1. The current in a circuit was $i = 4t^3$ amps. How many Coulombs were transmitted in 3 seconds?

$$q = \int i\,dt$$
$$q = \int 4t^3$$
$$q = \frac{4t^4}{4} + c$$
$$q = t^4 + C$$
$$q = (3)^4 + 0$$
$$\boxed{q = 81 \text{ coulombs}}$$ *

2. An $80\mu F$ capacitor is charged to 100 volts. We then apply a current $i_C = 0.04t^3$ amps in the same polarity as the initial charge. After how many seconds will the capacitor voltage reach 225 volts?

$$V_C = \frac{1}{C}\int i\,dt$$
$$225 = \frac{1}{80\mu F}\int 0.04t^3\,dt$$
$$225 = \frac{1}{80\mu F}\left(\frac{.04t^4}{4}\right) \begin{matrix} + C \\ + 100V \end{matrix}$$
$$225 = \frac{.04t^4}{320\times10^{-6}} + 100$$
$$225 - 100 = 125t^4$$
$$125 = 125t^4$$
$$\left(t^4 = 1\right)^{\frac{1}{4}}$$
$$\boxed{t = 1\text{ sec}}$$ *

3. The voltage applied to a circuit was $v = 2t + 1$ volts. If the current followed the equation $i = 0.03t$ amperes, find the energy w delivered from $t = 0$ to $t = 50$ seconds

$$w = \int p \, dt$$

$$w = \int (2t+1)(.03t)$$

$$w = \int .06t^2 + .03t$$

$$w = \frac{.06t^3}{3} + \frac{.03t^2}{2} \quad \overset{+C}{+0}$$

$$w = \frac{.06(50)^3}{3} + \frac{.03(50)^2}{2} + 0$$

$$w = 2.5\times10^3 + 37.5 + 0$$

$$\boxed{w = 2.5375 \, K \overset{\text{WATTS}}{\cancel{\text{joules}}}}$$

4. A 110 turn winding carries a flux of 0.8 webers. If we now want to vary the flux so that a voltage $v_{ind} = -5t^2$ volts appears in the winding, what equation must the flux through the winding follow?

$$\phi = \frac{-1}{N} \int v_{ind} \, dt$$

$$\phi = \frac{-1}{110} \int -5t^2$$

$$\phi = \frac{-1}{110}\left(\frac{-5t^3}{3}\right) + 0.8$$

$$\phi = \frac{5t^3}{330} + .8$$

$$\phi = 15.152\times10^{-3} t^3 + .8$$

OR

$$\boxed{\phi = \frac{t^3}{66} + .8}$$

5.) A DC current of 0.3 ampere flows in a 15 henry inductor. Superimposed on this DC is a varying current such that the voltage vind = $120t^{1/3}$ volts appears in the inductor. Find the instantaneous total current when t = 1 second. (assume that the DC and AC currents have the same polarity when t = 1 second).

$$i = \frac{-1}{L}\int v_{ind} \qquad t = 1 \text{ second}$$

$$i = \frac{-1}{15}\int 120t^{1/3}$$

$$i = \frac{-1}{15}\left(\frac{120t^{4/3}}{\frac{4}{3}}\right) + .3$$

same polarity

$$i = \frac{+3(120)(t)^{4/3}}{15(4)} + .3$$

$$i = 6 + .3$$

$$\boxed{i = 6.3 \text{ amps}}$$ ☆

6.) An inductance of 8 henrys is connected in series with a 12Ω resistor. Apply to this circuit a voltage $V = 20t^2$ volts. If i = 0 when t = 0, find an equation for i.

