RCET 2251

Systems Analog & Digital Theory

Advanced Review Copy
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Contents

Week 1

Oscilloscope & Test Equipment Familiarization

1.1 Objectives

Review Bench Equipment Specifications, General Operation, and Test and Measurement Procedures as Identified in the Operator's Manual.

1. **Equipment Specifications:** Students should be able to review and understand the specifications of bench equipment, including details such as voltage ranges, current limits, frequency response, and other relevant parameters as outlined in the operator's manual.

2. **General Operation:** Gain knowledge of the general operation principles of bench equipment, understanding functions, controls, and interface features described in the operator's manual.

3. **Test and Measurement Procedures:** Familiarize themselves with the recommended test and measurement procedures provided in the operator's manual for various bench equipment. This includes understanding how to perform accurate measurements, set appropriate parameters, and interpret results.

By achieving this theory objective, students will develop a solid theoretical foundation regarding the specifications, operation, and testing procedures for bench equipment. This knowledge is crucial for the effective and accurate utilization of the equipment in practical applications.

1.2 Reference Documents:

Tektronix XYZs of Oscilloscopes PDF [1]

Tektronix TDS Oscilloscope User Manual PDF [2]

*Tektronix AFG1022 Arbitrary/Function Generator Quick Start User Manual*PDF [3]

*Power Supply GPS-4303 User Manual*PDF [4]

Tektronix - Get more from your basic oscilloscope with the FFT function [5]

1.3 Tektronix XYZs of Oscilloscopes Part I

Review the Tektronix XYZs of Oscilloscopes and answer the following section questions:

1.3.1 Match the following:

1. ____ Acquisition
2. ____ Analog
3. ____ Bandwidth
4. ____ Digital Phosphor
5. ____ Frequency
6. ____ Glitch
7. ____ Period
8. ____ Phase
9. ____ Pulse
10. ____ Waveform Point
11. ____ Rise Time
12. ____ Sample Point
13. ____ Digital Storage
14. ____ Time Base
15. ____ Transient
16. ____ ADC Resolution
17. ____ Volt

A The unit of electric potential difference.

B A performance measurement indicating the precision of an ADC, measured in bits.

C Term used when referring to degree points of a signal's period.

D The number of times a signal repeats in one second.

E The amount of time it takes a wave to complete one cycle.

F A stored digital value that represents the voltage of a signal at a specific point in time on the display.

G A common waveform shape that has a rising edge, a width, and a falling edge.

H A performance measurement indicating the rising edge speed of a pulse.

I Oscilloscope circuitry that controls the timing of the sweep.

J An intermittent spike in a circuit.

K A signal measured by an oscilloscope that only occurs once.

L The oscilloscope's process of collecting sample points from the ADC, processing them, and storing them in memory.

M Something that operates with continuously changing values.

N Digital oscilloscope that captures 3 dimensions of signal information in real-time.

O Digital oscilloscope with serial processing.

P A sine wave frequency range, defined by the -3dB point.

Q The raw data from an ADC used to calculate and display waveform points.

1.3.2 Multiple Choice:

1. With an oscilloscope you can:
   (a) Calculate the frequency of a signal.
   (b) Find malfunctioning electrical components.
   (c) Analyze signal details.
   (d) All the above.

2. The difference between analog and digitizing oscilloscopes is:
   (a) Analog oscilloscopes do not have on-screen menus.
   (b) Analog oscilloscopes apply a measurement voltage directly to the display system, while digital oscilloscopes first convert the voltage into digital values.
   (c) Analog oscilloscopes measure analogs, whereas digitized oscilloscopes measure digits.
   (d) Analog oscilloscopes do not have an acquisition system.

3. An oscilloscope's vertical section does the following:
   (a) Acquires sample points with an ADC.
   (b) Starts a horizontal sweep.
   (c) Lets you adjust the brightness of the display.
   (d) Attenuates or amplifies the input signal.

4. The time base control of the oscilloscope does the following:
   (a) Adjusts the vertical scale.
   (b) Shows you the current time of day.

(c) Sets the amount of time represented by the horizontal width of the screen.

(d) Sends a clock pulse to the probe.

5. On an oscilloscope display:

   (a) Voltage is on the vertical axis and time is on the horizontal axis.

   (b) A straight diagonal trace means voltage is changing at a steady rate.

   (c) A flat horizontal trace means voltage is constant.

   (d) All the above.

6. All repeating waves have the following properties:

   (a) A frequency measured in Hertz.

   (b) A period measured in seconds.

   (c) a bandwidth measured in Hertz.

   (d) All the above.

7. If you probe inside a computer with an oscilloscope, you are likely to find the following types of signals:

   (a) Pulse trains.

   (b) Ramp waves.

   (c) Sine waves.

   (d) All the above.

8. When evaluating the performance of an analog oscilloscope, some things you might consider are:

   (a) The bandwidth.

   (b) The vertical sensitivity.

   (c) The ADC resolution.

   (d) The sweep speed.

9. The difference between digital storage oscilloscopes (DSO) and digital phosphor oscilloscopes (DPO) is:

   (a) The DSO has a higher bandwidth.

   (b) The DPO captures three dimensions of waveform information in real-time.

   (c) The DSO has a color display.

   (d) The DSO captures more signal details.

1.4 Tektronix XYZs of Oscilloscopes Part II

Review the Tektronix XYZs of Oscilloscopes and answer the following section questions:

1.4.1 Match the following:

1. ____ Averaging Mode
2. ____ Circuit Loading
3. ____ Compensation
4. ____ Coupling
5. ____ Earth Ground
6. ____ Equivalent-Time
7. ____ Graticule
8. ____ Interpolation
9. ____ Real Time
10. ____ Signal Generator
11. ____ Single Sweep
12. ____ Sensor

A The unintentional interaction of the probe and oscilloscope with the circuit being tested which distorts a signal.

B A conductor that connects electrical currents to the Earth.

C A sampling mode in which the digital oscilloscope collects as many samples as it can as the signal occurs, then constructs a display, using interpolation if necessary.

D A sampling mode in which the digital oscilloscope constructs a picture of a repetitive signal by capturing a little bit of information from each repetition.

E A device that converts a specific physical quantity such as sound, pressure, strain, or light intensity into an electrical signal.

F A test device for injecting a signal into a circuit input.

G A processing technique used by digital oscilloscopes to eliminate noise in a displayed signal.

H The method of connecting two circuits together.

I A "connect-the-dots" processing technique to estimate what a fast waveform looks like based on only a few sampled points.

J The grid lines on a screen for measuring oscilloscope traces.

K A trigger mode that triggers the sweep once, must be reset to accept another trigger event.

L A probe adjustment for 10X attenuator probes that balances the electrical properties of the probe with the electrical properties of the oscilloscope.

1.4.2 Multiple Choice:

1. To operate an oscilloscope safely, you should:
   (a) Ground the oscilloscope with the proper three-pronged power cord.
   (b) Learn to recognize potentially dangerous electrical components.
   (c) Avoid touching exposed connections in a circuit being tested even if the power is off.
   (d) All the above.
2. Grounding an oscilloscope is necessary:
   (a) For safety reasons.
   (b) To provide a reference point for making measurements.
   (c) To align the trace with the screen's horizontal axis.
   (d) All the above.
3. Circuit loading is caused by:
   (a) An input signal having too large a voltage.
   (b) The probe and oscilloscope interacting with the circuit being tested.
   (c) a 10X attenuator probe being uncompensated.
   (d) Putting too much weight on a circuit.
4. Compensating a probe is necessary to:
   (a) Ballance the electrical properties of the 10X attenuator probe with the oscilloscope.
   (b) Prevent damaging the circuit being tested.
   (c) Improve the accuracy of your measurements.
   (d) All the above.
5. The trace rotation control is useful for:
   (a) Scaling waveforms on the screen.
   (b) Detecting sine wave signals.
   (c) Aligning the waveform trace with the screen's horizontal axis on an analog oscilloscope.
   (d) Measuring pulse width.

6. The volts per division control is used to:(select all that apply)

   ☐ Scale a waveform vertically.

   ☐ Position a waveform vertically.

   ☐ Attenuate or amplify an input signal.

   ☐ Set the number of volts each division represents.

7. Setting the vertical input coupling to ground does the following:

   ☐ Disconnects the input signal from the oscilloscope.

   ☐ Causes a horizontal line to appear with auto trigger.

   ☐ Lets you see where zero volts is on the screen.

   ☐ All the above.

8. The trigger is necessary to:

   ☐ Stabilize repeating waveforms on the screen.

   ☐ Capture single-shot waveforms.

   ☐ Mark a particular point of an acquisition.

   ☐ All the above.

9. The difference between auto and normal trigger mode is:

   ☐ In normal mode the oscilloscope only sweeps once and then stops.

   ☐ In normal mode the oscilloscope only sweeps if the input signal reaches the trigger point; otherwise the screen is blank.

   ☐ Auto mode makes the oscilloscope sweep continuously even without being triggered.

   ☐ All the above.

10. The acquisition mode that best reduces noise in a repeating signal is:

    ☐ Sample mode.

    ☐ Peak detect mode.

    ☐ Envelope mode.

    ☐ Averaging mode.

11. The two most basic measurements you can make with an oscilloscope are:

    ☐ Time and frequency measurements.
    ☐ Time and voltage measurements.
    ☐ Voltage and pulse width measurements.
    ☐ Pulse width and phase shift measurements.

12. If the volts/division is set at 0.5, the largest signal that can fit on the screen (assuming an 8 x 10 division screen) is:

    ☐ 62.5 millivolts peak-to-peak.
    ☐ 8 volts peak-to-peak.
    ☐ 4 volts peak-to-peak.
    ☐ 0.5 volts peak-to-peak.

13. If the seconds/division is set at 0.1 ms, the amount of time represented by the width of the screen is:

    ☐ 0.1 ms.
    ☐ 1 ms.
    ☐ 1 second.
    ☐ 0.1 kHz.

14. By convention, pulse width is measured:

    ☐ At 10% of the pulse's peak-to-peak (pk-pk) voltage.
    ☐ At 50% of the pulse's peak-to-peak (pk-pk) voltage.
    ☐ At 90% of the pulse's peak-to-peak (pk-pk) voltage.
    ☐ At 10% and 90% of the pulse's peak-to-peak (pk-pk) voltage.

15. You attach a probe to your test circuit but the screen is blank. You should:

    ☐ Check that the screen intensity is turned up.
    ☐ Check that the oscilloscope is set to display the channel that the probe is connected to.
    ☐ Set the trigger mode to auto since norm mode blanks the screen.
    ☐ Set the vertical input coupling to AC and set the volts/division to its largest value since a large DC signal may go off the top or bottom of the screen.
    ☐ Check that the probe isn't shorted and make sure it is properly grounded.
    ☐ Check that the oscilloscope is set to trigger on the input channel you are using.
    ☐ All of the above.

1.5 Probe Compensation

Review the Tektronix XYZs of Oscilloscopes and answer the following section questions:

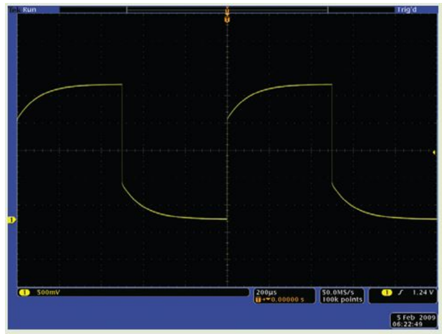

Figure 1.1: Probe Adjustment Signal

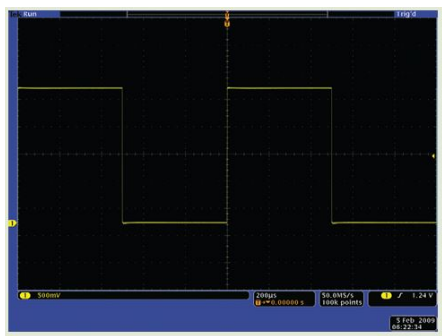

Figure 1.2: Probe Adjustment Signal

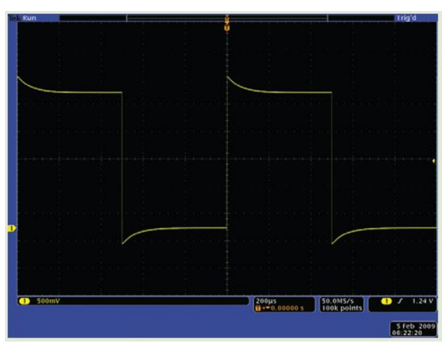

Figure 1.3: Probe Adjustment Signal

1. The Probe Adjustment Signal as seen in **Figure 1.1** represents a:
   - ☐ under-compensated probe.
   - ☐ properly compensated probe.
   - ☐ over-compensated probe.
2. The Probe Adjustment Signal as seen in **Figure 1.2** represents a:
   - ☐ under-compensated probe.
   - ☐ properly compensated probe.
   - ☐ over-compensated probe.
3. The Probe Adjustment Signal as seen in **Figure 1.3** represents a:
   - ☐ under-compensated probe.
   - ☐ properly compensated probe.
   - ☐ over-compensated probe.

1.6 AFG1022

Review the Tektronix AFG1022 Quick Start User Manual and answer the following section questions:

1. The AFG1022 can produce which of the following waveforms:

   ☐ Sine

   ☐ Square

   ☐ Ramp

   ☐ Pulse

   ☐ Noise

2. The Output impedance of the AFG1022 is ____________ .

   ☐ 50Ω

   ☐ 600Ω

   ☐ 1KΩ

   ☐ 1MΩ

   ☐ 10MΩ

3. Push the front-panel ____________ button to control the screen display. You can toggle between the two channels.

   ☐ On/Off

   ☐ Both

   ☐ Ch1

   ☐ Ch2

   ☐ Ch1/2

4. To enable CH1 signal output, push the front-panel On/Off with ____________ color.

   ☐ Red

   ☐ Green

   ☐ Blue

   ☐ Yellow

5. To enable CH2 signal output, push the front-panel On/Off with ____________ color.

   - ☐ Red
   - ☐ Green
   - ☐ Blue
   - ☐ Yellow

6. The Sweep outputs a waveform with the output signal frequency varying ____________ or ____________ .

   - ☐ continuously, non-continuously
   - ☐ linearly, logarithmically
   - ☐ synchronized, non-synchronized
   - ☐ sweep, digital pulse sweep

7. According to the AFG1022 Quick Start User Manual, What fuse should be used?

   - ☐ 250,F0.5AL
   - ☐ 250,F1AL
   - ☐ 250,F1.5AL
   - ☐ 250,F2AL

Week 2

RC Circuits

2.1 Objectives:

Understand and Identify Different Types and Characteristics of Waveforms.

- Students should be able to identify and understand different types of waveforms, including periodic and aperiod waveforms.

Understand Pulse Waveform Terminology and Formulas.

- Develop a solid understanding of pulse waveform terminology, including Period, Pulse Width, Pulse Space, Pulse Repetition Frequency, Duty Cycle, Tilt, Rise Time, Average Pulse Amplitude, Average Waveform Voltage, Capacitor Charge, Cycles to Stabilization, V_{Max}, and V_{Min}.

Understand Frequency Synthesis and the Process of Combining Sine Waves.

- Gain insight into the process of combining sine waves to produce complex waveforms. Understand the principles of frequency synthesis and its application.

Calculate Critical Frequencies using Tilt and Rise Time.

- Develop the ability to calculate critical frequencies using tilt and rise time measurements. Understand how series and parallel capacitance influence these parameters and the critical frequencies.

Identify Capacitor Charge Percentage in Terms of Time and τ.

- Students should be able to identify and calculate capacitor charge percentages concerning time and the time constant (τ) in RC circuits.

Calculate and Predict Waveforms for RC Integration Circuits

- Develop a comprehensive understanding of RC Integration circuits. Gain proficiency in calculating and predicting output waveforms for these circuits.

Calculate and Predict Waveforms for RC Differentiation Circuits

- Develop a comprehensive understanding of RC Differentiation circuits. Gain proficiency in calculating and predicting output waveforms for these circuits.

By achieving these objectives, students will acquire a deep understanding of waveform characteristics, pulse waveform terminology, frequency synthesis, critical frequencies, capacitor charge calculations, and the principles behind RC integration, differentiation, and sine wave analysis. These objectives aim to enhance their knowledge and proficiency in working with various waveforms and circuits.

2.2 References:

- Solid State Pulse Circuits [6]

2.3 Graphs and Waveforms

2.3.1 What is a Graph?

- **Horizontal Axis (X-Axis):** This axis runs horizontally from left to right and is usually used to represent the independent variable or input.
- **Vertical Axis (Y-Axis):** This axis runs vertically from bottom to top and is usually used to represent the dependent variable or output.
- The point where the two axes intersect is called the **Origin**. The coordinates of the origin are typically represented as (0,0). The horizontal and vertical distances from the origin to any point on the graph are called the x-coordinate and y-coordinate, respectively. The combination of these coordinates uniquely identifies a point in the coordinate system. This system is fundamental in graphing and visualizing mathematical functions and relationships.

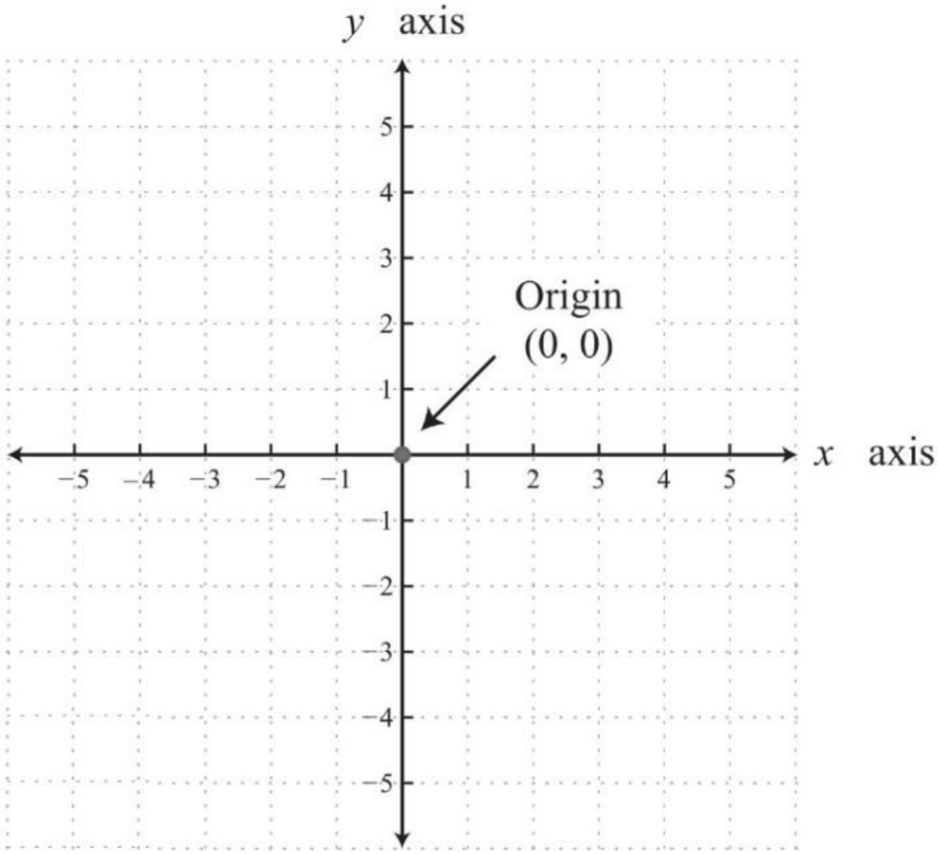

Figure 2.1: Graph

2.3.2 Periodic Waveforms

A **periodic waveform** is a type of waveform that repeats its shape over regular intervals of time. In other words, the waveform exhibits a regular and predictable pattern, and the pattern repeats itself after a specific period. The time it takes for one complete cycle to occur is called the period.

Key characteristics of periodic waveforms include:

1. **Frequency (f):** The frequency of a periodic waveform is the number of cycles that occur in one second. It is the reciprocal of the period and is measured in Hertz (Hz).

$$Frequency = \frac{1}{Time}$$

2. **Amplitude:** The amplitude of a periodic waveform represents the maximum displacement from the equilibrium position. It is a measure of the strength or intensity of the waveform. Amplitude is typically measured in *volts peak* or *volts peak to peak* using an oscilloscope.

3. **Phase:** The phase of a periodic waveform indicates the relative position of the waveform at a specific point in time within its cycle. It is often measured in degrees or radians.

Common examples of periodic waveforms include:

- **Sine Wave:** A smooth, oscillating waveform characterized by its sinusoidal shape.
- **Square Wave:** A waveform that alternates between two constant levels, resembling a square shape.
- **Triangular Wave:** A waveform that linearly increases and decreases in amplitude, forming a triangular shape.
- **Ramp Wave:** A waveform characterized by a linear increase or decrease in amplitude over time.
- **Sawtooth Wave:** A waveform that rises linearly and then falls abruptly, creating a sawtooth-like pattern.
- **Exponential Wave:** A waveform in which the amplitude changes exponentially over time.
- **Spike Wave:** A waveform characterized by a sudden, brief increase in amplitude, often appearing as a sharp spike in the signal.

