Assignment for Convex Optimization

1. Introduction

Convex optimization can be used to solve many problems that arise in control. We have a system with a state $x_t \in R_n$ that varies over the time steps t=0,…,T, and inputs or actions $u_t \in R_m$ we can use at each time step to affect the state. For example, x_t might be the position and velocity of a rocket, and u_t is the output of the rocket's thrusters. We model the evolution of the state as a linear dynamical system, i.e.

$$x_{t+1} = Ax_t + Bu_t$$

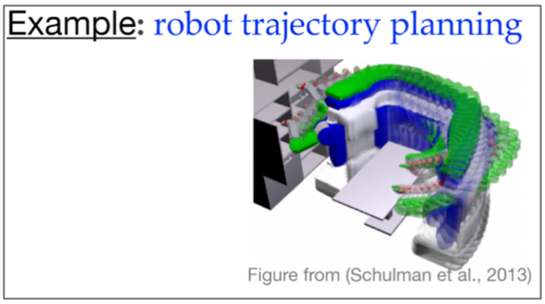
where $A \in R_{n\times n}$ and $B \in R_{n\times m}$ are known matrices.
Our goal is to find the optimal actions $u_t,\ldots, u_{T-1}$ by solving the optimization problems

$$\begin{array}{ll} \text{minimize} & \sum_{t=0}^{T-1} l(x_t, u_t) + l_T(x_t) \\ \text{subject to} & x_{t+1} = Ax_t + Bu_t \\ & (x_t, u_t) \in \mathrm{C}, x_t \in C_T, \end{array}$$

where ℓ: $R^n \times R^m \rightarrow R$ is the stage cost, ℓ_T is the terminal cost, C is the state/action constraints, and C_T is the terminal constraint. The optimization problem is convex if the costs and constraints are convex.

2. Assignment

Now, we want you to solve the example mentioned in our course using **CVXPY**.



The robot is now in a two-dimensional space.

Given robot state $x_t \in \mathrm{R}^2$ and control inputs $u_t \in \mathrm{R}^2$. The objective is

$$\text{Minimize} \quad \sum_{t=0}^{T-1} \|x_t - x_{t+1}\|_2^2 + \|u_t\|_2^2$$

The robot starts from point **(0.1, 5)** and wants to reach the point **(0, -5)**. The moving speed in the X direction shall not exceed **1**, and the moving speed in the Y direction shall not exceed **1**. The value of T shall not exceed **15**. The dynamics of the system is

$$x_{t+1} = x_t + u_t$$

The robot cannot collide with obstacles (when the robot is on the boundary of the obstacle, we think there is no collision).

Consider the three **conditions** below:

① There is no obstacle.

② There is a circular obstacle at point **(0, 0)** and its radius is **2**.

③ There is a circular obstacle at point **(1, 1)** and its radius is **1** and a circular obstacle at point **(-0.9, -1)** and its radius is **1**.

For each condition, you are required to design a few convex constraints and solve the convex problem.

Note that conditions 2 and 3 cannot be directly used as constraints since they are not convex. So you need to think out convex constraints yourself to avoid these obstacles. We expect your constraints to be more concise while requiring as fewer time steps as possible.

You are also required to **draw the resulting trajectory of the robot in a coordinate**.

3. Submission

You need to submit a detailed report and your code. Your report should be in PDF format

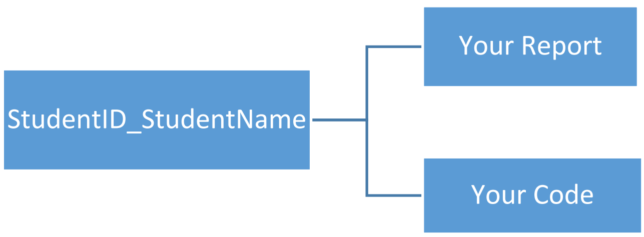


Figure 1 All the things you have to submit

4. Grading

This project will be counted towards 20% of your final course grade. The weights for different tasks are as follows:

- Complete the code [10% in total]

Complete the code of Condition 1	[5%]
Complete the code of Condition 2	[3%]
Complete the code of Condition 3	[2%]

- Complete the report [10% in total]

Describe the object and subject of the optimization problem	[5%]
Show the trajectory of the robot in a coordinate	[4%]
The report is required to be concise and direct	[1%]