$$i_g = C\frac{dv}{dt} + \frac{v}{R} + \frac{1}{L}\int v\,dt$$

$$i_g = \frac{V}{R} + \frac{1}{L}\int v\,dt$$

$$i_g = \frac{20t^2}{12} + \frac{1}{8}\left(\frac{20t^3}{3}\right) \begin{array}{c}+C\\+0\end{array}$$

$$i_g = \frac{5t^2}{3} + \frac{20t^3}{24} + 0$$

$$\boxed{i_g = \frac{5t^2}{3} + \frac{5t^3}{6}}$$ ☆

7. If we apply a voltage $v = 90t^{1/2}$ to a circuit consisting of a 30 henry inductance shunted by a 50 ohm resistance, what current flows when $t = 4$ seconds? (Let $i = 0$ when $t = 0$)

$$i_g = \frac{V}{R} + \frac{1}{L}\int v\,dt$$

$$i_g = \frac{90t^{1/2}}{50} + \frac{1}{30}\int 90t^{1/2}$$

$$i_g = \frac{90(4)^{1/2}}{50} + \frac{1}{30}\left(\frac{90t^{3/2}}{\frac{3}{2}}\right) + C$$

$$i_g = \frac{9(4)^{1/2}}{5} + \frac{2(90)(4)^{3/2}}{30(3)} + 0$$

$$i_g = 3.6 + 16 + 0$$

$$\boxed{i_g = 19.6 \text{ amps}}$$ ☆

8. If we apply a voltage $v = 20t^4$ volts across a parallel RL combination, where $R = 500\Omega$ and $L = 40$ henrys, find the total current when $t = 0.2$ seconds. Let $i = 4\mu A$ when $t = 0$.

$$i_g = \frac{V}{R} + \frac{1}{L}\int v\,dt$$

$$+ C$$

$$i_g = \frac{20t^4}{500} + \frac{1}{40}\left(\frac{20t^5}{5}\right) + 4\mu A$$

$$i_g = \frac{20(.2)^4}{500} + \frac{1}{40}\left(\frac{20(.2)^5}{5}\right) + 4\mu A$$

$$i_g = 64\mu A + 32\mu A + 4\mu A$$

$$\boxed{i_g = 100\mu A}$$ ☆

9. In a parallel RL circuit, $R = 5\,\Omega$ and $L = 0.2$ henrys. If a voltage $v = t^{3/2} + 2$ volts were applied, what would the current i be when $t = 4$ seconds? Assume $i = 0.4$ amps when $t = 0$.

$$i_g = \frac{v}{R} + \frac{1}{L}\int v\,dt$$

$$i_g = \frac{t^{\frac{3}{2}}+2}{5} + \frac{1}{0.2}\int t^{\frac{3}{2}}+2\ dt$$

$$i_g = \frac{4^{\frac{3}{2}}+2}{5} + \frac{1}{0.2}\left(\frac{t^{\frac{5}{2}}}{\frac{5}{2}} + 2t\right) + 0.4$$

$$i_g = 2 + \frac{1}{.2}\left(\frac{2(4^{\frac{5}{2}})}{5} + 2(4)\right) + 0.4$$

$$i_g = 2 + \frac{1}{.2}(12.8 + 8) + 0.4$$

$$i_g = 2 + 104 + 0.4$$

$$\boxed{i_g = 106.4 \text{ amps}}$$ ☆

10. A current $i = 0.005\,t^{1/2}$ amps flows in a parallel RC circuit where $R = 8.8\times10^4\,\Omega$ and $C = 1\,\mu f$. Find a formula for the voltage across the circuit as a function of time t. Assume the capacitor to be initially discharged.

$$v_g = Ri + \frac{1}{C}\int i\,dt$$

$$v_g = (8.8\times10^4)(0.005\,t^{1/2}) + \frac{1}{1\mu f}\left(\frac{.005\,t^{3/2}}{\frac{3}{2}}\right) + 0$$

$$v_g = 440\,t^{1/2} + \frac{10\times10^{-3}\,t^{3/2}}{3\times10^{-6}}$$

$$\boxed{v_g = 440\,t^{1/2} + 3.333\times10^{3}\,t^{3/2}}$$ ☆

11. The current function $i = 1\times10^{-3} t^{1/2}$ amperes is applied to a series RC circuit where $R = 8.8\times10^{4}\Omega$ and $C = 1\mu f$. Find a formula for the impressed voltage as a function of time t. (assume the initial capacitor charge to be 100v.)

$$v_g = Ri + \frac{1}{C}\int i\,dt = (8.8\times10^{4})(1\times10^{-3}t^{\frac{1}{2}}) + \frac{1}{1\mu f}\left(\frac{1\times10^{-3}t^{\frac{3}{2}}}{\frac{3}{2}}\right) + 100$$

$$\boxed{\begin{aligned} v_g &= 88t^{1/2} + 666.667t^{3/2} + 100 \\ v_g &= 88t^{1/2} + \frac{2\times10^{-3}t^{3/2}}{3\times10^{-6}} + 100 \end{aligned}}$$ ☆

