Week 10, HW Max-Mins, Differentials, & Higher Derivatives

1. The sum of two positive numbers is 56. Find the two numbers if their product is to be maximum.

$$y = 28$$

$$x = 28$$

2. An open box is to be made from a square piece of aluminum, 3cm on a side, by cutting equal squares from each corner and then folding up the sides. Determine the dimensions of the box that will have the largest volume.

$$2cm\ x\ 2cm\ x\ 0.5cm$$

3. A man wishes to fence in a rectangular plot lying next to a river. No fence is required along the river bank. If he has 800m of fence, and he wishes the maximum area to be fenced, find the dimensions of the desired enclosure.

$$200m\ x\ 400m$$

4. Find the maximum possible area of a rectangle whose perimeter is 36cm.

$$a = 81cm^2$$

5. A farmer wants to fence in 80,000m^2 of land and then divide it into three plots of equal area. Find the minimum amount of fence needed.

$$fence_{min} = 1600m$$

6. The charge transmitted through a circuit varies according to q=t^4-4t^3 coulombs. Find the time in seconds when the current i (in amps) i=(dq/dt) reaches a minimum.

$$t = 2sec$$

$$i = -16ma$$

7. A rectangle box, open at the top, with a square base, is to have a volume of 4000cm^3. Find the dimensions if the box is to contain the least amount of material.

$$20cm\ x\ 20cm\ x\ 10cm$$

8. The total cost C of making x units of a certain commodity is given by C=0.005x^3 + 0.45x^2 + 12.75x. All units made are sold at $ 36.75x - C. Find the number of units to make maximum profit.

$$20\ units$$

9. A cylindrical can with one end is to be made with 24πcm^2 of metal. Find the dimensions of the can that give the maximum volume.

$$r = 2.828cm, h = 2.828cm$$

10. The side of a square measures 12.00 cm with a maximum possible error of 0.05 cm. (a) Find the maximum possible error in the area using differentials. (b) Find the maximum possible error by substituting into the formula for the area of a square. (c) Find the percentage of error.

$$a.\ da = 1.2cm^2$$

$$b.\ 1.2025cm^2$$

$$c.\ 0.833\%$$

11. Suppose you want to build a spherical water tower with an inner diameter of 26.00 m and a side thickness of 4.0 cm. (a) Find the approximate volume of steel needed using differentials. (b) if the density of steel is 7800 kg/m^3, find the approximate volume of steel needed using differentials.

$$a.\ dv = 84.95m^3$$

$$b.\ 662{,}599kg$$

12. A freely falling body drops according to s=(1/2)gt^2, where s is the distance in meters, g = 9.80 m/s^2, and t is time in seconds. Approximate the distance, ds, that an object falls from t = 10.00 sec to t = 10.03 sec.

$$ds = 2.94m$$

13. The voltage Vin volts, varies according to $V=10p^{2/3}$, where p is the power in watts. Find the change dv when the power changes from 125w to 128w.

$$dv = 4v$$

14. The impedance Z in an ac circuit varies according to $Z=\sqrt{(R^2 + X^2)}$, where R is the resistance and X is the reactance. If R = 300Ω and X = 225Ω, find dz when R changes to 310Ω.

$$dz = 8\Omega$$

15. $y = x^5 + 3x^2$

$$y' = 5x^4 + 6x$$
$$y'' = 20x^3 + 6$$
$$y''' = 60x^2$$
$$y'''' = 120x$$

16. $y = 3x^6 - 8x^3 + 2$

$$y' = 18x^5 - 24x^2$$
$$y'' = 90x^4 - 48x$$
$$y''' = 360x^3 - 48$$
$$y'''' = 1080x^2$$

17. $y = 5x^5 + 2x^3 - 8x$

$$y' = 25x^4 + 6x^2 - 8$$
$$y'' = 100x^3 + 12x$$
$$y''' = 300x^2 + 12$$
$$y'''' = 600x$$

18. $y = 3x^2 + 4x - 7$

$$y' = 6x + 4$$
$$y'' = 6$$
$$y''' = 0$$
$$y''' = 0$$