Exercises 5.2

7. find the equation describing the motion of an object moving along a straight line when the acceleration is $a = 3t$, when the velocity at $t = 4s$ is $40 m/s$, and when the object has traveled $86 m$ from the origin at $t = 2s$.

motion of an object? $a = 3t$ velocity @ $t = 4s = 40 m/s$

distance $86m$ at $t = 2 sec$

$v = \int a\, dt$

$s = \int v\, dt$

$v = \int 3t\, dt$

$v = \left(\frac{3t^2}{2} + C\right.$

$40 m/sec = \frac{3(4)^2}{2} + C$

$C = 16 m/sec$

$v = \frac{3t^2}{2} + 16$

$s = \int \frac{3t^2}{2} + 16$

$s = \frac{3t^3}{2 \cdot 3} + 16t$

$s = \frac{t^3}{2} + 16t + C$

$86 m = \frac{t^3}{2} + 16t + C$

$86m = \frac{(2)^3}{2} + 16(2) + C$

$C = 50$

$$\boxed{s = \frac{t^3}{2} + 16t + 50}$$

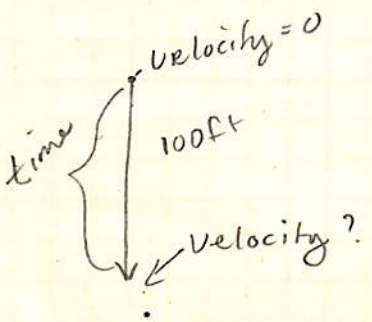
Exercises 5.2

9. A stone is dropped from a height of 100 ft. For a free-falling object, the acceleration is $a = -32\ ft/s^2$ (the effect of gravity). Find the distance the stone has traveled after 2 sec. Note that the initial velocity is 0 because the stone was dropped, not thrown down. Find also the velocity of the stone when it hits the ground.

$v = \int a\,dt$

$s = \int v\,dt$

Find distance



height of 100ft 2 sec

$v = \int -32\ ft/s^2\,dt$

$v = -32t + 0$

$s = \int v\,dt$

$s = \frac{-32t^2}{2} + C$

$s = \frac{-32(2)^2}{2} + 100$

$s = -64 + 100$

$s = 36$ ft from Ground

distance stone has traveled is

$= 100f - 36ft = \underline{64ft}$

$v = \int a\,dt$

$v = \int -32\ ft/sec\,dt$

$v = -32t + C$

$s = \frac{-32t^2}{2} + C$

(100)

$-100 = \frac{-32t^2}{2}$

$-100 = -16t^2$

$t^2 = 6.25$

$t = 2.5$ seconds

$v = -32(t) + 0$

$\boxed{v = -80\ ft/sec}$

Exercises 5.2.

11. An Airplane starts from rest and travels 3600 ft down the runway with constant acceleration before lifting off in 30 sec. Find its velocity at the moment of lift-off. $S = \frac{1}{2} a t^2$

$a = k$

$v = \int a\,dt$

$S = \int v$

$v = \int 8 ft/sec\, dt$

$v = 8t$

$v = 8(30)$

$\boxed{v = 240 ft/sec}$

$S = \frac{1}{2} a t^2$

$3600 ft = \frac{1}{2} a t^2$

$a t^2 = 7.2k$

$a = \frac{7.2k}{(30)}$

$a = 8 ft/sec$

13. A stone is hurled straight up from the ground at a velocity of 25 m/s. (a) Find the maximum height that the stone reaches. (B). How long does it take for the stone to hit the ground? (C) Find the speed at which the stone hits the ground.

$v = \int a\,dt$

$S = \int v\,dt$

Known Formula

acceleration in m/s^2 *

$a = -9.80 m/s^2\, dt$

$v = \int -9.80 m/s^2$

$v = -9.80t\ m/s + 25\ m/s$

$0 = -9.8t\ m/s + 25\ m/s$

$t = 2.551$ seconds

$v = -9.80(2.551) + 0$

C. $\boxed{v = -25 m/s}$

velocity = 25 m/s

$S = 25t + C$

$S = 25t + 0$

$S = \int -9.80t\ m/s + 25\ m/s$

$S = \frac{-9.8 m t^2}{2} + 25 m t$

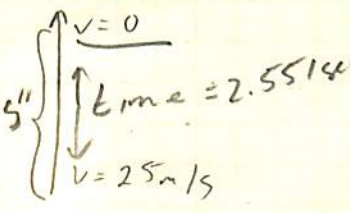
$S = \frac{-9.8 m (2.551)^2}{-31.887} + 25 m (2.551)$

