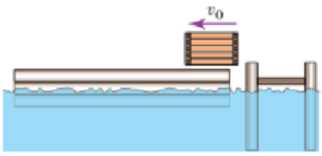
Problem 1

A 700 lb floating platform is at rest when a 200 lb crate is thrown onto it with a horizontal speed $v_0 = 12\,\mathrm{ft/s}$. Once the crate stops sliding relative to the platform, the platform and the crate move with a speed $v = 2.667\,\mathrm{ft/s}$. Neglecting the vertical motion of the system, as well as any resistance due to the relative motion of the platform with respect to the water, determine the distance that the crate slides relative to the platform if the coefficient of kinetic friction between the platform and the crate is $\mu_k = 0.25$.



Recall & Analysis

- The only force doing work here is the internal friction force between crate and platform.
- "Neglecting the vertical motion" means that there is no variation of the vertical position of the system, i.e. no variation of the potential energy.
- $T_1 + V_1 + U_{1-2} = T_2 + V_2$, where T is the kinetic energy, V is the potential energy, and U is the work done by the internal friction force.
  - $T_1 = \frac{1}{2} m_1 v_0^2$
  - $T_2 = \frac{1}{2}(m_1 + m_2)v^2$, where m_1 is the mass of the crate, m_2 is the mass of the platform, v_0 is the initial velocity of the crate, and v is the final velocity of the crate and the platform.
  - $U_{1-2} = \mu_k m_1 g d$, where d is the relative distance the crate moves on the platform.

Solution

$$\begin{aligned} T_1 &= \frac{1}{2} m_1 v_0^2 \\ &= \frac{1}{2} \times 200 \times 12^2 \\ &= 14400 \end{aligned}$$

$$\begin{aligned} T_2 &= \frac{1}{2}(m_1 + m_2)v^2 \\ &= \frac{1}{2} \times 900 \times 2.667^2 \\ &\approx 3200 \end{aligned}$$

Since no variation of the vertical position of the system, $V_1 = V_2$.

$$\begin{aligned} U_{1-2} &= \mu_k m_1 g d \\ &= 0.25 \times 200 \times 32.2 \times d \\ &= 1610d \end{aligned}$$

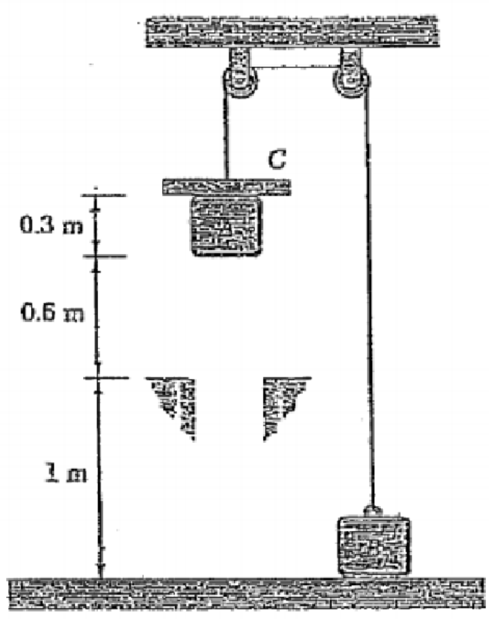
So, $T_1 + V_1 + U_{1-2} = T_2 + V_2$ becomes $14400 + V_1 + 1610d = 3200 + V_2$, then $d = 6.96$ ft..

> The unit is feet because everything is in the imperial system.

Problem 2

Work and Energy

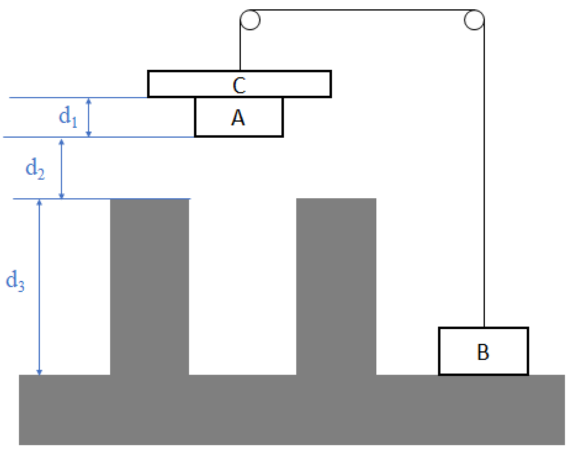
Example 3.12: Two block *A* and *B*, of mass 4-kg and 5-kg, respectively are connected by a cord which passes over pulleys as shown. A 3-kg collar *C* is placed on block *A* and the system is released from rest. After the blocks have moved 0.9 m, collar *C* is removed and blocks *A* and *B* continue to move. Determine the speed of block *A* just before it strikes the ground.



Analysis

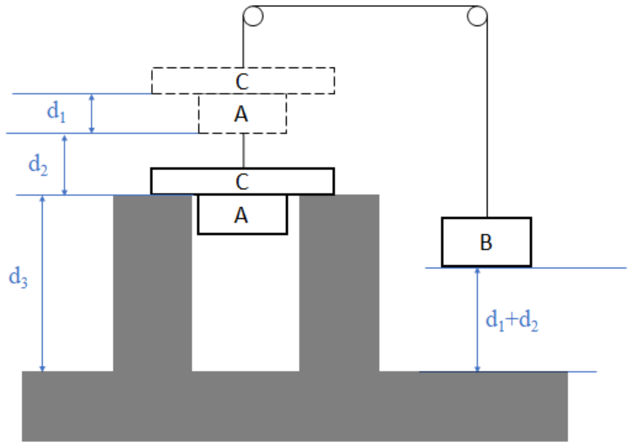
We can break it down into three stages, (1) the system starting from rest. (2) C was stopped and separated from the system. (3) A hits the ground.

