In [ ]:
```
from google.colab import drive
drive.mount('/content/drive')
```

SC2001 Project 1

In Mergesort, when the sizes of subarrays are small, the overhead of many recursive calls makes the algorithm inefficient. Therefore, in real use, we often combine Mergesort with Insertion Sort to come up with a hybrid sorting algorithm for better efficiency. The idea is to set a small integer **S** as a threshold for the size of subarrays. Once the size of a subarray in a recursive call of Mergesort is less than or equal to **S**, the algorithm will switch to Insertion Sort, which is efficient for small-sized input.

(a) Algorithm implementation

i. Insertion Sort

In [1]:
```
iarr = [0] * 10000001

def insertion_sort(l, r):
    count = 0
    for i in range(l, r + 1):
        for j in range(i, l, -1):
            if iarr[j] < iarr[j - 1]:
                tmp = iarr[j]
                iarr[j] = iarr[j - 1]
                iarr[j - 1] = tmp
                count += 1
            else: break
    return count
```

- store the array numbers into **iarr**, then use **insertion_sort(0, len - 1)** to sort the array and calculate the count.

ii. Merge Sort

In [2]:
```
marr = [0] * 10000001
mtmp = [0] * 10000001

def merge_sort(l, r):
    if(l == r): return 0
    mid = l + (r-l)//2
    count = 0
    count += merge_sort(l, mid)
    count += merge_sort(mid + 1, r)
    pl = l
    pr = mid + 1
    p = l
    while (pl <= mid) or (pr <= r):
        count += 1
        if (pr == r + 1) or ((pl != mid + 1) and (marr[pl] < marr[pr])):
            mtmp[p] = marr[pl]
            pl += 1
        else:
```

```
            mtmp[p] = marr[pr]
            pr += 1
        p += 1

    for i in range(l, r + 1):
        marr[i] = mtmp[i];
    return count
```

- store the array numbers into **marr**, then use **merge_sort(0, len - 1)** to sort the array and calculate the count.

iii. Hybrid Sort

In [3]:

```
harr = [0] * 10000001
htmp = [0] * 10000001

def hybrid_sort(l, r, threshold):
    if(r - l + 1 <= threshold):
        count = 0
        for i in range(l, r + 1):
            for j in range(i, l, -1):
                if harr[j] < harr[j - 1]:
                    tmp = harr[j]
                    harr[j] = harr[j - 1]
                    harr[j - 1] = tmp
                    count += 1
                else: break
        return count
    else:
        mid = (l + r) // 2
        count = 0
        count += hybrid_sort(l, mid, threshold)
        count += hybrid_sort(mid + 1, r, threshold)
        pl = l
        pr = mid + 1
        p = l
        while (pl <= mid) or (pr <= r):
            count += 1
            if (pr == r + 1) or ((pl != mid + 1) and (harr[pl] < harr[pr])):
                htmp[p] = harr[pl]
                pl += 1
            else:
                htmp[p] = harr[pr]
                pr += 1
            p += 1

        for i in range(l, r + 1):
            harr[i] = htmp[i];
        return count
```

- store the array numbers into **harr**, then use **hybrid_sort(0, len - 1, threshold)** to sort the array and calculate the count.

(b) Generate input data

In [ ]:

```
import random
def generate_data():
    m = 1000
```

```
    while m <= 1000000:
        for n in range(m, m * 10, m):
            arr = []
            for i in range(1, n + 1):
                arr.append(i)
            random.shuffle(arr)
            textfile = open("data/data_" + str(n) + ".txt", "w")
            textfile.write(str(arr))
            textfile.close()
        m *= 10;
    arr = []
    for i in range(1, m + 1):
        arr.append(i)
    random.shuffle(arr)
    textfile = open("data/data_" + str(m) + ".txt", "w")
    textfile.write(str(arr))
    textfile.close()

generate_data()
```

- The code is to generate the data:
  n = 1000, 2000, ... , 8000, 9000
  n = 10000, 20000, ... , 80000, 90000
  ...
  n = 1000000, 2000000, ... , 8000000, 9000000, 10000000
- The form of datasets is **data_\<size>.txt**

In [4]:

```
sample_size = 1000
file = open("data/data_" + str(sample_size) + ".txt", "r")
arr = (file.read())[1:-1].split(', ')
arr = [int(i) for i in arr]
# print(arr)
```

- The way of reading the data list from file.

