Problem 1

Problem 5.15

In an unfortunate incident, a 2.75 kg laptop computer is dropped onto the floor from a height of 1 m. Assuming that the laptop starts from rest, that it rebounds off the floor up to a height of 5 cm, and that the contact with the floor lasts 10^{-3} s, determine the impulse provided by the floor to the laptop and the average acceleration to which the laptop is subjected when in contact with the floor (express this result in terms of g, the acceleration of gravity).

Photo credit: © Chip Simons/Jupiter Images

Analysis

> Let us set y axis pointing vertically up and break down the process into 4 stages:
>
> 1. the laptop starts dropping, $T_1 = 0,\ V_1 = mgh$.
> 2. the moment before the laptop hits the ground, where: $T_2 = \frac{1}{2}mv_2^2,\ V_2 = 0$.
> 3. the moment after laptop bounce off the ground, where: $T_3 = \frac{1}{2}mv_3^2,\ V_2 = 0$.
> 4. The laptop rebounds to the highest point of $h_4 = 5cm$, where: $T_4 = 0,\ V_4 = mgh_4$.

We can use conservation of energy between 1 and 2, 3 and 4. And we can use impulse-momentum principle between 2 and 3.

Solution

Using conservation of energy between stage 1 and 2, 3 and 4. we can obtain these equations:

$$mgh = \frac{1}{2}mv_2^2 \tag{1}$$
$$\frac{1}{2}mv_3^2 = mgh_4 \tag{2}$$

Solving these 2 equations, we will have: $v_2 = -\sqrt{2gh} = (-4.429m/s)\hat{j}$ (negative implies velocity vector pointing down) and $v_3 = \sqrt{2gh_4} = (0.991m/s)\hat{j}$.

And Using impulse-momentum principle between stage 2 and 3, we have:

$$\vec{p_2} + I = \vec{p_3}$$

and $I = \vec{p_3} - \vec{p_2} = 2.75 \times [0.991 - (-4.429)] = 14.9\ \hat{j} N \cdot s$.

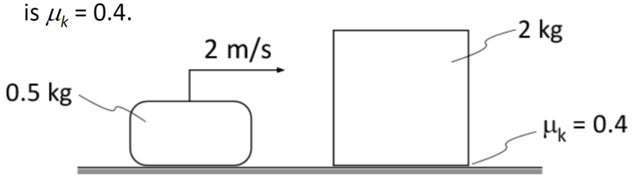
For average acceleration, we can use contact time $t = 10^{-3}s$ and change of velocity:

$$\bar{a} = \frac{v_3 - v_2}{t} = 5420\hat{j}\ m/s$$

Problem 2

Direct Central Impact

Example 4.6: The 0.5 kg block has a speed of 2 m/s at the instant it strikes a 2 kg block which is at rest. The coefficient of restitution between the two blocks is e = 0.4. The coefficient of friction between the blocks and the floor is μ_k = 0.4.



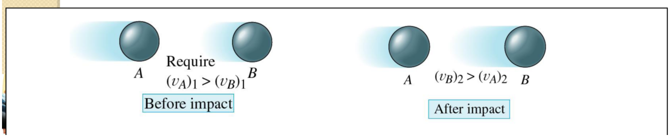
Determine

1. the speed of the blocks immediately after the impact.
2. the distance the block travels before it stops

Review

- Conversation of momentum(impact): $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$
- $$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

Defined along the line of impact (LOI)!



Solution

Part 1

Consider the impact between the two blocks, we have the following equations:

$$\begin{cases} m_A\vec{v}_{A-} + m_B\vec{v}_{B-} = m_A\vec{v}_{A+} + m_B\vec{v}_{B+} \\ e = \frac{v_{B+} - v_{A+}}{v_{A-} - v_{B-}} \end{cases}$$

where e is the coefficient of restitution, and v_{A-}, v_{B-}, v_{A+}, v_{B+} are the velocities of block A and B before and after the impact, respectively.

Plug in the given values, we have:

$$\begin{cases} 0.5 \times 2 + 2 \times 0 = 0.5 \times v_{A+} + 2 \times v_{B+} \\ 0.4 = \frac{v_{B+} - v_{A+}}{2 - 0} \end{cases}$$

So, we have: $v_{A+} = -0.24 \text{ m/s}$, $v_{B+} = 0.56 \text{ m/s}$.

Part 2

As for the friction and energy of block B, we have:

$$T_1 + V_1 + U_{1-2} = T_2 + V_2$$

where T is the kinetic energy, V is the potential energy, and U is the work done by friction.

There is no change in potential energy, so we have:

$$T_1 + U_{1-2} = T_2$$

- T_1 is the kinetic energy of block B after impact, which is $T_1 = \frac{1}{2}m_B v_{B+}^2$.
- T_2 is the kinetic energy of block B when it stops, so $T_2 = 0$.
- U_{1-2} is the work done by friction, which is $U_{1-2} = -\mu_k m_B g d$. The negative sign is because the friction force is in the opposite direction of the displacement.
- So, $d = \frac{1}{2\mu_k g} v_{B+}^2 = 0.04 \text{ m}$.