12. A series LC circuit where $L = 0.1$ henry and $C = 100\mu f$ has applied to it a current $i = 0.1a$ from $t=0$ onward. Find (a) the formula for the voltage across the circuit, and (b) the rate of change of the voltage at $t = 2$ sec. (assume $V_c = 0$ when $t = 0$)

$$V_g = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt \qquad i = 0.1a\,t^{0}$$

$$v_g = 0 + 0 + \frac{1}{100\mu f}\left(\frac{.1t^{1}}{1}\right) \qquad \frac{di}{dt} = 0$$

$$v_g = \frac{.1t}{100\times10^{-6}}$$

a) $\boxed{v_g = 1000t}$ ☆

Rate of Change $= \frac{dv}{dt}$ $\quad V_g = 1000t$

$$\frac{dv}{dt} = 1000t^{1-1} = 1000t^{0} = 1000$$

$$\boxed{\frac{dv}{dt} = 1000V}$$ ☆

13. A series circuit has these constants: $R = 5K\Omega$, $L = 200$ henry and $C = 20\mu f$. If we supply to the circuit a current $i = 0.02t^2$ amperes, at what rate does the voltage across the circuit change when $t = 0.2$ seconds?

$$v_g = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt \qquad i = 0.02t^2$$

$$v_g = 200(.04t) + 5K(.02t^2) + \frac{1}{20\mu f}\left(\frac{.02t^3}{3}\right) \qquad \frac{di}{dt} = .04t^1$$

$$v_g = 8t + 100t^2 + 333.33t^3$$

$$\frac{dv}{dt} = 8 + 200t + 999.999t^2$$

$$\frac{dv}{dt} = 8 + \underset{40}{200(.2)} + \underset{40}{1K(.2)^2}$$

$$\boxed{\frac{dv}{dt} = 88\,v/s}$$ ☆

14. In a series RCL circuit, let $R = 10\Omega$, $C = 10{,}000\mu f$, and $L = 10$ henrys. Through this circuit we pass a current $i = 1 - t^{1/2}$ amps. Find the total voltage v across this circuit when $t = 4$ seconds. Assume $v = 0.25$ volts when $t = 1$ second.

$$v_g = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt \qquad i = 1 - t^{1/2} \qquad \int 1 - t^{1/2}$$

$$.25v = 10\left(-\tfrac{1}{2}t^{-\frac{1}{2}}\right) + 10\left(1 - t^{\frac{1}{2}}\right) + 100\left(t^1 - \frac{2t^{3/2}}{3}\right) \qquad \frac{di}{dt} = -\frac{1}{2}t^{-\frac{1}{2}} \qquad = \frac{t^1}{1} - \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$.25v = -5 + 33.333 + C$$

$C = -28.083\,v$

$$v_g = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt$$

$$v_g = 10\left(-\frac{1}{2}t^{-1/2}\right) + 10\left(1 - t^{1/2}\right) + 100\left(t - \frac{2t^{3/2}}{3}\right) - 28.083$$

$$v_g = -2.5 - 10 - 133.333 - 28.083$$

$$\boxed{v_g = -173.9v}$$ ☆

Ch 14

$q = \int i\,dt$ coulombs

$v_c = \frac{1}{C}\int i\,dt$ volts

$w = \int p\,dt$ joules

$\phi = -\frac{1}{N}\int v_{ind}\,dt$ webers

$i_L = -\frac{1}{L}\int v_{ind}\,dt$ amperes

$i_1 = -\frac{1}{m}\int v_2\,dt$ amperes

Problems 14-1

1. The current in a circuit in a circuit was $i = 4t^3$ amperes. How many coulombs were transmitted in 3 seconds?

$q = \int i\,dt \qquad = \int 4t^3\,dt$

$q = \frac{4t^4}{4} = \frac{4(3)^4}{4} = 3^4 = 81$

$q = 81$ coulombs

3. An 80 µF capacitor is charged to 100v (K). We then supply to the capacitor a current $i_C = 0.04t^3$ amperes, in the same polarity as the initial charge. After what time interval in seconds does the capacitor voltage reach 225 volts?

