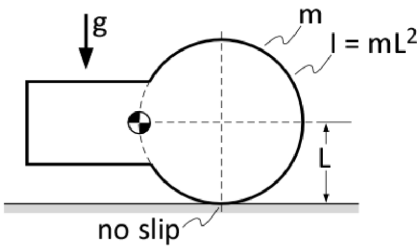
Problem 1

Example 6.4: the planar section shown is released from rest in the position shown. Immediately after release determine the following:



1. Angular acceleration
2. Minimum coefficient of friction to prevent slip

Review and Hints

- For the force analysis:
  - x-dimension: $-f = ma_x$
  - y-dimension: $N - mg = ma_y$
  - θ-dimension: $NL - fL = I\alpha = mL^2\alpha$
  - Hence, we have 3 equations but 5 unknowns: N, f, a_x, a_y, and α.
- Let's solve for a_x and a_y first:
  - $\vec{a}_c = \alpha(-L\hat{i})$
  - $\vec{a}_G = \vec{a}_c + \vec{\alpha} \times r_{\vec{G}/c} - \vec{\omega}^2 \times \vec{r}_{G/c}$
    - where $\vec{a}_G$ is the acceleration of point G, i.e. the center of mass
    - $\vec{a}_c$ is the acceleration of point c, i.e. the center of tthe circle
    - $\vec{\omega}$ is the angular velocity of relative rotation, which is **ZERO** in this case.
      - Because G and c are rotating together, so there is no relative rotation between them.
  - so, $\vec{a}_G = -\alpha L\hat{i} + \vec{\alpha} \times r_{\vec{G}/c} = -\alpha L\hat{i} + \alpha\hat{k} \times (-L\hat{i}) = -\alpha L\hat{i} - \alpha L\hat{j}$
  - $a_x = -\alpha L$, $a_y = -\alpha L$

Solution

$$f = mL\alpha$$
$$N - mg = -mL\alpha$$
$$NL - fL = mL^2\alpha$$

Hence, we have:

$$N = mg - mL\alpha$$
$$f = mL\alpha$$
$$\alpha = \frac{g}{3L}$$

for no slipping condition, we have: $f \leq \mu_s N$, so:

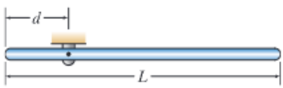
$$N = mg - mL\alpha = \frac{2}{3}mg$$
$$f = mL\alpha = \frac{1}{3}mg$$

And $f \leq \mu_s N$ becomes:

$$\frac{1}{3}mg \leq \mu_s \frac{2}{3}mg$$
$$\mu_s \geq \frac{1}{2}$$

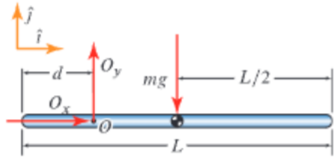
Problem 2

The uniform thin bar of length L and mass m is released from rest in the horizontal position shown. Determine the distance d at which the pin should be located from the end of the bar so that it has the maximum possible angular acceleration α_{max}. In addition, determine the value of α_{max}.



Solution

Based on the FBD shown, the Newton-Euler equations are



$$\sum F_x: \qquad O_x = ma_{Gx}, \tag{1}$$

$$\sum F_y: \qquad O_y - mg = ma_{Gy}, \tag{2}$$

$$\sum M_G: \quad -O_y\left(\frac{L}{2} - d\right) = I_G \alpha_{g\text{bar}}, \tag{3}$$

where α_{bar} is the angular acceleration of the bar and where

$$I_G = \frac{1}{12} mL^2, \tag{4}$$

is the mass moment of the bar about G. Since the bar is in a fixed axis rotation about O, we have

$$a_{Gx} = -\left(\frac{L}{2} - d\right)\omega_{\text{bar}}^2 = 0 \quad \text{and} \quad a_{Gy} = \left(\frac{L}{2} - d\right)\alpha_{\text{bar}}, \tag{5}$$

where ω_{bar} is the angular velocity of the bar. Substituting the kinematic equations and the expression for I_G into the Newton-Euler equations, we have a system of three equations in the three unknowns O_x, O_y, and α_{bar} whose solution is

$$O_x = 0, \quad O_y = \frac{1}{4}\frac{mgL^2}{3d^2 - 3dL + L^2}, \quad \alpha_{\text{bar}} = \frac{6dg - 3gL}{6d^2 - 6dL + 2L}. \tag{6}$$

We now proceed to find the value of d such that α_{bar} is maximum. Differentiating the expression for α_{bar} with respect to d and setting the result equal to zero, we have

$$-\frac{3}{2}g\frac{6d^2 - 6dL + L}{(3d^2 - 3dL + L^2)^2} = 0 \quad \Rightarrow \quad d_1 = \tfrac{1}{6}(3 - \sqrt{3})L \quad \text{and} \quad d_2 = \tfrac{1}{6}(3 + \sqrt{3})L. \tag{7}$$

There are two distinct values of d for which a relative extremum of α_{bar} is achieved. The values of α_{bar} corresponding to the two values of d just found are

$$\alpha_{\text{bar}} = \begin{cases} -\sqrt{3}g/L & \text{for } d = d_1, \\ \sqrt{3}g/L & \text{for } d = d_2. \end{cases}$$

Hence, the relative minimum and maximum values of α_{bar} are equal in absolute value and, if we do not distinguish between a clockwise and a counterclockwise rotation of the bar, both results are acceptable. Before providing the final answer, we need to verify whether or not absolute maximum values of α_{bar} are obtained for $d = 0$ and $d = L$. Indeed, for $d = 0$ and $d = L$, $|\alpha_{\text{bar}}| = 3g/(2L) < \sqrt{3}g/L$. Hence, the final result is

$$\boxed{|\alpha_{\text{bar}}|_{\text{max}} = \sqrt{3}\frac{g}{L} \quad \text{for } d = \tfrac{1}{6}(3 - \sqrt{3})L \text{ or } d = \tfrac{1}{6}(3 + \sqrt{3})L.}$$