Exercise 3.4

1. The sum of two positive numbers is 56. Find the two numbers if their product is to be maximum.

$x + y = 56 \rightarrow y = 56 - x$

$x \cdot y = \max \rightarrow x(56-x) = \max$

$x = 56 - y$

$\max = 56x - x^2$

$y \cdot (56-y) = \max$

$\frac{dm}{dx} = 56 - 2x$

$56y - y^2 = \max$

$0 = 56 - 2x$

$\frac{dm}{dx} = 56 - 2y$

$x = \frac{56}{2}$ $\boxed{x = 28}$

$0 = 56 - 2y$

$y = \frac{56}{2}$ $\boxed{y = 28}$

Graph

Exercise 3.4

1. The sum of two positive numbers is 56. Find the two numbers if their product is to be a maximum.

$1 + 55 = 56 \quad 1 \cdot 55 = 55$

$2 + 54 = 56 \quad 2 \cdot 54 = 108$

$3 + 53 = 56 \quad 3 \cdot 53 = 159$

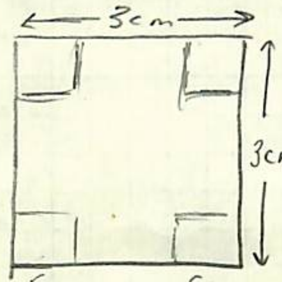
$\vdots \qquad\qquad \vdots$

$27 + 29 = 56 \quad 27 \cdot 29 = 783$

$\boxed{28 + 28 = 56 \quad 28 \cdot 28 = 784}$ $\boxed{28; 28}$

$29 + 27 = 56 \quad 29 \cdot 27 = 783$

3. An open box is to be made from a square piece of aluminum, 3cm on a side, by cutting equal squares from each corner and then folding up the sides. Determine the dementions of the box that will have the largest volume.



$A = x \cdot y \qquad P = 2x + 2y$

$A = (3-2s)(3-2s) = 9 - 6s - 6s + 4s^2$
$= 9 - 12s + 4s^2$

$V = (3-2s)^2(s) = 9s - 12s^2 + 4s^3$

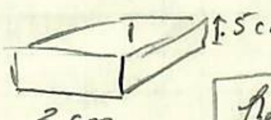
$max = \frac{dv}{ds} = 9 - 24s + 12s^2$

$0 = 9 - 24s + 12s^2$

$\underline{s = .5}$ or 1.5 (1.5 cm for would cut the square apart)

Graph S vs Volume

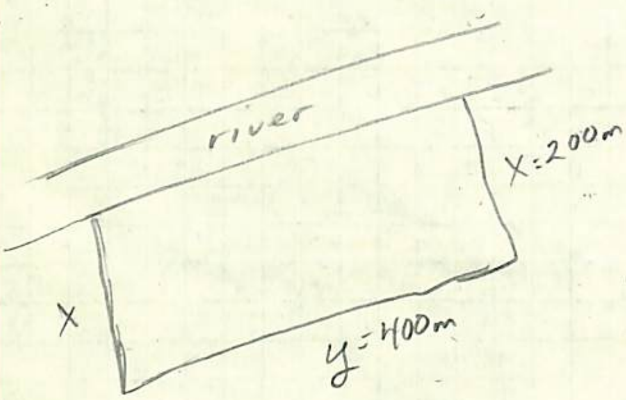
$s = .5$



$\boxed{\text{Dementions} = 2cm \times 2cm \times .5cm}$

Exercises 3.4

5. A man wishes to fence in a rectangular plot lying next to a river. No fenbe is requrred along the river bank. If he has 800 m of fence and he wishes the maximum area to be fenced, find the demensions of the desired enclosed plot.



