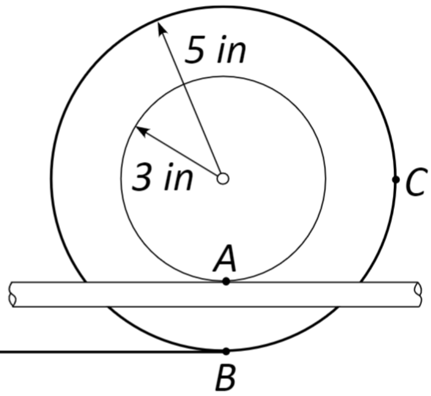
Problem 1

Linkage - Acceleration

Example 5.15: A 3-in. radius drum is rigidly attached to a 5-in. radius drum as shown. The 3-in drum rolls without sliding on the surface shown, and a cord is wound around 5-in. drum. At the instant shown end *D* of the cord has a velocity of 8 in/s and an acceleration of 30 in/s^2, both directed to the left



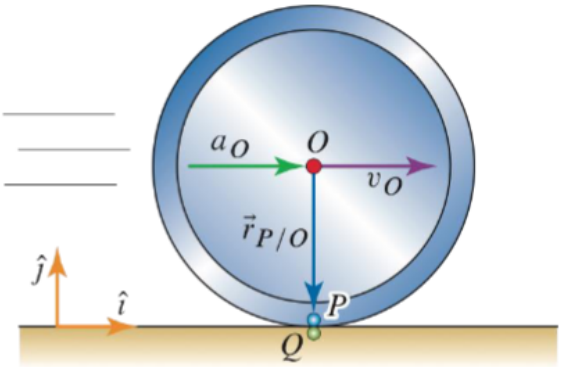
Determine the accelerations of points *A*, *B*, and *C* of the drum.

Set the drum center as point G.

Review

Given $\vec{v}_O$ and $\vec{a}_O$

We can find angular acceleration



- From the velocity analysis, we had already discovered that $\vec{v}_O = v_O\,\hat{\imath}$, $\vec{v}_P = \vec{0}$, and $\vec{\omega}_O = -(v_O/R)\,\hat{k}$.

- For the acceleration of P we can write

$$\vec{a}_P = \vec{a}_O + \alpha_O\,\hat{k} \times \vec{r}_{P/O} - \omega_O^2 \vec{r}_{P/O},$$

Solution

$$\vec{v}_B = \vec{v}_D = \omega \vec{r}_{B/A} \Rightarrow \omega = \frac{8}{2} = 4 \text{ rad/s}$$

$$\vec{\omega} = 4 \text{ rad/s}, \textit{clockwise}$$

$$\vec{a}_G = \vec{a}_A + \vec{\alpha} \times \vec{r}_{G/A} - \omega^2 \vec{r}_{G/A}$$

$$\vec{a}_G = a_G \hat{\imath} \quad \text{(Path of } G \text{ is horizontal)}$$

$$\vec{a}_A = a_A \hat{\jmath} \quad \text{(Path of } A \text{ at this instant is vertical, because the rolling is without slipping)}$$

$$a_G \hat{\imath} = a_A \hat{\jmath} + (-\alpha \hat{k} \times 3\hat{\jmath}) - (4^2)(3)\hat{\jmath}$$

$$a_G = 3\alpha$$

$$\boxed{\vec{a}_A = 48\hat{\jmath} \text{ in/s}^2} \uparrow$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times r_{B/A} - \omega^2 \vec{r}_{B/A}$$

$$\vec{a}_D = -30\hat{\imath} = \vec{\alpha} \times \vec{r}_{B/A} = -\alpha \hat{k} \times (-2\hat{\jmath}) = -2\alpha \hat{\imath}$$

$$-30\hat{\imath} = -2\alpha \hat{\imath}$$

$$\vec{\alpha} = 15 \text{ rad/s}^2$$

$$\vec{a}_B = 48\hat{\jmath} - 30\hat{\imath} - 4^2(-2\hat{\jmath})$$

$$\boxed{\vec{a}_B = -30\hat{\imath} + 80\hat{\jmath} \text{ in/s}^2}$$

$$a_G = 3\alpha = 45 \text{ in/s}^2$$

$$\boxed{\vec{a}_G = 45\hat{\imath} \text{ in/s}^2} \rightarrow$$

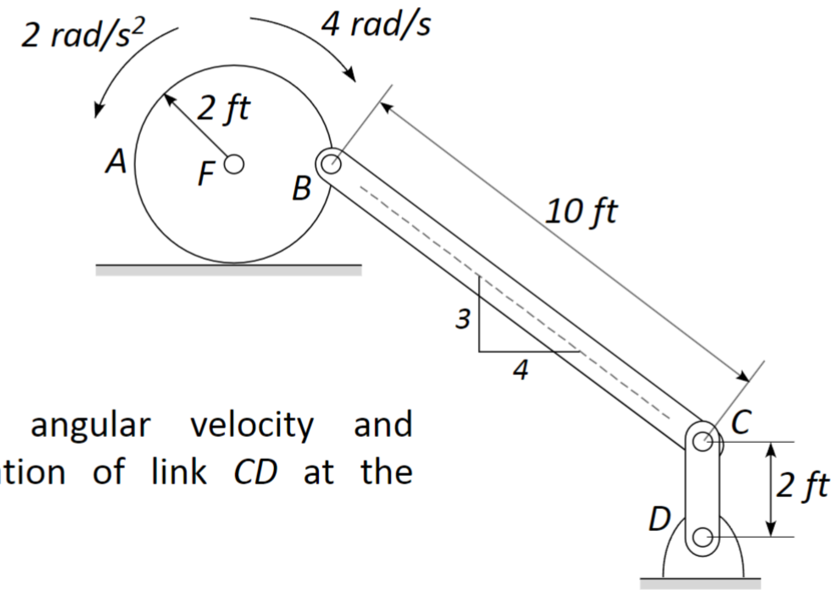
$$\vec{a}_C = \vec{a}_G + \vec{\alpha} \times \vec{r}_{C/G} - \omega^2 \vec{r}_{C/G} = 45\hat{\imath} + (-15\hat{k} \times (5\hat{\imath})) - (4^2)(5\hat{\imath})$$

