RCET 1372

Applied Calculus

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Getting Started

0.1 Welcome

Welcome to RCET 1372: Applied Calculus! I'm excited to have you in this course and look forward to helping you explore how calculus applies to real-world problems in electronics and engineering technology. This class is designed to give you practical tools for analyzing systems, modeling change, and solving problems you'll encounter in your future career. Whether you're working with signal processing, circuit behavior, or control systems, calculus plays a vital role in understanding how these systems function and respond. We'll focus on making these concepts approachable and directly relevant to your field, so come ready to engage, ask questions, and connect the math to real-world applications. Let's get started!

0.2 Optional Recommended Books

Optional but Recommended Books: While not required, I highly recommend supplementing your learning with a few excellent textbooks that align well with the topics we'll cover in RCET 1372. I encourage you to buy used copies if possible. The primary text I teach from is Calculus for Electronics, 4th Edition, by Gary R. Hecht and Anthony C. Richmond (ISBN 0-07-052355-X), which is tailored specifically to calculus applications in electronic systems. Another valuable resource is Technical Calculus, 4th Edition, by Dale Ewen, Joan S. Gary, James E. Trefzger (ISBN 0-13-093004-0), which offers a broader perspective on applied calculus across technical fields. Additionally, for students who would like to strengthen their foundational math skills, I recommend Basic Mathematics for Electronics, 6th Edition, by Nelson M. Cooke, Herbert F. Adams, and Peter B. Dell (ISBN 0-07-012521-X). These texts provide practical examples and explanations that reinforce the concepts discussed in class.

0.3 RCET 1372 Syllabus & RCET Student Handbook

- RCET 1372 Syllabus
- RCET Student Handbook

Week 1

Introduction to Derivatives

1.1 Desmos

1.1.1 What is Desmos?

Desmos is a free, web-based graphing calculator and mathematics tool designed to help students and educators visualize and explore mathematical concepts interactively. It allows users to plot functions, create tables, animate graphs, and manipulate variables in real time. With a user-friendly interface and powerful computational engine, Desmos supports learning across a range of math topics, from algebra to calculus. It's widely used in classrooms for its ability to make abstract mathematical ideas more accessible and engaging, fostering deeper understanding through exploration and experimentation.

1.1.2 Desmos initial test setup and assignment.

- Go to the website: http://www.desmos.com/calculator
  - Log In or Sign Up
  - Enter the equation "y=mx+b"
  - Select add slider "All" button.
  - Name the graph "(your name) Desmos Test" by clicking the Save button.
  - Verify the title of the graph in the upper left is "(your name) Desmos Test".
  - Adjust m to 1 and b to 0
  - Use the Snipping Tool to screenshot the graph, making sure to capture the Title, Equation, and Graph. Save as JPEG.
  - Submit Desmos JPEG screenshot in Canvas.

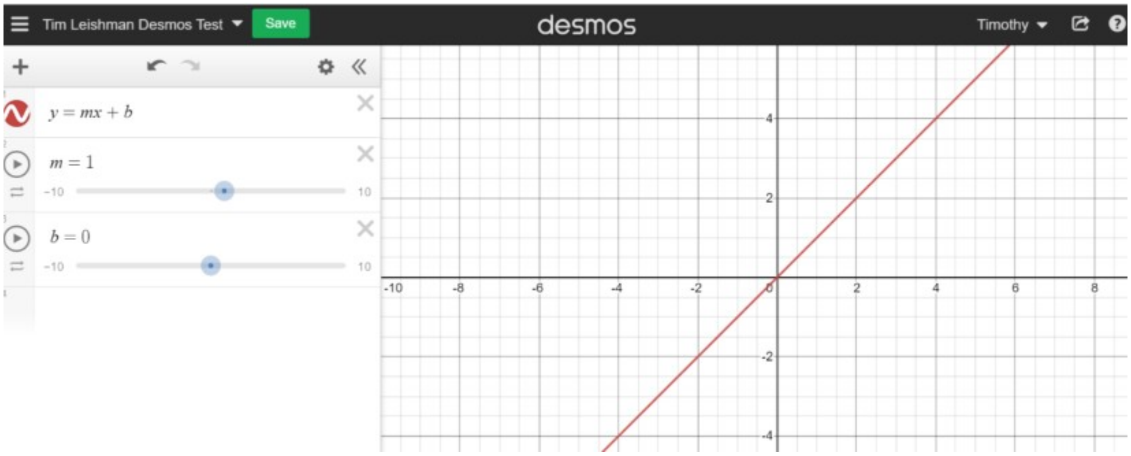

Figure 1.1: Desmos Assignment Example

1.2 The Derivative

1.2.1 What is a Derivative?

In the context of Applied Calculus, a derivative is a way to measure how a quantity changes in response to a change in another quantity.

Simple Definition: A derivative tells you the rate of change of a function.

For example, if you have a function that gives the position of an object over time, the derivative of that function gives the object's velocity—how fast and in what direction the position is changing.

In terms of graphical analysis: Think of a derivative as the slope of a function at a specific point—how steep the curve is at that exact spot. It's like looking at how fast your speedometer needle (y) is moving or changing at a given instant (x).

1.2.2 Slope Intercept Form

The Slope-Intercept Form is a way of writing the equation of a straight line using its slope and y-intercept.

$$y = mx + b$$

Where:

- y is the dependent variable
- x is the independent variable
- m is the slope of the line
- b is the y-intercept (where the line crosses the y-axis)

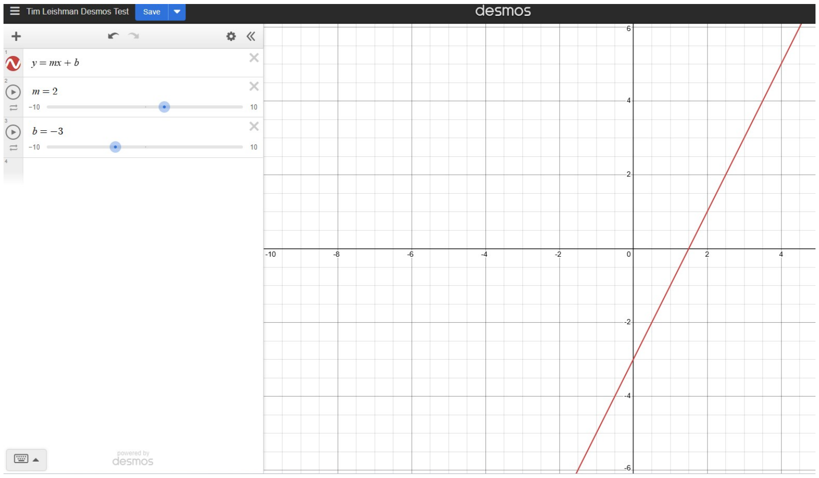

Figure 1.2: y=2x-3

We can observe the line $y = 2x - 3$ using Desmos. Notice that the y-intercept (b) is at y=-3.

How is the Slope (m) measured? Rise over Run is one method used to measure the slope of a line.

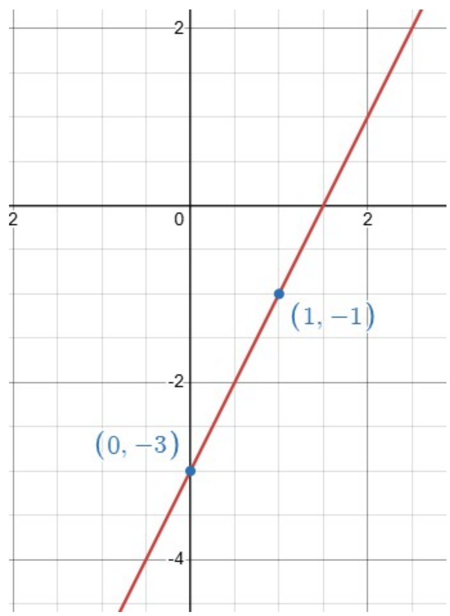

Figure 1.3: Rise over Run

- Rise over Run:

  Reading bottom to top, rise (from (0,-3) to (1,-1), y is moving in a positive direction from -3 to -1 which is +2. Therefore, rise is equal to 2.

  Reading left to right, run (from (0,-3) to (1,-1), x is moving in a positive direction from 0 to 1. Therefore, run is equal to 1.

$m = \frac{rise}{run} = \frac{2}{1} = 2$

- Another method for solving m is $\frac{\Delta y}{\Delta x}$ OR $\frac{y1-y2}{x1-x2}$.

  Points (0,-3) and (1,-1)

  $\frac{\Delta y}{\Delta x} = \frac{y1-y2}{x1-x2} = \frac{-3-(-1)}{0-1} = \frac{-3+1}{0-1} = \frac{-2}{-1} = 2$

Using Grapical Analysis, we have determined that the Slope of the line $y = 2x - 3$ is equal to 2 ($m = 2$).

1.2.3 What does the Slope of a line (m) have to do with calculus?

We have discovered that the slope m is equal to $\frac{rise}{run}$ and $\frac{\Delta y}{\Delta x}$. Additionally, the slope m is also equal to $\frac{dy}{dx}$ where $\frac{dy}{dx}$ represents the derivative of a function. Think of $\frac{dy}{dx}$ as an instantaneous slope.

- $m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

1.3 Derivatives of a Sum

1.3.1 How to take a Derivative of a function

If $y = ax^n$ then $\frac{dy}{dx} = anx^{n-1}$

- y is y and x is x
- a is the number in front of x
- n is the exponent of x

Yes, this is super confusing; it will become more apparent after practicing some example problems!

1.3.2 Example 1:

Given $y = 2x - 3$. Find the derivative $\frac{dy}{dx}$:

- $y = 2x - 3$
- $\frac{dy}{dx} = anx^{n-1}$
- In the first term $2x$, $a = 2$ and $n = 1$

  $\frac{dy}{dx} = anx^{n-1} = 2(1)x^{1-1} = 2x^0 = 2$

- In the second term -3, $a = -3$ and $n = 0$

  $\frac{dy}{dx} = anx^{n-1} = -3(0)x^{don't\ care}$ (anything times $0 = 0$).

- Therefore, $\frac{dy}{dx} = 2 + 0 = 2$
- $\frac{dy}{dx} = 2$

This shows that $m = \frac{rise}{run} = \frac{\Delta y}{\Delta x}$ and is also equal to $\frac{dy}{dx}$. Now what makes a derivative extra cool, is that $\frac{dy}{dx}$, in addition to its ability to find the slope of a line $y = mx + b$, $\frac{dy}{dx}$ can be used to find the instantaneous slope that is tangent to a point on a non-linear function or curve.

1.3.3 Example 2:

Given $y = 3x + 4$ *Technical Calculus* [2, page 104]

- Use Desmos to graph the function
- Using Desmos, measure the Slope of the Line
- Take the Derivative of the function $y = 3x + 4$

  $\frac{dy}{dx} = 3$

1.3.4 Example 3 (Non-Linear)

Given $y = 3x^2 + 4$

- Observe that the function $y = 3x^2 + 4$ is non-linear (the slope is changing as x changes).
- Take the Derivative of the function $y = 3x^2 + 4$.

  $\frac{dy}{dx} = (3)(2)(x^{(2-1)}) + 4(0)$
  $\frac{dy}{dx} = 6x$

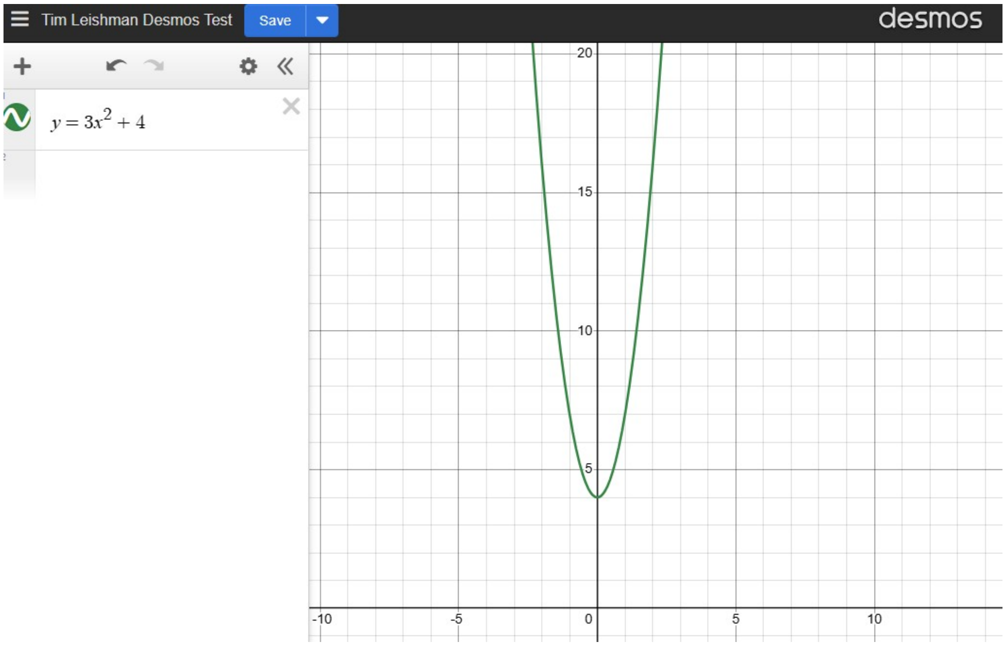

Figure 1.4: $y = 3x^2 + 4$

1.3.5 Example 4 (Fractions)

Given: $y = \frac{1}{x}$ *Technical Calculus* [2, page 104]

- rewrite the function $y = \frac{1}{x}$ in order to help make it easier to take the Derivative.
- $y = \frac{1}{x}$ is equivalent to $y = 1x^{-1}$
- Take the Derivative of $y = 1x^{-1}$

  $\frac{dy}{dx} = (1)(-1)(x^{-1-1})$

  $\frac{dy}{dx} = -1x^{-2}$

  Get rid of the negative exponent!

