Wideband Evaluation of Planar Finite Periodic Structures Using ASED Basis Function and Interpolation Technique

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***Abstract*—One efficient algorithm, using the accurate sub-entire-domain (ASED) basis functions with the matrix interpolation technique, is proposed to analyze wideband electromagnetic scattering by the planar finite periodic structures. Comparing with the conventional method of moments, the ASED basis function method with less unknowns is suitable for computing the large-scale finite periodic arrays. The interpolation technique can speed up the frequency sweep. Several numerical examples are carried out to validate the accuracy and efficiency of the proposed method. Simulation results validate the accuracy and efficiency.**

Keywords—accurate sub-entire domain basis function; periodic structure;interpolation

I. INTRODUCTION

Periodic structures, such as the photonic band-gap[1], meta-materials[2] and so on, have been studied by many researchers for decades. It's necessary to study some accurate and efficient methods for analyzing them.

Because the method of moments (MoM) with the RWG basis function is very accurate, it is often used to analyze electromagnetic problems [3]. However, it is unsuitable for analyzing large problems due to large memory requirement and long CPU time. Moreover, this method is very time consuming if it is used to wideband electromagnetic characteristics analysis without other techniques because impedance matrix has to be recomputed at each frequency point.

To accelerate single-frequency simulation, many researchers have proposed fast algorithms, such as the fast multi-pole method (FMM) [4], multilevel fast multi-pole algorithm (MLFMA) [5] and conjugate gradient-fast Fourier transform (CG-FFT) [6] have been proposed, by which large problems can be efficiently analyzed. In addition, some physics-based basis functions, i.e. the accurate sub-entire domain (ASED) basis function, have been proposed [7]. The ASED basis function method can be used to efficiently compute large-scale finite periodic structures.

To speed up the frequency sweep, the interpolation technique [8], and the asymptotic waveform evaluation (AWE) [9] and so on have been presented.

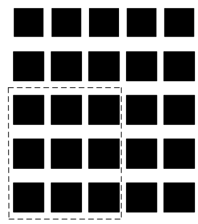

Fig. 1. A planar finite periodic structure (with N_0M cells). The 3×3 cells in the dashed line box is used to determine the ASED bases.

In this paper, we combine the ASED basis function method with the interpolation technique to analyze wideband electromagnetic scattering by planar periodic structures. In Section II, the ASED basis function method and the impedance matrix interpolation will be briefly introduced. In Section III, the accuracy and efficiency of the proposed algorithm will be validated by several examples. In section IV, we shall make some conclusions.

II. PRINCIPLE AND FORMULATION

A. ASED basis function method

A planar finite periodic structure is shown in Fig. 1, which consists of N_0 cells arranged in the *xoy* plane. We use the triangular patches to model the unit cells and get M RWG interior edges on each cell. The total number of unknowns is N_0M. The electric field integral equation with Galerkin's procedure is used.

First of all, it is to determine the ASED bases when using the method. According to the periodicity and the position, there are nine types of cells in the periodic structure if mutual couplings between cells are considered. Thus, there are nine types of ASED domain basis functions [7]. By solving a sub-array with 3×3 cells, the ASED bases can be obtained, which is shown Fig. 1. The ASED bases are expressed as

$$\boldsymbol{f}_n(\boldsymbol{r}) = \boldsymbol{J}_n(\boldsymbol{r}) = \sum_{m=1}^{M} \boldsymbol{\alpha}_{nm} \boldsymbol{f}_{nm}(\boldsymbol{r}) \tag{1}$$

where $\boldsymbol{J}_n(\boldsymbol{r})$ and α_{nm} are the induced electric current density and the expansion coefficient, respectively. $\boldsymbol{f}_{nm}(\boldsymbol{r})$ denotes the mth RWG basis function of the nth cell. Then, substitute the

