Constrained optimization - Practical Session

Master DSC/MLDM/CPS2

Assignment: Due date 26/03/2018 12:00 (on claroline - Assignments section)
It is recommended to post the assignment as soon as possible
This work will be done during 2 practical sessions - no more than 2 students can be associated to the same work

The goal of this practical is to learn to formulate (simplified) real-world problems as optimization problems in nice form, and use the software AMPL to solve them.
We will use the free-demo versions that are available on the AMPL Website:

http://ampl.com/try-ampl/download-a-free-demo/

This includes a Windows, Linux and Mac versions, accompanied with many (known) solvers (CPLEX, MINOS, LOQO, ...). Windows and Mac have a package with an IDE, but the command line versions can be sufficient for us. The size of command-line packages is between 40M and 75M, and it is more than 130M for those including the IDE. If this is too much for you, you can find on claroline some light command-line versions with AMPL and 2 solvers (LOQO and MINOS). Note that the licence permits unrestricted use subject to the following limits - for linear problems: 500 variables and 500 constraints plus objectives, for nonlinear problems: 300 variables and 300 constraints plus objectives (some additional restrictions can apply depending on the solver).

You are free to use any solver, note anyway that LOQO implements the interior-point approach seen in class and that MINOS implements a simplex method for linear programs, and a reduced-gradient approach for nonlinear problems. **Both of them are available on claroline**. Note that each solver has a specific documentation that can be found in the `doc` directory of each package.

Assignment: You must send a report on your work, detailing your methodology, including the results and the commands used. Personal comments are welcome. The work can be done by at most 2 students and must be posted on claroline (use `.zip` or `.tar.gz`).

1 Using AMPL

1.1 The AMPL program

AMPL works in command-line, although some GUI exists if you downloaded packages with IDE. Actually we will only need a few command-lines:

- First of all, type `option solver loqo;` so that AMPL uses LOQO to solve problems - this indicates that you want to use the LOQO software, by default we assume everything is in the current folder, consult document for any other option. Note that if you want to force LOQO to solve the dual problem and limit the number of iterations to 30, you can type:
  `option loqo_options 'iterlim 30 dual', solver loqo;`
- To load a model file, type `model <filename>;`
- To load a data file, type `data <filename>;`
- To solve the current problem, type `solve;`

- To display the value of a variable, type `display <varname>;`
- To reset (erase the current problem from memory before loading a new one): `reset;`

Note that you can put some of these commands directly in your model or data file. For instance, I recommend you start any model file by `reset;` to make sure that the problem is loaded from scratch (for instance, if you have an older version of your model still in memory, that can be an issue). You can also put at the end of your model or data file the commands to solve and display the variables if you like, so that you don't have to type them again each time you load the problem.

1.2 Syntax for model and data files

Please refer to the course slides for some examples of syntax. We shouldn't need much more than that for this practical. To get more information, you can use Google (look for AMPL Tutorial) or check the following resources:

http://www.ieor.berkeley.edu/~atamturk/ieor264/samples/ampl/ampldoc.pdf
https://www.tu-chemnitz.de/mathematik/part_dgl/teaching/WS2009_Grundlagen_der_Optimierung/amplguide.pdf
http://www.ampl.com/REFS/amplmod.pdf

2 Unconstrained optimization

We start with 3 unconstrained optimization problems that are easy to formulate to learn the basic features of AMPL.

2.1 Problem 1

Consider the following problem:

$$\min_{x_1,x_2\in\mathbb{R}} \quad 4x_1^2 + 7(x_2-4)^2 - 3x_1 + 4x_2$$

- Formulate the problem in AMPL (using a model file only) and solve it. Check that you get the correct solution by solving on your own.

2.2 Problem 2

Consider the following problem (known as the Rosenbrock function, seen in the convex optimization class):

$$\min_{x_1,x_2\in\mathbb{R}} \quad (1-x_1)^2 + 100(x_2-x_1^2)^2$$

- Formulate the problem in AMPL (using a model file only) and solve it. Try several starting points: do you always get the global minimum? To define the starting value of a variable, use the command `let <varname> := <startvalue>;` before the `solve;` command. By default, the starting value of variables is set to 0. Can you find the optimum.

2.3 Problem 3

Consider the following problem (known as the Rastrigin's function):

$$\min_{x_1,x_2\in\mathbb{R}} \quad 10 + x_1^2 + x_2^2 - 10(\cos 2\pi x_1 + \cos 2\pi x_2)$$

- The global minimum is actually located at $(0,0)$. Formulate the problem in AMPL (using a model file only) and solve it. Try several starting points: do you always get the global minimum?

3 Modeling constrained problems

We now focus on modeling some kind of "real" problems.

