RCET 1372

Applied Calculus

May 1, 2025

Timothy S. Leishman

Clinical Assistant Professor

College of Technology

Idaho State University

Contents

1 Introduction to Derivatives **1**

1.1 Homework 1

2 Derivatives: Product & Quotient Rules, Powers, and Implicit Differentiation **3**

2.1 Homework 3

3 Derivatives Applied part 1 **5**

3.1 Homework 5

4 Derivatives Applied part 2 **7**

4.1 Homework 7

5 Introduction to Integrals **11**

5.1 Homework 11

6 Integrals Applied **15**

6.1 Homework 15

7 Logarithms **19**

7.1 Homework 19

8 Circles **23**

8.1 Homework 23

9 Parabolas **27**

9.1 Homework 27

10 Max-Mins, Differentials, & Higher Derivatives **31**

10.1 Homework 31

11 Differentials Applied **35**
11.1 Homework 35

12 Limits **37**
12.1 Homework 37

13 Trigonometric Functions part 1 **39**
13.1 Homework 39

14 Trigonometric Functions part 2 **43**
14.1 Homework 43

Appendices **45**

Week 1

Introduction to Derivatives

1.1 Homework

Week 1 Questions and Answer Key

- Day 1, See Canvas
- Day 2, questions 1-4
- Day 3, questions 5-8
- Day 4, questions 9-12

1. $y = 3x^2$ $\quad \frac{dy}{dx} = 6x$

2. $y = x^2 - 2$ $\quad \frac{dy}{dx} = 2x$

3. $y = x^2 - 3x$ $\quad \frac{dy}{dx} = 2x - 3$

4. $y = \frac{1}{x}$ $\quad \frac{dy}{dx} = \frac{-1}{x^2}$

5. $y = \frac{2}{(x-3)}$ $\quad \frac{dy}{dx} = \frac{-2}{(x-3)^2}$

6. $y = \frac{1}{(4-x^2)}$ $\quad \frac{dy}{dx} = \frac{2x}{(4-x^2)^2}$

7. $y = \sqrt{x+1}$ $\quad \frac{dy}{dx} = \frac{1}{2\sqrt{x+1}}$

8. $y = \frac{1}{\sqrt{x-1}}$ $\frac{dy}{dx} = \frac{-1}{2(x-1)^{\frac{3}{2}}}$

9. $y = \frac{5}{2}x^8 - \frac{6}{5}x^5 + \frac{15}{2}x^4 - x^3 + \sqrt{2}$ $\frac{dy}{dx} = 20x^7 - 6x^4 + 30x^3 - 3x^2$

10. $y = 3x^2 + 2x - 1; a = -1$ $f' = -4$

11. $y = 2x^3 - 6x^2 + 2x + 9; a = -3$ $f' = 92$

12. Find the equation of the Tangent Line to the curve $y = x^3 + 4x^2 - x + 2$ at (-2,12).
    $y = -5x + 2$

Week 2

Derivatives: Product & Quotient Rules, Powers, and Implicit Differentiation

2.1 Homework

Week 2 Questions and Answer Key

- Day 1, MLK
- Day 2, questions 1-4
- Day 3, questions 5-8
- Day 4, questions 9-12
- Day 5, questions 13-18

1. $y = x^2(2x+1)$ $\quad \frac{dy}{dx} = 6x^2 + 2x$

2. $y = (2x+3)(5x-4)$ $\quad \frac{dy}{dx} = 20x + 7$

3. $y = (x^2+3x+4)(x^3-4x)$ $\quad \frac{dy}{dx} = 5x^4 + 12x^3 - 24x - 16$

4. *Find* f_2' *when* $f_x = (x^2 - 4x + 3)(x^3 - 5x)$ $\quad f_2' = -7$

5. $y = \frac{(x-1)}{(x^2+x+1)}$ $\quad \frac{dy}{dx} = \frac{-x^2+2x+2}{(x^2+x+1)^2}$

6. $y = \frac{4x^2+9}{3x^3-4x^2}$ $\quad \frac{dy}{dx} = \frac{-12x^4-81x^2+72x}{(3x^3-4x^2)^2}$

7. $y = \frac{3x-1}{2x+4}$ $\quad \frac{dy}{dx} = \frac{14}{(2x+4)^2}$ OR $\frac{7}{2(x+2)^2}$

8. *Find* f'_{-1} *when* $f_x = \frac{3x-4}{x+2}$ $\quad f'_{-1} = 10$

9. $y = x^3(x^3-x)^3$ $\quad \frac{dy}{dx} = (12x^5 - 6x^3)(x^3-x)^2$

10. $y = (3x+4)^{\frac{3}{4}}(4x^2+8)$ $\quad \frac{dy}{dx} = \frac{33x^2+32x+18}{(3x+4)^{\frac{1}{4}}}$

11. $y = \frac{(x^3+2)^4}{4x^2-3x}$ $\quad \frac{dy}{dx} = \frac{(x^3+2)^3(40x^4-33x^3-16x+6)}{(4x^2-3x)^2}$

12. $y = \frac{(3x+2)^5}{(2x-1)^3}$ $\quad \frac{dy}{dx} = \frac{(3x+2)^4(12x-27)}{(2x-1)^4}$

13. *Find the slope of the line tangent to the curve* $y = \frac{x-3}{2-5x}$ *at the point* $(2, \frac{1}{8})$.
$m_{tan} = \frac{-13}{64}$

14. *Find the equation of the tangent line at the given point in the previous question.*
$y = \frac{-13x}{64} + \frac{17}{32}$

15. $4x + 3y = 7$ $\quad y' = \frac{-4}{3}$

16. $x^2 - y^2 = 9$ $\quad y' = \frac{x}{y}$

17. $y^4 - y^2x + x^2 = 0$ $\quad y' = \frac{y^2-2x}{4y^3-2xy}$

18. $3x^2y^2 + 4y^5 + 8x^2y^3 + xy = 5$ $\quad y' = \frac{-6xy^2-16xy^3-y}{6x^2y+20y^4+24x^2y^2+x}$