ASED bases into the EFIE with the Galerkin's procedure, the impedance element can be obtained, which can be written as

$$Z_{pq} = \sum_{m=1}^{M} \sum_{n=1}^{M} \left[\alpha_{pm} \alpha_{qn} z_{pm,qn} \right] \tag{2}$$

with

$$\begin{aligned} z_{pm,qn} &= j\omega\mu_0 \int_{S_m} \boldsymbol{f}_{pm}(\boldsymbol{r}) \cdot d\boldsymbol{r} \int_{S_n} \frac{e^{-jkR}}{4\pi R} \boldsymbol{f}_{qn}(\boldsymbol{r}') \cdot d\boldsymbol{r}' \\ &+ \frac{1}{j\omega\varepsilon_0} \int_{S_m} \nabla_s \cdot \boldsymbol{f}_{pm}(\boldsymbol{r}) \cdot d\boldsymbol{r} \int_{S_n} \frac{e^{-jkR}}{4\pi R} \nabla'_s \cdot \boldsymbol{f}_{qn}(\boldsymbol{r}') \cdot d\boldsymbol{r}' \end{aligned} \tag{3}$$

where ω is the angular frequency, μ_0 and ε_0 are permeability and permittivity in the free-space, respectively. $e^{-jkR}/4\pi R$ stands for the Green's function in free space. k is the wavenumber. $R=|\boldsymbol{r} - \boldsymbol{r}'|$, where $\boldsymbol{r}$ and $\boldsymbol{r}'$ are the position vectors of the testing and the source functions, respectively. Generally speaking, the number of unknowns is not large in the first step. Thus, direct computation is used to determine the ASED bases over the entire frequency range. In the second step, the interpolation technique is used to obtain (2) and (3).

B. ASED with Interpolation technique

The element $z_{pm,qn}$ is a function of frequency. And it is affected by the rapid frequency variation of the factor exp($-jkR_{mn}$) with large R_{mn}. By factoring out the factor, the interpolation accuracy of the impedance element can be improved [8]. Thus, $z_{pm,qn}$ becomes $z'_{pm,qn}$.

The real part of the $z'_{pm,qn}$ is interpolated by using the quadratic interpolating function over the entire frequency band [8], which can be obtained by using the following

$$\mathrm{Re}[z'_{pm.qn}] = A_{mn} + B_{mn} f + C_{mn} f^2 \tag{4}$$

where f denotes the frequency. A_{mn}, B_{mn} and C_{mn} are the coefficients to be determined. Re[*] denotes the real part of a complex number.

When $R_{mn} > 0.5\lambda$, the imaginary part of z'_{mn} is also interpolated using (4). When $R_{mn} \le 0.5\lambda$, the imaginary part has to be interpolated by using the following [8]

$$\mathrm{Im}[z'_{pm,qn}] = A_{mn} + B_{mn} lnf + C_{mn} f \tag{5}$$

where Im [*] denotes the imaginary part of a complex number. Finally, $z'_{pm,qn}$ must be multiplied by exp($-jkR_{mn}$) to get $z_{pm,qn}$.

III. Numerical Results And Discussion

In this section, several example are executed to test the performance of the proposed technique. The simulation examples are carried out on a personal computer with Intel Core i7-4790 CPU @3.6GHz and 8G memory.

Firstly, we analyze a 4×4 planar periodic cross-dipole array. The array is located in *xoy* plane. The unit cell is shown in Fig. 2. The gap between the cells is 0.025 *m*. The number of unknown of each cell is 122. The total number of unknowns of the structure is 1952. The arrays are normally illuminated with a *x*-polarized plane wave.

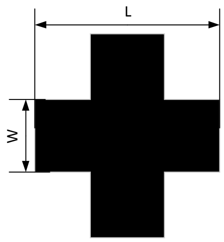

Fig. 2 Geometry of Cross-dipole (L=0.15m,W=0.075m)

The interpolation scheme is implemented from 300 to 1200 MHz. We get the frequency step size according to $\Delta f_M = f/2(L/\lambda)$, where L denote the largest value of R_{mn}, f and λ are the maximum frequency and the minimum wavelength over the frequency band, respectively [8].

The largest distance between the source and testing functions is about 3.51λ at the highest frequency and the maximum frequency step size is about 171MHz. So, we use 150MHz as the frequency step size to get the matching impedance matrices, which divide the entire band into three sub-bands. The impedance matrices at 300, 450, and 600MHz are calculated directly for the first sub-band, while matrices at 600, 750, and 900MHz are calculated directly for the second sub-band, and selecting 900,1050 and 1200MHz for the third sub-band.

