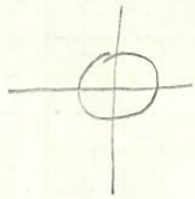
Circle Notes

Standard form (Center at origin)

$x^2 + y^2 = r^2$

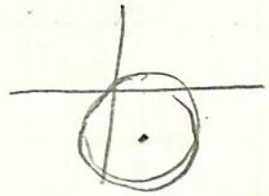
$x^2 + y^2 = 25 \quad (r = 5)$



Standard form (not at origin)

$(x-h)^2 + (y-k)^2 = r^2$

$(x-1)^2 + (y+2)^2 = 9$ =



General form = $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Typically we will use Standard form.

Example 3 pg 32

$C = (\overset{h}{3}, \overset{k}{-2})$

$P = (\overset{x}{-1}, \overset{y}{1})$

$(x-h)^2 + (y-k)^2 = r^2$

$(-1-3)^2 + (1+2)^2 = r^2$

$16 + 9 = r^2$

$25 = r^2$

$\boxed{r = 5}$

Equation = $\boxed{(x-3)^2 + (y+2)^2 = 25}$

Example 5 pg 33

$x^2+y^2-4x+2y-11=0$

$(x-h)^2+(y-k)^2=r^2$

$x^2-4x+y^2+2y=11$

$(x^2-4+4)+(y^2+2y+1)=11+4+1$

$(x-2)^2+(y+1)^2=16$

Center $=(2,-1)$

$r=4$

Example 6.

$x^2+y^2+6x-4y-12=0$

$x^2+6x+9+y^2-4y+4=12+9+4$

$\frac{6}{2}=3^2=9$ $\frac{4}{2}=2^2=4$

$(x+3)^2+(y-2)^2=25$

$C=(-3,+2)$

$r=5$

#12 pg 35 $x^2+y^2-4x-5=0$

$x^2-4x+y^2=5$ $+4$

$x^2-4x+4+y^2=9$

$(x-2)^2+(y+0)^2=9$

$C=2,0$

$R=3$

#14 pg 35. $x^2+y^2-6x+14y+42=0$

$x^2-6x+9+y^2+14y = -42+9+49$

$+9 \quad +49$

56

-42

16

$(x-3)^2+(y+7)^2=16$

$C=3,-7$

$r=4$

#20. $x^2+y^2-5x-8y=0$

$x^2-5x \quad +y^2-8y = 0 \quad +2.5^2 \quad +16$

6.25

$x^2-5x+2.5^2+y^2-8y+16=$

6.25

$(x-2.5)^2+(y-4)^2=22.25$

$C=2.5,4$

$r=\sqrt{22.25}$

$r=$

Ex 1.6 pg 34. Homework = 11-23 odds.

Homework Circles Ex 1.6 11-23 odds

Find the center and radius of the given circle.

11 $x^2 + y^2 = 16$

Standard form = $x^2 + y^2 = r^2$

Center = $(0, 0)$

radius = $r^2 = 16$

radius = 4

13. $x^2 + y^2 + 6x - 8y - 39 = 0$

$x^2 + 6x + 9 + y^2 - 8y + 16 = 39 + 9 + 16$

$(x+3)^2 + (y-4)^2 = 64$

Center = $(-3, 4)$

radius = 8 $\sqrt{64}$

15. $x^2 + y^2 - 8x + 12y - 8 = 0$

$x^2 - \frac{8}{2}x + 16 + y^2 + \frac{12}{2}y + 36 = 8 + 16 + 36$

$4 \cdot 4 = 16$ $6 \cdot 6 = 36$

$(x-4)^2 + (y+6)^2 = 60$

Center = $(4, -6)$

radius = $\sqrt{60} = \sqrt{4} \cdot \sqrt{15}$

radius = $2\sqrt{15}$

17. $x^2 + y^2 - 12x - 2y - 12 = 0$

$x^2 - 12x + 36 \; y^2 - 2y + 1 = 12 + 36 + 1$

$(x-6)^2 + (y-1)^2 = 49$

Center = $(6, 1)$

radius = 7

Homework Circle | Ex 1.6e 11-23 odd continued

19. $x^2+y^2+7x+3y-9=0$

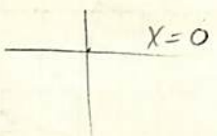
$x^2+7x+12.25+y^2+3y+2.25 = 9+12.25+2.25$