  $\frac{dy}{dx} = \frac{-1}{x^2}$

1.4 Derivative of a Power of a Function (U Substitution)

The derivative of a power of a function multiplied by a constant equals the product of the exponent of the original power times the constant multiplier times the function to a power one less times the derivative of the function. *Technical Calculus* [2, page 107]

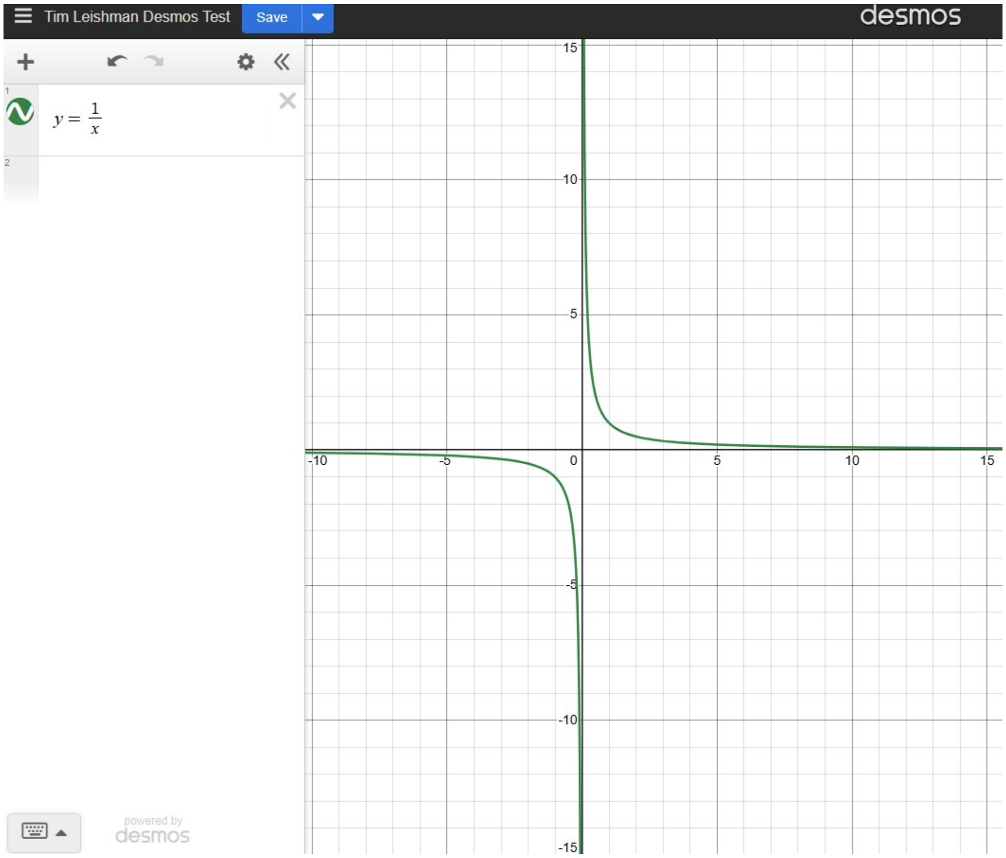

Figure 1.5: $y = \frac{1}{x}$

1.4.1 Example 1

Given: $y = \frac{1}{4-x^2}$ *Technical Calculus* [2, page 105], Find: $\frac{dy}{dx}$

- Rewrite the equation to make it easier to take the derivative.
- $y = (1)(4 - x^2)^{-1}$

$$n = -1$$
$$a = 1$$
$$u = 4 - x^2$$

- $\frac{du}{dx} = -2x$
- $\frac{dy}{du} = (1)(-1)(4 - x^2)^{-1-1} = -1(4 - x^2)^{-2}$
- Now using the substitution method we can solve for $\frac{dy}{dx}$

$$\frac{du}{dx} = -2x$$
$$\frac{dy}{du} = -1(4 - x^2)^{-2}$$

$\frac{du}{dx} \times \frac{dy}{du} = (-2x) \times (-1(4-x^2)^{-2})$
$\frac{dy}{dx} = 2x(4-x^2)^{-2}$, $\frac{du}{du} = 1$ and $(-2x) \times (-1) = 2x$
$\frac{dy}{dx} = \frac{2x}{(4-x^2)^2}$

1.4.2 Example 2

Given: $y = \sqrt{x+1}$ *Technical Calculus* [2, page 105], Find: $\frac{dy}{dx}$

- Rewrite the equation to make it easier to take the derivative.
- $y = (x+1)^{\frac{1}{2}}$

  $n = \frac{1}{2}$

  $a = 1$

  $u = x + 1$
- $\frac{du}{dx} = 1$
- $\frac{dy}{du} = (1)(\frac{1}{2})(x+1)^{\frac{1}{2}-1} = \frac{1}{2}(x+1)^{\frac{-1}{2}}$
- Now using substitution method we can solve for $\frac{dy}{dx}$

  $\frac{du}{dx} = 1$
  $\frac{dy}{du} = \frac{1}{2}(x+1)^{\frac{-1}{2}}$
  $\frac{du}{dx} \times \frac{dy}{du} = 1 \times \frac{1}{2}(x+1)^{\frac{-1}{2}}$
  $\frac{dy}{dx} = \frac{1}{2(x+1)^{\frac{1}{2}}}$
  $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}}$

1.5 (f') The slope of the tangent line of a curve at a given point.

Suppose we have the function $y = x^2$ and we want to find the slope of the tangent line at the specific point $(3, 9)$. See Figure **??**.

To find f' the slope m_{tan} of $y = x^2$ at the point (3,9) follow the steps below.

- Take the derivative of the function $y = x^2$.

  $\frac{dy}{dx} = 2x$
- Find m_{tan} at (3, 9)

  $m_{tan} = \frac{dy}{dx}$

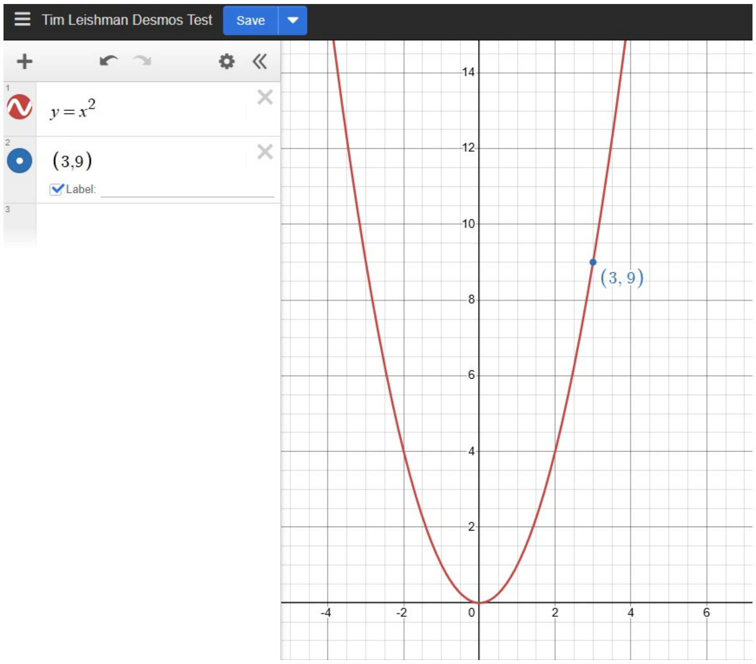

Figure 1.6: $y = x^2$

$$m_{tan} = 2x$$
$$m_{tan}(3,9) = 2(3)$$
$$m_{tan}(3,9) = 6$$

- Rewrite as f'

$$f'_x = m_{tan} \text{ at a specific point.}$$
$$f'_3 = 6$$

1.6 The equation of the Line Tangent to a Curve

Now we can solve for the equation of the line tangent to the curve $y = x^2$ at the point (3, 9).

- $y = mx + b$
  - Substitute known values $9 = (6)(3) + b$
  - Solve for b

$$9 = (6)(3) + b$$
$$b = 9 - 18$$
$$b = -9$$

  - Rewrite the tangent line equation.

$$y = 6x - 9$$

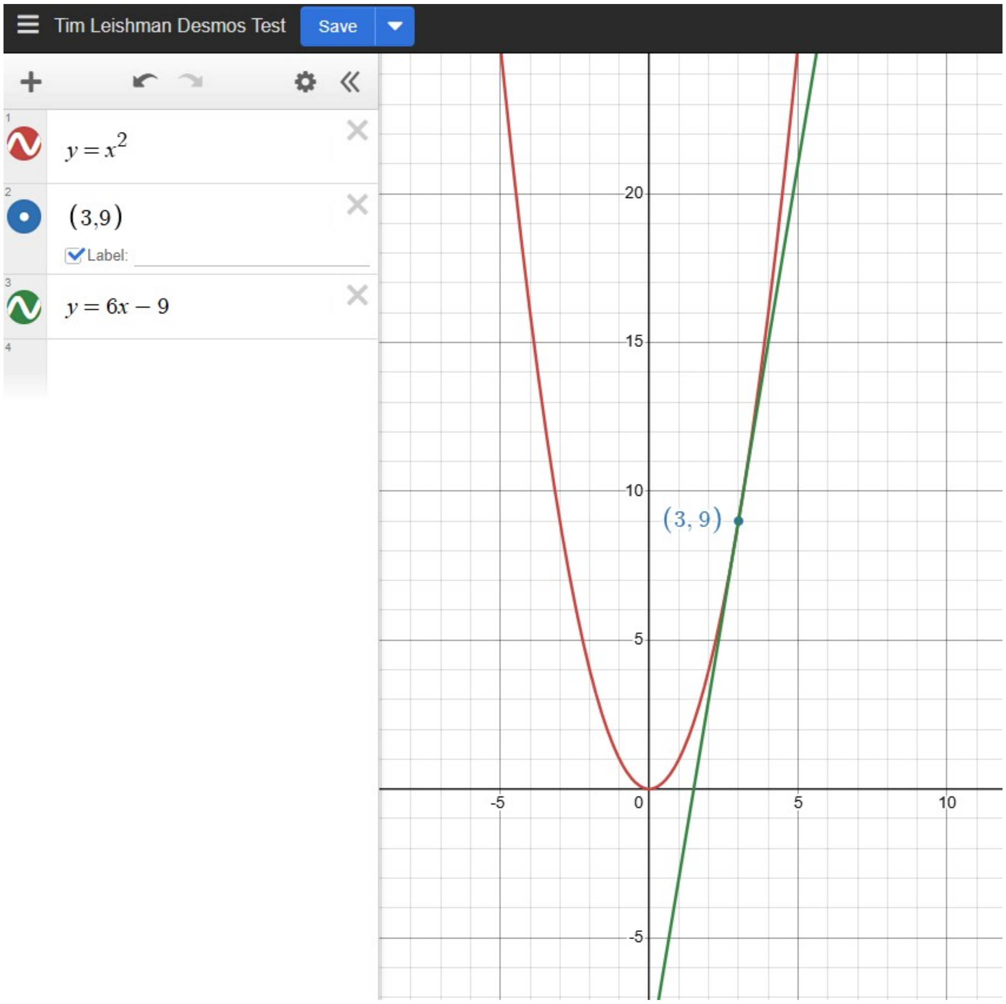

Figure 1.7: $y = x^2$

1.7 Video Instruction

- Slope of a Line, Khan Academy
- Derivative as a Concept, Khan Academy
- Slope Intercept Form & Derivative Introduction
- Derivative Problem Examples part 1
- Derivative Problem Examples part 2

1.8 Homework Week 1 - Questions with Answers

- Day 1, Desmos JPEG submission See page 1.
- Day 2, questions 1-4
- Day 3, questions 5-8
- Day 4, questions 9-12
- Day 5, Derivatives Test 1

