2. The voltage drop across a certain adjustable resistor is $v = Ir$ volts, where $I = 20$ milliamperes. In this formula for v, which symbol(s) represent variables? Which represent constants?

3. An electric signal is said to have an intensity level S dBm (decibels with respect to 1 milliwatt), given by $S = 30 + \log_{10} p$, where p is the signal power in watts. Which symbols represent variables? Which represent constants?

4. A dc ammeter whose internal resistance is r ohms has a full-scale reading of A amperes. To measure a current greater than A amperes, we need to connect across the ammeter a shunt resistor whose resistance is given by $R = Ar/(A_1 - A)$ ohms, where A_1 is the desired new ammeter range in amperes. Suppose we wish to use a 0.5-ohm ammeter having a range of 10 amperes to measure currents in various ranges. In the given formula, which symbols represent variables? Which represent constants?

5. The distance s meters traveled by a freely falling object is $s = gt^2/2$, where g is the acceleration produced by gravity and t is the time of fall in seconds. Which symbols represent variables? Which represent constants?

6. In a certain circuit the current varied according to $i = I_0e^{-0.5t}$, where I_0 is the initial current in amperes, e is the base of the natural logarithmic system, and t is the time interval in seconds. In this equation, which symbols represent variables? Which represent constants?

7. To override circuit noise, a computer signal must possess energy (in joules) greater than $E = kT(\log_e 2)$. Here k is Boltzmann's constant ($= 1.374 \times 10^{-23}$ joule per kelvin), T is the circuit temperature in kelvin, and $\log_e 2 = 0.69315$ approximately. In the equation for E, which symbols represent constants? Which represent variables?

8. An atom of a radioactive element has a mass m_1. Decay of this atom yields atoms of fission products, whose total mass is m_2, where m_2 is smaller than m_1. To the mass difference $m_1 - m_2$ we assign the symbol m. This decay of the original atom releases an amount of energy given by $E = mc^2$, where c is the speed of light in a vacuum. When we consider atoms of different radioactive elements, we get different values for m. In the expression $E = mc^2$, state which symbol or symbols indicate a variable or variables and which indicate a constant or constants.

9. If in a circuit the impedance Z depends upon the resistance R and the reactance X according to the formula $Z = \sqrt{R^2 + X^2}$, and if $R = 20$ ohms, which of these symbols represent variables? Which represent constants?

10. A certain particle has, at rest, a mass m_0. Relativity theory indicates that when this particle moves at a speed $\mathbf{v}$, its mass becomes $m_a = m_0/\sqrt{1 - (v/c)^2}$. Here c represents the speed of light in a vacuum. For a given particle moving at various speeds, which symbols represent variables? Which represent constants?

2-4 FUNCTIONS We often encounter statements such as "a quantity y is a *function* of a variable x," or "the power dissipated by a resistor is a *function* of the current in the resistor." What do such statements mean? What are we to think of when we encounter the term *function?*

> *If, for each value of a variable x, there exists a corresponding value of a quantity y, then we say that y is a* function *of x.*

One can formulate more rigorous statements, but the description above serves our purpose.

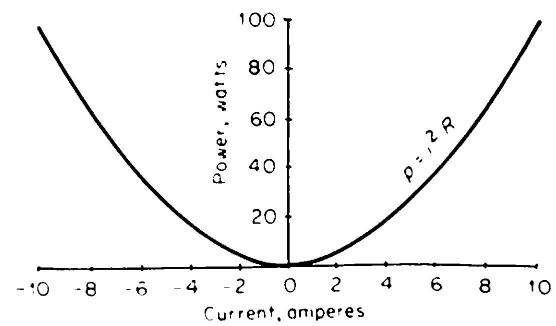

Fig. 2-1 A graph can show how values of a *function*, given by graph-curve height above the horizontal axis, relate to the independent variable. Here the function is power p versus current i.

Consider the example just mentioned—that of the resistor, where the text refers to the dissipated power as a *function* of the resistor current. Figure 2-1 illustrates this situation for the case where the resistance R equals 1 ohm. Does this situation meet the above description of a function? In other words, is it true that "for each value of the variable current i in the resistor there exists a corresponding value of power dissipation p"? If so, the situation fits the statement above so that we may call the resistor power a function of the resistor current.

To test this situation, we note that the power relates to the current by the formula $p = i^2R$. Assuming we know the value of R, we can substitute any actual current value (1 ampere, 2 amperes, etc.) into this formula and solve for a corresponding value of resistor power p. So far, then, our resistor arrangement fits the function description.

But how about the situation where we send no current at all (0 amperes) through the resistor? The power equals zero when the current is zero. But 0 watts is just as much a value as 1, 2, or 3 watts. Zero, in fact, is an integer, and is an even number. Therefore, for zero current as well as for other current values, "for each value of the variable current there is a corresponding value of the dissipated power." Thus, according to our statement above, the dissipated power is a *function* of the resistor current.

