# A Gentle Introduction to BLP[*]

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# 1 BLP

This exercise estimates the demand-side BLP model.

## 1.1 Logit model

The logit model without random coefficient is

$$u_{ij} = x_j\beta - \alpha p_j + \xi_j + \epsilon_{ij}$$

Inverting the market shares gives the mean utility of products,

$$\ln s_j - \ln s_0 = \delta_j = x_j\beta - \alpha p_j + \xi_j$$

We can estimate the equation using OLS or 2SLS depending on the assumptions on the correlation between $(x_j, p_j)$ and $\xi_j$.

For each market (defined by store-week), I calculate the market share by $s_j = q_j/M$, where $q_j$ denotes the sales for product $j$ and $M$ denotes the market size (proxied by count). Thus the market share for the outside good $s_0 = 1 - \sum_j s_j$.

Table 1 reports the regression results using the following five specifications.

Table 1: Logit Model Without Random Coefficients

| | (1) OLS | (2) OLS | (3) OLS | (4) IV | (5) IV |
|---|---|---|---|---|---|
| Price | 0.020 | $-0.189^{***}$ | $-0.145^{**}$ | $0.069^{***}$ | 0.169 |
| | (0.014) | (0.059) | (0.058) | (0.015) | (0.115) |
| Promotion | 0.121 | $0.187^{**}$ | $0.201^{***}$ | 0.149 | $0.308^{***}$ |
| | (0.093) | (0.074) | (0.073) | (0.093) | (0.082) |
| Constant | $-7.976^{***}$ | $-6.579^{***}$ | $-6.839^{***}$ | $-8.198^{***}$ | $-7.807^{***}$ |
| | (0.070) | (0.212) | (0.214) | (0.074) | (0.399) |
| Product FE | | Yes | Yes | | Yes |
| Store FE | | | Yes | | |
| Instrument | | | | Yes | Yes |

[*] This project is part of the problem set of the second-year PhD IO class taught by Prof. Marc Rysman.

## 1.2 Fixed point algorithm

Following BLP's random coefficient model, the utility to consumer $i$ from product $j$ in market $t$ is

$$u_{ijt} = \underbrace{x_{jt}\beta - \alpha p_{jt} + \gamma_j + \xi_{jt}}_{\delta_{jt}} + \underbrace{\sum_{\ell} x_{j\ell t}\sigma_{\ell t}\nu_{i\ell t}}_{\mu_{ijt}} + \epsilon_{ijt}$$

The last two terms $\mu_{ijt} + \epsilon_{ijt}$ captures the random coefficients. Assume that the variance of random coefficient is $E(\nu_{i\ell}^2) = 1$ and it is with mean zero, then the mean and variance of the marginal utility associated with characteristic $\ell$ is $\beta_\ell$ and $\sigma_\ell^2$. In this specific exercise, the random coefficient term is on the constant term, so $\ell = 1$.

The aggregate market share for each product $j$ is

$$s_{jt} = \int \frac{\exp(\delta_{jt} + \mu(x_{jt}, \nu_i, \sigma))}{1 + \sum_j \exp(\delta_{jt} + \mu(x_{jt}, \nu_i, \sigma))} f(\nu_i) d\nu_i$$

where $\nu \sim \mathcal{N}(0, 1)$. The integral has no analytical solution in the random coefficient model, so we need to compute the integral by simulation. One method to approximate the integral is to use

$$\hat{s}_{jt} = \frac{1}{ns} \sum_{i=1}^{ns} \frac{\exp(\delta_{jt} + x_{jt}\sigma_t\nu_{it})}{1 + \sum_{k=1}^{J} \exp(\delta_{kt} + x_{kt}\sigma_t\nu_{it})}$$

where $ns$ is the number of draws of $\nu_i$. BLP shows that we can use contraction mapping to invert the market shares to get the mean utility $\delta$,

$$\delta_{jt}^{'} = \delta_{jt} + \ln(s_{jt}) - \ln(\hat{s}_{jt}(\delta, \sigma))$$

The algorithm is as follows. First make $ns$ draws of $\nu_i$ from a one-dimensional standard normal distribution, guess $\delta$, compute the model prediction of $\hat{s}_{jt}$, and then solve for the contraction mapping problem for each market. The mean and standard deviation of $\delta$ is

$$mean(\hat{\delta}) = -8.364$$
$$sd(\hat{\delta}) = 0.854$$

## 1.3 GMM

After computing the mean utility $\delta$, we can compute the error term (also the unobserved product characteristics)

$$\xi_{jt} = \delta_{jt} - x_{jt}\beta + \alpha p_{jt} - \gamma_j = \delta_{jt}(\sigma) - X\theta_1 \equiv \omega(\theta)$$

where $\theta = (\beta, \alpha, \{\gamma\}_j, \sigma)$. Note that $\beta$, $\alpha$, and $\{\gamma\}_j$ enter this equation linearly, while $\sigma$ enters this equation nonlinearly. Let $Z$ be the set of instruments, then we have the moment condition

$$E[Z'\omega(\theta)] = 0$$

The GMM estimator is

$$\hat{\theta} = \arg\min_{\theta} \omega(\theta)' Z \Phi^{-1} Z' \omega(\theta)$$

where $\Phi = z'z/n$ is the weighting matrix. Nevo (2004) shows that the linear parameters can be written[1] as

$$\hat{\theta}_1 = [X_1' Z \Phi^{-1} Z' X_1]^{-1} X_1' Z \Phi^{-1} Z' \delta(\hat{\sigma})$$

[1]Note that $\omega(\theta) = \delta(\sigma) - X\theta_1$. Rewriting the GMM objective function and taking the first derivative yields the expression.

Instead of searching over the whole parameter space $\theta = (\beta, \alpha, \{\gamma\}_j, \sigma)$, we can just search over the nonlinear parameter $\sigma$ and then optimizer over the linear parameters $\theta_1 \equiv (\beta, \alpha, \gamma)$. The detailed algorithm is

- Guess $\sigma$ and compute $\delta(\sigma)$
- Compute the linear parameter $\hat{\theta}_1$ using $\hat{\theta}_1 = [X_1' Z \Phi^{-1} Z' X_1]^{-1} X_1' Z \Phi^{-1} Z' \delta(\hat{\sigma})$
- Compute the error term by plugging $\hat{\theta}_1$ and $\delta(\sigma)$ back into $\omega(\theta)$
- Compute the GMM objective function $\omega(\theta)' Z \Phi^{-1} Z' \omega(\theta)$ and find the minimum

Table 2 reports the model parameters.

Table 2: Estimated Parameters $\theta$

| | |
|---|---|
| $\sigma$ | 28.720 |
| Intercept | $-71.521$ |
| Price | $-1.085$ |
| Promotion | 0.222 |