Derivatives applied

1) Instantaneous Current.

The instantaneous current equals the derivative of the charge q in coulombs with respect to time t in seconds.

$$i = dq/dt$$

2) Instantaneous Power.

The instantaneous power in a circuit is the rate of change of energy (or work) w, at the instant in question.

$$p = dw/dt$$

3) Instantaneous Capacitor Current.

When the instantaneous voltage v across a capacitor varies at a rate dv/dt volts per second, the following equation gives the current in the capacitor at any instant t:

$$i_c = C\, dv/dt$$

4) Instantaneous Induced Voltage.

The induced voltage at any instant equals the product of the number of turns N times the rate of change dN/dt of the flux ϕ that links the winding.

$$v_{ind} = -N\, d\phi/dt$$

5) Instantaneous Inductor Voltage.

The voltage v_{ind} induced at any instant in an inductor of inductance L henrys is:

$$v_{ind} = -L\, di/dt$$

6) Mutual Inductance.

The voltage induced in winding 2 of two coupled windings with mutual inductance M henrys is:

$$v_2 = -M\, di_1/dt$$

Kirchoff's Laws Applied:

The current Law.

The sum of the currents toward any point in a circuit ant any instant equals zero.

$$i_g = C\,dv/dt + (1/R)\,v$$

The voltage Law.

The sum of the voltage drops around a circuit, at any instant, equals zero.

$$v_g = -L\,di/dt + Ri$$

Derivatives Applied

1) If the current in a 1μf capacitor is to be 0.1 ma, at what rate in volts per second must the applied voltage change?

2) The magnetic flux through a 500-turn winding varied according to $\Phi = 0.004t$ webers. Find the induced voltage in the winding (a) when $t = 0.01$ second and (b) when $t = 0.1$ second.

3) If the flux through a 150-turn winding varied according to the formula $\Phi = 0.01t - t^2 + 0.2$ webers, what voltage was induced when $t = 0.02$ second?

4) The magnetic flux N in a winding of 600 turns varied as $\Phi = 0.5t^{3/5}$ webers, where t was in seconds. Find the induced voltage v_{ind} when $t = 1$ second.

5) What formula expresses the voltage v_{ind} across a 100 mh inductor if the current i constantly equals 0.2 A? Neglect resistance.

6) How fast does the current in a 12 h winding change to cause an induced voltage of 3.6 v?

7) The mutual inductance between two windings is 0.2 henry. If a current $i_1 = 11t^{3/2}$ amps flows in one of the windings, how much voltage v_2 is induced in the second winding when $t = 0.001$ second?

8) The mutual inductance between two windings is $M = 6$ h. How fast must the current in one of the windings vary in amps per second to induce -4.8 volts in the other winding?

9) A winding linked a magnetic field that varied according to $\phi = 0.002t - 2t^2$ webers. When t was 0.0025 second, the voltage induced in the winding measured 8 volts. How many turns did the winding include?

10) If the current in a 30 h inductor changes according to $i = 0.02t^{5/3}$ amps, after what interval will the induced voltage measure -96 volts?

11) A voltage $v = t^3 + 1{,}000$ volts appears across a parallel RC combination, where $R = 300$ kΩ and $C = 20$ µf. Find the resulting current i_g at any time t.

12) A 50 kΩ bleeder resistor shunts a 4 µf filter capacitor. During a part of the charging process, the voltage across the capacitor varies approximately as $v_c = 1{,}000t^{2/3} + 100$ volts. Find the current i_g applied to the combination when $t = 0.001$ second.

13) A current $i = 3t^{1/3} + 2$ amps flows through a series RL circuit, where $R = 100\Omega$ and $L = 20$ h. Find the voltage v_g across this circuit when $t = 0.125$ second.

14) A relay winding has an inductance of 0.5 h and a resistance of 470Ω. If the winding current i equals $t^{1/2} + 0.02$ amps, find the voltage v_g across the winding when $t = 0.01$ seconds.