Week 3

Derivatives Applied part 1

3.1 Homework

Week 3 Questions and Answer Key

- Day 1, Review
- Day 2, Test
- Day 3, questions 1-4
- Day 4, questions 5-9
- Day 5, questions 10-13

1. If the current in a $1\mu F$ capacitor is to be $0.1mA$, at what rate in volts per second must the applied voltage change? $\frac{dy}{dx} = 100v/s$

2. The magnetic flux through a 500-turn winding varied according to $\phi = 0.004t$ webers. Find the induced voltage in the winding (a.) when $t = 0.01$ seconds and (b.) when $t = 0.1$ seconds. $v_{ind} = -2v$

3. If the flux through a 150-turn winding varied according to the formula $\phi = 0.01t - t^2 + 0.2$ webers, what voltage was induced when $t = 0.02$ seconds? $v_{ind} = 4.5v$

4. The magnetic flux N in a winding of 600 turns varied as $\phi = 0.5t^{\frac{3}{5}}$ webers, where t was in seconds. Find the induced voltage v_{ind} when $t = 1$ second. $v_{ind} = -180v$

5. What formula expresses the voltage v_{ind} across a $100mh$ inductor if the current i constantly equals $0.2A$? Neglect resistance. $v_{ind} = 0v$

6. How fast does the current in a $12h$ winding change to cause an induced voltage of $3.6v$? $\frac{di}{dt} = -300mA/sec$

7. The mutual inductance between two windings is 0.2 henrys. If a current $i_1 = 11t^{\frac{3}{2}}$ amps flows in the primary windings, how much voltage v_2 is induced in the secondary winding when $t = 0.001$ seconds? $v_2 = -104.355mV$

8. The mutual inductance between two windings is $M = 6h$. How fast must the current in one of the windings vary in amps per second to induce -4.8 volts in the other winding? $\frac{di}{dt} = 800mA/sec$

9. A winding linked a magnetic field that varied according to $\phi = 0.002t - 2t^2$ webers. When t was 0.0025 seconds, the voltage induced in the winding measured 8 volts. How many turns did the winding include? $N = 1000$ turns

10. If the current in a $30h$ inductor changes according to $i = 0.02t^{\frac{5}{3}}$ amps, after what interval will the induced voltage measure -96 volts? $t = 940.604$ seconds

11. A voltage, $v = t^3 + 1,000$ volts appears across a parallel RC combination, where $R = 300K\Omega$ and $C = 20\mu F$. Find the resulting current i_g at any time t. $i_g = 3.333 \times 10^{-6}t^3 + 60 \times 10^{-6}t^2 + 3.333 \times 10^{-3}$ amps

12. A $50K\Omega$ bleeder resistor shunts a $4\mu f$ filter capacitor. During a part of the charging process, the voltage across the capacitor varies approximately as $vc = 1,000t^{\frac{2}{3}} + 100$ volts. Find the current ig applied to the combination when $t = 0.001$ seconds. $ig = 28.867mA$

13. A current $i = 3t^{\frac{1}{3}} + 2$ amps flows through a series RL circuit, where $R = 100\Omega andL = 20h$. Find the voltage vg across this circuit when $t = 0.125$ seconds. $vg = 270v$

Week 4

Derivatives Applied part 2

4.1 Homework

Week 4 Questions and Answer Key

- Day 1, questions 14-17
- Day 2, questions 18-21
- Day 3, questions 22-25
- Day 4, questions 26-29
- Day 5, Review

14. A relay winding has an inductance of 0.5h and a resistance of 470Ω. If the winding current i equals $t^{\frac{1}{2}} + 0.02$ amps, find the voltage vg across the winding when $t = 0.01$ seconds. $vg = 53.9v$

15. A series circuit consists of a 22h inductor and a 68Ω resistor. A current $i = 2t^2 + t$ exists in this combination. After what time t does the voltage across the combination equal 375 volts? $t = 1.784$ seconds

16. A voltage $v = t^3 + 1,000$ volts appears across a parallel RC combination, where $R = 2M\Omega$ and $C = 1\mu F$. Find the resulting current ig at any time t. $ig = 3 \times 10^{-6}t^2 + 500 \times 10^{-9}t^3 + 500 \times 10^{-6}$

17. A transistor operates into a load resistance of $2.2K\Omega$. The shunt capacitance in the circuit equals $70pf$, as measured at the collector. Over a certain interval the output voltage supplied by the transistor equals $v = 1x10^7t + 30$ volts. Find the collector signal current when $t = 10\mu s$. $ic = 59.791mA$

18. A current $i = 10t^{\frac{1}{2}} + 0.1$ amps flows through a series RL circuit, where $R = 800\Omega$ and $L = 320h$. Find the voltage vg across this circuit when $t = 0.04$ seconds. $vg = -6.32Kv$

19. A transistor collector has a load resistor of $4.7K\Omega$ with a compensation inductor $L = 20mh$ in series with the resistor. The current i through the combination equals $2.5x10^4t + 0.01$ amps. Find the voltage across the RL circuit when $t = 25ns$.
$vg = -450.063v$

20. A $27K\Omega$ resistor shunts a $33\mu f$ capacitor. The applied voltage v equals $300t^2$ volts. At what time t does the total current i equal $84mA$?
$t = 1.999s$

21. The voltage applied across a capacitor of $0.2\mu f$ was $v = 5 - 3t^2$ volts. The energy stored in a capacitor is $w = \frac{Cv^2}{2}$ joules. Find a formula for $\frac{dw}{dt}$ in this capacitor.
$\frac{dw}{dt} = -1.2 \times 10^{-6}t(5 - 3t^2)$ OR $3.6 \times 10^{-6}t^3 - 6 \times 10^{-6}t$

22. The intensity I of light from a tungsten filament varies with the applied voltage according to $I = Av^{3.7}$, where A is a constant and v is the applied voltage. If $v = t - 2t^2$, find a formula for $\frac{dI}{dt}$.
$\frac{di}{dt} = 3.7A(t - 2t^2)^{2.7}(1 - 4t)$

