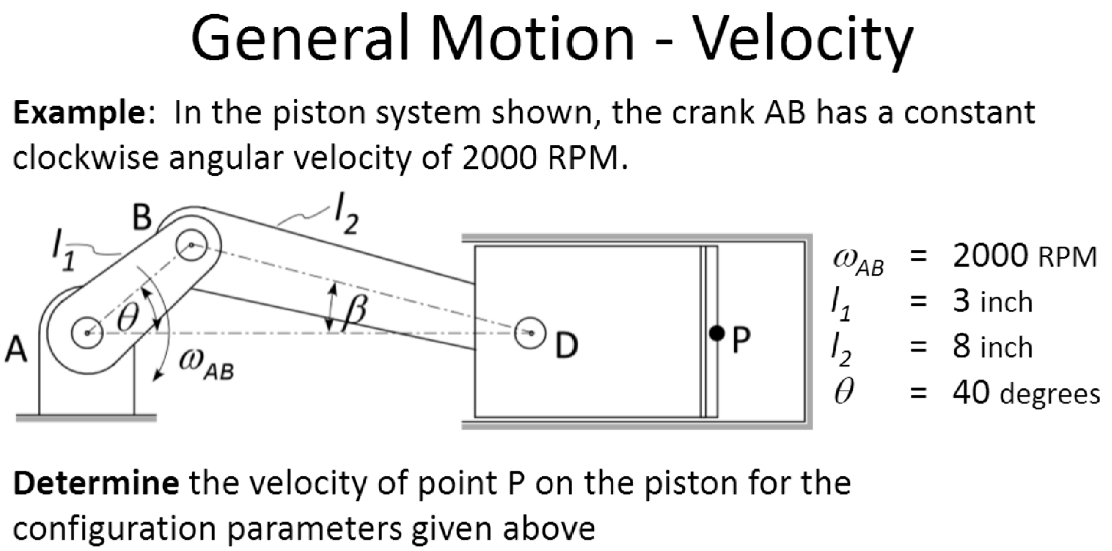
Problem 1



Review

The last lecture is about **Instantaneous center** (IC) of a rigid body. The key feature of instantaneous center is:

1. It is not constant in time, and not necessarily on the object.
2. Its velocity is zero at this instant. All points on the body can be described as:

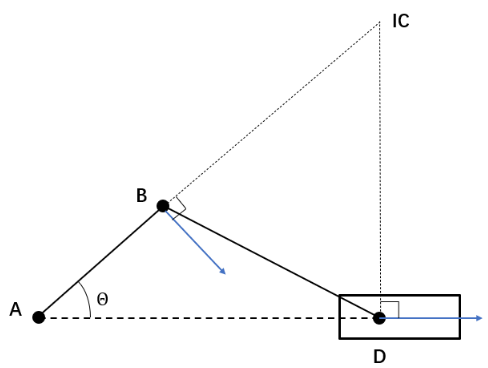
$$\overrightarrow{v_A} = \vec{\omega} \times \vec{r}_{A/IC}$$

   Where ω is the angular velocity around instantaneous center. Therefore, for every point on the rigid body, its velocity is perpendicular to the position vector from instantaneous center to itself.

We can find the instantaneous center by using physical features of the rigid body(rotating around a fixed point), or we can determine instantaneous center by know velocities, positions and angular velocities.

Analysis

For this problem, we have already done analysis and found $\vec{v}_D = \vec{v}_P$. To find $\vec{v}_D$, we can apply instantaneous center method on rod BD:



We used the feature "all points on the rigid body have perpendicular velocity to IC".

Solution

With the graph we have from analysis, we can first write down B and D's velocities described with IC:

$$\vec{v}_B = \vec{\omega} \times \vec{r}_{B/IC} \quad (1)$$
$$\vec{v}_D = \vec{\omega} \times \vec{r}_{D/IC} \quad (2)$$

And we can use geometric features of the system to solve $\vec{r}_{B/IC}$ and $\vec{r}_{D/IC}$

$$\begin{aligned}
\vec{r}_{B/IC} &= -(\bar{AD} - \frac{l_1}{cos\theta})\hat{i} - (\frac{\bar{AD}}{tan\theta} - l_1 sin\theta)\hat{j} \\
&= -7.76\hat{i} - 6.51\hat{j} \\
\vec{r}_{D/IC} &= -\frac{\bar{AD}}{tan\theta} \cdot \hat{j} \\
&= -8.44\hat{j}
\end{aligned}$$

Also, B is rotating around A, we have:

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A} \quad (3)$$

Where $\vec{v}_A = 0$ and $\vec{r}_{B/A} = l_1 cos\theta\hat{i} + l_1 sin\theta\hat{j}$. and we can solve that (just like last Friday) $\vec{v}_B = 403.9\hat{i} - 481.3\hat{j}(ft/s)$

Plugging into equation (1), we found that the remaining unknowns are: $\omega, \overrightarrow{v_D}$, with 2 equations and 2 unknowns, we can plug in to solve.

