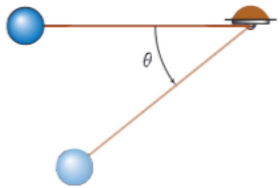
Problem 1

The pendulum is released from rest when $\theta = 0°$. If the string holding the pendulum bob breaks when the tension is twice the weight of the bob, at what angle does the string break? Treat the pendulum as a particle, ignore air resistance, and let the string be inextensible and massless.



Review

- FBD: two forces: F_c (the string tension) and mg (the weight of the ball)
- N-T coordinate system:
  - N dimension: $F_c - mg\sin\theta = ma_n$, where $a_n = v^2/L$
  - T dimension: $mg\cos\theta = ma_t$, where $a_t = \dot{v}$
- Energy conservation: since F_c is always perpendicular to the velocity, it does not do work. Therefore, the kinetic energy and the potential energy are conserved.
  - $T_0 + V_0 = T_1 + V_1$, where T is the kinetic energy and V is the potential energy.
  - Initial condition:
    - $T_0 = \frac{1}{2}mv_0^2 = 0$, because the ball is released from rest.
    - $V_0 = mg\triangle h = 0$, where $\triangle h$ is the height difference between the initial position and the current position.
    - $T_1 = \frac{1}{2}mv_1^2$, $V_1 = mg\triangle h_1$, where $\triangle h_1 = -L\sin\theta$.

Solution

Energy conservation:

$$
\begin{aligned}
\frac{1}{2}mv_1^2 + mg\triangle h_1 &= \frac{1}{2}mv_0^2 + mg\triangle h_0 \\
\frac{1}{2}mv_1^2 + mg\triangle h_1 &= 0 \\
\frac{1}{2}mv_1^2 &= -mg\triangle h_1 \\
\frac{1}{2}mv_1^2 &= mgL\sin\theta \\
v_1 &= \sqrt{2gL\sin\theta}
\end{aligned}
$$

N-T coordinate system:

N dimension:

$$
\begin{aligned}
F_c - mg\sin\theta &= ma_n \\
F_c - mg\sin\theta &= m\frac{v_1^2}{L} \\
F_c &= mg\sin\theta + m\frac{v_1^2}{L} \\
F_c &= mg\sin\theta + m\frac{2gL\sin\theta}{L} \\
F_c &= 3mg\sin\theta
\end{aligned}
$$

The tension threshold is $F_c = 2mg$, so:

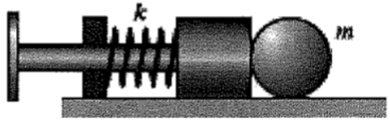
$$
\begin{aligned}
3mg\sin\theta &= 2mg \\
\sin\theta &= \frac{2}{3} \\
\theta &= \sin^{-1}\frac{2}{3} \\
\theta &= 41.8^{\circ}
\end{aligned}
$$

Problem 2

— Short derivation of work done by gravity + work done by springs

Dyncamics Discussion – 2/26

Assuming that the plunger of a pinball machine has negligible mass and that friction is negligible, determine the spring constant k such that a 2.85 oz ball is released with a speed $v = 15$ ft/s, after pulling back the plunger 2 in. from its rest position, i.e., from the position in which the spring is uncompressed.



Recall and Analysis

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

For springs, its potential energy is:

$$V = \frac{1}{2}k\Delta x^2$$

We can recall the equation of kinetic energy from 14.1:

$$T = \frac{1}{2}mv^2$$

Solution

Given $m = 2.85oz$

The ball starts from rest ($v_1 = 0$) with plunger compression of 2 inch. So the initial condition can be listed as: $v_1 = 0, \Delta x_1 = 2\ inch$

$$T_1 = \frac{1}{2}mv_1^2 = 0, \quad V_1 = \frac{1}{2}k\Delta x_1^2$$

The ball is released with speed $v_2 = v$, when the spring returns its rest position (and no longer)

Final Condition $v_2 = 15ft/s, \Delta x_2 = 0$

$$T_2 = \frac{1}{2}mv_2^2, \quad V_2 = \frac{1}{2}k\Delta x_2^2 = 0$$

List the Conservation of Energy equation,

$$T_1 + V_1 = T_2 + V_2$$

and plug in the terms of potential and kinetic energy, we have:

$$\frac{1}{2}k\Delta x_1^2 = \frac{1}{2}mv^2 \tag{1}$$

$$k = \frac{mv^2}{\Delta x_1^2} \tag{2}$$

$$k = 44.81lb/ft \tag{3}$$