1. $y = 3x^2$ $\frac{dy}{dx} = 6x$
2. $y = x^2 - 2$ $\frac{dy}{dx} = 2x$
3. $y = x^2 - 3x$ $\frac{dy}{dx} = 2x - 3$
4. $y = \frac{1}{x}$ $\frac{dy}{dx} = \frac{-1}{x^2}$
5. $y = \frac{2}{(x-3)}$ $\frac{dy}{dx} = \frac{-2}{(x-3)^2}$
6. $y = \frac{1}{(4-x^2)}$ $\frac{dy}{dx} = \frac{2x}{(4-x^2)^2}$
7. $y = \sqrt{x+1}$ $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}}$
8. $y = \frac{1}{\sqrt{x-1}}$ $\frac{dy}{dx} = \frac{-1}{2(x-1)^{\frac{3}{2}}}$
9. $y = \frac{5}{2}x^8 - \frac{6}{5}x^5 + \frac{15}{2}x^4 - x^3 + \sqrt{2}$ $\frac{dy}{dx} = 20x^7 - 6x^4 + 30x^3 - 3x^2$
10. $y = 3x^2 + 2x - 1; a = -1$ $f' = -4$
11. $y = 2x^3 - 6x^2 + 2x + 9; a = -3$ $f' = 92$
12. Find the equation of the Tangent Line to the curve $y = x^3 + 4x^2 - x + 2$ at (-2,12).
    $y = -5x + 2$

Week 2

Derivatives: Product & Quotient Rules, Powers, and Implicit Differentiation

2.1 Product Rule

The derivative of the product of two functions equals the first times the derivative of the second, plus the second times the derivative of the first. *Calculus For Electronics* [2, page 109]

The derivative of a product of two functions is the product of the first function and the derivative of the second function, plus the product of the second function and the derivative of the first function. *Technical Calculus* [1, page 111]

Simplified - The derivative of the first times the second PLUS the derivative of the second times the first. Tim Leishman

$$y = g(x) \times h(x)$$

Rewritten (substitution for simplification):

$$y = u \times v$$

The derivative (where u is the first and v is the second):

$$\frac{dy}{dx} = \frac{du}{dx}(v) + \frac{dv}{dx}(u)$$

2.1.1 Example 1

Given: $y = 2x(4x^2 + 3x - 5)$, Find: $\frac{dy}{dx}$

- Solve:

$$
\begin{aligned}
y &= 2x(4x^2+3x-5) \\
\tfrac{dy}{dx} &= (2)(4x^2+3x-5)+(8x+3)(2x) \\
\tfrac{dy}{dx} &= (8x^2+6x-10)+(16x^2+6x) \\
\tfrac{dy}{dx} &= 24x^2+12x-10
\end{aligned}
$$

2.1.2 Example 2

Given: $y = (4x+7)(x^2-1)$, Find: $\frac{dy}{dx}$

- Solve:

$$
\begin{aligned}
y &= (4x+7)(x^2-1) \\
\tfrac{dy}{dx} &= (4)(x^2-1)+(2x)(4x+7) \\
\tfrac{dy}{dx} &= (4x^2-4)+(8x^2+14x) \\
\tfrac{dy}{dx} &= 12x^2+14x-4
\end{aligned}
$$

2.2 Quotient Rule

The derivative of a quotient (or fraction) equals the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the denominator squared. *Calculus For Electronics* [2, page 113]

The derivative of a quotient of two functions is the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator *Technical Calculus* [1, page 112]

Simplified - The derivative of the numerator times the denominator MINUS the derivative of the denominator times the numerator ALL divided by the denominator squared. Tim Leishman

$$y = \frac{g(x)}{h(x)}$$

Rewritten (substitution for simplification):

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{\frac{du}{dx}(v) - \frac{dv}{dx}(u)}{v^2}$$

2.2.1 Example 1

Given: $y = \frac{x}{2x+5}$, Find: $\frac{dy}{dx}$

- Solve

$$y = \frac{x}{2x+5}$$
$$\frac{dy}{dx} = \frac{(1)(2x+5)-(2)(x)}{(2x+5)^2}$$
$$\frac{dy}{dx} = \frac{(2x+5)-(2x)}{(2x+5)^2}$$
$$\frac{dy}{dx} = \frac{5}{(2x+5)^2}$$

2.2.2 Example 2

Given: $y = \frac{5x-1}{3x+2}$, Find: $\frac{dy}{dx}$

- Solve

$$y = \frac{5x-1}{3x+2}$$
$$\frac{dy}{dx} = \frac{(5)(3x+2)-(3)(5x-1)}{(3x+2)^2}$$
$$\frac{dy}{dx} = \frac{(15x+10)-(15x-3)}{(3x+2)^2}$$
$$\frac{dy}{dx} = \frac{15x+10-15x+3}{(3x+2)^2}$$
$$\frac{dy}{dx} = \frac{13}{(3x+2)^2}$$

2.3 Product Rule Powers

2.3.1 Example 1

Given: $y = (2x+1)^2(x^2+1)^2$ *Technical Calculus* [1, page 118]

- Find: $\frac{dy}{dx}$
- Solve:

$$y = (2x+1)^2(x^2+1)^2$$
$$\frac{dy}{dx} = 2(2x+1)(2)(x^2+1)^2 + (2)(x^2+1)(2x)(2x+1)^2$$
$$\frac{dy}{dx} = 4(2x+1)(x^2+1)^2 + (4x)(x^2+1)(2x+1)^2$$
$$\frac{dy}{dx} = 4(2x+1)(x^2+1)[(x^2+1) + (x)(2x+1)]$$
$$\frac{dy}{dx} = 4(2x+1)(x^2+1)[(x^2+1) + (2x^2+1x)]$$
$$\frac{dy}{dx} = 4(2x+1)(x^2+1)[x^2+1+2x^2+1x]$$
$$\frac{dy}{dx} = 4(2x+1)(x^2+1)(3x^2+x+1)$$

2.3.2 Example 2

Given: $y = (x^2+1)\sqrt{9x^2-2x}$ *Technical Calculus* [1, page 118]

- Find: $\frac{dy}{dx}$
- Solve:

$$y = (x^2+1)\sqrt{9x^2-2x}$$
$$\text{Rewrite: } y = (x^2+1)(9x^2-2x)^{\frac{1}{2}}$$
$$\frac{dy}{dx} = (1)(1)(2x)(9x^2-2x)^{\frac{1}{2}} + (\tfrac{1}{2})(9x^2-2x)^{\frac{-1}{2}}(18x-2)(x^2+1)$$
$$\frac{dy}{dx} = (2x)(9x^2-2x)^{\frac{1}{2}} + (9x^2-2x)^{\frac{-1}{2}}(9x-1)(x^2+1)$$
$$\frac{dy}{dx} = (9x^2-2x)^{\frac{-1}{2}}[(2x)(9x^2-2x)^1 + (9x-1)(x^2+1)]$$
$$\frac{dy}{dx} = (9x^2-2x)^{\frac{-1}{2}}[18x^3-4x^2+9x^3+9x-x^2-1)]$$
$$\frac{dy}{dx} = (9x^2-2x)^{\frac{-1}{2}}(27x^3-5x^2+9x-1)$$
$$\frac{dy}{dx} = \frac{27x^3-5x^2+9x-1}{(9x^2-2x)^{\frac{1}{2}}}$$
$$\frac{dy}{dx} = \frac{27x^3-5x^2+9x-1}{\sqrt{9x^2-2x}}$$

2.4 Quotient Rule Powers

2.4.1 Example 1

Given:$y = \frac{(x^3+2)^4}{4x^2-3x}$ *Technical Calculus* [1, page 118]

- Find: $\frac{dy}{dx}$
- Solve:

$$y = \frac{(x^3+2)^4}{4x^2-3x}$$
$$\frac{dy}{dx} = \frac{4(x^3+2)^3(3x^2)(4x^2-3x)-[(8x-3)(x^3+2)^4]}{(4x^2-3x)^2}$$
$$\frac{dy}{dx} = \frac{(x^3+2)^3[4(3x^2)(4x^2-3x)-[(8x-3)(x^3+2)^1]]}{(4x^2-3x)^2}$$
$$\frac{dy}{dx} = \frac{(x^3+2)^3[(12x^2)(4x^2-3x)-[(8x-3)(x^3+2)^1]]}{(4x^2-3x)^2}$$
$$\frac{dy}{dx} = \frac{(x^3+2)^3[(48x^4-36x^3)-[(8x-3)(x^3+2)^1]]}{(4x^2-3x)^2}$$
$$\frac{dy}{dx} = \frac{(x^3+2)^3[(48x^4-36x^3)-[(8x^4+16x-3x^3-6)]]}{(4x^2-3x)^2}$$
$$\frac{dy}{dx} = \frac{(x^3+2)^3[48x^4-36x^3-8x^4-16x+3x^3+6)]}{(4x^2-3x)^2}$$
$$\frac{dy}{dx} = \frac{(x^3+2)^3(40x^4-33x^3-16x+6)}{(4x^2-3x)^2}$$

2.4.2 Example 2

Given:$y = (\frac{1+x}{1-x})^4$ *Technical Calculus* [1, page 118]

- Find: $\frac{dy}{dx}$
- Solve:

$y = (\frac{1+x}{1-x})^4$

Rewrite: $y = \frac{(1+x)^4}{(1-x)^4}$

$\frac{dy}{dx} = \frac{4(1+x)^3(1)(1-x)^4-[4(1-x)^3(-1)(1+x)^4]}{((1-x)^4)^2}$

$\frac{dy}{dx} = \frac{4(1+x)^3(1-x)^4-[-4(1-x)^3(1+x)^4]}{(1-x)^8}$

$\frac{dy}{dx} = \frac{(1+x)^3(1-x)^3[4(1-x)^1-[-4(1+x)^1]]}{(1-x)^8}$

$\frac{dy}{dx} = \frac{(1+x)^3(1-x)^3[(4-4x)-[-4-4x]]}{(1-x)^8}$

$\frac{dy}{dx} = \frac{(1+x)^3(1-x)^3[4-4x+4+4x]}{(1-x)^8}$

$\frac{dy}{dx} = \frac{(1+x)^3(1-x)^3[8]}{(1-x)^8}$

$\frac{dy}{dx} = \frac{(1+x)^3[8]}{(1-x)^5}$

$\frac{dy}{dx} = \frac{8(1+x)^3}{(1-x)^5}$

2.5 Video Instruction

- Derivative Product Rule
- Derivative Quotient Rule
- Derivative of a Power, Product Rule
- Derivative of a Power, Quotient Rule
- Implicit Differentiation

2.6 Homework Week 2 - Questions with Answers

- Day 1, MLK (Spring)
- Day 2, questions 1-4
- Day 3, questions 5-8
- Day 4, questions 9-12
- Day 5, questions 13-18

1. $y = x^2(2x+1)$ $\quad \frac{dy}{dx} = 6x^2 + 2x$

2. $y = (2x+3)(5x-4)$ $\quad \frac{dy}{dx} = 20x + 7$

3. $y = (x^2+3x+4)(x^3-4x)$ $\quad \frac{dy}{dx} = 5x^4 + 12x^3 - 24x - 16$

4. *Find* f_2' *when* $f_x = (x^2-4x+3)(x^3-5x)$ $\quad f_2' = -7$

5. $y = \frac{(x-1)}{(x^2+x+1)}$ $\quad \frac{dy}{dx} = \frac{-x^2+2x+2}{(x^2+x+1)^2}$

6. $y = \frac{4x^2+9}{3x^3-4x^2}$ $\quad \frac{dy}{dx} = \frac{-12x^4-81x^2+72x}{(3x^3-4x^2)^2}$

7. $y = \frac{3x-1}{2x+4}$ $\quad \frac{dy}{dx} = \frac{14}{(2x+4)^2}$ OR $\frac{7}{2(x+2)^2}$

8. *Find* f_{-1}' *when* $f_x = \frac{3x-4}{x+2}$ $\quad f_{-1}' = 10$

9. $y = x^3(x^3-x)^3$ $\quad \frac{dy}{dx} = (12x^5 - 6x^3)(x^3-x)^2$

10. $y = (3x+4)^{\frac{3}{4}}(4x^2+8)$ $\quad \frac{dy}{dx} = \frac{33x^2+32x+18}{(3x+4)^{\frac{1}{4}}}$

11. $y = \frac{(x^3+2)^4}{4x^2-3x}$ $\quad \frac{dy}{dx} = \frac{(x^3+2)^3(40x^4-33x^3-16x+6)}{(4x^2-3x)^2}$

12. $y = \frac{(3x+2)^5}{(2x-1)^3}$ $\quad \frac{dy}{dx} = \frac{(3x+2)^4(12x-27)}{(2x-1)^4}$