15) A series circuit consists of a 22 h inductor and a 68Ω resistor. A current $i = 2t^2 + t$ exists in this combination. After what time t does the voltage across the combination equal 375 volts?

16) A voltage $v = t^3 + 1{,}000$ volts appears across a parallel RC combination, where $R = 2$ MΩ and $C = 1\mu$f. Find the resulting current i_g at any time t.

17) A transistor operates into a load resistance of 2.2 kΩ. The shunt capacitance in the circuit equals 70 pf, as measured at the collector. Over a certain interval the output voltage supplied by the transistor equals $v = 1\text{x}10^7 t + 30$ volts. Find the collector signal current when $t = 10$ μs.

18) A current $i = 10t^{1/2} + 0.1$ amps flows through a series RL circuit, where $R = 800\Omega$ and $L = 320$ h. Find the voltage v_g across this circuit when $t = 0.04$ second.

19) A transistor collector has a load resistor of 4.7 kΩ with a compensation inductor $L = 20$ mh in series with the resistor. The current i through the combination equals $2.5\text{x}10^4 t + 0.01$ amps. Find the voltage across the RL circuit when $t = 25$ ns.

20) A 27 kΩ resistor shunts a 33μf capacitor. The applied voltage v equals $300t^2$ volts. At what time t does the total current i equal 84 mA?

Derivatives - the Chain Rule

21) The voltage applied across a capacitor of 0.2 μf was $v = 5 - 3t^2$ volts. The energy stored in a capacitor is $w = Cv^2/2$ joules. Find a formula for dw/dt in this capacitor.

22) The intensity I of light from a tungsten filament varies with the applied voltage according to $I = Av^{3.7}$, where A is a constant and v is the applied voltage. If $v = t - 2t^2$, find a formula for dI/dt.

23) When a length l meters of a conductor moves at a speed of **v** meters per second in a magnetic field of uniform flux density β teslas, a voltage is induced equal to $v = -\beta l\mathbf{v}$ volts. If **v** = 10 meters per second, $l = 0.3$ meter, and β varies over a certain interval according to $\beta = 1/t^2$, find dv/dt when $t = 0.5$ seconds.

24) The frequency of a certain crystal oscillator varies with temperature T according to $f = f_a[1 + k(T - T_a)]$, where f_a is the frequency at an initial temperature T_a and k is a constant of the crystal. If T varies with time (t minutes) according to $T = 55 + 0.01t^2$, how fast does f change when $t = 10$?

25) The wavelength λ meters of a radio wave traveling at a speed $c = 3 \times 10^8$ meters per second varies with the frequency according to $\lambda = c/f$. If $f = 1 \times 10^8 + (5 \times 10^7)t^{1/2}$ hertz find a formula for $d\lambda/dt$.

Derivatives - the Chain Rule (Cont.)

26) The voltage v across a varying resistor r, carrying a fixed current I, is $v = Ir$. If r varies with time t according to $r = t^3 + 5$, find a formula for dv/dt in this capacitor.

27) The mutual inductance between two windings is $M = N_2\phi_2/i_1$, where i_1 is the current in one of the windings and N_2 and ϕ_2 are the number of turns of the second winding and the flux linking it to the first winding. If i_1 and N_2 are constant, and if the second winding moves so that ϕ_2 varies with time t seconds according to $\phi_2 = t^3 - 2t$, find a formula for dM/dt.

28) A copper wire of diameter d and length s has a resistance of $r = ks/d^2$, where k is a constant. Suppose a sliding wire changes the length so that $s = t^2 - 0.6t$, where t is in seconds. Find a formula for dr/dt.

29) The force between two charged particles having fixed charges Q_1 and Q_2 varies with the distance separating them according to $\mathbf{F} = Q_1Q_2/4\pi\varepsilon s^2$. If ε is a constant, and if s varies with time as $s = 6t^{3/2}$, find a formula for $d\mathbf{F}/dt$.