

Problem 1

Problem 6.27

At the instant shown, the angle $\phi = 30^\circ$, $|\vec{v}_A| = 292 \text{ ft/s}$, and the turbine is rotating clockwise. Letting $\overline{OA} = R$, $\overline{OB} = R/2$, $R = 182 \text{ ft}$, and treating the blades as being equally spaced, determine the velocity of point B at the given instant and express it using the component system shown.

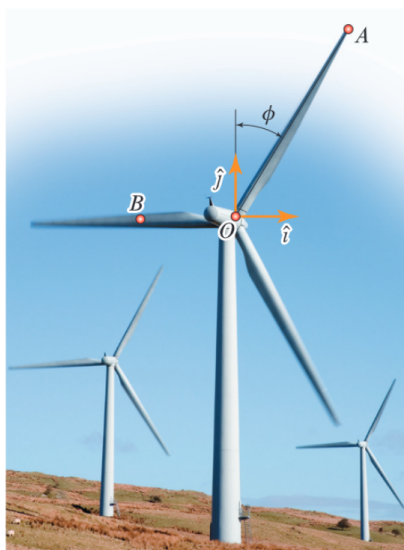


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Recall

We first review the cross product of vectors.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y) \hat{i} + (a_z b_x - a_x b_z) \hat{j} + (a_x b_y - a_y b_x) \hat{k}$$

Where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors of positive x, y, z direction. And by definition, we know that: $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$, and $\vec{a} \times \vec{a} = \vec{0}$.

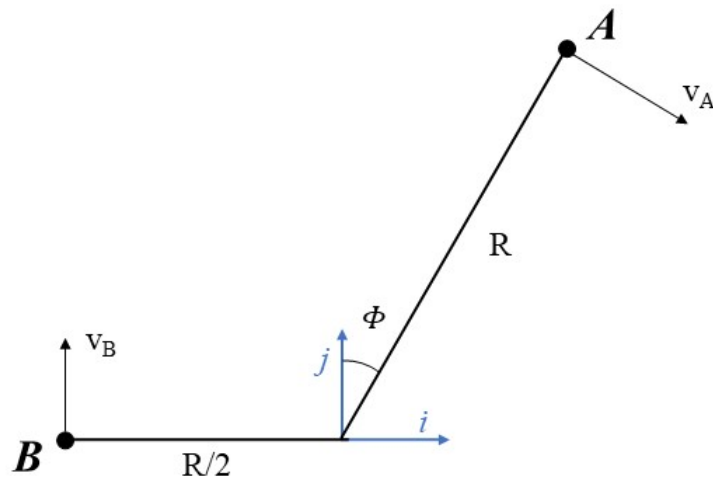
Specifically, we have:

$$\begin{aligned} \vec{i} \times \vec{j} &= \vec{k} \\ \vec{j} \times \vec{k} &= \vec{i} \\ \vec{k} \times \vec{i} &= \vec{j} \end{aligned}$$

For rigid body rotating to a fixed axis at A, for any point B on the object, we have:

$$\vec{v}_{B/A} = \vec{\omega}_{AB} \times \vec{R}_{B/A}$$

Solution



For this problem, we first find \vec{r} and $\vec{\omega}$ for the fixed rotation

$$\begin{aligned}\vec{r}_{A/O} &= R(\sin\Phi \cdot \hat{i} + \cos\Phi \cdot \hat{j}) \\ \vec{r}_{B/O} &= -R/2 \cdot \hat{i}\end{aligned}$$

We can find ω using given velocity of A and radius. Since: $v_A = |\omega|R$, z axis is pointing outwards of the plane, and the turbine is rotating clockwise (angular vector pointing inwards), we have

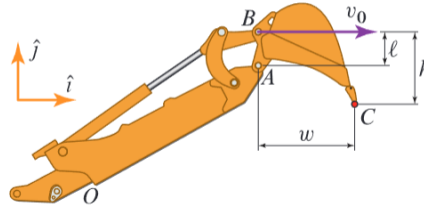
$$\vec{\omega} = -v_A/R \cdot \hat{k}$$

therefore,

$$\begin{aligned}\vec{v}_B &= \vec{\omega} \times \vec{R} \\ &= -v_A/R \cdot \hat{k} \times \left(-\frac{R}{2} \cdot \hat{i}\right) \\ &= \frac{v_A}{2} \hat{j} \\ &= 146 \hat{j} \text{ ft/s}\end{aligned}$$

Problem 2

The bucket of a backhoe is being operated while holding the arm OA fixed. At the instant shown, point B has a horizontal component of velocity $v_0 = 0.25$ ft/s and is vertically aligned with point A . Letting $\ell = 0.9$ ft, $w = 2.65$ ft, and $h = 1.95$ ft, determine the velocity of point C . In addition, assuming that, at the instant shown, point B is not accelerating in the horizontal direction, compute the acceleration of point C . Express your answers using the component system shown.