$$\boxed{\vec{a}_C = -35\hat{\imath} - 75\hat{k} \text{ in/s}^2}$$

Problem 2

Linkage - Acceleration

Example 5.16: The disk at *A* is subjected to the angular motion (velocity and acceleration) shown.



Determine the angular velocity and angular acceleration of link *CD* at the instant shown.

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Recall

- In non-slipping rolling, contact point has zero-velocity $\vec{v}_E = 0$, hence is the instantaneous center at the instant.
- The acceleration of contact point can be different from zero, but only in the direction perpendicular to the contact. $\vec{a}_E = \omega^2 R \cdot \hat{j}$.
- Acceleration analysis of different points on a rigid body:

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}.$$

Solution

Angular Velocity of CD

There are 3 rigid bodies in this problem: disk A, link BC, and link CD. Assume their angular velocities as ω_A, ω_{BC} and ω_{CD}. ω_{CD} is what we need to solve.

To analyze the motion of disk A, set Point E as the contact point, which is also instantaneous center of the disk.

> This is commonly used in non-slipping rolling motions. The contact point is also IC at the time instant.

$$\vec{v}_B = \vec{\omega}_A \times \vec{r}_{B/E} \tag{1}$$

Instantaneous center of link CD is D, since Link CD is spinning around D. velocity of C can be derived as:

$$\vec{v}_C = \vec{\omega}_{CD} \times \vec{r}_{C/D} \tag{2}$$

B and C is connected through link BC, so velocity of B can be represented by relative motion to C:

$$\vec{v}_B = \vec{v}_C + \vec{\omega}_{BC} \times \vec{r}_{B/C} \tag{3}$$

(You can also use $\vec{v}_C = \vec{v}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B}$, to represent velocity of B, but they are equivalent somehow.)

Combining equations $(1), (2), (3)$, we have:

$$\vec{\omega}_A \times \vec{r}_{B/E} = \vec{\omega}_{CD} \times \vec{r}_{C/D} + \vec{\omega}_{BC} \times \vec{r}_{B/C}$$

Where $\omega_A = 4\hat{k},\ \ \vec{r}_{B/E} = 2\hat{\imath} + 2\hat{\jmath},\ \vec{r}_{C/D} = 2\hat{\jmath}, \vec{r}_{B/C} = -8\hat{\imath} + 6\hat{\jmath}$.

Plug in and solve:

$$\begin{cases} 8\hat{\imath} = (-2\omega_{CD} - 6\omega_{BC})\hat{\imath} \\ -8\hat{\jmath} = (-8\omega_{BC})\hat{\jmath}. \end{cases}$$

Solve for $\omega_{BC} = 1 rad/s \cdot \hat{k}$, $\omega_{CD} = 7 rad/s \cdot \hat{k}$

Angular acceleration of CD

Using the angular acceleration equation, we have:

$$\vec{a}_B = \vec{a}_F + \vec{\alpha}_A \times \vec{r}_{B/F} - \omega_A^2 \vec{r}_{B/F} \tag{4}$$

By recalling acceleration of F is always perpendicular to contact surface and is $\omega^2 r$, $\vec{a}_F = \omega_A^2 R_A \cdot \hat{\jmath} = 32\hat{\jmath}$.

Also, by analyzing B and C on link BC, we have acceleration of B described by C:

$$\vec{a}_B = \vec{a}_C + \vec{\alpha}_{BC} \times \vec{r}_{B/C} - \omega_{BC}^2 \vec{r}_{B/C} \tag{5}$$

Where $\omega_{BC} = 1\hat{k},\ \vec{r}_{B/C} = -8\hat{\imath} + 6\hat{\jmath}$

Acceleration of C is :

$$\vec{a}_C = \vec{a}_D + \vec{\alpha}_{CD} \times \vec{r}_{C/D} - \omega_{CD}^2 \vec{r}_{C/D} \tag{6}$$

Where $\vec{a}_D = 0$, $\omega_{CD} = 7\hat{k},\ \vec{r}_{C/D} = 2\hat{j}$.

Combining $(4), (5)$ and (6), we have:

$$\omega_A^2(R \cdot \hat{j} - \vec{r}_{B/F}) + \vec{\alpha}_A \times \vec{r}_{B/F} = (\vec{\alpha}_{CD} \times \vec{r}_{C/D} - \omega_{CD}^2 \vec{r}_{C/D}) + \vec{\alpha}_{BC} \times \vec{r}_{B/C} - \omega_{BC}^2 \vec{r}_{B/C}$$

Separating i and j terms, we have

$$\begin{cases} (-4 - 32)\hat{i} = (-2\alpha_{CD} - 6\alpha_{BC} + 8)\hat{i} \\ -8\hat{j} = (-98 - 8\alpha_{BC} - 6)\hat{j}. \end{cases}$$

Solve for $\alpha_{BC} = 13.5 rad/s \cdot \hat{k}$, $\alpha_{CD} = 62.5 rad/s \cdot \hat{k}$.