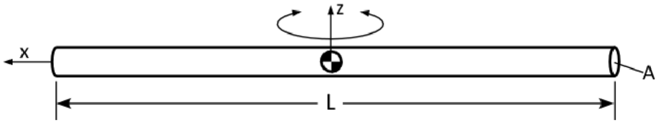
Problem 1

Planar Rigid Body Dynamics

Example 6.1 (a): Determine the moment of inertia and radius of gyration about the z-axis of the uniform slender rod of mass *m* shown.



Review

- For the moment of inertia about an axis:
  - $I_{\text{axis}} = \int r^2 dm$
    - r is the distance from the axis of rotation to the mass element
    - dm is the mass element
- Radius of gyration:
  - $k^2 = \frac{I_{\text{axis}}}{m}$
    - k is the radius of gyration
    - I_{axis} is the moment of inertia about the axis
    - m is the mass of the object

Solution

$$I_G = \int r^2 dm = \int_{-L/2}^{L/2} r^2 dm$$

As for the mass element, we can use the linear density ρ to represent it:

$$dm = \rho dV = \rho A dr = \rho \pi b^2 dr$$

where A is the cross-sectional area of the rod, b is the radius of the cross-section, and dx is the length of the mass element. Therefore, we can rewrite the moment of inertia as:

$$\begin{aligned}
I_G &= \int_{-L/2}^{L/2} r^2 dm \\
&= \int_{-L/2}^{L/2} r^2 \rho \pi b^2 dr \\
&= \rho \pi b^2 \int_{-L/2}^{L/2} r^2 dr \\
&= \rho \pi b^2 \left[\frac{r^3}{3}\right]_{-L/2}^{L/2} \\
&= \rho \pi b^2 \left[\frac{L^3}{24} - \frac{(-L)^3}{24}\right] \\
&= \rho \pi b^2 \left[\frac{L^3}{12}\right] \\
&= \frac{1}{12} \rho \pi b^2 L^3
\end{aligned}$$

As for the mass of the rod, we can use the linear density ρ to represent it:

$$m = \rho V = \rho A L = \rho \pi b^2 L$$

Therefore, we can rewrite the inertia as:

$$\begin{aligned}
I_G &= \frac{1}{12} \rho \pi b^2 L^3 \\
&= \frac{1}{12} \rho \pi b^2 L L^2 \\
&= \frac{1}{12} m L^2
\end{aligned}$$

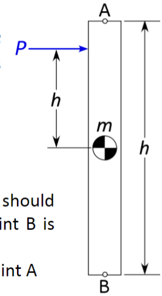
As for the radius of gyration, we have $k^2 = \frac{I_{\text{axis}}}{m}$, so:

$$\begin{aligned} k = \sqrt{\frac{I_{\text{axis}}}{m}} &= \sqrt{\frac{\frac{1}{12}mL^2}{m}} \\ &= \sqrt{\frac{1}{12}L^2} \\ &= \frac{L}{\sqrt{12}} \\ &= \frac{L}{2\sqrt{3}} \end{aligned}$$

Problem 2

Planar Rigid Body Dynamics

Example 6.2: A uniform slender rod *AB* of length *h* feet rests on a frictionless horizontal surface. A force *P* is applied in a direction perpendicular to the rod. The mass of the rod is given as *m*.



Determine

1. the point of the rod *AB* at which *P* should be applied if the acceleration of point B is to be zero.
2. the corresponding acceleration of point A

> ATTENTION:
>
> - Denote the center of mass as **G** and the pivot point as **B**.
> - There are two h in the figure: the left one is the distance from the center of mass to the force application point, and the right one is the height of the block. Now, we are going to use l to represent the height of the block.

Review

- For the force and moment equilibrium:
  - (x) $P = ma_x$
  - (y) $0 = ma_y \rightarrow a_y = 0$, because the block is not moving in the y-direction
  - (θ) $-Ph = I\alpha = \frac{1}{12}ml^2\alpha$
- $\vec{a_B} = \vec{a_G} + \vec{\alpha} \times \vec{r_{B/G}} - \vec{\omega}^2 \times \vec{r_{B/G}}$
  - $\vec{a_B}$ is the acceleration of point B
  - $\vec{a_G}$ is the acceleration of point G, i.e. the center of mass
  - $\vec{\alpha}$ is the angular acceleration of the block
  - $\vec{r_{G/B}}$ is the position vector from point B to point G
  - $\vec{\omega}$ is the angular velocity of the block, which is **ZERO** in this case.
    - Because G and B are rotating together, so there is no relative rotation between them.

