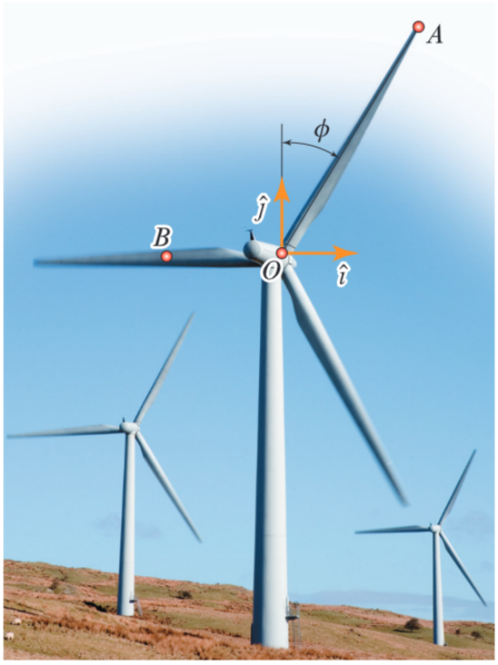
Problem 1

Problem 6.27

At the instant shown, the angle $\phi = 30°$, $|\vec{v}_A| = 292\,\text{ft/s}$, and the turbine is rotating clockwise. Letting $\overline{OA} = R$, $\overline{OB} = R/2$, $R = 182\,\text{ft}$, and treating the blades as being equally spaced, determine the velocity of point B at the given instant and express it using the component system shown.



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Recall

We first review the cross product of vectors.

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\hat{\imath} + (a_z b_x - a_x b_z)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}$$

Where $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit vectors of positive x, y, z direction. And by definition, we knows that: $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$, and $\vec{a} \times \vec{a} = \vec{0}$.

Specifically, we have:

$$\begin{gathered} \vec{\imath} \times \vec{\jmath} = \vec{k} \\ \vec{\jmath} \times \vec{k} = \vec{\imath} \\ \vec{k} \times \vec{\imath} = \vec{\jmath} \end{gathered}$$

For rigid body rotating to a fixed axis at A, for any point B on the object, we have:

$$\vec{v}_{B/A} = \vec{\omega}_{AB} \times \vec{R}_{B/A}$$

Solution

For this problem, we first find $\vec{r}$ and $\vec{\omega}$ for the fixed rotation

$$\vec{r}_{A/O} = R(sin\Phi \cdot \hat{i} + cos\Phi \cdot \hat{j})$$
$$\vec{r}_{B/O} = -R/2 \cdot \hat{i}$$

We can find ω using given velocity of A and radius. Since: $v_A = |\omega|R$, z axis is pointing outwards of the plane, and the turbine is rotating clockwise (angular vector pointing inwards), we have

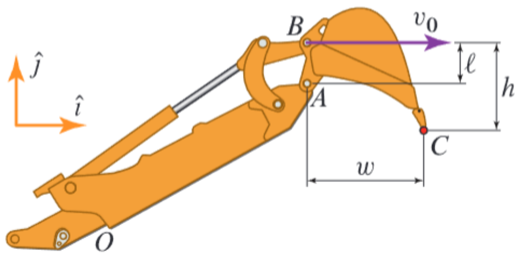
$$\vec{\omega} = -v_A/R \cdot \hat{k}$$

therefore,

$$\begin{aligned}
\vec{v}_B &= \vec{\omega} \times \vec{R} && (1)\\
&= -v_A/R \cdot \hat{k} \times (-\frac{R}{2} \cdot \hat{i}) && (2)\\
&= \frac{v_A}{2}\hat{j} && (3)\\
&= 146\hat{j}\ ft/s && (4)
\end{aligned}$$

Problem 2

The bucket of a backhoe is being operated while holding the arm OA fixed. At the instant shown, point B has a horizontal component of velocity $v_0 = 0.25\,\mathrm{ft/s}$ and is vertically aligned with point A. Letting $\ell = 0.9\,\mathrm{ft}$, $w = 2.65\,\mathrm{ft}$, and $h = 1.95\,\mathrm{ft}$, determine the velocity of point C. In addition, assuming that, at the instant shown, point B is not accelerating in the horizontal direction, compute the acceleration of point C. Express your answers using the component system shown.



