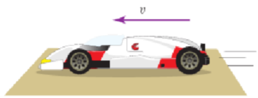
Problem 1

A 3850 lb sports car (driver's weight included), driving along a horizontal rectilinear stretch of road, goes from 0 to 62 mph in 4.2 s.

If the magnitude of the force propelling the car has the form $F_0\left(1 - e^{-t/\tau}\right)$, with $\tau = 0.5$ s, determine F_0.

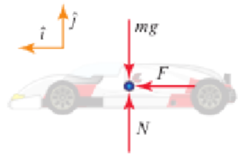


Review

- The impulse-momentum principle: $m v_{x1} + \int_{t_1}^{t_2} F dt = m v_{x2}$
  - v_{x1} and v_{x2} are the velocities of the particle at initial and final instants, respectively.

Solution

Solution



We model the car as a particle subject only to its weight mg, the normal reaction with the ground N, and the propelling force F. We will use the impulse-momentum principle to compute the average value of F over the time interval $t_1 \leq t \leq t_2$, where $t_1 = 0$ and $t_2 = 4.2\,\mathrm{s}$. We use subscripts 1 and 2 to denote quantities at t_1 and t_2, respectively.

Balance Principles. Applying the impulse-momentum principle in the x direction direction, we have

$$m v_{x1} + \int_{t_1}^{t_2} F\,dt = m v_{x2}, \tag{1}$$

where v_x denotes the x component of the velocity of the car.

Force Laws. The force propelling the car has the form

$$F = F_0\big(1 - e^{-t/\tau}\big). \tag{2}$$

Kinematic Equations. Since the car moves in the positive x direction, recalling that the car starts from rest, and letting $v_2 = 62\,\mathrm{mph}$, we have

$$v_{x1} = 0 \quad \text{and} \quad v_{x2} = v_2. \tag{3}$$

Computation. Substituting Eqs. (2) and (3) into Eq. (1), and carrying out the integration, we have

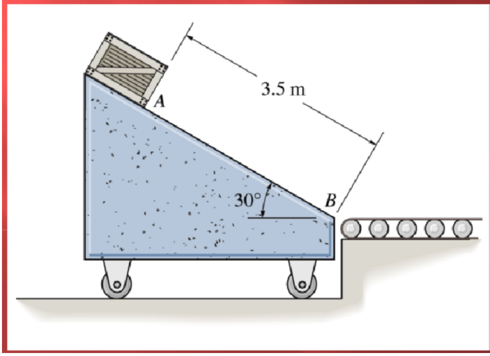
$$F_0\big[t_2 + \tau\big(e^{-t_2/\tau} - 1\big)\big] = m v_2 \quad \Rightarrow \quad F_0 = \frac{m v_2}{t_2 + \tau\big(e^{-t_2/\tau} - 1\big)}, \tag{4}$$

where we have accounted for the fact that $t_1 = 0$. Recalling that $m = 3850\,\mathrm{lb}/g$ $g = 32.2\,\mathrm{ft/s^2}$, $v_2 = 62\,\mathrm{mph} = 62\frac{5280}{3600}\,\mathrm{ft/s}$, $t_2 = 4.2\,\mathrm{s}$, and $\tau = 0.5\,\mathrm{s}$, we can evaluate the last of Eqs. (4) to obtain

$$\boxed{F_{\mathrm{avg}} = 2938\,\mathrm{lb}.}$$

Problem 2

Example 3: The ramp below has a mass of 40 kg. Assume its wheels are not locked and it is free to roll along the floor. A 10-kg crate is released from rest at A and slides down 3.5 m to point B. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches B. Also, what is the velocity of the crate?



Review

- Energy: $T_1 + V_1 = T_2 + V_2$
    - There is no U_{1-2} because there is no non-conservative force like friction.
    - $T_1 = 0$ because they are at rest initially.
    - $V_2 = 0$ because crate reaches the bottom of the ramp in the end.
    - $V_1 = m_c g d \sin\theta$
    - $T_2 = \frac{1}{2} m_c v_{c_2}^2 + \frac{1}{2} m_R v_{R_2}^2$
- Momentum: $m_c v_{c_{1_x}} + m_R v_{R_{1_x}} = m_c v_{c_{2_x}} + m_R v_{R_{2_x}}$ because the system is balanced in the x-direction.
    - $v_{c_{1_x}} = 0$ because the crate is at rest initially.
    - $v_{R_{1_x}} = 0$ because the ramp is at rest initially.
    - $v_{R_{2_x}} = v_{R_2}$ because the ramp moves in the x-direction only.
    - $v_{c_{2_x}} = v_{c/R_2} \cos\theta + v_{R_{2_x}}$
        - because the crate moves in both x and y directions.
        - $\vec{v}_{c_2} = \vec{v}_{c/R_2} + \vec{v}_{R_2} = v_{c/R_2} \cos\theta \hat{i} - v_{c/R_2} \sin\theta \hat{j} + v_{R_2} \hat{i}$

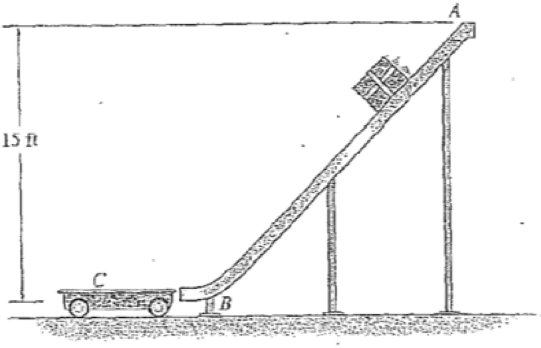
Solution

$$
\begin{aligned}
m_c g d \sin\theta &= \frac{1}{2} m_c v_{c_2}^2 + \frac{1}{2} m_R v_{R_2}^2 \\
0 &= m_c (v_{c/R_2} \cos\theta + v_{R_2}) + m_R v_{R_2} \\
\vec{v}_{c_2} &= v_{c/R_2} \cos\theta \hat{\imath} - v_{c/R_2} \sin\theta \hat{\jmath} + v_{R_2} \hat{\imath}
\end{aligned}
$$

Plug the values into the equations above and solve for v_{c_2} and v_{R_2}. So, $v_{c_2} = 4.41\hat{\imath} - 3.17\hat{\jmath}\ \mathrm{m/s}$ and $v_{R_2} = -1.09\hat{\imath}\ \mathrm{m/s}$.

Problem 3

A 40-lb box slides from rest at A down the smooth ramp onto the surface of a 20-lb cart. Determine the speed of the box at the instant it stops sliding on the cart. If someone ties the cart to the ramp at B, determine the horizontal impulse the box will exert at C in order to stop its motion. Neglect the size of the box.



Solution

(a) :Energy: $T_1 + V_1 = T_2 + V_2$, where $T_1 = 0$, $V_2 = 0$

$$mgh = \frac{1}{2}mv_2^2$$

Solve to get: $v_2 = 31.08 ft/s$

Momentum: $mv_2 = (m_c + m)v_3$, so $v_3 = 20.72 ft/s$

(b) :Assume the impulse $I = \int Fdt$, and $m_b v_2 - \int Fdt = 0$

Just plug the result in (a) and: $I = m_b v_2 = \frac{40}{32.2} * 31.08 = 38.6093(Lb \cdot s)$