63.775

(a) $\boxed{S = 31.888 m}$

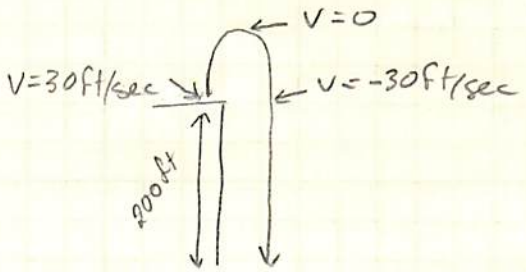
(B) $t \times 2 \uparrow\downarrow = 2.551 sec \times 2$

$\boxed{(B) = 5.102 seconds}$



Exercises 5.2

15. A stone is thrown vertically upward from the roof of a 200 ft tall building with an initial velocity of 30 ft/sec (a) Find the equation describing the altitude of the stone from the ground (b) How long does it take for the stone to hit the ground.



$\uparrow a = -32\ ft/s^2$

$v = \int a\,dt$

$\uparrow v = -32t + C$

$v = -32t + 30$

at peak arch $V = 0$

$0 = -32t + 30$

$\uparrow time = \frac{-30}{-32}$

$\uparrow time = .937\ sec$

$\uparrow s = \int v\,dt$

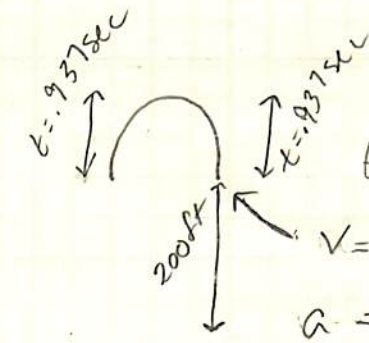
$\uparrow s = \frac{-32t^2}{2} + 30t + C$ (0 from Roof ↑)

$\uparrow s = \frac{-32(.937)^2}{2} + 30(.937)$

$-14.0475 + 28.11$

$\uparrow s = 14.0625\ ft$ from Building

$\uparrow s = 214.0625\ ft$ from Ground.



time of arch = 1.874 sec

$V = -30\ ft/s$ (w/ Gravity)

$a = -32\ ft/s^2$

$v = \int a\,dt$

$v = -32t + 30$

$s = \int v\,dt$

$s = \frac{-32t^2}{2} - 30t + C$ (0 roof to ground)

$-200ft = \frac{-32t^2}{2} - 30t$

$-200 = -16t^2 - 30t$

$t = 2.72$ sec Roof to ground

time total = 2.72 sec roof to ground
+1.874 sec Arch time

time total = 4.594 sec

Exercises 5.2

17. A flywheel is turning at a rate given by $w = 80 - 12t + 3t^2$ where w is the angular speed in revolutions per minute (rpm). Find the number of revolutions that the flywheel makes in the first 3s:

$$w = \frac{d\theta}{dt} \quad (\text{assume that } \underset{c}{\theta = 0} \text{ when } t = 0)$$

$t = 3s.$

$$\theta = \int w\, dt$$

$$\theta = \int 80 - 12t + 3t^2\, dt$$

$$\theta = 80t - \frac{12t^2}{2} + \frac{3t^3}{3} + C$$

$$\theta = 80(3) - 6(3)^2 + (3)^3 + 0$$

$$\theta = 240 - 54 + 27$$

$$\boxed{\theta = 213 \text{ revolutions}}$$

19. A capacitor with capacitance 10^{-4}F has a voltage of 100v across it. At a given instant ($t = 0$) the capacitor is connected to a source that sends a current $i = \frac{1}{2}\sqrt{t} + .02A$ through the circuit. Find the voltage across the capacitor when $t = 0.16$ sec

$$C = 1 \times 10^{-4}F \qquad \underset{(C)}{V_c = 100v} \qquad i = \frac{1}{2}(t)^{\frac{1}{2}} + .2A \qquad t = 0.16 \text{ sec}$$

$$V_c = \frac{1}{C}\int i\, dt$$

$$V_c = \frac{1}{1 \times 10^{-4}} \int \frac{1}{2}(t)^{\frac{1}{2}} + .02$$

$$V_c = \frac{1}{1 \times 10^{-4}} \left(\left(\frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + .2t \right) + C \right)$$

$$V_c = \frac{1}{1 \times 10^{-4}} \left(\left(\frac{2t^{3/2}}{6} \right) + .2t \right) + 100$$

$$V_c = \frac{1}{1 \times 10^{-4}} \left(\frac{t^{3/2}}{3} \right) + .2t + 100 = \frac{1}{1 \times 10^{-4}} \left(\left(\frac{.16^{3/2}}{3} \right) + .2(.16) \right) + 100$$

$$213.333 + 320 + 100$$

$$\boxed{V_c = 633.333v}$$