13. *Find the slope of the line tangent to the curve* $y = \frac{x-3}{2-5x}$ *at the point* $(2, \frac{1}{8})$.
    $m_{tan} = \frac{-13}{64}$

14. *Find the equation of the tangent line at the given point in the previous question.*
    $y = \frac{-13x}{64} + \frac{17}{32}$

15. $4x + 3y = 7$ $\quad y' = \frac{-4}{3}$

16. $x^2 - y^2 = 9$ $\quad y' = \frac{x}{y}$

17. $y^4 - y^2 x + x^2 = 0$ $\quad y' = \frac{y^2 - 2x}{4y^3 - 2xy}$

18. $3x^2y^2 + 4y^5 + 8x^2y^3 + xy = 5$ $\quad y' = \frac{-6xy^2 - 16xy^3 - y}{6x^2y + 20y^4 + 24x^2y^2 + x}$

Week 3

Derivatives Applied Electronics part 1

3.1 Derivatives Applied Electronics Formulas

1. **Instantaneous Current**: *The instantaneous current equals the derivative of the charge q in coulombs with respect to time t in seconds.*

$$i = \frac{dq}{dt}$$

2. **Instantaneous Power**: *The instantaneous power in a circuit is the rate of change of energy (or work) w, at the instant in question.*

$$p = \frac{dw}{dt}$$

3. **Instantaneous Capacitor Current**: *When the instantaneous voltage v across a capacitor varies at a rate dv/dt volts per second, the following equation gives the current in the capacitor at any instant t:*

$$i_C = C\frac{dv}{dt}$$

4. **Instantaneous Induced Voltage**: *The induced voltage at any instant equals the product of the number of turns N times the rate of change of the flux ϕ that links the winding.*

$$v_{ind} = -N\frac{d\phi}{dt}$$

5. **Instantaneous Inductor Voltage**: *The induced voltage in an inductor with an inductance in henrys and a change in current is:*

$$v_{ind} = -L\frac{di}{dt}$$

6. **Mutual Inductance**: *The voltage induced in winding 2 of two coupled windings with mutual inductance M henrys is:*

$$v_2 = -M\frac{di_1}{dt}$$

7. **Kirchhoff's Current Law**: *The sum of the currents toward any point in a circuit, at any instant, equals zero.*

$$i = C\frac{dv}{dt} + \frac{V}{R}$$

8. **Kirchhoff's Voltage Law**: *The sum of the voltage drops around a circuit, at any instant, equals zero*

$$v = -L\frac{di}{dt} + Ri$$

3.2 Video Instruction

- Derivatives Applied Electronics Part 1
- Derivatives Applied Electronics Part 2
- Derivatives Applied Electronics Part 3
- Derivatives Applied Electronics Part 4

3.3 Homework Week 3 - Questions with Answers

- Day 1, Review
- Day 2, Derivatives Test 2
- Day 3, questions 1-4
- Day 4, questions 5-9
- Day 5, questions 10-13

1. If the current in a $1\mu F$ capacitor is to be $0.1mA$, at what rate in volts per second must the applied voltage change? $\frac{dy}{dx} = 100v/s$

2. The magnetic flux through a 500-turn winding varied according to $\phi = 0.004t$ webers. Find the induced voltage in the winding (a.) when $t = 0.01$ seconds and (b.) when $t = 0.1$ seconds. $v_{ind} = -2v$

3. If the flux through a 150-turn winding varied according to the formula $\phi = 0.01t - t^2 + 0.2$ webers, what voltage was induced when $t = 0.02$ seconds? $v_{ind} = 4.5v$

4. The magnetic flux N in a winding of 600 turns varied as $\phi = 0.5t^{\frac{3}{5}}$ webers, where t was in seconds. Find the induced voltage v_{ind} when $t = 1$ second. $v_{ind} = -180v$

5. What formula expresses the voltage v_{ind} across a $100mh$ inductor if the current i constantly equals $0.2A$? Neglect resistance. $v_{ind} = 0v$

6. How fast does the current in a $12h$ winding change to cause an induced voltage of $3.6v$? $\frac{di}{dt} = -300mA/sec$

7. The mutual inductance between two windings is 0.2 henrys. If a current $i_1 = 11t^{\frac{3}{2}}$ amps flows in the primary windings, how much voltage v_2 is induced in the secondary winding when $t = 0.001$ seconds? $v_2 = -104.355mV$

8. The mutual inductance between two windings is $M = 6h$. How fast must the current in one of the windings vary in amps per second to induce -4.8 volts in the other winding? $\frac{di}{dt} = 800mA/sec$

9. A winding linked a magnetic field that varied according to $\phi = 0.002t - 2t^2$ webers. When t was 0.0025 seconds, the voltage induced in the winding measured 8 volts. How many turns did the winding include? $N = 1000$ turns

10. If the current in a $30h$ inductor changes according to $i = 0.02t^{\frac{5}{3}}$ amps, after what interval will the induced voltage measure -96 volts? $t = 940.604$ seconds

11. A voltage, $v = t^3 + 1,000$ volts appears across a parallel RC combination, where $R = 300K\Omega$ and $C = 20\mu F$. Find the resulting current i_g at any time t. $i_g = 3.333 \times 10^{-6}t^3 + 60 \times 10^{-6}t^2 + 3.333 \times 10^{-3}$ amps

12. A $50K\Omega$ bleeder resistor shunts a $4\mu f$ filter capacitor. During a part of the charging process, the voltage across the capacitor varies approximately as $vc = 1,000t^{\frac{2}{3}} + 100$ volts. Find the current ig applied to the combination when $t = 0.001$ seconds. $ig = 28.867mA$

13. A current $i = 3t^{\frac{1}{3}} + 2$ amps flows through a series RL circuit, where $R = 100\Omega andL = 20h$. Find the voltage vg across this circuit when $t = 0.125$ seconds. $vg = 270v$

Week 4

Derivatives Applied Electronics part 2

4.1 Homework Week 4 - Questions with Answers

- Day 1, questions 14-17
- Day 2, questions 18-21
- Day 3, questions 22-25
- Day 4, questions 26-29
- Day 5, Review

14. A relay winding has an inductance of 0.5h and a resistance of 470Ω. If the winding current i equals $t^{\frac{1}{2}} + 0.02$ amps, find the voltage vg across the winding when $t = 0.01$ seconds. $vg = 53.9v$

15. A series circuit consists of a 22h inductor and a 68Ω resistor. A current $i = 2t^2 + t$ exists in this combination. After what time t does the voltage across the combination equal 375 volts? $t = 1.784$ seconds

16. A voltage $v = t^3 + 1,000$ volts appears across a parallel RC combination, where $R = 2M\Omega$ and $C = 1\mu F$. Find the resulting current ig at any time t.
$ig = 3 \times 10^{-6}t^2 + 500 \times 10^{-9}t^3 + 500 \times 10^{-6}$

17. A transistor operates into a load resistance of $2.2K\Omega$. The shunt capacitance in the circuit equals $70pf$, as measured at the collector. Over a certain interval the output voltage supplied by the transistor equals $v = 1 \times 10^7 t + 30$ volts. Find the collector signal current when $t = 10\mu s$. $ic = 59.791mA$

18. A current $i = 10t^{\frac{1}{2}} + 0.1$ amps flows through a series RL circuit, where $R = 800\Omega$ and $L = 320h$. Find the voltage vg across this circuit when $t = 0.04$ seconds.
$vg = -6.32Kv$

19. A transistor collector has a load resistor of $4.7K\Omega$ with a compensation inductor $L = 20mh$ in series with the resistor. The current i through the combination equals $2.5 \times 10^4 t + 0.01$ amps. Find the voltage across the RL circuit when $t = 25ns$.
$vg = -450.063v$

20. A $27K\Omega$ resistor shunts a $33\mu f$ capacitor. The applied voltage v equals $300t^2$ volts. At what time t does the total current i equal $84mA$?
$t = 1.999s$

21. The voltage applied across a capacitor of $0.2\mu f$ was $v = 5 - 3t^2$ volts. The energy stored in a capacitor is $w = \frac{Cv^2}{2}$ joules. Find a formula for $\frac{dw}{dt}$ in this capacitor.
$\frac{dw}{dt} = -1.2 \times 10^{-6} t(5 - 3t^2)$ OR $3.6 \times 10^{-6} t^3 - 6 \times 10^{-6} t$

22. The intensity I of light from a tungsten filament varies with the applied voltage according to $I = Av^{3.7}$, where A is a constant and v is the applied voltage. If $v = t - 2t^2$, find a formula for $\frac{dI}{dt}$.
$\frac{di}{dt} = 3.7A(t - 2t^2)^{2.7}(1 - 4t)$

23. When a length l meters of a conductor moves at a speed of v meters per second in a magnetic field of uniform flux density β teslas, a voltage is induced equal to $v = -\beta l v$ volts. If $v = 10$ meters per second, $l = 0.3$ meter, and β varies over a certain interval according to $\beta = \frac{1}{t^2}$, find $\frac{dv}{dt}$ when $t = 0.5$ seconds.
$\frac{dv}{dt} = 48v/sec$

24. The frequency of a certain crystal oscillator varies with temperature T according to $f = fa[1 + k(T - Ta)]$, where fa is the frequency at an initial temperature Ta and k is a constant of the crystal. If T varies with time (t minutes) according to $T = 55 + 0.01t^2$, how fast does f change when $t = 10$?
$\frac{df}{dt} = fak(0.2)$

25. The wavelength λ meters of a radio wave traveling at a speed $c = 3 \times 10^8$ meters per second varies with the frequency according to $\lambda = \frac{c}{f}$. If $f = 1 \times 10^8 + (5 \times 10^7)t^{\frac{1}{2}}$ hertz find a formula for $\frac{d\lambda}{dt}$.
$\frac{d\lambda}{dt} = \frac{-7.5 \times 10^{15}}{(1 \times 10^8 + 5 \times 10^7 t^{\frac{1}{2}})^2 t^{\frac{1}{2}}}$ seconds

26. The voltage v across a varying resistor r, carrying a fixed current I, is $v = Ir$. If r varies with time t according to $r = t^3 + 5$, find a formula for $\frac{dv}{dt}$ in this capacitor.
$\frac{dv}{dt} = 3t^2 I$

27. The mutual inductance between two windings is $M = \frac{N_2 \phi_2}{1_1}$, where i_1 is the current in one of the windings and N_2 and ϕ_2 are the number of turns of the second winding and the flux linking it to the first winding. If i_1 and N_2 are constant, and if the second winding moves so that ϕ_2 varies with time t seconds according to $\phi_2 = t^3 - 2t$, find a formula for $\frac{dm}{dt}$.
$\frac{dm}{dt} = \frac{n_2}{i_1}(3t^2 - 2)$

28. A copper wire of diameter d and length s has a resistance of $r = \frac{ks}{d^2}$, where k is a constant. Suppose a sliding wire changes the length so that $s = t^2 - 0.6t$, where t is in seconds. Find a formula for $\frac{dr}{dt}$.
$\frac{dr}{dt} = \frac{k}{d^2}(2t - 0.6)$

29. The force between two charged particles having fixed charges Q_1 and Q_2 varies with the distance separating them according to $f = \frac{Q_1 Q_2}{4\pi\epsilon s^2}$. If ϵ is a constant, and if s varies with time as $s = 6t^{\frac{3}{2}}$, find a formula for $\frac{df}{dt}$.
$\frac{df}{dt} = \frac{-Q_1 Q_2}{48\pi\epsilon t^4}$

Week 5

Introduction to Integrals

5.1 The Integral

5.1.1 What is a Integral?

- **Simple Definition:** An integral adds up small pieces to find the total amount.
- **Everyday Terms:** If a derivative tells you how fast something is changing (like speed), an integral tells you the total accumulated result of that change (like distance traveled).
- **Examples:**
  - If a car's speed is changing over time, the integral of that speed function gives you the total distance the car has traveled.
  - If a curve represents a rate (like flow, speed, or growth), the area under the curve found by integration gives the total amount over time or space.

Often we have the derivative function and need to find the original function, which requires that we perform the inverse operation of differentiation. Now, consider finding the derivative in reverse; that is, given the derivative of a function, find the function. This is called antidifferentiation or integration. [1, p. 224].

The process of finding a function that has a given differential (or derivative) we call integration or antidifferentiation. [2, p.175].

5.2 Breaking Down the Integral or Anti-Derivative

5.2.1 The Integral Decoder Ring

$$y = \int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

This is generally confusing at first glance. However, if you work through the practice problems it will become perfectly clear.

5.2.2 Steps

The mission of the integral is to convert a known derivative $\frac{dy}{dx}$ back to its original function y. Suppose that we are given the derivative $\frac{dy}{dx} = x^7$.