23. When a length l meters of a conductor moves at a speed of v meters per second in a magnetic field of uniform flux density β teslas, a voltage is induced equal to $v = -\beta lv$ volts. If $v = 10$ meters per second, $l = 0.3$ meter, and β varies over a certain interval according to $\beta = \frac{1}{t^2}$, find $\frac{dv}{dt}$ when $t = 0.5$ seconds.
$\frac{dv}{dt} = 48v/sec$

24. The frequency of a certain crystal oscillator varies with temperature T according to $f = fa[1+k(T-Ta)]$, where fa is the frequency at an initial temperature Ta and k is a constant of the crystal. If T varies with time (t minutes) according to $T = 55+0.01t^2$, how fast does f change when $t = 10$?
$\frac{df}{dt} = fak(0.2)$

25. The wavelength λ meters of a radio wave traveling at a speed $c = 3 \times 10^8$ meters per second varies with the frequency according to $\lambda = \frac{c}{f}$. If $f = 1 \times 10^8 + (5 \times 10^7)t^{\frac{1}{2}}$ hertz find a formula for $\frac{d\lambda}{dt}$.
$\frac{d\lambda}{dt} = \frac{-7.5\times10^{15}}{(1\times10^8+5\times10^7t^{\frac{1}{2}})^2t^{\frac{1}{2}}}$ seconds

26. The voltage v across a varying resistor r, carrying a fixed current I, is $v = Ir$. If r varies with time t according to $r = t^3 + 5$, find a formula for $\frac{dv}{dt}$ in this capacitor.
$\frac{dv}{dt} = 3t^2I$

27. The mutual inductance between two windings is $M = \frac{N_2\phi_2}{1_1}$, where i_1 is the current in one of the windings and N_2 and ϕ_2 are the number of turns of the second winding and the flux linking it to the first winding. If i_1 and N_2 are constant, and if the second winding moves so that ϕ_2 varies with time t seconds according to $\phi_2 = t^3 - 2t$, find a formula for $\frac{dm}{dt}$.
$\frac{dm}{dt} = \frac{n_2}{i_1}(3t^2 - 2)$

28. A copper wire of diameter d and length s has a resistance of $r = \frac{ks}{d^2}$, where k is a constant. Suppose a sliding wire changes the length so that $s = t^2 - 0.6t$, where t is in seconds. Find a formula for $\frac{dr}{dt}$.
$\frac{dr}{dt} = \frac{k}{d^2}(2t - 0.6)$

29. The force between two charged particles having fixed charges Q_1 and Q_2 varies with the distance separating them according to $f = \frac{Q_1 Q_2}{4\pi\epsilon s^2}$. If ϵ is a constant, and if s varies with time as $s = 6t^{\frac{3}{2}}$, find a formula for $\frac{df}{dt}$.
$\frac{df}{dt} = \frac{-Q_1 Q_2}{48\pi\epsilon t^4}$

Week 5

Introduction to Integrals

5.1 Homework

Week 5 Questions and Answer Key

- Day 1, Test
- Day 2, questions 1-5
- Day 3, questions 6-10
- Day 4, questions 11-13
- Day 5, questions 14-16

1. $y = \int x^7 \, dx$

   $y = \frac{x^8}{8} + C$

2. $y = \int \frac{6}{x^3} \, dx$

   $y = \frac{-3}{x^2} + C$

3. $y = \int \sqrt{6x+2} \, dx$

   $y = \frac{(6x+2)^{\frac{3}{2}}}{9} + C$

4. $y = \int x\sqrt[3]{5x^2-1} \, dx$

$y = \frac{3(5x^2-1)^{\frac{4}{3}}}{40} + C$

5. $y = \int (3x^2+2)(x^3+2x)^3 \ dx$

   $y = \frac{(x^3+2x)^4}{4} + C$

6. $y = \int (10x-1)\sqrt{5x^2-x} \ dx$

   $y = \frac{2(5x2-x)^{\frac{3}{2}}}{3} + C$

7. $y = \int (2x+3)^2 \ dx$

   $y = \frac{(2x+3)^3}{6} + C$

8. $y = \int 4x(x^2+1)^3 \ dx$

   $y = \frac{(x^2+1)^4}{2} + C$

9. $y = \int (6x^2+6)(x^3+3x)^{\frac{-1}{3}} \ dx$

   $y = 3(x^3+3x)^{\frac{2}{3}} + C$

10. $\int (x-1)(x)^{-3} \ dx$

    $y = \frac{-1}{x} + \frac{1}{2x^2} + C$

11. Find the equation describing the distance of an object moving along a straight line when the acceleration is $a = 3t$, when the velocity at $t = 4s$ is $40m/s$, and when the object has traveled 86m from the origin at $t = 2s$.

    $s = \frac{t^3}{2} + 16t + 50$

12. A stone is dropped from a height of 100ft. For a free-falling object, the acceleration is $a = -32ft/s^2$ (gravity). A. Find the distance the stone has traveled after 2 seconds. Note that the initial velocity is 0 because the stone was dropped, not thrown. B. Find also the velocity of the stone when it hits the ground.

    $v = -80ft/sec$

13. A stone is hurled straight up from the ground at a velocity of 25m/sec. A. Find the maximum height that the stone reaches. B. How long does it take for the stone to hit the ground? C. Find the speed at which the stone hits the ground.

    a. $s = 31.888m$
    b. $t = 5.102sec$
    c. $v = -25m/sec$

14. A stone is thrown vertically upward from the roof of a 200ft tall building with an initial velocity of 30ft/sec. A. Find the equation describing the altitude of the stone from the ground. B. How long does it take for the stone to hit the ground?

    a. $s = -16t^2 + 30t + 200$
    b. $t = 4.594sec$

15. A stone is thrown straight down from an 80-meter-tall building with an initial velocity of 10m/sec. A. Find the equation describing the height of the stone from the ground. B. How long does it take for the stone to hit the ground?

    a. $s = -4.9t^2 + 10t + 80$
    b. $t = 3.147sec$

16. An object is dropped from a stationary ballon at 500m. A. Express the objects height above the ground as a function of time. B. How long does it take the object to hit the ground?

    a. $s = -4.9t^2 + 500$
    b. $t = 10.102sec$

Week 6

Integrals Applied

6.1 Homework

Week 6 Questions and Answer Key

- Day 1, Presidents Day
- Day 2, Integrals Test
- Day 3, questions 1-4
- Day 4, questions 5-9
- Day 5, questions 10-14

1. The current in a circuit was $i = 4t^3$ amps. How many coulombs were transmitted in 3 seconds?

   $q = 81$ coulombs

2. A $80\mu f$ capacitor is charged to 100 volts. We then apply a current $i_c = 0.04t^3$ amps in the same polarity as the initial charge. After how many seconds will the capacitor voltage reach 225 volts?

