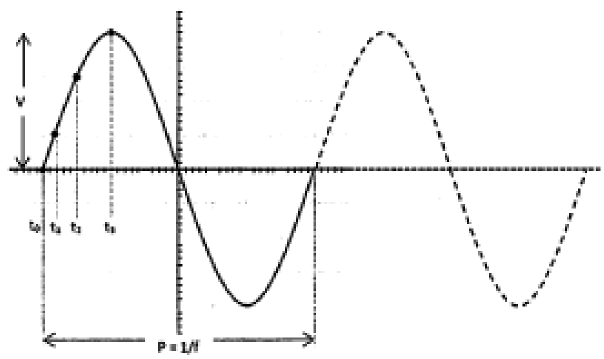
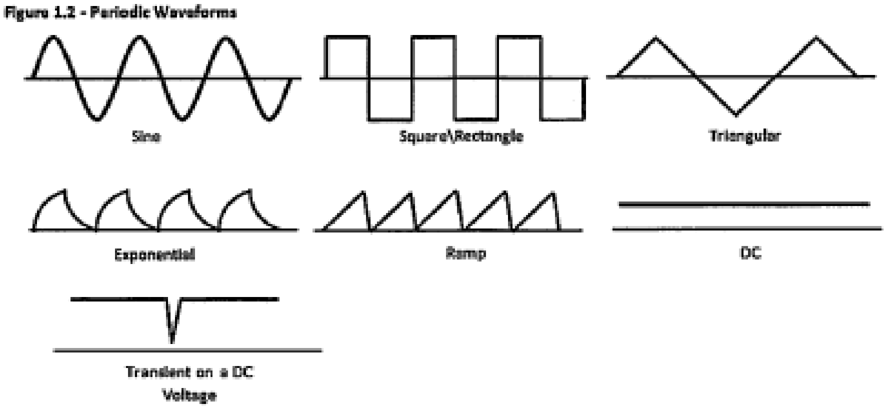
Waveforms

Whenever two quantities vary in relation to each other, the relationship between those values can be plotted on a graph. Graphs may be used to plot the relationship of voltage and current or the relationship of amplitude with respect to frequency. A **waveform** is a plot of the instantaneous value of voltage with respect to time. Direct current voltage with respect to time produces a straight line. This shows that as time increases, the voltage remains at a constant value. A plot of alternating current voltage in respect to time would produce a graph that indicates the instantaneous value of voltage increases and decreases with time. Figure 1.1 shows a sinusoidal waveform. From the graph we can see that at t_0 the instantaneous voltage is close zero, at t_1 the voltage has increased, at t_2, the voltage has increased again, and at t_3 the voltage has reached its peak value. As time continues, the voltage decreases to zero and continues.



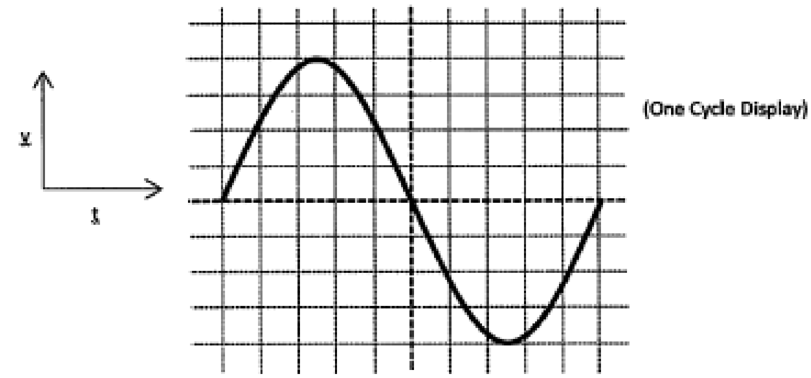
The sine wave is a repeating cycle of voltage with a sinusoidal relationship to time. Waveforms that are composed of identical cycles that keep repeating are termed **repetitive waveforms** or **periodic waveforms.** Repetitive waveforms can be comprised of sine waves, square waves which are an alternating DC value, triangle waves which are an alternating quantity with an equal rate of change increasing and decreasing, a ramp is also an alternating quantity, but the rate of change increasing is different than the rate of change decreasing. When successive cycles of alternating quantities are not identical, it is an **aperiodic waveform.** Figure 1.2 shows different examples of periodic waveforms.