1. T_1=0, $V_1 = V_A + V_B + V_C$

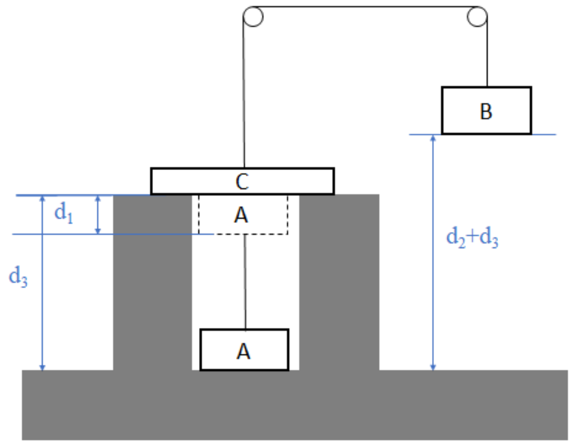


2. for A, B, and C: $T_2 = \frac{1}{2}(m_A + m_B + m_C)v_2^2$, $V_2 = V_{A2} + V_{B2} + V_{C2}$,

   or for A and B: $T_2' = \frac{1}{2}(m_A + m_B)v_2^2$, $V_2' = V_{A2} + V_{B2}$.



3. For A and B: $T_3 = \frac{1}{2}(m_A + m_B)v_3^2$, $V_3 = V_{A3} + V_{B3}$



Solution

Since the external force in the system is gravity (conservative force), we can solve it in a kinetic-potential energy fashion: $T_1 + V_1 + U_{1\to 2} = T_2 + V_2$. Since C was removed in the second stage, we need to break it down into 2 stages.

1. C drops along with A and B, and stopped at some height, where: $v_A = v_C = -v_B$. The system travelled $d_1 + d_2$

   With: $T_1 + V_1 + U_{nc} = T_2 + V_2$, and $U_{nc} = 0$, we have:

   $0 + [m_A g(d_2 + d_3) + m_C g(d_1 + d_2 + d_3)] = \frac{1}{2}(m_A + m_B + m_C)v_2^2 + (m_A g(d_3 - d_1) + m_b g(d_1 + d_2) + m_C g d_3)$

   we can use it to solve v_2:

   $$v_2^2 = \frac{2g(m_A + m_C - m_B)(d_1 + d_2)}{m_A + m_B + m_C}$$

   or $v_2 = 1.72m/s$.

2. A and B continues to move, where $v_A = -v_B$,and $T_2' + V_2' + U_{2\to 3} = T_3' + V_3'$. The system travelled a distance of $d_3 - d_1$.

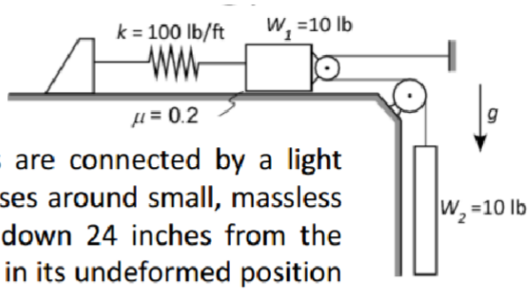
   $\frac{1}{2}(m_A + m_B)v_2^2 + [m_A g(d_3 - d_1) + m_b g(d_1 + d_2)] = \frac{1}{2}(m_A + m_B)v_3^2 + (m_b g(d_2 + d_3))$

   We use it to solve v_3:

   $$v_3^2 = \frac{(m_A + m_B)v_2^2 + 2g(m_A - m_B)(d_3 - d_1)}{m_A + m_B}$$

   where $v_3 = 1.19m/s^2$

Problem 3



Example 3.14: Two blocks are connected by a light inextensible cord which passes around small, massless pulleys. Block 2 is pulled down 24 inches from the position where the spring is in its undeformed position and then released from rest.

Determine the speed of both blocks when block 2 has rebounded from its initial position by 12 inches

Solution

> W_1, W_2 are forces.
> $x_2 = 2x_1$

$T_1 + V_1 + U_{1-2} = T_2 + v_2$, where T is the kinetic energy, V is the potential energy, and U is the work done by the internal friction force.

- $T_1 = 0$, because the it's released from rest.
- $T_2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$, where v_{1f} is the final velocity block 1, and v_{2f} is the final velocity of block 2.
- $U_{1-2} = \mu_k m_1 gd$, where $d = 1ft$
- $V_1 = \frac{1}{2}kx_1^2$, where $x_1 = 1ft$ is the initial displacement of block 1.
- $V_2 = \frac{1}{2}kx_{1f}^2 + m_2g(x_2 - x_{2f})$, where $x_{1f} = 0.5ft$ is the final displacement of block 1, and $x_{2f} = 1ft$ is the final displacement of block 2, $x_2 = 2ft$ is the initial displacement of block 2.

Assemble all the equations above, we have: $v_{1f}^2 + v_{2f}^2 = 170.64$.
So, $v_{1f} = 5.84ft/s$, and $v_{2f} = 11.68ft/s$.

> - Students have 1 week to submit a regrade request to the grader. Students must write a document outlining why they need a regrade and email it or bring it to office hours. Warning: regrades may result in a lower score.
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> - Q2 Josh Murwin
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