$(x+3.5)^2+(y+1.5)^2 = 23.5$

Center = $(-3.5, -1.5)$

(?) radius = $\sqrt{23.5} = 4.84768$

21. Find the equation of the circle or circles whose center is on the y axis and that contain the points $(1,4)$ & $(-3,2)$. Give the center & the radius.



$(x-h)^2+(y-k)^2 = r^2$

$(1-0)^2+(4-k)^2 = r^2$

$(-3-0)^2+(2-k)^2 = r^2$

$(1-0)^2+(4-k)^2 = (-3-0)^2+(2-k)^2$

$1+(4-k)^2 = 9+(2-k)^2$

$1+(4-k)(4-k) = 9+(2-k)(2-k)$

$1+16-4k-4k+k^2 = 9+4-2k-2k+k^2$

$17-8k+k^2 = 13-4k+k^2$

$-13+8k \quad\quad -13+8k$

$4 = 4k$

$k=1$

center = $(0,1)$

$(x-h)^2+(y-k)^2 = r^2$

$(1-0)^2+(4-1)^2 = r^2$

$1+9 = r^2$

$10 = r^2$

$r = \sqrt{10}$

#23. Find the equation of the circle containing the points (3,1), (0,0), and (8,4). Give its center and radius.

$(x-h)^2 + (y-k)^2 = r^2$

$*\quad (3-h)^2 + (1-k)^2 = r^2$

$*\quad (0-h)^2 + (0-k)^2 = r^2$

$(3-h)(3-h) + (1-k)(1-k) = (0-h)(0-h) + (0-k)(0-k)$

$9 - 3h - 3h + h^2 + 1 - k - k + k^2 = 0 + h^2 + 0 + k^2$

$**\quad 10 - 6h - 2k = 0$

$(0-h)^2 + (0-k)^2 = (8-h)^2 + (4-k)^2$

$(0-h)(0-h) + (0-k)(0-k) = (8-h)(8-h) + (4-k)(4-k)$

$h^2 + k^2 = 64 - 8h - 8h + h^2 + 16 - 4k - 4k + k^2$

$**\quad 80 - 16h - 8k = 0$

$**\quad 10 - 6h - 2k = 0 \quad \times(-4)$

