# Structure preserving low-rank algorithms for plasma simulations: Exercises

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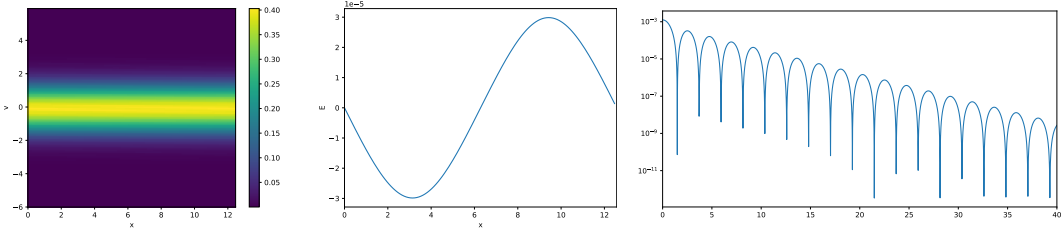
Link to slides: `http://www.einkemmer.net/training.html`

# Lab Period I

# Exercise 1.1

Based on the template `exercise1/lr_template.py` develop a projector splitting based dynamical low-rank algorithm for the Vlasov–Poisson equation.

You should get the following results (Landau damping).



# Exercise 1.2

Try a more challenging problem, e.g. the two-stream instability given by

$$f(0,x,v) = \frac{1}{2}(1+10^{-3}\cos(0.2x))\left(\mu(v-2.4)+\mu(v+2.4)\right), \qquad \mu(v) = \frac{\exp(-v^2/2)}{\sqrt{2\pi}}$$

on $\Omega = [0, 10\pi]$.

What rank $r$ do you need to get good results? Compare this to the Landau damping problem in exercise 1.

# Lab Period II

# Exercises

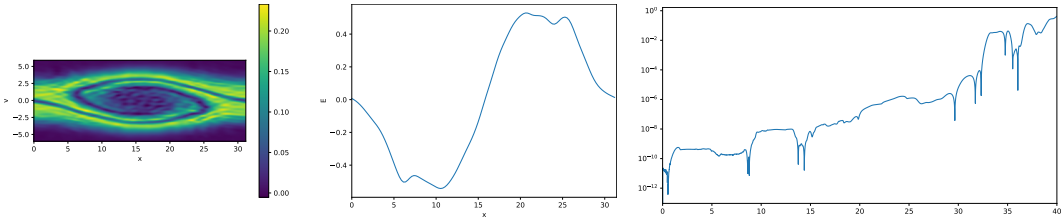
I would suggest you make a choice between

- **Exercise 2** focus on algorithmic aspects and conservation.
- **Exercise 3** focuses on efficient implementation using our low-rank framework Ensign.

# Exercise 2.1

Based on the template `exercise2/lr-conservative_template.py` implement the augmented BUG integrator in the function `time_step_augBUG`.

Check how well mass is conserved for the two-stream instability with $r = 20$.

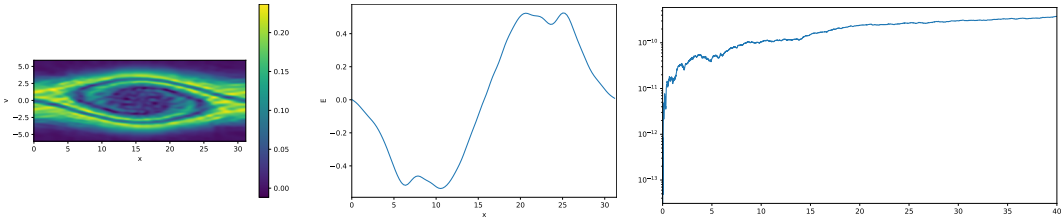



**Mass error becomes large!**

# Exercise 2.2

Based on the template `exercise2/lr-conservative_template.py` implement the conservative BUG integrator in the function `time_step_consBUG`.

Check how well mass is conserved for the two-stream instability with $r = 20$.




**Mass error is close to machine precision!**

# Exercise 3

Based on the template `exercise3/main_template.cpp` develop a projector splitting based dynamical low-rank algorithm for the Vlasov–Poisson equation.

- Rename `main_template.cpp` to `main.cpp`

Instructions of how to build the software can be found in `exercise3/Readme.md`.

- This downloads Ensign and all dependencies automatically.