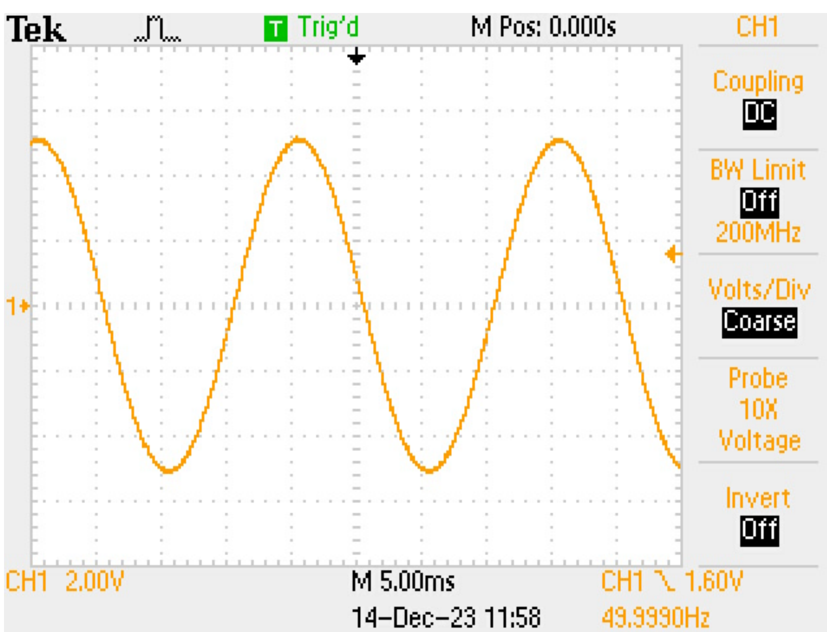

Figure 2.2: Sine Wave

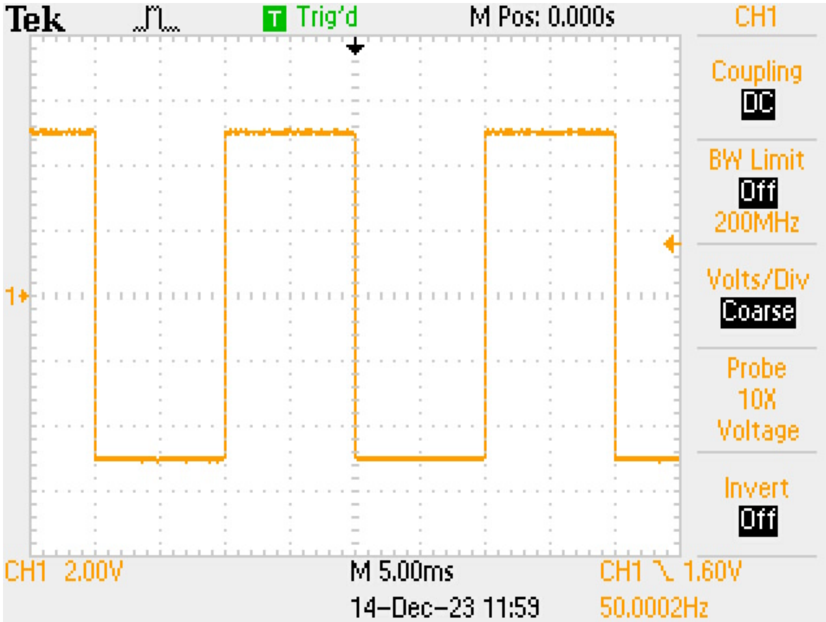

Figure 2.3: Square Wave

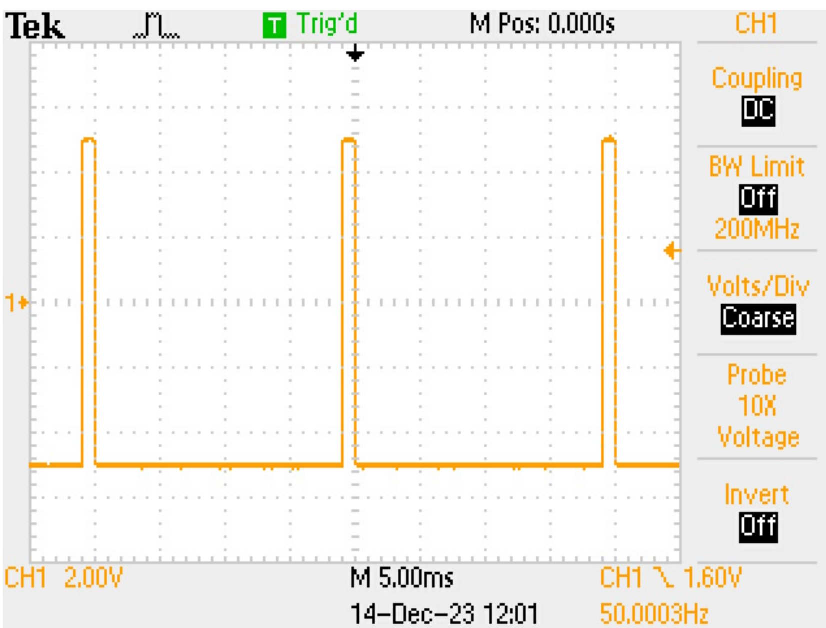

Figure 2.4: Pulse Wave

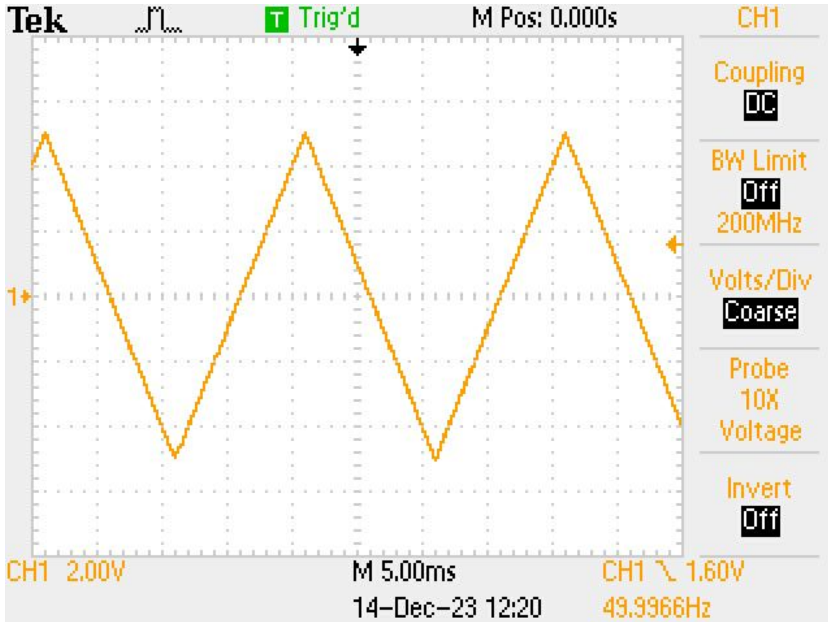

Figure 2.5: Triangle Wave

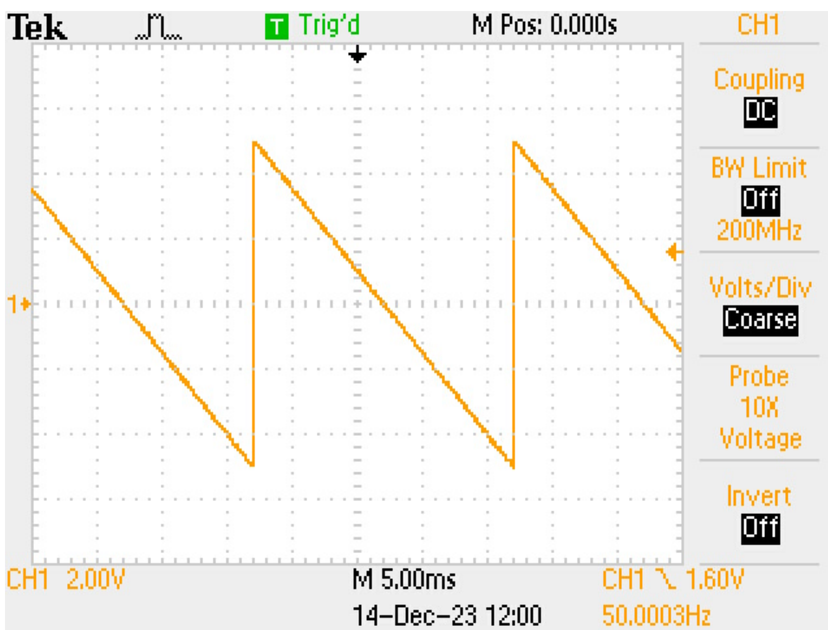

Figure 2.6: Sawtooth Wave

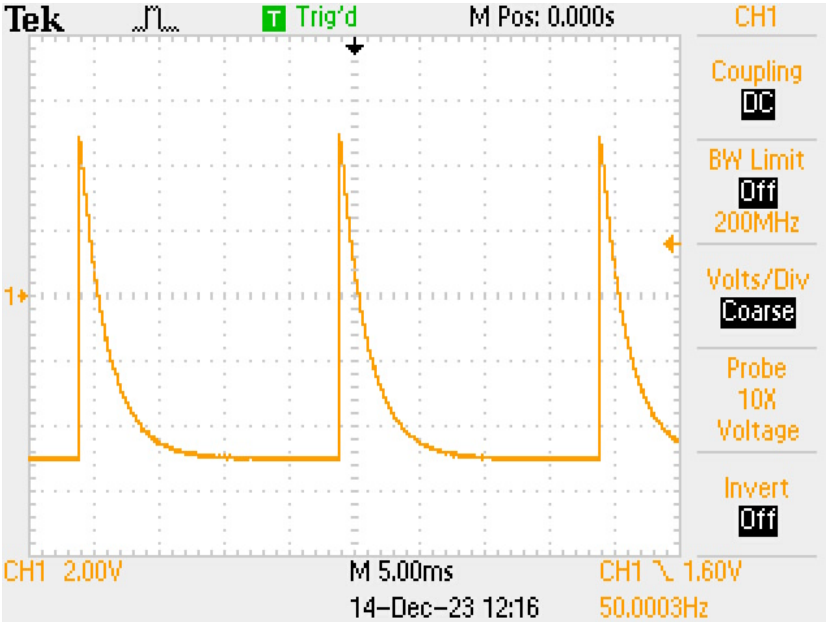

Figure 2.7: Exponential Spike Wave

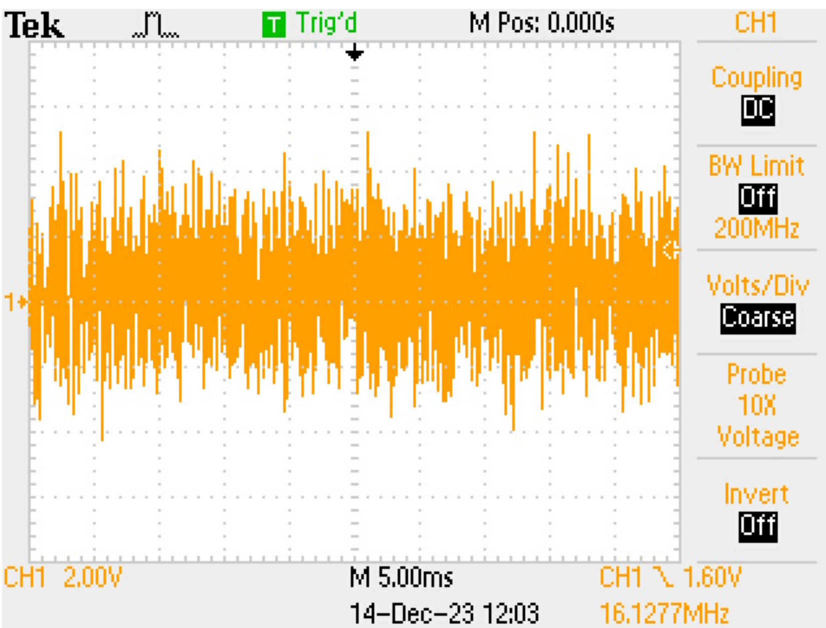

Figure 2.8: Noise Wave

2.3.3 Aperiodic Waveforms

Aperiodic waveforms, in contrast to periodic waveforms, do not exhibit a regular and repeating pattern over time. These waveforms lack a well-defined period, and their shapes do not recur in a predictable manner. Aperiodic waveforms are often associated with transient or non-repetitive signals.

Examples and characteristics:

1. **Impulse or Spike:** A waveform characterized by an instantaneous, brief increase in amplitude, representing an abrupt change.
2. **Step Function:** A waveform that changes abruptly from one constant level to another, creating a step-like pattern.
3. **Random Noise:** A waveform characterized by random variations in amplitude over time, without a discernible pattern.
4. **Exponential Decay:** A waveform where the amplitude decreases exponentially over time.
5. **Chirp Signal:** A waveform with a continuously changing frequency, often used in radar and communication systems.
6. **Pulse Train:** A series of individual pulses, where the intervals between pulses may not be constant.

2.4 Pulse Waveform Characteristics

2.4.1 Ideal Pulse Waveform

The **Ideal Pulse Waveform** has perfectly vertical leading and lagging edges (instantaneous rise and fall times) and perfectly flat tops and bottoms.

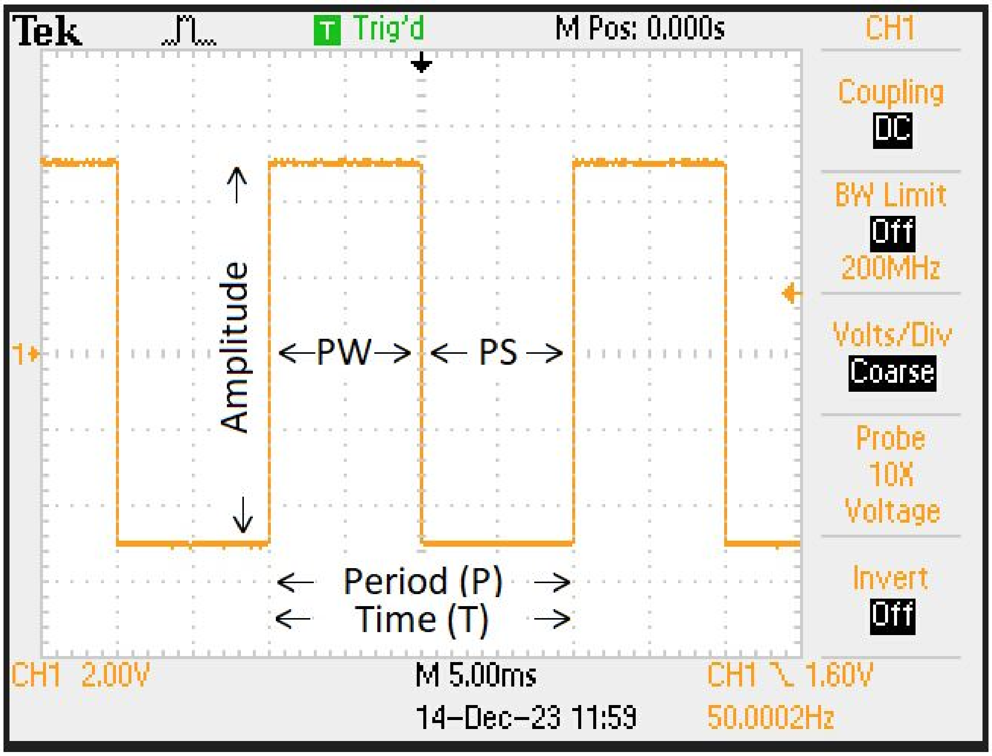

Figure 2.9: Ideal Pulse Waveform

2.4.2 Pulse Waveform Terminology:

- **Period (P):** The time it takes for one complete cycle, from one rising edge to the next rising edge.
- **Pulse Width (PW):** Also known as Time High/On, it is the duration of time during which the waveform is at its maximum amplitude. Typically measured at 50% of the waveform amplitude.
- **Pulse Space (PS):** Also known as Time Low/Off, it is the duration of time during which the waveform is at its minimum amplitude. Also measured at 50% of the waveform amplitude.
- **Pulse Repetition Frequency (PRF):** The reciprocal of the period, representing the number of pulses per unit time.

$$PRF = \frac{1}{Period}$$

- **Duty Cycle (DC%):** The ratio of the pulse width to the period, expressed as a percentage.

$$DC\% = \frac{PW}{Period} \times 100$$

2.4.3 Practical Pulse Waveform

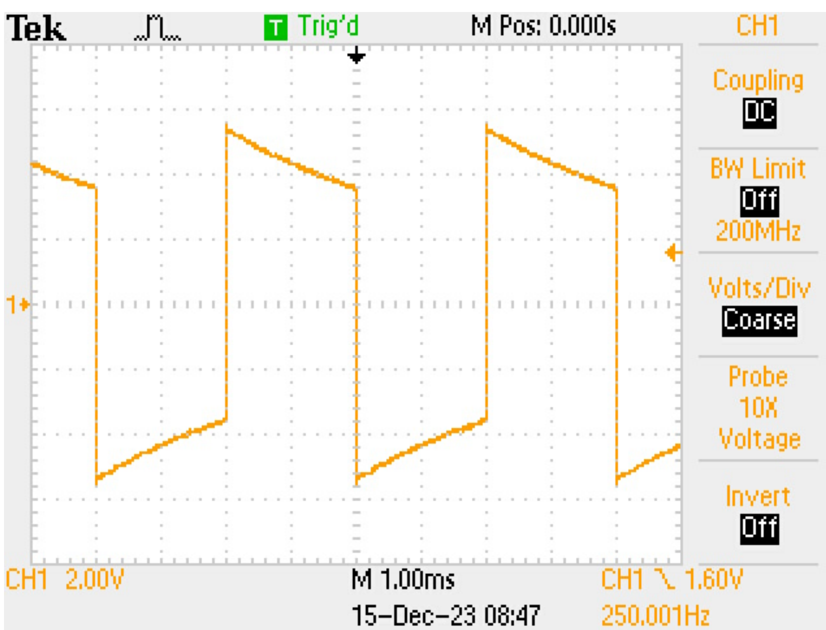

Figure 2.10: Practical Pulse Waveform

The RC circuit practical pulse waveform as seen in Fig 2.10 at 250Hz is outputting a square wave with significant tilt. Tilt represents circuit low-frequency attenuation.

2.4.4 Vmax & Vmin

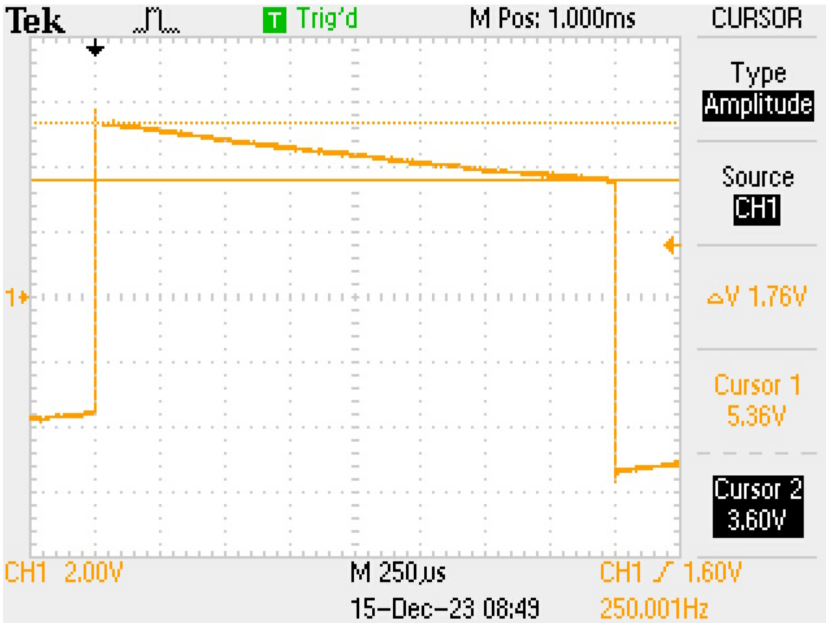

Figure 2.11: Measuring Vmax & Vmin on a Tilt waveform

Defined

- **Vmax** is the highest point of the positive peak.
- **Vmin** is the lowest point of the positive peak.

Measurement Steps

1. Lower the square-wave test signal frequency until tilt is observable.
2. Expand the waveform across the display of the oscilloscope using the horizontal control, see Fig 2.11.
3. Use the cursors to measure Vmax and Vmin of the pulse, see Fig 2.11. Cursor 1 (Vmax) is 5.56V and Cursor 2 (Vmin) is 3.6V.

2.4.5 Rise & Fall Time

Defined

- Rise Time (t_r) is defined as the time required for the voltage to go from 10% to 90% of the average pulse amplitude (APA).

  $APA = \frac{Vmax+Vmin}{2}$

- Fall Time (t_f is defined as the time required for the voltage to go from 90% to 10% of the average pulse amplitude (APA).

  $APA = \frac{Vmax+Vmin}{2}$

Measurements Steps

1. Increase the square-wave test signal frequency until the tilt is mostly gone, see Fig 2.12.
2. Use the **Fine** Volts/Div to adjust the signal to have exactly 5 major divisions peak to peak, see Fig 2.12.
3. Center the waveform vertically, trigger on the rising edge, and set the trigger point on the y-axis.
4. Zoom in on the rising edge of the waveform, see Fig 2.13
5. Measure the rise time at the -2 divisions (10%) and +2 divisions (90%), see Fig 2.13.

   $t_r = 410nS$

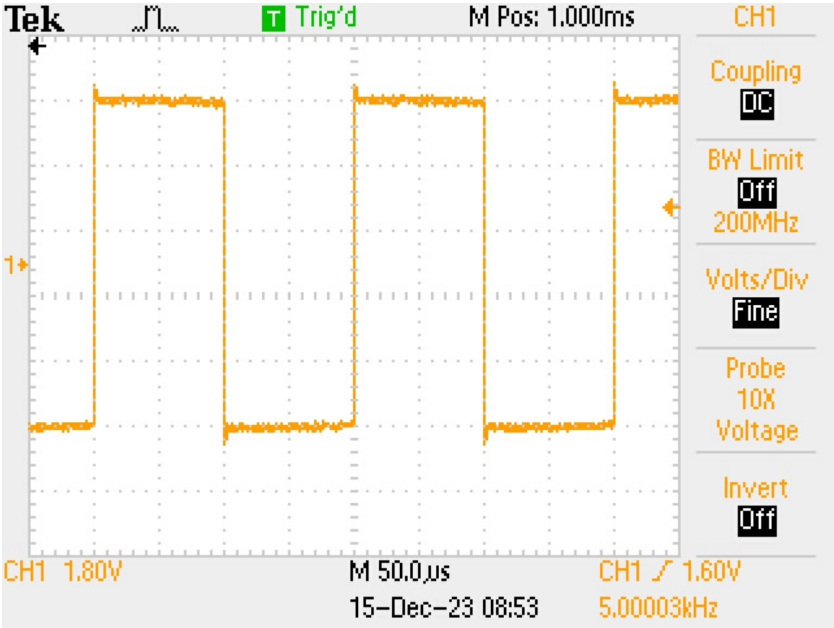

Figure 2.12: Rise Time Set-up Waveform

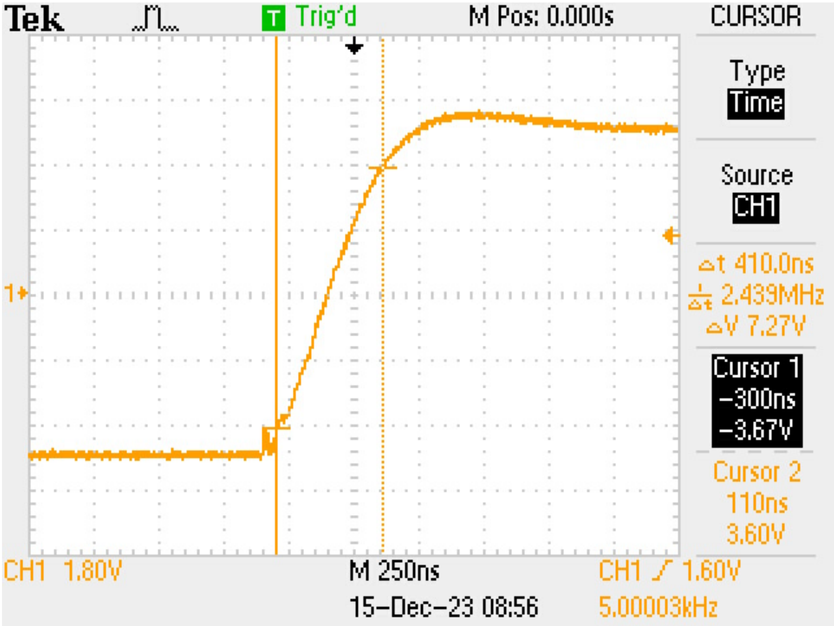

Figure 2.13: Rise Time Measurement

2.4.6 APA (Average Pulse Amplitude) formula

$$APA = \frac{Vmax + Vmin}{2}$$

2.4.7 Tilt formulas

$$Tilt = \frac{Vmax - Vmin}{APA}$$

$$Tilt\% = \frac{Vmax - Vmin}{APA} \times 100$$

2.4.8 AWV (Average Waveform Voltage)

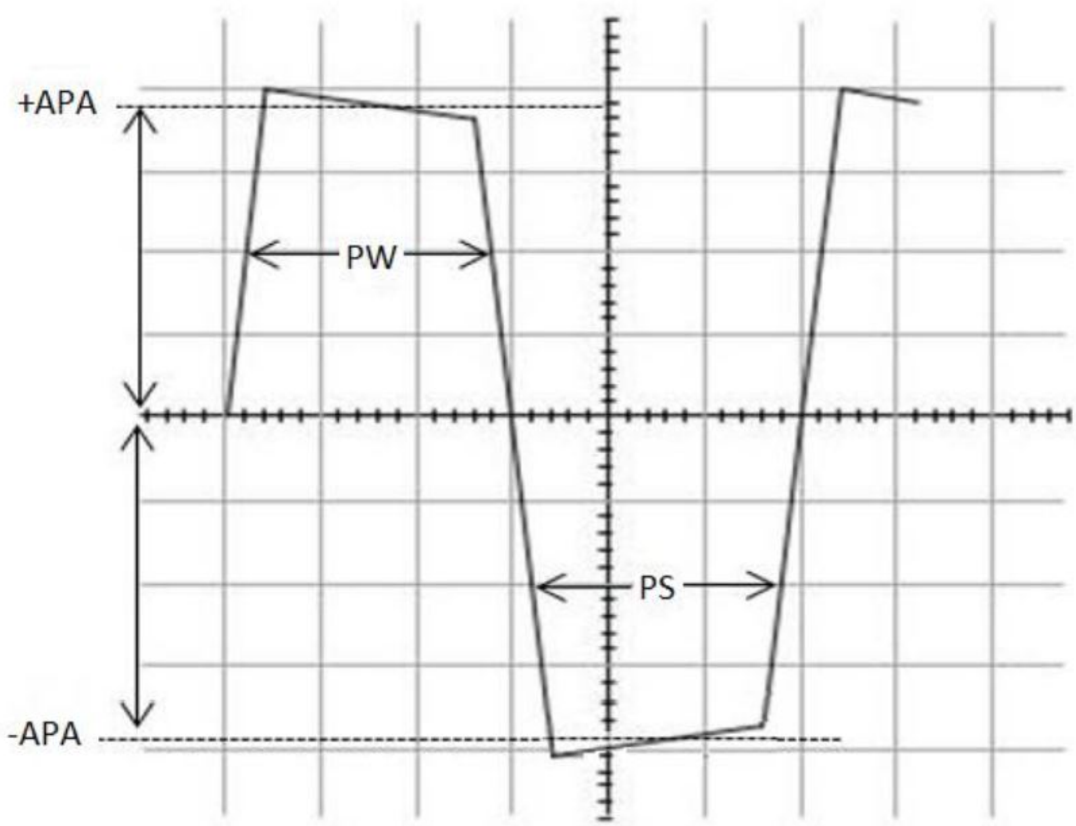

Figure 2.14: AWV (Average Waveform Voltage)

AWV formula

$$AWV = \frac{(+APA \times PW) + (-APA \times PS)}{Period}$$

AWV explained

If the positive peak and negative peak are equal in amplitude, and the pulse width and pulse space are equal, then the waveform is symmetric, and the average voltage is indeed zero. This is because, over a complete cycle, the positive and negative voltages cancel each other out.

However, if the positive peak and negative peak amplitudes are different, or if the pulse width and pulse space are different, the waveform is no longer symmetric, and the average voltage will not be zero. In this case, you would use the above formula to calculate the average voltage.

AWV examples

1. AWV Practice Problem

   Given:

   - $+APA = 8V$
   - $-APA = -10V$
   - $PW\&PS = 50\mu S$

$$AWV = \frac{(+APA \times PW) + (-APA \times PS)}{Period}$$

$$AWV = \frac{(8V \times 50\mu S) + (-10V \times 50\mu S)}{100\mu S}$$

$$AWV = \frac{400 \times 10^{-6} + -500 \times 10^{-6}}{100 \times 10^{-6}}$$

$$AWV = \frac{-100 \times 10^{-6}}{100 \times 10^{-6}}$$

$$AWV = -1V$$

2. AWV Practice Problem

   Given:

   - $+APA = 8V$
   - $-APA = -10V$
   - $PW = 30\mu S$
   - $PS = 10\mu S$

$$AWV = \frac{(+APA \times PW) + (-APA \times PS)}{Period}$$

$$AWV = \frac{(8V \times 30\mu S) + (-10V \times 10\mu S)}{40\mu S}$$

$$AWV = \frac{240 \times 10^{-6} + -100 \times 10^{-6}}{40 \times 10^{-6}}$$

$$AWV = \frac{140 \times 10^{-6}}{40 \times 10^{-6}}$$

$$AWV = 3.5V$$

2.5 Frequency Synthesis & Analysis

2.5.1 Frequency Synthesis

Frequency synthesis involves combining two or more signals to create a new waveform. Just like an AC signal will superimpose on a DC signal, two AC signals present at the same time will also combine to form a new waveform. When this occurs it is referred to as Frequency Synthesis.

> **synthesis** noun: the composition or combination of parts or elements so as to form a whole. Merriam Webster
>
> **Frequency Synthesis** is the process of combining multiple sine waves to produce a new desired waveform.
>
> **Harmonics**
>
> ✓ A harmonic is a multiple of the fundamental.
>
> ✓ Harmonics are numbered according to their ratio to the fundamental.
>
> ✓ The number of harmonics is infinite; however, the amplitude of each harmonic will successively decrease as frequency increases.

Table 2.1: Sinusoidal Harmonics

Harmonic Number	Frequency	Amplitude
Fundamental	F	V
2^{nd} harmonic	2F	$\frac{V}{2}$
3^{rd} harmonic	3F	$\frac{V}{3}$
4^{th} harmonic	4F	$\frac{V}{4}$
5^{th} harmonic	5F	$\frac{V}{5}$
6^{th} harmonic	6F	$\frac{V}{6}$
7^{th} harmonic	7F	$\frac{V}{7}$
100^{th} harmonic	100F	$\frac{V}{100}$

2.5.2 Perfect Square Waves

A **Perfect Square Wave** is comprised of an infinite number of odd harmonic sine waves.

Sawtooth, Exponential, and Triangle Waveforms are comprised of a combination of odd and even harmonic sine waves.

Can a Square Wave really be created using Sine Wave Frequency Synthesis?

To test this theory we can use Desmos Graphing Calculator [7]. Link to final graph https://www.desmos.com/calculator/bbjnjmrxyz.

