Week 10 Questions and Answer Key

- Day 1, Max-Mins 1-3
- Day 2, 4-6
- Day 3, 7-9
- Day 4, 10-14
- Day 5, 15-18

1. The sum of two positive numbers is 56. Find the two numbers if their product is to be maximum.

   $y = 28$ & $x = 28$

2. An open box is to be made from a square piece of aluminum, 3cm on a side, by cutting equal squares from each corner and then folding up the sides. Determine the dimensions of the box that will have the largest volume.

   $2cm \times 2cm \times 0.5cm$

3. A man wishes to fence in a rectangular plot lying next to a river. No fence is required along the river bank. If he has 800m of fence, and he wishes the maximum area to be fenced, find the dimensions of the desired enclosure.

   $200m \times 400m$

4. Find the maximum possible area of a rectangle whose perimeter is 36cm.

   $a = 81cm^2$

5. A farmer wants to fence in $80,000m^2$ of land and then divide it into three plots of equal area. Find the minimum amount of fence needed.

   $fence_{min} = 1600m$

6. The charge transmitted through a circuit varies according to $q = t^4 - 4t^3$ coulombs. Find the time in seconds when the current i (in amps) $i = (dq/dt)$ reaches a minimum.

   $t = 2sec,\ i = -16ma$

7. A rectangle box, open at the top, with a square base, is to have a volume of $4000cm^3$. Find the dimensions, if the box is to contain the least amount of material.

   $20cm \times 20cm \times 10cm$

8. The total cost C of making x units of a certain commodity is given by $C = 0.005x^3 + 0.45x^2 + 12.75x$. All units made are sold at $\$36.75x - C$. Find the number of units to make maximum profit.

   $20\ units$

9. A cylindrical can with one end is to be made with $24\pi cm^2$ of metal. Find the dimensions of the can that give the maximum volume.

   $r = 2.828cm,\ h = 2.828cm$

10. The side of a square measures $12.00cm$ with a maximum possible error of $0.05cm$. (a) Find the maximum possible error in the area using differentials. (b) Find the maximum possible error by substituting into the formula for the area of a square. (c) Find the percentage of error.

    a. $da = 1.2cm^2$
    b. $1.2025cm^2$
    c. 0.833%

11. Suppose you want to build a spherical water tower with an inner diameter of $26.00m$ and a side thickness of $4.0cm$. (a) Find the approximate volume of steel needed using differentials. (b) if the density of steel is $7800kg/m^3$, find the approximate volume of steel needed using differentials.

    a. $dv = 84.95m^3$
    b. $662,599kg$

12. A freely falling body drops according to $s = (1/2)gt^2$, where s is the distance in meters, $g = 9.80m/s^2$, and t is time in seconds. Approximate the distance, ds, that an object falls from $t = 10.00$ sec to $t = 10.03$ sec.

    $ds = 2.94m$

13. The voltage Vin volts, varies according to $V = 10p^{2/3}$, where p is the power in watts. Find the change dv when the power changes from 125w to 128w.

    $dv = 4v$

14. The impedance Z in an ac circuit varies according to $Z = \sqrt{R2 + X2}$, where R is the resistance and X is the reactance. If $R = 300\Omega$ and $X = 225\Omega$, find dz when R changes to 310Ω.

    $dz = 8\Omega$

    Find the first four derivatives of each of the following functions.

15. $y = x^5 + 3x^2$

    $y' = 5x^4 + 6x$
    $y'' = 20x^3 + 6$
    $y''' = 60x^2$
    $y'''' = 120x$

16. $3x^6 - 8x^3 + 2$

    $y' = 18x^5 - 24x^2$
    $y'' = 90x^4 - 48x$
    $y''' = 360x^3 - 48$
    $y'''' = 1080x^2$

17. $5x^5 + 2x^3 - 8x$

$y' = 25x^4 + 6x^2 - 8$
$y'' = 100x^3 + 12x$
$y''' = 300x^2 + 12$
$y'''' = 600x$

18. $3x^2 + 4x - 7$

$y' = 6x + 4$
$y'' = 6$
$y''' = 0$
$y'''' = 0$