$80 - 16h - 8k = 0$

$-40 + 24h + 8k = 0$

$40 + 8h + 0 = 0$

$8h = -40$

$h = -5$

$80 - 16(-5) - 8k = 0$

$80 + 80 - 8k = 0$

$160 - 8k = 0$

$-8k = -160$

$k = 20$

$10 - 6(-5) - 2k = 0$

$10 + 30 - 2k = 0$

$-2k = -40$

$k = 20$

h k

Center = $(-5, 20)$

$r = \sqrt{425}$ or 20.615

$(3-h)^2 + (1-k)^2 = r^2 =$

$64 + 361 = 425 = 20.615$

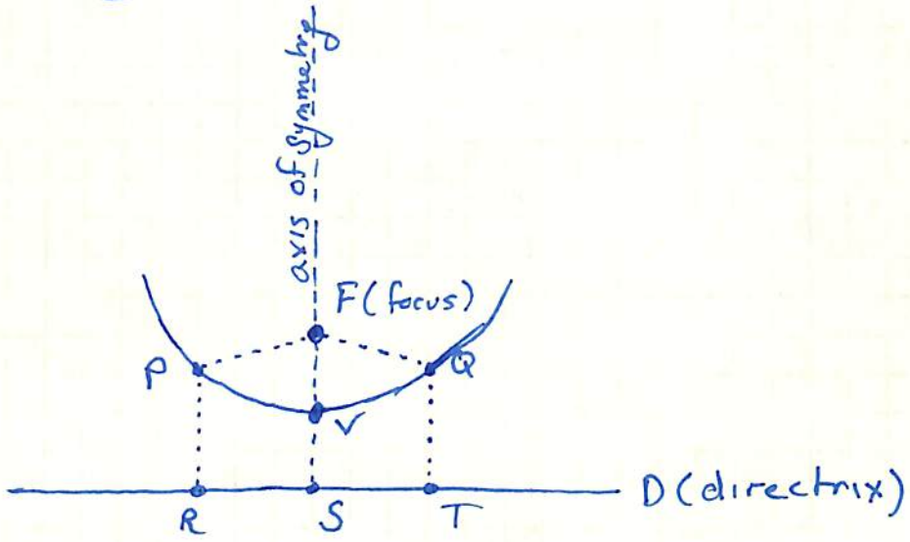
$(0-(-5))^2 + (0-(20))^2 = 425 = 20.615$

$25 + 400$

$(8-(-5))^2 + (4-20)^2 =$

$169 + 256 = 425$

THE PARABOLA



$RP = PF$
$SV = VF$
$TQ = QF$

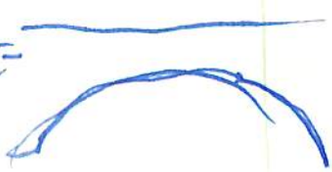
- Parabola consists of all points that are the same distance from a given fixed point and a given fixed line.
- The fixed point is called the focus.
- The fixed line is called the directrix.
- The point V, midway between the directrix is called the Vertex
- The vertex and the focus lie on a line perpendicular to the directrix, which is called the <u>axis of symmetry</u>.

There are two standard forms:

$y^2 = 4px$ = 

$y^2 = 4(-p)x$ = 

$x^2 = 4py$ = 

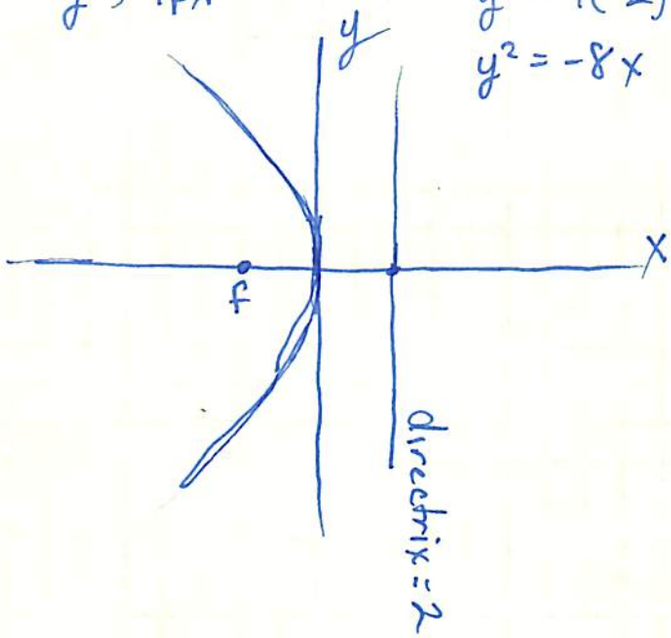
$x^2 = 4(-p)y$ = 

Example 3. Find the equation of the parabola with focus at $(-2, 0)$ and directrix $x = 2$.

