Exercise 3.6

19. The side of a square measures 12.00 cm with a maximum possible error of 0.05 cm. (A) Find the maximum possible error in the area using differentials. (B) Find the maximum possible error by substituting into the formula for the area of a square. (C) Find the percentage of error.

Side = 12.00cm to 12.05cm

area = Side x Side

$area = s^2$

$\frac{da}{ds} = 2s$

$da = 2(s)(ds)$

$da = 2(12)(.05)$

a). $\boxed{da = 1.2\,cm^2}$

b). $12.05^2 - 12.00^2 = \boxed{1.2025\,cm^2}$

c). $area = 12cm \times 12cm = 144\,cm^2$ $\quad da = 1.2\,cm^2$

$\frac{1.2\,cm^2}{144\,cm^2} = .00833 \quad .00833 \times 100 = \boxed{.833\%}$

21. Suppose you want to build a spherical water tower with an inner diameter of 26.00m and sides of thickness 4.00 cm. a) Find the approximate volume of steel needed using differentials b) If the density of steel is 7800 Kg/m³, find the approximate volume of steel needed using differentials.

Diameter = 26.00m Thickness = 4.00cm Volume = ?

$V = \frac{4}{3}\pi r^3$ (volume of a sphere)

$\frac{dv}{dr} = 4\pi r^2 \quad dv = 4\pi r^2 dr = dv = 4\pi(13^2).04$

a.) $\boxed{dv = 84.95\,m^3}$ b. = dv · density $= (84.95)(7800)\,Kg$

$\boxed{b = 662{,}599\,Kg}$

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Exercise 3.6.

22. The current in a resistor varies according to $i = 0.08t^6 - 0.04t^2$. Find the approximate change in current using differentials as t changes from 2.00 seconds to 2.10 seconds.

$$i = 0.08t^6 - 0.04t^2$$

$$\frac{di}{dt} = .48t^5 - .08t$$

$$di = (.48(2^5) - .08(2)).10$$

$$\boxed{di = 1.52 \text{ amps}}$$

23. The horsepower of an internal combustion engine is given by $p = nd^2$, where n is the number of cylinders and d is the diameter of each bore. Find the approximate increase in horsepower using differentials for an Engine with eight cylinders when the bore of each cylinder is increased from 3.750 in to 3.755 in.

$$p = 8d^2$$

$$\frac{dp}{dt} = 16d$$

$$dp = 16(3.750)(.005)$$

$$\boxed{dp = .3hp}$$

24. A freely falling body drops according to $s = \frac{1}{2}gt^2$, where s is the distance in meters, $g = 9.80\ m/s^2$, and t is time in seconds. Approximate the distance, ds, that an object falls from $t = 10.00$ sec to $t = 10.03$ sec.

$$s = \tfrac{1}{2}gt^2$$

$$\frac{ds}{dt} = 9.80t \qquad ds = 9.8(10)(.03)$$

$$\boxed{ds = 2.94 \text{ m}}$$

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Exercise 3.6

25. The voltage V in volts varies according to $V = 10p^{2/3}$, where p is the power in watts. Find the change dv when the power changes from 125 w to 126 w.

$$v = 10p^{2/3\left(\frac{3}{3}\right)}$$

$$\frac{dv}{dp} = \frac{20}{3}p^{-1/3}$$

$$dv = \frac{20}{3}(125w^{\frac{1}{3}})(3\ )$$

$$\boxed{dv = 4v}$$

26. The impedance Z in an ac circuit varies according to $Z = \sqrt{R^2 + X^2}$, where R is the resistance and X is the reactance. If $R = 300\Omega$ and $X = 225\Omega$ find dZ when R changes to 310Ω.

$$Z = (R^2 + X^2)^{\frac{1}{2}}$$

$$\frac{dz}{dR} = \frac{1}{2}(R^2 + X^2)^{-1/2}(2R)$$

$$dz = \frac{1}{2}(300^2 + 225^2)^{-1/2}(2 \cdot 300)(10)$$

$$\boxed{dz = 8\Omega}$$

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