Sine-wave formulas:

$$Fundamental = \frac{4}{\pi}\frac{1}{1}\sin(1\pi x)$$

$$3^{rd} harmonic = \frac{4}{\pi}\frac{1}{3}\sin(3\pi x)$$

$$5^{th} harmonic = \frac{4}{\pi}\frac{1}{5}\sin(5\pi x)$$

$$7^{th} harmonic = \frac{4}{\pi}\frac{1}{7}\sin(7\pi x)$$

$$9^{th} harmonic = \frac{4}{\pi}\frac{1}{9}\sin(9\pi x)$$

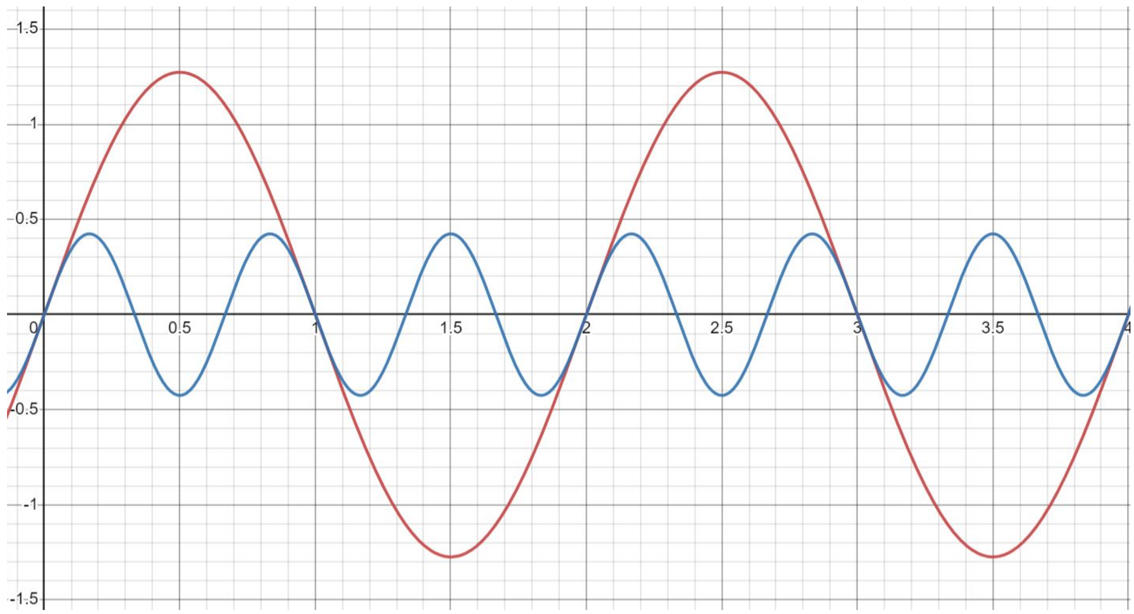

Figure 2.15: Fundamental & Third Harmonic

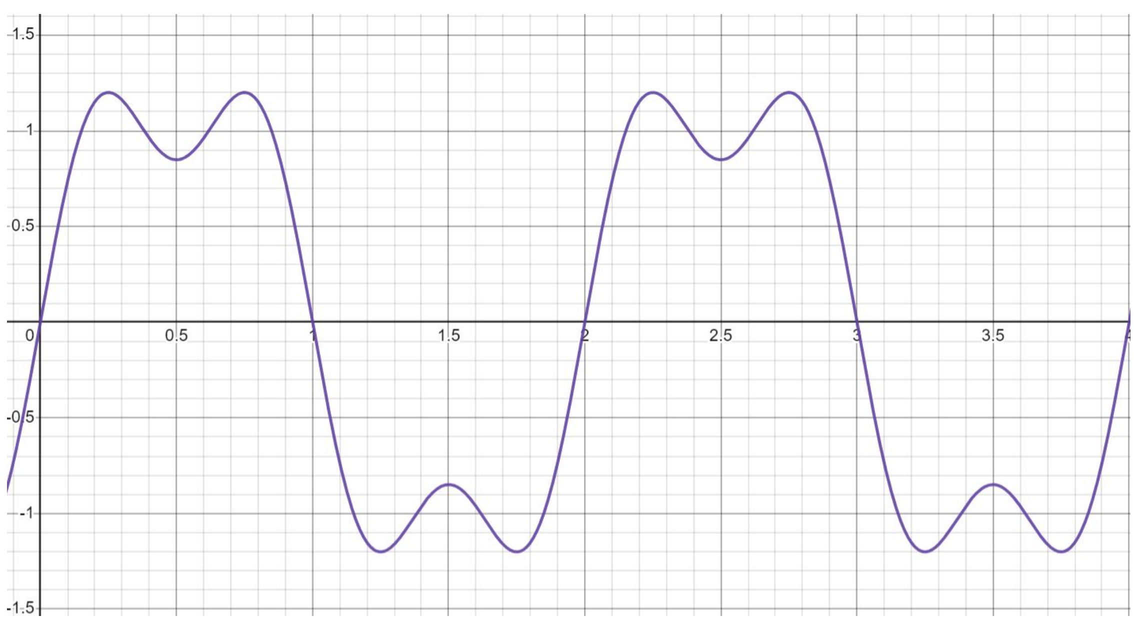

Figure 2.16: Fundamental & Third Harmonic Synthesis

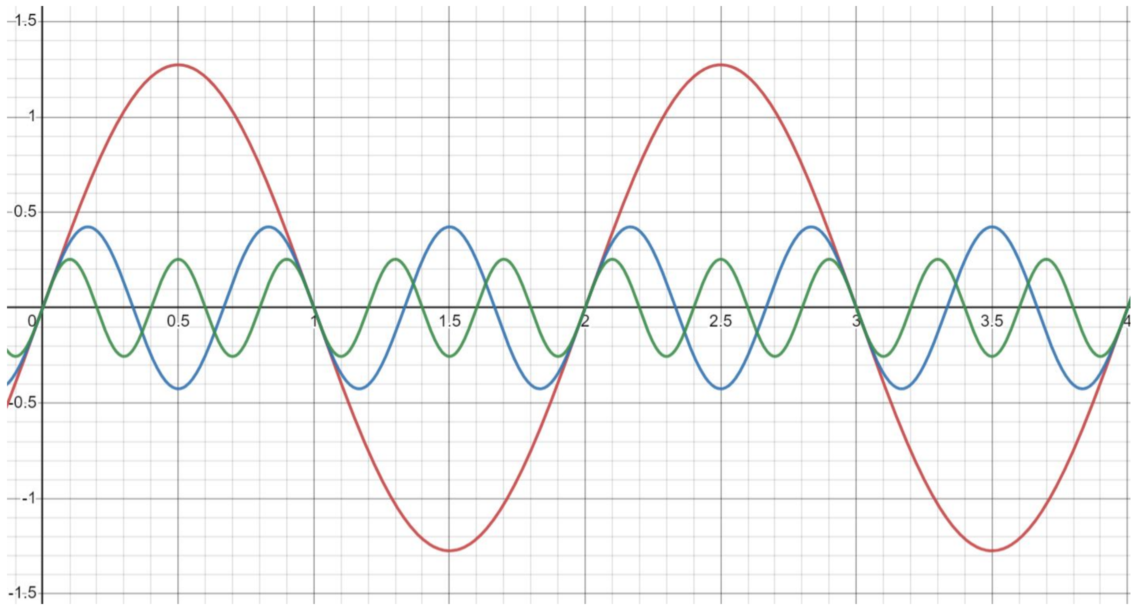

Figure 2.17: Fundamental, Third & Fifth Harmonics

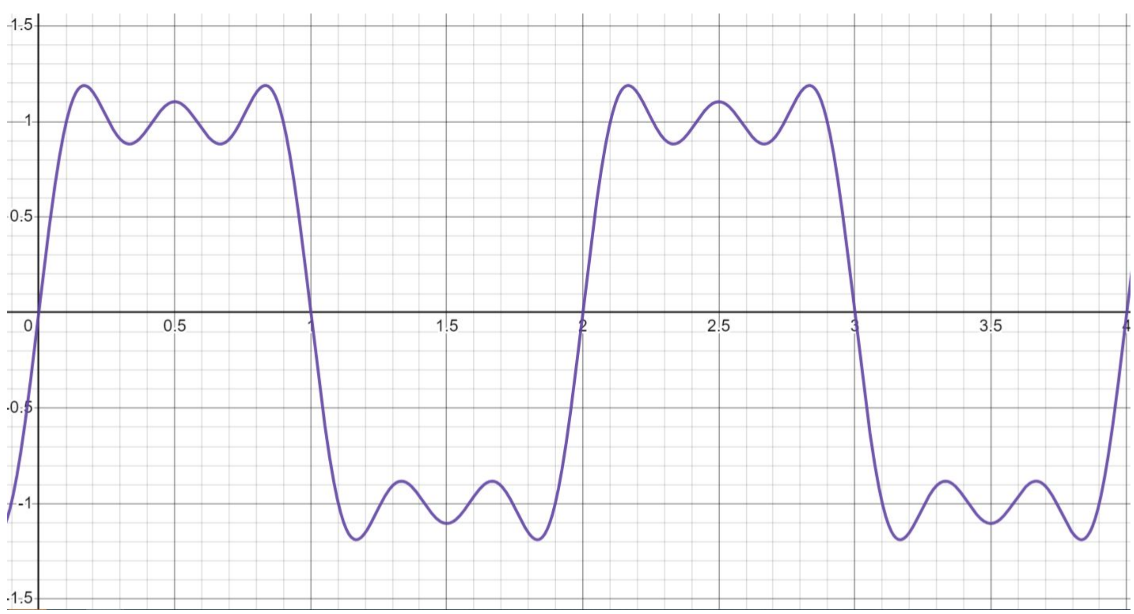

Figure 2.18: Fundamental, Third & Fifth Harmonics Synthesis

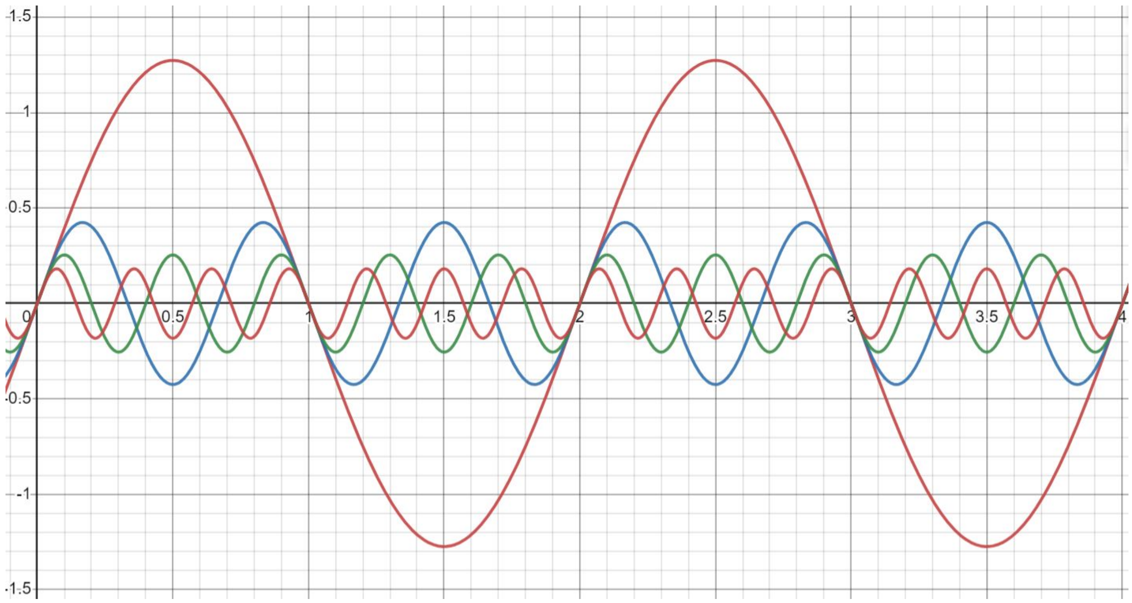

Figure 2.19: Fundamental, Third, Fifth, & Seventh Harmonics

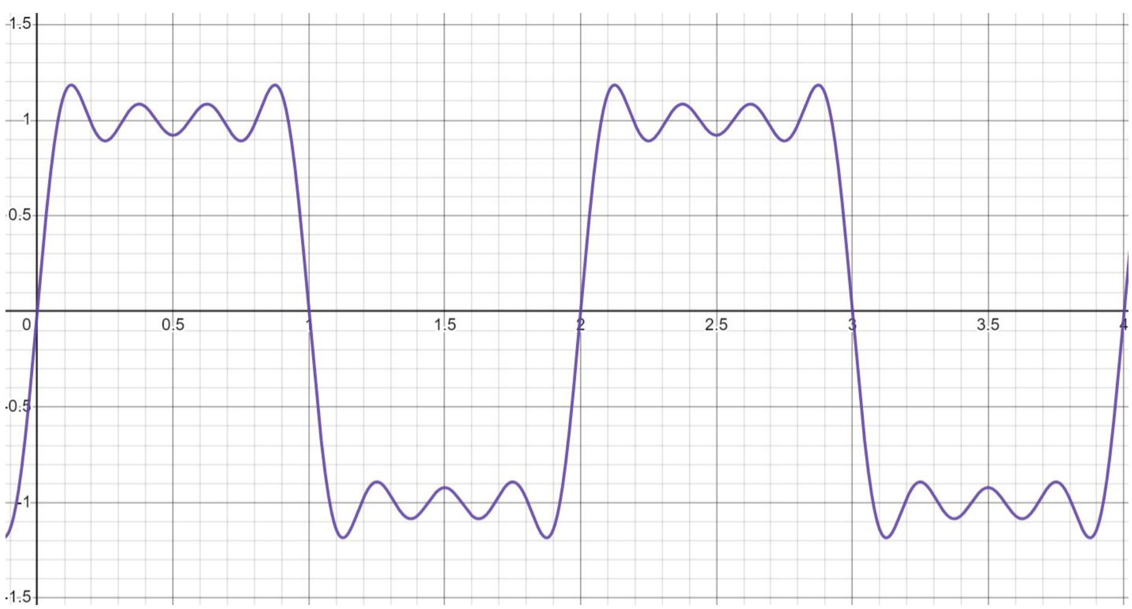

Figure 2.20: Fundamental, Third, Fifth, & Seventh Harmonics Synthesis

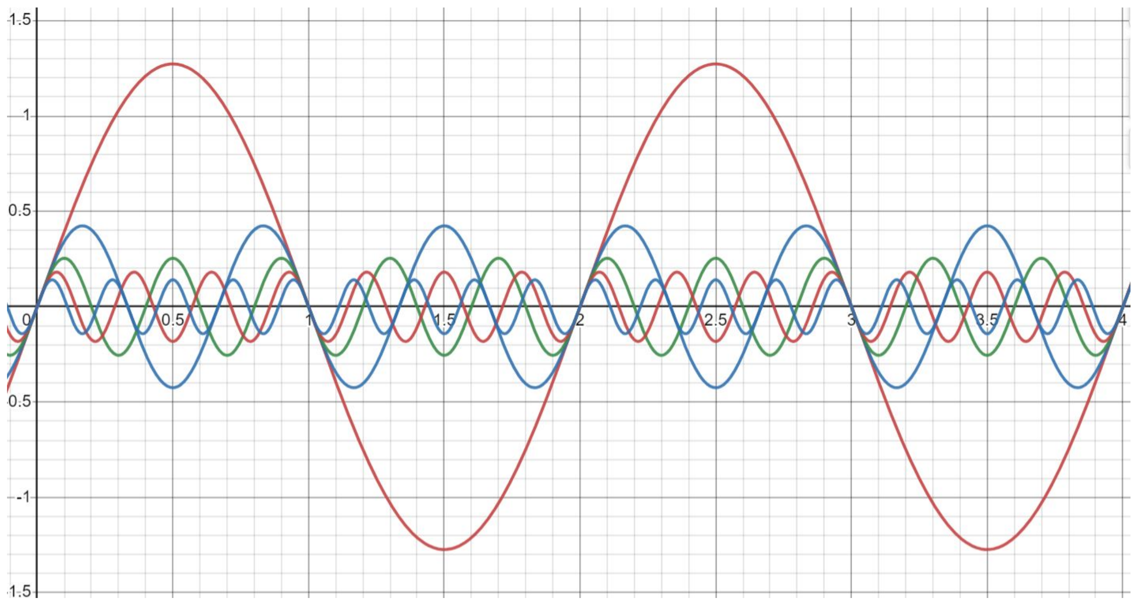

Figure 2.21: Fundamental, Third, Fifth, Seventh, & Ninth Harmonics

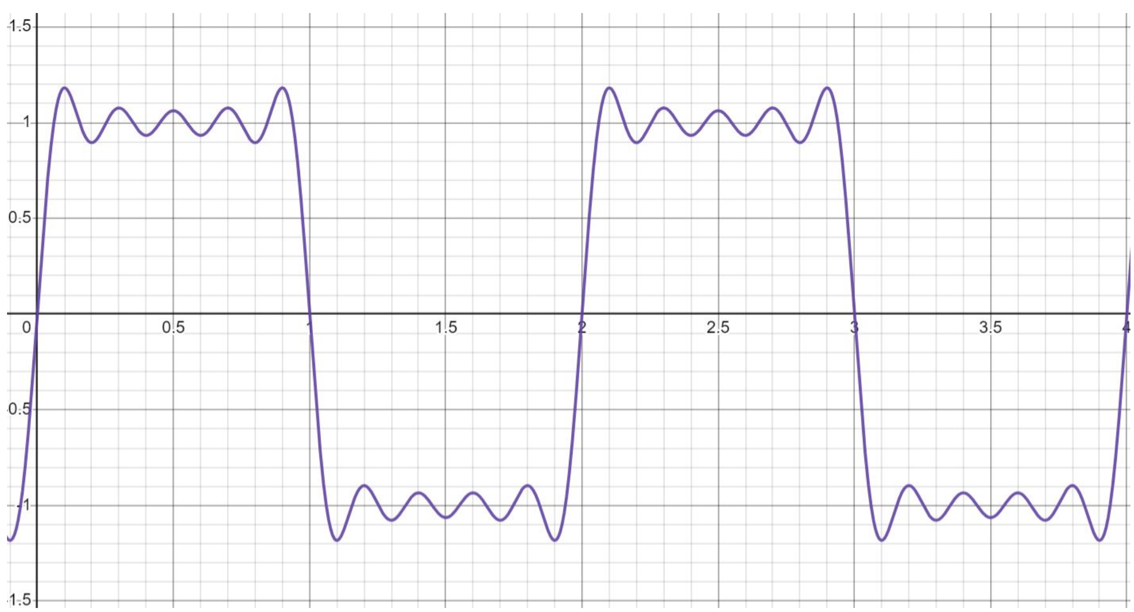

Figure 2.22: Fundamental, Third, Fifth, Seventh, & Ninth Harmonics Synthesis

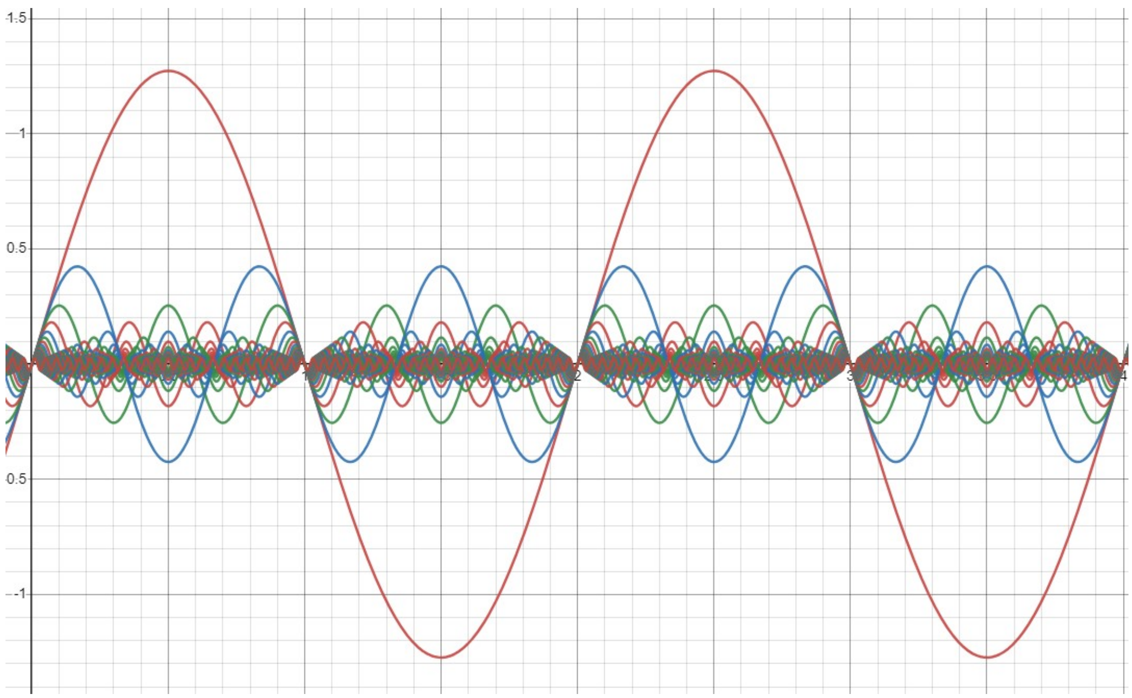

Figure 2.23: Fundamental with odd harmonics up to harmonic 49

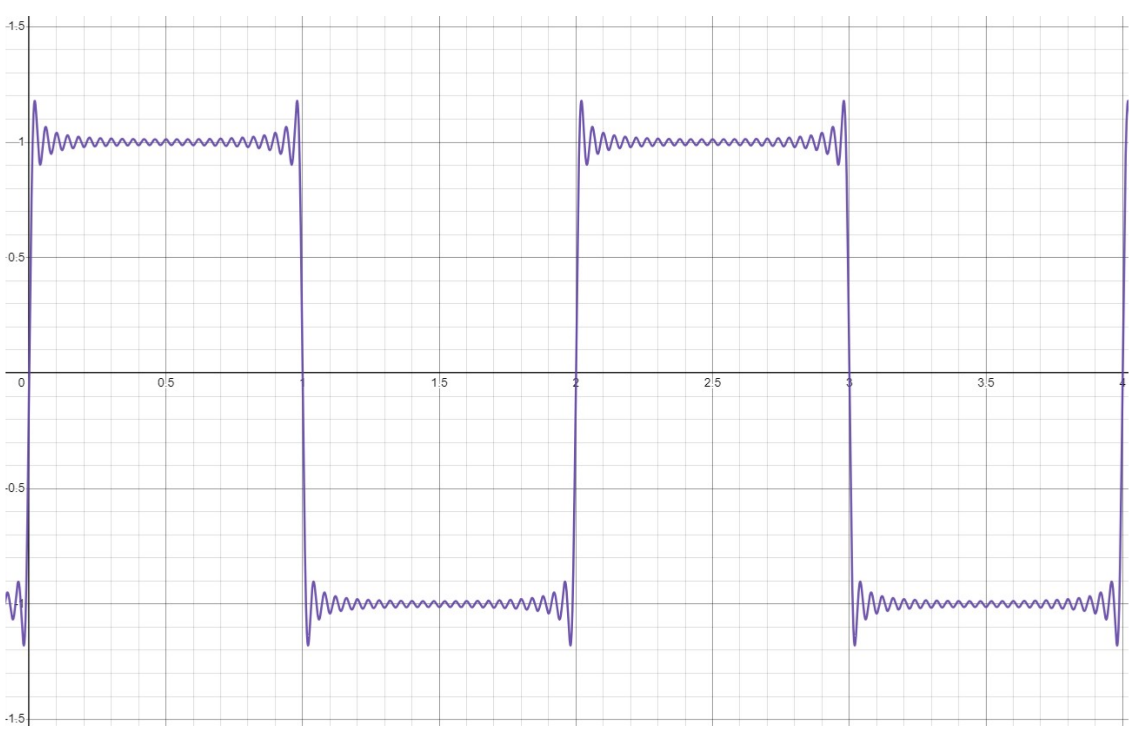

Figure 2.24: Fundamental with odd harmonics up to harmonic 49 Synthesis

2.5.3 Harmonic Analysis

Harmonic Analysis could be thought of as the opposite of, or inversely related to, Frequency Synthesis.

Harmonic Analysis involves breaking down a complex waveform into its individual sinusoidal components or harmonics and is typically achieved through techniques like Fourier Analysis.

Fourier Analysis is a mathematical technique used to decompose a complex waveform into its individual sinusoidal components, representing different frequencies.

2.5.4 Fast Fourier Transformation FFT

The "Fast Fourier Transform" (FFT) is an important measurement method in the science of audio and acoustics measurement. It converts a signal into individual spectral components and thereby provides frequency information about the signal. FFTs are used for fault analysis, quality control, and condition monitoring of machines or systems. NTI Audio [8]

Strictly speaking, the FFT is an optimized algorithm for the implementation of the "Discrete Fourier Transformation" (DFT). A signal is sampled over a period of time and divided into its frequency components. These components are single sinusoidal oscillations at distinct frequencies each with their own amplitude and phase. This transformation is illustrated in figure 2.25 FFT Time, Frequency, Amplitude - Image from NTI Audio [8]. Over the time period measured, the signal contains 3 distinct dominant frequencies. NTI Audio [8].

There are a variety of uses that can benefit from viewing the frequency spectrum of a signal. Using the FFT math function on a time domain signal provides the user with frequency domain information and can provide the user a different view of the signal quality, resulting in improved measurement productivity when troubleshooting a device-under-test. Tektronix[9]

Examples include:

- Analyze harmonics in power lines
- Measure harmonic content and distortion in systems
- Characterize noise in DC power supplies
- Test impulse response of filters and systems
- Analyze vibration Tektronix[9]

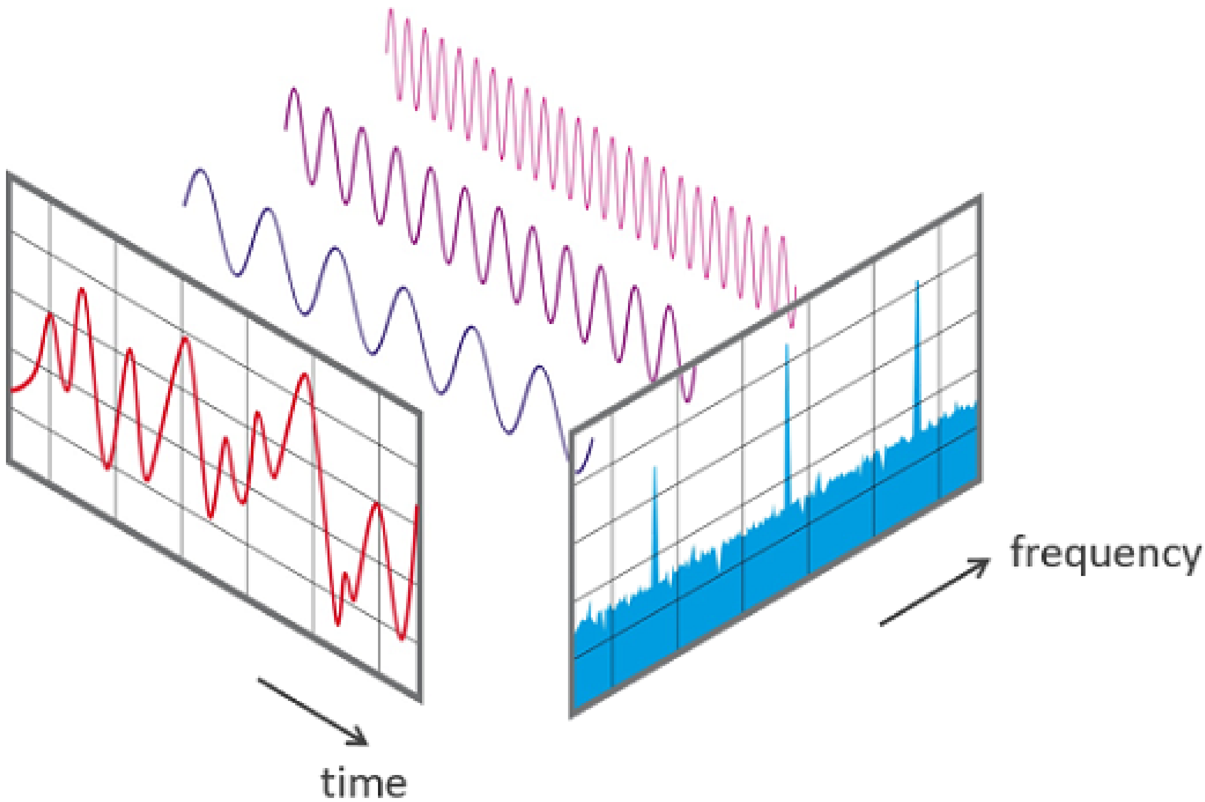

Figure 2.25: FFT Time, Frequency, Amplitude - Image from NTI Audio [8]

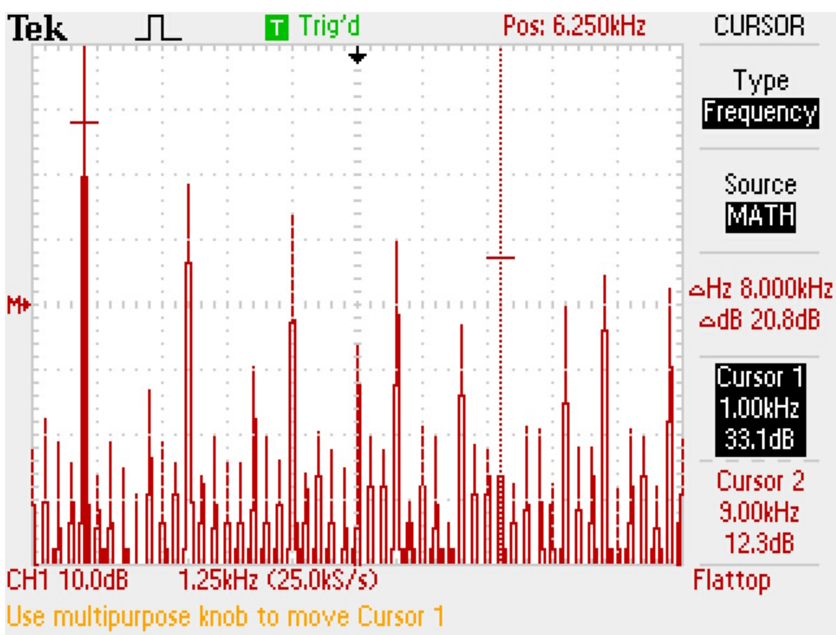

Figure 2.26: Oscilloscope FFT Measurement

2.6 Waveform Distortion

Consider an amplifier or filter circuit with a particular set of band-pass frequencies. What will happen if a square wave is applied to a circuit that limits the amplitude of the square wave harmonic content of the frequencies outside the pass-band? The answer is **waveform distortion**. The two types of square wave distortion that are useful for circuit frequency response analysis are Tilt and Rise Time (t_r).

