# A Method for Calculating the Purity of Vortex Electromagnetic Waves

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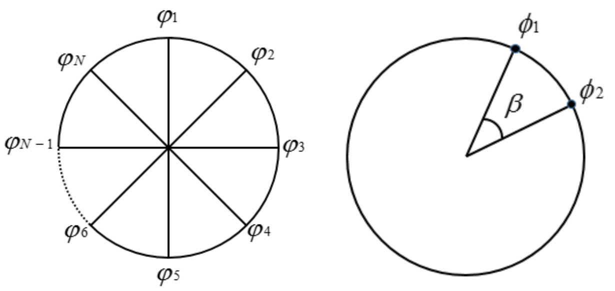
***Abstract*—This paper describes a criterion and calculation method for the purity of vortex electromagnetic wave modes. The factors affecting the purity are discussed on the basis of orbital angular momentum(OAM) waves generated by a dipole uniform circular array. Simulations show that the property of the vortex electromagnetic wave in the propagation process will distort with the decrease of propagation distance r as well as the increase of $\theta$ in spherical coordinate system. In addition, a threshold of variance is defined to judge whether the purity of vortex is acceptable or not.**

***Keywords—Orbital angular momentum, vortex wave, uniform circular array, purity***

## I. INTRODUCTION

Vortex detection has always been a concern, and actually a particular mode of vortex electromagnetic waves generated by antenna will take along different degrees of distortion[1,2]. In order to determine the degree of distortion, in this paper, a criterion to judge the purity of vortex electromagnetic waves and the corresponding calculation method is proposed. At first, take a specific circle($r = r_0, \theta = \theta_0$) in spherical coordinate system and sample from $-\pi$ to $\pi$ as the uniform distance along the direction $\varphi$ ,as is shown in Figure1(a). Each two adjacent sampling points can be used to sequentially calculate the local order of the OAM mode according to the phase gradient method[3,4]. Then make the calculated order minus the expected order and take the variance, the variance criterion of the degree of distortion can be finally obtained. Simulations show that the property of the vortex electromagnetic wave produced by uniform circular array(UCA) will distort with the decrease of propagation distance r as well as the increase of $\theta$ in spherical coordinate system in the propagation process.




(a) (b)

Fig.1 (a) Sketch map of variance criterion method
(b)Sketch map of phase gradient method

The phase gradient method is a simple and effective method to determine the OAM order. Since the isophase of the OAM electromagnetic wave is a helicoid, the mode of the OAM can be determined by measuring the phase jump[5]. As shown in Fig.1(b), when the two measurement points have a circle center angle of $\beta$, the measured phases are $\phi_1$and$\phi_2$ ,the OAM mode can be calculated by $(\phi_1 - \phi_2)/\beta$. It should be noted that the central angle of the sample points must satisfy the condition $\beta < \pi/|l|$, otherwise, the OAM module value cannot be accurately determined[3].

## II. VORTEX MODE PURITY CALCULATION METHOD

From the above description, it can be seen that when the electromagnetic vortex wave propagates, there are various factors that will cause the wave being distorted. Therefore, it is necessary to quantify the boundary of distortion to detect the purity of the vortex wave. The following describes a statistical method to detect the purity of an electromagnetic vortex wave with order $l$.

For vortex wave with order $l$, when two planes in spherical coordinate system $r = r_0$ and $\theta = \theta_0$ have a common boundary circle, N sampling points are evenly located on this circle with the phase $\varphi_i$ and detected vortex wave phase $\phi_i$ for each sampling point i(i=1,2…N, $\varphi_i \in [-\pi, \pi]$, $\phi_i \in [-\pi, \pi]$). With a special treatment to the points that span from $\pi$ to $-\pi$ and utilizing all the phase information, the variance with standard order $l$ can be derived as

$$\sigma^2 = \frac{(\frac{\phi_1-\phi_N}{\varphi_1-\varphi_N}-l)^2+\sum_2^N(\frac{\phi_n-\phi_{n-1}}{\varphi_n-\varphi_{n-1}}-l)^2}{N} \qquad (1)$$

Through this variance, the vortex phase distribution of the wave can be quantified as a basis for evaluating the property. It is also possible to define the maximum variance value that can maintain the nature of the vortex wave to determine whether a wave to be evaluated is an vortex wave or not.

## III. PARAMETER ANAYSIS

For electromagnetic vortex wave generated by uniform circular array, due to the limited number of array elements and mutual coupling, non-ideal vortex wave is generated. Actually, the generated vortex can be evaluated by the order variance. In the calculation cases below, correlation between vortex mode purity and propagation distance r as well as θ in spherical coordinate system is verified by using the above method.

