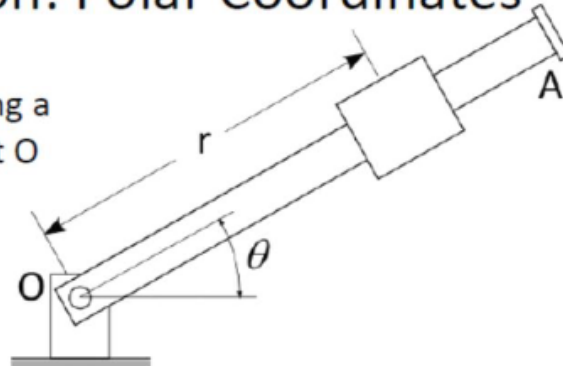


Problem 1

Curvilinear Motion: Polar Coordinates

Example: A collar slides along a rod. The distance from point O to the center of the collar, r , and the angle of the rod relative to horizontal, θ , are given as a function of time.



$$\theta = 0.15t^2$$

$$r = 0.9 - 0.12t^2$$

When the rod angle, θ , reaches 30° , determine the velocity and acceleration of the collar.

Solution

The question is to find the \vec{v} and \vec{a} at $\theta = 30^\circ$ in radial-transverse coordinates.

Recall the position vector in radial-transverse coordinates taught in class:

Polar Coordinates: Position and Velocity

- The position is simply the radial coordinate

$$\vec{r} = r \hat{u}_r.$$

where:

$$\vec{r} = r \hat{e}_r = (0.9 - 0.12t^2) \hat{e}_r$$

Recall the velocity vector in radial-transverse coordinates taught in class:

$$\begin{aligned}\vec{v} &= \dot{r} \hat{u}_r + r\dot{\theta} \hat{u}_\theta \\ &= v_r \hat{u}_r + v_\theta \hat{u}_\theta,\end{aligned}$$

$$v_r = \dot{r} \quad \text{and} \quad v_\theta = r\dot{\theta}.$$

where:

$$\vec{v} = \dot{r} \hat{e}_r + r\dot{\theta} \hat{e}_\theta$$

to get \dot{r} and $\dot{\theta}$, we need to do some differentiation:

$$\begin{cases} \dot{r} = -0.24t \\ \dot{\theta} = 0.3t \end{cases}$$

therefore velocity is:

$$\vec{v} = -0.24t\hat{e}_r + (0.9 - 0.12t^2)(0.3t)\hat{e}_\theta$$

When $\theta = 30^\circ$:

recall $\theta = 0.15t^2$, so $t^2 = \theta/0.15$, i.e.: $t = \sqrt{\frac{\theta}{0.15}} = 1.87s$

$$\vec{v} = -0.45\hat{e}_r + 0.27\hat{e}_\theta$$

for acceleration, recall the formula taught in class:

$$\begin{aligned}\vec{a} &= (\ddot{r} - r\dot{\theta}^2)\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta \\ &= a_r \hat{u}_r + a_\theta \hat{u}_\theta,\end{aligned}$$

$$\begin{aligned}a_r &= \ddot{r} - r\dot{\theta}^2, \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta}.\end{aligned}$$

$$\vec{a} = (\ddot{r} - r\omega^2)\hat{e}_r + (r\alpha + 2\dot{r}\omega)\hat{e}_\theta$$

where:

$$\begin{cases} r = 0.9 - 0.12t^2 \\ \dot{r} = -0.24t \\ \omega = \dot{\theta} = 0.3t \end{cases}$$

In acceleration, we know everything except \ddot{r} and α , to get them, we need to do some differentiation:

$$\ddot{r} = \frac{d\dot{r}}{dt} = -0.24, \alpha = \frac{d\omega}{dt} = 0.3$$

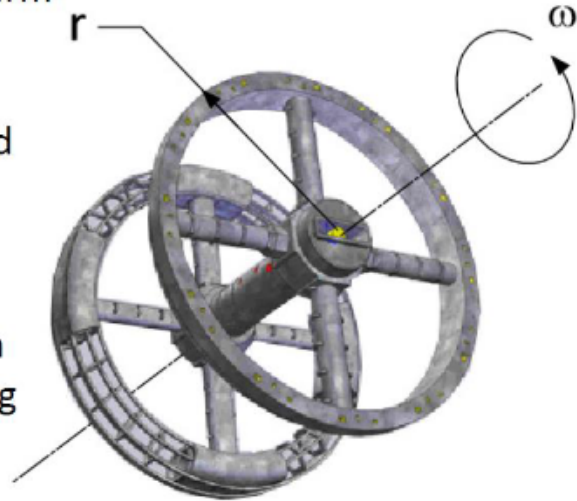
Now, we can get the acceleration:

$$\vec{a} = -0.391\hat{e}_r - 0.358\hat{e}_\theta$$

Problem 2

Example: A spinning spacecraft is to be designed to support long-term space voyage by simulating the effect of gravity. Determine the range of suitable spin rate (ω) and spacecraft radius, r , such that:

- simulated gravity is $> g/2$
- lateral acceleration $< g/20$ when moving from a sitting to standing position ($v = 2 \text{ m/s}$)



Solution

The acceleration of radial-transverse coordinates:

$$\vec{a} = (\ddot{r} - r\omega^2)\hat{e}_r + (r\alpha + 2\dot{r}\omega)\hat{e}_\theta$$

where: $\alpha = 0, \ddot{r} = 0$

therefore:

$$\vec{a} = -r\omega^2\hat{e}_r + 2\dot{r}\omega\hat{e}_\theta$$

The first term stands for simulated gravity term: g_s (since the direction of \hat{e}_r is outward, while the gravity is downward, so we need to add a negative sign):

$$g_s = -(-r\omega^2) > \frac{g}{2}$$

The second term stands for lateral acceleration a_L :

$$a_L = 2\dot{r}\omega < \frac{g}{20}$$

given the conditions $\dot{r} = 2 \text{ m/s}$, we have:

$$\begin{cases} r\omega^2 > g/2 \\ \omega < g/80 \end{cases}$$

$$\begin{cases} r > g/(2\omega^2) \\ \omega < g/80 \end{cases}$$

To get the minimum radius, we need to maximize ω because the radius is inversely proportional to ω^2 :

$$r_{min} = \frac{g}{2\omega^2} = \frac{g}{2(\frac{g}{80})^2} = \frac{3200}{g} \approx 326m (i.e. 1070ft)$$