Figure 1.2 - Periodic Waveforms



Sometimes a DC voltage will momentarily increase or decrease in voltage, and then return to its original value. This may happen for example when a load is suddenly switched into a power supply. These momentary non-repetitive waveforms are called **transients**.
The oscilloscope is one tool used in electronics, specifically in the study of electrical waveforms. The oscilloscope plots an instantaneous voltage value in respect to time. This provides a waveform graph that can be used to analyze a circuit.

Figure 1.3 Oscilloscope Display

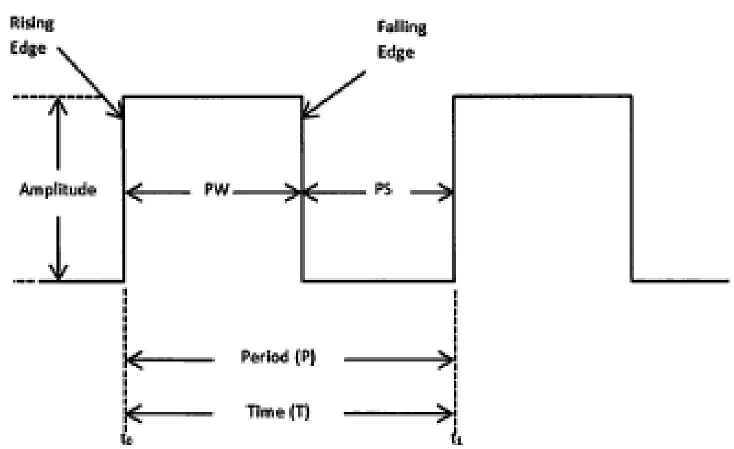


Waveforms are displayed on and oscilloscope vertically by the instantaneous voltage and horizontally in proportion to time.

IDEAL PULSE WAVEFORM

The ideal pulse waveform shown in figure 1.4 has positive pulses with respect to ground. The **pulse amplitude** is the voltage measured from the top of the waveform to the bottom of the waveform. The first edge of the pulse at t_0 is referred to as the **rising edge, leading edge** or **positive-going edge**. The next edge is termed the **falling edge**, **trailing edge** or **negative-going edge**.

Figure 1.4 Ideal Pulse Waveform



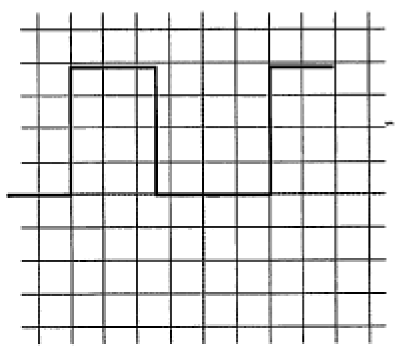
An ideal pulse waveform has perfectly vertical rising and falling edges and perfectly flat tops and bottoms.

The **period**(P) or time (T) is the time measured from the rising edge of one pulse(t_0) to the rising edge of the next pulse (t_1). If P = 1 second, the **pulse repetition frequency (PRF)** is 1/P= 1 cycle per second (Hz) or 1 **pulse per second** (pps).

$$DutyCycle(DC\%) = \frac{PW}{P} * 100\%$$

Example 1
Determine the PW, PRF, amplitude, and duty cycle.

Figure 1.5 Ideal pulse waveform displayed on oscilloscope



Pulse amplitude = 3.8 div x 1V/div
= 3.8V

P = 6 div x 0.1ms/div
= 0.6ms

PRF = 1/P = 1/0.6ms = 1.667kHz

PW = 2.5 div x 0.1ms/div
= 0.25ms
PS = 3.5 div x 0.1ms/div
= 0.35ms

$$DC\% = \frac{PW}{P} * 100\%$$

$$DC\% = \frac{0.25ms}{0.6ms} * 100\%$$

= 41.7%

The time measured from the rising edge to the falling edge of one pulse is the **pulse width** (PW). The time between each pulse is the **pulse space** (PS). The ratio of the pulse width to the period is defined as the duty cycle, and is given in percentage.

RISE TIME, FALL TIME, AND TILT

The waveform displayed in figure 1.5 shows a pulse waveform with perfectly flat tops and bottoms and perfectly vertical rising and falling edges. Examining a pulse carefully will show that the rising and falling edges are not perfectly vertical, nor are the top of the pulse and bottom of the pulse perfectly flat.

Careful examination of a pulse waveform shows that the voltage at the falling edge is normally less than the voltage at the leading edge. In many cases the difference in voltage from the rising to falling edges is so small that it cannot be measured, in other cases the difference is very obvious as seen in figure 1.6.