(c) Analyze time complexity

Let's assume the **threshold** = S

- For **insertion sort** :

Each subarray with the length of S, the best case of count $O(S)$ and the worst case $O(S^2)$

As there's $\frac{n}{S}$ blocks, the best case of total cost $O(S \times \frac{n}{S}) = O(n)$ and the worst case $O(S^2 \times \frac{n}{S}) = O(Sn)$

- For **merge sort** :

Since the length of threshold is S, there's would be $\log(\frac{n}{S})$ interations of merge sort

For each iteration, the cost of merging is $O(n)$, thus total cost of merge sort part is $O(n \times \log(\frac{n}{S}))$

- For **hybrid sort** :

The best case : $O(n + n \times \log(\frac{n}{S}))$

The worst case : $O(Sn + n \times \log(\frac{n}{S}))$

In [5]:

```
import math
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

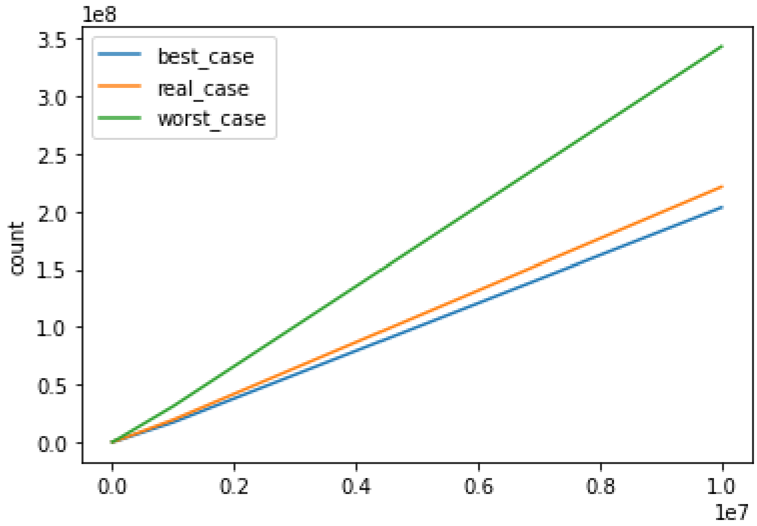
i. With the value of S fixed, plot the number of key comparisons over different sizes of the input list n. Compare your empirical results with your theoretical analysis of the time complexity.

- here we fix S (threshold) at 15, and consider the dataset_size at 1000, 10000, ... , 10000000

In [6]:

```
datasets = [1000, 10000, 100000, 1000000, 10000000]

best_case = []
real_case = []
worst_case = []

for size in datasets:
    file = open("data/data_" + str(size) + ".txt", "r")
    arr = (file.read())[1:-1].split(', ')
    arr = [int(i) for i in arr]
    harr = arr
    best_case.append(int(size + size * math.log2(size / 15)))
    real_case.append(hybrid_sort(0, size - 1, 15))
    worst_case.append(int(size * 15 + size * math.log2(size / 15)))

df = pd.DataFrame({'dataset_size': datasets, 'best_case': best_case, 'real_ca
print(df)
plt.plot(datasets, best_case, label="best_case")
plt.plot(datasets, real_case, label="real_case")
plt.plot(datasets, worst_case, label="worst_case")
plt.ylabel('count')
plt.legend()
plt.show()
```