   $t = 1$ second

3. The voltage applied to a circuit was $v = 2t + 1$ volts. If the current followed the equation $i = 0.03t$ amperes, find the energy w delivered from $t = 0$ to $t = 50$ seconds.

   $w = 2.5375$K watts

4. A 110 turn winding caries a flux of 0.8 webers. If we now want to vary the flux so that a voltage $v_{ind} = -5t^2$ volts appears in the winding, what equation must the flux through the winding follow?

   $\phi = \frac{t^3}{66} + 0.8$ webers

5. A DC current of 0.3 ampere flows in a 15 henry inductor. Superimposed on this DC is a varying current such that the voltage $v_{ind} = 120t^{\frac{1}{3}}$ volts appears in the inductor. Find the instantaneous total current when $t = 1$ second (assume that the DC and AC currents have the same polarity when $t = 1$ second).

   $i = 6.3$ or -6.3 amps

6. An inductance of 8 henrys is connected in parallel with a 12Ω resistor. Apply to this circuit a voltage $v = 20t^2$ volts. If $i = 0$ when $t = 0$, find an equation for i.

   $ig = \frac{5t^2}{3} - \frac{5t^3}{6}$ amps

7. If we apply a voltage $v = 90t^{\frac{1}{2}}$ to a circuit consisting of a 30 henry inductance shunted by a 50Ω resistance, what current flows when $t = 4$ seconds? (let $i = 0$ when $t = 0$).

   $ig = -12.4$ amps

8. If we apply a voltage $v = 20t^4$ volts across a parallel RL combination, where $R = 500\Omega$ and $L = 40$ henrys, find the total current when $t = 0.2$ seconds. Let $i = 4\mu$A when $t = 0$.

   $ig = 36\mu$A

9. In a parallel RL circuit, $R = 5\Omega$ and $L = 0.2$ henrys. If a voltage $v = t^{\frac{3}{2}} + 2$ volts were applied, what would the current i be when $t = 4$ seconds? Assume $i = 0.4$ amps when $t = 0$.

   $ig = 101.6$ amps

10. A current $i = 0.005t^{\frac{1}{2}}$ amps flows in a parallel RC circuit where $R = 8.8 \times 10^4\Omega$ and $C = 1\mu f$. Find a formula for the voltage across the circuit as a function of time t. Assume the capacitor to be initially discharged.

$vg = 440t^{\frac{1}{2}} + 3.333 \times 10^{3}t^{\frac{3}{2}}$ volts

11. The current function $i = 1 \times 10^{-3}t^{\frac{1}{2}}$ amperes is applied to a series RC circuit where $R = 8.8 \times 10^{4}\Omega$ and $C = 1\mu f$. Find a formula for the impressed voltage as a function of time t. (Assume the initial capacitor charge to be 100v.)

$vg = 88t^{\frac{1}{2}} + 666.667t^{\frac{3}{2}} + 100$ volts

12. A series LC circuit where $L = 0.1$ henry and $C = 100\mu f$ has applied to it a current $i = 0.1$A from $t = 0$ onward. Find (a) the formula for the voltage across the circuit, and (b) the rate of change at $t = 2$ sec. (assume $v_c = 0$ when $t = 0$)

    a. $vg = 1000t$

    b. $\frac{dv}{dt} = 1000v/s$

13. A series circuit has these constants: $R = 5K\Omega$, $L = 200$ henrys, and $C = 20\mu f$. If we supply to the circuit a current $i = 0.02t^2$ amperes, at what rate does the voltage across the circuit change when $t = 0.2$ seconds?

    $\frac{dv}{dt} = 72v/s$

14. In a series RCL circuit, let $R = 10\Omega$, $C = 10,000\mu f$, and $L = 10$ henrys. Through this circuit we pass a current $i = 1 - t^{\frac{1}{2}}$ amps. Find the total voltage v across this circuit when $t = 4$ seconds. Assume $v = 0.25$ volts when $t = 1$ second.

    a. $vg = -178v$

Week 7

Logarithms

7.1 Homework

Week 7 Questions and Answer Key

- Day 1, Review
- Day 2, Test (Integrals Applied)
- Day 3, questions 1-5
- Day 4, questions 6-10
- Day 5, questions 11-15

1. Given the equation $m^{2.3} = 25$, solve for m.

   $m = 4.053$

2. Given the equation $x = log_3 2187$, solve for x.

   $x = 7$

3. Given the equation $L_1 = ((L_2)^2)^{\frac{1}{3}}$, solve for L_2.

   $L_2 = \sqrt{(L_1)^3}$

4. Given the equation $I = (\frac{V}{L})te^{sct}$, solve for sc.

$sc = \frac{LN(\frac{IL}{Vt})}{t}$

5. Given the equation $I_K = AT^2e^{\frac{-B}{t}}$, solve for A and B.

$A = \frac{I_K}{T^2e^{\frac{-B}{t}}}$
$B = -t(LN(\frac{I_K}{AT^2}))$

6. Given the equation $L_1 = ((L_2)^2)^{\frac{1}{3}}$, solve for L_2.

$L_2 = \sqrt{(L_1)^3}$

7. An amplifier is rated as having a $90dB$ gain. What power ratio does this represent?

$\frac{P_{out}}{P_{in}} = 10^9$

8. An amplifier has a gain of $60dBm$. What is the output power?

$P_{out} = 1KW$

9. The manufacturer of a high fidelity $100w$ power amplifier claimed that hum and noise in the amplifier is $90dB$ below the full power output. How much hum and noise power does this represent?

noise and hum $= 100nW$

10. A network has a loss of $80dB$. What power ratio corresponds to this loss?

$\frac{P_{out}}{P_{in}} = 10^{-8}$

11. An amplifier has a input impedance of 600Ω and a output impedance of $6K\Omega$. The power out is $30W$ when $1.9v$ is applied across the input:

    (a) What is the voltage gain of the amplifier?