$y^2 = 4px$

$y^2 = 4(-2)x$

$y^2 = -8x$

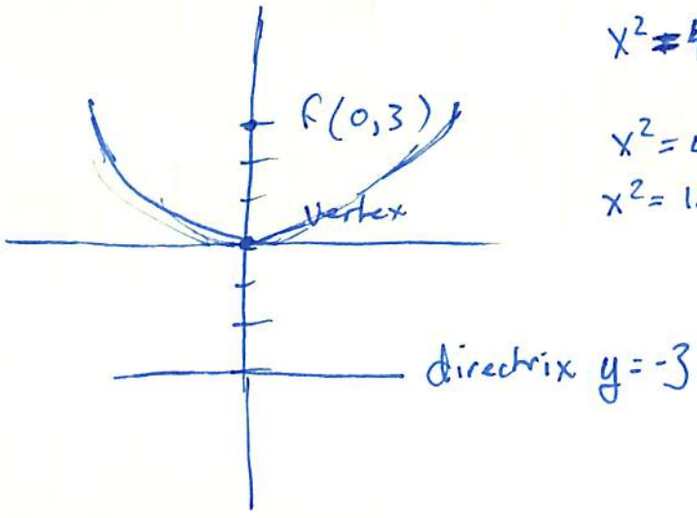


Example 4. Find the equation of the Parabola with focus at $(0, 3)$ and with directrix $y = -3$.

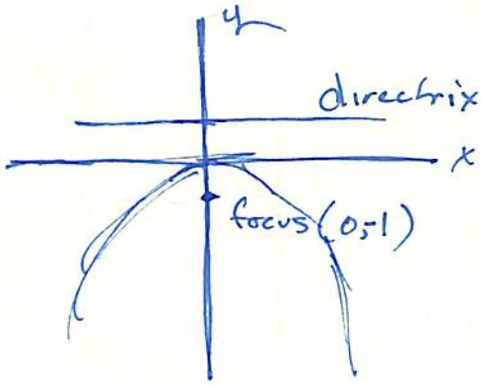
$x^2 = 4py$

$x^2 = 4(3)y$

$x^2 = 12y$



Example 5. Find the equation of the parabola with focus at $(0,-1)$ and with directrix $y=1$.