$$2x + 1y = 800m$$

$$y = 800m - 2x$$

$$A = x \cdot y$$

$$A = x(800m - 2x)$$

$$A = 800x - 2x^2$$

$$\frac{da}{dx} = 800 - 4x$$

$$0 = 800 - 4x$$

$$x = \frac{800}{4}$$

$$\underline{x = 200}$$

$$2x = 800 - y$$

$$x = 400 - \frac{y}{2}$$

$$A = \left(400 - \frac{y}{2}\right)(y)$$

$$A = 400y - \frac{y^2}{2}$$

$$\frac{da}{dy} = 400 - y$$

$$0 = 400 - y$$

$$\underline{y = 400}$$

$X = 200m$
$y = 400m$

7. Find the <u>maximum possible area</u> of a rectange whose perimeter is 36cm.

$$P = 2x + 2y \qquad P = 36\,cm$$

$$36 = 2x + 2y \qquad x = \frac{36}{2} - y \qquad y = \frac{36}{2} - x$$

$$A = x \cdot y \qquad A = x \cdot y$$

$$A = (18 - y)y \qquad A = x(18 - x)$$

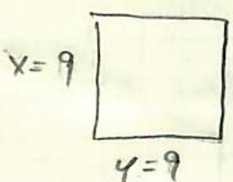
$$A = 18y - y^2 \qquad A = 18x - x^2$$

$$\frac{da}{dy} = 18 - 2y \qquad \frac{da}{dx} = 18 - 2x$$

$$0 = 18 - 2y \qquad 0 = 18 - 2x$$

$$y = \frac{18}{2} \qquad x = \frac{18}{2}$$

$$y = 9 \qquad x = 9$$



$A = x \cdot y$
$A = 9cm \cdot 9cm$
$A = 81\,cm\,cm$
$\boxed{A = 81\,cm^2}$

TOPS 35502

Exercise 3.4

a. A farmer wants to fence in 80,000 m^2 of land and then divide it into three plots of equal area as shown in fig 3.28. Find the minimum amount of fence needed.

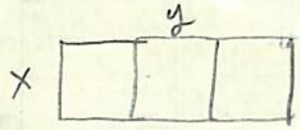


Fig 3.28

$x \cdot y = 80{,}000 m^2 \Rightarrow x = \frac{80{,}000}{y}$

$P = 4x + 2y$

$P = 4\left(\frac{80{,}000}{y}\right) + 2y$

$P = 320 \times 10^3 y^{-1} + 2y$

$0 = -320 \times 10^3 y^{-2} + 2$

$\frac{320 \times 10^3}{y^2} = +2$

$\frac{320 \times 10^3}{2} = y^{+2}$

$160 \times 10^3 = y^2$

$* \; y = 400$

$x \cdot y = 80{,}000 m \quad y = \frac{80{,}000}{x}$

$P = 4x + 2y$

$P = 4x + 2\left(\frac{80{,}000}{x}\right)$

$P = 4x + 160{,}000 x^{-1}$

$\frac{dP}{dx} = 4 + -160{,}000 x^{-2}$

$0 = 4 - \frac{160{,}000}{x^2}$

$\frac{160{,}000}{x^2} = 4 \rightarrow \frac{160{,}000}{4} = x^2 \rightarrow 40{,}000 = x^2$

$* \; X = 200$

total Fence =

$P = 4x + 2y$

$P = 4(200) + 2(400)$

$P = 800 + 800$

$\boxed{P = 1600 m}$

Exercise 3.4

21. The charge transmitted through a circuit varies according to $q = t^4 - 4t^3$ Coulombs. Find the time in seconds when the current i (in amperes) $i = \frac{dq}{dt}$ reaches a minimum.

$q = t^4 - 4t^3 \qquad i = \frac{dq}{dt}$

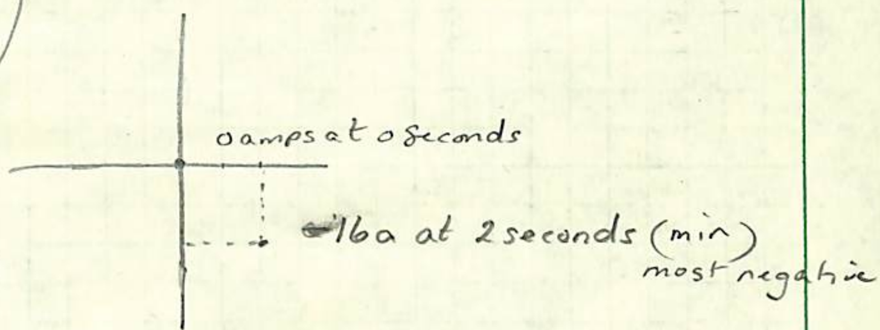
$\frac{dq}{dt} = 4t^3 - 12t^2$

$i = 4t^3 - 12t^2$

$\frac{di}{dt} = 12t^2 - 24t$

$0 = 12t^2 - 24t$

$t = 0 \text{ or } 2$



$i = 4(2^3) - 12(2^2)$

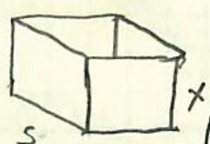
$i = 32 - 48$

$i = -16a$

$\boxed{\begin{array}{l} t = 2 \text{ seconds} \\ i = -16mA \end{array}}$

23. A rectangle box, open at the top, with a square base is to have a volume of 4000 cm³. Find the dimensions of the box is to contain the least amount of material.