    $\Delta_V = 223.297$

(b) What is the power gain of the amplifier in dB?

$\Delta_P = 36.978dB$

(c) What is the input power?

$P_{in} = 6.017mW$

12. The noise level of a telephone line used for wired music programs is $60dB$ down from the program level of $12.5mW$. How much noise power is represented by this level?

$noise = 12.5nW$

13. A crystal microphone is rated at $-80dB$. There is onhand a final AF amplifier rated at $60dB$. How much gain must be provided by a preamp in order to drive the final amplifier to full output if a attenuator pad between the microphone and the preamp has a loss of $20dB$?

$preamp\ gain = 100dB$

14. An amplifier has a normal output of $30W$. A selector switch is arranged to reduce the output in $5dB$ steps. What power output corresponds to the reduction of 5, 10, 15, 20, 25, and $30dB$?

$-5dB = 9.487W$

$-10dB = 3W$

$-15dB = 0.9487W$

$-20dB = 0.3W$

$-25dB = 0.09487W$

$-20dB = 0.03W$

$noise = 12.5nW$

15. A two-stage video RF amp has a $300\mu V$ input signal into 75Ω. The second stage has a gain of $50dB$. When matched input-output impedances are used, the voltage output of the second stage must be $4.22V$ to allow distribution of the signal. Determine the following:

(a) The input voltage of the second stage.

$Vin_{second\ stage}$=13.345mv

(b) The dB gain of the first stage.

$\Delta_{dB} = 32.964dB$

(c) The overall gain of the two amplifiers when all impedances are 75Ω.

$Total\ Gain_{dB} = 82.964_{dB}$

Week 8

Circles

8.1 Homework

Week 8 Questions and Answer Key

- Day 1, Logarithms 16-21
- Day 2, Review (Logarithms)
- Day 3, Test Logarithms
- Day 4, Circles 1-5
- Day 5, Circles 6 & 7

16. A video tuner amplifier has an input impedance of 300 ohms and an output impedance of 3,500 ohms. When a $300mV$ signal is applied at the input, a $250V$ signal appears at the output.

    (a) What is the power output of the amplifier?
    $P_{out} = 17.857$ watts

    (b) What is the power gain in dB?
    $\Delta_{dB} = 47.75dB$

    (c) What is the voltage gain of the amplifier?
    $\Delta V = 833.333$

17. Given the following specifications for a 2N45 transistor, What is the power input?

- Collector Voltage $= -20V$
- Emitter Current $= 5mA$
- Input Impedance $= 10\Omega$
- Source Impedance $= 50\Omega$
- Load Impedance $= 4.5K\Omega$
- Power Output $= 45mW$
- Power Gain $= 23dB$

$P_{In} = 225.534\mu W$

18. The input power to a 50Km line is $10mW$. The output of this line is $40\mu W$ What is the attenuation (dB) of the line per kilometer?

    $Attenuation = -0.4796db/Km$

19. What is the dB gain necessary to produce a $60\mu W$ signal in a 600Ω telephone if the received signal supplies $9\mu V$ to the 80Ω line that feeds the receiver?

    $Gain_{necessary} = 77.727dB$

20. In problem 19, if the overall gain is increased to $96dB$ what received signal will produce the $60\mu W$ signal in the telephone?

    $Signal_{Voltage} = 1.098\mu V$

21. The voltage across a 600Ω telephone is adjusted to 1.73 volts. When an audio filter is installed in the circuit, the voltage drops to 1.44 volts. What is the insertion loss of the filter?

    $-1.594dB$

Find the Center and Radius for the following Circles:

1. $x^2 + y^2 = 16$

   Center=(0,0) & Radius = 4

2. $x^2 + y^2 + 6x - 8y - 39 = 0$

   Center=(-3,4) & Radius = 8

3. $x^2 + y^2 - 8x + 12y - 8 = 0$
   Center=(4,-6) & Radius = $2\sqrt{15}$ or $\sqrt{60}$

4. $x^2 + y^2 - 12x - 2y - 12 = 0$
   Center=(6,1) & Radius = 7

5. $x^2 + y^2 + 7x + 13y - 9 = 0$
   Center=(-3.5,-6.5) & Radius = $\sqrt{63.5}$

6. The Center is on the y-axis, with Points (1,4) & (-3, 2).
   Center=(0,1) & Radius = $\sqrt{10}$

7. Points (3,1) & (0,0) & (8,4).
   Center=(-5,20) & Radius = $\sqrt{425}$ or 20.615

Week 9

Parabolas

9.1 Homework

Week 9 Questions and Answer Key

- Day 1, Parabolas 8-12
- Day 2, 13-17
- Day 3, 18-21
- Day 4, Review
- Day 5, Circles and Parabolas Test

Find the Focus and the Directrix for the following Parabolas:

8. $x^2 = 4y$

   Focus =(0,1) & Directrix y=-1

9. $y^2 = -16x$

   Focus =(-4,0) & Directrix x=4

10. $y^2 = x$

    Focus =($\frac{1}{4}$,0) & Directrix x=$\frac{-1}{4}$

11. $x^2 = 16y$

    Focus =(0,4) & Directrix y=-4

12. $y^2 = 8x$

    Focus =(2,0) & Directrix x=-2

Given the focus and directrix for the following, find the equation for the parabola.