## A. *Propagation distance $r$*

For a certain $\theta = \theta_0$, by utilizing the variance calculation method on different propagation distance r from $\lambda$ to $10\lambda$ stepped by $\lambda$, correlation between vortex mode purity and propagation distance can be obtained. The electromagnetic vortex wave of order $l$=1 and $l$=2 is generated by an 8-element dipole uniform circular array(UCA) at 3GHz. Figure.2 and Figure.3 correspond to $\theta = 40°$ and $\theta = 70°$ respectively.

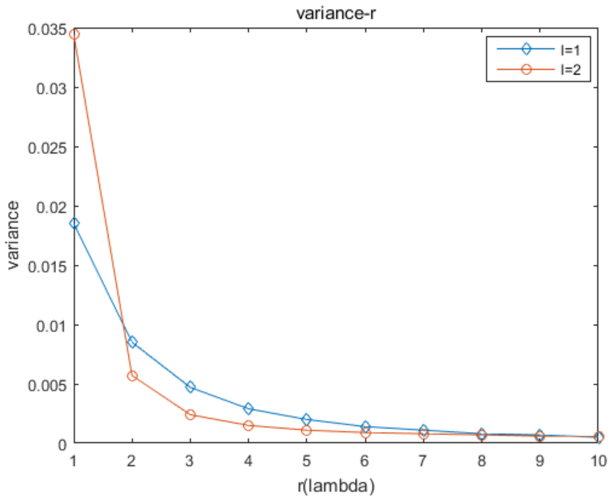



Fig.2 The relation between the order variance and distance($\theta = 40°$)

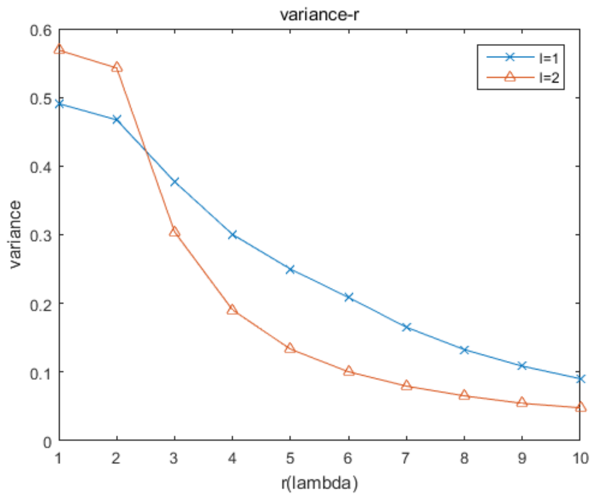



Fig.3 The distribution of local order values above the circle($\theta = 70°$)

From above results, it derives that when the propagation distance $r$ increases, the phase variance decreases and the vortex properties improved.

## B. *Coordinate value $\theta$*

For a certain propagation distance $r = r_0$, by utilizing the variance calculation method on different $\theta$ from 1° to 90° stepped by 1°, correlation between vortex mode purity and coordinate value $\theta$ can also be obtained. The electromagnetic vortex wave is generated by an 8-element dipole uniform circular array(UCA) at 3GHz. Figure.4 to Figure.6 stands for $l$=1, $l$=2 and $l$=3 respectively.

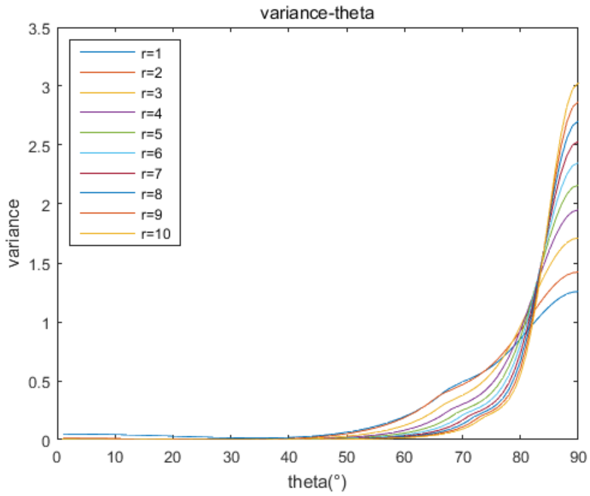



Fig.4 The relation between the order variance and $\theta$ ($l = 1$)

