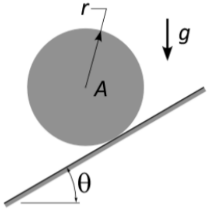
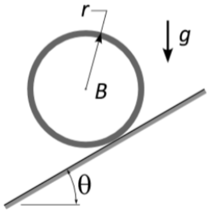
Problem 1

Work and Energy: Planar Rigid Bodies

Example 7.1: A thin-walled cylinder, *A*, and solid cylinder, *B*, of radius *r* are released from rest on a ramp included at an angle of θ. They both roll without slip. The mass of both cylinders is given as *m*.



Determine:
The velocity of both cylinders when they have rolled down the ramp a distance *b*.



Review and Hints

- Both cases are: $T_1 + V_1 + W_{1-2} = T_2 + V_2$
  - W_{1-2} is the work done by the unconservative force, like friction. Here, we set $W_{1-2} = 0$.
  - T is the kinetic energy, including the rotational kinetic energy and the translational kinetic energy.
- For the moment of inertia, we have:
  - $I_A = \frac{1}{2}mR^2$
  - $I_B = mR^2$

Solution

Part (a)

$$
\begin{aligned}
T_1 + V_1 &= T_2 + V_2 \\
mgb\sin\theta + 0 &= \frac{1}{2}mv_2^2 + \frac{1}{2}I_A\omega_2^2 \\
mgb\sin\theta &= \frac{1}{2}mv_2^2 + \frac{1}{2}(\frac{1}{2}mr^2)(\frac{v_2}{r})^2 \\
mgb\sin\theta &= \frac{1}{2}mv_2^2 + \frac{1}{4}mv_2^2 \\
mgb\sin\theta &= \frac{3}{4}mv_2^2 \\
v_2 &= \sqrt{\frac{4}{3}gb\sin\theta}
\end{aligned}
$$

Part (b)

$$
\begin{aligned}
T_1 + V_1 &= T_2 + V_2 \\
mgb\sin\theta + 0 &= \frac{1}{2}mv_2^2 + \frac{1}{2}I_B\omega_2^2 \\
mgb\sin\theta &= \frac{1}{2}mv_2^2 + \frac{1}{2}(mr^2)(\frac{v_2}{r})^2 \\
mgb\sin\theta &= \frac{1}{2}mv_2^2 + \frac{1}{2}mv_2^2 \\
mgb\sin\theta &= mv_2^2 \\
v_2 &= \sqrt{gb\sin\theta}
\end{aligned}
$$

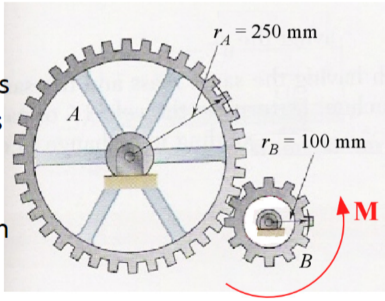
Problem 2

Example 7.4: Gear A has a mass of 10 kg and a radius of gyration of 200 mm, and gear B has a mass of 3 kg and a radius of gyration of 80 mm. The system is at rest when a couple M of magnitude 6 Nm is applied to gear B. Neglect friction.

Determine

(a) the number of revolutions executed by gear B before its angular velocity reaches 600 rpm

(b) the tangential force which gear B exerts on gear A.



Solution

(a) Find number of revolutions of B when $\omega_B = 600rpm$.(Where $600rpm = 20\pi\ rad/s$)

Again, we can apply work energy equation in this problem.

$$T_1 + V_1 + U_{unc} = T_2 + V_2$$

Where T_1, V_1 and V_2 are all 0. And $T_2 = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2$.

For work done by unconservative force: $U_{1\to2} = M \cdot \theta_B + F_T\theta_A r_A - F_T\theta_B r_B$.

As we learnt the gear equations: $\omega_A r_A = \omega_B r_B$, to integrate w.r.t. t on both sides, we have:

$$\theta_A r_A = \theta_B r_B$$

Plugging in the terms into the equation, we have:

$$M \cdot \theta_B = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2$$

Here $\omega_A = \omega_B \frac{r_B}{r_A} = 5\pi\ rad/s$, and $I_A = m_A k_A^2$, $I_B = m_B k_B^2$.

$$M\theta_B = \frac{1}{2}[m_A k_A^2(\frac{r_B}{r_A}) + m_B k_B^2]\omega_B^2$$

Plug in the values to have: $\theta_B = 27.34rad \approx 4.35r$

(b) This time we look at Gear B only.

$$T_1 + V_1 + U_{unc} = T_2 + V_2$$

Here $T_2 = \frac{1}{2}I_B\omega_B^2 = \frac{1}{2}m_B k_B^2\omega_B^2$

And $U_{1\to2} = M \cdot \theta_B - F_T\theta_B r_B$, where $\theta_B = 27.34rad$ as solved in part (a). Plug in the values into the equation:

$$M \cdot \theta_B - F_T\theta_B r_B = \frac{1}{2}m_B k_B^2\omega_B^2$$

And solving for $F_t = 46.15N$