$x^2 = 4py$

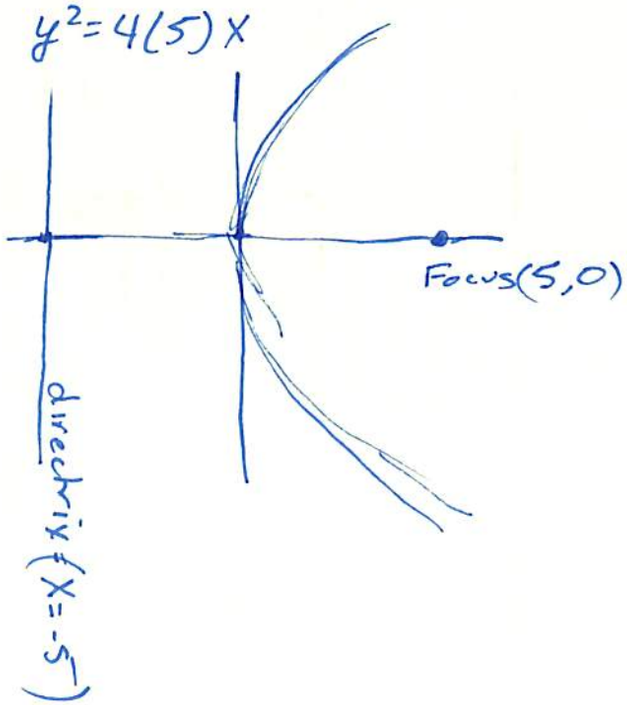
$x^2 = 4(-1)y$

$x^2 = -4y$

Example 6. $y^2 = 20x$

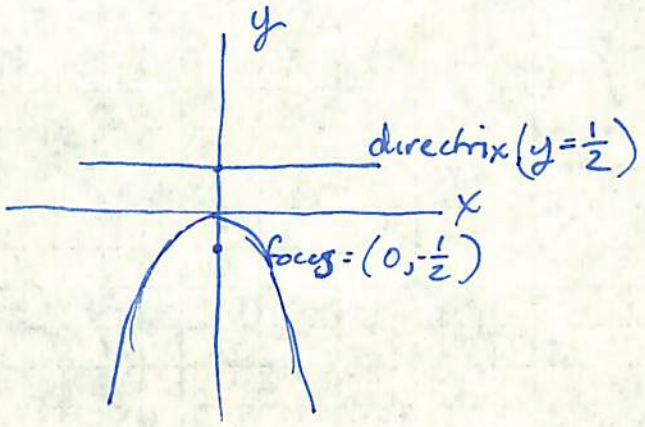
~~$y^2 = 4(5)y$~~

$y^2 = 4(5)x$

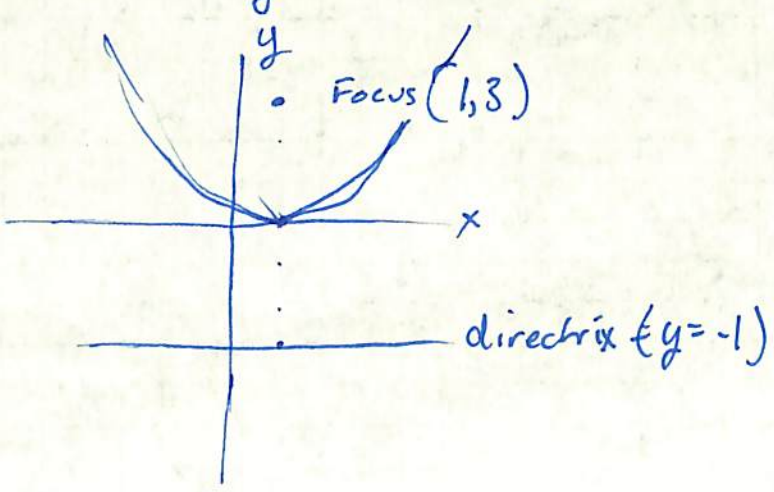


Example 7 $x^2 = -2y$

$x^2 = 4\left(-\frac{1}{2}\right)y$



Example 8. Find the equation of the Parabola with focus at $(1,3)$ and with the line $y=-1$ as directrix.



$PF = PQ$

$\sqrt{(x-p)^2+(y-F)^2} = \sqrt{(x-x)^2+[y-(-D)]^2}$

Focus, directrix

$\sqrt{(x-1)^2+(y-3)^2} = \sqrt{(x-x)^2+[y-(-1)]^2}$

$(x-1)^2+(y-3)^2 = (x-x)^2+(y+1)^2$

$x^2-x-x+1+y^2-3y-3y+9 = x^2-x^2-x^2+x^2+y^2+y+y+1$ $\quad (x-x)(x-x)$

$x^2-x^2-x^2+x^2$

$x^2-2x+1+y^2-6y+9 = y^2+2y+1$

$-y^2 \quad -1 \quad -y^2 \quad -2y-1$

$x^2-2x+1-8y+8=0$

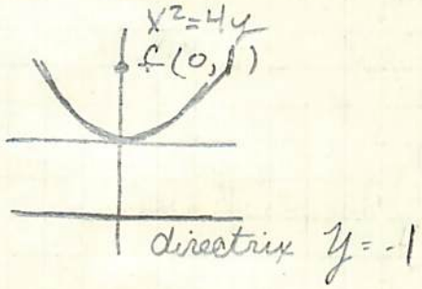
$\boxed{x^2-2x-8y+9=0}$

Exercise 1.7 | Parabola

find the focus and the directrix of each parabola. Sketch each graph.