- **Tilt** represents low frequency amplitude attenuation and can be used to determine Frequency Critical Low (FC_L).
- **Rise Time (t_r)** represents high-frequency amplitude attenuation and can be used to determine Frequency Critical High (FC_H).

2.7 Tilt and Frequency Critical Low (FC_L)

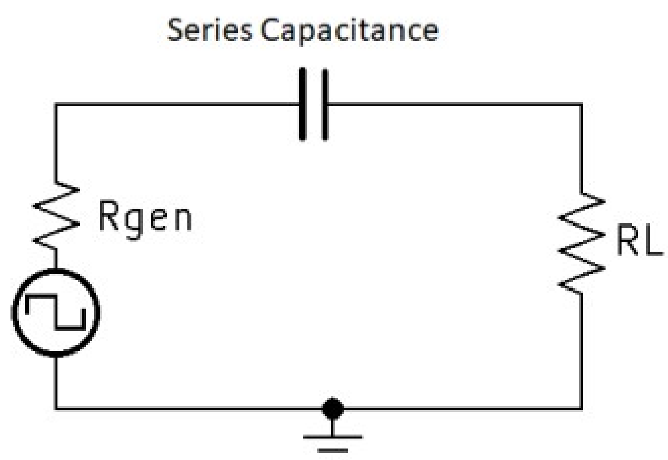

Figure 2.27: Series Capacitance, Tilt, & FC_L

Consider the circuit of figure 2.27 Series Capacitance, Tilt, & FC_L.

- Series capacitance will allow high frequencies through but will attenuate frequencies below FC_L.
- The horizontal component of the square wave represents higher to lower (left to right) frequency amplitude of the odd harmonics.
- X_C of the series capacitance increases as frequency decreases.
- Tilt represents the attenuation of low frequencies.

2.7.1 FC_{Low} formula using Tilt

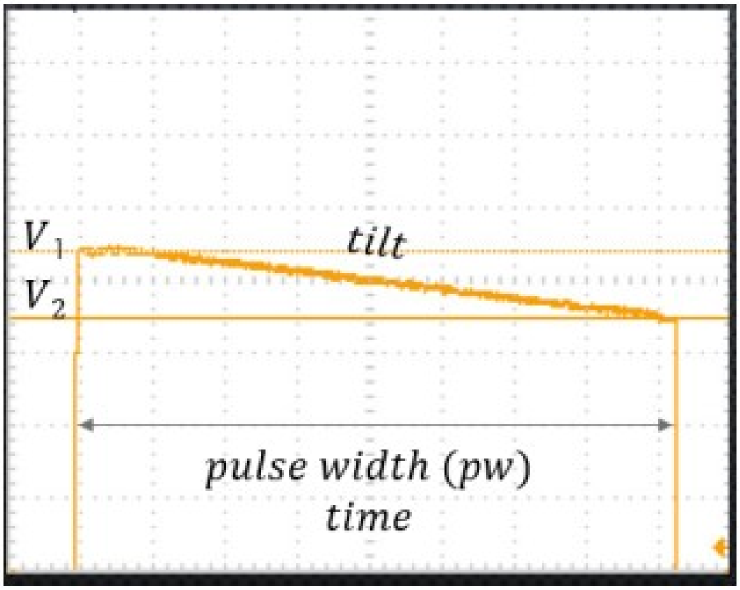

Figure 2.28: Tilt Measurement

$$FC_{Low} = \frac{fractional\ tilt}{2\pi PW}$$

$$fractional\ tilt = \frac{V1 - V2}{APA}$$

$$APA = \frac{V1 + V2}{2}$$

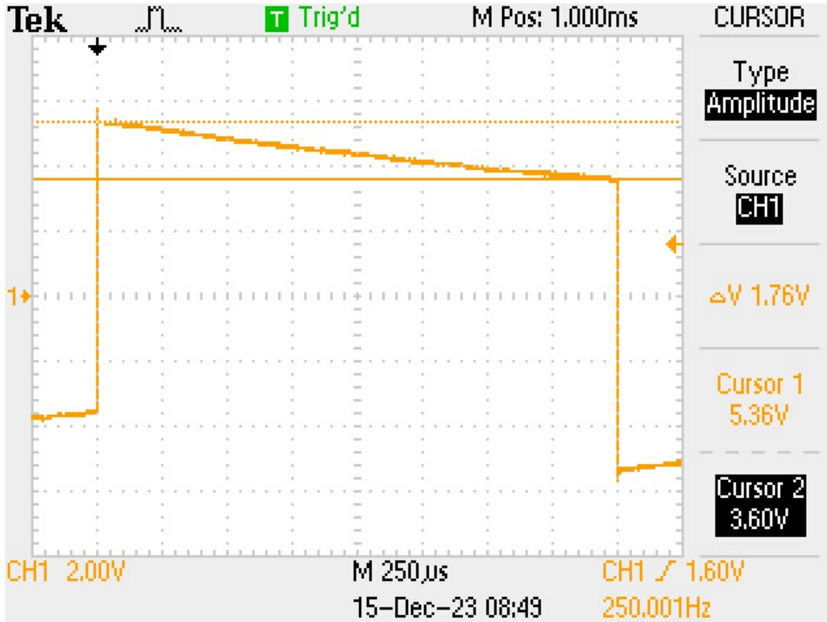

Figure 2.29: Measuring Vmax & Vmin on a Tilt waveform

2.7.2 FC_{Low} Tilt Example

Example 1. See Figure 2.29 Measuring Vmax & Vmin on a Tilt waveform.

- $V_{Cursor1} = 5.36V$
- $V_{Cursor2} = 3.6V$
- $Frequency = 250Hz$

Solution Steps:

1. Find APA:

$$APA = \frac{V1 + V2}{2}$$

$$APA = \frac{5.36V + 3.6V}{2}$$

$$APA = \frac{8.96V}{2}$$

$$\boxed{APA = 4.48V}$$

2. Find Fractional Tilt:

$$fractional\ tilt = \frac{V1 - V2}{APA}$$

$$fractional\ tilt = \frac{5.36V - 3.6V}{4.48V}$$

$$fractional\ tilt = \frac{1.76V}{4.48V}$$

$$\boxed{fractional\ tilt = 0.393}$$

3. Find Pulse Width:

$$PW = \frac{Period}{2}$$

$$Period = \frac{1}{Frequency}$$

$$Period = \frac{1}{250HZ}$$

$$Period = 4mS$$

$$PW = \frac{4mS}{2}$$

$$\boxed{PW = 2mS}$$

4. Find Frequency Critical Low:

$$FC_{Low} = \frac{fractional\ tilt}{2\pi PW}$$

$$FC_{Low} = \frac{0.393}{2\pi(2mS)}$$

$$FC_{Low} = \frac{0.393}{12.566 \times 10^{-3}}$$

$$\boxed{FC_{Low} = 31.274HZ}$$

2.8 Rise Time t_r and Frequency Critical High FC_H

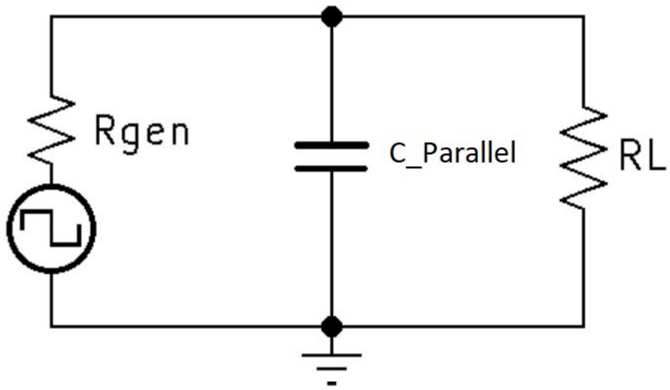

Figure 2.30: Parallel Capacitance, Rise Time, & FC_H

Consider the circuit of figure 2.30 Parallel Capacitance, Rise Time, & FC_H.

- **High Frequency Distortion** is caused by **parallel capacitance**.
- **Parallel capacitance** can include stray capacitance, probe capacitance, generator capacitance, and device capacitance.
- The attenuation of high frequencies caused by parallel capacitance will slow or decrease the **Rise Time** t_r of a measured square wave.

2.8.1 FC_{High} formula using Rise Time t_r

Using Pulse Theory or Square-Wave Analysis, we can calculate the circuit's high critical frequency using the following formula:

$$FC_{High} = \frac{0.35}{t_r}$$

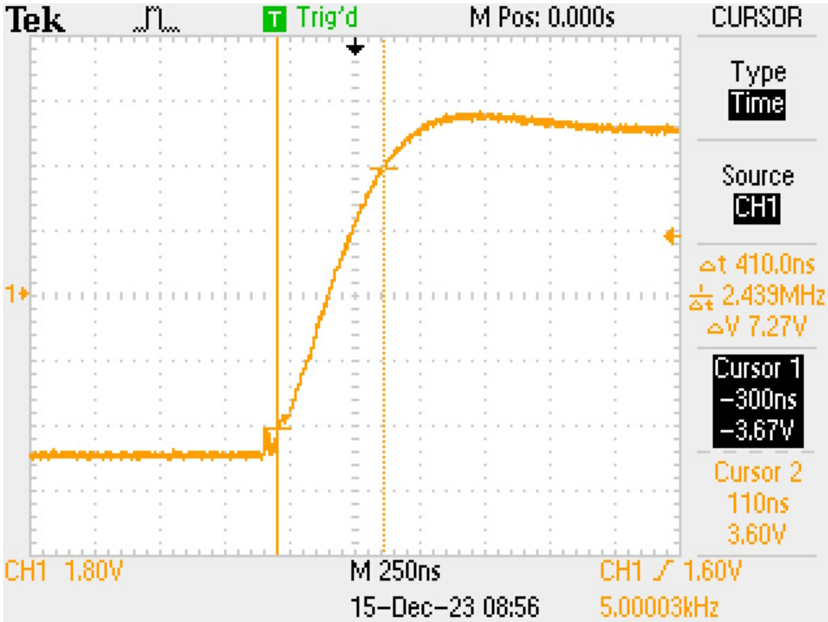

Figure 2.31: Rise Time Measurement

2.8.2 FC_{High} and Rise Time t_r Example

See Figure 2.31 Rise Time Measurement.

1. Measure Rise Time

$$t_r = 410nS$$

2. Calculated FC_{High}

$$FC_{High} = \frac{0.35}{t_r}$$

$$FC_{High} = \frac{0.35}{410nS}$$

$$\boxed{FC_{High} = 853.659Khz}$$

2.9 Capacitor Charge Formula

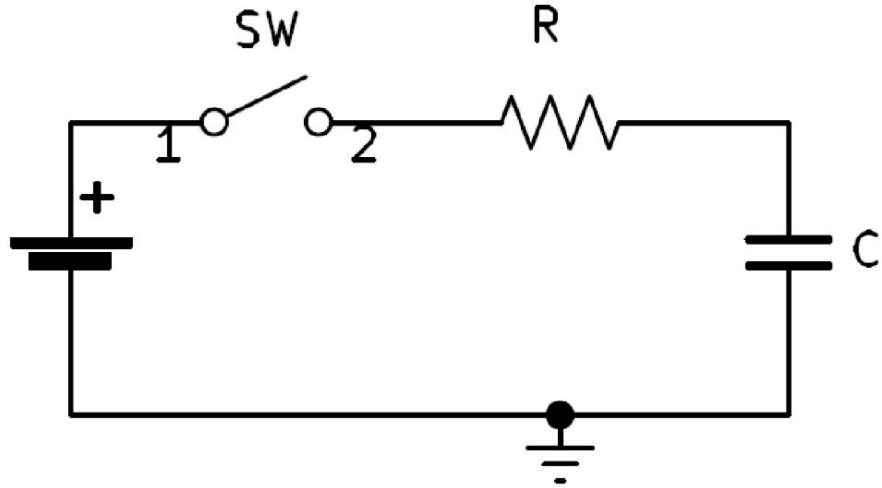

Figure 2.32: Capacitor Charge

After the switch is closed, how long will it take for the capacitor to fully charge?

$$V_{Capacitor} = V_{Final} - (V_{Final} - V_{Initial})e^{(\frac{-t}{\tau})}$$

* V_{Final} is the voltage that the capacitor would charge or discharge to if time is greater than 5τ.
* $V_{Initial}$ is the initial voltage that the capacitor is initially at.
* t is the amount of charge time.
* Tau (τ) is RC.

2.9.1 Capacitor Charge Formula in terms of % and time vs. τ

Percent of Charge at time equals 1τ:

$$V_C = 100\%V_{Fin} - (100\%V_{Fin} - 0V_{init})e^{(\frac{-1}{1})}$$

* $V_C = 100\%V_{Fin} - (100\%V_{Fin})e^{-1}$
* $V_C = 100\%V_{Fin} - (100\%V_{Fin})(0.36787944)$
* $V_C = 100\%V_{Fin} - 36.787944\%V_{Fin}$

  * $V_C = 63.212056\% V_{Fin}$

✓ At 1τ, the capacitor will charge to 63.212% of the final voltage.

Percent of Charge at time equals 2τ:

$V_C = 100\% V_{Fin} - (100\% V_{Fin} - 0V_{init})e^{(\frac{-2}{1})}$

  * $V_C = 100\% V_{Fin} - (100\% V_{Fin})e^{-2}$
  * $V_C = 100\% V_{Fin} - (100\% V_{Fin})(0.13533528)$
  * $V_C = 100\% V_{Fin} - 13.533528\% V_{Fin}$
  * $V_C = 86.466471\% V_{Fin}$

✓ At 2τ, the capacitor will charge to 86.466% of the final voltage.

Percent of Charge at time equals 3τ:

$V_C = 100\% V_{Fin} - (100\% V_{Fin} - 0V_{init})e^{(\frac{-3}{1})}$

  * $V_C = 100\% V_{Fin} - (100\% V_{Fin})e^{-3}$
  * $V_C = 100\% V_{Fin} - (100\% V_{Fin})(0.049787068)$
  * $V_C = 100\% V_{Fin} - 4.9787068\% V_{Fin}$
  * $V_C = 95.02129316\% V_{Fin}$

✓ At 3τ, the capacitor will charge to 95.021% of the final voltage.

Percent of Charge at time equals 4τ:

$V_C = 100\% V_{Fin} - (100\% V_{Fin} - 0V_{init})e^{(\frac{-4}{1})}$

  * $V_C = 100\% V_{Fin} - (100\% V_{Fin})e^{-4}$

* $V_C = 100\%V_{Fin} - (100\%V_{Fin})(0.018315639)$
* $V_C = 100\%V_{Fin} - 1.8315639\%V_{Fin}$
* $V_C = 98.1684361\%V_{Fin}$

✓ At 4τ, the capacitor will charge to 98.168% of the final voltage.

Percent of Charge at time equals 5τ:

$$V_C = 100\%V_{Fin} - (100\%V_{Fin} - 0V_{init})e^{(\frac{-5}{1})}$$

* $V_C = 100\%V_{Fin} - (100\%V_{Fin})e^{-5}$
* $V_C = 100\%V_{Fin} - (100\%V_{Fin})(0.006737947)$
* $V_C = 100\%V_{Fin} - 0.6737947\%V_{Fin}$
* $V_C = 99.3262\%V_{Fin}$

✓ At 5τ, the capacitor will charge to 99.326% of the final voltage.

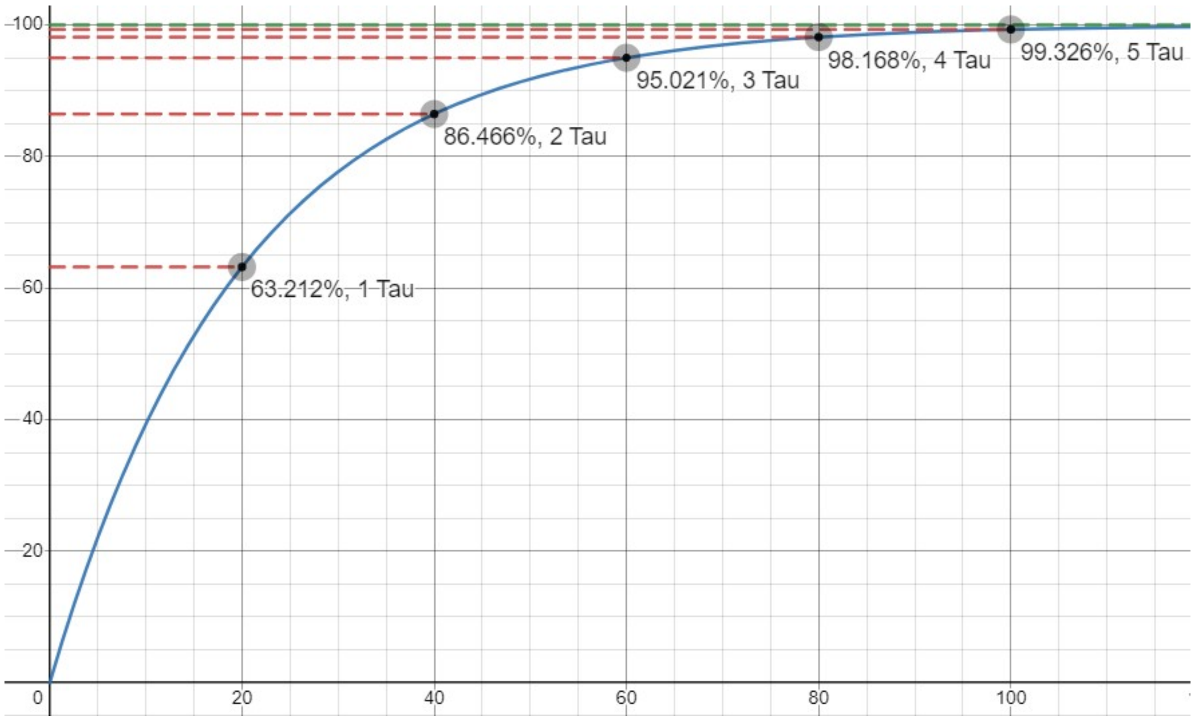

Figure 2.33: Capacitor % of Charge vs. Tau

Table 2.2: RC Circuits Capacitor % of Charge vs. Tau

Tau	VC % of Charge
1	63.212%
2	86.466%
3	95.021%
4	98.168%
5	99.326%

2.10 RC Circuits Stability, Vmax & Vmin Calculations

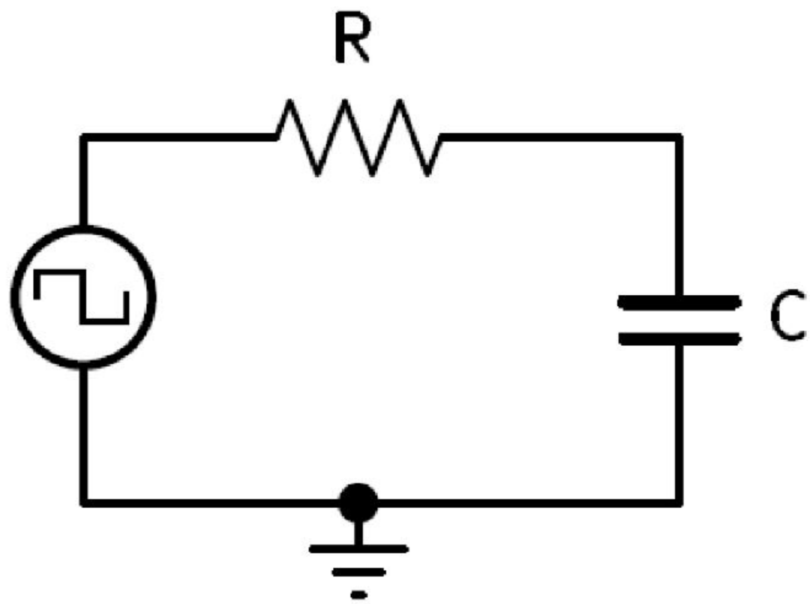

Figure 2.34: RC Circuit Stability

Consider the circuit of Figure 2.34 RC Circuit Stability. What happens to the capacitor voltage if the generator pulse width and pulse space are less than 5τ? In other words, there is not enough time to fully charge or fully discharge the capacitor. If the time on/off is less than 5 tau the capacitor voltage will not be able to charge up to the full generator voltage, likewise, Vc will not be able to fully discharge during the time off. Assuming the Duty Cycle is 50%, time on & off time are equal, the Vc waveform will appear to center itself vertically between the peak pulse width voltage and the peak pulse space voltage, as though superimposed on an average DC voltage.

$$V_{Capacitor} = V_{Final} - (V_{Final} - V_{Initial})e^{(\frac{-t}{\tau})}$$

The $V_{Capacitor}$ charge formula could be used to calculate cycle after cycle until the charge and discharge voltages stabilized. Next the number of cycles could be counted to find Cycles to Stabilization. This method can be tedious and mathematically error-prone if there are many cycles to stabilization.

2.10.1 Cycles to Stabilization Formula

Stability $= 5\tau$ (Stable?)

How many cycles to Stabilization?

If:

1 Cycle = Period

Stability $= 5\tau$

Then:

$$\frac{Stability}{5\tau(Sec)} = \frac{1Cycle}{Period(Sec)}$$

$$Stability = \frac{1Cycle \times 5\tau(Sec)}{Period(Sec)}$$

Notice that the seconds/time unit cancels and we are left with cycles.

$$\checkmark \boxed{Stability_{Cycles} = \frac{5\tau}{Period}}$$

The $Stability_{Cycles}$ formula only works if the Duty Cycle is 50%. If the generator signal is not at 50% Duty Cycle, the capacitor charge formula long method must be used to calculate the number of cycles to stability.

2.10.2 Vmax and Vmin formulas

Now that we can predict the number of cycles to stabilization,What will the stable capacitor voltage waveform voltage be? What will the maximum and minimum voltage be?

$$V_C = V_{fin} - (V_{fin} - V_{In})e^{\frac{-t}{RC}}$$

$$t = PW$$

$$V_C = V_{Max}$$

$$V_{gen+} = V_{fin}$$

$$V_{In} = V_{Min}$$

$$V_{Min} = V_{gen+} - V_{Max}$$

$$V_{In} = V_{gen+} - V_{Max}$$

$$V_{Max} = V_{gen+} - (V_{gen+} - (V_{gen+} - V_{Max}))e^{\frac{-PW}{RC}}$$

$$V_{Max} = V_{gen+} - (V_{gen+} - V_{gen+} + V_{Max})e^{\frac{-PW}{RC}}$$

$$V_{Max} = V_{gen+} - (V_{Max})e^{\frac{-PW}{RC}}$$

$$V_{Max} + (V_{Max})e^{\frac{-PW}{RC}} = V_{gen+}$$

$$V_{Max}(1 + e^{\frac{-PW}{RC}}) = V_{gen+}$$

$$\boxed{V_{Max} = \frac{V_{gen+}}{1 + e^{\frac{-PW}{RC}}}}$$

$$\boxed{V_{Min} = V_{gen+} - V_{Max}}$$

The V_{Max} & V_{Min} formulas only works if the Duty Cycle is 50%.

2.10.3 RC Circuit Stability Example Problem:

Refer to Figure 2.34 RC Circuit Stability schematic.

Given:

- V_{gen} is 0 to 10v, 1Khz, 50%DC, square-wave.
- $R = 1K\Omega$
- $C = 1\mu F$
- $\tau = RC = 1mS$
- $5\tau = 5 \times 10^{-3} seconds\ (5mS)$
- $Period = \frac{1}{1Khz} = 1mS$
- $PW\ and\ PS = \frac{Period}{2} = 0.5mS$

Solving using the Capacitor Charge Formula:

$$V_{Capacitor} = V_{Final} - (V_{Final} - V_{Initial})e^{(\frac{-t}{\tau})}$$

- $time = 0\ to\ 0.5mSec$

  $V_C = 10V - (10V - 0V)e^{(\frac{-0.5mS}{1mS})}$

  $V_C = 3.935V$

- $time = 0.5\ to\ 1.0mSec$

  $V_C = 0V - (0V - 3.935V)e^{(\frac{-0.5mS}{1mS})}$

  $V_C = 2.387V$

- $time = 1\ to\ 1.5mSec$

  $V_C = 10V - (10V - 2.387V)e^{(\frac{-0.5mS}{1mS})}$

  $V_C = 5.382V$

- $time = 1.5\ to\ 2.0mSec$

  $V_C = 0V - (0V - 5.382V)e^{(\frac{-0.5mS}{1mS})}$

  $V_C = 3.265V$

- $time = 2.0\ to\ 2.5mSec$

$$V_C = 10V - (10V - 3.265V)e^{(\frac{-0.5mS}{1mS})}$$
$$V_C = 5.915V$$

- $time = 2.5 \ to \ 3.0mSec$

$$V_C = 0V - (0V - 5.915V)e^{(\frac{-0.5mS}{1mS})}$$
$$V_C = 3.587V$$

- $time = 3.0 \ to \ 3.5mSec$

$$V_C = 10V - (10V - 3.587V)e^{(\frac{-0.5mS}{1mS})}$$
$$V_C = 6.111V$$

- $time = 3.5 \ to \ 4.0mSec$

$$V_C = 0V - (0V - 6.111V)e^{(\frac{-0.5mS}{1mS})}$$
$$V_C = 3.706V$$

- $time = 4.0 \ to \ 4.5mSec$

$$V_C = 10V - (10V - 3.706V)e^{(\frac{-0.5mS}{1mS})}$$
$$V_C = 6.182V$$

- $time = 4.5 \ to \ 5.0mSec$

$$V_C = 0V - (0V - 6.182V)e^{(\frac{-0.5mS}{1mS})}$$
$$V_C = 3.740V$$

- $time = 5.0 \ to \ 5.5mSec$

$$V_C = 10V - (10V - 3.740V)e^{(\frac{-0.5mS}{1mS})}$$
$$V_C = 6.209V$$

- $time = 5.5 \ to \ 6.0mSec$

$$V_C = 0V - (0V - 6.209V)e^{(\frac{-0.5mS}{1mS})}$$
$$V_C = 3.766V$$

- $time = 6.0 \ to \ 6.5mSec$

$$V_C = 10V - (10V - 3.766V)e^{(\frac{-0.5mS}{1mS})}$$
$$V_C = 6.219V$$

- $time = 6.5 \ to \ 7.0mSec$

$$V_C = 0V - (0V - 6.219V)e^{(\frac{-0.5mS}{1mS})}$$
$$V_C = 3.772V$$

- $time = 7.0\ to\ 7.5mSec$

$$V_C = 10V - (10V - 3.772V)e^{(\frac{-0.5mS}{1mS})}$$
$$V_C = 6.223V$$

- $time = 7.5\ to\ 8.0mSec$

$$V_C = 0V - (0V - 6.223V)e^{(\frac{-0.5mS}{1mS})}$$
$$V_C = 3.774V$$

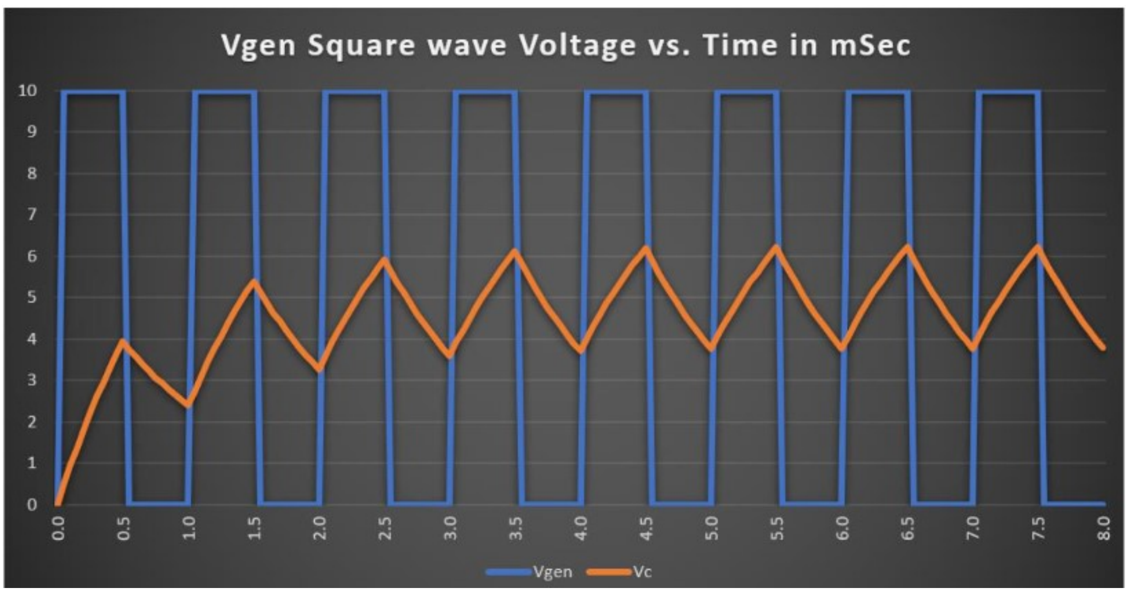

Figure 2.35: VC Stability

See Figure 2.35, observe that the capacitor waveform voltage is stable at 5τ ($5mS$) & 5 *cycles*.