3.1 Water resources

A city needs 500,000 liters of water per day, which can be drawn from either a reservoir or a stream. Characteristics of these two sources are found in Table 1. No more than 100,000 liters per day can be drawn from the stream, and the concentration of pollutants in the water served to the city cannot exceed 100 ppm [1]. The problem is to find how much water the city should draw from each source (subject to the constraints) so as to minimize the cost.

	Reservoir	**Stream**
Cost (€ per 1,000L)	100	50
Upper limit ($\times$ 1,000L)	∞	100
Pollution (ppm)	50	250

Table 1: Reservoir and stream information

3.2 Good-smelling perfume design

A perfume company located in the south of France is trying to develop new perfumes based on mixes of 4 blends of essential oils developed in their lab. Each of these 4 blends embeds some essences of real flowers (rose, bergamot orange, lily of the valley, thymus). The table below describes the composition of each blend and the cost to produce each of them. The company wants to develop its own trademark and imposes some

	blend 1	blend 2	blend 3	blend 4
rose	30	20	40	20
bergamot orange	35	60	35	40
lily of the valley	20	15	5	30
thymus	15	5	20	10
Cost (€/liter)	55	65	35	85

contraints on the possible mixes to design a new perfume:

- the percentage of blend 2 in the perfume must be at least 5% and cannot exceed 20%,
- the percentage of blend 3 has to be at least 30%,
- the percentage of blend 1 has to be between 10% and 25%,
- the final percentage of bergamot orange content in the perfume must be at most 50%,
- the final percentage of thymus content has to be between 8% to 13%,
- the final percentage of rose content must be at most 35%,
- finally, the percentage of lily of the valley content has to be at least 19%.

We are looking for the least costly way of mixing the 4 blends of essential oils to produce a new perfume subject to the constraints given above.
Questions:

1. Model this problem as an constrained optimization problem.
2. Formulate it in AMPL (using a model file only) and solve it. Check that the optimal solution makes sense. What is the optimal cost?

[1] The proportion of pollutants is measured by the quantity of pollutants coming from the reservoir and from the stream divided by the quantity in stream and reservoir.

3.3 Roadway expenses

3.3.1 Rural/Urban case

France has 200M€ to spend on roadway improvements this coming year, and the government has to decide how much to spend on rural projects, and how much to spend on urban projects. Let x_{rural} and x_{urban} represent the amount of money spent on these two categories, in millions of euros. The benefit from spending x_{rural} on rural projects is:

$$B_{rural} = 7000 \log(1 + x_{rural}),$$

and the benefit from spending x_{urban} on urban projects is

$$B_{urban} = 5000 \log(1 + x_{urban}).$$

We want to maximize the net benefit to the state, that is, $B_{rural} + B_{urban} - x_{rural} - x_{urban}$.

Questions:

1. Model this problem as a constrained optimization problem.
2. Formulate it in AMPL (using a model file only) and solve it. Check that the optimal solution makes sense. What is the optimal benefit?

3.3.2 General case

Now suppose we have a set I of categories of roads in which we can invest. The benefit from spending x_i on projects of category i is:

$$B_i = C_i \log(1 + x_i),$$

where C_i's are parameters of the model (not variables). The available budget (in millions of euros) to be spent is given by parameter *budget*. The net benefit to the state is now: $\sum_{i \in I} B_i - \sum_{i \in I} x_i$. In the previous case, we had $budget = 200$, $I = \{rural, urban\}$, $C_{rural} = 7000$ and $C_{urban} = 5000$.

Questions:

1. Model this general case using an AMPL model file where no numbers appear (except for the non-negativity constraints). You must use parameters for C_i's and *budget*, and a set for the road types. You can define a set *myset* and a parameter *myparam* for each element of the set in the following way:

```
set myset;
param myparam{i in my_set};
```

2. Write an AMPL data file that assigns the value of the previous section. Solve the problem and make sure that you get the same result as before.
3. Write a new AMPL data file that adds a third road category to the problem (*suburban*) with $C_{suburban} = 7000$ and solve the problem.

3.4 Design you own optimization problem

Now it's your turn! Propose a problem that could be realistic. It can be related to your personal interests, hobbies (or not). To do:

1. You must clearly define the problem to solve and the data associated with it.
2. Write the optimization problem that can be used to find the solution.
3. Provide the AMPL code associated with it. If relevant, do not hesitate to precise how general and smart your implementation is.

4 Data Analysis

Choose a method you saw in the context of the data analysis or the machine learning classes that can be expressed as an optimization problem.

1. Propose an implementation of this method with AMPL.
2. Propose several variants of this problem by using different norms, constraints, formulations when appropriate. The diversity of the fomulations will be taken into account.