap methods, represented by the gray area.

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