```
   dataset_size  best_case  real_case  worst_case
0          1000       7058       9041       21058
1         10000     103808     121572      243808
2        100000    1370274    1579631     2770274
3       1000000   17024677   19295669    31024677
4      10000000  203466060  221405031   343466060
```



ii. With the input size n fixed, plot the number of key comparisons over different values of S. Compare your empirical results with your theoretical analysis of the time complexity.

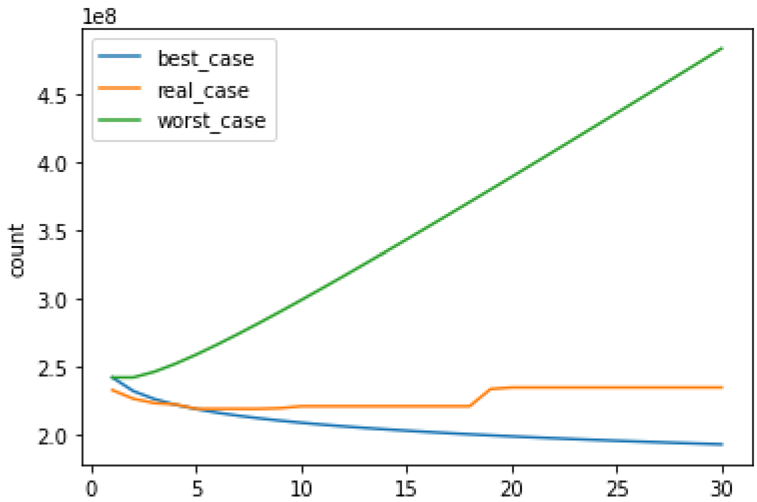
- here we fix the dataset size at 10_000_000, and consider the threshold at [1, 30]

In [7]:

```
def getCountsPerThreshold(size: int, printDataFrame: bool) -> list[int]:
    best_case = []
    real_case = []
    worst_case = []
    file = open("data/data_" + str(size) + ".txt", "r")
    arr = (file.read())[1:-1].split(', ')
    arr = [int(i) for i in arr]
    for threshold in range(1, 31):
        global harr
        harr = [i for i in arr]
        best_case.append(int(size + size * math.log2(size / threshold)))
        real_case.append(hybrid_sort(0, size - 1, threshold))
        worst_case.append(int(size * threshold + size * math.log2(size / thre

    df = pd.DataFrame({'threshold': range(1, 31), 'best_case': best_case, 're
    if printDataFrame:
        print(df)
    plt.plot(range(1, 31), best_case, label="best_case")
    plt.plot(range(1, 31), real_case, label="real_case")
    plt.plot(range(1, 31), worst_case, label="worst_case")
    plt.ylabel('count')
    plt.legend()
    plt.show()
    return real_case