Solution

(1) Solving for $\vec{a}_B = 0$, we start from analyzing the acceleration equation of B.

$$\vec{a_B} = \vec{a_G} + \vec{\alpha} \times \vec{r_{B/G}} - \vec{\omega}^2 \times \vec{r_{B/G}}$$

Plugging in $\vec{\omega} = 0$, $\vec{a}_B = 0$, we first have:

$$\vec{a_G} = -\vec{\alpha} \times \vec{r_{B/G}} \tag{1}$$

$$\vec{a_G} = -\alpha\hat{k} \times (-\frac{l}{2})\hat{j} \tag{2}$$

$$\vec{a_G} = -\frac{\alpha l}{2}\hat{i} \tag{1}$$

In this equation, $\vec{a_G}$ and α remain unknown, so we need to derive their form using our analysis. We have:

$$\vec{a_G} = a_x\hat{i} + a_y\hat{j} \tag{3}$$

$$\vec{a_G} = \frac{P}{m}\hat{i} \tag{2}$$

And:

$$-Ph = \frac{1}{12}ml^2\alpha \tag{4}$$

$$\alpha = -\frac{12Ph}{ml^2} \tag{3}$$

By plugging in (2) and (3) into (1). We then have an equation containing h and other known values.

$$P/m = 6Ph/ml$$
$$h = l/6$$

Which is independent to mass and force.

(2) Using similar equation:

$$\vec{a_A} = \vec{a_G} + \vec{\alpha} \times \vec{r_{A/G}} - \vec{\omega}^2 \times \vec{r_{A/G}} \tag{5}$$

$$\vec{a_A} = P/m\hat{i} + -\frac{12Ph}{ml^2}\hat{i} \times (l/2 - l/6) + 0 \tag{6}$$

And we have: $\vec{a_A} = 2P/m\hat{i}$

Problem 3

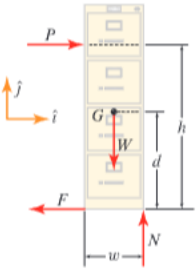
Problem 7.17

A file cabinet weighing 230 lb is being pushed to the right with a horizontal force P applied a distance h from the floor. The width of the file cabinet is $w = 15$ in., its mass center G is a distance $d = 2$ ft above the floor, and static friction is insufficient to prevent slipping between the cabinet and the floor.

If $P = 70$ lb and the coefficient of kinetic friction between the cabinet and the floor is $\mu_k = 0.28$, determine the maximum height h at which the cabinet can be pushed so that it does not tip over, and find the corresponding acceleration of the cabinet.

Solution

An FBD of the cabinet is shown on the right, where F is the friction force between the cabinet and the floor. Since we want to find the maximum height h for no tipping (i.e., tip is impending), we have placed the normal force at the corner of the cabinet.



Balance Principles. The Newton-Euler equations corresponding to this FBD are

$$\sum F_x: \qquad P - F = \frac{W}{g} a_{Gx},$$

$$\sum F_y: \qquad N - W = \frac{W}{g} a_{Gy},$$

$$\sum M_G: \quad N\frac{w}{2} - P(h - d) - Fd = I_G \alpha_c,$$

where α_c is the angular acceleration of the cabinet.

Force Laws. Since the cabinet is sliding, the force law is

$$F = \mu_k N.$$

Kinematic Equations. Since tip is only impending, the kinematics equations are

$$a_{Gy} = 0 \quad \text{and} \quad \alpha_c = 0.$$

Computation. Substituting the force law and the kinematics equations into the Newton-Euler equations, we obtain

$$P - \mu_k N = \frac{W}{g} a_{Gx},$$

$$N - W = 0,$$

$$N\frac{w}{2} - P(h - d) - \mu_k N d = 0,$$

which are three equations for the unknowns N, a_{Gx}, and h. Solving, we obtain

$$N = W, \quad a_{Gx} = \frac{g}{W}(P - \mu_k W), \quad \text{and} \quad h = \frac{2d(P - \mu_k W) + wW}{2P}.$$

Evaluating a_{Gx} and h at $W = 230\,\text{lb}$, $w = \frac{15}{12}\,\text{ft}$, $d = 2\,\text{ft}$, $P = 70\,\text{lb}$, $\mu_k = 0.28$, and $g = 32.2\,\text{ft/s}^2$, we obtain

$$\boxed{a_{Gx} = 0.7840\,\text{ft/s}^2 \quad \text{and} \quad h = 2.214\,\text{ft}.}$$