1. $x^2 = 4y$ $x^2 = 4(1)y$

focus = $(0, 1)$

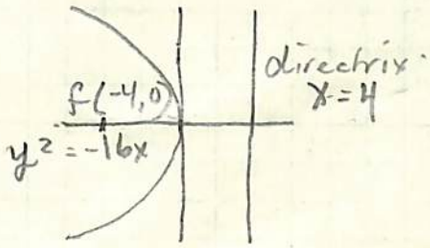


3. $y^2 = -16x$

~~focus = $(-16, 0)$~~

$y^2 = 4(-4)x$

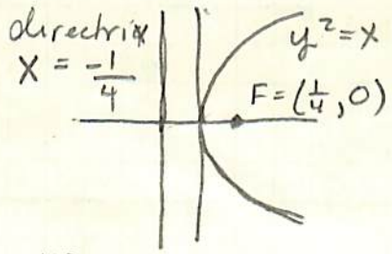
focus = $(-4, 0)$



5. $y^2 = x$

$y^2 = 4\left(\frac{1}{4}\right)x$

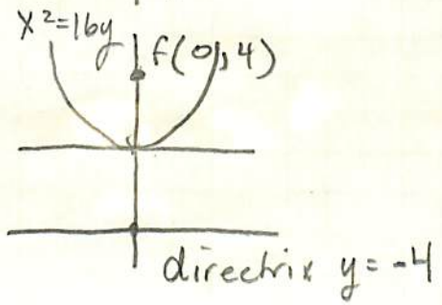
focus = $\left(\frac{1}{4}, 0\right)$



7. $x^2 = 16y$

$x^2 = 4(4)y$

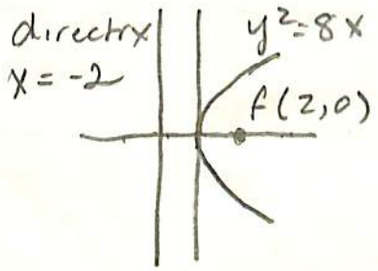
focus = $(0, 4)$



9. $y^2 = 8x$

$y^2 = 4(2)x$

focus = $(2, 0)$

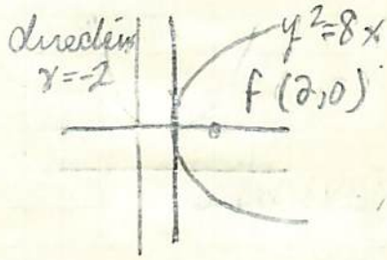


Find the equation of the parabola with given focus and directrix.