1. $\frac{dy}{dx} = x^7$
2. Solve for dy:
   - $dy = x^7\ dx$
3. Taking the integral of the entire equation:
   - $\int(dy = x^7\ dx)$
   - OR $\int(dy) = \int(x^7\ dx)$
   - Simplify $y = \int(x^7\ dx)$
   - Rewritten $y = \int x^7\ dx$
     Technically, the dx would turn to 1. However, dx is kept as a reference so that we know what the original rate was. The dx tells us what variable to apply to the "Decoder", which is necessary if there are multiple variables.
4. Review Decoder: $\int x^n\ dx = \frac{x^{n+1}}{n+1} + C$
5. Complete integral of $y = \int x^7\ dx$:
   - $x = x$
   - $n = 7$
   - $y = \frac{x^{(7+1)}}{(7+1)} + C$
   - Simplify: $y = \frac{x^8}{8} + C$
     C represents the original constant that was not associated with an x variable (x^0). This constant disappeared when the derivative was taken and there is no way for the Integral to know what it was. Therefore, we must represent the mystery value using C when performing an integral.
6. Check your work by taking the derivative of function $y = \frac{x^8}{8} + C$.
   - $y = \frac{x^8}{8} + Cx^0$.
   - $\frac{dy}{dx} = \frac{1}{8} \times 8 \times x^{(8-1)} + 0$
   - $\frac{dy}{dx} = \frac{8}{8} \times x^{(7)}$
   - $\frac{dy}{dx} = x^7$ (back to the original derivative!)

5.3 Acceleration, Velocity, Distance, and the Integral

5.3.1 Acceleration to Velocity

Acceleration is the rate of change of velocity.

$$a(t) = \frac{dv}{dt}$$

Acceleration tells us how velocity is changing over time. So when you integrate acceleration, you're adding up all those small changes in velocity over a time interval — and the total gives you the change in velocity.

- $a(t) = \frac{dv}{dt}$
- (solve for dv) $dv = a(t)dt$
- (integrate to solve for v) $\int(dv = a(t)dt)$
- (simplify) $\int dv = \int a(t)\ dt$
- (simplify) $v = \int a(t)\ dt + C$

$$v(t) = \int a(t)\ dt + C$$

- $v(t)$: velocity at time t
- $a(t)$: acceleration at time t
- C: constant of integration (initial velocity)

5.3.2 Velocity to Distance

Velocity is the rate of change of position or distance.

$$v(t) = \frac{ds}{dt}$$

Velocity tells us how position (or distance) is changing over time. So when you integrate velocity, you're adding up all those small changes in position — giving you the total displacement or distance traveled.

- $v(t) = \frac{ds}{dt}$
- (solve for ds) $ds = v(t)dt$

- (integrate to solve for s) $\int (ds = v(t)dt)$
- (simplify) $\int ds = \int v(t)\ dt$
- (simplify) $s = \int v(t)\ dt + C$

$$s(t) = \int v(t)\ dt + C$$

- $s(t)$: position (or distance) at time t
- $v(t)$: velocity at time t
- C: constant of integration (initial position)

5.4 Acceleration due to Gravity

Metric Units:

$$a(g) = 9.8m/s^2$$

This means velocity increases by 9.8 meters per second every second.

5.4.1 Imperial Units:

$$a(g) \approx 32ft/s^2$$

This means that velocity increases by 32 feet per second every second.

5.5 Video Instruction

- Integrals Part 1
- Integrals Part 2
- Integrals Part 3
- Integrals Applied Part 1
- Integrals Applied Part 2
- Integrals Applied Part 3

5.6 Homework Week 5 - Questions with Answers

- Day 1, Derivatives Applied Test
- Day 2, questions 1-5
- Day 3, questions 6-10
- Day 4, questions 11-13
- Day 5, questions 14-16

1. $y = \int x^7 \; dx$

   $y = \frac{x^8}{8} + C$

2. $y = \int \frac{6}{x^3} \; dx$

   $y = \frac{-3}{x^2} + C$

3. $y = \int \sqrt{6x+2} \; dx$

   $y = \frac{(6x+2)^{\frac{3}{2}}}{9} + C$

4. $y = \int x\sqrt[3]{5x^2-1} \; dx$

   $y = \frac{3(5x^2-1)^{\frac{4}{3}}}{40} + C$

5. $y = \int (3x^2+2)(x^3+2x)^3 \; dx$

   $y = \frac{(x^3+2x)^4}{4} + C$

6. $y = \int (10x-1)\sqrt{5x^2-x} \; dx$

   $y = \frac{2(5x^2-x)^{\frac{3}{2}}}{3} + C$

7. $y = \int (2x+3)^2 \; dx$

   $y = \frac{(2x+3)^3}{6} + C$

8. $y = \int 4x(x^2+1)^3 \; dx$

   $y = \frac{(x^2+1)^4}{2} + C$

9. $y = \int (6x^2+6)(x^3+3x)^{\frac{-1}{3}} \; dx$

   $y = 3(x^3+3x)^{\frac{2}{3}} + C$

10. $\int (x-1)(x)^{-3} \; dx$

    $y = \frac{-1}{x} + \frac{1}{2x^2} + C$

11. Find the equation describing the distance of an object moving along a straight line when the acceleration is $a = 3t$, when the velocity at $t = 4s$ is $40m/s$, and when the object has traveled 86m from the origin at $t = 2s$.

    $s = \frac{t^3}{2} + 16t + 50$

12. A stone is dropped from a height of 100ft. For a free-falling object, the acceleration is $a = -32ft/s^2$ (gravity). A. Find the distance the stone has traveled after 2 seconds. Note that the initial velocity is 0 because the stone was dropped, not thrown. B. Find also the velocity of the stone when it hits the ground.

    $v = -80ft/sec$

13. A stone is hurled straight up from the ground at a velocity of 25m/sec. A. Find the maximum height that the stone reaches. B. How long does it take for the stone to hit the ground? C. Find the speed at which the stone hits the ground.

    a. $s = 31.888m$
    b. $t = 5.102sec$
    c. $v = -25m/sec$

14. A stone is thrown vertically upward from the roof of a 200ft tall building with an initial velocity of 30ft/sec. A. Find the equation describing the altitude of the stone from the ground. B. How long does it take for the stone to hit the ground?

    a. $s = -16t^2 + 30t + 200$
    b. $t = 4.594sec$

15. A stone is thrown straight down from an 80-meter-tall building with an initial velocity of 10m/sec. A. Find the equation describing the height of the stone from the ground. B. How long does it take for the stone to hit the ground?

    a. $s = -4.9t^2 - 10t + 80$
    b. $t = 3.147sec$

16. An object is dropped from a stationary balloon at 500m. A. Express the objects height above the ground as a function of time. B. How long does it take the object to hit the ground?

    a. $s = -4.9t^2 + 500$
    b. $t = 10.102sec$

Week 6

Integrals Applied Electronics

6.1 Integrals Applied Electronic Formulas

6.1.1 Review Derivatives Applied Electronics Formulas

1. **Instantaneous Current**: *The instantaneous current equals the derivative of the charge q in coulombs with respect to time t in seconds.*

$$i = \frac{dq}{dt}$$

2. **Instantaneous Power**: *The instantaneous power in a circuit is the rate of change of energy (or work) w, at the instant in question.*

$$p = \frac{dw}{dt}$$

3. **Instantaneous Capacitor Current**: *When the instantaneous voltage v across a capacitor varies at a rate dv/dt volts per second, the following equation gives the current in the capacitor at any instant t:*

$$i_C = C\frac{dv}{dt}$$

4. **Instantaneous Induced Voltage**: *The induced voltage at any instant equals the product of the number of turns N times the rate of change of the flux ϕ that links the winding.*

$$v_{ind} = -N\frac{d\phi}{dt}$$

5. **Instantaneous Inductor Voltage**: *The induced voltage in an inductor with an inductance in henrys and a change in current is:*

$$v_{ind} = -L\frac{di}{dt}$$

6. **Mutual Inductance**: *The voltage induced in winding 2 of two coupled windings with mutual inductance M henrys is:*

$$v_2 = -M\frac{di_1}{dt}$$

7. **Kirchhoff's Current Law**: *The sum of the currents toward any point in a circuit, at any instant, equals zero.*

$$i = C\frac{dv}{dt} + \frac{V}{R}$$

8. **Kirchhoff's Voltage Law**: *The sum of the voltage drops around a circuit, at any instant, equals zero*

$$v = -L\frac{di}{dt} + Ri$$

6.1.2 How to manipulate the formulas to convert the rate to a quantity function using integrals.

1. $i = \frac{dq}{dt}$

   $dq = i\ dt$

   $\int(dq = i\ dt)$

   $q = \int i\ dt$ (coulombs q is equal to the integral of i in terms of change of time in seconds.

   $$q = \int i\ dt\ + C\ (coulombs)$$

2. $p = \frac{dw}{dt}$

   $dw = p\ dt$

   $\int(dw = p\ dt)$

   $w = \int p\ dt$ (energy w is equal to the integral of power p in terms of change of time in seconds. The unit is joules.

   $$w = \int p\ dt\ + C\ (joules)$$

3. $i_C = C\frac{dv}{dt}$

   $dv = \frac{1}{C}\ i_C\ dt$

$\int(dv = \frac{1}{C}\ i_C\ dt)$

$v = \int \frac{1}{C}\ i_C\ dt$

$v = \frac{1}{C}\int\ i_C\ dt$ (Voltage of the capacitor is equal to the inverse of capacitance multiplied to the integral of the current through the capacitor in terms of change of time in seconds. The unit is volts.

$$v_C = \frac{1}{C}\int\ i_C\ dt\ + C\ (volts)$$

4. $v_{ind} = -N\frac{d\phi}{dt}$

$d\phi = \frac{1}{-N}\ v_{ind}\ dt$

$\int(d\phi = \frac{1}{-N}\ v_{ind}\ dt)$

$\phi = \int \frac{1}{-N}\ v_{ind}\ dt$

$\phi = \frac{-1}{N}\int v_{ind}\ dt$ (The flux density is equal to the inverse of number of turns multiplied to the integral of the induced voltage in terms of change of time in seconds. The unit is webers.

$$\phi = \frac{-1}{N}\int v_{ind}\ dt\ + C\ (webers)$$

5. $v_{ind} = -L\frac{di}{dt}$

$di = \frac{1}{-L}\ v_{ind}\ dt$

$\int(di = \frac{1}{-L}\ v_{ind}\ dt)$

$i = \int \frac{1}{-L}\ v_{ind}\ dt$

$i = \frac{-1}{L}\int v_{ind}\ dt$ (Current is equal to the inverse of the inductance in henrys multiplied to the integral of the induced voltage in terms of change of time in seconds. The unit is Amps.

$$i = \frac{-1}{L}\int v_{ind}\ dt\ + C\ (Amps)$$

6. $v_2 = -M\frac{di_1}{dt}$

$di_1 = \frac{1}{-M}\ v_2\ dt$

$\int(di_1 = \frac{1}{-M}\ v_2\ dt)$

$i_1 = \int \frac{1}{-M}\ v_2\ dt$

$i_1 = \frac{-1}{M}\int v_2\ dt$ (The primary current is equal to the inverse of the mutual inductance in henrys multiplied to the integral of the secondary voltage in terms of change of time in seconds. The unit is Amps.

$$i_1 = \frac{-1}{M}\int v_2\ dt\ + C\ (Amps)$$

7. **Kirchhoff's Current Law**: *The sum of the currents toward any point in a circuit, at any instant, equals zero.*

$$i = C\frac{dv}{dt} + \frac{V}{R} - \frac{1}{L}\int v_{ind}\ dt$$

8. **Kirchhoff's Voltage Law**: *The sum of the voltage drops around a circuit, at any instant, equals zero*

$$v = -L\frac{di}{dt} + Ri + \frac{1}{C}\int\ i_C\ dt$$

6.2 Video Instruction

- Integrals Applied Electronics Part 1
- Integrals Applied Electronics Part 2
- Integrals Applied Electronics Part 3

6.3 Homework Week 6 - Questions with Answers

- Day 1, Presidents Day (Spring)
- Day 2, Integrals Test
- Day 3, questions 1-4
- Day 4, questions 5-9
- Day 5, questions 10-14

1. The current in a circuit was $i = 4t^3$ amps. How many coulombs were transmitted in 3 seconds?

   $q = 81$ coulombs

2. A $80\mu f$ capacitor is charged to 100 volts. We then apply a current $i_c = 0.04t^3$ amps in the same polarity as the initial charge. After how many seconds will the capacitor voltage reach 225 volts?

   $t = 1$ second

3. The voltage applied to a circuit was $v = 2t + 1$ volts. If the current followed the equation $i = 0.03t$ amperes, find the energy w delivered from $t = 0$ to $t = 50$ seconds.