First, $\omega = 403.9/6.51 = 62.03 rad/s$

Then plug in (2), $\vec{v}_D = 62.03\hat{k} \times (-8.44\hat{j}) = 523.4 in/s\hat{i}$

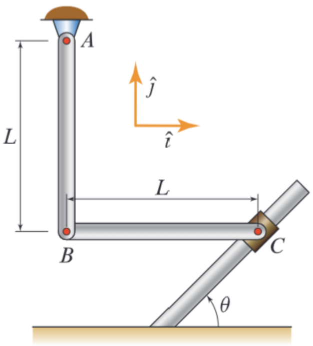
Which is the same with our solution from last Friday.

Problem 2

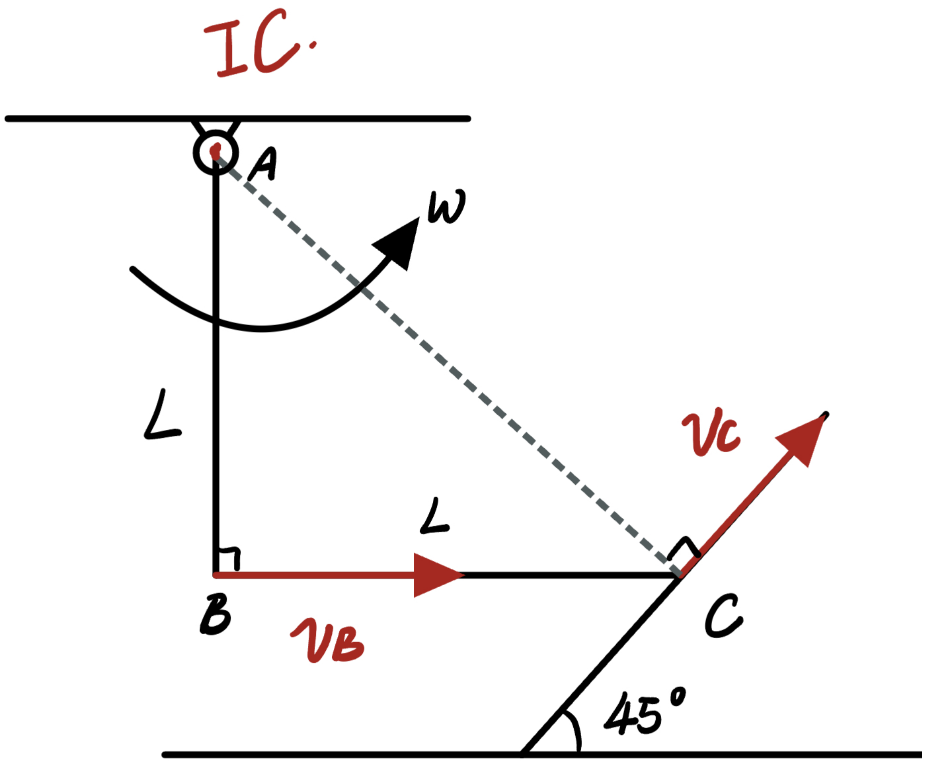
818 *Solutions Manual*

Problem 6.41

At the instant shown bars AB and BC are perpendicular to each other, and bar BC is rotating counterclockwise at 20 rad/s. Letting $L = 2.5$ ft and $\theta = 45°$, determine the angular velocity of bar AB as well as the velocity of the slider C.



Analysis



Repeat what we have done for the first problem: find IC. In this case. Since $\theta = 45^o$, velocity of C is perpendicular to $A\bar{C}$, and B is on rod AB which is also rotating around A, so we can find out IC in this case is A.

Solution

The angular velocity of C with respect to the instantaneous center is the same as the angular velocity of BC, which is $\omega_{BC} = 20 rad/s \cdot \hat{k}$

$$\begin{aligned}\vec{v}_C &= \vec{v}_A + \vec{\omega} \times \vec{r}_{C/A} \\ &= 0 + 20\hat{k} \times (2.5\hat{\imath} - 2.5\hat{\jmath}) \\ &= 50\hat{\imath} + 50\hat{\jmath}\end{aligned}$$

For B, we have

$$\vec{v}_B = \vec{v}_{IC} + \vec{\omega} \times \vec{r}_{B/IC}$$
$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

It is obvious that $\vec{\omega}_{AB} = \vec{\omega} = 20 rad/s \cdot \hat{k}$