Solving using the *Stability*, V_{Max}, & V_{Min} using formulas derived in Sections 2.10.1 & 2.10.2:

Refer to Figure 2.34 RC Circuit Stability schematic.

Given:

- V_{gen} is 0 to 10v, 1Khz, 50%DC, square-wave.
- $R = 1K\Omega$
- $C = 1\mu F$
- $\tau = RC = 1mS$
- $5\tau = 5 \times 10^{-3} seconds\ (5mS)$
- $Period = \frac{1}{1Khz} = 1mS$
- $PW\ and\ PS = \frac{Period}{2} = 0.5mS$

Formulas:

$$Stability_{Cycles} = \frac{5\tau}{Period}$$

$$V_{Max} = \frac{V_{gen+}}{1 + e^{\frac{-PW}{RC}}}$$

$$V_{Min} = V_{gen+} - V_{Max}$$

Calculating The Number of Cycles to Stability:

$$Stability_{Cycles} = \frac{5\tau}{Period}$$

$$Stability_{Cycles} = \frac{5(RC)}{1mS}$$

$$Stability_{Cycles} = \frac{5(1K\Omega \times 1\mu F)}{1mS}$$

$$Stability_{Cycles} = \frac{5(1mS)}{1mS}$$

$$\boxed{Stability_{Cycles} = 5\ cycles}$$

Calculating V_{Max}:

$$V_{Max} = \frac{V_{gen+}}{1 + e^{\frac{-PW}{RC}}}$$

$$V_{Max} = \frac{10V}{1 + e^{\frac{-0.5mS}{1K\Omega \times 1\mu F}}}$$

$$V_{Max} = \frac{10V}{1 + 0.606531}$$

$$\boxed{V_{Max} = 6.225vp}$$

Calculating V_{Min}:

$$V_{Min} = V_{gen+} - V_{Max}$$

$$V_{Min} = 10V - 6.225vp$$

$$\boxed{V_{Min} = 3.775vp}$$

Additionally, we can check our work by taking the average voltage of V_{Max} and V_{Min}. We know that the waveform voltage should be centered around the average generator voltage, in this case 5volts, ($5v = \frac{0+10}{2}$).

Calculating V_{avg}:

$$V_{avg} = \frac{V_{Max} + V_{Min}}{2}$$

$$V_{avg} = \frac{6.225v + 3.775v}{2}$$

$$V_{avg} = \frac{10v}{2}$$

$$\boxed{V_{avg} = 5v}$$

2.11 RC Integration

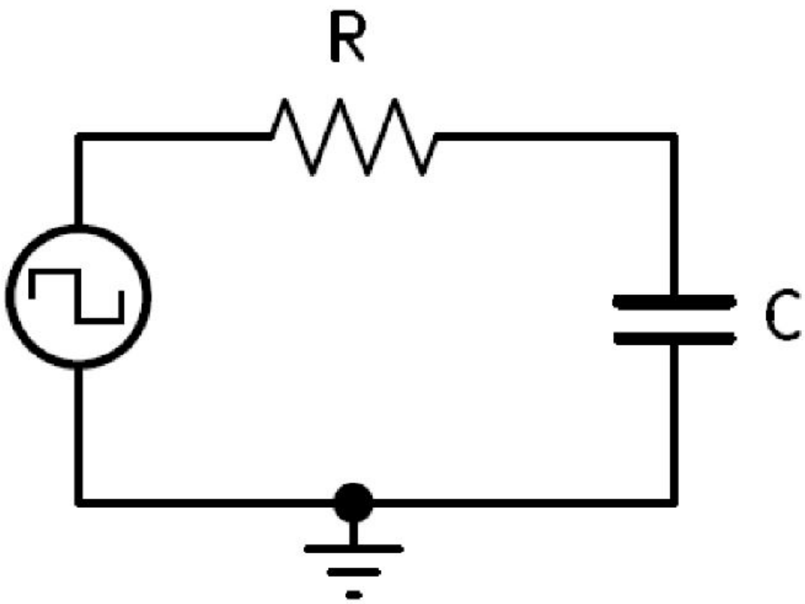

Figure 2.36: RC Integration

2.11.1 Characteristics of RC Integration Circuits

- Think Averaging!
- The output is across the capacitor
- There is not enough time for the capacitor to fully charge or discharge.
- PW and PS is less than 5τ
- $RC \geq (10 \times PW)$
- $PW(time) \leq \frac{RC}{10}$ (not enough time)
- Additional Formulas:
  * $Stability_{Cycles} = \frac{5\tau}{Period}$
  * $V_{Max} = \frac{V_{gen+}}{1+e^{\frac{-PW}{RC}}}$
  * $V_{Min} = V_{gen+} - V_{Max}$

2.11.2 RC Integration Circuit Example

Calculate an RC Integration Circuit using a $1Khz$ frequency and a $1\mu F$ capacitor.

Given:

- V_{gen} = 0vp to 10vp, 50% DC, square-wave.
- $F_{gen} = 1Khz$
- $C = 1\mu F$

Find charge/discharge time($time$):

$$Period = \frac{1}{Frequency}$$

$$Period = \frac{1}{1Khz}$$

$$Period = 1mS$$

$$time = \frac{Period}{2}$$

$$time = \frac{1mS}{2}$$

$$\boxed{time = 0.5mS}$$

Find the series resistance (R):

$$PW(time) \leq \frac{RC}{10}$$

$$0.5mS \leq \frac{R(1\mu F)}{10}$$

$$5mS \leq R(1\mu F)$$

$$\frac{5mS}{1\mu F} \leq R$$

$$R \geq \frac{5mS}{1\mu F}$$

$$\boxed{R \geq 5K\Omega}$$

Find the number of cycles to stabilization ($Stability_{Cycles}$):

$$Stability_{Cycles} = \frac{5\tau}{Period}$$

$$Stability_{Cycles} = \frac{5(5K\Omega \times 1\mu F)}{1mS}$$

$$Stability_{Cycles} = \frac{5(5mS)}{1mS}$$

$$Stability_{Cycles} = \frac{25mS}{1mS}$$

$$\boxed{Stability_{Cycles} = 25_{Cycles}}$$

Find V_{Max}:

$$V_{Max} = \frac{V_{gen+}}{1 + e^{\frac{-PW}{RC}}}$$

$$V_{Max} = \frac{10vp}{1 + e^{\frac{-0.5mS}{5K\Omega \times 1\mu F}}}$$

$$V_{Max} = \frac{10vp}{1 + e^{\frac{-0.5mS}{5K\Omega \times 1\mu F}}}$$

$$V_{Max} = \frac{10vp}{1 + 0.905}$$

$$\boxed{V_{Max} = 5.250vp}$$

Find V_{Min}:

$$V_{Min} = V_{gen+} - V_{Max}$$

$$V_{Min} = 10vp - 5.250vp$$

$$\boxed{V_{Min} = 4.750vp}$$

******Add here Measured Image Integrator Gen=10vp squarewave 1KHZ Waveform, R=5K, C=1microF******

2.12 RC Differentiation

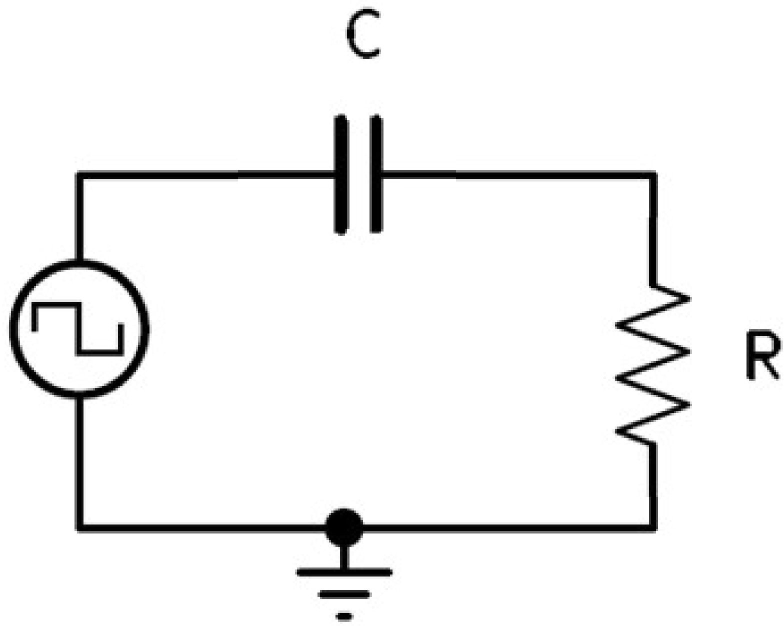

Figure 2.37: RC Differentiation

2.12.1 Characteristics of RC Differentiation Circuits

- When RC is less than one-tenth of the pulse width, the capacitor is charged very rapidly. Only a brief pulse of current is necessary to charge and discharge the capacitor at the beginning and end of the pulse. The resultant waveform of the resistor voltage is a series of positive and negative spikes at the pulse leading and lagging edges, respectively. Bell p.56[6]
- The output is across the resistor.
- There is more than enough time for the capacitor to fully charge or discharge.
- PW and PS is more than 5τ
- $RC \leq \frac{PW}{10}$
- $PW(time) \geq 10 \times RC$ (lots of time)

2.12.2 RC Differentiation Circuit Example

Calculate an RC Differentiation Circuit using a $1Khz$ frequency and a $1\mu F$ capacitor.

Given:

- V_{gen} = 0vp to 10vp, 50% DC, square-wave.
- $F_{gen} = 1Khz$
- $R = 5K\Omega$

Find charge/discharge time($time$):

$$Period = \frac{1}{Frequency}$$

$$Period = \frac{1}{1Khz}$$

$$Period = 1mS$$

$$time = \frac{Period}{2}$$

$$\boxed{time = 0.5mS}$$

Find the series capacitance (C):

$$PW(time) \geq 10 \times RC$$

$$0.5mS \geq 10 \times 5K\Omega \times C$$

$$0.5mS \geq 50K\Omega \times C$$

$$\frac{0.5mS}{50K\Omega} \geq C$$

$$\boxed{C = 10nF \ or \ 10,000pF}$$

Calculating the capacitor voltage based on Tau:

In Section 2.9 Capacitor Charge Formula the capacitor charge formula was simplified in terms of Tau. We can now apply this to our current example to find the capacitor charge voltage in terms of Tau and percentage of charge. Additionally, the same percentages also apply to the discharge time of a capacitor. For this example the capacitor was initially charged to 10V and will discharge 63.212% at 1τ leaving the capacitor voltage at 3.679V. Table **?? ??** has the compiled data and Figure 2.38 RC Differentiation, Generator and Capacitor Waveforms shows a graph of the generator and capacitor waveform voltages.

Once the capacitor voltages are calculated, the resistor voltage can be determined using Kirchhoff's Voltage Law. A tricky spot to pay attention to is the points where the generator voltage is vertical, 0 to 10v & 10v to 0. For example, you will need to do two Kirchhoff calculations at time equals 0.5mS, one where the generator voltage is equal to 0V and one where the generator voltage is equal to 10V. See Figure 2.39 RC Differentiation, Generator, Capacitor, and Resistor Waveforms.Notice how the resistor voltage goes negative, this is because at the instance that the generator voltage goes to zero, the capacitor voltage is still 10V, therefor to Kirchhoff, the resistor voltage must go negative.

$$V_R = V_{Gen} - V_C$$

Table 2.3: RC Circuits Capacitor Voltage

Capacitor Charge				
Tau	**Time**	**Generator Voltage**	**VC % of Charge**	**Capacitor Voltage**
0	**0μS**	**10V**	**0%**	**0V**
1	**50μS**	**10V**	**63.212%**	**6.321V**
2	**100μS**	**10V**	**86.466%**	**8.647V**
3	**150μS**	**10V**	**95.021%**	**9.502V**
4	**200μS**	**10V**	**98.168%**	**9.817V**
5	**250μS**	**10V**	**99.326%**	**9.933V**
$\geq 5\tau$	**500μS**	**10V**	**100%**	**10V**
250μS to 500μS the capacitor is fully charged to 10V				
Capacitor Discharge				
Tau	**Time**	**Generator Voltage**	**VC % of Discharge**	**Capacitor Voltage**
0	**500μS**	**0V**	**0%**	**10V**
1	**550μS**	**0V**	**63.212%**	**3.679V**
2	**600μS**	**0V**	**86.466%**	**1.353V**
3	**650μS**	**0V**	**95.021%**	**0.498V**
4	**700μS**	**0V**	**98.168%**	**0.183V**
5	**750μS**	**0V**	**99.326%**	**0.067V**
$\geq 5\tau$	**1mS**	**0V**	**100%**	**0V**

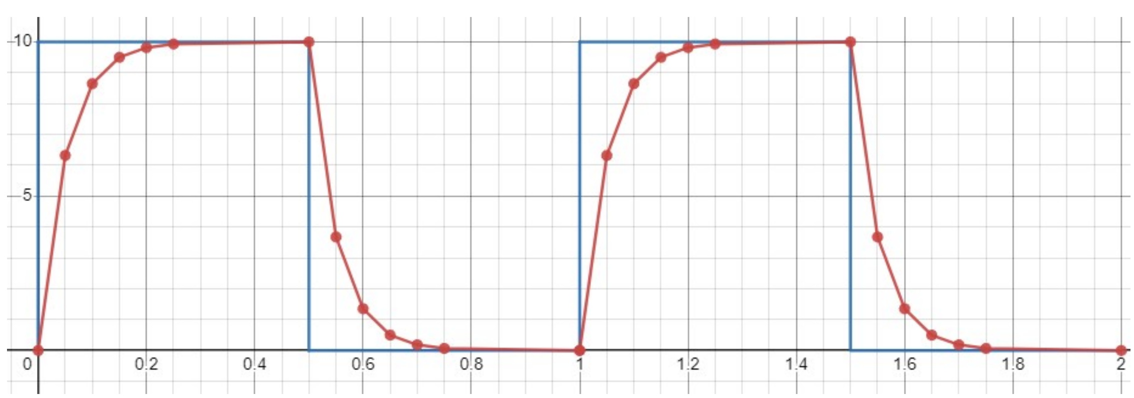

Figure 2.38: RC Differentiation, Generator and Capacitor Waveforms

Table 2.4: RC Circuits Capacitor & Resistor waveform voltages

Capacitor Charge					
Tau	**Time**	**Generator Voltage**	**VC % of Charge**	**Capacitor Voltage**	**Resistor Voltage**
0	**0μS**	**10V**	**0%**	**0V**	**10V**
1	**50μS**	**10V**	**63.212%**	**6.321V**	**3.679V**
2	**100μS**	**10V**	**86.466%**	**8.647V**	**1.353V**
3	**150μS**	**10V**	**95.021%**	**9.502V**	**0.498V**
4	**200μS**	**10V**	**98.168%**	**9.817V**	**0.183V**
5	**250μS**	**10V**	**99.326%**	**9.933V**	**0.067V**
$\geq 5\tau$	**500μS**	**10V**	**100%**	**10V**	**0V**
250μS to 500μS the capacitor is fully charged to 10V					
Capacitor Discharge					
Tau	**Time**	**Generator Voltage**	**VC % Discharge**	**Capacitor Voltage**	**Resistor Voltage**
0	**500μS**	**0V**	**0%**	**10V**	**-10V**
1	**550μS**	**0V**	**63.212%**	**3.679V**	**-3.679V**
2	**600μS**	**0V**	**86.466%**	**1.353V**	**-1.353V**
3	**650μS**	**0V**	**95.021%**	**0.498V**	**-0.498V**
4	**700μS**	**0V**	**98.168%**	**0.183V**	**-0.183V**
5	**750μS**	**0V**	**99.326%**	**0.067V**	**-0.067V**
$\geq 5\tau$	**1mS**	**0V**	**100%**	**0V**	**0V**

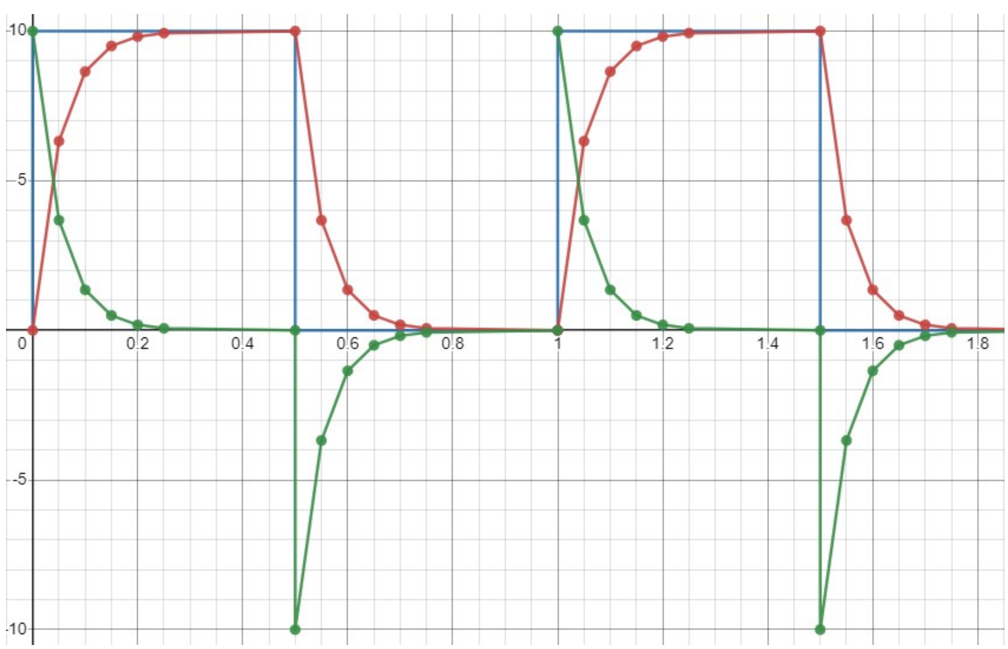

Figure 2.39: RC Differentiation, Generator, Capacitor, and Resistor Waveforms

2.13 Sine-waves and Instantaneous Voltage

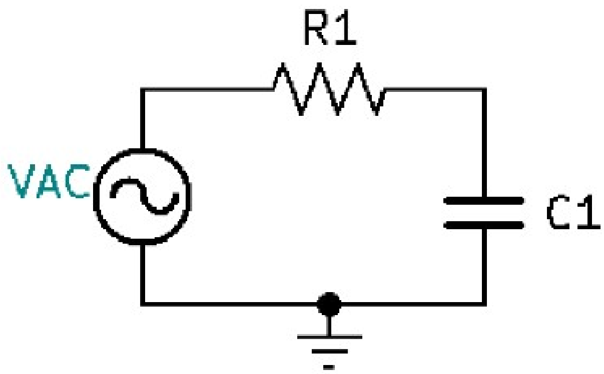

Figure 2.40: RC Circuit Sine-wave

2.13.1 Sine-wave analysis at resonance (F_C):

Consider Figure 2.40 RC Circuit Sine-wave.

Formulas:

- At F_C, $X_{C1} = R1$
- $X_{C1} = \frac{1}{2\pi F_C C1}$
- $R1 = \frac{1}{2\pi F_C C1}$
  - ✓ $F_C = \frac{1}{2\pi R1C1}$
  - ✓ $Z_T = \sqrt{(R1)^2 + (X_{C1})^2}$, $\angle = \tan^{-1} \frac{X_{C1}}{R1}$
  - ✓ $I_T = \frac{V_{gen}}{Z_T}$
  - ✓ $V_{R1} = I_T \times R_1$
  - ✓ $V_{C1} = I_T \times X_{C1}$
  - ✓ $V_{inst} = V_{Max} \sin(360Ft \pm \theta)$

2.13.2 Sine-wave Instantaneous Example 1

Consider circuit Figure 2.40, and let's assume that V_{gen} is set to 10vp at 1Khz, and X_{C1} and $R1$ both are equal to $1K\Omega$.

Given:

- $V_{gen} = 10vp\ \angle 0°,\ 1Khz$
- $X_{C1} = 1K\Omega\ \angle -90°$
- $R1 = 1K\Omega\ \angle\ 0°$

Find: $Z_T,\ I_T,\ V_R,\ V_C,\ VGen_{inst},\ VR_{inst},\ VC_{inst}$

Solve:

$$Z_T = \sqrt{(R1)^2 + (X_{C1})^2}, \angle = \tan^{-1}\frac{X_{C1}}{R1}$$

$$Z_T = \sqrt{(1K)^2 + (1K)^2}, \angle = \tan^{-1}\frac{-1K}{1K}$$

$$\boxed{Z_T = 1.414K\Omega, \angle = -45°}$$

$$I_T = \frac{V_{gen}}{Z_T}$$

$$I_T = \frac{10vp\angle 0°}{1.414K\Omega\ \angle -45°}$$

$$\boxed{I_T = 7.071mAp\ \angle 45°}$$

$$V_{R1} = I_T \times R_1$$

$$V_{R1} = 7.071mAp\ \angle 45° \times 1K\Omega\ \angle\ 0°$$

$$\boxed{V_{R1} = 7.071vp\ \angle 45°}$$

$$V_{C1} = I_T \times X_C 1$$

$$V_{C1} = 7.071mAp\ \angle 45^\circ \times 1K\Omega\ \angle\ -90^\circ$$

$$\boxed{V_{C1} = 7.071vp\ \angle{-45^\circ}}$$

$$V_{inst} = V_{Max} \sin(360Ft \pm \theta)$$

$$\boxed{VGen_{inst} = 10vp \sin(360 \times 1Khz \times time\ \pm\ 0^\circ)}$$

$$\boxed{VR_{inst} = 7.071vp \sin(360 \times 1Khz \times time\ +\ 45^\circ)}$$

$$\boxed{VC_{inst} = 7.071vp \sin(360 \times 1Khz \times time\ -\ 45^\circ)}$$

Table 2.5: Instantaneous Voltages for Example 1. section 2.13.2

Time	VGen	VR1	VC1
0μS	$0.000vp$	$5.000vp$	$-5.000vp$
50μS	$3.090vp$	$6.300vp$	$-3.210vp$
100μS	$5.878vp$	$6.984vp$	$-1.106vp$
150μS	$8.090vp$	$6.984vp$	$1.106vp$
200μS	$9.511vp$	$6.300vp$	$3.210vp$
250μS	$10.00vp$	$5.000vp$	$5.000vp$
300μS	$9.511vp$	$3.210vp$	$6.300vp$
350μS	$8.090vp$	$1.106vp$	$6.984vp$
400μS	$5.878vp$	$-1.106vp$	$6.984vp$
450μS	$3.090vp$	$-3.210vp$	$6.300vp$
500μS	$0.000vp$	$-5.000vp$	$5.000vp$
550μS	$-3.090vp$	$-6.300vp$	$3.210vp$
600μS	$-5.878vp$	$-6.984vp$	$1.106vp$
650μS	$-8.090vp$	$-6.984vp$	$-1.106vp$
700μS	$-9.511vp$	$-6.300vp$	$-3.210vp$
750μS	$-10.00vp$	$-5.000vp$	$-5.000vp$
800μS	$-9.511vp$	$-3.210vp$	$-6.300vp$
850μS	$-8.090vp$	$-1.106vp$	$-6.984vp$
900μS	$-5.878vp$	$1.106vp$	$-6.984vp$
950μS	$-3.090vp$	$3.210vp$	$-6.300vp$
$1m$S	$0.000vp$	$5.000vp$	$-5.000vp$

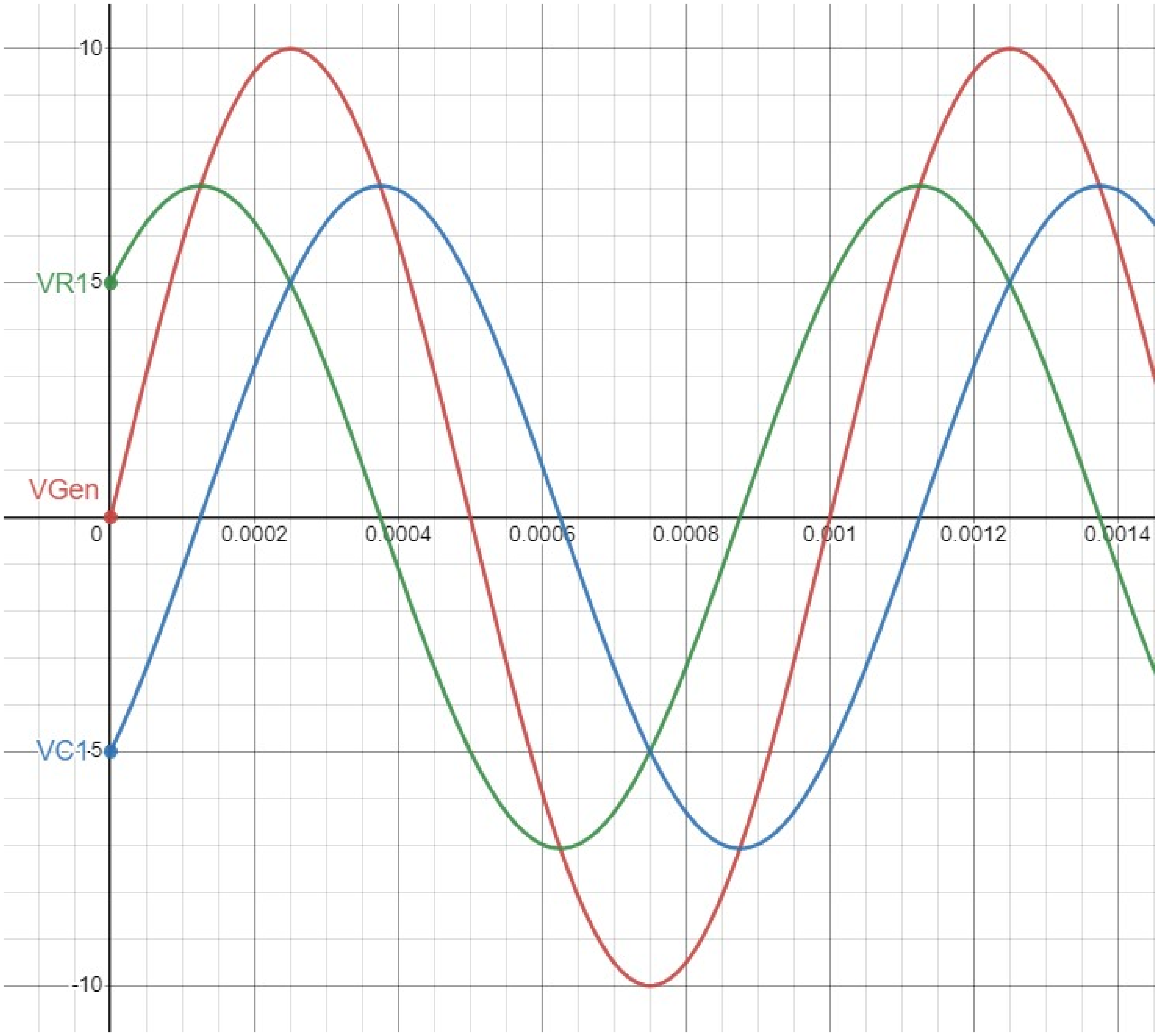

Figure 2.41: RC Circuit Instantaneous Waveforms Example 1. section 2.13.2

2.13.3 Sine-wave Instantaneous Example 2

What happens if double the resistance of the previous example?

