Exercises 5.1 $\int u^n du = \frac{u^{n+1}}{n+1} + C$

1. $\int x^7 dx = \frac{x^8}{8} + C$

3. $\int 3x^8 dx = \frac{3x^9}{9} + C = \frac{x^9}{3} + C$

5. $\int 4 dx = 4x + C$

7. $\int 9\sqrt[6]{x^5}\, dx = \int 9(x)^{\frac{5}{6}} = \frac{9x^{\frac{11}{6}}}{\frac{11}{6}} = \frac{54x^{\frac{11}{6}}}{11} + C$

9. $\int \frac{6dx}{x^3}$ $\int 6x^{-3} dx = \frac{6x^{-2}}{-2} = -3x^{-2} + C = \frac{-3}{x^2} + C$

11. $\int (5x^2 - 12x + 8)\, dx = \frac{5x^3}{3} - \frac{12x^2}{2} + 8x = \frac{5x^3}{3} - 6x^2 + 8x + C$

13. $\int \left(3x^2 - x + \frac{5}{x^3}\right) dx = \int (3x^2 - x + 5x^{-3})\, dx = \frac{3x^3}{3} - \frac{x^2}{2} + \frac{5x^{-2}}{-2} + C$

$= x^3 - \frac{x^2}{2} - \frac{5}{2x^2} + C$

15. $\int (2x^2-3)^2 dx \int (2x^2-3)(2x^2-3)$ $\int 4x^4 - 6x^2 - 6x^2 + 9$

$\int 4x^4 - 12x^2 + 9\, dx = \frac{4x^5}{5} - \frac{12x^3}{3} + 9x + C$

$= \frac{4x^5}{5} - 4x^3 + 9x + C$

17. $\int \sqrt{6x+2}\, dx = \int (6x+2)^{\frac{1}{2}} dx = \frac{1}{6}\int 6(6x+2)^{\frac{1}{2}}$

$= \frac{1}{6}\frac{(6x+2)^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2(6x+2)^{\frac{3}{2}}}{18} = \frac{(6x+2)^{\frac{3}{2}}}{9} + C$

19. $\int 8x(x^2+3)^3 dx = 4\int (2x)(x^2+3)^3 dx$ $\frac{8}{4} = 2$

$= \frac{4(x^2+3)^4}{4} = (x^2+3)^4 + C$

Exercises 5.1 Continued

21. $\int x\sqrt[3]{5x^2-1}\,dx \qquad \int x(5x^2-1)^{1/3}\,dx$

$= \frac{1}{10}\int 10x(5x^2-1)^{1/3}\,dx = \frac{1}{10}\frac{(5x^2-1)^{4/3}}{\frac{4}{3}} = \frac{3(5x^2-1)^{4/3}}{40} + C$

23. $\int x(x^2-1)^4\,dx = \frac{1}{2}\int 2x(x^2-1)^4\,dx = \frac{1}{2}\frac{(x^2-1)^5}{5} = \frac{(x^2-1)^5}{10} + C$

25. $\int \frac{2x\,dx}{\sqrt{x^2+1}} = \int 2x(x^2+1)^{-1/2}\,dx = \frac{(x^2+1)^{1/2}}{\frac{1}{2}} = 2(x^2+1)^{1/2} + C$ or $2\sqrt{x^2+1} + C$

27. $\int (3x^2+2)(x^3+2x)^3\,dx$

$=$ (derivative of) $= \boxed{\frac{(x^3+2x)^4}{4} + C}$

29. $\int \frac{x^2\,dx}{(x^3-4)^2} = \int x^2(x^3-4)^{-2}$

$= \frac{1}{3}\int 3x^2(x^3-4)^{-2} = \frac{1}{3}\frac{(x^3-4)^{-1}}{-1} + C$

$= -\frac{1}{3}(x^3-4)^{-1} + C$

$= \frac{-1}{3(x^3-4)} + C$

31. $\int (10x-1)\sqrt{5x^2-x}\,dx = \int (10x-1)(5x^2-x)^{1/2}\,dx$

(10x−1 is the derivative of 5x²−x)

$= \frac{(5x^2-x)^{3/2}}{\frac{3}{2}} = \frac{2(5x^2-x)^{3/2}}{3} + C$

33. $\int \frac{(2x+1)\,dx}{\sqrt{x^2+x}} = \int (2x+1)(x^2+x)^{-1/2} = \frac{(x^2+x)^{1/2}}{\frac{1}{2}} + C =$

(2x+1 is the derivative of x²+x)

$2(x^2+x)^{1/2} + C$

$= 2\sqrt{x^2+x} + C$

Exercises 5.1 Continued

35. $\int (2x+3)^2 dx = \int 2(2x+3)^2 = \frac{1}{2}\int 2(2x+3)^2$

$= \frac{1}{2}\frac{(2x+3)^3}{3} = \frac{(2x+3)^3}{6} + C$

37. $\int (2x-1)^4 dx = \frac{1}{2}\int 2(2x-1)^4 dx$

$= \frac{1}{2}\frac{(2x-1)^5}{5} = \frac{(2x-1)^5}{10} + C$

39. $\int (x^2+1)^3 dx = \int (x^2+1)(x^2+1)(x^2+1)$

$= \int (x^4+x^2+x^2+1)(x^2+1)$

$= \int (x^4+2x^2+1)(x^2+1)$

$= \int x^6+2x^4+x^2+x^4+2x^2+1$

$= \int x^6+3x^4+3x^2+1$

$= \frac{x^7}{7} + \frac{3x^5}{5} + \frac{3x^3}{3} + x + C$

$= \frac{x^7}{7} + \frac{3x^5}{5} + x^3 + x + C$

41. $\int 4x(x^2+1)^3 dx = 2\int 2x(x^2+1)^3$

$= \frac{2(x^2+1)^4}{4} = \frac{(x^2+1)^4}{2} + C$

43. $\int 30x^2(5x^3+1)^4 dx = 2\int 15x^2(5x^3+1)^4$

$= \frac{2(5x^3+1)^5}{5} + C$

Exercises 5.1 Continued

45. $\int \frac{6x^2 dx}{\sqrt{x^3+1}} = \int 6x^2 (x^3+1)^{-1/2} dx$

$= 2\int 3x^2 (x^3+1)^{-1/2} dx$

$= \frac{2(x^3+1)^{1/2}}{\frac{1}{2}} = 4(x^3+1)^{1/2} + C$

47. $\int (6x^2+6)(x^3+3x)^{-1/3} dx$

$= 2\int (3x^2+3)(x^3+3x)^{-1/3} dx$

$= \frac{2(x^3+3x)^{2/3}}{\frac{2}{3}} = \frac{6(x^3+3x)^{2/3}}{2} = 3(x^3+3x)^{2/3} + C$

49. $\int (x-1)(x)^{-3} dx = \int x^{-2} - x^{-3}$

$= \frac{x^{-1}}{-1} - \frac{x^{-2}}{-2} = -x^{-1} + \frac{x^{-2}}{2} + C = \frac{-1}{x} + \frac{1}{2x^2} + C$