2-5 DOMAIN; RANGE Consider for a further moment the power dissipated by a resistor as a function of its current. For simplicity, suppose the resistor current is a direct current. Now think of all the possible values that this current might take: zero, weak, and strong currents; positive currents (current through the circuit in that sense we assume to be the positive sense); and negative currents (in a sense the reverse of the assumed positive sense).

But the functional relationship might exist *for only certain values* of the current. Such a limitation might appear for either practical or theoretical reasons. And it might arise either through our own choice or despite our wishes.

For example, suppose that the resistor cannot safely carry currents exceeding 10 amperes. We then have a scale of actual or practical current values extending from -10 through 0 to $+10$ amperes. This example illustrates the fact that p (in practice at least) exists as a function of i for some values of i but *not for other values.*

We say that the *domain* of the functional relationship, or the *domain of definition* of the function, consists of all possible values of i from -10 to $+10$ amperes.

The domain *of a functional relationship is that set of values of the variable (x in the statement of Sec. 2-4) for which the functional relationship applies.*

In the resistor example, the power is always $p = i^2R$. Suppose that $R =$ 1 ohm. We see that the power varies from 0 (for a current of zero) to 100 watts (for currents of either -10 or $+10$ amperes). We say that the *range* of the functional relationship consists of all possible power values from 0 to 100 watts.

The range *of a functional relationship is that set of values (y in the statement of Sec. 2-4) for which the functional relationship applies.*

Some other examples of the use of the terms *domain* and *range* follow:

1. When the base current of a certain transistor varies over the domain from 0.3 to 2.0 milliamperes, the collector current, as a function of the base current, varies over the range from 27 to 180 milliamperes.
2. When the test input-signal frequency applied to a certain audio amplifier varies over the domain from 10 to 150,000 hertz, the corresponding amplifier-gain values vary over the range from 37 to 40 decibels.

2-6 CONSTANT FUNCTION

In the preceding example involving resistor power, the dissipated power changed whenever we varied the current. That is, the *function*—the resistor power dissipation—was a variable, just as the resistor current was a variable.

But we note that the function statement of Sec. 2-4 doesn't say anything about whether the function (in that case, y) varies or not. Are there functions that *do not vary?*

To answer this question, consider the example of a particularly well-regulated electronic power supply. Suppose that as we change the output-current drain imposed on this supply from 50 to 100 milliamperes, the output voltage from the supply remains (so far as we can measure) at its rated value of $+30$ volts. Is the output voltage a function of the output current?

The answer is yes. The function statement (Sec. 2-4) deliberately does not exclude such instances. In the example of the regulated power supply, then, for each value of the varying load current i there exists a corresponding value of the output voltage v. The fact that all these values are the same doesn't change the situation. Accordingly we say that v is a function of i.

Observe that, so far as we carried our measurements on this power supply, the above-mentioned functional relationship exists for a load-current *domain* from 50 to 100 milliamperes, and for an output-voltage *range* restricted to the single value of $+30$ volts.

In summary, the function statement of Sec. 2-4 applies even if the function retains a constant value over all or a part of the domain of the variable.

QUESTIONS 2-2

1. If we are given that the current in a circuit is a function of the applied voltage, what meaning do we attach to that statement?

2. The charge in ampere-hours accumulated in a storage battery is a function of the length of time during which we charge the battery. What does this statement mean?

3. The radio-frequency voltage between the conductors of a certain transmission line is a function of the distance of the point of measurement from the source end of the line. Interpret this statement.

4. We connect an initially discharged capacitor through a switch and a resistor in series to a battery. For each value of elapsed time after a switch closure, there exists a value of capacitor voltage. Express this latter sentence in *function* terms.

5. State in function terms the fact that for each value of base current in a certain transistor there exists a corresponding value of collector current.

6. If a quantity y is a function of a variable x and if x undergoes a change, does y necessarily change also?

7. State what we mean by the term *function.*

8. If in a circuit that includes a certain field-effect transistor we can change the drain-to-source voltage from V_1 to V_2 volts without altering the drain current, can we nevertheless properly say that the drain current is a function of the drain-to-source voltage?

9. By means of a field rheostat we can change the field current of a certain dc generator from 0 to 4 amperes. This field-current adjustment changes the output voltage from 22 to 150 volts. What is the domain of the functional relationship of output voltage to field current? What is the range of this relationship?

2-7 GRAPHS

You are of course familiar with graphs. In particular you know about graphs in which points on a horizontal axis indicate values of one quantity, while the height of a curve above (or below) this horizontal axis indicates corresponding values of a function of that first quantity.

Figure 2-1 illustrates such a graph. This graph shows the functional relationship between the power in a 1-ohm resistor and the current in that resistor, as treated in Secs. 2-4 and 2-5. We call Fig. 2-1 a graph of p *versus* i. Note that *domain* of the variable i—from -10 to $+10$ amperes, determined in this example by the practical current-carrying ability of the resistor. And note the *range* of the function p—from 0 to 100 watts, corresponding in every case to a value or values of current.