Recall

- Rotation of a Rigid Body About a Fixed Axis:

$$\boxed{\vec{v}_P = \vec{\omega} \times \vec{r}}$$

$$\boxed{\vec{a}_P = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \vec{\alpha} \times \vec{r} - \omega^2 \vec{r}}$$

- For this problem, we regard the rotation around the A axis as the fixed axis rotation. And the rotation positive direction is counter-clockwise.
  - For example:

$$\vec{v}_{B/A} = \vec{\omega}_{AB} \times \vec{r}_{B/A}$$
$$v_0\hat{i} = \omega_{AB}\hat{k} \times l\hat{j}$$

where $\vec{v}_{B/A}$ is the velocity of point B relative to point A, $\vec{\omega}_{AB}$ is the angular velocity of the rigid body, and $\vec{r}_{B/A}$ is the position vector of point B relative to point A.

Solution

Since the bucket is in a fixed axis rotation about point A, we have:

$$\vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A} = v_0\hat{i}$$
$$v_0\hat{i} = \omega_{AB}\hat{k} \times l\hat{j}$$
$$-\omega_{AB}l\hat{i} = v_0\hat{i}$$
$$\omega_{AB} = -\frac{v_0}{l} = -0.2778 rad/s$$

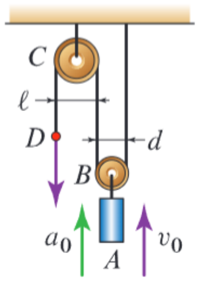
Plug in the value of ω_{AB}, we have:

$$\vec{v}_C = \omega_{AB}\hat{k} \times \vec{r}_{C/A} = \omega_{AB}\hat{k} \times [w\hat{i} - (h-l)\hat{j}]$$
$$\vec{v}_C = (h-l)\omega_{AB}\hat{i} + w\omega_{AB}\hat{j}$$
$$\vec{v}_C = (-0.292\hat{i} - 0.736\hat{j}) ft/s$$

Since $\alpha_{AB} = 0$, $\vec{a}_C = -\omega_{AB}^2\vec{r}_{C/A} = -v_0^2[w\hat{i} - (h-l)\hat{j}]$,which means that:
$\vec{a}_C = (-0.204\hat{i} + 0.081\hat{j}) ft/s^2$

Problem 3

At the instant shown, A is moving upward with a speed $v_0 = 5\,\mathrm{ft/s}$ and acceleration $a_0 = 0.65\,\mathrm{ft/s^2}$. Assuming that the rope that connects the pulleys does not slip relative to the pulleys and letting $\ell = 6\,\mathrm{in.}$ and $d = 4\,\mathrm{in.}$, determine the angular velocity and angular acceleration of pulley C.



Recall

We can define the length of the rope as:

$$L = y_D + 2y_A$$

Taking two time derivatives of this equation, and remembering that the overall length isconstant because the rope is inextensible, we have:

$$0 = \dot{y}_D + 2\dot{y}_A \rightarrow \dot{y}_D = -2\dot{y}_A = 2v_0 \tag{5}$$
$$0 = \ddot{y}_D + 2\ddot{y}_A \rightarrow \ddot{y}_D = -2\ddot{y}_A = 2a_0 \tag{6}$$

Solution

Because the point Q is contacting the rope, and the rope is not slipping, point Q has thesame velocity and vertical component of acceleration as D.

$$\vec{v}_Q = \vec{v}_D = 2v_0 = \omega_C \frac{l}{2} \rightarrow \vec{\omega}_C = \frac{4v_0}{l} = 40.0\hat{k} rad/s \tag{7}$$
$$\vec{a}_{Qy} = \vec{a}_D = 2a_0 = \alpha_C \frac{l}{2} \rightarrow \vec{\alpha}_C = \frac{4a_0}{l} = 5.20\hat{k} rad/s^2 \tag{8}$$