   $w = 2.5375\text{K watts}$

4. A 110 turn winding caries a flux of 0.8 webers. If we now want to vary the flux so that a voltage $v_{ind} = -5t^2$ volts appears in the winding, what equation must the flux through the winding follow?

   $\phi = \frac{t^3}{66} + 0.8$ webers

5. A DC current of 0.3 ampere flows in a 15 henry inductor. Superimposed on this DC is a varying current such that the voltage $v_{ind} = 120t^{\frac{1}{3}}$ volts appears in the inductor. Find the instantaneous total current when $t = 1$ second (assume that the DC and AC currents have the same polarity when $t = 1$ second).

   $i = 6.3$ or -6.3 amps

6. An inductance of 8 henrys is connected in parallel with a 12Ω resistor. Apply to this circuit a voltage $v = 20t^2$ volts. If $i = 0$ when $t = 0$, find an equation for i.

   $ig = \frac{5t^2}{3} - \frac{5t^3}{6}$ amps

7. If we apply a voltage $v = 90t^{\frac{1}{2}}$ to a circuit consisting of a 30 henry inductance shunted by a 50Ω resistance, what current flows when $t = 4$ seconds? (let $i = 0$ when $t = 0$).

   $ig = -12.4$ amps

8. If we apply a voltage $v = 20t^4$ volts across a parallel RL combination, where $R = 500\Omega$ and $L = 40$ henrys, find the total current when $t = 0.2$ seconds. Let $i = 4\mu\text{A}$ when $t = 0$.

   $ig = 36\mu\text{A}$

9. In a parallel RL circuit, $R = 5\Omega$ and $L = 0.2$ henrys. If a voltage $v = t^{\frac{3}{2}} + 2$ volts were applied, what would the current i be when $t = 4$ seconds? Assume $i = 0.4$ amps when $t = 0$.

$ig = 101.6$ amps

10. A current $i = 0.005t^{\frac{1}{2}}$ amps flows in a series RC circuit where $R = 8.8 \times 10^4\Omega$ and $C = 1\mu f$. Find a formula for the voltage across the circuit as a function of time t. Assume the capacitor to be initially discharged.

    $vg = 440t^{\frac{1}{2}} + 3.333 \times 10^3 t^{\frac{3}{2}}$ volts

11. The current function $i = 1 \times 10^{-3} t^{\frac{1}{2}}$ amperes is applied to a series RC circuit where $R = 8.8 \times 10^4\Omega$ and $C = 1\mu f$. Find a formula for the impressed voltage as a function of time t. (Assume the initial capacitor charge to be 100v.)

    $vg = 88t^{\frac{1}{2}} + 666.667t^{\frac{3}{2}} + 100$ volts

12. A series LC circuit where $L = 0.1$ henry and $C = 100\mu f$ has applied to it a current $i = 0.1$A from $t = 0$ onward. Find (a) the formula for the voltage across the circuit, and (b) the rate of change at $t = 2$ sec. (assume $v_c = 0$ when $t = 0$)

    a. $vg = 1000t$

    b. $\frac{dv}{dt} = 1000v/s$

13. A series circuit has these constants: $R = 5K\Omega$, $L = 200$ henrys, and $C = 20\mu f$. If we supply to the circuit a current $i = 0.02t^2$ amperes, at what rate does the voltage across the circuit change when $t = 0.2$ seconds?

    $\frac{dv}{dt} = 72v/s$

14. In a series RCL circuit, let $R = 10\Omega$, $C = 10,000\mu f$, and $L = 10$ henrys. Through this circuit we pass a current $i = 1 - t^{\frac{1}{2}}$ amps. Find the total voltage v across this circuit when $t = 4$ seconds. Assume $v = 0.25$ volts when $t = 1$ second.

    a. $vg = -178v$

Week 7

Logarithms

7.1 Logarithm Introduction

In applied electronics, logarithms are essential for analyzing circuits and signals that involve exponential relationships, such as those found in RC (resistor-capacitor) circuits, decibel calculations, and semiconductor behavior. A logarithm tells us the power to which a base, often 10 or the natural base (e) must be raised to get a given number. For example, the natural logarithm $\ln(x)$ is used to describe how voltage decays over time in a discharging capacitor, following an exponential pattern. Logarithmic functions also simplify the math behind signal strength and gain, such as when calculating decibels (dB) in amplifier design. In applied calculus, logarithms allow us to solve equations, integrate exponential decay functions, and linearize nonlinear data, making them crucial tools in electronics engineering and analysis.

7.2 Logarithm and Exponential - Relationships and Conversions

7.2.1 Base 10:

$$Log_{10}(N) = X$$
$$10^X = N$$

7.2.2 Base 3:

$$Log_3(N) = X$$
$$3^X = N$$

7.2.3 Natural Logs:

$$LN(N) = X$$
$$e^X = N$$

7.3 Video Instruction

- Logarithms Part 1
- Logarithms Part 2
- Logarithms Part 3

7.4 Homework Week 7 - Questions with Answers

- Day 1, Review
- Day 2, Test (Integrals Applied)
- Day 3, questions 1-5
- Day 4, questions 6-10
- Day 5, questions 11-15

1. Given the equation $m^{2.3} = 25$, solve for m.

   $m = 4.053$

2. Given the equation $x = log_3 2187$, solve for x.

   $x = 7$

3. Given the equation $L_1 = ((L_2)^2)^{\frac{1}{3}}$, solve for L_2.

   $L_2 = \sqrt{(L_1)^3}$

4. Given the equation $I = (\frac{V}{L})te^{sct}$, solve for sc.

   $sc = \frac{LN(\frac{IL}{Vt})}{t}$

5. Given the equation $I_K = AT^2 e^{\frac{-B}{t}}$, solve for A and B.

   $A = \frac{I_K}{T^2 e^{\frac{-B}{t}}}$ $\quad$ $B = -t(LN(\frac{I_K}{AT^2}))$

6. Given the equation $L_1 = ((L_2)^2)^{\frac{1}{3}}$, solve for L_2.

   $L_2 = \sqrt{(L_1)^3}$

7. An amplifier is rated as having a $90dB$ gain. What power ratio does this represent?

   $\frac{P_{out}}{P_{in}} = 10^9$

8. An amplifier has a gain of $60dBm$. What is the output power?

   $P_{out} = 1KW$

9. The manufacturer of a high fidelity $100w$ power amplifier claimed that hum and noise in the amplifier is $90dB$ below the full power output. How much hum and noise power does this represent?

   noise and hum $= 100nW$

10. A network has a loss of $80dB$. What power ratio corresponds to this loss?

    $\frac{P_{out}}{P_{in}} = 10^{-8}$

11. An amplifier has a input impedance of 600Ω and a output impedance of $6K\Omega$. The power out is $30W$ when $1.9v$ is applied across the input:

    (a) What is the voltage gain of the amplifier?

    $\Delta_V = 223.297$

    (b) What is the power gain of the amplifier in dB?

    $\Delta_P = 36.978dB$

(c) What is the input power?

$P_{in} = 6.017mW$

12. The noise level of a telephone line used for wired music programs is $60dB$ down from the program level of $12.5mW$. How much noise power is represented by this level?

$noise = 12.5nW$

13. A crystal microphone is rated at $-80dB$. There is onhand a final AF amplifier rated at $60dB$. How much gain must be provided by a preamp in order to drive the final amplifier to full output if a attenuator pad between the microphone and the preamp has a loss of $20dB$?

$preamp\ gain = 100dB$

14. An amplifier has a normal output of $30W$. A selector switch is arranged to reduce the output in $5dB$ steps. What power output corresponds to the reduction of 5, 10, 15, 20, 25, and $30dB$?

$$-5dB = 9.487W$$
$$-10dB = 3W$$
$$-15dB = 0.9487W$$
$$-20dB = 0.3W$$
$$-25dB = 0.09487W$$
$$-30dB = 0.03W$$

15. A two-stage video RF amp has a $300\mu V$ input signal into 75Ω. The second stage has a gain of $50dB$. When matched input-output impedances are used, the voltage output of the second stage must be $4.22V$ to allow distribution of the signal. Determine the following:

(a) The input voltage of the second stage.

$Vin_{second\ stage}$=13.345mv

(b) The dB gain of the first stage.

$\Delta_{dB} = 32.964dB$

(c) The overall gain of the two amplifiers when all impedances are 75Ω.

$Total\ Gain_{dB} = 82.964_{dB}$

Week 8

Circles

8.1 Circles Introduction

In applied calculus with a focus on electronics, circles are far more than simple geometric shapes; they are essential tools for modeling and understanding cyclical and oscillatory behavior in electronic systems. Circular motion forms the basis of sine and cosine functions, which describe the voltage and current waveforms in alternating current (AC) circuits. Concepts like phasors, impedance, and signal phase shifts are rooted in circular representations on the complex plane, where calculus helps us analyze how these quantities change over time. By studying circles and their mathematical properties, we lay the groundwork for understanding key electronic applications such as oscillators, filters, amplifiers, and signal processing systems.

8.2 Circle Formulas

Standard Form with Center at Origin:

$$x^2 + y^2 = r^2$$

- r equals the radius of the circle

Standard Form with offset (center not at Origin):

$$(x - h)^2 + (y - k)^2 = r^2$$

- Center is at h & k
- r is the radius

General Form

$$x^2 + y^2 + Dx + Ey + F = 0$$

When to use Standard Form and when to use General Form

The **standard form** is clearer for understanding and graphing. While the **general form** has practical advantages in analysis, algebraic manipulation, and system solving. In control systems, robotics, and electronics simulations, the **general form** gives an implicit equation of a curve. Implicit forms are often more efficient for computation and are used in simulators and SPICE models.

8.3 Video Instruction

- Circles

8.4 Homework Week 8 - Questions and Answers

- Day 1, Logarithms 16-21
- Day 2, Review (Logarithms)
- Day 3, Test Logarithms
- Day 4, Circles 1-5
- Day 5, Circles 6 & 7

16. A video tuner amplifier has an input impedance of 300 ohms and an output impedance of 3,500 ohms. When a $300mV$ signal is applied at the input, a $250V$ signal appears at the output.

    (a) What is the power output of the amplifier?
    $P_{out} = 17.857$ watts

    (b) What is the power gain in dB?
    $\Delta_{dB} = 47.75dB$

    (c) What is the voltage gain of the amplifier?
    $\Delta V = 833.333$

17. Given the following specifications for a 2N45 transistor, What is the power input?

    - Collector Voltage $= -20V$
    - Emitter Current $= 5mA$

- Input Impedance = 10Ω
- Source Impedance = 50Ω
- Load Impedance = $4.5K\Omega$
- Power Output = $45mW$
- Power Gain = $23dB$

$P_{In} = 225.534\mu W$

18. The input power to a 50Km line is $10mW$. The output of this line is $40\mu W$ What is the attenuation (dB) of the line per kilometer?

    $Attenuation = -0.4796db/Km$

19. What is the dB gain necessary to produce a $60\mu W$ signal in a 600Ω telephone if the received signal supplies $9\mu V$ to the 80Ω line that feeds the receiver?

    $Gain_{necessary} = 77.727dB$

20. In problem 19, if the overall gain is increased to $96dB$ what received signal will produce the $60\mu W$ signal in the telephone?

    $Signal_{Voltage} = 1.098\mu V$

21. The voltage across a 600Ω telephone is adjusted to 1.73 volts. When an audio filter is installed in the circuit, the voltage drops to 1.44 volts. What is the insertion loss of the filter?

    $-1.594dB$

Find the Center and Radius for the following Circles:

1. $x^2 + y^2 = 16$

   Center=(0,0) & Radius = 4

2. $x^2 + y^2 + 6x - 8y - 39 = 0$

   Center=(-3,4) & Radius = 8

3. $x^2 + y^2 - 8x + 12y - 8 = 0$

   Center=(4,-6) & Radius = $2\sqrt{15}$ or $\sqrt{60}$

4. $x^2 + y^2 - 12x - 2y - 12 = 0$
   Center=(6,1) & Radius = 7

5. $x^2 + y^2 + 7x + 13y - 9 = 0$
   Center=(-3.5,-6.5) & Radius = $\sqrt{63.5}$

6. The Center is on the y-axis, with Points (1,4) & (-3, 2).
   Center=(0,1) & Radius = $\sqrt{10}$

7. Points (3,1) & (0,0) & (8,4).
   Center=(-5,20) & Radius = $\sqrt{425}$ or 20.615

Week 9

Parabolas

9.1 Video Instruction

- Parabolas Part 1
- Parabolas Part 2
- Parabolas Applied Part 1
- Parabolas Applied Part 2

9.2 Homework Week 9 - Questions with Answers

- Day 1, Parabolas 8-12
- Day 2, 13-17
- Day 3, 18-21
- Day 4, Review
- Day 5, Circles and Parabolas Test

Find the Focus and the Directrix for the following Parabolas:

8. $x^2 = 4y$

   Focus =(0,1) & Directrix y=-1

9. $y^2 = -16x$

   Focus =(-4,0) & Directrix x=4

10. $y^2 = x$

    Focus =($\frac{1}{4}$,0) & Directrix x=$\frac{-1}{4}$

11. $x^2 = 16y$

    Focus =(0,4) & Directrix y=-4

12. $y^2 = 8x$

    Focus =(2,0) & Directrix x=-2

Given the focus and directrix for the following, find the equation for the parabola.