Figure 1.6 shows that if the PW is measured at the bottom of the waveform it would be very different than if it were measured at the top of the waveform. This difference in PW measurements indicates that the pulse voltage does not instantaneously go from one level to another and that there is a definite **rise time** (t_r) and **fall time** (t_f) at the leading and lagging edges. Because of the rise time and fall time of the pulse, the PW is defined as the average pulse width and is measured at 50% of the waveforms amplitude or ½ of the waveform_s average amplitude. The pulse space is measured at the same voltage level as the pulse width. The period is the sum of the PW and PS.

$$P = PW + PS$$

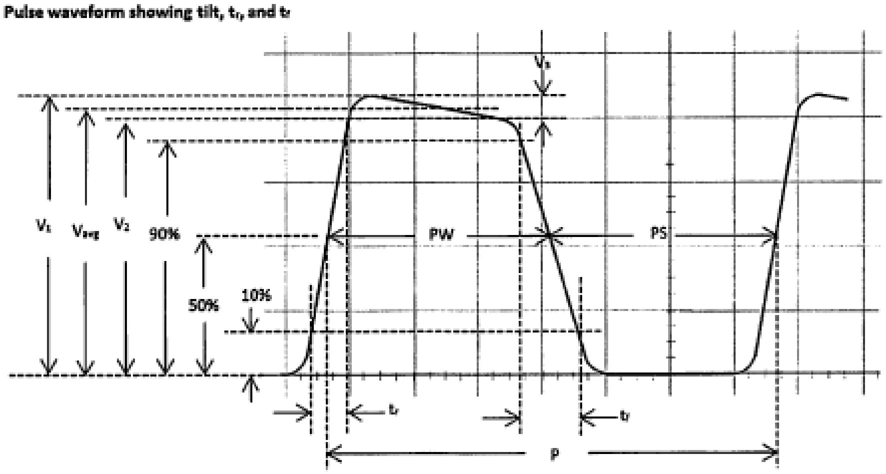
The waveform_s Average Pulse Amplitude (APA) is the average voltage of the pulse width.

$$APA = \frac{V_{max} + V_{min}}{2}$$

The rise time (t_r) is defined as the time required for the voltage to increase from 10% of the average amplitude, to 90% of the average amplitude. Similarly the fall time (t_f) is the time required for the voltage to decrease from 90% to 10% of the average amplitude.
Tilt is the difference between the maximum amplitude and the minimum amplitude divided by the average pulse amplitude. .

$$tilt\% = \frac{V_{max} - V_{min}}{APA} * 100\%$$

Figure 1.6 Pulse waveform showing tilt, t_r, and t_f



V_{avg} = Average Pulse Amplitude (APA)

V_1 = Maximum pulse amplitude

V_2 = Minimum amplitude

$$APA = \frac{V_1 + V_2}{2}$$

$$V_3 = V_1 - V_2$$

$$tilt\% = \frac{V_3}{APA} * 100\%$$

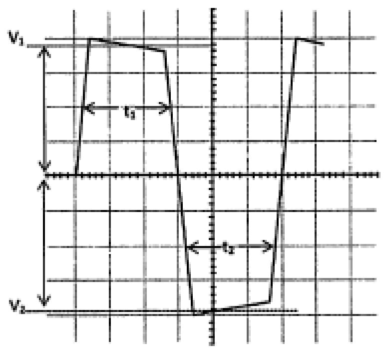
Tilt is normally given in percentages but in some cases it needs to be represented as a ratio. This is termed fractional tilt.

$$tilt_{fractional} = \frac{V_3}{APA}$$

Average Waveform Voltage

If a pulse waveform is symmetrical above and below ground, with the positive and negative peaks at equal amplitudes, the average voltage of the waveform is zero. This means that the voltage measured with a DC voltmeter would read zero.

Figure 1.7 Average Waveform Voltage



$$AWV = \frac{(V_1 * t_1) + (V_2 * t_2)}{(t_1 + t_2)}$$

*Where V_1 and V_2 are the average amplitude as measured from ground

Example 2

Refer to figure 1.7. If the average amplitude of V_1 is +6V, and the average amplitude of V_2 is -4V as measured from ground, and t_1 = 6ms, and t_2 = 2ms, find the average waveform voltage.

$$AWV = \frac{(V_1 * t_1) + (V_2 * t_2)}{(t_1 + t_2)}$$

$$AWV = \frac{(6 * 6ms) + (-4 * 2ms)}{8ms}$$

$$AWV = +3.5V$$

HARMONIC CONTENT OF WAVEFORMS

The process of building up a particular waveform by combining several sine waves of different frequencies and amplitudes is known as **frequency synthesis**. As sine waves are added in series with each other all at different amplitudes and frequencies, the result is a waveform that resembles a square wave. For example if two signal generators producing a sine wave are connected in series, where the first generator is producing a sine wave, and the second generator is producing a sine wave but at three times the frequency of the first, and one-third the amplitude of the first, the waveform produced is the larger amplitude and lower frequency, with the smaller amplitude signal super imposed. This combination resembles a square wave as shown in figure 1.4a.

