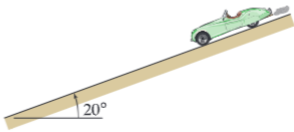
Problem 1

A classic car is driving down a 20° incline at 45 km/h when the brakes are applied. Treat the car as a particle, and neglect all forces except gravity and friction.

Using the work-energy principle, determine the stopping distance if the tires slide and the coefficient of kinetic friction between the tires and the road is 0.7.



Recall and Analysis

- Variance of kinetic energy: $U_{1\to2} = T_2 - T_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$
- Work done by the force: $W_{1\to2} = \int_1^2 \vec{F} \cdot d\vec{r}$
- The work done by the forces acting on the particle as it moves from position 1 to position 2 is equal to the change in the particle's kinetic energy between those two positions, i.e., $W_{1\to2} = U_{1\to2}$
- Here, it's about stopping distance, so $v_2 = 0$, $v_1 = v$, $U_{1\to2} = -\frac{1}{2}mv^2$
- For the work done by the force, we have gravity and friction. (*Direction*)

Solution

The vehicle is driving down, the gravity has a projection on the direction of motion, so it's positive. The friction is against the direction of motion, so it's negative.

External forces: $\vec{F} = \vec{F}_g + \vec{F}_f = mg\sin\theta - \mu_k mg\cos\theta$
So:

$$
\begin{aligned}
W_{1\to 2} &= \int_1^2 \vec{F} \cdot d\vec{r} \\
&= (mg\sin\theta - \mu_k mg\cos\theta)\int_1^2 (1) \cdot d\vec{r} \\
&= (mg\sin\theta - \mu_k mg\cos\theta)(r_2 - r_1) \\
&= (mg\sin\theta - \mu_k mg\cos\theta)d
\end{aligned}
$$

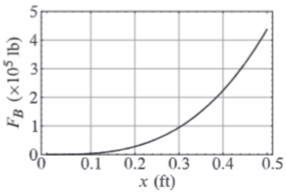
where d is the stopping distance.

Since $W_{1\to 2} = U_{1\to 2}$, we have:

$$
\begin{aligned}
W_{1\to 2} &= U_{1\to 2} \\
(mg\sin\theta - \mu_k mg\cos\theta)d &= -\frac{1}{2}mv^2 \\
d &= \frac{-\frac{1}{2}mv^2}{mg\sin\theta - \mu_k mg\cos\theta} \\
&= \frac{v^2}{2g(\mu_k\cos\theta - \sin\theta)} \\
&= 25.22 \text{ m}
\end{aligned}
$$

Problem 2

Rubber bumpers are commonly used in marine applications to keep boats and ships from getting damaged by docks. Treating the boat C as a particle, neglecting its vertical motion, and neglecting the drag force between the water and the boat C, what is the maximum speed of the boat at impact with the bumper B so that the deflection of the bumper is limited to 6 in.? The weight of the boat is 70,000 lb, and the force compression profile for the rubber bumper is given by $F_B = \beta x^3$, where $\beta = 3.5\times 10^6\ \mathrm{lb/ft^3}$ and x is the compression of the bumper.



Recall and Analysis

- $W_{1\to 2} = U_{1\to 2}$:
  - where $W_{1\to 2}$ is the work done by the impact force, and $U_{1\to 2}$ is the change in kinetic energy.
  - The bumper is to slown down the boat until it stops, so $v_2 = 0$.

Solution

- External forces: $\vec{F} = \vec{F}_i = -\beta x^3$, the impact force is negative because it's against the direction of motion.
- So:

$$
\begin{aligned}
W_{1\to 2} &= \int_1^2 \vec{F} \cdot d\vec{r} \\
&= -\beta \int_1^2 x^3 \cdot d\vec{r} \\
&= -\beta \frac{1}{4} x^4 \Big|_0^d \\
&= -\frac{\beta}{4} d^4
\end{aligned}
$$

  where d is the stopping distance.
- For the change in kinetic energy, we have:

$$
\begin{aligned}
U_{1\to 2} &= T_2 - T_1 \\
&= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \\
&= 0 - \frac{1}{2}mv_1^2
\end{aligned}
$$

where v_1 is the initial velocity.

- Since $W_{1\to 2} = U_{1\to 2}$, we have:

$$
\begin{aligned}
W_{1\to 2} &= U_{1\to 2} \\
-\frac{\beta}{4}d^4 &= -\frac{1}{2}mv_1^2 \\
v &= \sqrt{\frac{2\beta}{4m}d^4} \\
&= 7.093 \text{ ft/s}
\end{aligned}
$$