13. (2,0), x=-2

    $y^2 = 8x$

14. (-8,0), x=8

    $y^2 = -32x$

15. (0,6), y=-6

    $x^2 = 24y$

16. (-1,3), x=3

    $y^2 - 6y + 8x + 1 = 0$ or $(y-3)^2 = -8(x-1)$

17. (2,-5), y=-1

    $x^2 - 4x + 8y + 28 = 0$ or $(x-2)^2 = -8(y+3)$

Given the focus and the vertex, find the equation for the parabola.

18. *focus*=(-4,0), *vertex*=(0,0)

    $y^2 = -16x$

19. The shape of a wire hanging between two poles closely approximates a parabola. Find the equation of a wire that is suspended between two poles 40m apart and whose lowest point is 10 m below the level of the insulators.

    $y^2 = 40y$

20. A suspension bridge is supported by two cables that hang between two supports. The curve of these cables is approximately parabolic. Find the equation of this curve if the focus lies 8m above the lowest point of the cable.

    $y^2 = 32y$

21. A culvert is shaped like a parabola, 120cm across the top and 80cm deep. How wide is the culvert 50cm from the top?

    width at -50cm =73.485cm

Week 10

Max-Mins, Differentials, & Higher Derivatives

10.1 Homework

Week 10 Questions and Answer Key

- Day 1, Max-Mins 1-3
- Day 2, 4-6
- Day 3, 7-9
- Day 4, 10-14
- Day 5, 15-18

1. The sum of two positive numbers is 56. Find the two numbers if their product is to be maximum.

   $y = 28$ & $x = 28$

2. An open box is to be made from a square piece of aluminum, 3cm on a side, by cutting equal squares from each corner and then folding up the sides. Determine the dimensions of the box that will have the largest volume.

   $2cm \times 2cm \times 0.5cm$

3. A man wishes to fence in a rectangular plot lying next to a river. No fence is required along the river bank. If he has 800m of fence, and he wishes the maximum area to be fenced, find the dimensions of the desired enclosure.

   $200m \times 400m$

4. Find the maximum possible area of a rectangle whose perimeter is 36cm.

   $a = 81cm^2$

5. A farmer wants to fence in $80,000m^2$ of land and then divide it into three plots of equal area. Find the minimum amount of fence needed.

   $fence_{min} = 1600m$

6. The charge transmitted through a circuit varies according to $q = t^4 - 4t^3$ coulombs. Find the time in seconds when the current i (in amps) $i = (dq/dt)$ reaches a minimum.

   $t = 2sec, \ i = -16ma$

7. A rectangle box, open at the top, with a square base, is to have a volume of $4000cm^3$. Find the dimensions, if the box is to contain the least amount of material.

   $20cm \times 20cm \times 10cm$

8. The total cost C of making x units of a certain commodity is given by $C = 0.005x^3 + 0.45x^2 + 12.75x$. All units made are sold at $\$36.75x - C$. Find the number of units to make maximum profit.

   $20 \ units$

9. A cylindrical can with one end is to be made with $24\pi cm^2$ of metal. Find the dimensions of the can that give the maximum volume.

   $r = 2.828cm, \ h = 2.828cm$

10. The side of a square measures $12.00cm$ with a maximum possible error of $0.05cm$. (a) Find the maximum possible error in the area using differentials. (b) Find the maximum possible error by substituting into the formula for the area of a square. (c) Find the percentage of error.

    $a. \ da = 1.2cm^2$
    $b. \ 1.2025cm^2$
    $c. \ 0.833\%$

11. Suppose you want to build a spherical water tower with an inner diameter of $26.00m$ and a side thickness of $4.0cm$. (a) Find the approximate volume of steel needed using differentials. (b) if the density of steel is $7800kg/m^3$, find the approximate volume of steel needed using differentials.

    $a. \ dv = 84.95m^3$
    $b. \ 662,599kg$

12. A freely falling body drops according to $s = (1/2)gt^2$, where s is the distance in meters, $g = 9.80m/s^2$, and t is time in seconds. Approximate the distance, ds, that an object falls from $t = 10.00$ sec to $t = 10.03$ sec.

    $ds = 2.94m$

13. The voltage Vin volts, varies according to $V = 10p^{2/3}$, where p is the power in watts. Find the change dv when the power changes from 125w to 128w.

    $dv = 4v$

14. The impedance Z in an ac circuit varies according to $Z = \sqrt{R2 + X2}$, where R is the resistance and X is the reactance. If $R = 300\Omega$ and $X = 225\Omega$, find dz when R changes to 310Ω.

    $dz = 8\Omega$

    Find the first four derivatives of each of the following functions.

15. $y = x^5 + 3x^2$

    $y' = 5x^4 + 6x$
    $y'' = 20x^3 + 6$
    $y''' = 60x^2$
    $y'''' = 120x$

16. $3x^6 - 8x^3 + 2$

    $y' = 18x^5 - 24x^2$
    $y'' = 90x^4 - 48x$
    $y''' = 360x^3 - 48$
    $y'''' = 1080x^2$

17. $5x^5 + 2x^3 - 8x$

    $y' = 25x^4 + 6x^2 - 8$
    $y'' = 100x^3 + 12x$
    $y''' = 300x^2 + 12$
    $y'''' = 600x$

18. $3x^2 + 4x - 7$

    $y' = 6x + 4$
    $y'' = 6$
    $y''' = 0$
    $y'''' = 0$

Week 11

Differentials Applied

11.1 Homework

Week 11 Questions and Answer Key

- Day 1, Review
- Day 2, Test
- Day 3, Differentials, 1-5
- Day 4, 6-10
- Day 5, Review

1. An electron (whose mass is M_e) moves at a speed V. Its momentum is $p = mV$. Find a formula for the approximate change dp in momentum resulting from a small increase dv, in speed.

   $dp = m\ dv$

2. The low-frequency inductance of a single-layer solenoid is approximately $L = kDn^2$, where k is a form factor, D is the diameter in centimeters, and n is the number of turns. Find a formula for the approximate change dl in the inductance resulting from the addition of a small part of a turn dn.