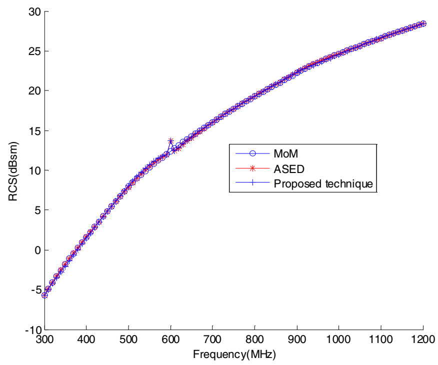

Fig. 3 Wideband monostatic RCS from a 4×4 planar Cross-dipole array.

The monostatic RCSs are computed by using the conventional MoM, the ASED basis function method, and the proposed technique. The frequency increment is 0.1GHz. The results are shown in Fig. 3. From Fig. 3, it can be seen that good agreement is achieved between the results of these three methods. The CPU time is about 786.68 *s* for the ASED basis function, while it is only about 222.56 *s* for the proposed technique.

Next, a 5×5 split ring resonator (SRR) array is analyzed. The unit cell is shown in Fig. 4. The gap between cells is 0.012m. There are 96 RWG edges on each cell. The total number of unknowns is 2400. The frequency range is from 3GHz to 15GHz. Δf=1.5GHz is used as interpolation frequency step size to get the matching impedance matrix. The entire frequency band is divided into four sub-bands, which the impedance matrices are calculated at 3GHz, 4.5GHz, and

6GHz directly for the first sub-band. The impedance matrices are calculated directly at 6GHz, 7.5GHz and 9GHz for the second sub-band, while matrices at 9GHz,10.5GHz and 12GHz are calculated directly for the third sub-band, and the matrices at 12GHz,13.5GHz and 15GHz are directly calculated for the last sub-band.

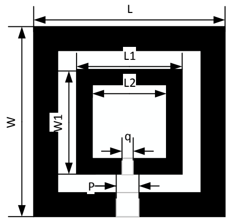

Fig. 4 Geometry unit of split ring resonator(L==0.008m ,W=0.008m, L1=0.004m,W1=0.004m,L2=0.002m,p=0.0008m,q=0.0006m).

The monostatic RCS calculated using three different methods are shown in Fig. 5. The frequency increment is 0.15GHz during the frequency sweep. One can see that the results agree well. The relative error norm of expansion coefficients is shown in Fig. 6, which is defined by norm(I^{ased}-I^{Prop}) /norm(I^{mom}) ×100%. The relative errors of the proposed technique is almost the same to the ASED bases. There is a little discrepancy between 5GHz and 6GHz for interpolation technique, which may not choose a optimum interpolation frequency step size. The CPU time of the proposed technique is 345.06 *s*, while it is 1049.55 *s* for the ASED basis function method.

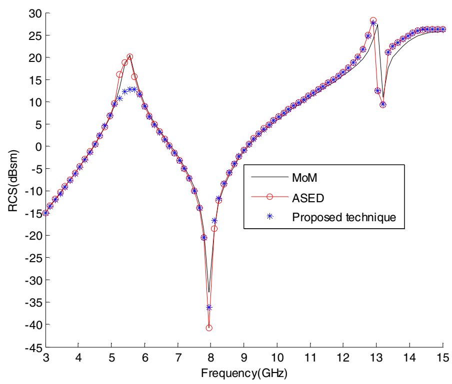

Fig. 5 Wideband monostatic RCS from the 5×5 planar split ring resonator periodic array obtained by three algorithms.

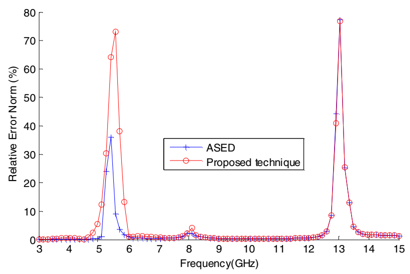

Fig 6 Relative error norm of expansion coefficients

IV. CONCLUSION

In this paper, the ASED basis function together with the matrix interpolation technique has been presented to analyze the wideband scattering by the finite periodic arrays. The accuracy and efficiency of the proposed technique have been validated by numerical examples.

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