$v_C = \frac{1}{C}\int i\,dt \quad = \quad 225v = \frac{1}{80\mu F}\int 0.04t^3\,dt$

$\frac{225v}{1} \cdot \frac{80\mu F}{1} = \int 0.04t^3\,dt$

$18\times10^{-3} = \int 0.04t^3\,dt$

$18\times10^{-3} = \frac{.04t^4}{4} + 100v \; (+K)$

$18\times10^{-3} - 100 = \frac{.04t^4}{4}$

$-99.982 = \frac{.04t^4}{4}$

$-399.928 = .04t^4$

$-9.9982\times10^3 = t^4$ →

Problems 14-1
#3 Continued.

$C = 80\mu F$ initial Charge $= \overset{(K)}{100v}$ $ic = 0.04t^3$

$vc = 225V$ $t = ?$

$vc = \frac{1}{C}\int i\,dt$

$225v = \frac{1}{80\mu f}\int .04t^3\,dt$

$225v = \frac{1}{80\mu f}\left(\frac{.04t^4}{4}\right) + 100v$

-100 $\qquad -100$

$125v = \frac{1}{80\mu f} \times \frac{.04t^4}{4}$

$125v = \frac{.04t^4}{80\mu f \times 4}$

$125v = \frac{.04t^4}{320 \times 10^{-6}}$

$125 = 125t^4$

$t^4 = \frac{125}{125}$

$(t^4 = 1)^{1/4}$

$t = 1^{1/4}$

$\boxed{t = 1}$

#5. The voltage applied to a circuit was $v = 2t + 1$ volts. If the current followed the equation ~~is~~ $i = 0.03t$ amperes, find the energy w delivered from $t = 0$ to $t = 50$ seconds.

$v = 2t + 1$ $i = 0.03t$ find w $t = 0$ $t = 50$ seconds

$w = \int p\,dt$ $\qquad p = I \cdot V$ $(.03t)(2t + 1)$

$p = .06t^2 + .03t$

$w = \int .06t^2 + .03t$

$w = \frac{.06t^3}{3} + \frac{.03t^2}{2}$

$w = 2.5K + 37.5$

$\boxed{w = 2{,}537.5 \text{ joules}}$

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Problems 14-1

#7. A 110 turn winding carries a flux of (0.8) webers. If we now want to vary the flux so that a voltage $v_{ind} = -5t^2$ volts appears in the winding, what equation must the flux through the winding follow?

$N = 110 \quad K = .8\text{ webers} \quad v_{ind} = -5t^2$

$$\phi = \frac{-1}{N}\int v_{ind}\, dt$$

$$\phi = \frac{-1}{110}\int -5t^2$$

$$\phi = \frac{-1}{110}\int -5t^2$$

$$\phi = -\frac{1}{110}\left(\frac{-5t^3}{3}\right) + .8$$

$$\phi = \frac{+5t^3}{330} + .8$$

$$\boxed{\phi = \frac{t^3}{66} + .8}$$

#9. A direct current of 0.3 ampere flows in a 15 henry inductor. Superimposed on this direct current is a varying current such that a voltage $v_{ind} = 120t^{1/3}$ volts appears in the inductor. Find the instantaneous total current when $t = 1$ second (assume that the steady and the varying currents have the same polarity when $t = 1$ second.)

.3 amp flows = K $\quad L = 15\text{ henry} \quad v_{ind} = 120t^{1/3}\text{ volts} \quad t = 1\text{ second}$

find total current.

$$i_L = -\frac{1}{L}\int v_{ind}\, dt$$

$$i_L = \frac{-1}{15}\int 120t^{1/3}\, dt$$

$$i_L = -\frac{1}{15}\left(\frac{120t^{4/3}}{\left(\frac{4}{3}\right)}\right) + K$$

Same polarity

$$i_L = \frac{+1}{15}\left(\frac{3 \cdot 120t^{4/3}}{4}\right) + .3$$

$$i_L = +6 + .3$$

$$\boxed{i_L = 6.3\text{ amps or } -6.3\text{ amps}}$$