Given:

- $V_{gen} = 10vp\ \angle 0°,\ 1Khz$
- $X_{C1} = 1K\Omega\ \angle -90°$
- $R1 = 2K\Omega\ \angle\ 0°$

Find: $Z_T,\ I_T,\ V_R,\ V_C,\ VGen_{inst},\ VR_{inst},\ VC_{inst}$

Solve:

$$Z_T = \sqrt{(R1)^2 + (X_{C1})^2}, \angle = \tan^{-1}\frac{X_{C1}}{R1}$$

$$Z_T = \sqrt{(2K)^2 + (1K)^2}, \angle = \tan^{-1}\frac{-1K}{2K}$$

$$\boxed{Z_T = 2.236K\Omega, \angle = -26.565°}$$

$$I_T = \frac{V_{gen}}{Z_T}$$

$$I_T = \frac{10vp\angle 0°}{2.236K\Omega\ \angle -26.565°}$$

$$\boxed{I_T = 4.472mAp\ \angle 26.565°}$$

$$V_{R1} = I_T \times R_1$$

$$V_{R1} = 4.472mAp\ \angle 26.565° \times 2K\Omega\ \angle\ 0°$$

$$\boxed{V_{R1} = 8.944vp\ \angle 26.565°}$$

$$V_{C1} = I_T \times X_C 1$$

$$V_{C1} = 4.472mAp\ \angle 26.565^\circ \times 1K\Omega\ \angle\ -90^\circ$$

$$\boxed{V_{C1} = 4.472vp\ \angle{-63.435^\circ}}$$

$$V_{inst} = V_{Max} \sin(360Ft \pm \theta)$$

$$\boxed{VGen_{inst} = 10vp \sin(360 \times 1Khz \times time\ \ \pm\ \ 0^\circ)}$$

$$\boxed{VR_{inst} = 8.944vp \sin(360 \times 1Khz \times time\ \ +\ \ 26.565^\circ)}$$

$$\boxed{VC_{inst} = 4.472vp \sin(360 \times 1Khz \times time\ \ -\ \ 63.435^\circ)}$$

Table 2.6: Instantaneous Voltages for Example 2. section 2.13.3

Time	**VGen**	**VR1**	**VC1**
0μS	$0.000vp$	$4.000vp$	$-4.000vp$
50μS	$3.090vp$	$6.276vp$	$-3.186vp$
100μS	$5.878vp$	$7.938vp$	$-2.060vp$
150μS	$8.090vp$	$8.823vp$	$-0.733vp$
200μS	$9.511vp$	$8.844vp$	$0.666vp$
250μS	$10.00vp$	$8.000vp$	$2.000vp$
300μS	$9.511vp$	$6.372vp$	$3.138vp$
350μS	$8.090vp$	$4.121vp$	$3.969vp$
400μS	$5.878vp$	$1.466vp$	$4.412vp$
450μS	$3.090vp$	$-1.332vp$	$4.422vp$
500μS	$0.000vp$	$-4.000vp$	$4.000vp$
550μS	$-3.090vp$	$-6.276vp$	$3.186vp$
600μS	$-5.878vp$	$-7.938vp$	$2.060vp$
650μS	$-8.090vp$	$-8.823vp$	$0.733vp$
700μS	$-9.511vp$	$-8.844vp$	$-0.666vp$
750μS	$-10.00vp$	$-8.000vp$	$-2.000vp$
800μS	$-9.511vp$	$-6.372vp$	$-3.138vp$
850μS	$-8.090vp$	$-4.121vp$	$-3.969vp$
900μS	$-5.878vp$	$-1.466vp$	$-4.412vp$
950μS	$-3.090vp$	$1.332vp$	$-4.422vp$
$1m$S	$0.000vp$	$4.000vp$	$-4.000vp$

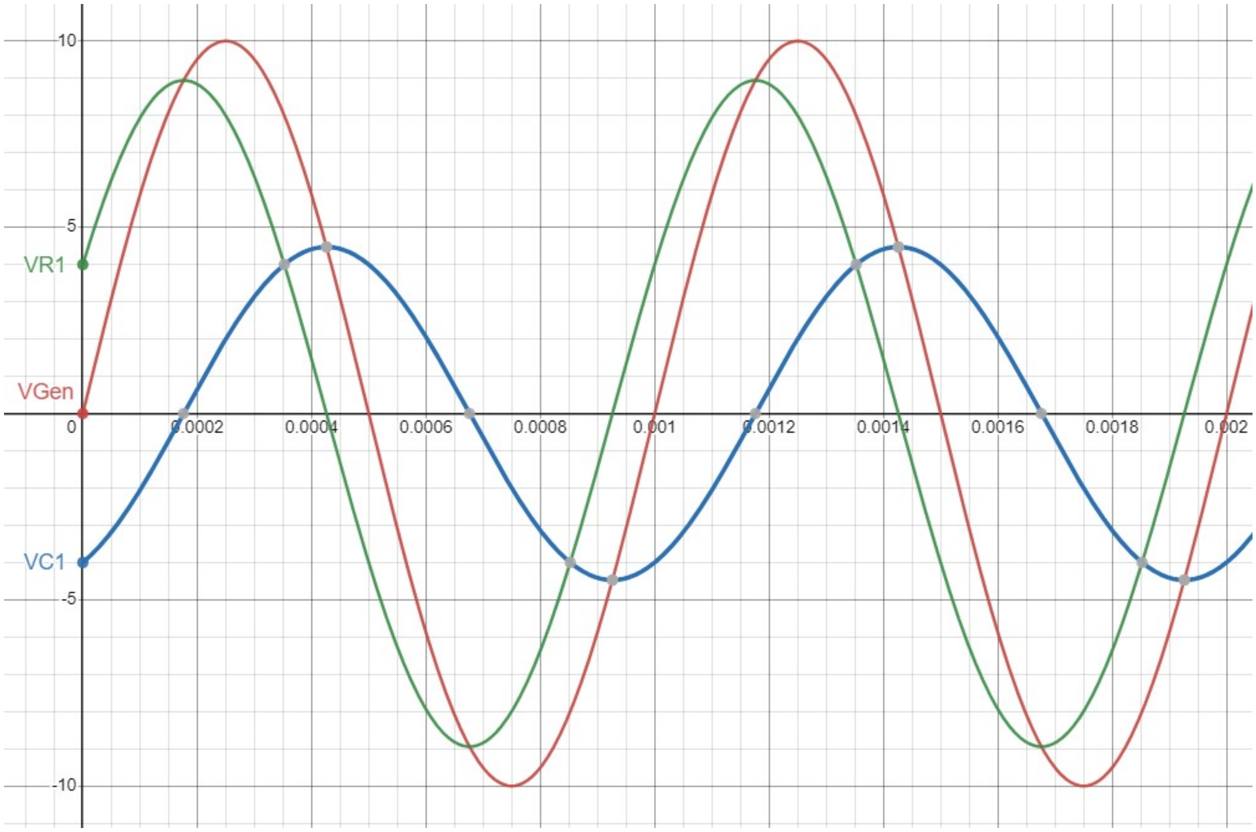

Figure 2.42: RC Circuit Instantaneous Waveforms Example 2. section 2.13.3

2.14 RC Circuits Questions

2.3 Graphs and Waveforms

1. On the graph, this axis represents the independent variable. (select all that apply)

   ☐ X-Axis

   ☐ Horizontal-Axis

   ☐ Y-Axis

   ☐ Vertical-Axis

2. This axis on the graph is used to represent the dependent variable. (select all that apply)

   ☐ X-Axis

   ☐ Horizontal-Axis

   ☐ Y-Axis

   ☐ Vertical-Axis

3. The point on the graph, where the two axis intersect is called the ____________ .

4. A ____________ waveform is a type of waveform that repeats its shape over regular intervals of time.

5. ____________ waveforms do not exhibit a regular and repeating pattern over time.

6. Common examples of periodic waveforms include which of the following waveforms?

   ☐ Sine

   ☐ Ramp

   ☐ Noise

   ☐ Sawtooth

   ☐ Exponetial

2.4 Pulse Waveform Characteristics

Matching:

1. ____ Period
2. ____ PW
3. ____ PS
4. ____ PRF
5. ____ DC%
6. ____ t_r
7. ____ t_f
8. ____ APA
9. ____ Tilt%
10. ____ AWV

(a) $=\frac{(+APA\times PW)+(-APA\times PS)}{Period}$

(b) The time it takes for one cycle.

(c) $=\frac{Vmax-Vmin}{APA}\times 100$

(d) Time High/On

(e) $=\frac{Vmax+Vmin}{2}$

(f) Time Low/Off

(g) The time required for the voltage to go from 90% to 10% of APA.

(h) $=\frac{1}{Period}$

(i) $=\frac{PW}{Period}\times 100$

(j) The time required for the voltage to go from 10% to 90% of APA.

Questions:

1. The ____________ ____________ waveform has perfectly vertical leading and lagging edges and perfectly flat tops and bottoms.
2. PW and PS are measured at ____________ of the waveform amplitude.
3. What is the DC% of a 1Khz waveform having with a pulse width equal to 250μS? Answer ____________ %.
4. What is the APA of a pulse that has a Vmax equal to 1.5V and a Vmin equal to 1V? Answer ____________ V.
5. What is the Tilt% of a pulse that has a Vmax equal to 1.5V and a Vmin equal to 1V? Answer ____________ %.
6. What is the AWV if +APA is equal to 10V and -APA is equal to -8V and PW is equal to 10μS and PS is equal to 20μS? Answer ____________ V.

2.5 Frequency Synthesis & Analysis

1. ____________ ____________ involves combining two or more signals to create a new waveform.
2. Select each true statement concerning Harmonics:
   - ☐ A harmonic is a multiple of the fundamental.
   - ☐ Harmonics are numbered according to their ratio to the fundamental.
   - ☐ The number of harmonics is infinite.
   - ☐ The 5^{th} harmonic frequency is equal to 5 times the Fundamental Frequency and the 5^{th} harmonic amplitude is equal to the Fundamentals Amplitude divided by 5.
3. A Perfect Square Wave is comprised of an infinite number of ____________ harmonic sine waves.
4. ____________ ____________ involves breaking down a complex waveform into its individual sinusoidal components or harmonics.
5. ____________ ____________ is a mathematical technique used to decompose a complex waveform into its individual sinusoidal components, representing different frequencies.
6. FFT stands for ____________ ____________ ____________ .
7. FFT can be used to analyze which of the following:
   - ☐ Analyze harmonics in power lines
   - ☐ Measure harmonic content and distortion in systems
   - ☐ Characterize noise in DC power supplies
   - ☐ Test impulse response of filters and systems
   - ☐ Analyze vibration

2.6 Waveform Distortion

1. The two types of square wave distortion that are useful for circuit frequency response analysis are ____________ and ____________ ____________ .

2.7 Tilt and Frequency Critical Low (FC_L)

1. ____________ capacitance will attenuate frequencies below FC_L.
2. ____________ represents the square wave distortion and attenuation of low frequencies.
3. FC_{Low} is equal to ____________ ____________ divided by 2π____________ .
4. What is FC_{Low} if the measured Tilt is 49.867% on a 630hz square wave.
   Answer: FC_{Low}=____________ hz.

2.8 Rise Time t_r and Frequency Critical High FC_H

1. The attenuation of high frequencies is caused by ____________ capacitance and will affect the ____________ ____________ of a measured square wave.

2. FC_{High} is equal to ____________ divided by ____________ .

3. If the measured Rise Time is 35μS, FC_{High} =____________ Khz.

2.9 Capacitor Charge Formula

1. At what percent of the Final Voltage will a capacitor charge in 1τ?
   Answer ____________ %

2. At what percent of the Final Voltage will a capacitor charge in 2τ?
   Answer ____________ %

3. At what percent of the Final Voltage will a capacitor charge in 3τ?
   Answer ____________ %

4. At what percent of the Final Voltage will a capacitor charge in 4τ?
   Answer ____________ %

5. At what percent of the Final Voltage will a capacitor charge in 5τ?
   Answer ____________ %

2.10 RC Circuits Stability, Vmax & Vmin Calculations

Matching:

1. ____ $V_{Capacitor}$
2. ____ $Stability_{Cycles}$
3. ____ V_{Max}
4. ____ V_{Min}

(a) $= V_{gen+} - V_{Max}$

(b) $= \frac{V_{gen+}}{1+e^{\frac{-PW}{RC}}}$

(c) $= \frac{5\tau}{Period}$

(d) $= V_{Final} - (V_{Final} - V_{Initial})e^{\frac{-t}{\tau}}$

Questions:

1. How many cycles will it take for the output to stabilize if the square wave frequency is 1Khz and τ is equal to 2mS? Answer ____________ cycles.

2. If the square wave voltage in question 1. is set at 0 to 10v, what is the stabilized V_{Max}? Answer V_{Max}=____________ V.

3. Considering the previous questions, What is V_{Min}? Answer V_{Min}= ____________ V.

2.11 RC Integration

1. Correctly select the **RC Integration Formula(s)** from the following:

   ☐ $RC \geq (10 \times PW)$
   ☐ $RC \leq (10 \times PW)$
   ☐ $PW \geq \frac{RC}{10}$
   ☐ $PW \leq \frac{RC}{10}$

2. Calculate the correct capacitance for an integrator circuit operating at 5Khz with a series resistance of 4.7KΩ. Select the correct answer:

   ☐ 0.220μF
   ☐ 0.210μF
   ☐ 0.120μF
   ☐ 0.110μF

3. For the previous question, Calculate the Cycles to Stabilization, V_{Max}, and V_{Min} with a generator voltage that is 0V to 10Vp.

   $Stabity =$ ____________ cycles
   $V_{Max} =$ ____________ V
   $V_{Min} =$ ____________ V

2.12 RC Differentiation

1. Correctly select the **RC Differentiation Formula(s)** from the following:

   ☐ $PW \geq (10 \times RC)$
   ☐ $PW \leq (10 \times RC)$
   ☐ $RC \geq \frac{PW}{10}$
   ☐ $RC \leq \frac{PW}{10}$

2. Calculate the correct capacitance for a differentiating circuit operating at 5Khz with a series resistance of 4.7KΩ. Select the correct answer:

   ☐ 0.022μF
   ☐ 0.021μF
   ☐ 2,200pF
   ☐ 2,100pF

2.13 Sine-waves and Instantaneous Voltage

Matching:

1. ____ At F_C,
2. ____ X_C
3. ____ F_C
4. ____ Z_T
5. ____ $\angle$
6. ____ I_T
7. ____ V_{Inst}

(a) $= V_{Max} sin(360Ft \pm \theta)$

(b) $\sqrt{(R)^2 + (X_C)^2}$

(c) $X_C = R$

(d) $= tan^{-1} \frac{X_C}{R}$

(e) $= \frac{V_T}{Z_T}$

(f) $= \frac{1}{2\pi RC}$

(g) $= \frac{1}{2\pi FC}$

Questions:

1. If a series RC circuit is operating at resonance with a generator voltage is $10vp$, $1Khz$ and circuit resistance equal to 2KΩ. Find the generator, resistor, and capacitor instantaneous voltage after 0.375mSec.

$VGen_{0.375mS} =$ ____________ V

$VR_{0.375mS} =$ ____________ V

$VC_{0.375mS} =$ ____________ V

Week 3

Multi-Stage Amplifier: Design, Circuit Analysis, and Low-Frequency Response

3.1 Objectives:

Multi-Stage Amplifier Design and Analysis:

DC Biasing Calculations - Kirchhoff's and Thevenin Review.

- Kirchhoff's Laws: Students should be able to apply Kirchhoff's laws for DC biasing calculations in multi-stage amplifiers, considering voltage and current relationships in transistor amplifier circuits.
- Thevenin Equivalent: Understand and utilize Thevenin's theorem in transistor DC biasing calculations for simplifying complex circuits into simpler equivalent circuits.

AC Gain Calculations.

- AC Gain Analysis: Develop proficiency in calculating AC voltage gain for multi-stage amplifiers, considering the configuration and characteristics of each stage.

Load-Lines, Vout Max, and Vin Max.

- Load-Line Analysis: Understand load-line concepts and how to use them for analyzing the performance of multi-stage amplifiers.
- Vout Max Calculation: Calculate the maximum output voltage (Vout Max) for a multi-stage amplifier, considering various factors and load conditions.
- Vin Max Calculation: Calculate the maximum input voltage (Vin Max) that can be applied to a multi-stage amplifier without distortion or clipping.

Critical Frequencies.

- Low Critical Frequency (FCL): Understand and calculate the low critical frequency (FCL) for multi-stage amplifiers, considering the impact of coupling and bypass capacitors.
- High Critical Frequency (FCH): Understand and calculate the high critical frequency (FCH) for multi-stage amplifiers, taking into account the internal capacitances and parasitic elements in the amplifier stages.

By achieving these objectives, students will develop a comprehensive understanding of DC biasing calculations, AC gain, load-line analysis, maximum output and input voltage calculations, and critical frequencies for multi-stage amplifiers. These objectives aim to enhance their proficiency in designing and analyzing multi-stage amplifier circuits, particularly in terms of frequency response.

3.2 Circuit Design, DC Biasing Stage 2:

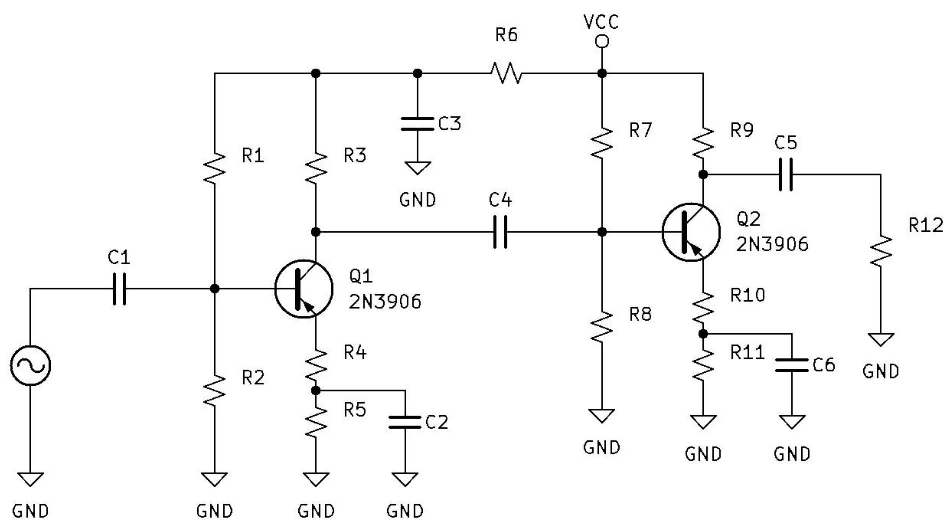

Figure 3.1: Two Stage Amplifier

Determine the Maximum Power for the 2N3906 Transistor

See Figure 3.2. P_D Total Device Dissipation.

DC Redraw for Stage 2:

Complete a DC redraw of the second stage of the amplifier making note of the DC biasing voltage polarities for each resistor and the transistor. See Figure 3.3.

Thermal Characteristics

Values are at $T_A = 25°C$ unless otherwise noted.

Symbol	Parameter	Maximum			Unit
		2N3906[3]	MMBT3906[2]	PZT3906[3]	
P_D	Total Device Dissipation	625	350	1,000	mW
	Derate Above 25°C	5.0	2.8	8.0	mW/°C
$R_{\theta JC}$	Thermal Resistance, Junction to Case	83.3			°C/W
$R_{\theta JA}$	Thermal Resistance, Junction to Ambient	200	357	125	°C/W

Figure 3.2: Maximum Power for the 2n3906 transistor

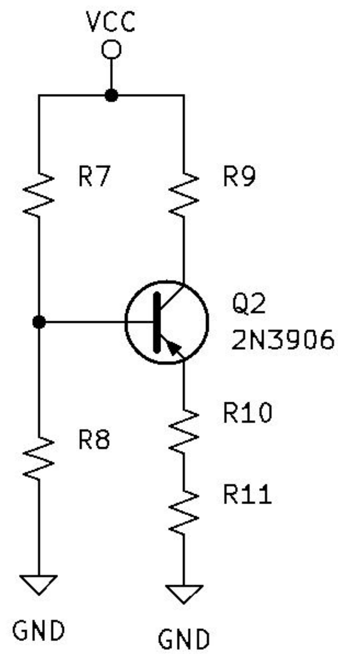

Figure 3.3: Second Stage DC Redraw

Determine Optimized Voltages and Transistor Power:

Optimize the biasing voltage using the 45, 45, 10 VCC ratios.

Voltage Optimization:

$V_{RC} = 45\% VCC$

$VCE = 45\% VCC$

$V_{RE} = 10\% VCC$

Make the transistor power ≈ half of the Maximum Power.

Q2 Power:

$$P_{Q2} = \frac{P_{MAX}}{2} = 0.5P_{MAX}$$

Collector Current:

$$P_{Q2} = IC \times V_{CE}$$

$$IC = \frac{P_{Q2}}{V_{CE}}$$

$$IC = \frac{P_{MAX}}{2V_{CE}}$$

$$IC = \frac{P_{MAX}}{2 \times 45\% VCC}$$

$$IC = \frac{P_{MAX}}{0.9VCC}$$

Base Current:

$$IB = \frac{IC}{Beta}$$

$$Beta_{2N3906} \approx 200$$

$$IB = \frac{P_{MAX}}{0.9VCC \times 200}$$

$$IB = \frac{P_{MAX}}{180VCC}$$

Emitter Current:

$$IE = IB \times (Beta + 1)$$

$$IE = \frac{P_{MAX}}{180VCC} \times (200 + 1)$$

$$IE = \frac{P_{MAX} \times 201}{180VCC}$$

$$IE = \frac{67P_{MAX}}{60VCC}$$

R9:

$$R9 = \frac{V_{R8}}{I_{R8}}$$

$$R9 = \frac{45\% VCC}{I_C}$$

$$R9 = \frac{45\% VCC}{\frac{P_{MAX}}{0.9VCC}}$$

$$R9 = \frac{45\% VCC \times 0.9VCC}{P_{MAX}}$$

$$R9 = \frac{0.405VCC^2}{P_{MAX}}$$

Power R9:

$$P_{R9} = IC \times V_{R8}$$

$$P_{R9} = IC \times 45\%VCC$$

$$P_{Q2} = IC \times 45\%VCC$$

$$P_{R9} = P_{Q2}$$

$$P_{R9} = \frac{P_{MAX}}{2} = 0.5P_{MAX}$$

RE = R10 + R11:

$$RE = \frac{V_{RE}}{I_{R9}}$$

$$RE = \frac{V_{RE}}{I_E}$$

$$RE = \frac{10\%VCC}{I_E}$$

$$RE = \frac{10\%VCC}{\frac{67P_{MAX}}{60VCC}}$$

$$RE = \frac{6VCC^2}{67P_{MAX}}$$

Power RE $(P_{R10} + P_{R11})$

$$P_{RE} = IE \times V_{R9}$$

$$P_{RE} = \frac{67P_{MAX}}{60VCC} \times 10\%VCC$$

$$P_{RE} = \frac{6.7P_{MAX} \times VCC}{60VCC}$$

$$P_{RE} = \frac{6.7P_{MAX}}{60}$$

R8:

$$R8 = \frac{V_{R8}}{I_{R8}}$$

$$V_{R8} = V_{RE} + V_{BE}$$

$$V_{R8} = 10\%VCC + 0.7V$$

$$I_{R8} = IB \times 10$$

$$I_{R8} = \frac{P_{MAX}}{180VCC} \times 10$$

$$I_{R8} = \frac{10P_{MAX}}{180VCC}$$

$$I_{R8} = \frac{P_{MAX}}{18VCC}$$

$$R8 = \frac{10\%VCC+0.7V}{\frac{P_{MAX}}{18VCC}}$$

$$R8 = \frac{18VCC(10\%VCC+0.7V)}{P_{MAX}}$$

$$R8 = \frac{1.8VCC^2+12.6VCC}{P_{MAX}}$$

Power R8:

$$P_{R8} = I_{R8} \times V_{R8}$$

$$P_{R8} = \frac{P_{MAX}}{18VCC} \times (10\%VCC + 0.7V)$$

$$P_{R8} = \frac{P_{MAX}(10\%VCC+0.7V)}{18VCC}$$

R7:

$$R7 = \frac{V_{R7}}{I_{R7}}$$

$$V_{R7} = VCC - V_{R8}$$

$$V_{R7} = VCC - (10\%VCC + 0.7V)$$

$$V_{R7} = VCC - 10\%VCC - 0.7V$$

$$V_{R7} = 0.9VCC - 0.7V$$

$$I_{R7} = I_{R7} + IB$$

$$I_{R7} = \frac{P_{MAX}}{180VCC} + \frac{P_{MAX}}{18VCC}$$

$$I_{R7} = \frac{P_{MAX}}{180VCC} + \frac{10P_{MAX}}{180VCC}$$

$$I_{R7} = \frac{P_{MAX}+10P_{MAX}}{180VCC}$$

$$I_{R7} = \frac{P_{MAX}(1+10)}{180VCC}$$

$$I_{R7} = \frac{11P_{MAX}}{180VCC}$$

$$R7 = \frac{0.9VCC-0.7V}{\frac{11P_{MAX}}{180VCC}}$$

$$R7 = \frac{180VCC(0.9VCC-0.7V)}{11P_{MAX}}$$

$$R7 = \frac{162VCC^2-126VCC}{11P_{MAX}}$$

Power R7:

$$P_{R7} = I_{R7} \times V_{R7}$$

$$P_{R7} = \frac{11P_{MAX}}{180VCC} \times (0.9VCC - 0.7V)$$

$$P_{R7} = \frac{P_{MAX}(9.9VCC+7.7V)}{180VCC}$$

3.3 Gain Calculations for Stage 2

R12

Make R12 ten times larger than R9.

$$R12 = R9 \times 10$$

Determine the Voltage Gain for Stage 2

$$\Delta V = \frac{V_{OUT}}{V_{IN}}$$

$$\Delta V = \frac{IC(R9//R12)}{IB(R10+r'e)(B+1)}$$

$$\Delta V = \frac{IC}{IB} \times \frac{(R9//R12)}{(R10+r'e)(B+1)}$$

$$\Delta V = \beta \times \frac{(R9//R12)}{(R10+r'e)(B+1)}$$

$$\Delta V = \frac{\beta}{\beta+1} \times \frac{(R9//R12)}{(R10+r'e)}$$

$$\Delta V = \alpha \times \frac{(R9//R12)}{(R10+r'e)}$$

$$\Delta V = \frac{\alpha(R9//R12)}{(R10+r'e)}$$

Calculating R10 to set the Desired Gain for Stage 2

Choose the desired gain, $\Delta V_{desired}$ and solve for a $R10$ value.

$$\Delta V_{desired} = \frac{\alpha(R9//R12)}{(R10+r'e)}$$

$$R10 + r'e = \frac{\alpha(R9//R12)}{\Delta V_{desired}}$$

$$R10 = \frac{\alpha(R9//R12)}{\Delta V_{desired}} - r'e$$

R11

$$RE = R10 + R11$$

$$R11 = RE - R10$$

3.4 Circuit Design, DC Biasing Stage 1:

DC Redraw for Stage 1:

Complete a DC redraw of the second stage of the amplifier making note of the DC biasing voltage polarities for each resistor and the transistor. See Figure 3.4.