countsPerThreshold_10m = getCountsPerThreshold(10_000_000, True)
```

```
   threshold  best_case  real_case  worst_case
0          1  242534966  233222784   242534966
1          2  232534966  226930625   242534966
2          3  226685341  223708672   246685341
3          4  222534966  222735783   252534966
4          5  219315685  219513361   259315685
5          6  216685341  219513361   266685341
6          7  214461417  219513361   274461417
7          8  212534966  219513361   282534966
8          9  210835716  219996164   290835716
9         10  209315685  221405031   299315685
```

10	11	207940650	221405031	307940650
11	12	206685341	221405031	316685341
12	13	205530569	221405031	325530569
13	14	204461417	221405031	334461417
14	15	203466060	221405031	343466060
15	16	202534966	221405031	352534966
16	17	201660338	221405031	361660338
17	18	200835716	221405031	370835716
18	19	200055691	234020557	380055691
19	20	199315685	235172174	389315685
20	21	198611792	235172174	398611792
21	22	197940650	235172174	407940650
22	23	197299347	235172174	417299347
23	24	196685341	235172174	426685341
24	25	196096404	235172174	436096404
25	26	195530569	235172174	445530569
26	27	194986091	235172174	454986091
27	28	194461417	235172174	464461417
28	29	193955156	235172174	473955156
29	30	193466060	235172174	483466060



- You may find that when threshold is very small, the real_case of key comparison may be lower than best_case of expectation. That's because when the data is randomly generated, the real case may be less than expected case.

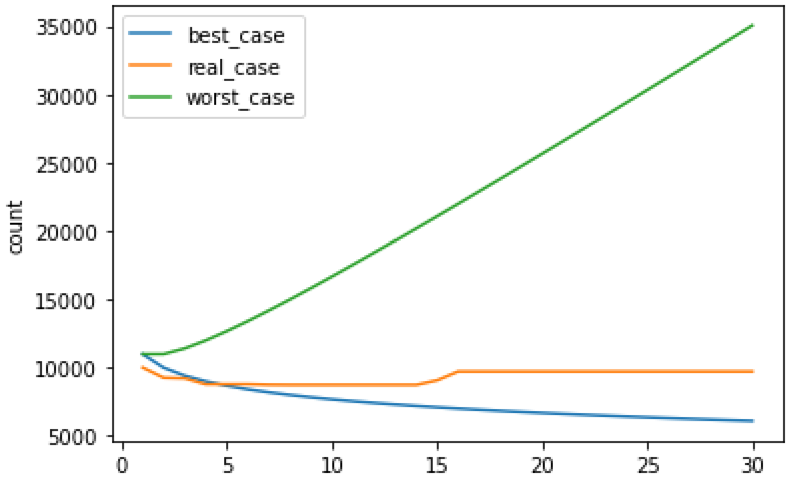
iii. Using different sizes of input datasets, study how to determine an optimal value of S for the best performance of this hybrid algorithm.

- Plot values of s(threshold) against count(number of comparisons) for size of N in [1000, 10_000, 100_000, 1_000_000, 10_000_000].

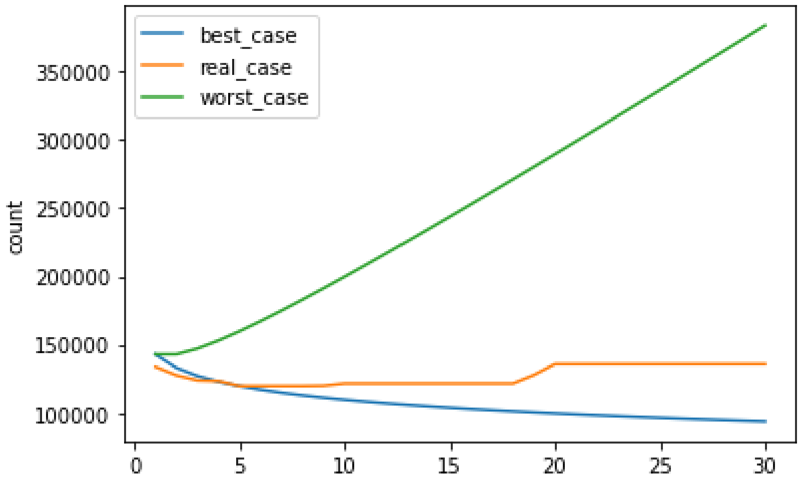
In [8]:

```
# omit input size of 10million as it is already calculated above
inputSizes = [1000, 10_000, 100_000, 1_000_000]
result_dict = {}

def getMinThresholdRange(counts: list[int]) -> (int,int):
    minThreshold, maxThreshold = 0, 0
    minCount = float('infinity')

    for index, count in enumerate(counts):
        if count < minCount:
            minCount, minThreshold = count, index + 1
        if count == minCount:
```

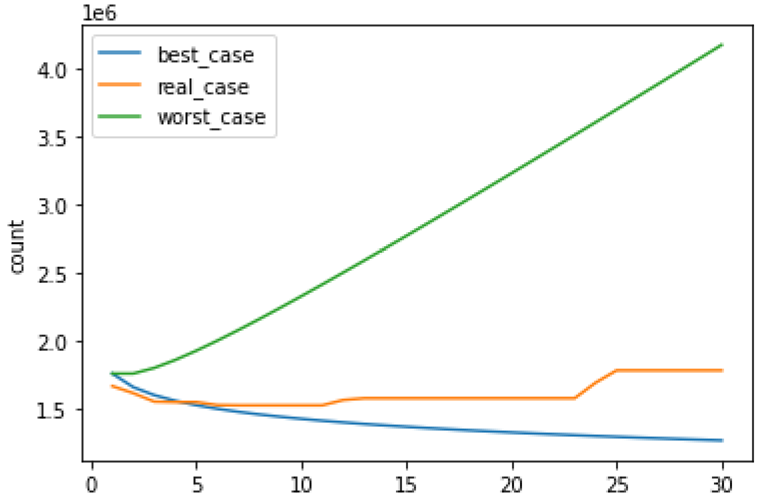
```
            maxThreshold = index + 1

    return (minThreshold, maxThreshold)


for size in inputSizes:
    counts = getCountsPerThreshold(size, False)
    minThreshold, maxThreshold = getMinThresholdRange(counts)
    result_dict[size] = counts
    print(f"For input size {size}, threshold which yields the minimum number 