13. (2,0), x=-2

    $y^2 = 8x$

14. (-8,0), x=8

    $y^2 = -32x$

15. (0,6), y=-6

    $x^2 = 24y$

16. (-1,3), x=3

    $y^2 - 6y + 8x + 1 = 0$ or $(y - 3)^2 = -8(x - 1)$

17. (2,-5), y=-1

    $x^2 - 4x + 8y + 28 = 0$ or $(x - 2)^2 = -8(y + 3)$

Given the focus and the vertex, find the equation for the parabola.

18. *focus*=(-4,0), *vertex*=(0,0)

    $y^2 = -16x$

19. The shape of a wire hanging between two poles closely approximates a parabola. Find the equation of a wire that is suspended between two poles 40m apart and whose lowest point is 10 m below the level of the insulators.

    $x^2 = 40y$

20. A suspension bridge is supported by two cables that hang between two supports. The curve of these cables is approximately parabolic. Find the equation of this curve if the focus lies 8m above the lowest point of the cable.

    $x^2 = 32y$

21. A culvert is shaped like a parabola, 120cm across the top and 80cm deep. How wide is the culvert 50cm from the top?

    width at -50cm =73.485cm

Week 10

Max-Mins, Differentials, & Higher Derivatives

10.1 Video Instruction

- Max Min Introduction
- Max Min Applied Part 1
- Max Min Applied Part 2
- Max Min Applied Part 3
- Max Min Applied Electronics Part 1
- Max Min Applied Electronics Part 2
- Max Min Applied Electronics Part 3
- Differentials Applied Part 1
- Differentials Applied Part 2
- Higher Derivatives

10.2 Homework Week 10 - Questions with Answers

- Day 1, Max-Mins 1-3
- Day 2, 4-6
- Day 3, 7-9
- Day 4, 10-14

- Day 5, 15-18

1. The sum of two positive numbers is 56. Find the two numbers if their product is to be maximum.

   $y = 28$ & $x = 28$

2. An open box is to be made from a square piece of aluminum, 3cm on a side, by cutting equal squares from each corner and then folding up the sides. Determine the dimensions of the box that will have the largest volume.

   $2cm \times 2cm \times 0.5cm$

3. A man wishes to fence in a rectangular plot lying next to a river. No fence is required along the river bank. If he has 800m of fence, and he wishes the maximum area to be fenced, find the dimensions of the desired enclosure.

   $200m \times 400m$

4. Find the maximum possible area of a rectangle whose perimeter is 36cm.

   $a = 81cm^2$

5. A farmer wants to fence in $80,000m^2$ of land and then divide it into three plots of equal area. Find the minimum amount of fence needed.

   $fence_{min} = 1600m$

6. The charge transmitted through a circuit varies according to $q = t^4 - 4t^3$ coulombs. Find the time in seconds when the current i (in amps) $i = (dq/dt)$ reaches a minimum.

   $t = 2sec, \ i = -16A$

7. A rectangle box, open at the top, with a square base, is to have a volume of $4000cm^3$. Find the dimensions, if the box is to contain the least amount of material.

   $20cm \times 20cm \times 10cm$

8. The total cost C of making x units of a certain commodity is given by $C = 0.005x^3 + 0.45x^2 + 12.75x$. All units made are sold at $\$36.75x - C$. Find the number of units to make maximum profit.

   $20 \ units$

9. A cylindrical can with one end is to be made with $24\pi cm^2$ of metal. Find the dimensions of the can that give the maximum volume.

   $r = 2.828cm, \ h = 2.828cm$

10. The side of a square measures $12.00cm$ with a maximum possible error of $0.05cm$. (a) Find the maximum possible error in the area using differentials. (b) Find the maximum possible error by substituting into the formula for the area of a square. (c) Find the percentage of error.

a. $da = 1.2cm^2$
b. $1.2025cm^2$
c. 0.833%

11. Suppose you want to build a spherical water tower with an inner diameter of $26.00m$ and a side thickness of $4.0cm$. (a) Find the approximate volume of steel needed using differentials. (b) if the density of steel is $7800kg/m^3$, find the approximate volume of steel needed using differentials.

    a. $dv = 84.95m^3$
    b. $662,599kg$

12. A freely falling body drops according to $s = (1/2)gt^2$, where s is the distance in meters, $g = 9.80m/s^2$, and t is time in seconds. Approximate the distance, ds, that an object falls from $t = 10.00$ sec to $t = 10.03$ sec.

    $ds = 2.94m$

13. The voltage Vin volts, varies according to $V = 10p^{2/3}$, where p is the power in watts. Find the change dv when the power changes from 125w to 128w.

    $dv = 4v$

14. The impedance Z in an ac circuit varies according to $Z = \sqrt{R^2 + X^2}$, where R is the resistance and X is the reactance. If $R = 300\Omega$ and $X = 225\Omega$, find dz when R changes to 310Ω.

    $dz = 8\Omega$

    Find the first four derivatives of each of the following functions.

15. $y = x^5 + 3x^2$

    $y' = 5x^4 + 6x$
    $y'' = 20x^3 + 6$
    $y''' = 60x^2$
    $y'''' = 120x$

16. $3x^6 - 8x^3 + 2$

    $y' = 18x^5 - 24x^2$
    $y'' = 90x^4 - 48x$
    $y''' = 360x^3 - 48$
    $y'''' = 1080x^2$

17. $5x^5 + 2x^3 - 8x$

    $y' = 25x^4 + 6x^2 - 8$
    $y'' = 100x^3 + 12x$
    $y''' = 300x^2 + 12$
    $y'''' = 600x$

18. $3x^2 + 4x - 7$

    $y' = 6x + 4$
    $y'' = 6$
    $y''' = 0$
    $y'''' = 0$

Week 11

Differentials Applied

11.1 Video Instruction

- Derivatives Applied Electronics Part 1

11.2 Homework Week 11 - Questions with Answers

- Day 1, Review
- Day 2, Test - Differentials, Higher Derivatives, & Max-Mins
- Day 3, Differentials, 1-5
- Day 4, 6-10
- Day 5, Limits 1-5

1. An electron (whose mass is M_e) moves at a speed V. Its momentum is $p = mV$. Find a formula for the approximate change dp in momentum resulting from a small increase dv, in speed.

   $dp = m\ dv$

2. The low-frequency inductance of a single-layer solenoid is approximately $L = kDn^2$, where k is a form factor, D is the diameter in centimeters, and n is the number of turns. Find a formula for the approximate change dl in the inductance resulting from the addition of a small part of a turn dn.

   $dL = 2KDn\ dn$

3. The power in a circuit was $p = t - 5$ watts. What was the approximate energy dw in joules expended from $t = 4$ sec to $t = 4.002$ sec?

   $dw = -2mW$

4. The induced voltage in an 8-henry inductor varied according to $vind = 3t^2 - t$. About how much change di occurred in the inductor current from $t = 2$ sec to $t = 2.01$ sec?

   $di = -12.5mA$

5. The power in a circuit is given by $p = Ri^2$, where $R = 100\Omega$ and i is the current in amperes. If i changes from 12 amps to 12.005 amps, approximately what change dp occurs in power in watts?

   $dp = 12\ watts$

6. The current i amperes in a circuit varied with time t seconds according to $i = t^2 + 3t$. About what current change di occurred as t changed from 0.98 sec to 1 sec?

   $di = 99.2mA$

7. The intensity J of the heat radiation from a transmitting tube plate varies with its absolute temperature according to $J = \sigma T^4$ where σ is a constant and T is the temperature in $°C$. If $J = 50$ units when $T = 1200°C$, approximately what change dJ in J results from a change in T to $1205°C$?

   $dJ = 833.345 \times 10^{-3}$ units of heat radiation

8. If the resistance r ohms in a circuit varies with time t seconds according to $r = 100 + t^{\frac{1}{2}}$, what approximate change dr in r occurs as t changes from 4 sec to 4.001 sec?

   $dr = 250\mu\Omega$

9. A right circular cone used in constructing a broadband antenna has a volume $v = \frac{\pi r^2 h}{3}$, where r is the radius of the base and h is the altitude of the cone. If $r = 10$ cm, and $h = 24$ cm, what approximate change dv in the volume occurs when r changes to 10.052 cm?

   $dv = 13.069cm^3$

10. An increase in the apparent mass Ma of a moving particle occurs in accord with $Ma = \frac{Mo}{[1-(\frac{V}{C})^2]^{\frac{1}{2}}}$ where Mo is the mass of the particle at reset, V is its speed, and C is the speed of light in a vacuum. What approximate change d_{Ma} occurs in the apparent mass as a result of a small change d_V in the speed of the particle? Express your answer as a formula.

    $dma = \frac{Vmo}{c^2[1-(\frac{v}{c})^2]^{\frac{3}{2}}}\ dv$

Week 12

Limits

12.1 Video Instruction

- Limits Part 1

12.2 Homework Week 12 - Questions with Answers

- Day 1, Limits, 6-11
- Day 2, Review
- Day 3, Test - Differentials Applied Electronics & Limits
- Day 4, Intro to Trig Functions
- Day 5, Intro to Trig Functions

1. $\lim\limits_{x\to 2} \dfrac{x^2-4}{x-2} = 4$

2. $\lim\limits_{x\to \infty} \dfrac{3x+2}{x} = 3$

3. $\lim\limits_{x\to 0} \dfrac{Sinx}{x} = 1$

4. $\lim\limits_{x\to 2}(x^2-5x) = -6$

5. $\lim\limits_{x\to -1}(2x^3+5x^2-2) = 1$

6. $\lim\limits_{x\to 1} \dfrac{x^2-1}{x-1} = 2$

7. $\lim\limits_{x\to \frac{3}{2}} \dfrac{4x^2-9}{2x+3} = 0$

8. $\lim\limits_{x\to -1} \sqrt{2x+3} = 1$

9. $\lim\limits_{x\to 6} \sqrt{4-x} = \textit{no limit}$

10. $\lim\limits_{x\to \infty} \dfrac{1}{2x} = 0$

11. $\lim\limits_{x\to \infty} \dfrac{3x^2-5x+2}{4x^2+8x-11} = \dfrac{3}{4}$

Week 13

Trigonometric Functions part 1

13.1 Trigonometric Functions Introduction

13.1.1 SOH, CAH, TOA

SOH-CAH-TOA is a helpful mnemonic for remembering the definitions of the sine, cosine, and tangent functions in right triangles—key tools in trigonometry and applied calculus. It stands for:

- Sine(θ) = Opposite / Hypotenuse
- Cosine(θ) = Adjacent / Hypotenuse
- Tangent(θ) = Opposite / Adjacent

These relationships allow us to connect angles with side lengths and are essential when analyzing forces, signals, and motion in electrical and mechanical systems. Whether determining the projection of a vector or calculating phase differences in AC circuits, SOH-CAH-TOA provides a foundational link between geometry and real-world electronic applications.

13.1.2 Angle, Angular Speed, and Angular Acceleration

In the study of trigonometric functions, angle, angular speed, and angular acceleration are key concepts for modeling rotational motion, which is common in electronics and electromechanical systems like motors and oscillators. An angle measures rotational position, typically in radians, while angular speed describes how fast that angle changes over time. When the rate of rotation itself changes, we describe it using angular acceleration. These rotational quantities are directly linked to trigonometric functions like sine and cosine, which describe the circular paths of rotating objects, phasors, and AC waveforms in a calculus-based electronics context.

13.1.3 Angular Formulas

- **Angle** θ, in radians, is the distance s along the arc divided by the radius r of the arc.

$$\theta = \frac{s}{r}$$

- **Angular Speed** ω is equal to the change in radians divided by the change in time.

$$\omega = \frac{d\theta}{dt}$$

- **Angular Acceleration** α is equal to the change in angular speed divided by the change in time, OR the second derivative of the angle θ.

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

13.1.4 Arc Length, Linear Speed, and Linear Acceleration

When an object moves along a circular path, we describe its motion using arc length, linear speed, and linear acceleration. The arc length is the linear distance the object travels along the curve of the circle, closely related to the angle in radians and the radius of the circle. Linear speed measures how fast the object is moving along that arc, while linear acceleration describes how the speed is changing over time. In applied calculus, these quantities connect directly to trigonometric and rotational functions, making them essential for analyzing motion in rotating systems like motors, pulleys, and signal phase tracking in electronics.