If a third generator is connected in series with the first two with its amplitude one-fifth the first and its frequency is five times the first, then the resulting waveform more closely resembles a square wave (figure 1.4b). If this process of combining a large amplitude sine wave with a number of smaller amplitude, higher frequency sine waves is continued and higher frequency sine waves are added each time the output waveform more closely resemble a square wave.

Figure 1.8 Frequency Synthesis

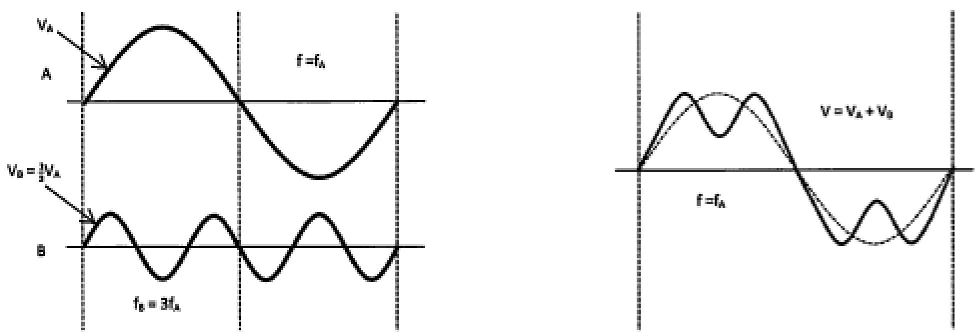
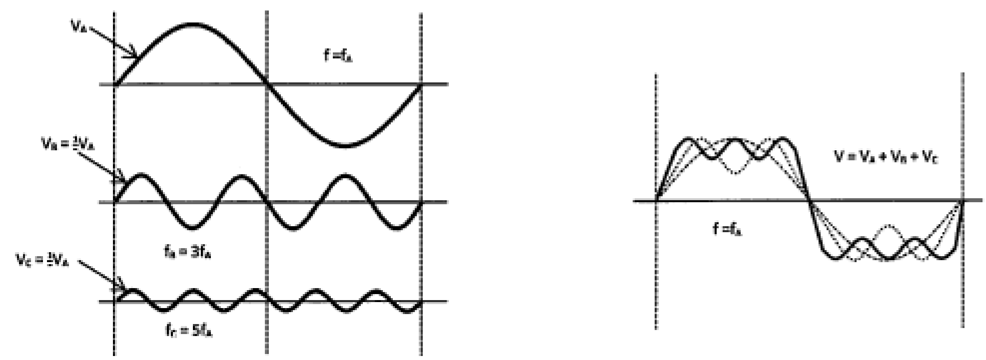


Figure 1.8a Fundamental and third harmonic.

Figure 1.4b Fundamental, third harmonic, and fifth harmonic.

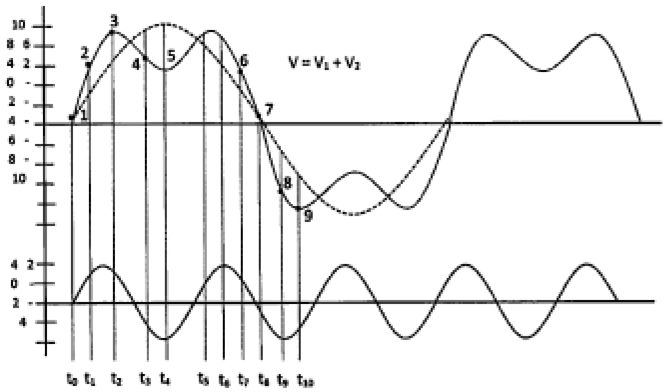


The construction of this approximate square wave can be shown by analyzing the instantaneous voltages of each generator output. At each time period, the instantaneous voltage of generator 1 is added to the instantaneous voltage of generator 2.