    result_dict[10_000_000] = countsPerThreshold_10m

def highlight_min(s, props=''):
    return np.where(s == np.amin(s.values), props, '')

result_df = pd.DataFrame(result_dict)
result_df.index = np.arange(1, len(result_df)+1)
presentable_result = result_df.style.apply(highlight_min, props='color:white;
presentable_result
```



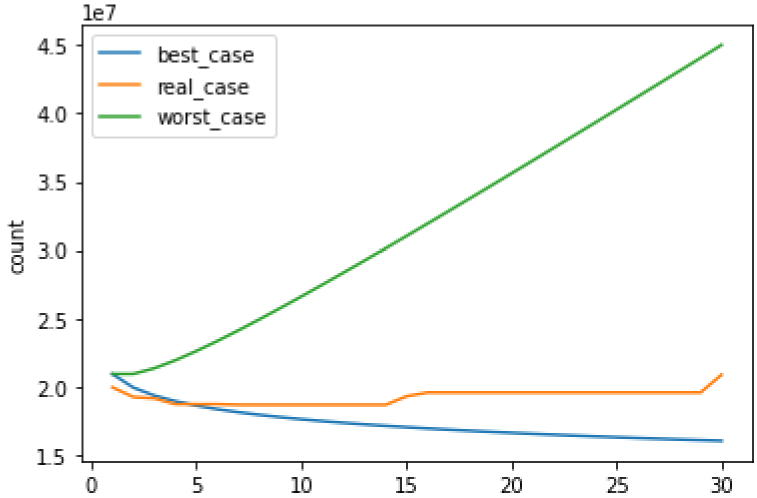

```
For input size 1000, threshold which yields the minimum number of comparisons
is [8, 14]
```




```
For input size 10000, threshold which yields the minimum number of comparisons
is [5, 8]
```




```
For input size 100000, threshold which yields the minimum number of comparison
s is [7, 11]
```




