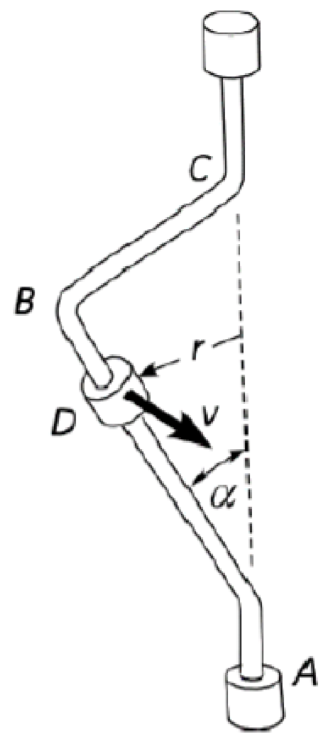
Problem 1

Newton's 2nd Law: Polar Coordinates

Example 2.13: A small 8-oz collar *D* can slide on portion *AB* of a rod which is bent as shown. Knowing that the rod rotates about the vertical *AC* at a constant rate and that α = 40 degrees and r = 24 inches,

Determine the range of values of the speed *v* for which the collar will not slide on the rod if the coefficient of static friction between the rod and collar is 0.35.

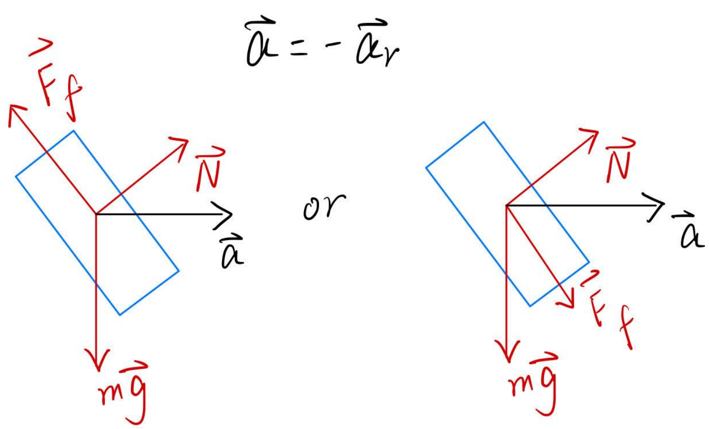


Recall

- Curvilinear motion
  - $a_r = \ddot{R} - R\dot{\theta}^2$, direction is outward the center of curvature
  - $a_\theta = R\ddot{\theta} + 2\dot{R}\dot{\theta}$, direction is perpendicular to the radial direction
- Friction: $F_f \leq \mu F_N$, where F_N is the normal force

Analysis

- The rod and the collar is in rotation. The faster they rotate, the further the collar is away from the center of rotation. The question is about `the range of speed`, i.e. the **maximum** speed when the collar is going up and the **minimum** speed when the collar is going down.
- $\sum F_\theta = 0$, due to constant angular velocity
- $\sum F_Z = ma_Z$, where $a_Z = 0$ because the question is about collar will not slide along the rod.
- $\sum F_r = a_r$,
- Draw FBD



> Note: the normal force is perpendicular to the surface, and the friction force is parallel to the surface which is either up or down.
> When the object tends to slide down but is not sliding, the friction force is up. When the object tends to slide up but is not sliding, the friction force is down.

Solution

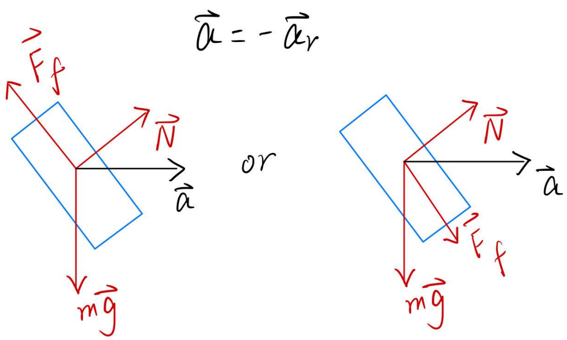
- $\sum F_Z = ma_Z$:
  For first case, the friction is up:

$$F_f \cos\alpha + N \sin\alpha - mg = ma_Z = 0$$

  For second case, the friction is down:

$$-F_f \cos\alpha + N \sin\alpha - mg = ma_Z = 0$$

  So, $F_f = \pm\mu N$, $N = \frac{mg}{\sin\alpha \pm \mu\cos\alpha}$, where $+$ is for the first case and $-$ is for the second case.



- $\sum F_r = a_r$:

$$F_f \sin\alpha - N\cos\alpha = ma_r$$
$$\pm\mu N \sin\alpha - N\cos\alpha = ma_r$$
$$a_r = \frac{N}{m}(\pm\mu\sin\alpha - \cos\alpha)$$
$$a_r = \frac{1}{m}\frac{mg}{\sin\alpha \pm \mu\cos\alpha}(\pm\mu\sin\alpha - \cos\alpha) = \frac{g}{\sin\alpha \pm \mu\cos\alpha}(\pm\mu\sin\alpha - \cos\alpha)$$

Since the collar is not sliding, no change in radius i.e. $\ddot{r} = 0$, so $a_r = \ddot{r} - r\dot{\theta}^2 = -r\dot{\theta}^2$. And $\dot{\theta} = \sqrt{-\frac{a_r}{r}}$

