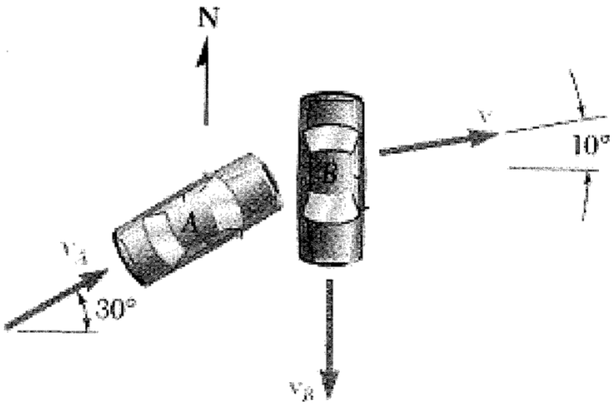
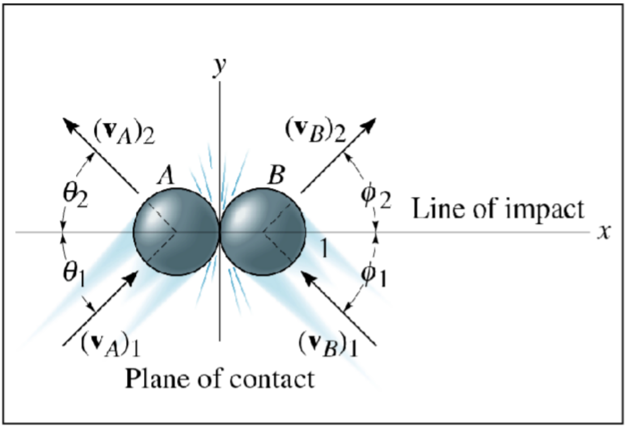
Problem 1

At an intersection car B was traveling south and car A was traveling 30° north of east when they slammed into each other. Upon investigation it was found that after the crash the two cars got stuck and skidded off at an angle of 10° north of east. Each driver claimed that he was going at the speed limit of 30 mi/h and that he tried to slow down but couldn't avoid the crash because the other driver was going a lot faster. Knowing that the weights of cars A and B were 3600 lb and 2800 lb, respectively, determine (a) which car was going faster, (b) the speed of the faster of the two cars if the slower car was traveling at the speed limit.



Recall & Analysis



Some Key concepts and equations for oblique impact.

Plane of contact: The plane where two objects contact. Perpendicular to line of impact.

Line of impact: where internal deformation and restitution impulses takes place, hence results obtained in the previous lecture (about 1-D impact) can be applied along LOI. Line of impact is set as local $x - axis$ in the following equations.

(1) conservation of momentum:

$$m_A(v_{Ax})_1 + m_A(v_{Bx})_1 = m_A(v_{Ax})_1 + m_A(v_{Bx})_2$$

(2) relative speed before and after the impact is related through the coefficient of restitution:

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$$

(3) There is no impulsive forces perpendicular to the line of impact, so momentum in plan of impact direction is conserved for each object:

$$m_A(v_{Ay})_1 + m_A(v_{By})_1 = m_A(v_{Ay})_1 + m_A(v_{By})_2$$

Or in most cases without friction and other types of forces on plane of impact:

$$(v_{Ay})_1 = (v_{Ay})_2$$
$$(v_{By})_1 = (v_{By})_2$$

In today's problem, the speed of car A and B before and after collision can be described as:

$$(v_A)_1 = v_A cos30^o \cdot \hat{i} + v_A sin30^o \cdot \hat{j} \tag{1}$$
$$(v_B)_1 = 0 \cdot \hat{i} + -v_B \cdot \hat{j} \tag{2}$$
$$(v_A)_2 = (v_B)_2 = v\ cos10^o \cdot \hat{i} + v\ sin10^o \cdot \hat{j} \tag{3}$$

Applying the conclusions above to this problem, we can have these equations:

momentum conservation along LOI:

$$m_A(v_{Ax})_1 + m_A(v_{Bx})_1 = (m_A + m_B)(v_x)_2$$

and momentum conservation on plane of impact:

$$m_A(v_{Ay})_1 + m_A(v_{By})_1 = (m_A + m_B)(v_y)_2$$

Solution

(1) Plugging in the known values to our equations, we have:

x-direction: $m_A v_A cos30^o = (m_A + m_B)v\ cos10^o$ (1)

y-direction: $m_A v_A sin30^o - m_B v_B = (m_A + m_B)v\ sin10^o$ (2)