```
For input size 1000000, threshold which yields the minimum number of compariso
ns is [8, 14]
```

Out[8]:

	1000	10000000	10000	100000	1000000
1	9976	233222784	133616	1668928	19951424
2	9229	226930625	127457	1617136	19238250
3	9188	223708672	123813	1554921	19140972
4	8754	222735783	123350	1551526	18714142
5	8754	219513361	119718	1551526	18714142
6	8754	219513361	119718	1529474	18714142
7	8712	219513361	119718	1527731	18665227
8	8701	219513361	119718	1527731	18663969
9	8701	219996164	119918	1527731	18663969
10	8701	221405031	121572	1527731	18663969
11	8701	221405031	121572	1527731	18663969
12	8701	221405031	121572	1566832	18663969
13	8701	221405031	121572	1579631	18663969
14	8701	221405031	121572	1579631	18663969
15	9041	221405031	121572	1579631	19295669

	1000	10000000	10000	100000	1000000
16	9689	221405031	121572	1579631	19567477
17	9689	221405031	121572	1579631	19567477
18	9689	221405031	121572	1579631	19567477
19	9689	234020557	127761	1579631	19567477
20	9689	235172174	135743	1579631	19567477
21	9689	235172174	135743	1579631	19567477
22	9689	235172174	135743	1579631	19567477
23	9689	235172174	135743	1579631	19567477
24	9689	235172174	135743	1693948	19567477
25	9689	235172174	135743	1784304	19567477
26	9689	235172174	135743	1784304	19567477
27	9689	235172174	135743	1784304	19567477
28	9689	235172174	135743	1784304	19567477
29	9689	235172174	135743	1784304	19567477
30	9689	235172174	135743	1784304	20868589

(d) Compare with original Mergesort

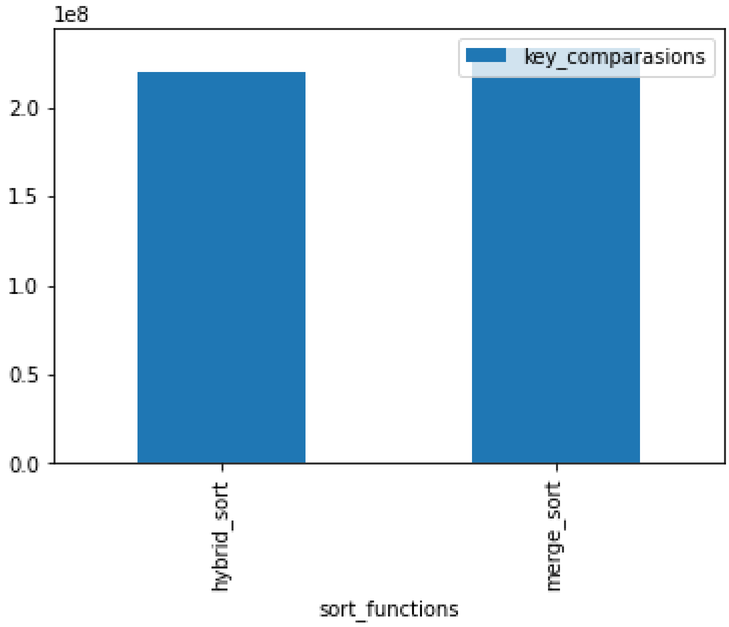
In [11]:

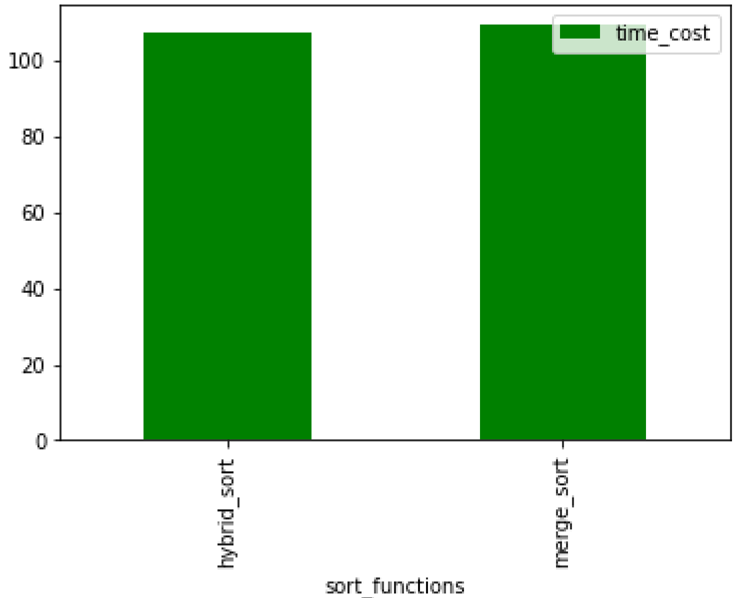
```
import timeit
threshold = getMinThresholdRange(countsPerThreshold_10m)[0]
size = 10_000_000

file = open("data/data_" + str(size) + ".txt", "r")
arr = (file.read())[1:-1].split(', ')

harr = [int(i) for i in arr]
time_start_hybrid = timeit.default_timer()
count_hybrid = hybrid_sort(0, size - 1, threshold)
time_end_hybrid = timeit.default_timer()

marr = [int(i) for i in arr]
time_start_merge = timeit.default_timer()
count_merge = merge_sort(0, size - 1)
time_end_merge = timeit.default_timer()

frame = pd.DataFrame(np.array([['hybrid_sort', count_hybrid, time_end_hybrid
                               ['merge_sort', count_merge, time_end_merge - t
                       columns=['sort_functions', 'key_comparasions', 'time_cost'
print(frame)
frame[['key_comparasions','time_cost']] = frame[['key_comparasions',"time_cos

frame[['key_comparasions','sort_functions']].plot(kind='bar',y='key_comparasi
frame[['time_cost','sort_functions']].plot(kind='bar',y='time_cost',x='sort_f
```

```
  sort_functions key_comparasions           time_cost
0    hybrid_sort        219513361   107.1745278340004
1     merge_sort        233222784  109.33150066599956
```

Out[11]:

```
<AxesSubplot:xlabel='sort_functions'>
```


1e8
key_comparasions
2.0
1.5
1.0
0.5
0.0
hybrid_sort
merge_sort
sort_functions


time_cost
100
80
60
40
20
0
hybrid_sort
merge_sort
sort_functions