At t_0 the instantaneous voltage of waveform 1 is 0V, and the instantaneous voltage of waveform 2 is 0V, so the resulting voltage is zero (point 1). At t_1 the instantaneous voltage of waveform 1 is approximately 3V and the instantaneous voltage of waveform 2 is approximately 3V so the resulting voltage is 6V (point 2). At t_3, the instantaneous voltage of waveform 1 is 6V and the instantaneous voltage of waveform 2 is approximately 3V, so the resulting voltage is 9V. As this process continues, it is seen how the final waveform is constructed.

HARMONIC ANALYSIS

Figure 1.5 Addition of instantaneous levels of a fundamental frequency sine wave and a third harmonic.



The instantaneous voltages of 2 generators are added together to produce a waveform that resembles a square wave with dented peaks

Harmonic analysis is a process by which a waveform is analyzed to discover the individual sine wave frequencies it contains. Periodic non-sinusoidal waveforms such as square waves are made up of combinations of pure sine waves. These combinations of sine waves are the **fundamental**, which is a large amplitude sine wave of the same frequency as the periodic waveform under consideration, and many other sine waves each at a frequency and amplitude that is a multiple of the fundamental. These other sine waves are termed **harmonics**. These harmonics are numbered according to the ratio of their frequencies to that of the fundamental. For example a harmonic that has a frequency that is exactly double the fundamental frequency would be the second harmonic. The frequency of the third harmonic would be exactly three times the frequency of the fundamental.

By a process known as Fourier analysis, which is a mathematical operation to determine the harmonic content of a waveform, a perfect square wave that is symmetrical above and below ground can be shown to have a fundamental component, and a number of odd harmonics. This symmetrical square wave contains no even harmonics and no DC component. Fourier analysis shows that a non-symmetrical pulse waveform contains both odd and even harmonics, as well as some DC components. Saw tooth waveforms, triangular waveforms, and rectified sine waves are made up of more complicated combinations of harmonics. In all cases, the number of harmonics is infinite, and the amplitude of each harmonic will decrease as the frequency increases.

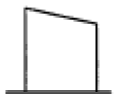
The information derived from harmonic analysis becomes important when considering the circuitry through which various waveforms are processed.

Harmonic number	Frequency	Amplitude
Fundamental	f	V
2nd Harmonic	2f	1/2V
3rd Harmonic	3f	1/3V
4th Harmonic	4f	1/4V
5th Harmonic	5f	1/5V
7th Harmonic	7f	1/7V
100th Harmonic	100f	1/100V

WAVEFORM DISTORTIONS

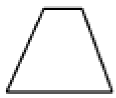
3 main types

Tilt



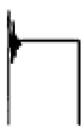
Low Frequency Distortion – Caused by parallel capacitance

Distorted rise time, distorted fall time



High Frequency Distortion – Caused by series capacitance

Ringing



Over-emphasis of high frequency components – Ringing occurs when the circuit oscillates for a short time due to stray inductance and stray capacitance.

RISE-TIME AND UPPER CRITICAL FREQUENCY

When a square wave with no measurable rise time is applied to a circuit, all harmonic components with frequencies above the upper cutoff frequencies are attenuated. Thus the rise and fall times of the output waveform are limited by the upper cutoff frequency of the circuit. The relationship is given by:

$$t_r = t_f = \frac{0.35}{fch}$$

The upper cutoff frequency can be predicted using the above equation.

Derivation of this equation shows how tr and fch are related:

1. $V_c = V_f - (V_f - V_{ini})e^{\frac{-t}{\tau}}$ Solve this equation for t:

2. $V_c - V_f = -(V_f - V_{ini})\varepsilon^{\frac{-t}{\tau}}$

3. $\dfrac{V_c - V_f}{-(V_f - V_{ini})} = \varepsilon^{\frac{-t}{\tau}}$

4. $\dfrac{V_c - V_f}{V_{ini} - V_f} = \varepsilon^{\frac{-t}{\tau}}$

5. $\dfrac{V_{ini} - V_f}{V_c - V_f} = \varepsilon \dfrac{t}{\tau}$

6. $\tau = RC$

7. $t = RC\ell\eta \dfrac{V_{ini} - V_f}{V_c - V_f}$

T_r is determined by 10% Vf to 90%Vf. Substitute 10%Vf for V_{ini} and 90%Vf for Vc. So the equation becomes:

$$t = RC\ell\eta \frac{0.1V_f - V_f}{0.9V_f - V_f}$$ Factor out Vf $$t = RC\ell\eta \frac{0.1 - 1}{0.9 - 1}$$

$$t = RC\ell\eta \frac{-0.9}{-0.1} \quad \Rightarrow \quad t = RC2.197$$

So $t \approx 2.2RC$ and $RC \approx \dfrac{t}{2.2}$

If $f_c = \dfrac{1}{2\pi RC}$ then $f_c = \dfrac{1}{2\pi \frac{t}{2.2}} \Rightarrow f_c = \dfrac{2.2}{2\pi * t} \Rightarrow f_c = \dfrac{0.35}{t}$

Where fc is the upper critical frequency and t is the rise time of the circuit.