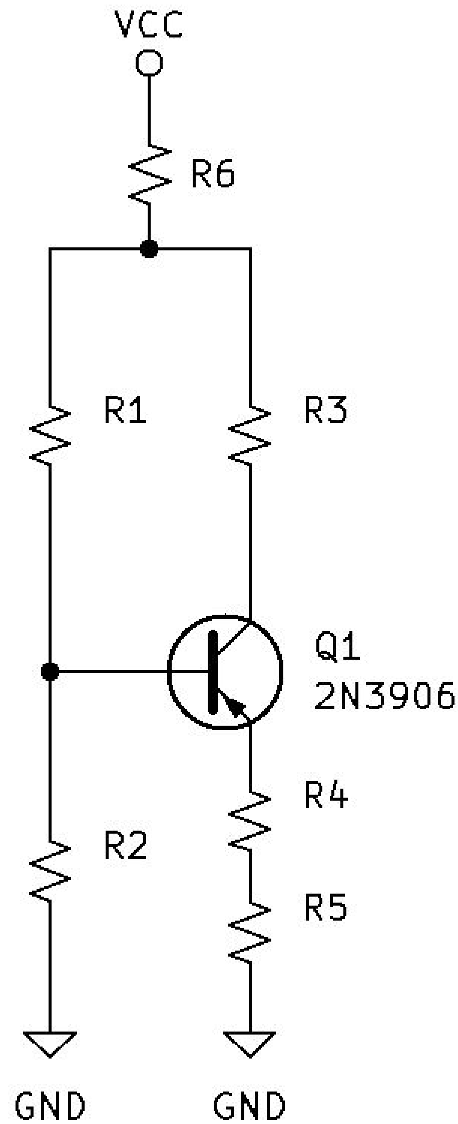

Figure 3.4: First Stage DC Redraw with polarities

Decoupling Resistor R6

The primary objective of the Decoupling Resistor R6 is to prevent oscillations by eliminating any positive feedback from the output of Q2 back to the input of Q1. C2 of Figure 3.1 will act like an AC short to ground allowing any potential feedback signal to be dropped on R6.

Additionally, R6 can be used to significantly lower the power for Q1 if needed.

Find the equivalent load resistance RL_{EqQ1} for Q1:

$$RL_{EqQ1} = ((R10 + r'e_{Q2}) \times (\beta_{Q2} + 1))//R7//R8$$

R3:

Make $R3$ ten times smaller than RL_{EqQ1}.

$$R3 = \frac{RL_{EqQ1}}{10}$$

Make P_{R3} equal to half the maximum power of Q1.

$$P_{R3} = \frac{PMAX_{Q1}}{2}$$

Calculate I_{R3}.

$$P = I^2 \times R$$

$$P_{R3} = I_{R3}^2 \times R3$$

$$I_{R3}^2 = \frac{P_{R3}}{R3}$$

$$I_{R3} = \sqrt{\frac{P_{R3}}{R3}}$$

Calculate V_{R3}.

$$P = \frac{V^2}{R}$$

$$P_{R3} = \frac{V_{R3}^2}{R3}$$

$$V_{R3}^2 = P_{R3} \times R3$$

$$V_{R3} = \sqrt{P_{R3} \times R3}$$

Q1:

$$IC = I_{R3}$$

$$IB = \frac{IC}{\beta}$$

$$IE = IB \times (\beta + 1)$$

Make V_{CE} of Q1 equal to V_{R3}.

$$V_{CE} = V_{R3}$$

$$P_{Q1} = P_{R3} = \frac{PMAX_{Q1}}{2}$$

RE:

$$RE = R4 + R5$$

Following the 45, 45, 10 optimization, Find VRE.

$$V_{CE} = .45X$$

$$X = \frac{V_{CE}}{.45}$$

$$V_{RE} = .10X$$

$$X = \frac{V_{RE}}{.10}$$

if X=X, then:

$$\frac{V_{CE}}{.45} = \frac{V_{RE}}{.10}$$

$$V_{RE} = \frac{.10V_{CE}}{.45}$$

$$V_{RE} = \frac{V_{CE}}{4.5}$$

R2:

$$V_{R2} = V_{BE} + V_{RE}$$

$$V_{R2} = 0.7v + V_{RE}$$

Make IR2 ten times larger than the base current of Q1.

$$I_{R2} = IB \times 10$$

$$R2 = \frac{V_{R2}}{I_{R2}}$$

R1:

Find VR1.

$$V_{R1} + V_{R2} = V_{R3} + V_{CE} + V_{RE}$$

$$V_{R1} = V_{R3} + V_{CE} + V_{RE} - V_{R2}$$

Calculate IR1.

$$I_{R1} = I_{R2} + I_B$$

Solve for R1.

$$R1 = \frac{V_{R1}}{I_{R1}}$$

R6:

Find VR6.

$$V_{R2} + V_{R1} + V_{R6} - V_{CC} = 0$$

$$V_{R6} = V_{CC} - V_{R2} - V_{R1}$$

Calculate IR6.

$$I_{R6} = I_{R1} + I_{R3}$$

Solve for R6.

$$R6 = \frac{V_{R6}}{I_{R6}}$$

3.5 Gain Calculations for Stage 1

Find the equivalent load resistance RL_{EqQ1} for Q1:

$$RL_{EqQ1} = ((R10 + r'e_{Q2}) \times (\beta_{Q2} + 1))//R7//R8$$

Find r'e for Q1

$$r'e = \frac{0.026}{IE}$$

Determine the Voltage Gain for Stage 1

$$\Delta V = \frac{V_{OUT}}{V_{IN}}$$

$$\Delta V = \frac{IC(R3//RL_{EqQ1})}{IB(R4+r'e)(B+1)}$$

$$\Delta V = \frac{IC}{IB} \times \frac{(R3//RL_{EqQ1})}{(R4+r'e)(B+1)}$$

$$\Delta V = \beta \times \frac{(R3//RL_{EqQ1})}{(R4+r'e)(B+1)}$$

$$\Delta V = \frac{\beta}{\beta+1} \times \frac{(R3//RL_{EqQ1})}{(R4+r'e)}$$

$$\Delta V = \alpha \times \frac{(R3//RL_{EqQ1})}{(R4+r'e)}$$

$$\Delta V = \frac{\alpha(R3//RL_{EqQ1})}{(R4+r'e)}$$

Calculating R4 to set the Desired Gain for Stage 1

Choose the desired gain, $\Delta V_{desired}$ and solve for a $R4$ value.

$$\Delta V_{desired} = \frac{\alpha(R3//RL_{EqQ1})}{(R4+r'e)}$$

$$R4 + r'e = \frac{\alpha(R3//RL_{EqQ1})}{\Delta V_{desired}}$$

$$R4 = \frac{\alpha(R3//RL_{EqQ1})}{\Delta V_{desired}} - r'e$$

R5

$$RE = R4 + R4$$

$$R5 = RE - R4$$

Q2, DC Loadline

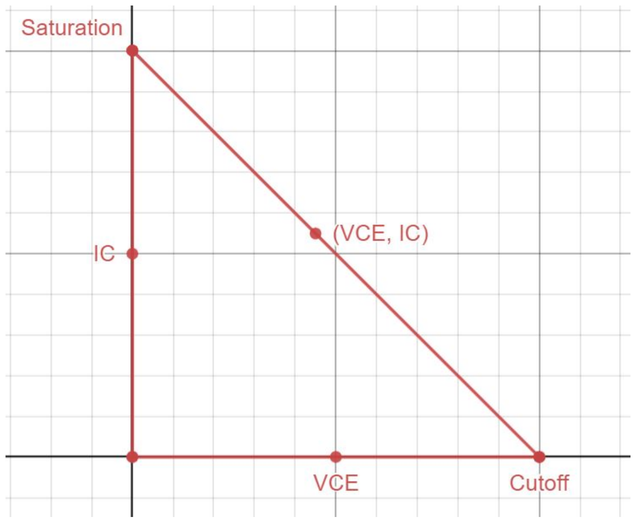

Figure 3.5: Second Stage DC Loadline

IC Saturation in terms of VCC and Max Power:

$$IC_{Sat} = \frac{VCC}{R9+RE}$$

$$IC_{Sat} = \frac{VCC}{\frac{0.405VCC^2}{P_{MAX}}+\frac{6VCC^2}{67P_{MAX}}}$$

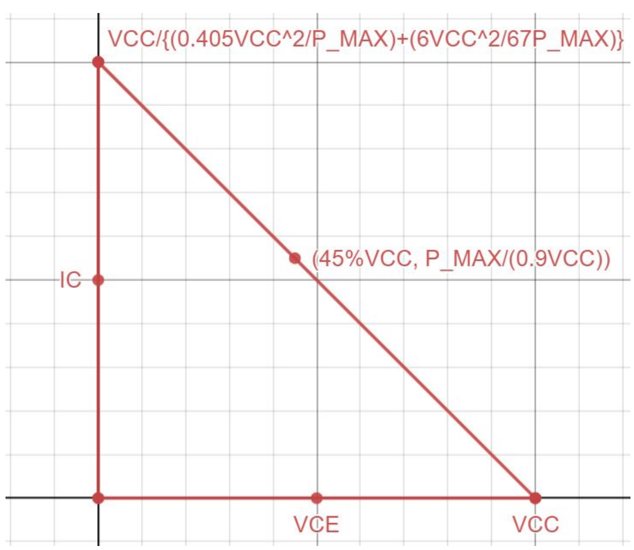

Figure 3.6: Second Stage DC Loadline in Terms of VCC and Max Power

Q2, AC Loadline

Because the load resistor R12 is ten times larger than R9 and the biasing is optimized, $Vout_{MAXp}$ will be just less than VCE.

$$Vout_{MAXp} \approx 0.9(V_{CE\_Q2})$$

$$V_{CE\_AC\_Cutoff} = V_{CE\_Q2} + I_{R9}(R9//R12) \qquad IC_{AC\_Sat} = I_{R9} + \frac{V_{CE\_Q2}}{R9//R12}$$

$$Vout_{MAXp} = V_{CE\_AC\_Cutoff} - V_{CE\_Q2}$$

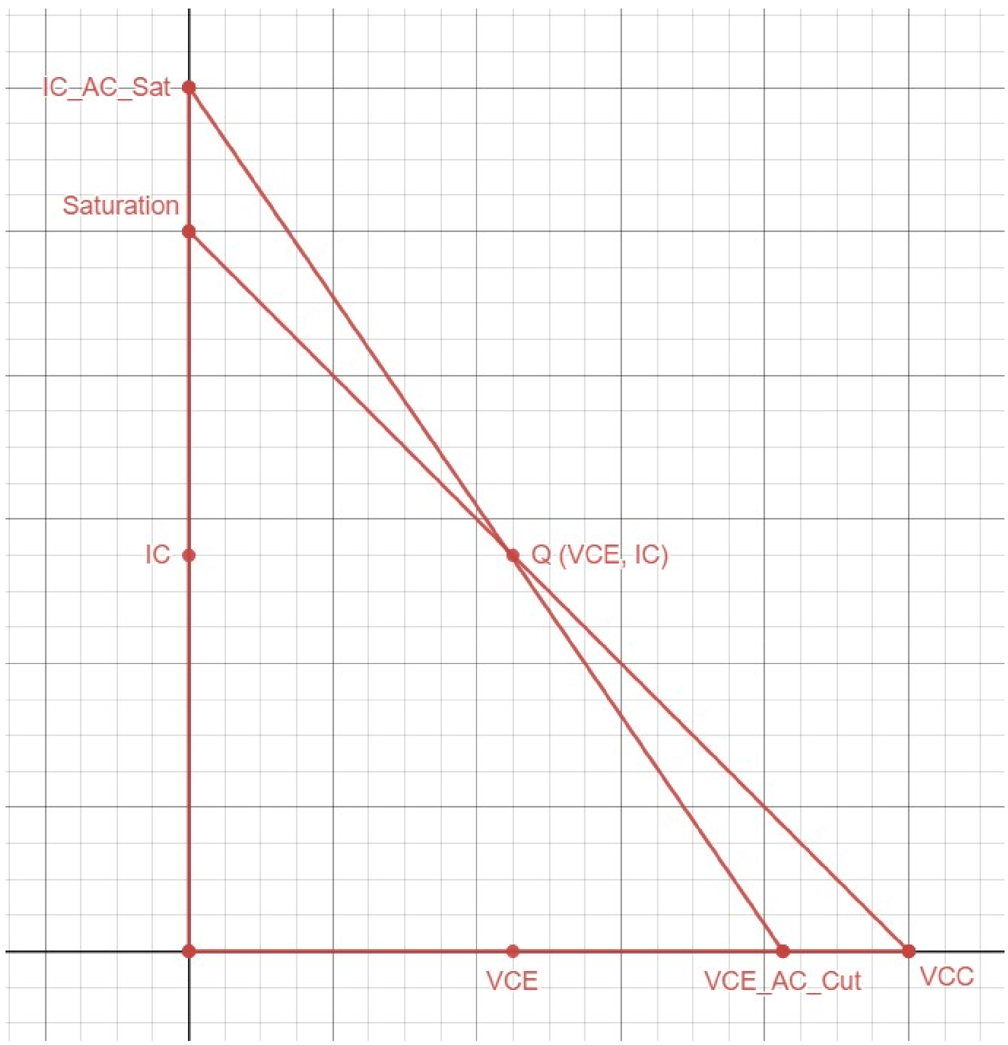

Figure 3.7: Second Stage DC & AC Loadline

Q1, Loadline

Because of the Decoupling Resistor R6, the DC Load Line is not Optimized at 45, 45 10 and will need to be calculated. However, because the equivalent load resistance of Q1 is ten times larger than R3, stage 1 maximum peak output should also be just below Q1's VCE.

$$Vout_{MaxP\_Q1} \approx 0.9(V_{CE\_Q1})$$

Additionally, if the voltage gain of the first stage is approximately equal to or less than the voltage gain of the second stage, the actual maximum peak voltage out will be determined by the second stage. This can be verified by using the following steps:

1. Divide the second stage Vout max peak voltage by the voltage gain of the second stage.

   $$X = \frac{Vout_{Q2\_MAXp}}{\Delta V_{Q2}}$$

2. Verify that Q1 VCE is greater than X.

   ✓ $V_{CE\_Q1} > X$

3. If X is greater than or equal to Q1 VCE, biasing could be adjusted by redesigning the circuit to raise Q1's VCE and lower the decoupling voltage. However, it may be quicker and easier to lower Q1's voltage gain and raise Q2's voltage gain by the same factor.

3.6 Circuit Analysis

Universal Biasing

The Universal Bias Circuit can be analyzed using either Thevenin or Kirchhoff analysis. Kirchhoff's analysis is based on Kirchhoff's Voltage and Current Laws and will always produce an accurate analysis. Thevenin Analysis is based on Thevenin's Theorem and produces accurate analysis as long as certain circuit parameters are met.

Universal Bias, Thevenin Analysis

Thevenin Analysis converts a complex circuit into a simple, easier-to-analyze, series circuit see Figure 3.8.

Thevenin Analysis Steps:

Find the Thevenin Voltage V_{Thev}.

V_{Thev} is the maximum voltage that the load will see. Consider Q2 and RE as the load, see Figure 3.1.

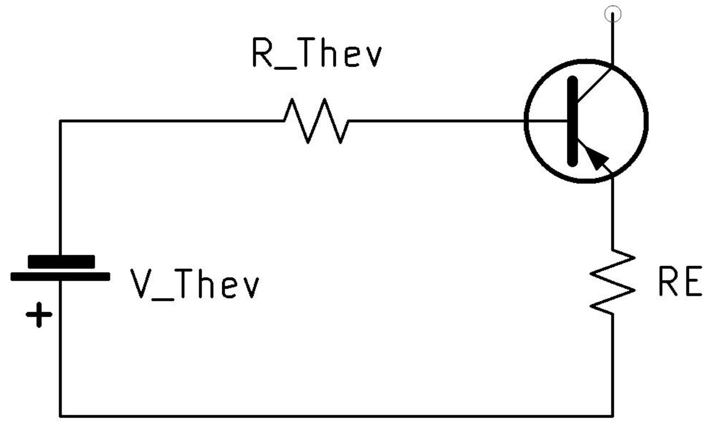

Figure 3.8: Thevenin Equivalent Circuit for Q2

Imagine that Q2 and RE (R10 and R11) are removed from the circuit.

R7 and R8 now make a series circuit, and V_{Thev} is equal to V_{R8}. See Figure 3.9

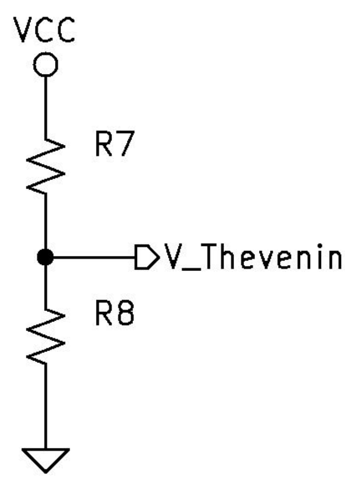

Figure 3.9: Thevenin Voltage

Find the Thevenin Resistance R_{Thev}.

R_{Thev} is equal to the total resistance that the load sees, for our example, R7 in parallel with R8.

$R_{Thev} = R7//R8$

Find the base current IB of the transistor.

Write a loop equation:

$V_{Thev} - V_{R\_Thev} - V_{BE} - V_{RE} = 0$

$$V_{R\_Thev} + V_{RE} = V_{Thev} - V_{BE}$$
$$IB(R\_Thev) + IE(RE) = V_{Thev} - V_{BE}$$
$$IB(R\_Thev) + (IB(\beta+1))(RE) = V_{Thev} - V_{BE}$$
$$IB\left((R\_Thev) + (\beta+1)(RE)\right) = V_{Thev} - V_{BE}$$

$$IB = \frac{V_{Thev} - V_{BE}}{(R\_Thev)+(\beta+1)(RE)}$$

Verify that IR8 is greater than or equal to IB. If IR8 is not greater than or equal to ten times IB, the circuit will need to be analyzed using Kirchhoff Analysis.

✓ $I_{R8} \geq 10 \times IB$

Universal Bias, Kirchhoff Analysis

Kirchhoff's Analysis of Universal Biasing requires two loop equations.

Kirchhoff Loop Equation 1 (solve for IR8 in terms of IR8 and IB):

$$VCC - V_{R7} - V_{R8} = 0$$
$$VCC - I_{R7}(R7) - I_{R8}(R8) = 0$$
$$VCC - (IB + I_{R8})(R7) - I_{R8}(R8) = 0$$
$$VCC - IB(R7) - I_{R8}(R7) - I_{R8}(R8) = 0$$
$$VCC - IB(R7) - I_{R8}(R7 + R8) = 0$$
$$I_{R8}(R7 + R8) = VCC - IB(R7)$$

✓ $I_{R8} = \frac{VCC - IB(R7)}{(R7+R8)}$

Kirchhoff Loop Equation 2 (solve for IR8 in terms of IB):

$$VCC - V_{R7} - V_{BE} - V_{RE} = 0$$
$$VCC - I_{R7}(R7) - 0.7 - IE(RE) = 0$$
$$VCC - (IB + I_{R8})(R7) - 0.7 - IB(\beta+1)(RE) = 0$$
$$VCC - IB(R7) - I_{R8}(R7) - 0.7 - IB(\beta+1)(RE) = 0$$
$$VCC - IB(R7) - 0.7 - IB(\beta+1)(RE) = I_{R8}(R7)$$
$$I_{R8}(R7) = VCC - IB(R7) - 0.7 - IB(\beta+1)(RE)$$
$$I_{R8}(R7) = VCC - 0.7 - IB(R7 + (\beta+1)(RE))$$

✓ $I_{R8} = \frac{VCC - 0.7 - IB(R7+(\beta+1)(RE))}{R7}$

Use the substitution method to solve for IB by setting each equation equal to the other.

$$\frac{VCC - IB(R7)}{(R7+R8)} = \frac{VCC - 0.7 - IB(R7+(\beta+1)(RE))}{R7}$$

$$R7(VCC - IBR7) = (R7 + R8)(VCC - 0.7 - IB(R7 + (\beta + 1)(RE))$$

$$R7VCC - IBR7^2 = (R7 + R8)(VCC - 0.7 - IBR7 - IB(\beta + 1)(RE))$$

$$R7VCC - IBR7^2 = R7VCC - R7(0.7) - IBR7^2 - IBR7(\beta+1)(RE) + R8VCC - R8(0.7) - IBR7R8 - IBR8(\beta + 1)(RE)$$

$$0 = -R7(0.7) - IBR7(\beta + 1)(RE) + R8VCC - R8(0.7) - IBR7R8 - IBR8(\beta + 1)(RE)$$

$$IBR7(\beta+1)(RE) + IBR7R8 + IBR8(\beta+1)(RE) = -R7(0.7) + R8VCC - R8(0.7)$$

$$IB\{R7(\beta + 1)(RE) + R7R8 + R8(\beta + 1)(RE)\} = R8(VCC - 0.7) - R7(0.7)$$

$$IB = \frac{R8(VCC-0.7)-R7(0.7)}{(R7)(\beta+1)(RE)+R7R8+R8(\beta+1)(RE)}$$

$$\checkmark\ IB = \frac{R8(VCC-0.7)-R7(0.7)}{(R7)(\beta+1)(RE)+R8(R7+(\beta+1)(RE))}$$

Universal Bias with Decoupling Resistor, Kirchhoff Analysis

Kirchhoff's Analysis of Universal Biasing requires two loop equations.

Kirchhoff Loop Equation 1 (solve for IR2 in terms of IB):

$$VCC - VR6 - VR1 - VR2 = 0$$
$$VCC - I_{R6}R6 - I_{R1}R1 - I_{R2}R2 = 0$$
$$VCC - (IC + I_{R1})R6 - I_{R1}R1 - I_{R2}R2 = 0$$
$$VCC - (IB(\beta) + I_{R2} + IB)R6 - (I_{R2} + IB)R1 - I_{R2}R2 = 0$$
$$VCC - IB(\beta)R6 - I_{R2}R6 - IBR6 - I_{R2}R1 - IBR1 - I_{R2}R2 = 0$$
$$I_{R2}R6 + I_{R2}R1 + I_{R2}R2 = VCC - IB(\beta)R6 - IBR6 - IBR1$$
$$I_{R2}(R6 + R1 + R2) = VCC - (IB)((\beta)R6 + R6 + R1)$$

✓ $I_{R2} = \frac{VCC-(IB)((\beta)R6+R6+R1)}{R6+R1+R2}$

Kirchhoff Loop Equation 2 (solve for IR2 in terms of IB):

$$-V_{BE} - V_{RE} + V_{R2} = 0$$
$$-0.7V - (IE)RE + I_{R2}R2 = 0$$
$$-0.7V - IB(\beta + 1)RE + I_{R2}R2 = 0$$
$$I_{R2}R2 = 0.7V + IB(\beta + 1)RE$$

✓ $I_{R2} = \frac{0.7V+IB(\beta+1)RE}{R2}$

Use the substitution method to solve for IB by setting each equation equal to the other.

$$\frac{VCC-(IB)((\beta)(R6+R6+R1)}{R6+R1+R2} = \frac{0.7V+IB(\beta+1)RE}{R2}$$

$$R2[VCC - (IB)((\beta)(R6 + R6 + R1)] = (R6 + R1 + R2)(0.7V + IB(\beta + 1)(RE)$$

$$VCC(R2) - (IB)(R2)((\beta)R6 + R6 + R1) = 0.7V(R6 + R1 + R2) + IB(\beta + 1)(RE)(R6 + R1 + R2)$$

$$VCC(R2) - 0.7V(R6 + R1 + R2) = (IB)(R2)((\beta)R6 + R6 + R1) + IB(\beta + 1)(RE)(R6 + R1 + R2)$$

$$VCC(R2) - 0.7V(R6+R1+R2) = (IB)\{(R2)((\beta)R6+R6+R1)+(\beta+1)(RE)(R6+R1 + R2)\}$$

$$\frac{VCC(R2)-0.7V(R6+R1+R2)}{\{(R2)((\beta)R6+R6+R1)+(\beta+1)(RE)(R6+R1+R2)\}} = (IB)$$

✓ $IB = \frac{VCC(R2)-0.7V(R1+R2+R6)}{R2(R1+R6+(\beta\times R6))+RE(\beta+1)(R1+R2+R6)}$

➥ Find all resistor voltages and powers.