Though we have 3 unknowns(v_A, v_B and v), we only need the ratio v_B/v_A to determine who is faster.

by (1)/(2) and rearrange to solve v_B/v_A, we have:

$$\frac{v_B}{v_A} = 0.3473\frac{m_B}{m_A} = 0.4465$$

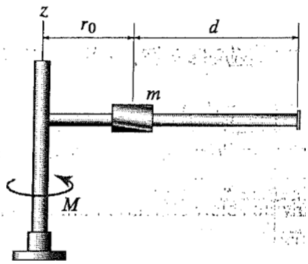
Therefore, $v_A > v_B$.

(2) Given $v_B = 30mph$, we have $v_A = v_B/0.4465 = 67.2mph$

Problem 2

A collar of mass m is initially at rest on a horizontal arm when a constant moment M is applied to the system to make it rotate. Assume that the mass of the horizontal arm is negligible and that the collar is free to slide without friction.

Problem 5.87 Derive the equations of motion of the system, taking advantage of the angular impulse-momentum principle. *Hint:* Applying the angular impulse momentum principle yields only one of the needed equations of motion.



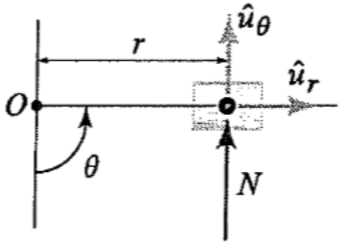
Recall & Analysis

- The ***equations of motion*** are functions of time that describe the position, velocity, and acceleration. In this problem, it's in the 2-D polar coordinate system, so we need **two** variables: r and θ to describe the position of the particle. To solve for two variables, we need two equations of motion.
- Equation 1: it's a curvilinear motion, so we could consider the acceleration equation:
  - $a_r = \ddot{R} - R\dot{\theta}^2$, direction is outward the center of curvature
  - $a_\theta = R\ddot{\theta} + 2\dot{R}\dot{\theta}$, direction is perpendicular to the radial direction

  > **Note**: we don't need both equations. Please analyze the problem and choose the one.

- Equation 2: impulse-momentum principle:$M = \dot{h}$, where h is the angular momentum of the particle about the center of curvature.

Solution



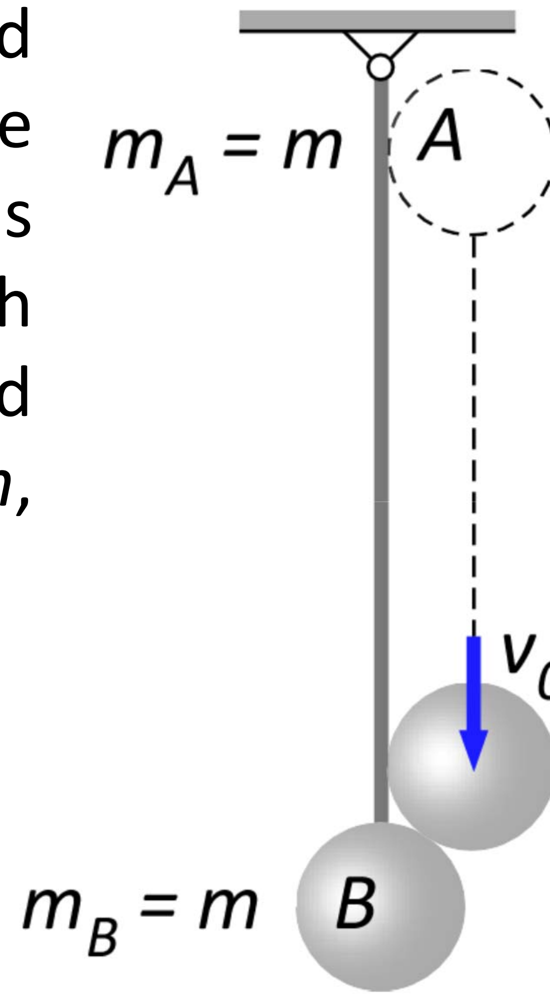
- Impulse momentum principle: $M = \dot{h}$, where $\vec{h} = \vec{r} \times m\vec{v}$
  - $\vec{r} = R\hat{u}_r$
  - $\vec{v} = \dot{R}\hat{u}_r + R\dot{\theta}\hat{u}_\theta$
  - $h = mR^2\dot{\theta}$, because $\vec{r} \times \vec{v} = mR^2\dot{\theta}\hat{u}_z$ where $\hat{u}_r \times \hat{u}_\theta = \hat{u}_z$ and $\hat{u}_r \times \hat{u}_r = 0$
  - $\dot{h} = mR^2\ddot{\theta} + 2mR\dot{R}\dot{\theta}$
  - $M = mR^2\ddot{\theta} + 2mR\dot{R}\dot{\theta}$
- Curvilinear motion: observe that the collar is not subject to any force in the r direction, so $a_r = 0$, this then implies: $\ddot{R} - R\dot{\theta}^2 = 0$