It is possible to determine the upper cutoff for a circuit that must pass a pulse with an acceptable amount of high-frequency distortion if the rise & fall times are approximately 0.1 of the pulse width. If the rise time and fall time is greater than 0.1 of the PW then the pulse becomes distorted.

There is always a time difference between the start of a pulse waveform and when that waveform reaches its maximum amplitude. Usually a requirement of the circuitry that processes the waveform is that it does not add significantly to the input rise and fall times.

If an oscilloscope has an upper cutoff frequency that produces rise and fall times in apparently perfect input pulses, the rise time for the scope should be much smaller than the rise and fall time of the input pulse. If the circuit produced rise time (trc) is less than 0.1 of the signal rise time, then the output rise time is not significantly affected.

$$t_{ro} = \sqrt{(t_{rs})^2 + (t_{rc})^2}$$

TILT AND LOWER CUTOFF FREQUENCY

When tilt is 10% or less of the average pulse amplitude, it is directly proportional to the pulse width and the lower cutoff frequency.

If $f_c = \dfrac{1}{2\pi RC}$ with R=input resistance
And C=coupling capacitor

Solving the exponential equation for time

$$t = RC\ell\eta\frac{V_{ini} - V_f}{V_c - V_f}$$

with a 10% tilt,
Vini = 0 and Vc= 10%Vf,

$$t = 0.1RC$$

Remember that $tilt_{fractional} = \dfrac{V_3}{APA}$ where V_3 is the change in capacitor voltage,

$$FractionalTilt = \frac{0.1}{V}$$

If PW=0.1RC then $\dfrac{PW}{RC} = fractionaltilt$

So $f_c = \dfrac{1}{2\pi RC} \Rightarrow f_c = \dfrac{1}{2\pi \dfrac{PW}{FracTilt}} \Rightarrow$

$f_c = \dfrac{FracTilt}{2\pi PW}$ This equation describes the relationship between the fractional tilt and the lower cutoff frequency

RC CIRCUIT OPERATION

Refer the the circuit of Figure 2.1a and 2.1b.

Figure 2.1a Capacitor Charge Circuit

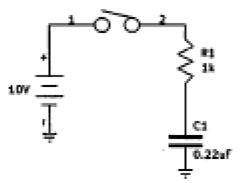
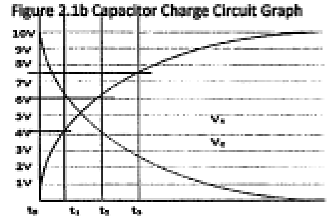


Figure 2.1b Capacitor Charge Circuit Graph



The voltage across the resistor R_1 is given by:

$VR_1 = V_s - V_c$ Where V_{R1} is the voltage across R_1
And V_s is the source voltage
And V_c is the voltage across C_1

The current through the circuit is given by:

$$ic = \frac{V_R}{R} = \frac{V_s - V_c}{R}$$

The charge on the capacitor at t_0=0 and $ic = \frac{10V - 0V}{1K} = 10mA$

This current causes the capacitor C_1 to charge.
At t_1 when the capacitor is charged to approximately 3V, the voltage on R_1:

$V_{R1} = 10V - 3V = 7V$ Which makes the current in the circuit:

$ic = \frac{10V - 3V}{1K} = 7mA$ As C_1 continues to charge to a voltage of 6V at t_2 then the voltage across R_1 now is

$V_{R1} = 10V - 6V = 4V$ and the current in the circuit is $ic = \frac{10V - 6V}{1K} = 4mA$

At each time the current in the circuit is reduced, and it takes longer to charge the capacitor. The capacitor does not receive its charge at a constant rate, as V_C increases, V_R decreases and the charge current is also reduced.