   $dL = 2KDn\ dn$

3. The power in a circuit was $p = t - 5$ watts. What was the approximate energy dw in joules expended from $t = 4$ sec to $t = 4.002$ sec?

   $dp = -2mW$

4. The induced voltage in an 8-henry inductor varied according to $vind = 3t^2 - t$. About how much change di occurred in the inductor current from $t = 2$ sec to $t = 2.01$ sec?

   $di = -12.5mA$

5. The power in a circuit is given by $p = Ri^2$, where $R = 100\Omega$ and i is the current in amperes. If i changes from 12 amps to 12.005 amps, approximately what change dp occurs in power in watts?

   $dp = 12\ watts$

6. The current i amperes in a circuit varied with time t seconds according to $i = t^2 + 3t$. About what current change di occurred as t changed from 0.98 sec to 1 sec?

   $di = 99.2mA$

7. The intensity J of the heat radiation from a transmitting tube plate varies with its absolute temperature according to $J = \sigma T^4$ where σ is a constant and T is the temperature in $°C$. If $J = 50$ units when $T = 1200°C$, approximately what change dJ in J results from a change in T to $1205°C$?

   $dJ = 833.345 \times 10^{-3}$ units

8. If the resistance r ohms in a circuit varies with time t seconds according to $r = 100 + t^{\frac{1}{2}}$, what approximate change dr in r occurs as t changes from 4 sec to 4.001 sec?

   $dr = 250\mu\Omega$

9. A right circular cone used in constructing a broadband antenna has a volume $v = \pi r^2 h$, where r is the radius of the base and h is the altitude of the cone. If $r = 10$ cm, and $h = 24$ cm, what approximate change dv in the volume occurs when r changes to 10.052 cm?

   $dv = 78.414cm^3$

10. An increase in the apparent mass Ma of a moving particle occurs in accord with $Ma = \frac{Mo}{[1-(\frac{V}{C})^2]^{\frac{1}{2}}}$ where Mo is the mass of the particle at reset, V is its speed, and C is the speed of light in a vacuum. What approximate change d_{Ma} occurs in the apparent mass as a result of a small change d_V in the speed of the particle? Express your answer as a formula.

    $dma = \frac{Vmo}{c^2[1-(\frac{v}{c})^2]^{\frac{3}{2}}}\ dv$

Week 12

Limits

12.1 Homework

Week 12 Questions and Answer Key

- Day 1, Limits, 1-5
- Day 2, 6-11
- Day 3, Test
- Day 4,
- Day 5,

1. $\lim_{x\to 2} \frac{x^2-4}{x-2} = 4$

2. $\lim_{x\to\infty} \frac{3x+2}{x} = 3$

3. $\lim_{x\to 0} \frac{Sinx}{x} = 1$

4. $\lim_{x\to 2}(x^2-5x) = -6$

5. $\lim_{x\to -1}(2x^3+5x^2-2) = 1$

6. $\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$

7. $\lim_{x\to \frac{3}{2}} \frac{4x^2-9}{2x+3} = 0$

8. $\lim_{x\to -1} \sqrt{2x+3} = 1$

9. $\lim_{x\to 6} \sqrt{4-x} = \textit{no limit}$

10. $\lim_{x\to\infty} \frac{1}{2x} = 0$

11. $\lim_{x\to\infty} \frac{3x^2-5x+2}{4x^2+8x-11} = \frac{3}{4}$

Week 13

Trigonometric Functions part 1

13.1 Homework

Week 13 Questions and Answer Key

- Day 1, Trigonometric Functions, 1-4
- Day 2, 5-8
- Day 3, 9-12
- Day 4, 13-16
- Day 5, 17-20

1. In a directional broadcast antenna, tower 2 is located 212 feet from tower 1, in a direction 38° north of east. How far to the north of tower 1 is tower 2 situated?

   North = 130.52 ft

2. A target-practice object is observed on a radar screen at an airline distance of 18,000 feet and at an elevation angle of 72°. If the object were shot down, at what horizontal distance from the observer would the wreckage fall, assuming a vertical fall?

   Horizontal distance = 5,562.31 ft

3. A radar screen shows an object 70 miles from the observer at an angle of 50° east of north. How far to the east of the observer is the object?

   East distance = 53.623 miles

4. A broadcast antenna is 100 ft tall and it is recommended that the guy wires to the top be anchored into the ground at an elevation angle to the horizon of 50°. Find (a) the length of each guy wire and (b) how far the anchor points are from the antenna base?

   Guy Wire = 130.541 ft
   Anchor distance = 83.91 ft

5. A microphone diaphragm intercepts 6.75×10^-9watts of acoustic power when turned broadside to a sound source. What will be the theoretical intercept power if the diaphragm is turned at an angle of 55° to the source?

   Intercept power $= 3.872nW$

6. Light radiations having a plane wave strike a photosensitive surface at an angle of 30°. If the surface were turned to face the light directly, by what factor would the amount of received light energy increase?

   Factor of increase $= 2$

7. A "Curtain" receiving antenna is broadside to a distant transmitter. If it is now turned through an angle of 21°, by what factor will the radiated power impinging upon the curtain be reduced? (This calculation does not include the effect of the antenna directional pattern)

   factor of reduction $= 0.9336$

8. A radar screen shows an object 40° East of North. A second radar screen located 100 miles directly East of the first screen locates the same object at 55° West of North at a distance of 76.9 miles. How far to the east of the first screen is the object?

