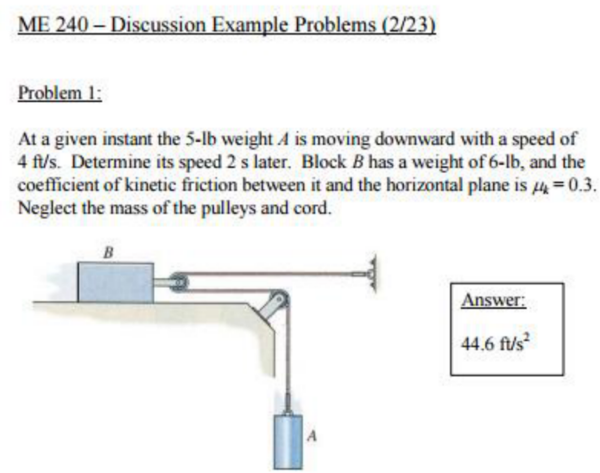
Discussion_3_3_Sol

Problem 1



ME 240 – Discussion Example Problems (2/23)

Problem 1:

At a given instant the 5-lb weight A is moving downward with a speed of 4 ft/s. Determine its speed 2 s later. Block B has a weight of 6-lb, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.3$. Neglect the mass of the pulleys and cord.

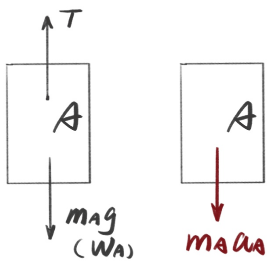
Answer:

44.6 ft/s^2

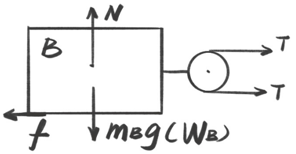
Recall & Analysis

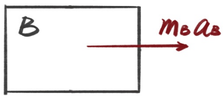
- The tension along the rope is uniform.(set as T)
- friction can be calculated as: $f = \mu N$
- The length of the string is constant.

We can try to draw Full Body Diagram of both A and B, and define their direction of acceleration. The external forces applied on A are gravity and tension on the rope.

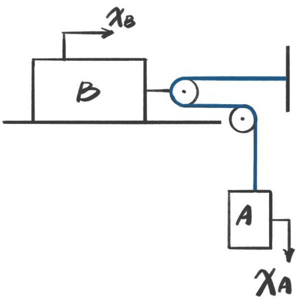


The external forces applied on B are gravity, Normal force by the plan, friction and Tension on the rope.





We can also use kinematic analysis to determine the relationship between displacement of A and B. Where: $\Delta L = 2x_A - x_B = 0$



Solution

First applied Newton 2nd Law on both A and B:

for A:

$$m_A a_A = m_A g - T \quad (1)$$

for B (Here we skipped the equations $N = m_B g$ and $f = \mu N$):

$$m_B a_B = 2T - \mu m_B g \quad (2)$$

Here we have 2 equations and 3 unknown variables: a_A, a_B, T, so we still need to analyze the kinematics of the system to find other constraints.

Give the fixed length of the rope, we have:

$$\Delta L = x_A - 2x_B = 0$$

$$x_A = 2x_B$$

the equation's second derivate of time yields:

$$a_A = 2a_B \quad (3)$$

With 3 equations and 3 unknows, we can solve all of them, but most importantly, a_A

$$a_A = \frac{4m_A - 2m_B}{m_B + 4m_A}\mu g = 20.3ft/s^2$$

Therefore, the velocity of A should be:

$$v_A(t) = v_0 + a_A t \quad (1)$$
$$v_A(2) = 4 + 20.3 * 2 \quad (2)$$
$$v_A(2) = 44.6ft/s \quad (3)$$

And it is moving downwards.

Problem 2

Problem 2:

To unload a bound stack of plywood from a truck, the driver first tilts the bed of the truck and then accelerates from rest. Knowing that the coefficients of friction between the bottom sheet of plywood and the bed are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine *(a)* the smallest acceleration of the truck which will cause the stack of plywood to slide, *(b)* the acceleration of the truck which causes corner A of the stack to reach the end of the bed in 0.9 s.

