Math 309 Fall 2023 Final Project due: 5PM on Fri November 17

Project Directions

- Include a report on every group member's contribution.
- Submit the group's well commented code used for the project with instructions on how to compile and run.
- Make a **10** to **20** minute video presentation of your results.

The project consists of three problems. One from each section.

Section 1

The problem in this section is the same for all groups.

The following "Miniopoly" is a very rough approximation to the game of Monopoly. The round board has 40 "squares," or landing places. Squares 0 and 40 are the same; and this square is called "Go." A player starts with $200 on Go, and on each turn, rolls a pair of dice to determine a move, which will be equal to the sum rolled. Squares 2, 7, 17, 22, 33, and 36 are "follow instructions on card" squares. The instructions are receive $50, or $100, or $200 or pay $100 or $150 and are equally likely. Squares 0, 10, and 20 are "free": nothing happens there. Square 30 is "Go to jail." Passing or landing on Go pays $200. Being in jail means that on one's turn, if a roll is "doubles" (i.e. two 1's or two 2's or ... or two 6's) then the player "gets out of jail" and moves the value of the doubles from square 10; otherwise, the player pays $10, stays in jail and doesn't move. On any other square, like 29 for instance, pay $29 (to the "bank," an infinite source or sink of money). If at any time a player's money falls to $0 or below, that player loses (and the game is over for that player). **To be done**: simulate a game of Miniopoly for 20 rolls, or bankruptcy, and observe the amount of money, the fortune, of the player at that time. Bankruptcy means fortune equal to or less than $0. Simulate 10000 games and histogram the fortune.

Section 2

1. One hundred passengers line up to board an airplane with 100 seats. Each passenger is. to board the plan individually, and must take their seat before the next passenger may board. However, the passenger first in line has lost their pass and takes a random seat instead. This passenger randomly selects another unoccupied seat each time it appears that they are not occupying their assigned seat. Simulate the process 10000000 times to find the probability of the passenger changing seats five or more times before getting to their assigned seat. *Hint*: number the passengers in line as 1, 2, ..., 100 and number their assigned seats accordingly.

2. Each of seven dwarfs has his own bed in a common dormitory. Every night, they retire to bed one at a time, always in the same sequential order. On a particular evening, the youngest dwarf, who always retires first, has had too much to drink. He randomly chooses one of the seven beds to fall asleep on. As each of the other dwarfs retires,

he chooses his own bed if it is not occupied, and otherwise randomly chooses another unoccupied bed. Write a simulation to find for $k = 1, 2, \ldots, 7$, the probability that the kth dwarf to retire can sleep in his own bed. Simulate 10000000 times to find the probability.

3. An early detection test for the coronavirus is 98% accurate. In a certain community, 3% of the population has the virus. What is the probability that a person with a positive test for the coronavirus actually has the disease? Simulate this by selecting a person at random from the community and deciding whether the person has the disease. Then administer the test and decide whether it was positive. Keep a counter for the times the test is positive and for the times the test is positive and the person has the disease. Simulate the process a billion times to compute the probability.

4. A queue of 50 people is waiting at a box office in order to buy a ticket. The tickets cost five dollars each. For any person, there is a probability of 1/2 that she/he will pay with a five-dollar note and a probability of 1/2 that she/he will pay with a ten-dollar note. When the box opens there is no money in the till. If each person just buys one ticket, what is the probability that none of them will have to wait for change? Simulate the process 10000000 times to compute the probability.

Section 3

The problem in this section is the same for all groups.

The length of the arc between any two points a and b for a function $f(x)$ is defined as

$$S = \int_a^b \sqrt{1 + (f'(x))^2}\, dx.$$

Write a program to that takes as inputs a function $f(x)$, an interval $[a, b]$ and outputs the arc length over $[a, b]$. The program should use centered difference to approximate $f'(x)$ and trapezoidal rule to approximate the integral. Hence, the program should also take in n, the number of sub-divisions of the interval $[a, b]$ as input. Use the same n for the approximations of the derivative and the integral.