Summarize the equations of motion:

$$\begin{cases} \ddot{R} - R\dot{\theta}^2 = 0 \\ mR^2\ddot{\theta} + 2mR\dot{R}\dot{\theta} = M \end{cases}$$

Oblique Central Impact - Constrained

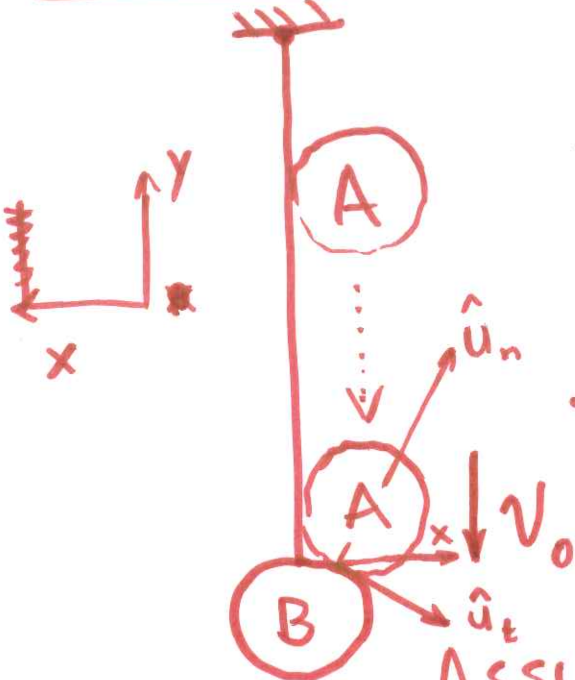
Example 4.9: Ball *A* is dropped and strikes ball *B* with a velocity v_0. The impact is *perfectly elastic*. Ball *B* is rigidly attached to a massless rod which is pinned at point O. The radius and mass of both balls are given as *r* and *m*, respectively.



Determine

The velocity of ball A and B immediately after impact.

Example: Impact with Constraints



- Ball A dropped and strikes ball B at speed v_0
- Find velocity of B and A after impact

Assumptions:

- radius of A and B equals r
- mass of A and B = m
- no friction
- Ball B constrained to move in circular arc by inextensible rod

Solving for unknows:

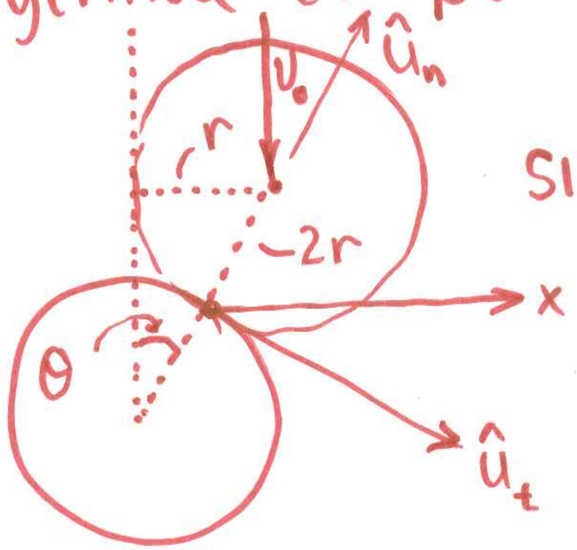
$$V_A^+ \Rightarrow (V_A^+)_n \text{ and } (V_A^+)_t$$

