Calculus FOR ELECTRONICS | Differentials

Problems 10-2

1). An Electron (whose mass is m_e) moves at a speed v. Its momentum is $p = mv$. FIND a formula for the approximate change dp in momentum resulting from a small increase dv in the speed.

$$p = mv$$

$$\frac{dp}{dv} = m$$

$$\boxed{dp = m\,dv}$$

3). The Low-frequency inductance of a single-layer solenoid is approximately $L = KDn^2$, where K is a form factor, D is the diameter in centimeters, and n is the number of turns. Find a formula for the approximate change dL in the inductance resulting from the addition of a small part of a turn dn.

$$L = kDn^2$$

$$\frac{dL}{dn} = 2KDn$$

$$\boxed{dL = 2KDn\,dn}$$

5). The power in a circuit was $p = t - 5$ watts. What was the approximate energy dw in joules expended from $t = 4$ to $t = 4{,}002$ seconds.

$$dp = t - 5 \text{ watts}$$

$$\frac{dp}{dt} = (t - 5)$$

$$dp = (t-5)dt$$

$$dp = (4-5)(.002)$$

$$dp = -2m \text{ Joules}(W/s)$$

$$\boxed{dp = 2m\,W/s}$$ Joules = W/s

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7.) The induced voltage in an 8-henry inductor varied according to $v_{ind} = 3t^2 - t$. About how much change di occurred in the inductor current from t=2 to t=2.01 seconds?

$$v_{ind} = 3t^2 - t$$

$$v_{ind} = -L\frac{di}{dt}$$

$$3t^2 - t = -8\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{3t^2 - t}{-8}$$

$$di = \left(\frac{3t^2 - t}{-8}\right) dt$$

$$di = \left(\frac{3(2^2) - 2}{-8}\right)(.01)$$

$$\boxed{di = -12.5\,mA}$$

9.) The power in a circuit is given by $p = Ri^2$ watts, where $R = 100\,\Omega$ and i is the current in amperes. If i changes from 12 to 12.005, approximately what change dp occurs in the power in watts?

$$p = Ri^2$$

$$p = 100i^2$$

$$\frac{dp}{di} = 200i$$

$$dp = 200i\,(di)$$

$$dp = 200(12)(.005)$$

$$\boxed{dp = 12\,watts}$$

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11.) The current i amperes in a circuit varied with time t seconds according to $i = t^2 + 3t$. About what current change di occurred as t changed from .98 to 1 second?

$$i = t^2 + 3t$$

$$\frac{di}{dt} = 2t + 3$$

$$di = (2t+3)dt$$

$$di = (2(.98)+3)(.02)$$

$$\boxed{di = 99.2\,mA}$$

13.) The intensity J of the heat radiation from a transmitting-tube plate varies with its absolute temperature according to $J = \sigma T^4$ where σ is a constant and T is the temperature in °C. If J=50 units when T = 1200°C, approximately what change d_J in J results from a change in T to 1205°C?

$$J = \sigma T^4 \qquad \sigma = \frac{J}{T^4}$$

$$\frac{dJ}{dT} = 4\sigma T^3 \qquad \sigma = \frac{50}{(1200)^4}$$

$$dJ = 4\sigma T^3(dT) \qquad \sigma = 24.113 \times 10^{-12}$$

$$d_J = 4(24.113 \times 10^{-12})(1200^3)(5)$$

$$\boxed{d_J = 833.345 \times 10^{-3}\ \text{units}}$$

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15.) If the resistance r ohms in a circuit varies with time t seconds according to $r = 100 + t^{1/2}$, what approximate change dr in r occurs as t changes from 4 to 4.001?

$$r = 100 + t^{1/2}$$

$$\frac{dr}{dt} = \frac{1}{2} t^{-1/2}$$

$$dr = \frac{1}{2}(t^{-1/2})(dt)$$

$$dr = \frac{1}{2}(4^{-1/2})(.001)$$

$$\boxed{dr = 250\,\mu\Omega}$$

17.) A right circular cone used in constructing a broadband antenna has a volume $V = \pi r^2 h$, where r is the radius of the base and h is the altitude of the cone. If r = 10 centimeters and h = 24 centimeters, what approximate change dV in the volume occurs when r changes to 10.052 centimeters?

$$V = \pi r^2 h$$

$$\frac{dv}{dr} = 2\pi h r$$

$$dv = 2\pi h r (dr)$$

$$dv = 2\pi(24)(10)(.052)$$

$$\boxed{dv = 78.414 \text{ Cubic Centimeters}}$$

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19). An increase in the apparent mass m_a of a moving particle occurs in accord with $m_a = \frac{m_o}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}$, where m_o is the mass of the particle at rest, v is its speed, and c is the speed of light in a vacuum. What approximate change dm_a occurs in the apparent mass as a result of a small change dv in the speed of the particle? Express your answer as a formula?

$$m_a = \frac{m_o}{\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}} = m_o\left[1-\left(\frac{v}{c}\right)^2\right]^{-\frac{1}{2}} = m_o\left[1-\left(\frac{1}{c^2}\right)\frac{v^2}{1}\right]^{-\frac{1}{2}}$$

$$\frac{dm_a}{dv} = \left(\frac{-1}{\cancel{2}}\right)m_o\left[1-\left(\frac{v}{c}\right)^2\right]^{-\frac{3}{2}}\left[-\frac{\cancel{2}v}{c^2}\right]$$

$$\frac{dm_a}{dv} = \frac{v\,m_o}{c^2\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{3}{2}}}$$

$$\boxed{dm_a = \frac{v\,m_o}{c^2\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{3}{2}}}(dv)}$$