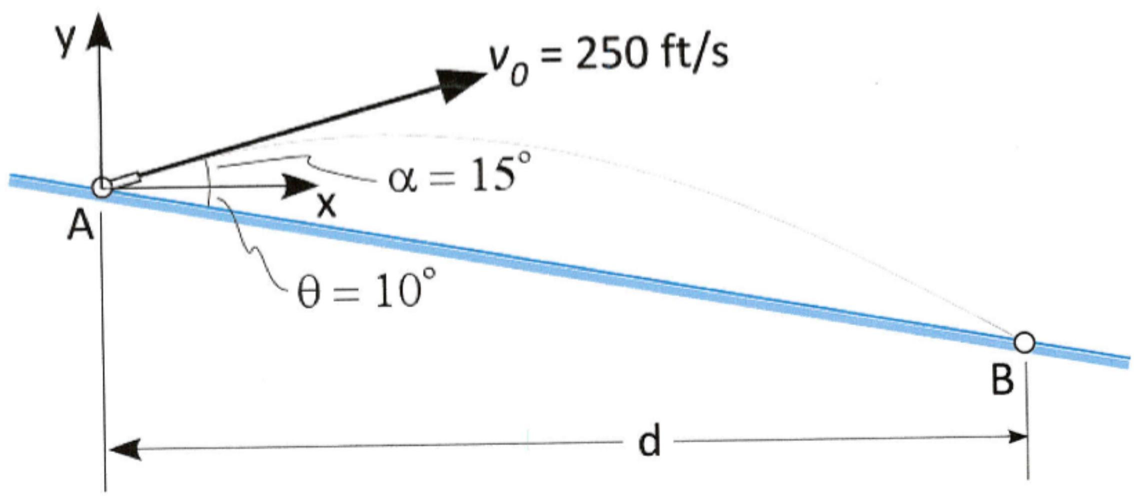
Problem 1

Example: A projectile is fired from a cannon with a muzzle velocity of 250 feet per second at an inclination of 15°. The cannon is placed on a hillside with slope of 10°. Determine the horizontal distance, *d*, traveled by the projectile before it strikes the ground at B



Solution

First, split the velocity and acceleration into x and y components:

$$a_x = 0$$
$$a_y = -g = -32.2\frac{ft}{s^2}$$
$$v_{0x} = v_0 \cos\alpha$$
$$v_{0y} = v_0 \sin\alpha$$

Then, we can work for the displacement in x and y directions:
For y direction:

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$
$$y = 0 + v_0 \sin\alpha t - \frac{1}{2}gt^2$$

For x direction:

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
$$x = 0 + v_0 \cos\alpha t + 0$$

Then, what about the relationship between x and y?
At point **B**, it should be:

$$y(t) = -x(t)\tan\theta$$

that is:

$$v_0 \sin\alpha t - \frac{1}{2}gt^2 = -v_0\cos\alpha t \tan\theta$$

so, the time t can be solved as:

$$t = \frac{2v_0 \sin\alpha}{g(1+\tan^2\theta)} \approx 6.6636s$$

Then, we can plug the time t into the equation of $x(t)$ to get the distance x:

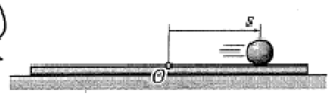
$$x = v_0\cos\alpha t = 250\cos 15 \times 6.6636 \approx 1609.1ft$$

Problem 2

In a physics experiment, a sphere with a given electric charge is constrained to move along a rectilinear guide with the following acceleration: $\boxed{a = a_0 \sin(2\pi s/\lambda)}$, where $a_0 = 8\,\mathrm{m/s^2}$, π is measured in radians, s is the position of the sphere measured in meters, $-\lambda \le s \le \lambda$, and $\lambda = 0.25\,\mathrm{m}$.

Suppose that the velocity of the sphere is equal to zero for $s = \lambda/4$. Determine the range of motion of the sphere, that is, the interval along the s axis within which the sphere moves. *Hint:* Determine the speed of the sphere and the interval along the s axis within which the speed has admissible values.