11. $(2, 0)$, $x = -2$

$y^2 = 4(2)x$

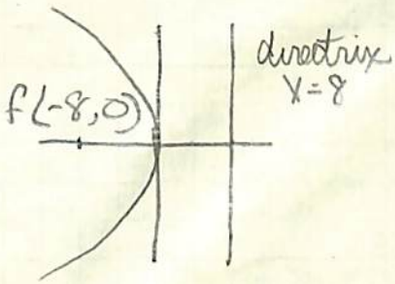
$y^2 = 8x$



13. $(-8, 0)$, $x = 8$

$y^2 = 4(-8)x$

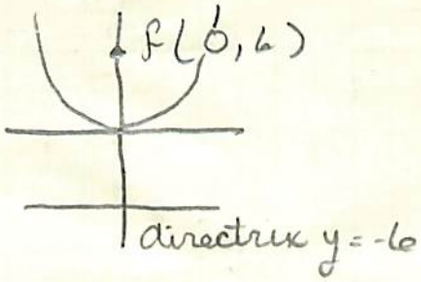
$y^2 = -32x$



15. $(0, 6)$, $y = -6$

$x^2 = 4(6)y$

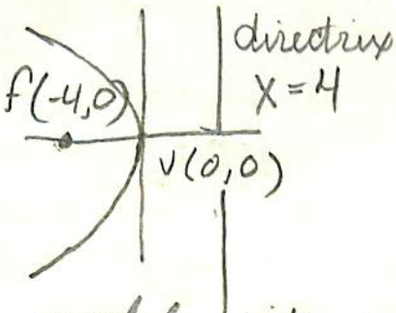
$x^2 = 24y$



17. Find the equation of the parabola with focus at $(-4, 0)$ and vertex at $(0, 0)$

$y^2 = 4(-4)x$

$y^2 = -16x$

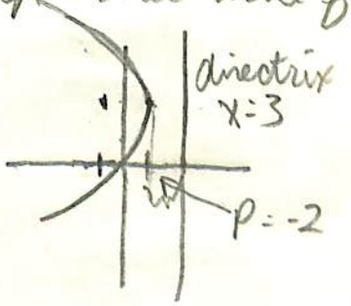


$x \quad y$
$h \quad k$

19. Find the equation of the parabola with focus $(-1, 3)$ and directrix $x = 3$.

$y^2 = 4(p)x$

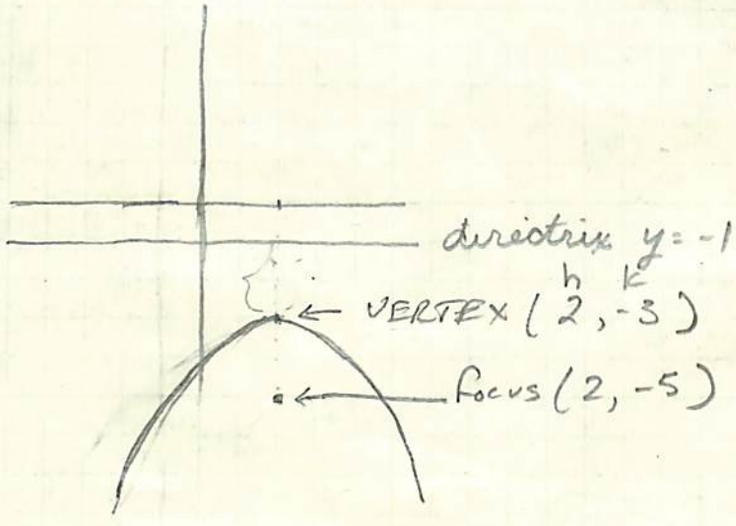
$(y-3)^2 = 4(-2)(x+1)$



$(y-3)(y-3) = -8x + 4$

$y^2 - 6y + 9 = -8x + 8$ = $(y^2 - 6y + 8x + 1 = 0)$ General form

20. Find the equation of the parabola with focus $(2,-5)$ and directrix $y=-1$



$x^2 = 4py$

$(x-h)^2 = 4(p)(y-k)$

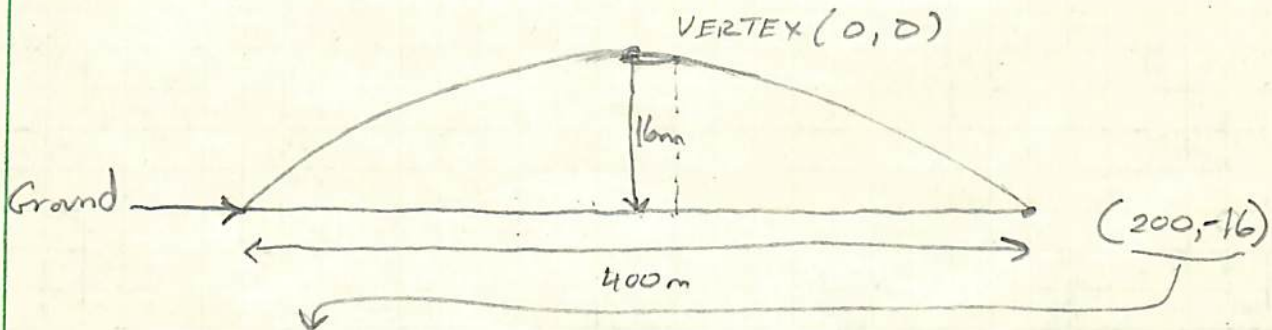
$(x-2)^2 = 4(-2)(y--3)$

$(x-2)(x-2) = -8(y+3)$

$x^2-4x+4 = -8y-24.$

$\boxed{x^2-4x+8y+28=0}$

21. The surface of a roadway over a bridge follows a parabolic curve with vertex at the middle of the bridge. The span of the bridge is 400m. The roadway is 16m higher in the middle than at the ends. How far above the end supports (Ground) is a point 50 m from the middle? 150m from the middle?



