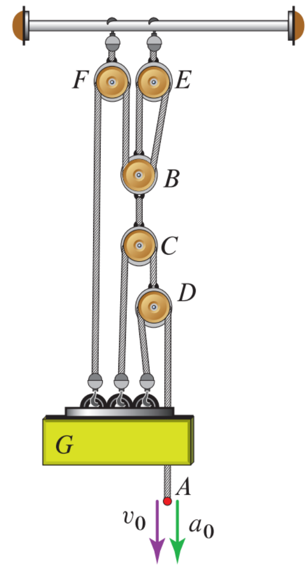
# Problem 1

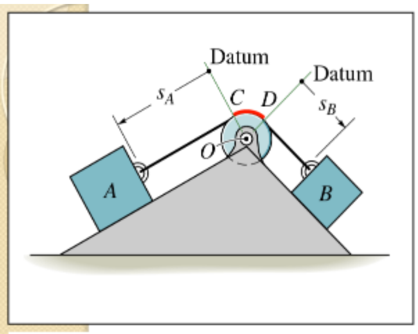
*Dynamics 1e* **259**

Example 2-21

Assuming that all ropes are vertically aligned, determine the velocity and acceleration of the load $G$ if $v_0 = 3\,\text{ft/s}$ and $a_0 = 1\,\text{ft/s}^2$.




## Recall & Analysis




In this example, think the cable length $l_{AB}$ as subdivided in three:

- the length in contact with the pulley, *CD*
- the length *CA*
- the length *DB*

In this geometry, no matter how *A* and *B* move, the *quantity* in contact with the pulley is constant. We could write:

$$s_A + l_{CD} + s_B = l_{AB} = \text{constant}$$

Differentiating with respect to time,

$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0 \quad \ldots\text{or}\ldots \quad v_B = -v_A$$

The length of all ropes are constant, so we can set up the equations with them.

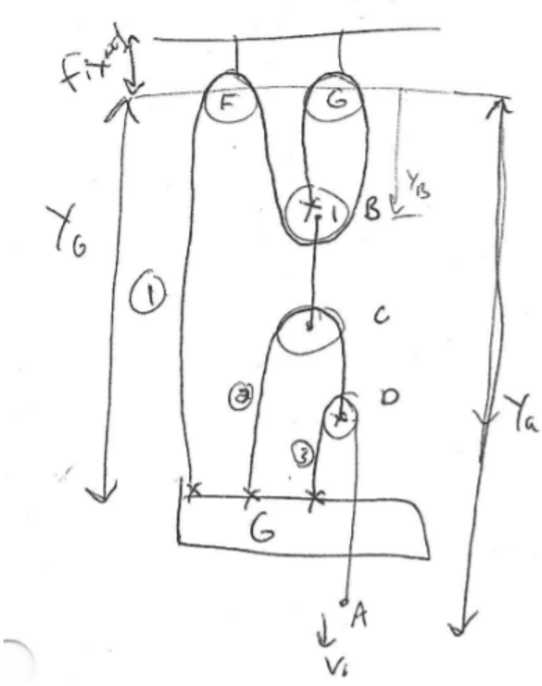
Here, we have three ropes: $L_1, L_2, L_3$ , $L_1$ is $G-F-B-E-B$, $L_2$ is $G-C-D$, $L_3$ is $G-D-A$. And their lengths are constant, so we can set up the equations with them.

# Solution

In case to find $v_G$ and $a_G$, we should first determine the expression of $y_G$. But we don't need the the explicit form of it, since our final goal is velocity and acceleartion.

First we need to set up our coordinates system, where $y$ axis points downwards.