Plug in the expression of a_r:

$$\dot{\theta} = \sqrt{-\frac{g}{r}\frac{\pm\mu\sin\alpha - \cos\alpha}{\sin\alpha \pm \mu\cos\alpha}}$$

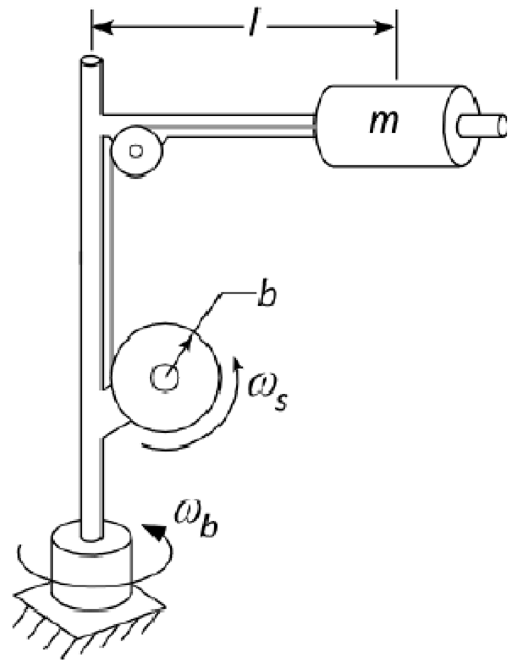
Plug the value of r and α: $\dot{\theta} = 3.09$ or $\dot{\theta} = 6.52 rad/s$

$$\vec{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta$$
$$\vec{v} = 0\hat{u}_r + r\dot{\theta}\hat{u}_\theta$$
$$\vec{v}_{min} = 6.18\hat{u}_\theta ft/s$$
$$\vec{v}_{max} = 13.04\hat{u}_\theta ft/s$$

Problem 2

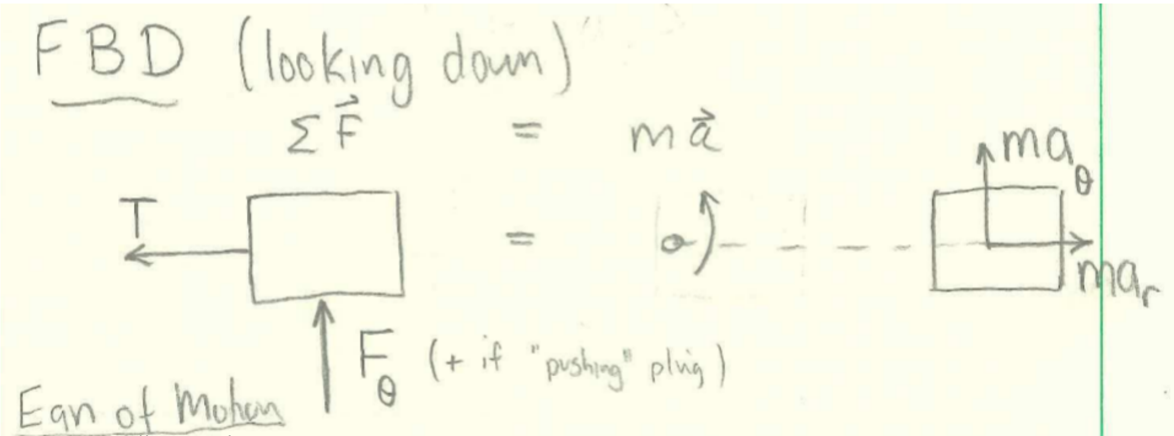
Newton's 2nd Law: Polar Coordinates

Example 2.12: A cylinder of mass m slides on a horizontal rod which is fixed to a vertical rod. The vertical rod is rotating at a fixed angular velocity, ω_b. The cylinder is prevented from moving outward with a cable. The cable length is changed by operating a winch which rotates with an angular velocity, ω_s, and has a drum radius, b. The initial cable length is l.



Determine the tension in the cable and the reaction force between the cylinder and the horizontal rod as a function of time.

Recall and Analysis



- $a_r = \ddot{R} - R\dot{\theta}^2$, direction is outward the center of curvature
- $a_\theta = R\ddot{\theta} + 2\dot{R}\dot{\theta}$, direction is perpendicular to the radial direction

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Solution

- Transverse direction: $\sum F_\theta = ma_\theta = m(R\ddot{\theta} + 2\dot{R}\dot{\theta})$
- Radial direction: $\sum F_r = ma_r = m(\ddot{R} - R\dot{\theta}^2)$

For radical direction, since T is inwards and a_r is outward, so:

$$
\begin{aligned}
-T &= ma_r \\
T &= -ma_r \\
T &= m(R\dot{\theta}^2 - \ddot{R})
\end{aligned}
$$

where, $R = l - w_s bt$ and $\dot{R} = -w_s b$, $\ddot{R} = 0$; also $\dot{\theta} = w_b$
So, $T = m(l - w_s bt)w_b^2$

For transverse direction:

$$F_\theta = ma_\theta$$

$$F_\theta = m(R\ddot{\theta} + 2\dot{R}\dot{\theta})$$

where, $R = l - w_s bt$ and $\dot{R} = -w_s b$, $\ddot{R} = 0$; also $\dot{\theta} = w_b$, $\ddot{\theta} = 0$
So, $F_\theta = -2mw_s bw_b$, if w_s is positive, then it moves inward, otherwise it moves outward.