$x^2 = 4py$

$(200)^2 = 4(p)(-16)$

$40 \times 10^3 = -64(p)$

$p = -625$

$x = 50m$

$x^2 = 4py$

$50^2 = 4(-625)\,y$

$2.5 \times 10^3 = -2.5 \times 10^3\, y$

$y = -1$ therefor @ 50m from Center the Height is 16m - 1m

@ 50m from Center the Height is 15m

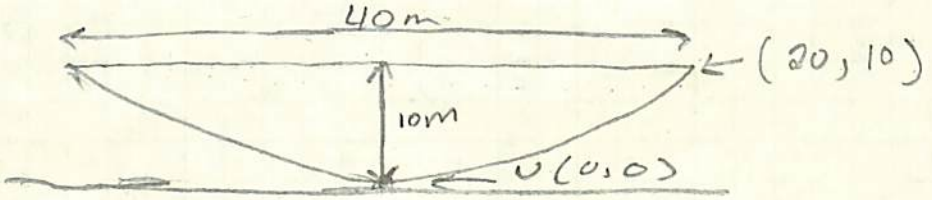
$x = 150m$

$x^2 = 4py$ $\quad 150^2 = 4(-625)y$ $\quad 22.5 \times 10^3 = -2.5 \times 10^3\, y$

$y = -9m$ $\quad$ Height = 16 - 9 = 7m

@ 150m from Center the Height is 7m

22 The shape of a wire hanging between two poles closely approximates a parabola. Find the equation of a wire that is suspended between two poles 40m apart and whose lowest point is 10m below the level of the insulators.



$x^2 = 4py$

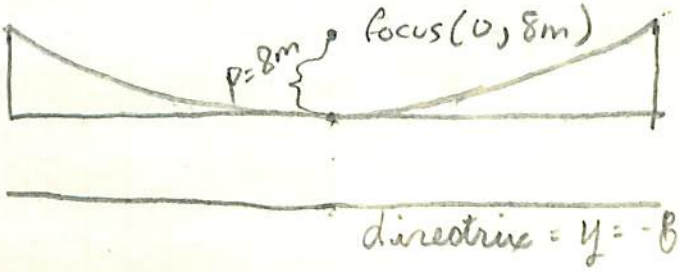
$20^2 = 4(p)(10)$

$p = \frac{20^2}{40} = \frac{400}{40}$

$p = 10$

$x^2 = 4(10)y$ $\boxed{x^2 = 40y}$

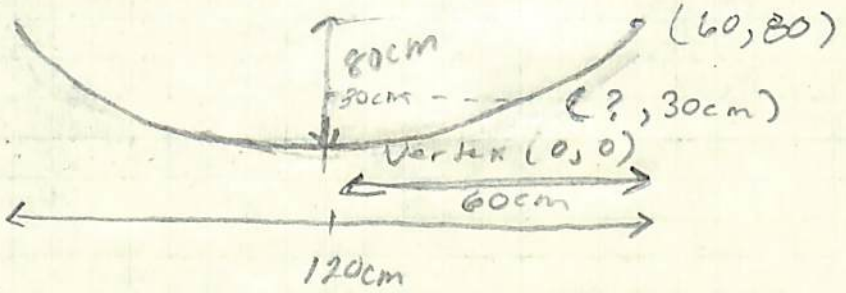
23 A suspension bridge is supported by two cables that hang between two supports. The curve of these cables is approximately parabolic. Find the equation of this curve if the focus lies 8m above the lowest point of the cable.



$x^2 = 4(8)y$

$\boxed{x^2 = 32y}$

24. A culvert is shaped like a parabola, 120 cm across the top and 80 cm deep. How wide is the culvert 50 cm from the top?



$x^2 = 4(P)y$

$60^2 = 4(P)80$

$P = \frac{60^2}{4 \cdot 80}$

$P = 11.25$

50 cm from the top = 30 cm from Vertex y

$x^2 = 4(P)(y)$

$x^2 = 4(11.25)(30)$

$x^2 = 1.35 \times 10^3$

$x = 36.742$

width @ -50cm $2x = 2(36.742)$

width @ -50cm = 73.485 cm