Volume = 4000 cm^3



$4000 = s \cdot s \cdot x$

Volume $4000 = s^2 x$

Area $= s \cdot s + s \cdot x + s \cdot x + s \cdot x + s \cdot x$

Area $= s^2 + 4s \cdot x$

$x = 4000/s^2$

Area $= s^2 + 4s\left(\frac{4000}{s^2}\right)$

Area $= s^2 + \frac{16000}{s}$

Exercise 3.4

23. (continued)

$$\text{Area} = S^2 + 16000 s^{-1}$$

$$\frac{da}{ds} = 2s - 16000 s^{-2}$$

$$0 = 2s\left(1 - 8000 s^{-3}\right)$$

$$2s = 0$$

$$* \; s = 0$$

$$1 - 8000 s^{-3} = 0$$

$$\frac{8000}{s^3} = 1$$

$$s^3 = 8000$$

$$* \; s = 20$$

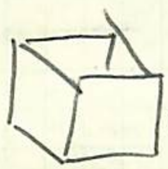
$$\text{Volume} = 4000$$

$$\text{Volume} = S^2 X$$

$$4000 = 20^2 (x)$$

$$x = \frac{4000}{20^2}$$

$$* \; x = 10$$



$x = 10$ cm

$s = 20$ cm

$\boxed{20\,\text{cm} \times 20\,\text{cm} \times 10\,\text{cm}}$

25. The total cost C of making x units of a certain commodity is given by $C = 0.005x^3 + 0.45x^2 + 12.75x$. All units made are sold at \$36.75 per unit. The profit P is then given by $P = 36.75x - C$. Find the number of units to be make maximum profit.

$$P = 36.75x - C \qquad C = 0.005x^3 + 0.45x^2 + 12.75x$$

$$P = 36.75x - (0.005x^3 + 0.45x^2 + 12.75x)$$

$$P = 36.75x - 0.005x^3 - 0.45x^2 - 12.75x$$

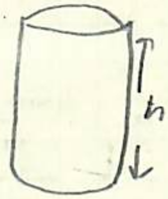
$$\frac{dP}{dx} = 36.75 - .015x^2 - .9x - 12.75$$

$$0 = \overset{C}{24} - \overset{B}{.9x} - \overset{A}{.015x^2}$$

$$x = \underline{20} \text{ or } -80 \qquad \boxed{x = 20 \text{ units}}$$

Exercise 3.4.

27. A cylindrical can with one end is to be made with 24π cm^2 of metal. Find the dementions of the can that give the maximum volume.



$$\begin{cases} \text{Volume} = \pi r^2 h \\ \text{area} = \pi r^2 + 2\pi r h \end{cases}$$

$$24\pi\,cm^2 = \pi r^2 + 2\pi r h$$

$$h = \frac{24\pi cm^2 - \pi r^2}{2\pi r}$$

$$V = \pi r^2 \frac{(24\pi cm^2 - \pi r^2)}{2\pi r}$$

$$V = \frac{r(24\pi cm^2 - \pi r^2)}{2}$$

$$V = \frac{24\pi r - \pi r^3}{2}$$

$$V = \frac{1}{2}(24\pi r - \pi r^3)$$

$$V = 12\pi r - \frac{1}{2}\pi r^3$$

$$\frac{dv}{dr} = 12\pi - \frac{3}{2}\pi r^2$$

$$0 = 12\pi - \frac{3}{2}\pi r^2$$

$$\frac{3}{2}\pi r^2 = 12\pi$$

$$r^2 = \frac{2(12\pi)}{3\pi}$$

$$r^2 = 8$$

$$r = \sqrt{8}$$

$$r = 2.828$$

$$h = \frac{24\pi - \pi(8)}{2\pi\sqrt{8}}$$

$$h = \frac{75.398 - 25.133}{17.772}$$

$$h = \frac{50.265}{17.772}$$

$$h = 2.828$$