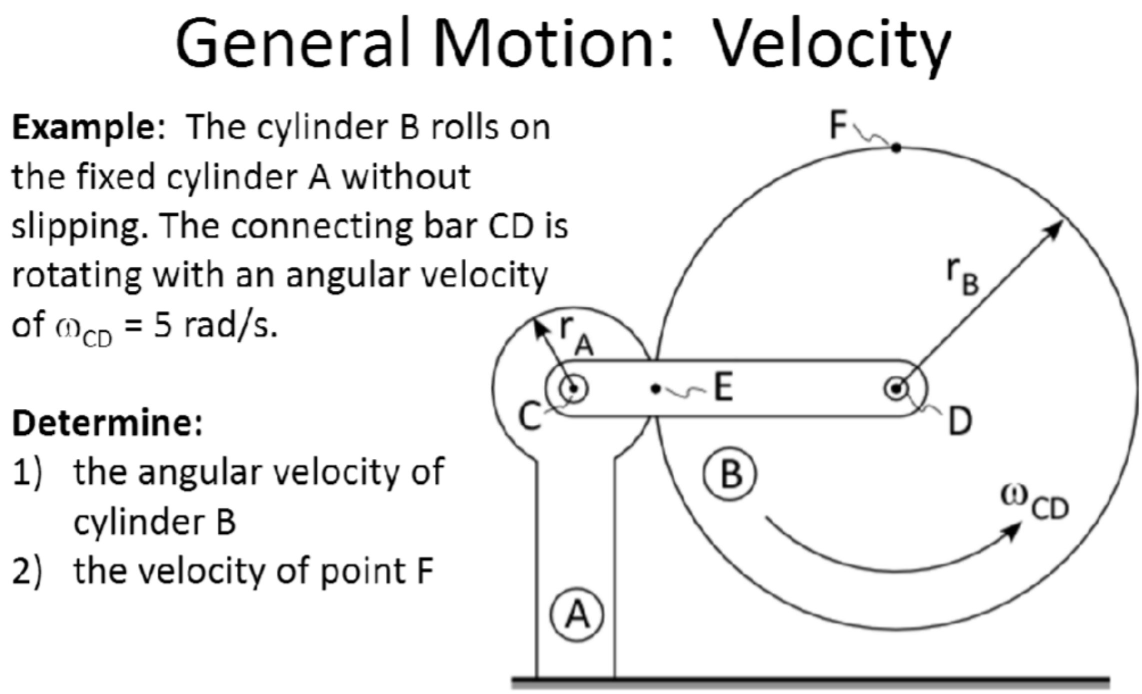
Problem 1



- $r_A = 0.1\,m$
- $r_B = 0.3\,m$

Review

- Point D is both on bar CD and cylinder B. Therefore, there are two ways to find the velocity of point D. **One is using the angular velocity of bar CD and the other is using the angular velocity of cylinder B. The two ways should give the same result.**
- Find V_D from bar CD: $\vec{V}_D = \vec{V}_C + \vec{\omega}_{CD} \times \vec{r}_{D/C}$
    - where $\vec{V}_C = 0$ as cylinder A is fixed, and $\vec{r}_{D/C} = (r_A + r_B)\hat{\imath}$
- Find V_D from cylinder B: $\vec{V}_D = \vec{V}_E + \vec{\omega}_{ED} \times \vec{r}_{D/E}$
    - where point E is on cylinder A, and it's the contact point of cylinder A and B. Therefore, $\vec{V}_E = 0$ as cylinder A is fixed, and $\omega_{ED} = \omega_B$ as there is a rotation between cylinders.
        - If we set point E is on cylinder B, then $\omega_{ED} = 0$
    - $\vec{r}_{D/E} = r_B\hat{\imath}$

Solution

Find V_D from bar CD:

$$\begin{aligned}\vec{V}_D &= \vec{V}_C + \vec{\omega}_{CD} \times \vec{r}_{D/C} \\ &= 0 + (\omega_{CD}\hat{k}) \times [(r_A + r_B)\hat{i}] \\ &= 5(0.1 + 0.3)\hat{j} \\ &= 2\hat{j}\end{aligned}$$

Find V_D from cylinder B:

$$\begin{aligned}\vec{V}_D &= \vec{V}_E + \vec{\omega}_{ED} \times \vec{r}_{D/E} \\ &= 0 + (\omega_B\hat{k}) \times [r_B\hat{i}] \\ &= 0.3\omega_B\hat{j}\end{aligned}$$

Since the two ways should give the same result, we have:

$$\begin{aligned}\vec{V}_D &= 2\hat{j} \\ &= 0.3\omega_B\hat{j} \\ \omega_B &= \frac{2}{0.3} \\ &= 6.67\ rad/s\end{aligned}$$

