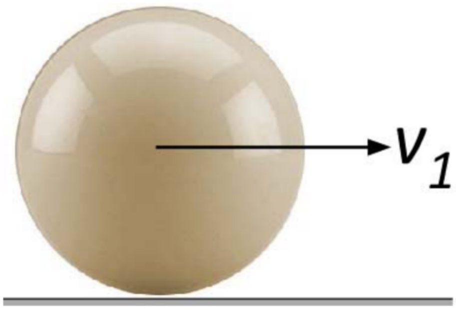
Problem 1

Impulse & Momentum–Planar Rigid Bodies

Example 8.2: A uniform sphere of mass *m* and radius *r* is projected along a rough horizontal surface with a linear velocity v_1 and no angular velocity. The coefficient of kinetic friction between the sphere and the surface is denoted by μ_k



Determine

(a) time t_2 when the sphere will start rolling (without sliding)

(b) the linear and angular velocities of the sphere at time t_2.

8-5

Review

- Angular Momentum: $\vec{h}_P = I_G\vec{\omega}_B + \vec{r}_{G/P} \times m\vec{v}_G$.
- Angular Impulse-Momentum Principle(2 equivalent forms):

$$\vec{M}_p = \dot{\vec{h}}_p + \vec{v}_p \times m\vec{v}_G \tag{1}$$

$$\vec{h}_{P1} + \int_{t_1}^{t_2} \vec{M}_p dt = \vec{h}_{P2} \tag{2}$$

- Compare with linear momentum we have learnt:

$$\vec{F} = \dot{\vec{p}}$$
$$\vec{p}_1 + \int_{t_1}^{t_2} \vec{F} dt = \vec{p}_2$$

Solution

(a) Initial State: $v_1, \omega_1 = 0$, Final state: v_2, ω_2,

With rolling without slipping condition, and the ball is rolling clockwise and to $+x$ direction, we have: $v_2 = -\omega_2 r$

- Linear Momentum principle:

$$
\begin{aligned}
mv_{1x} + (-\mu_k N)t_2 &= mv_{2x} && \text{(x-direction)} \\
mv_{1y} + (N - mg)t_2 &= mv_{2y} && \text{(y-direction)}
\end{aligned}
$$

Knowing the ball have no velocity on y direction, we can know that $N = mg$. Plug in to the x direction equation to have:

$$mv_1 - \mu_k mgt_2 = mv_2 \tag{1}$$

- Angular Momentum principle:

$$I_G\omega_1 - \mu_k Nt_2 = I_G\omega_2$$

Plug in $I_G = \frac{2}{5}mr^2$, $\omega_1 = 0, \omega_2 = -v_2/r$ and $N = mg$, we have:

$$\mu_k mgt_2 = I_G\frac{v_2}{r} \tag{2}$$

Solving equations (1) and (2) for:

$$t_2 = \frac{2v_2}{7\mu_k g}$$

(b) By plugging in value of t_2 to equation (1), we have:

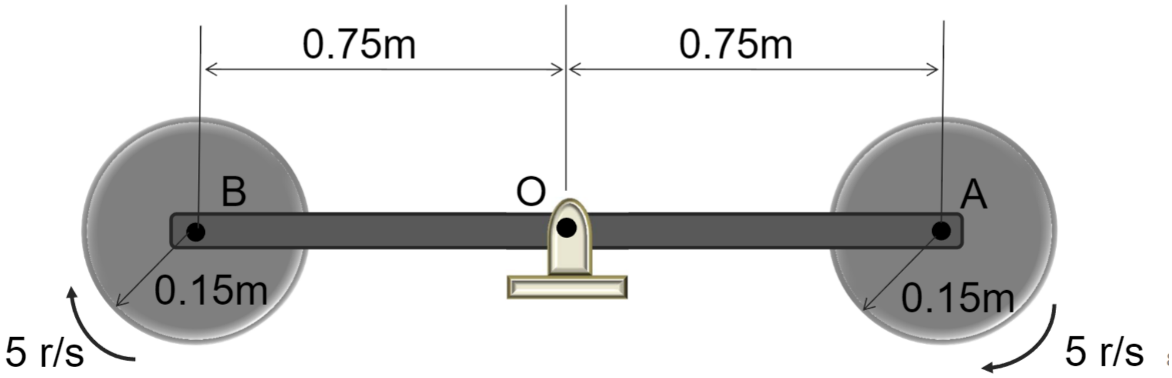
$$v_2 = \frac{5}{7}v_1$$

And Using $\omega_2 r = v_2$ we have:

$$\omega_2 = \frac{-5v_1}{7r}$$

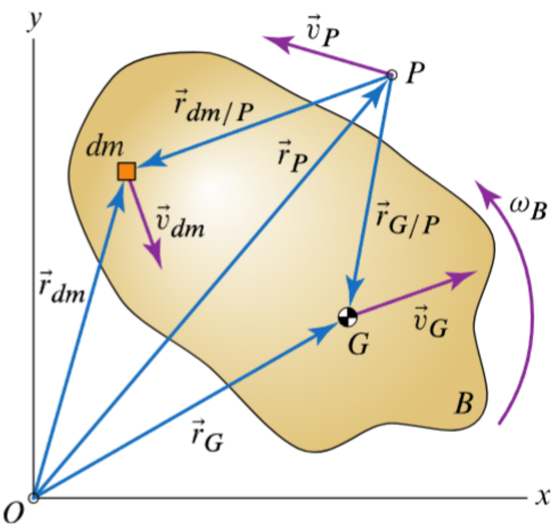
Problem 2

Example 8.5: The 2 kg rod ACB supports two 4 kg disks at its ends. If both disks are given a clockwise angular velocity of 5 rad/s while the rod is held stationary and then released, determine the angular velocity of the rod after both disks have stopped spinning relative to the rod due to frictional resistance at the pins A and B. Motion is in the horizontal plane. Neglect friction at pin O.



Review and Hints

- For the angular impulse momentum: to convert the angular impulse momentum from point P to point G, we have:
  - $\vec{h}_P = I_G\vec{\omega}_B + m\vec{r}_{G/P} \times \vec{v}_G$



- Conservation of angular impulse monmentum:
  - It's conserved when there is no external moment applied to the system. In this case, there is no external moment applied to the system due to the negligible friction.
  - $\vec{h}_{P1} = \vec{h}_{P2}$
    - $\vec{h}_{P1}$ is the angular impulse momentum at point P before the mition.
    - $\vec{h}_{P2}$ is the angular impulse momentum at point P after the motion.

Solution

Before the motion, we have:

$$\begin{aligned}\vec{h}_{P1} &= (I_{GB}\vec{\omega}_1 + m_B\vec{r}_{B/O} \times \vec{v}_{B1}) + (I_{GA}\vec{\omega}_1 + m_A\vec{r}_{A/O} \times \vec{v}_{A1}) + (I_{OC}\vec{\omega}_C + 0)\\ &= \underline{DiskB} + \underline{DiskA} + \underline{RodC}\\ &= (I_{GB}\vec{\omega}_1 + 0) + (I_{GA}\vec{\omega}_1 + 0) + (I_{OC}\vec{\omega}_C)\end{aligned}$$

In the initial instant, the $\vec{v}_{B1}$, $\vec{v}_{A1}$ and $\vec{\omega}_C$ are all 0 because the system is at rest.

After the motion, we have:

$$\begin{aligned}\vec{h}_{P2} &= (I_{GB}\vec{\omega}_2 + m_B\vec{r}_{B/O} \times \vec{v}_{B2}) + (I_{GA}\vec{\omega}_2 + m_A\vec{r}_{A/O} \times \vec{v}_{A2}) + (I_{OC}\vec{\omega}_C + 0)\\ &= \underline{DiskB} + \underline{DiskA} + \underline{RodC}\\ &= (I_{GB}\vec{\omega}_2 + m_B\omega_2 r_{B/O}^2\hat{k}_r) + (I_{GA}\vec{\omega}_2 + m_A\omega_2 r_{A/O}^2\hat{k}_r) + (I_{OC}\vec{\omega}_2)\end{aligned}$$

In the final instant, the $\vec{v}_{B2} = \omega_2\vec{r}_{B/O}$, $\vec{v}_{A2} = \omega_2\vec{r}_{A/O}$ and $\vec{\omega}_C = \omega_2\hat{k}$ because the system is rotating at the same angular velocity ω_2.

Fill in the moment of inertia:

$$\begin{aligned}I_{GA} &= \frac{1}{2}m_A r_{A/O}^2\\ &= 0.045kg \cdot m^2\\ I_{GB} &= \frac{1}{2}m_B r_{B/O}^2\\ &= 0.045kg \cdot m^2\\ I_{OC} &= \frac{1}{12}m_C L^2\\ &= 0.375kg \cdot m^2\end{aligned}$$

Therefore, we have:

$$0.045 \cdot (-5\hat{k}) + 0.045 \cdot (-5\hat{k}) = 0.045 \cdot (\omega_2\hat{k}) + 2.25 \cdot (\omega_2\hat{k}) + 0.045\dot{(}\omega_2\hat{k}) + 2.25 \cdot (\omega_2\hat{k}) + 0.375 \cdot (\omega_2\hat{k})$$

So, $\omega_2 = -0.0906\ rad/s$. The negative sign indicates that the rotation is clockwise.