Determine v(s)



Solution

Recall the chain rule:

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$$

So, we can get:

$$a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v$$

Then, we can get:

$$ads = vdv$$

Now, we can integrate both sides, but what about the bounds?
Recalling $v = 0$ for $s = \frac{\lambda}{4}$, so we can integrate from $v = 0$ to $v = v$ and from $s = \frac{\lambda}{4}$ to $s = s$:

$$\int_0^v vdv = \int_{\frac{\lambda}{4}}^s ads$$

$$\int_0^v v dv = \int_{\frac{\lambda}{4}}^s a_0 \sin(2\pi s/\lambda) ds$$

which gives:

$$\frac{1}{2}v^2 = -\frac{a_0\lambda}{2\pi}\cos(2\pi s/\lambda)\bigg|_{\frac{\lambda}{4}}^{s}$$
$$v^2 = -\frac{a_0\lambda}{\pi}\cos(2\pi s/\lambda) - (-\frac{a_0\lambda}{\pi}\cos(\pi/2))$$
$$v^2 = -\frac{a_0\lambda}{\pi}\cos(2\pi s/\lambda)$$

Since $v^2 \geq 0$, so we can get: $\cos(2\pi s/\lambda) \leq 0$
Then, we can get:

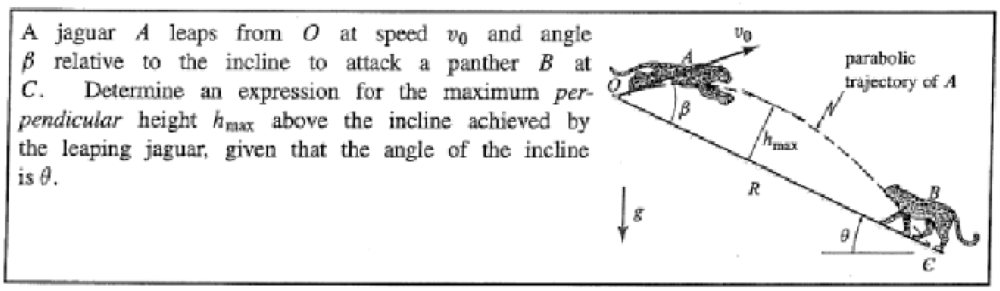
$$\frac{1}{2}\pi + 2\pi n \leq 2\pi s/\lambda \leq \frac{3}{2}\pi + 2\pi n$$
$$\frac{1}{4}\lambda + n \leq s \leq \frac{3}{4}\lambda + 3n, n = \pm 1, \pm 2, \pm 3, ...$$

> **Note**: The sphere CAN NOT jump from one admissible range to the next. So, the sphere can only be in ***one*** range.

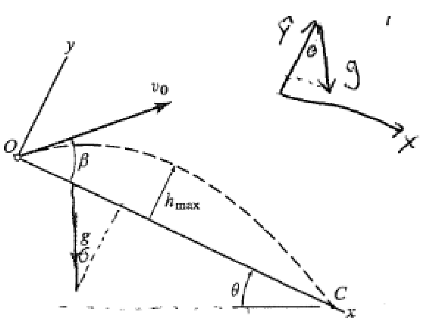
Observing that $a = a_0 = 8 > 0$ for $s = \frac{\lambda}{4}$, so the sphere is accelerating in the right direction. So, the sphere is in the range of $s \in [\frac{1}{4}\lambda, \frac{3}{4}\lambda]$.

So, the final result is: $s \in [0.0625m, 0.1875m]$

Problem 3


A jaguar A leaps from O at speed v_0 and angle β relative to the incline to attack a panther B at C. Determine an expression for the maximum *perpendicular* height h_{max} above the incline achieved by the leaping jaguar, given that the angle of the incline is θ.

Solution



First, we can get the acceleration of the block:

$$a = g \sin\theta \hat{i} - g\cos\theta \hat{j}$$
$$a_x = g\sin\theta$$
$$a_y = -g\cos\theta$$

Then, we can get:

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

When jaguar A reaches the `h_{max}` , $v_y = 0$, so we can get:

$$0 = (v_0 \sin\beta)^2 - 2g\cos\theta(h_{max} - h_0), h_0 = 0$$

So, $h_{max} = \frac{(v_0 \sin\beta)^2}{2g\cos\theta}$