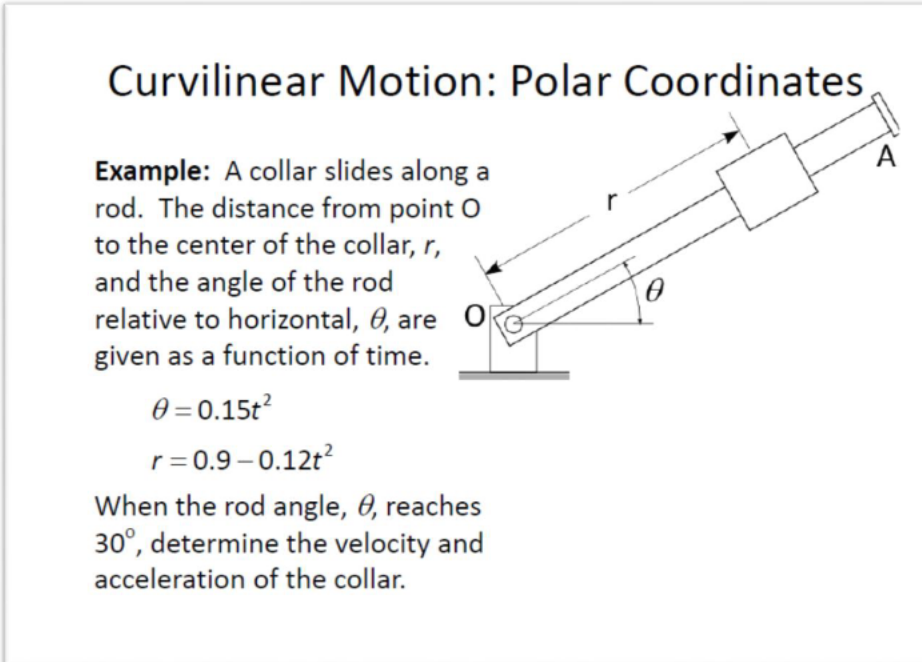
Problem 1



Solution

The question is to find the $\vec{v}$ and $\vec{a}$ at $\theta = 30^o$ in radial-transverse coordinates.

Recall the position vector in radial-transverse coordinates taught in class:

Polar Coordinates: Position and Velocity

- The position is simply the radial coordinate $\boxed{\vec{r} = r\,\hat{u}_r.}$

where:

$$\vec{r} = r\hat{e}_r = (0.9 - 0.12t^2)\hat{e}_r$$

Recall the velocity vector in radial-transverse coordinates taught in class:

$$\boxed{\begin{aligned}\vec{v} &= \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta \\ &= v_r\,\hat{u}_r + v_\theta\,\hat{u}_\theta,\end{aligned}}$$

$$\boxed{v_r = \dot{r} \quad \text{and} \quad v_\theta = r\dot{\theta}.}$$

where:

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

to get $\dot{r}$ and $\dot{\theta}$, we need to do some differentiation:

$$\begin{cases} \dot{r} = -0.24t \\ \dot{\theta} = 0.3t \end{cases}$$

therefore velocity is:

$$\vec{v} = -0.24t\hat{e}_r + (0.9 - 0.12t^2)(0.3t)\hat{e}_\theta$$

When $\theta = 30^o$:
recall $\theta = 0.15t^2$, so $t^2 = \theta/0.15$, i.e.: $t = \sqrt{\frac{\theta}{0.15}} = 1.87s$

$$\vec{v} = -0.45\hat{e}_r + 0.27\hat{e}_\theta$$

for acceleration, recall the formula taught in class:

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{u}_\theta = a_r\,\hat{u}_r + a_\theta\,\hat{u}_\theta,$$

$$a_r = \ddot{r} - r\dot{\theta}^2, \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}.$$

$$\vec{a} = (\ddot{r} - r\omega^2)\hat{e}_r + (r\alpha + 2\dot{r}\omega)\hat{e}_\theta$$

where:

$$\begin{cases} r = 0.9 - 0.12t^2 \\ \dot{r} = -0.24t \\ \omega = \dot{\theta} = 0.3t \end{cases}$$

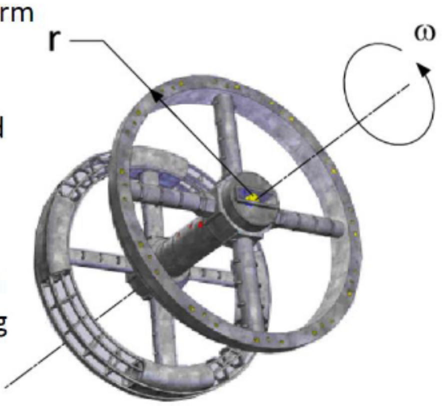
In acceleration, we know everything except $\ddot{r}$ and α, to get them, we need to do some differentiation:
$\ddot{r} = \frac{d\dot{r}}{dt} = -0.24$, $\alpha = \frac{d\omega}{dt} = 0.3$
Now, we can get the acceleration:

$$\vec{a} = -0.391\hat{e}_r - 0.358\hat{e}_\theta$$

Problem 2

Example: A spinning spacecraft is to be designed to support long-term space voyage by simulating the effect of gravity. Determine the range of suitable spin rate (ω) and spacecraft radius, r, such that:

- simulated gravity is > g/2
- lateral acceleration < g/20 when moving from a sitting to standing position (v = 2 m/s)



Solution

The acceleration of radial-transverse coordinates:

$$\vec{a} = (\ddot{r} - r\omega^2)\hat{e}_r + (r\alpha + 2\dot{r}\omega)\hat{e}_\theta \tag{1}$$

where: $\alpha = \emptyset, \ddot{r} = \emptyset$

therefore:

$$\vec{a} = -r\omega^2\hat{e}_r + +2\dot{r}\omega\hat{e}_\theta$$

The first term stands for simulated gravity term: g_s (since the direction of $\hat{e_r}$ is outward, while the gravity is downward, so we need to add a negative sign):

$$g_s = -(-r\omega^2) > \frac{g}{2}$$

The second term stands for lateral acceleration a_L:

$$a_L = 2\dot{r}\omega < \frac{g}{20}$$

given the conditions $\dot{r} = 2m/s$, we have:

$$\begin{cases} r\omega^2 > g/2 \\ \omega < g/80 \end{cases}$$

$$\begin{cases} r > g/(2\omega^2) \\ \omega < g/80 \end{cases}$$

To get the minimum radius, we need to maximize ω because the radius is inversely proportional to ω^2:

$$r_{min} = \frac{g}{2\omega^2} = \frac{g}{2(\frac{g}{80})^2} = \frac{3200}{g} \approx 326m(i.e. 1070ft)$$