Another functional relationship is shown in the graph of Fig. 2-2. This graph shows how the drain current I_D of a certain field-effect transistor corresponds to the drain-to-source voltage V_{DS}. Here we have a graph of I_D versus V_{DS}. You can think of many other useful graphs. Examples are graphs that show variations in output voltage as the field current of a generator varies, or that show changes in resonant frequency as the capacitance in a tuned circuit varies.

Observe in Fig. 2-2 that the drain current is a function of the drain-to-source voltage. That is, at least over the domain shown (voltages from 0 to +20 volts), there is a value of drain current corresponding to each drain-to-source voltage. The drain-current range is from 0 to 12 milliamperes. Graphs, as we see here, display functional relationships with excellent effect.

In general discussions, x often represents horizontal distance on a graph, while y represents vertical distance. Thus, as in Fig. 2-3, we might have a graph of y versus x. The horizontal x axis and the vertical y axis intersect (usually at right angles) at a point O (or 0) that we call the *origin*.

We identify and locate any given point (say, a point P) by giving its *distances from the origin* along the x and y axes thus: $P(x, y)$. Consider, for example, a point A that lies 3 units to the right of the origin (that is, where $x = +3$) and 2 units above the origin (that is, where $y = +2$). As indicated in Fig. 2-3, we identify

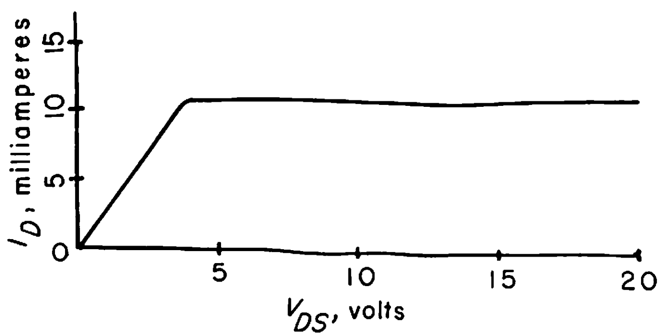

Fig. 2-2 In this field-effect transistor, drain current I_D remains virtually constant over a wide domain of drain-to-source voltage V_{DS}. Even in such an instance we can take I_D to be a function of V_{DS}.

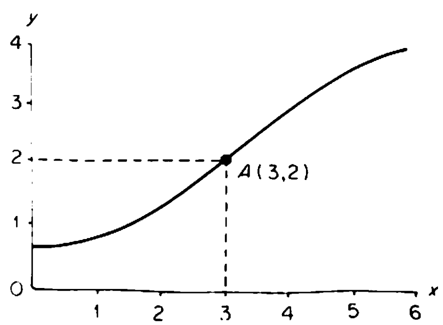

Fig. 2-3 This graph shows y as a function of the independent variable x. That is, we have "a graph of y versus x." Point A has a position given by $A(3, 2)$, which is 3 units to the right and 2 units upward, all measured from the origin O (or 0).

this point as $A(3, 2)$. We refer to the x and y distances (here 3 and 2 units, respectively) as the *coordinates* of the point A.

2-8 INDEPENDENT AND DEPENDENT VARIABLES

To some variables, we can arbitrarily assign any law of variation whatsoever. For example, imagine a 100-ohm variable resistor. By simply turning the control knob we can (at least hypothetically) increase and decrease the resistance r of this resistor in any way we like, over the domain from 0 to 100 ohms. We are likely to consider such a variable as this r to be an *independent* variable.

> *An* independent variable *is one that may change in any manner whatsoever.*

A *dependent* variable, on the other hand, is a *function* of some independent variable. Suppose we connect the 100-ohm variable resistor just mentioned across a fixed 10-volt power supply. Then the resulting current i in the resistor corresponds in a definite way to the resistor setting r. (In this example, this correspondence accords with Ohm's law.) We are likely to say that such a variable as this i is a *dependent* variable.

> *A* dependent variable *is a variable function whose values correspond to the values of an independent variable, according to a rule or law.*

In graphing, we commonly use the horizontal axis to portray values of the independent variable. Then we use the height of the curve, corresponding to distance along the vertical axis, to indicate the associated values of the dependent variable. Accordingly, in the preceding example we would probably let distances along the horizontal axis represent values of the resistance r. And we would use the height of the graph line above the horizontal axis to represent values of the dependent variable, which is the current i. Figure 2-4 shows the result.

In general x and y problems, of course, x-axis distances show the independent-variable values. The height of the graph line, corresponding to y-axis distances, indicates values of the dependent variable.

Later we will sometimes switch the dependent and independent variables around if it meets our mathematical convenience to do so—regardless of which variable may be cause and which an effect.

QUESTIONS 2-3

1. Define the term *independent variable.*
2. Define the term *dependent variable.*

PROBLEMS 2-2

1. The current i in a dc circuit of resistance 5 ohms is $i = v/R$ amperes, where v is the applied voltage. Which quantity, i or v, would you likely choose as the independent variable? The dependent variable?