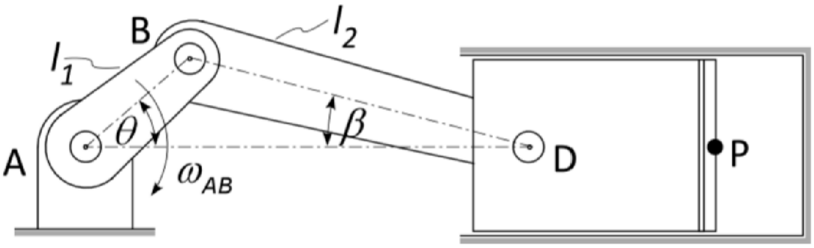
Find the velocity of point F:

$$\begin{aligned}\vec{V}_F &= \vec{V}_D + \vec{\omega}_B \times \vec{r}_{F/D} \\ &= 2\hat{j} + (6.67\hat{k}) \times [0.3\hat{j}] \\ &= 2\hat{j} - 2\hat{i} \\ &= -2\hat{i} + 2\hat{j}\end{aligned}$$

Problem 2

General Motion - Velocity

Example: In the piston system shown, the crank AB has a constant clockwise angular velocity of 2000 RPM.



ω_{AB} = 2000 RPM
l_1 = 3 inch
l_2 = 8 inch
θ = 40 degrees

Determine the velocity of point P on the piston for the configuration parameters given above

Analysis

Velocity of B is easy to obtain since B is rotating at constant rate to A.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \quad (i)$$

As A is fixed, we have: $\vec{v}_B = \vec{v}_{B/A}$

Also, with D is rotating with respect to B, and the velocity of D can be described with this equation:

$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B} \quad (ii)$$

As D and P are 2 points constrainted on the piston, their velocity are the same:

$$\vec{v}_D = \vec{v}_P$$

Our strategy to solve this problem can be: Break the problem down to a chain of relative motions. B with respect to A, and D with respect to B, then use the equations above to solve them.

Solution

Given values: $\theta = 40^o$, $l_1 = 3inch$, $l_2 = 8inch$, $\omega_{AB} = 2000rpm$

Unknow variables: β, all velocities.

B is rotating with respect to A, its speed is:

$$\vec{v}_B = \vec{v}_{B/A} = \vec{\omega}_{AB} \times \vec{r}_{AB} = (-\omega_{AB}) \cdot \hat{k} \times (l_1 cos\theta \cdot \hat{\imath} + l_1 sin\theta \cdot \hat{\jmath}) \quad (1)$$

By plugging in the known values, we have:

$$\begin{aligned} \vec{v}_B &= \omega_{AB} l_1 cos\theta \cdot \hat{\jmath} - \omega_{AB} l_1 sin\theta \cdot \hat{\imath} \\ &= 403.9\hat{\imath} - 481.3\hat{\jmath}(ft/s) \end{aligned}$$

As $\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$, we still need $\vec{v}_{D/B}$, which is:

$$\vec{v}_{D/B} = \vec{\omega}_{BD} \times \vec{r}_{BD} = \omega_{BD} \cdot \hat{k} \times (l_2 cos\beta \cdot \hat{\imath} - l_1 sin\beta \cdot \hat{\jmath}) \quad (2)$$

(hint: here we used B as the origin of the relative motion, therefore the $\hat{\jmath}$ component is negative).

In this equation, ω_{BD} and β are also unknown, so we still need other constraints to solve them, for β, we can use law of sines:

$$\frac{l_1}{\beta} = \frac{l_2}{\theta} \Rightarrow \beta = sin^{-1}(\frac{l_1}{l_2} sin\theta) \quad (3)$$

Solve that $\beta = 0.24rad$, or 14 degrees.

For ω_{BD} and $\vec{v}_{D/B}$, we can use another constraint, that is D is moving with the pistol, which means that $\vec{v}_D$ doesn't have a vertical component. Which indicates:

$$\begin{cases} v_{Dx} = v_D \\ v_{Dy} = 0 \end{cases}$$

Using $\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$, and (1), we have:

$$\vec{v}_D = (\omega_{AB} l_1 cos\theta + \omega_{BD} l_2 cos\beta) \cdot \hat{\jmath} - (\omega_{AB} l_1 sin\theta - \omega_{BD} l_2 sin\beta) \cdot \hat{\imath}$$

Plug in the equations, we have:

$$\begin{cases} v_D = -\omega_{AB} l_1 sin\theta + \omega_{BD} l_2 sin\beta \\ \quad 0 = \omega_{AB} l_1 cos\theta + \omega_{BD} l_2 cos\beta \end{cases}$$

With 2 equations and 2 unknowns, we can solve $v_D = 523.4in/s$