13.1.5 Linear Formulas

- **Linear Distance** s is equal to the angle θ, in radians, multiplied to the radius r of the arc.

$$s = \theta r$$

- **Linear Speed** or velocity v is equal to the change of speed divided by the change of time which is also equal to the radius of the arc multiplied to the quantity change of angle divided by the change of time, which is equal to the radius multiplied to angular speed.

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$$

- **Linear Acceleration** is the change of velocity divided by the change of time. Additionally, Linear Acceleration can be found using the second derivative of distance s.

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = r\frac{dw}{dt} = r\frac{d^2\theta}{dt^2} = r\alpha$$

13.1.6 Rectangular Velocity

In two-dimensional motion, especially involving circular or oscillatory paths, rectangular velocity refers to the object's velocity expressed in horizontal (x) and vertical (y) components. Instead of focusing only on speed along a curve, rectangular velocity breaks motion into vx and vy, showing how fast the object is moving in each direction at a given moment. These components are often modeled using trigonometric functions like sine and cosine, especially in systems involving circular motion or AC signals. In applied calculus, analyzing rectangular velocity helps us understand and predict the behavior of objects or signals in complex motion—critical in robotics, signal analysis, and electromechanical systems.

13.1.7 Rectangular Velocity Formulas

- The velocity v_x is the speed of the x or horizontal component of the angular speed.

$$v_x = \frac{dx}{dt} = r\omega \sin\theta$$

- The velocity v_y is the speed of the y or vertical component of the angular speed.

$$v_y = \frac{dy}{dt} = r\omega \cos\theta$$

13.1.8 Conversions: Degrees to Radians and Radians to Degrees

The ability to convert between degrees and radians is essential for working with angles in both practical and mathematical contexts. While degrees are more intuitive and commonly used in everyday measurements and mechanical settings, radians are the standard unit in calculus, especially when working with trigonometric functions, angular velocity, and calculus-based formulas.

- One complete rotation is equal to 360°
- One complete rotation is equal to $2\pi\ rad$
- $360° = 2\pi\ rad$
- Converting Radians to Degrees:
  - $\frac{X\ degrees°}{known\ radians} = \frac{360°}{2\pi\ radians}$
  - $X\ degrees° = \frac{360° \times\ known\ radians}{2\pi\ radians}$ (radians cancel leaving only degrees!)

- Converting Degrees to Radians:
  - $\frac{X\ radians}{known\ degrees^{\circ}} = \frac{2\pi\ radians}{360^{\circ}}$
  - $X\ radians = \frac{2\pi\ radians \times\ known\ degrees^{\circ}}{360^{\circ}}$ (degrees cancel leaving only radians!)

13.1.9 Derivatives of Sine and Cosine Functions

Transcendental functions, such as exponential, logarithmic, and trigonometric functions, cannot be expressed using only algebraic operations. In applied calculus, they are essential for modeling real-world systems involving growth, decay, and oscillation. These functions appear throughout electronics, from capacitor charging (exponentials) to semiconductor behavior (logarithms) and AC signals (trigonometric functions). Understanding them is key to analyzing dynamic systems and solving differential equations in electronic applications.

The Derivative of a Sine Function:

- $y = \sin u$
- $\frac{d}{dx}\sin u = \cos u \frac{du}{dx}$

Example: $y = \sin 2x$

$\frac{dy}{dx} = (\cos 2x)(2)$

$\frac{dy}{dx} = 2(\cos 2x)$

The Derivative of a Cosine Function:

- $y = \cos u$
- $\frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$

Example: $y = 2\cos 3t^3$

$\frac{dy}{dt} = -2(\sin 3t^3)(9t^2)$

$\frac{dy}{dt} = -18t^2(\sin 3t^3)$

13.2 Video Instruction

- Trigonometric Functions Part 1
- Trigonometric Functions Part 2
- Trigonometric Functions Part 3
- Trigonometric Functions Part 4

13.3 Homework Week 13 - Questions with Answers

- Day 1, Trigonometric Functions, 1-4
- Day 2, 5-8
- Day 3, 9-12
- Day 4, 13-16
- Day 5, 17-20

1. In a directional broadcast antenna, tower 2 is located 212 feet from tower 1, in a direction 38° north of east. How far to the north of tower 1 is tower 2 situated?

   North = 130.52 ft

2. A target-practice object is observed on a radar screen at an airline distance of 18,000 feet and at an elevation angle of 72°. If the object were shot down, at what horizontal distance from the observer would the wreckage fall, assuming a vertical fall?

   Horizontal distance = 5,562.31 ft

3. A radar screen shows an object 70 miles from the observer at an angle of 50° east of north. How far to the east of the observer is the object?

   East distance = 53.623 miles

4. A broadcast antenna is 100 ft tall and it is recommended that the guy wires to the top be anchored into the ground at an elevation angle to the horizon of 50°. Find (a) the length of each guy wire and (b) how far the anchor points are from the antenna base?

   Guy Wire = 130.541 ft
   Anchor distance = 83.91 ft

5. A microphone diaphragm intercepts 6.75×10^-9watts of acoustic power when turned broadside to a sound source. What will be the theoretical intercept power if the diaphragm is turned at an angle of 55° to the source?

   Intercept power = $3.872nW$

6. Light radiations having a plane wave strike a photosensitive surface at an angle of 30°. If the surface were turned to face the light directly, by what factor would the amount of received light energy increase?

   Factor of increase = 2

7. A "Curtain" receiving antenna is broadside to a distant transmitter. If it is now turned through an angle of 21°, by what factor will the radiated power impinging upon the curtain be reduced? (This calculation does not include the effect of the antenna directional pattern)

   factor of reduction = 0.9336

8. A radar screen shows an object 40° East of North. A second radar screen located 100 miles directly East of the first screen locates the same object at 55° West of North at a distance of 76.9 miles. How far to the east of the first screen is the object?

   East distance = 37.007 miles

9. How many radians correspond to each of the following angles?

   (a) $180° = \pi$ radians
   (b) $90° = \frac{\pi}{2}$ radians
   (c) $45° = \frac{\pi}{4}$ radians
   (d) $60° = \frac{\pi}{3}$ radians
   (e) $30° = \frac{\pi}{6}$ radians
   (f) $15° = \frac{\pi}{12}$ radians
   (g) $20° = \frac{\pi}{9}$ radians
   (h) $54° = \frac{3\pi}{10}$ radians

10. How many degrees correspond to each of the following angles expressed in radians?

    (a) $\frac{\pi}{4}$ radians $= 45°$
    (b) $\frac{3\pi}{2}$ radians $= 270°$
    (c) $\frac{\pi}{9}$ radians $= 20°$
    (d) $\frac{2\pi}{3}$ radians $= 120°$
    (e) $\frac{5\pi}{3}$ radians $= 300°$
    (f) $\frac{\pi}{10}$ radians $= 18°$
    (g) $\frac{4\pi}{3}$ radians $= 240°$
    (h) $\frac{3\pi}{4}$ radians $= 135°$

11. An Instrument pointer moves through an arc of 270°. To how many radians is this equivalent?

    $\frac{3\pi}{2}$ *radians*

12. The radiation pattern of an antenna has a minimum value in a direction 24° off the antenna axis, express this angle in radians.

    $\frac{2\pi}{15}$ *radians*

13. An armature turns at 1,800 revolutions per minute. To what value ω, in radians per second, does this correspond?

    $\omega = 60\pi\ rad/\ sec$

14. The coil of an instrument rotates at a rate of 0.005 radians per millisecond. Express this angular speed in degrees per second.

    $\omega = 286.479°/\ sec$

15. A motor accelerates at a rate of 600 revolutions per minute per second. To how many radians per second squared is this equal?

    $\alpha = 20\pi\ rad/\ sec^2$

16. An instrument pointer is 2.1 inches long. The tip of the pointer moves over a scale 2.4 inches long. What angle does this describe in radians?

    $\theta = 1.14286\ radians$

17. An alternator has a rotating field that is 32 inches in diameter. When the field is turned at 120 revolutions per minute, what is the linear speed of a point on its circumference?

    $v = 201.062\ inches/second$

18. If the field assembly in question 17 is accelerated at 12 revolutions/min/sec, what linear acceleration is applied to a point on its circumference?

    $a = 20.1062\ inches/s^2$

19. If the field assembly in question 17 turns in a counter-clockwise direction, (a) what is the upward component of the velocity at a point P on its circumference when P is at an angle of 45° above the horizon? (b) what is the horizontal component of the velocity at the same point?

    $(a)\ v_y = 142.172\ inches/sec$
    $(b)\ v_x = 142.172\ inches/sec$

20. An airplane propeller has a radius of 3 feet to the blade tip. It is desired to keep the tip velocity below the speed of sound (769.5 miles per hour). What number of revolutions per minute would correspond to this limit?

    $\omega = 3.59245 \times 10^3\ rev/\ min$

Week 14

Trigonometric Functions part 2

14.1 Homework

14.2 Homework Week 14 - Questions with Answers

- Day 1, 21-22
- Day 2, 23-28
- Day 3, 29-35
- Day 4, Review
- Day 5, Test Trigonometric Functions

21. It can be shown that, when an armature of radius r rotates at ω radians per second, a point on its circumference is given a constant normal acceleration toward the center equal to $a_n = r\omega^2$. If an armature 0.3 meters in diameter is rotated at 2,000 revolutions per minute, (a.) to what normal acceleration will a conductor on the surface be subjected? Using $f = ma$, (b.) what centrifugal force in newtons will be applied to a conductor of mass 0.04Kg located at the circumference?

    $a_n = 6.5797Km/s^2$

    $F = 263.189\ newtons$

22. When power is applied to a motor, the shaft speed during initial power-up corresponds to $10t + 4t^2$ revolutions per second. (a.) Write an equation for the angular speed ω, in radians per second of the shaft. (b.) Find an equation for the angular acceleration α of the shaft at any time. (c.) Find an equation for the angle θ, in radians, of the shaft position at any time. (d.) Find the angular position θ, in degrees, at t1 = 0.1 sec & t2 = 0.2 sec. (assume $\theta = 0$ when t = 0).

$\omega = (10t + 4t^2)2\pi \ radians/sec$

$\alpha = 2\pi(10 + 8t) \ radians/sec^2$

$\theta = 10\pi t^2 + \frac{8\pi t^3}{3} + C \ radians$

$\theta_{t1} = 18.48°$

$\theta_{t2} = 75.84°$

Perform the derivative for the following functions.

23. $y = \sin 2x$
    $\frac{dy}{dx} = 2(\cos 2x)$
24. $y = 3 \sin x$
    $\frac{dy}{dx} = 3(\cos x)$
25. $y = 12 \sin 14t$
    $\frac{dy}{dt} = 168(\cos 14t)$
26. $y = 10 \sin 10t^{\frac{1}{2}}$
    $\frac{dy}{dt} = 50t^{\frac{-1}{2}}(\cos 10t^{\frac{1}{2}})$
27. $y = \sin t^2$
    $\frac{dy}{dt} = 2t(\cos t^2)$
28. $y = 2 \cos 3t^3$
    $\frac{dy}{dt} = -18t^2(\sin 3t^3)$
29. $y = 500 \cos(t^2 - t)^{\frac{1}{2}}$
    $\frac{dy}{dt} = \frac{-250(2t-1)}{(t^2-t)^{\frac{1}{2}}} \sin(t^2 - t)^{\frac{1}{2}}$
30. $y = 10t^3 + \cos t$
    $\frac{dy}{dt} = 30t^2 - \sin t$
31. $y = \sin^2 t$
    $\frac{dy}{dt} = 2 \sin t \cos t$
32. $y = -\cos^2 t^{-1}$
    $\frac{dy}{dt} = \frac{-2}{t^2}(\cos t^{-1})(\sin t^{-1})$

33. $y = 2\sin^2 t^2$

    $\frac{dy}{dt} = 8t(\sin t^2)(\cos t^2)$

34. Let the primary current in a transformer be $i_1 = I_{Max}\sin\omega t$, Where I_{Max} is the crest value of the current. Write a formula for the induced secondary emf v_2.

    $v_2 = -m(I_{Max} \times \omega)(\cos\omega t)$

35. A voltage $v = 2,000\sin 500t$ is impressed across a 20μF capacitor. Find a formula for the resulting current.

    $Ic = 20\cos 500t$

References

[1] James E. Trefzger Dale Ewen, Joan S. Gary. *Technical Calculus*. Pearson Education, Upper Saddle River, NJ, 4th edition, 2002.

[2] A.E. Richmond and Gary W. Hecht. *Calculus For Electronic*. McGraw-Hill, Westville, OH, 1989.