Answer:

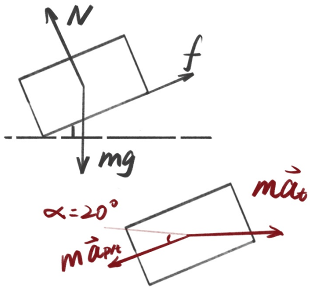
(a) 0.31 m/s^2

(b) 4.17 m/s^2

Recall & Analysis

Newton 2nd Law in Cartesian Coordinate system:

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$



Solution

(a). With Full Body diagram of the plywood, we can apply Newton 2nd Law of it in Cartesian Coordinate system.(Here we set $\alpha = 20^o$)

On $x - axis, \sum F_x = ma_x$:

$$f cos\alpha - N sin\alpha = ma_t - ma_{p/t} cos\alpha$$

On $y - axis, \sum F_y = ma_y$:

$$N cos\alpha - f sin\alpha = -ma_{p/t} sin\alpha$$

for this case, at the critical state where the plywood started sliding,:

- the friction would be static friction ($\mu = \mu_s$),
- the relative acceleration would be 0 ($ma_{p/t} = 0$).

With $f = \mu_s N$, we have the equations:

$$\begin{cases} \mu_s N cos\alpha - N sin\alpha = ma_t & (1)(4) \\ N cos\alpha - \mu_s N sin\alpha = 0 & (2)(5) \end{cases}$$

where 2 equations and 2 unknowns a_t, N, we can solve the minimum acceleration a_t to be:

$$a_t = g\frac{\mu_s cos20^o - sin20^o}{cos20^o + \mu_s sin20^o}$$

and $a_t = 0.31 m/s^2$

(b). The analysis are similar to **(a)**, but the differences are:

- friction would be kinetic friction, so $f = \mu_k N$
- $a_{p/t}$ is an unknown value.

But before we solve a_t, we might use given conditions to solve $a_{p/t}$. Note that starting position and velocity are all 0.

$$\begin{aligned} x_{p/t}(t) &= x_{p/t}|_{t=0} + v_{p/t}|_{t=0}t + \frac{1}{2}a_{p/t}t^2 && (6) \\ &= \frac{1}{2}a_{p/t}t^2 && (7) \end{aligned}$$

Plug in $x_{p/t}(t) = 2m$ and $t = 0.9s$, we can find $a_{p/t} = 4.94m/s^2$

This time, the dynamics equations should be:

$$\begin{cases} \mu_k N cos\alpha - N sin\alpha = ma_t - ma_{p/t}cos\alpha & (1)(8) \\ N cos\alpha - \mu_k N sin\alpha = -ma_{p/t}sin\alpha & (2)(9) \end{cases}$$

With 2 equations and 2 unknown variables (a_t and N), we can solve:

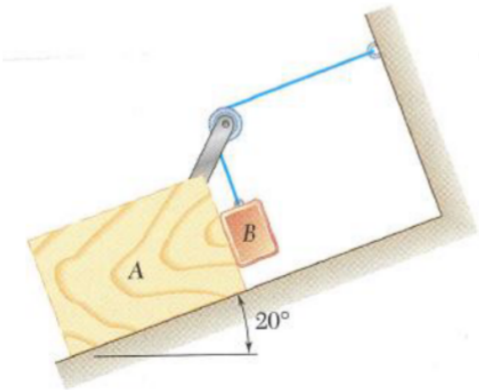
$$a_t = \frac{\mu_k cos20^o - sin20^o}{cos20^o + \mu_k sin20^o}(g - a_{p/t}sin20^o) + a_{p/t}cos20^o$$

So, $a_t = 4.17m/s^2$.

Problem 3

Problem 3:

A 50-lb block A rests on an inclined surface, and a 30-lb counterweight B is attached to a cable which passes over the pulley and is fixed to the wall. The pulley is attached to a bracket fixed to block A. Neglecting friction, determine *(a)* the acceleration of A and *(b)* the tension in the cable immediately after the system is released from rest.