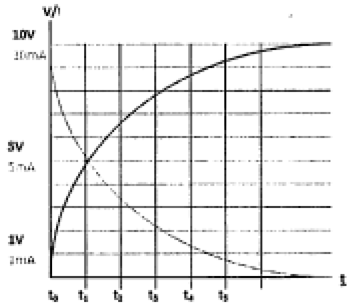
The capacitor voltage follows the exponential equation:

$Vc = Vf - (Vf - Vini)\varepsilon\frac{-t}{\tau}$ Where:

Vc = instantaneous capacitor voltage at time t
Vf = maximum charge voltage
Vini = initial charge on the capacitor
ε = exponential constant ≈ 2.718
t = time from commencement of charge
C = capacitance in the circuit
R = resistance in the circuit

CIRCUIT TIME CONSTANT

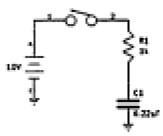
Figure 2.2 Plot of capacitor voltage and charge current with respect to time



The graph of figure 2.2 shows the current and the capacitor voltage for each point in time as given by the exponential equation. This shows that as the voltage across the capacitor increases, the current in the circuit decreases.

Refer to figure 2.3:

Figure 2.3 Circuit to show the instantaneous capacitor voltage.



In the circuit of figure 2.3 the product of R_1 and C_1 = 4 mS

When $t = 4mS$ $\quad$ $Vc = 6.32V$ $\quad$ or $63.2\% V_{max}$

When t=RC then V_C = 63.2% of V_{max} regardless of the value of Vmax.

The product RC is known as the circuit's time constant. τ is the symbol used to represent the circuit's time constant. After *t=RC*, the charging current is reduced by 63.2%.

5τ is the time required for the capacitor to complete a full charge.

$$V_C = Vf - (Vf - Vini)\varepsilon^{\frac{-5\tau}{\tau}}$$

$$Vc = 10V - (10V - Vini)\varepsilon^{\frac{-5*4mS}{4mS}} = 9.93V$$

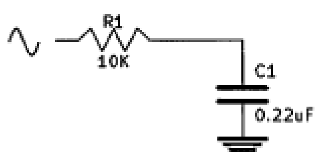
After *5RC* the capacitor voltage is $99.3\% V_{max}$

The exponential equation $Vc = Vf - (Vf - Vini)\varepsilon^{\frac{-t}{\tau}}$ can be applied to any RC circuit with a constant DC voltage applied to the input.

RC CIRCUIT RESPONSE TO SINE WAVES

Figure 2.4 shows a low pass filter.
The capacitor is reactive with a sine wave input, and has a reactance: $Xc = \dfrac{1}{2\pi * f * C}$

Figure 2.4 Low Pass Filter



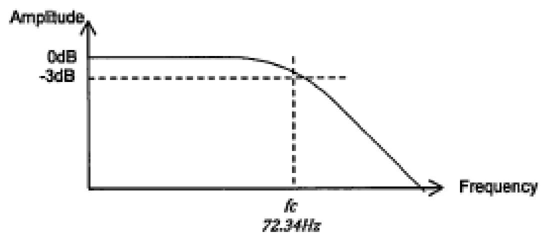
The cutoff frequency for the given circuit is found by:

$$fc = \frac{1}{2\pi * RC}$$

$$fc = \frac{1}{2\pi * 10k * 0.22uF} = 72.34KHz$$

The bode plot is a graph of the amplitude vs. frequency:

Figure 2.5 Bode Plot for the Circuit of Figure 2.4



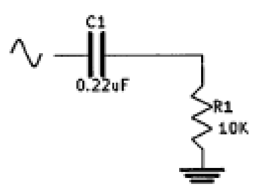
A low pass filter passes all frequencies below the cutoff frequency with an amplitude of 0dB.

X_c= R at the critical frequency.

$$R = \frac{1}{2\pi * f * c} \quad \text{and} \quad f = \frac{1}{2\pi * R * C}$$

Given the high pass filter of Figure 2.6:

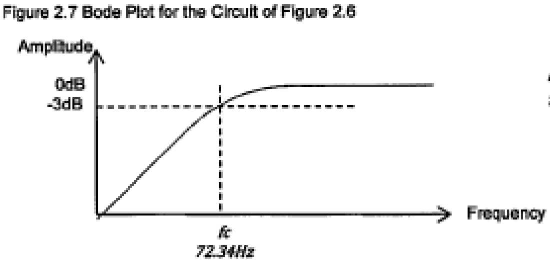
Figure 2.6 High Pass Filter



$$fc = \frac{1}{2\pi * RC}$$

$$fc = \frac{1}{2\pi * 10k * 0.22uF} = 72.34Hz$$

Figure 2.7 Bode Plot for the Circuit of Figure 2.6

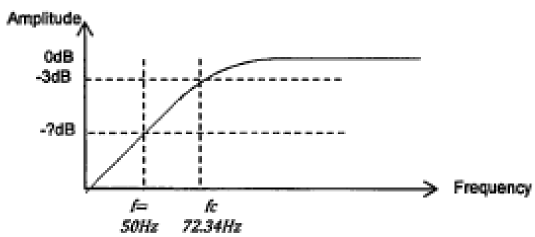


All frequencies above fc are passed with an amplitude of 0dB

RC CIRCUIT RESPONSE TO SINE WAVES

What happens to Vout when the frequency is decreased below cutoff. At f=50Hz:

Figure 2.8 Bode Plot for frequency below cutoff.



50Hz is below the cutoff for this RC circuit.

$$Xc = \frac{1}{2\pi * f * C} = \frac{1}{2\pi * 50Hz * 0.22uF} = 14.468k\Omega$$

$$Z_T = \sqrt{R^2 + X_C^{\ 2}} = \sqrt{10K^2 + 14.468K^2} = 17.587K$$

$$\theta = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{-14.468K}{10K}\right) = -55.348°$$

$$Z_T = 17.587K\angle -55.348°$$

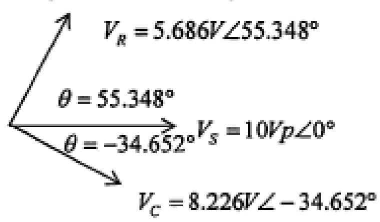
$$I_T = \frac{V_S}{Z_T} = \frac{10Vp\angle 0°}{17.587K\angle -55.348°} = 568.601\mu A\angle 55.348°$$

$$V_R = (10k\angle 0°)(568.601\mu A\angle 55.348°) = 5.686V\angle 55.348°$$

$$V_C = (14.468k\angle -90°)(568.601\mu A\angle 55.348°) = 8.226V\angle -34.652°$$

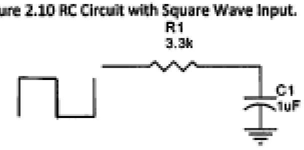
Voltage vector diagram shows the relationship of each components voltage:

Figure 2.9 Voltage Vector for circuit of Figure 2.6.



RC CIRCUIT RESPONSE TO SQUARE WAVES

Figure 2.10 RC Circuit with Square Wave Input.

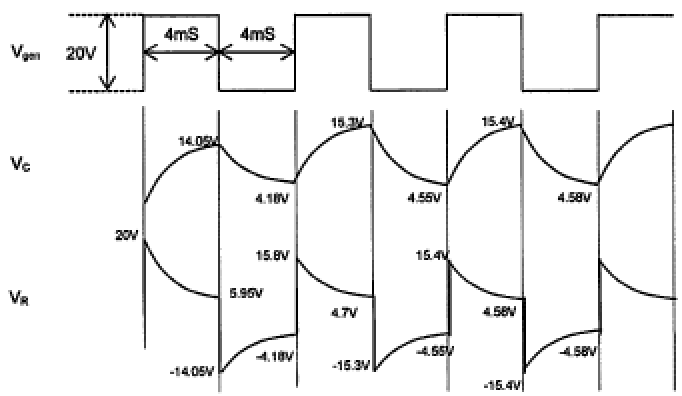


RC Circuit response to a square wave input.

The capacitor voltage builds up from zero over several cycles of the input reaching maximum and minimum settled levels known as **steady state**.

$\tau = R * C = 3.3ms$

$V_C = V_f - (V_f - V_{ini})e^{\frac{-t}{RC}}$ The Exponential equation gives the capacitor voltage after any given time period.



During the pulse width, the capacitor is charging toward 20V. During the pulse spacing, the cap discharges toward zero. After each time period, the cap voltage is the starting voltage for the exponential equation.

The resistor can be found at any given point by $V_S - V_C$. When $V_S = 0$, The cap is discharging and current through the resistor is reversed.

INTEGRATION OF PULSE CIRCUITS

Integration is defined as a summation of area. An integrating circuit produces and output that is proportional to the area enclosed by the input waveform.

With the output taken across the capacitor,
The shape of the output waveform is dependent on the relationship between the time constant and the pulse width.
The circuit will function as and integrator when the time constant is equal to or greater than the Pulse Width.