   East distance $= 37.007$ miles

9. How many radians correspond to each of the following angles?

   (a) $180° = \pi$ radians
   (b) $90° = \frac{\pi}{2}$ radians
   (c) $45° = \frac{\pi}{4}$ radians
   (d) $60° = \frac{\pi}{3}$ radians
   (e) $30° = \frac{\pi}{6}$ radians
   (f) $15° = \frac{\pi}{12}$ radians
   (g) $20° = \frac{\pi}{9}$ radians
   (h) $54° = \frac{3\pi}{10}$ radians

10. How many degrees correspond to each of the following angles expressed in radians?

   (a) $\frac{\pi}{4}$ radians $= 45°$
   (b) $\frac{3\pi}{2}$ radians $= 270°$
   (c) $\frac{\pi}{9}$ radians $= 20°$
   (d) $\frac{2\pi}{3}$ radians $= 120°$
   (e) $\frac{5\pi}{3}$ radians $= 300°$
   (f) $\frac{\pi}{10}$ radians $= 18°$
   (g) $\frac{4\pi}{3}$ radians $= 240°$

(h) $\frac{3\pi}{4}$ radians = $135°$

11. An Instrument pointer moves through an arc of 270°. To how many radians is this equivalent?

    $\frac{3\pi}{2}$ *radians*

12. The radiation pattern of an antenna has a minimum value in a direction 24° off the antenna axis, express this angle in radians.

    $\frac{2\pi}{15}$ *radians*

13. An armature turns at 1,800 revolutions per minute. To what value ω, in radians per second, does this correspond?

    $\omega = 60\pi \; rad/\; sec$

14. The coil of an instrument rotates at a rate of 0.005 radians per millisecond. Express this angular speed in degrees per second.

    $\omega = 286.479°/\; sec$

15. A motor accelerates at a rate of 600 revolutions per minute per second. To how many radians per second squared is this equal?

    $\alpha = 20\pi \; rad/\; sec^2$

16. An instrument pointer is 2.1 inches long. The tip of the pointer moves over a scale 2.4 inches long. What angle does this describe in radians?

    $\theta = 1.14286 \; radians$

17. An alternator has a rotating field that is 32 inches in diameter. When the field is turned at 120 revolutions per minute, what is the linear speed of a point on its circumference?

    $v = 201.062 \; inches/second$

18. If the field assembly in question 17 is accelerated at 12 revolutions/min/sec, what linear acceleration is applied to a point on its circumference?

    $a = 20.1062 \; inches/s^2$

19. If the field assembly in question 17 turns in a counter-clockwise direction, (a) what is the upward component of the velocity at a point P on its circumference when P is at an angle of 45° above the horizon? (b) what is the horizontal component of the velocity at the same point?

    (a) $v_y = 142.172 \; inches/sec$
    (b) $v_x = 142.172 \; inches/sec$

20. An airplane propeller has a radius of 3 feet to the blade tip. It is desired to keep the tip velocity below the speed of sound (769.5 miles per hour). What number of revolutions per minute would correspond to this limit?

    $\omega = 3.59245 \times 10^3 \; rev/\; min$

Week 14

Trigonometric Functions part 2

14.1 Homework

Week 14 Questions and Answer Key

- Day 1, 21-22
- Day 2, 23-28
- Day 3, 29-35
- Day 4, Review
- Day 5, Test

21. It can be shown that, when an armature of radius r rotates at ω radians per second, a point on its circumference is given a constant normal acceleration toward the center equal to $a_n = r\omega^2$. If an armature 0.3 meters in diameter is rotated at 2,000 revolutions per minute, (a.) to what normal acceleration will a conductor on the surface be subjected? Using $f = ma$, (b.) what centrifugal force in newtons will be applied to a conductor of mass 0.04Kg located at the circumference?

    $a_n = 6.5797Km/s^2$

    $F = 263.189\ newtons$

22. When power is applied to a motor, the shaft speed during initial power-up corresponds to $10t + 4t^2$ revolutions per second. (a.) Write an equation for the angular speed ω, in radians per second of the shaft. (b.) Find an equation for the angular acceleration α of the shaft at any time. (c.) Find an equation for the angle θ, in radians, of the shaft position at any time. (d.) Find the angular position θ, in degrees, at t1 = 0.1 sec & t2 = 0.2 sec. (assume $\theta = 0$ when t = 0).

$\omega = (10t + 4t^2)2\pi \; radians/sec$

$\alpha = 2\pi(10 + 8t) \; radians/sec^2$

$\theta = 10\pi t^2 + \frac{8\pi t^3}{3} + C \; radians$

$\theta_{t1} = 18.48°$

$\theta_{t2} = 75.84°$

Perform the derivative for the following functions.

23. $y = \sin 2x$

    $\frac{dy}{dx} = 2(\cos 2x)$

24. $y = 3 \sin x$

    $\frac{dy}{dx} = 3(\cos x)$

25. $y = 12 \sin 14t$

    $\frac{dy}{dt} = 168(\cos 14t)$

26. $y = 10 \sin 10t^{\frac{1}{2}}$

    $\frac{dy}{dt} = 50t^{\frac{-1}{2}}(\cos 10t^{\frac{1}{2}})$

27. $y = \sin t^2$

    $\frac{dy}{dt} = 2t(\cos t^2)$

28. $y = 2 \cos 3t^3$

    $\frac{dy}{dt} = -18t^2(\sin 3t^3)$

29. $y = 500 \cos(t^2 - t)^{\frac{1}{2}}$

    $\frac{dy}{dt} = \frac{-250(2t-1)}{(t^2-t)^{\frac{1}{2}}} \sin(t^2 - t)^{\frac{1}{2}}$

30. $y = 10t^3 + \cos t$

    $\frac{dy}{dt} = 30t^2 - \sin t$

31. $y = \sin^2 t$

    $\frac{dy}{dt} = 2 \sin t \cos t$

32. $y = -\cos^2 t^{-1}$

    $\frac{dy}{dt} = \frac{-2}{t^2}(\cos t^{-1})(\sin t^{-1})$

33. $y = 2 \sin^2 t^2$

    $\frac{dy}{dt} = 8t(\sin t^2)(\cos t^2)$

34. Let the primary current in a transformer be $i_1 = I_{Max} \sin \omega t$, Where I_{Max} is the crest value of the current. Write a formula for the induced secondary emf v_2.

    $v_2 = -m(I_{Max} \times \omega)(\cos \omega t)$

35. A voltage $v = 2,000 \sin 500t$ is impressed across a 20μF capacitor. Find a formula for the resulting current.

    $Ic = 20 \cos 500t$

References