Q2, Power and Load-line Analysis

Find V_{CE_Q2}

$$V_{CE\_Q2} = VCC - V_{R9} - V_{R10} - V_{R11}$$

Find IC

$$IC_{R9} = \frac{V_{R9}}{R9}$$

Find P_{Q2}

$$P_{Q2} = V_{CE\_Q2} \times IC_{R9}$$

Find IC_{Sat_Q2}

$$IC_{Sat\_Q2} = \frac{VCC}{R9+R10+R11}$$

Find V_{cut_Q2}

$$V_{cut\_Q2} = VCC$$

Find $v_{cut\_ac\_Q2}$

$$v_{cut\_ac\_Q2} = V_{CE\_Q2} + IC_{R9}(R12//R9)$$

Find ic_{ac_Q2}

$$ic_{ac\_Q2} = IC_{Sat\_Q2} + \frac{V_{CE\_Q2}}{(R12//R9)}$$

Depending on the biasing (Q point), the smaller of the two following equations will represent $Vout_{MaxP\_Q2}$. If the amplifier is designed for a Z_{out} (R9) that is ten times smaller than the load (R12) and the biasing is optimized, Vout Max Peak will be approximately equal to 90% of VCE.

$$Vout_{MaxP\_Q2} \approx 0.9(V_{CE\_Q2})$$

Vout max peak actual will be the smaller of the following:

$$VoutP_{Max\_Q2} = v_{cut\_ac\_Q2} - V_{CE\_Q2}$$

OR

$$VoutP_{Max\_Q2} = V_{CE\_Q2}$$

Q1, Power and Load-line Analysis

Find V_{CE_Q1}, $V_{CE\_Q1} = VCC - V_{R6} - V_{R3} - V_{R4} - V_{R5}$

Find IC, $IC_{R3} = \frac{V_{R3}}{R3}$

Find P_{Q1}, $P_{Q1} = V_{CE\_Q1} \times IC_{R3}$

Find $VoutP_{Max\_Q1}$, $VoutP_{Max\_Q1} \approx 0.9(V_{CE\_Q1})$

Q2, Voltage Gain Analysis

Find ΔV_{Q2}, $\Delta V_{Q2} \approx \frac{R9}{R10}$

$$\Delta V_{Q2} = \frac{V_{OUT}}{V_{IN}}$$

$$\Delta V_{Q2} = \frac{IC(R9//R12)}{IB(\beta+1)(R10+r'e)}$$

$$\Delta V_{Q2} = \frac{IC(R9//R12)}{IE(R10+r'e)}$$

$$\Delta V_{Q2} = \alpha\frac{(R9//R12)}{R10+r'e}$$

Q1, Voltage Gain Analysis

Find ΔV_{Q1}, $\Delta V_{Q1} \approx \frac{R3}{R4}$

$$\Delta V_{Q1} = \frac{V_{OUT}}{V_{IN}}$$

$$\Delta V_{Q1} = \frac{IC(R3//R7//R8//((R10+r'e)(\beta+1))}{IB(\beta+1)(R4+r'e)}$$

$$\Delta V_{Q1} = \frac{IC(R3//R7//R8//((R10+r'e)(\beta+1))}{IE(R4+r'e)}$$

$$\Delta V_{Q1} = \alpha\frac{R3//R7//R8//((R10+r'e)(\beta+1))}{R4+r'e}$$

Total Voltage Gain Analysis

Find ΔV_{Total},

$$\Delta V_{Total} = \Delta V_{Q1} \times \Delta V_{Q2}$$

Maximum Peak Input Voltage

Find $VinP_{Max}$,

$$VinP_{Max} = \frac{VoutP_{Max}}{\Delta V_{Total}}$$

3.7 Frequency Response Low

Rules and Steps for Calculating Frequency Response Low

Rules:

1. Treat all capacitors like opens.
2. Find $R_{Thevenin}$ for each capacitor.
3. Calculate Fc_{Low_C} for each capacitor:

$$Fc_{Low\_C} = \frac{1}{2\pi Rthev_C C}$$

4. Calculate Fc_{LOW} Total

$$FC_{Low} = \sqrt{(FC_{C1})^2 + (FC_{C2})^2 + (FC_{C3})^2 ...}$$

Steps:

Calculate the Capacitor Thevenin Resistances for Figure 3.1.

$$Rthev_{C1} = \{(R5 + R4 + r'e_{Q1})(\beta + 1)//R2//(R1 + R5)\} + R_{Gen}$$

$$Rthev_{C2} = \{(\frac{R2//(R1+R6)}{\beta+1}) + r'e_{Q1} + R4\}//R5$$

$$Rthev_{C3} = R6//\{R1 + (R2//((\beta + 1)(r'e_{Q1} + R4 + R5))\}$$

$$Rthev_{C4} = \{((R11 + R10 + r'e_{Q2})(\beta + 1))//R7//R8\} + \{(((((R5 + R4 + r'e_{Q1})(\beta + 1))//R2) + R1)//R6) + R3\}$$

$$Rthev_{C5} = R9 + R12$$

$$Rthev_{C6} = ((\frac{R7//R8}{\beta+1}) + r'e_{Q2} + R10)//R11$$

Design: Calculating capacitor values for a desired Frequency Critical Low

Choose a desired Frequency Critical Low $Fc_{Low\_Desired}$.

$$Fc_{Low\_Desired} = \sqrt{x^2 + x^2 + x^2 ...}$$

$$(Fc_{Low\_Desired})^2 = x^2 + x^2 + x^2 ...$$

$$(Fc_{Low\_Desired})^2 = (NumberOfCaps)x^2$$

$$x^2 = \frac{(Fc_{Low\_Desired})^2}{NumberOfCaps}$$

$$xhz = \sqrt{\frac{(Fc_{Low\_Desired})^2}{NumberOfCaps}}$$

Calculate the capacitance value for each capacitor using its $R_{Thevinin}$ and the previously calculated Xhz for the desired Frequency Critical Low.

$$C_X = \frac{1}{2\pi(Rthev_{Cx})(xhz)}(farads)$$

Week 4

Multi-Stage Amplifier: High-Frequency Response & Peaking

4.1 Objectives:

Multi-Stage Amplifier Design and Analysis:

High Frequency Response

- High Critical Frequency Fc_{High}: Understand and calculate the high critical frequency Fc_{High} for a multi-stage amplifier, taking into account the internal device capacitance and parasitic circuit elements in the amplifier stages.

Emitter Peaking

- Investigate the theory of Emitter Peaking.
- Calculate the Emitter Peaking capacitor and the Improvement Factor for a given circuit.

Shunt Peaking

- Investigate the theory of Shunt Peaking.
- Calculate the Shunt Peaking Inductor and the Improvement Factor for a given circuit.

Series Peaking

- Investigate the theory of Series Peaking.
- Calculate the Series Peaking Inductor and the Improvement Factor for a given circuit.

By achieving these objectives, students will develop a comprehensive understanding of an amplifier's High-Frequency Response and how to modify a circuit to improve frequency response using Emitter, Shunt, and Series Peaking.

4.2 Frequency Critical High:

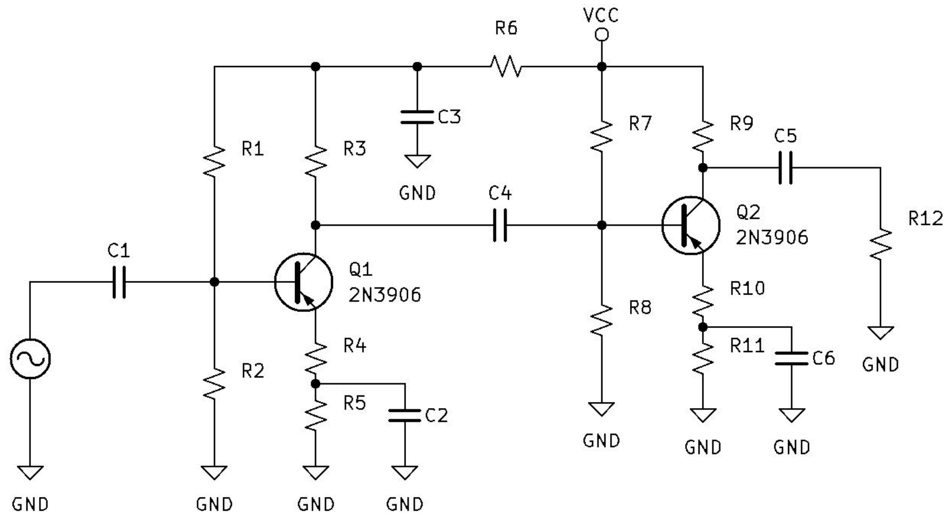

Figure 4.1: Two Stage Amplifier

4.2.1 Frequency Critical High Formula

For the two-stage amplifier, Frequency Critical High $FcHigh$ can be found using the following formula:

$$Fc_{High} = \frac{0.35}{\sqrt{(\frac{0.35}{FcH_{In}})^2 + (\frac{0.35}{FcH_{Mid}})^2 + (\frac{0.35}{FcH_{Out}})^2}}$$

SMALL–SIGNAL CHARACTERISTICS

Current–Gain – Bandwidth Product	(I_C = 10 mAdc, V_{CE} = 20 Vdc, f = 100 MHz)	f_T	250	–	MHz
Output Capacitance	(V_{CB} = 5.0 Vdc, I_E = 0, f = 1.0 MHz)	C_{obo}	–	4.5	pF
Input Capacitance	(V_{EB} = 0.5 Vdc, I_C = 0, f = 1.0 MHz)	C_{ibo}	–	10	pF
Input Impedance	(I_C = 1.0 mAdc, V_{CE} = 10 Vdc, f = 1.0 kHz)	h_{ie}	2.0	12	kΩ
Voltage Feedback Ratio	(I_C = 1.0 mAdc, V_{CE} = 10 Vdc, f = 1.0 kHz)	h_{re}	0.1	10	X 10^{-4}
Small–Signal Current Gain	(I_C = 1.0 mAdc, V_{CE} = 10 Vdc, f = 1.0 kHz)	h_{fe}	100	400	–
Output Admittance	(I_C = 1.0 mAdc, V_{CE} = 10 Vdc, f = 1.0 kHz)	h_{oe}	3.0	60	μmhos
Noise Figure	(I_C = 100 μAdc, V_{CE} = 5.0 Vdc, R_S = 1.0 kΩ, f = 1.0 kHz)	NF	–	4.0	dB

Figure 4.2: ON Semiconductor 2n3906 Data Sheet Excerpt

4.2.2 Frequency Critical High In

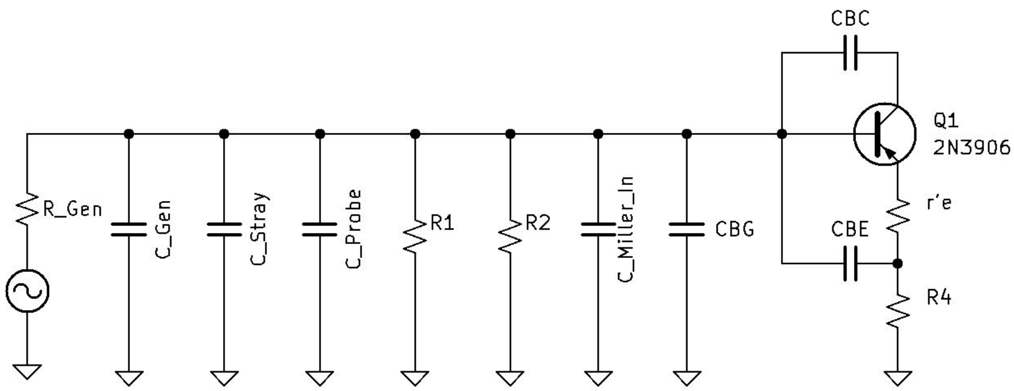

Figure 4.3: Input AC equivalent circuit at High Frequencies

Calculate the total capacitance at the input of the amplifier.

$$Ctotal_{IN} = C_{Gen} + C_{Stray} + C_{Probe} + C_{MillerIN} + C_{BG}$$

$$C_{Gen} = Specification\ in\ Manual$$
$$C_{Stray} \approx 10pF$$
$$C_{Probe} \approx 16pF, Specification\ in\ Manual$$
$$C_{MillerIN} = C_{OBO}(1 + \Delta V_{CE(Q1)})$$
$$C_{OBO} = transistor\ datasheet\ specification$$
$$\Delta V_{CE} = \frac{Vout}{Vin} = \frac{ic(RC//RL)}{ie(r'e+RE)} = \alpha\frac{RC//RL}{r'e+RE} = \alpha\frac{R3//RL_{eq}}{r'e+R4}$$
$$C_{BG} = C_{BE}(1 - \Delta V_{CC(Q1)})$$
$$C_{BE} = \frac{1}{2\pi f_\tau r'e}$$
$$f_\tau = transistor\ datasheet\ specification$$
$$\Delta V_{CC} = \frac{RE}{r'e+RE} = \frac{R4}{r'e+R4}$$

Calculate the Thevenin resistance for C_Total_{In}.

$$R\_Thevenin_{C\_TotalIn} = (R4 + r'e)(\beta + 1)//R2//R1//R_{Gen}$$

$$R\_Thevenin_{C\_TotalIn} \approx R_{Gen}$$

Calculate the Input High Critical Frequency.

$$FcH_{In} = \frac{1}{2\pi \times R\_Thev_{C\_TotalIn} \times C\_Total_{IN}}$$

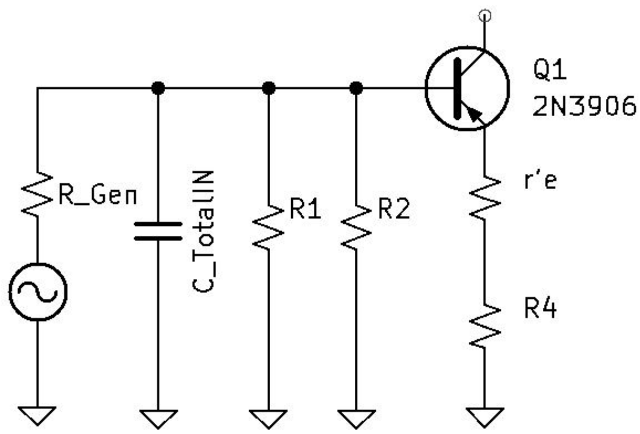

Figure 4.4: Capacitance Total In equivalent circuit at High Frequencies

4.2.3 Frequency Critical High Middle

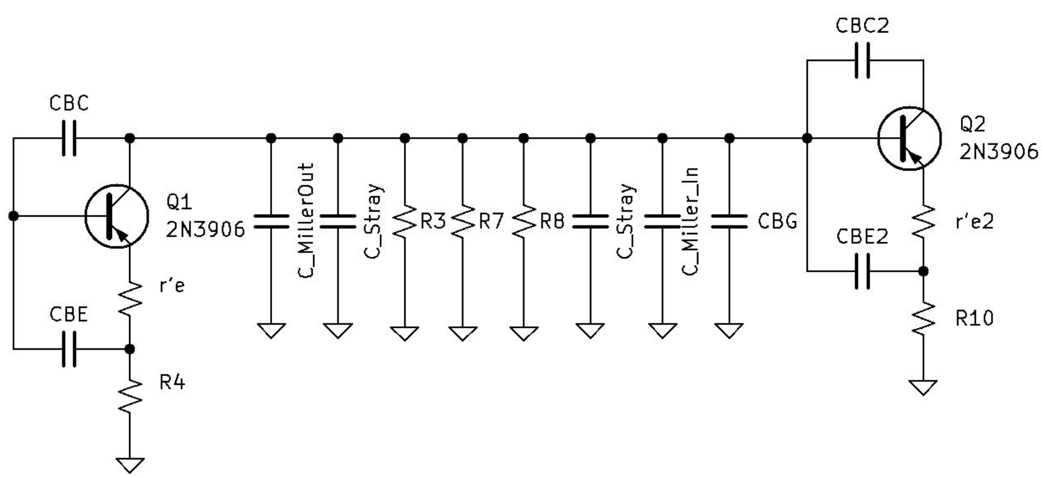

Figure 4.5: Middle AC equivalent circuit at High Frequencies

Calculate the total capacitance at the middle stage of the amplifier.

$$Ctotal_{Mid} = C_{MillerOut} + C_{Stray} + C_{Stray} + C_{MillerIN} + C_{BG}$$

$$C_{MillerOut} = CBC(\frac{1+\Delta V_{CE(Q1)}}{\Delta V_{CE(Q1)}})$$

$$CBC \approx C_{OBO}(Data\ Sheet)$$

$$C_{Stray} \approx 10pF$$

$$C_{MillerIN} = C_{OBO}(1 + \Delta V_{CE(Q2)})$$

$$C_{OBO} = transistor\ datasheet\ specification$$
$$\Delta V_{CE(Q2)} = \frac{Vout}{Vin} = \frac{ic(RC//RL)}{ie(r'e+RE)} = \alpha\frac{RC//RL}{r'e+RE} = \alpha\frac{R9//R12}{r'e+R10}$$
$$C_{BG} = C_{BE}(1 - \Delta V_{CC})$$
$$C_{BE} = \frac{1}{2\pi f_{\tau} r'e}$$
$$f_{\tau} = transistor\ datasheet\ specification$$
$$\Delta V_{CC(Q2)} = \frac{RE}{r'e+RE} = \frac{R10}{r'e+R10}$$

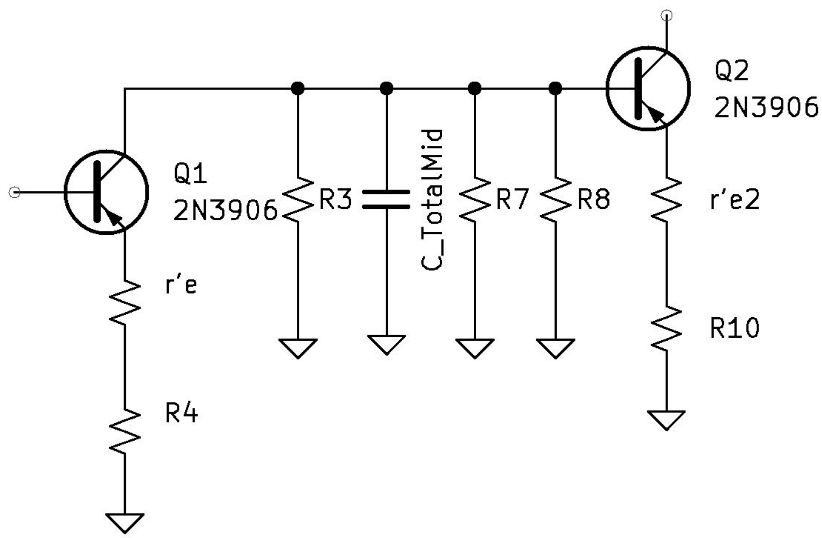

Figure 4.6: Capacitance Total Middle AC equivalent circuit at High Frequencies

Calculate the Thevenin resistance for C_Total_{Mid}.

$$R\_Thevenin_{C\_TotalMid} = (R10 + r'e2)(\beta + 1)//R3//R7//R8$$

Calculate the Input High Critical Frequency.

$$FcH_{Mid} = \frac{1}{2\pi \times R\_Thev_{C\_TotalMid} \times C\_Total_{Mid}}$$

4.2.4 Frequency Critical High Out

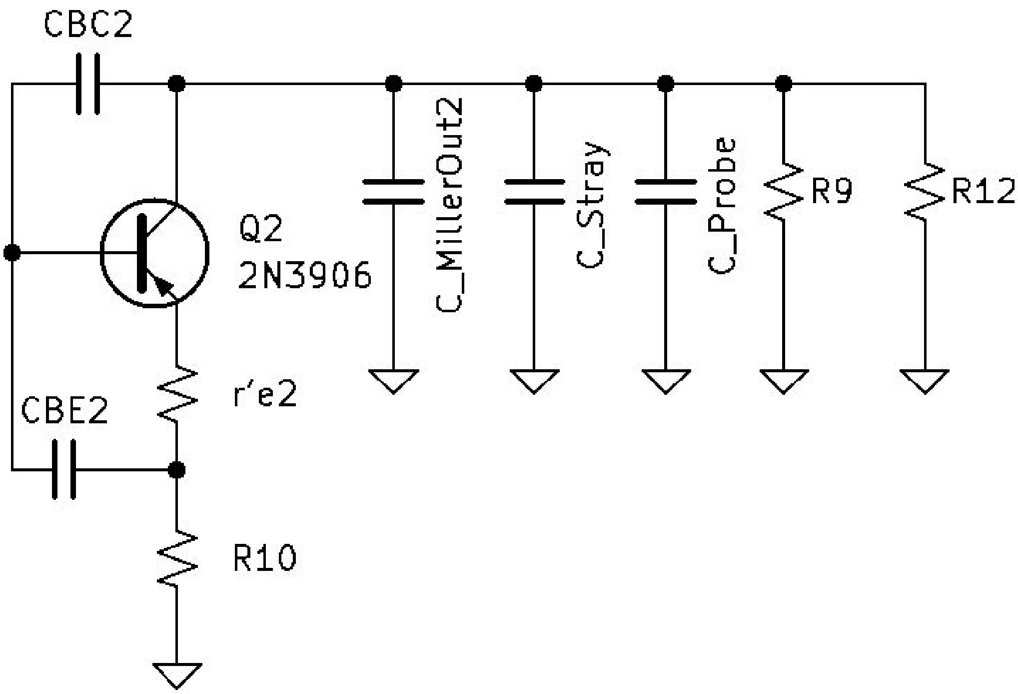

Figure 4.7: Output AC equivalent circuit at High Frequencies

Calculate the total capacitance at the output stage of the amplifier.

$$Ctotal_{Out} = C_{MillerOut2} + C_{Stray} + C_{Probe}$$

$$C_{MillerOut2} = CBC(\frac{1+\Delta V_{CE(Q2)}}{\Delta V_{CE(Q2)}})$$

$$CBC \approx C_{OBO}(Data\ Sheet)$$

$$C_{Stray} \approx 10pF$$

$$C_{Probe} \approx 16pF, Specification\ in\ Manual$$

Calculate the Thevenin resistance for C_Total_{Out}.

$$R\_Thevenin_{C\_TotalOut} = R9//R12$$

Calculate the Out High Critical Frequency.

$$FcH_{Mid} = \frac{1}{2\pi \times R\_Thev_{C\_TotalMid} \times C\_Total_{Mid}}$$

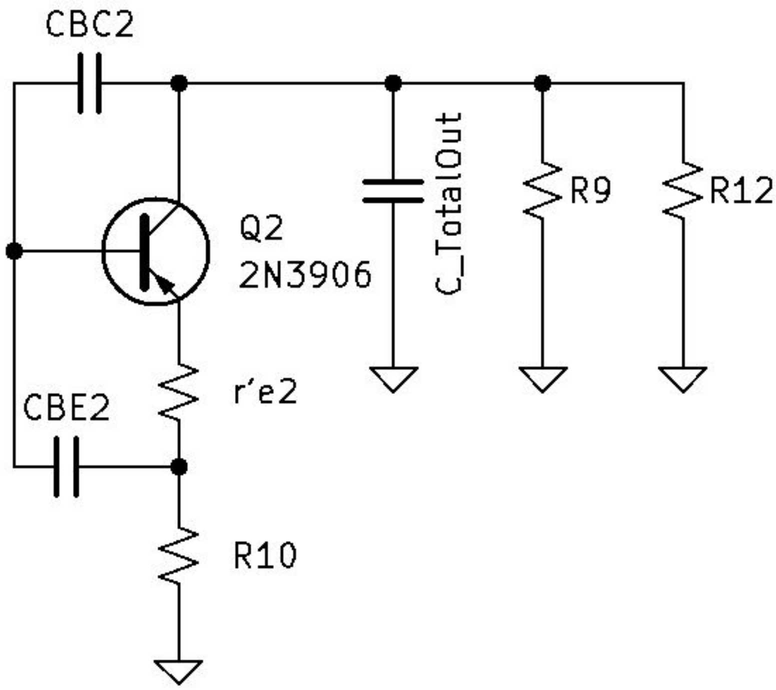

Figure 4.8: Capacitance Total Out AC equivalent circuit at High Frequencies

4.2.5 Frequency Critical High final calculation

Calculate the High Critical Frequency for the circuit using the previously calculated FcH_{In}, FcH_{Mid}, and FcH_{Out}.

$$Fc_{High} = \frac{0.35}{\sqrt{(\frac{0.35}{FcH_{In}})^2 + (\frac{0.35}{FcH_{Mid}})^2 + (\frac{0.35}{FcH_{Out}})^2}}$$

4.3 Emitter Peaking

4.3.1 Voltage Gain at Frequency Critical High

See figure 4.8 from the previous section. If we evaluate the ΔV formula we can see that at the FcH the numerator of the formula begins to roll off due to the parallel capacitive reactance.

$\Delta V = \frac{Vout}{Vin}$

$\Delta V_{MidBand} = \alpha \frac{RL//RC}{RE+r'3} = \alpha \frac{R9//R12}{R10+r'e2}$

$\Delta V_{FcHigh} = \alpha \frac{R9//R12//XC_{C\_Out}}{R10+r'e2}$

4.3.2 Emitter Peaking Explained

Knowing that the numerator is rolling off at a 20db/decade rate at the High Critical Frequency and that delta-V gap or difference is shrinking between the output voltage and the input voltage, how can we extend or prolong the difference between the output and the input?

One way is to apply Emitter Peaking. If the numerator is rolling off at FcHigh, can we also roll off the denominator of the formula at the same time? As both the numerator and the denominator roll the difference between them stays the same maintaining the gain. This can be achieved by placing an Emitter Peaking Capacitor in parallel R10 in our formula.

4.3.3 Solving for Emitter Peaking Capacitance

$\Delta V_{FcHigh} = \alpha \frac{R9//R12//XC_{C\_Out}}{(XC_{EP}//R10)+r'e2}$

Solve for the Emitter Peaking Capacitance value:

$XC_{EP} = R10$ at Fc_{High}

$C_{EP} = \frac{1}{2\pi \times R10 \times Fc_{High}}$

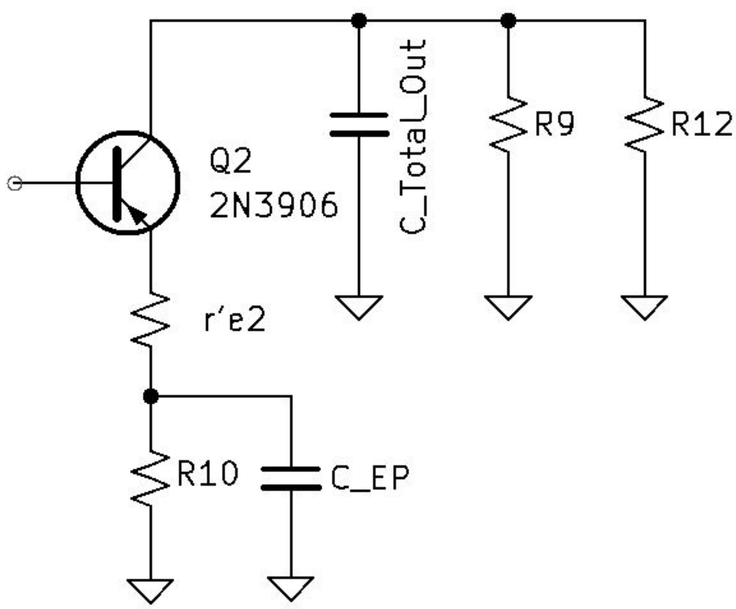

Figure 4.9: Emitter Peaking AC equivalent circuit

References

[1] Tektronix, "Tektronix xyzs of oscilloscopes." https://download.tek.com/document/03W_8605_7_HR_Letter.pdf. (accessed Sept. 18, 2023).

[2] "Tektronix tds user manual." https://download.tek.com/manual/071273300web_0.pdf. Accessed: 2023-09-18.

[3] "Tektronix afg1022 user manual." https://download.tek.com/manual/AFG1022-Quick-Start-User-Manual-EN.pdf. Accessed: 2023-09-18.

[4] "Power supply gps4303 user manual." https://www.gwinstek.com/en-global/products/downloadSeriesDownNew/8083/1210. Accessed: 2023-09-18.

[5] "Get more from your basic oscilloscope with the fft function." https://www.tek.com/en/blog/get-more-your-basic-oscilloscope-fft-function. Accessed: 2023-09-18.

[6] *Solid State Pulse Circuits 4Th Ed.* Prentice-Hall Of India Pvt. Limited, 2006.

[7] "Desmos graphing calculator." https://www.desmos.com/calculator. Accessed: 2023-12-16.

[8] "Fast fourier transformation fft - basics." https://www.nti-audio.com/en/support/know-how/fast-fourier-transform-fft. Accessed: 2023-12-15.

[9] "What is fft - tektronix." https://www.tek.com/en/support/faqs/what-fft-fast-fourier-transform-math-function-oscilloscope-useful. Accessed: 2023-12-19.