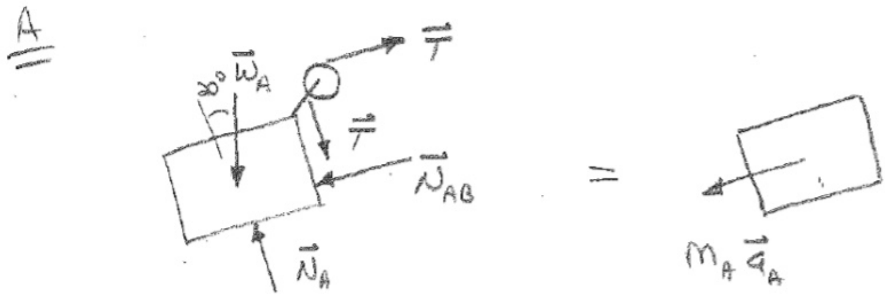
Answer:

(a) $\boldsymbol{a_A} = 0.243 \text{ m/s}^2 \nearrow$

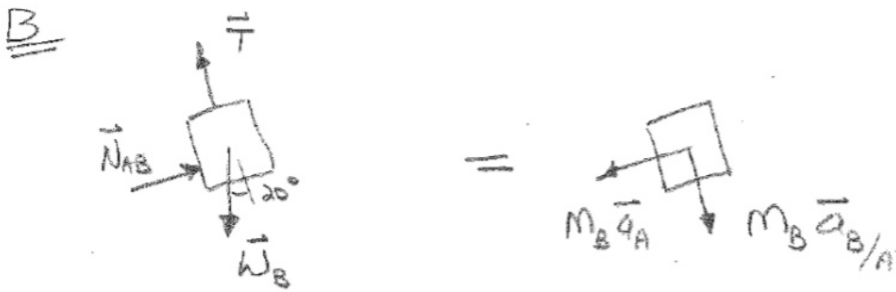
(b) T = 27.96 lb

Recall and Analysis

- For this system, it has two contacted objects on the incline and one cable-pulley.
  - Objects keep contacting with the incline, so they have the same acceleration in the direction of the incline.
  - The cable-pulley is frictionless, so the tension is the same on both sides of the cable. As usual, we could use **"the length of the cable is constant"**.
- Build a coordinate system with **x-axis along the incline and y-axis perpendicular to the incline**.
- For B object, its motion has two parts:
  - $a_{B/A}$: the acceleration of B object relative to A object, i.e. **y-axis**.
  - $a_{incline}$: the acceleration of B object sliding on the incline, i.e. **x-axis**.



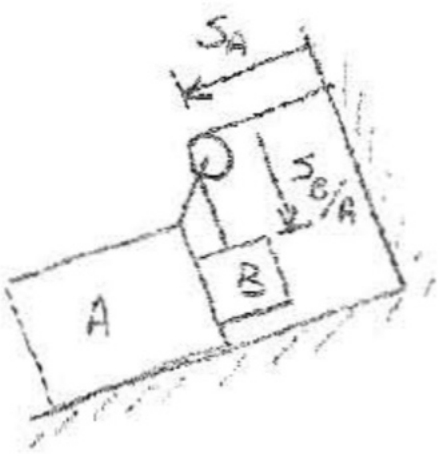
- Obj A:
  - **y-axis**: $\sum F_y = ma_y = 0$, with normal force from incline, cabel tension and gravity's component.
  - **x-axis**: $\sum F_x = ma_x$, with gravity's component, cabel tension and normal force from B.



- Obj B:
  - **y-axis**: $\sum F_y = ma_y = 0$, with cabel tension and gravity's component.
  - **x-axis**: $\sum F_x = ma_x$, with gravity's component and normal force from A.

Solution

- Obj A:
  - **y-axis**: $N_A - T - w_A \cos 20^\circ = 0$
  - **x-axis**: $w_A \sin 20^\circ + N_{AB} - T = m_A a_A$
- Obj B:
  - **y-axis**: $w_B \cos 20^\circ - T = m_B a_{B/A}$
  - **x-axis**: $w_B \sin 20^\circ - N_{AB} = m_B a_A$, because B has the same acceleration as A in the direction of the incline.
- 4 equations and 5 unknowns: $N_A, N_{AB}, T, a_A, a_{B/A}$.
- Cable length is constant: $S_A + S_{B/A} = constant$, so $a_A + a_{B/A} = 0$, COOL! Now we have 4 equations and 4 unknowns. So we could solve them.



$$a_A = \left(\frac{w_A \sin 20^\circ + w_B(\sin 20^\circ - \cos 20^\circ)}{w_A + 2w_B}\right)g = -0.243 ft/s^2$$
$$T = \left(\frac{w_A + w_B}{g}\right)(g \sin 20^\circ - a_A) = 27.96 lb$$

a_A is negative, which means A is moving upwards.