Recall

- Rotation of a Rigid Body About a Fixed Axis:

$$\vec{v}_P = \vec{\omega} \times \vec{r}$$

$$\vec{a}_P = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \vec{\alpha} \times \vec{r} - \omega^2 \vec{r}$$

- For this problem, we regard the rotation around the A axis as the fixed axis rotation. And the rotation positive direction is counter-clockwise.
 - For example:

$$\begin{aligned}\vec{v}_{B/A} &= \vec{\omega}_{AB} \times \vec{r}_{B/A} \\ v_0 \hat{i} &= \omega_{AB} \hat{k} \times l \hat{j}\end{aligned}$$

where $\vec{v}_{B/A}$ is the velocity of point B relative to point A, $\vec{\omega}_{AB}$ is the angular velocity of the rigid body, and $\vec{r}_{B/A}$ is the position vector of point B relative to point A.

Solution

Since the bucket is in a fixed axis rotation about point A, we have:

$$\begin{aligned}\vec{v}_B &= \vec{\omega}_{AB} \times \vec{r}_{B/A} = v_0 \hat{i} \\ v_0 \hat{i} &= \omega_{AB} \hat{k} \times l \hat{j} \\ &= \omega_{AB} l \hat{i} = v_0 \hat{i} \\ \omega_{AB} &= -\frac{v_0}{l} = -0.2778 \text{ rad/s}\end{aligned}$$

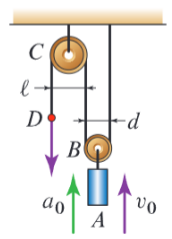
Plug in the value of ω_{AB} , we have:

$$\begin{aligned}\vec{v}_C &= \omega_{AB} \hat{k} \times \vec{r}_{C/A} = \omega_{AB} \hat{k} \times [w \hat{i} - (h - l) \hat{j}] \\ \vec{v}_C &= (h - l) \omega_{AB} \hat{i} + w \omega_{AB} \hat{j} \\ \vec{v}_C &= (-0.292 \hat{i} - 0.736 \hat{j}) \text{ ft/s}\end{aligned}$$

Since $\alpha_{AB} = 0$, $\vec{a}_C = -\omega_{AB}^2 \vec{r}_{C/A} = -v_0^2 [w \hat{i} - (h - l) \hat{j}]$, which means that:
 $\vec{a}_C = (-0.204 \hat{i} + 0.081 \hat{j}) \text{ ft/s}^2$

Problem 3

At the instant shown, A is moving upward with a speed $v_0 = 5 \text{ ft/s}$ and acceleration $a_0 = 0.65 \text{ ft/s}^2$. Assuming that the rope that connects the pulleys does not slip relative to the pulleys and letting $\ell = 6 \text{ in.}$ and $d = 4 \text{ in.}$, determine the angular velocity and angular acceleration of pulley C .



Recall

We can define the length of the rope as:

$$L = y_D + 2y_A$$

Taking two time derivatives of this equation, and remembering that the overall length is constant because the rope is inextensible, we have:

$$\begin{aligned} 0 &= \dot{y}_D + 2\dot{y}_A \rightarrow \dot{y}_D = -2\dot{y}_A = 2v_0 \\ 0 &= \ddot{y}_D + 2\ddot{y}_A \rightarrow \ddot{y}_D = -2\ddot{y}_A = 2a_0 \end{aligned}$$

Solution

Because the point Q is contacting the rope, and the rope is not slipping, point Q has the same velocity and vertical component of acceleration as D.

$$\begin{aligned} \vec{v}_Q = \vec{v}_D = 2v_0 &= \omega_C \frac{l}{2} \rightarrow \vec{\omega}_C = \frac{4v_0}{l} = 40.0 \hat{k} \text{ rad/s} \\ \vec{a}_{Qy} = \vec{a}_D = 2a_0 &= \alpha_C \frac{l}{2} \rightarrow \vec{\alpha}_C = \frac{4a_0}{l} = 5.20 \hat{k} \text{ rad/s}^2 \end{aligned}$$