$$V_B^+ \Rightarrow (V_B^+)_x$$

⇨ Impulse and Momentum – Ball A

$$m\vec{V}_A^- + F\Delta t = m\vec{V}_A^+$$

tangential component:



$$\sin\theta = \frac{r}{2r} = \frac{1}{2}$$

$$\theta = 30°$$

$$m(V_A^-)_t + 0 = m(V_A^+)_t$$

impulse is zero in tangential direction (friction = 0)

$$(V_A^+)_t = (V_A^-)_t = V_0 \sin 30 = \frac{1}{2}V_0$$

$$(V_A^+)_t = \frac{1}{2}V_0 \quad (1)$$

⇨ Impulse and Momentum – Balls A + B

$$m\vec{V}_A^- + m\vec{V}_B^- + \underbrace{T\Delta t}_{\text{impulse due to force in rod}} = m\vec{V}_A^+ + m\vec{V}_B^+$$

X – components:

$$\underbrace{m(V_A^-)_x}_{0} + \underbrace{(T\Delta t)_x}_{\phi} = m(V_A^+)_x + m(V_B^+)_x$$

ϕ (in horizontal direction) } due to constraint rod only (internal forces cancel)

$$0 = m(V_A^+)_x + m(V_B^+)_x$$

$$= m(V_A^+)_t \cos 30^\circ + m(V_A^+)_n \sin 30^\circ + m(V_B^+)_x$$

sub in (1) $(V_A^+)_t = \frac{1}{2}v_0$

$$0 = m\left(\tfrac{1}{2}v_0\right)\cos 30 + m(V_A^+)_n \sin 30^\circ + m(V_B^+)_x$$

$$\frac{1}{2}(V_A^+)_n + (V_B^+)_x = -0.433\,v_0 \quad (2)$$

⇨ Restitution Equation (normal direction)

$$e = 1$$

$$(V_B^+)_n - (V_A^+)_n = (V_A^-)_n - (V_B^-)_n$$

$$(V_B^+)_x \sin 30° - (V_A^+)_n = -v_0 \cos 30 - \phi$$

$$\frac{1}{2}(V_B^+)_x - (V_A^+)_n = -0.866\, v_0 \quad (3)$$

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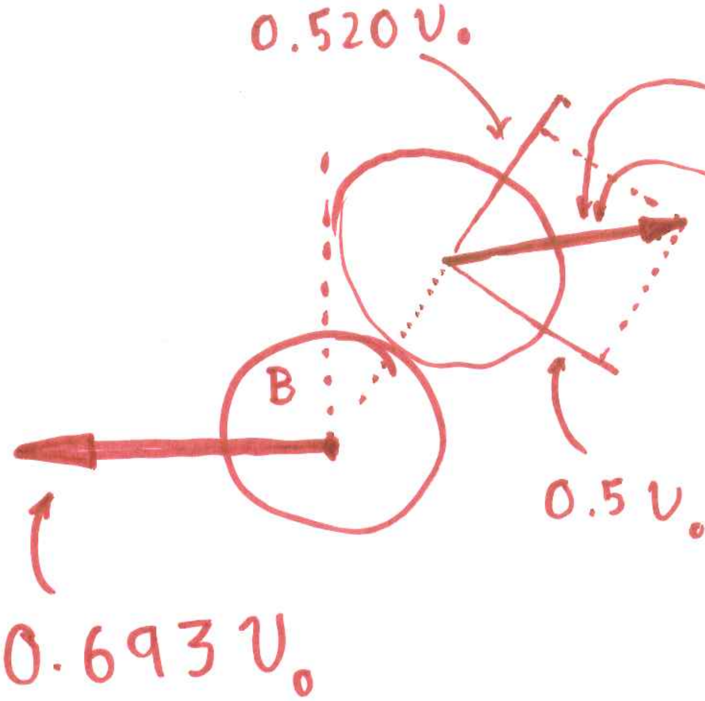
solving (2) and (3) for $(V_B^+)_x$ + $(V_A^+)_n$

$$(V_A^+)_n \cong +0.520\, v_0$$

$$(V_B^+)_x \cong -0.693\, v_0$$

motion (velocity) after impact



$$\vec{V}_A^+ = 0.520\, v_0\, \hat{u}_n + 0.5\, v_0\, \hat{u}_t$$

$$|\vec{V}_A^+| \cong 0.721\, v_0$$