As the lengths of all ropes($L_1, L_2, L_3$) don't change over time, we can set up the equations with them and coordinates of all pulleys.




$$\begin{cases} L_1 = y_G + 3y_B \\ L_2 = (y_G - y_C) + (y_D - y_C) \\ L_3 = (y_G - y_D) + (y_A - y_D) \\ L_4 = y_C - y_B \end{cases}$$

$$\begin{cases} L_1 = y_G + 3y_B \\ L_2 = y_G + y_D - 2y_C \\ L_3 = y_A + y_G - 2y_D \\ L_4 = y_C - y_B \end{cases}$$

Do derivative w.r.t. time, we have:

$$\begin{cases} 0 = \dot{y_G} + 3\dot{y_B} \\ 0 = \dot{y_G} + \dot{y_D} - 2\dot{y_C} \\ 0 = \dot{y_G} + \dot{y_A} - 2\dot{y_D} \\ 0 = \dot{y_C} - \dot{y_B} \end{cases}$$

So, $\dot{y_B} = \dot{y_C}$, and plug it into the first and second equation, we have: $\dot{y_D} = -\frac{5}{3}\dot{y_G}$, then plug it into the third equation, we have:

$$\dot{y_G} = -\frac{3}{13}\dot{y_A}$$

Placing $\dot{y_a} = v_0 = 3ft/s$ inside, we have $\dot{y_G} = -0.692ft/s$

As for acceleration, do time derivative, We have

$$\ddot{y_G} = -\frac{3}{13}\ddot{y_A}$$

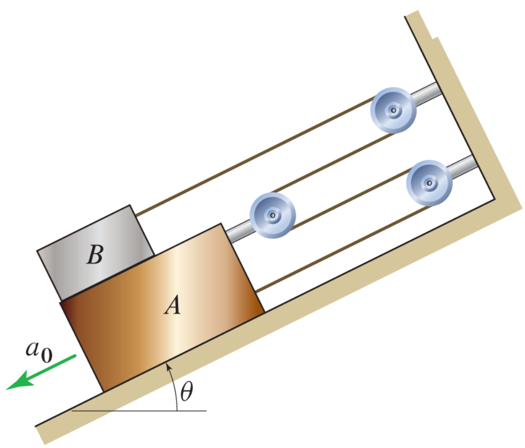
Where $\ddot{y_A} = a_0 = aft/s$, and $\ddot{y_G} = 0.231ft/s^2$

# Problem2



Example 2-22

Block $A$ is released from rest and starts sliding down the incline with an acceleration $a_0 = 3.7\,\mathrm{m/s^2}$. Determine the acceleration of block $B$ relative to the incline. Also, determine the time needed for $B$ to move a distance $d = 0.2\,\mathrm{m}$ relative to $A$.




## Recall & Analysis

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

Same as the previous problem, we can set up the equations with the lengths of all ropes.

## Solution

make $\vec{x}$ pointing downwards the slope.

$$L = 3x_A + x_B$$
$$0 = 3\dot{x}_A + \dot{x}_B$$
$$0 = 3\ddot{x}_A + \ddot{x}_B$$

using the fact $\ddot{x}_A = a_0$, we have

$$\vec{a}_B = -3\vec{a}_A = -3a_0 = -11.10m/s^2$$

where the positive direction is pointing downwards the incline.

$$\vec{x}_{B/A}(t) = \vec{x}_{B/A}(0) + \dot{\vec{x}}_{B/A}(0)t + \frac{1}{2}\vec{a}_{B/A}t^2$$

$$\vec{x}_{B/A}(t) = \vec{x}_{B/A}(0) + \dot{\vec{x}}_{B/A}(0)t + \frac{1}{2}(\vec{a}_B - \vec{a}_A)t^2$$
$$\vec{x}_{B/A}(t) = \vec{x}_{B/A}(0) + \dot{\vec{x}}_{B/A}(0)t + \frac{1}{2}(-3a_0 - a_0)t^2$$

Since the block is released from rest, we have $\dot{\vec{x}}_{B/A}(0) = 0$, and $\vec{x}_{B/A}(0) = 0$, so we have:

$$\vec{x}_{B/A}(t) = \frac{1}{2}(-4a_0)t^2$$
$$\vec{x}_{B/A}(t) = -2a_0t^2$$

When the distance $d = 0.2m$, we have:

$$-0.2 = -2a_0t^2$$
$$t = 0.1644s$$