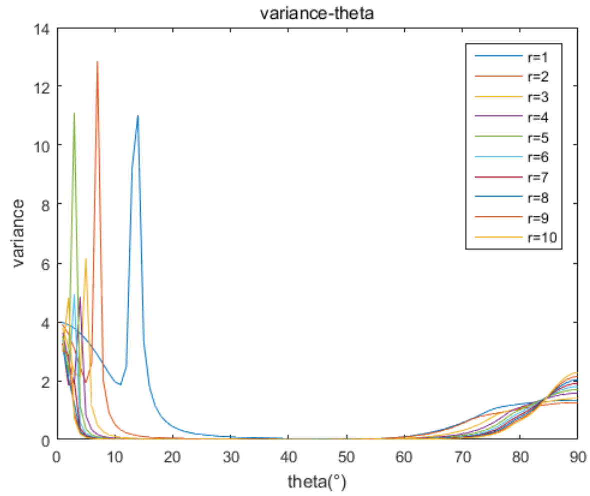



Fig.5 The relation between the order variance and $\theta$ ($l = 2$)

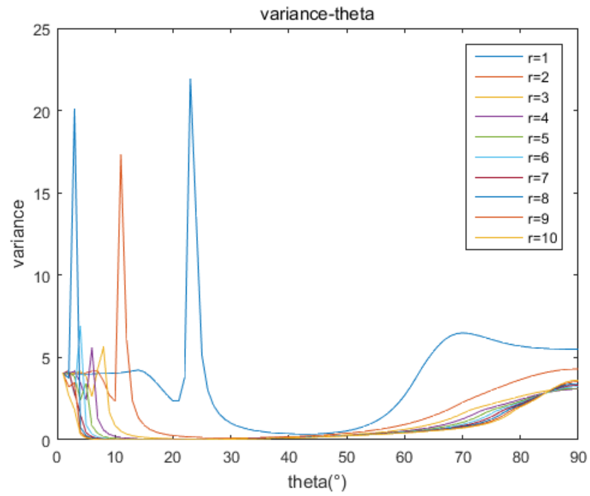
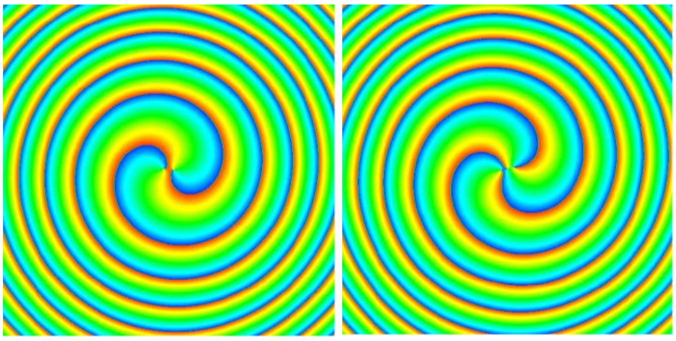



Fig.6 The relation between the order variance and $\theta$ ($l = 3$)

Results above show that the phase variance increases and the vortex properties deteriorate with the increase of $\theta$ when

$l$=1 for different propagation distance, as is shown in Figure.4. However, for $l$=2 and $l$=3, peak values are observed in the variance-$\theta$ curve in Figure.5 and Figure.6. In addition, it can also be observed that the start points of vortex do not coincide when OAM mode $l$=2 and $l$=3. There are 2 phase singularities in Figure.7(a) while 3 in Figure.7(b).Through analysis it is verified that the $\theta$-coordinate value at the peak for $r = 10\lambda$ in Figure.5 and Figure.6 is the same as the $\theta$-coordinate value at the singular point in the phase distribution as shown in Figure.7(a) and Figure.7(b) respectively.



(a) $l$=2 (b) $l$=3

Fig.7 The X-component phase distribution of wavefront of vortex wave with different OAM mode of E field in the cross-section perpendicular to the beam axis. The cross-section is 10λ away from the array plane and the phase changes from -π (blue) to π (red).

Therefore, when the coordinate value $\theta$ increases, the phase variance increases and the vortex properties deteriorated after excluding the influence of the phase singularity. This conclusion can be drawn from Figure(4) to Figure(6) that the phase variance increases with the increase of $\theta$ in the condition of $\theta > 40°$ no matter what OAM order number $l$ is.

### *C. Threshold of variance*

Based on the variance criterion and calculation method, a threshold value of variance need to be defined to judge whether the purity of vortex is acceptable or not. For a certain calculation of two adjacent sampling points with expected order $l$ and calculated order $l_m$, use $0 \le |l - l_m| < 0.5$ as an acceptance criteria for vortex purity. Considering the entire sampling circle, due to the average value of $l_m$ on entire sampling circle is $l$, a threshold of variance can be obtained by:

$$variance|_{max} = |l - l_m|^2_{max} = 0.25 \quad (2)$$

If the calculated variance value satisfies $variance \ge 0.25$ , the degree of vortex property deterioration is unacceptable. If the calculated variance value satisfies $0 < variance < 0.25$, the vortex property is still relatively normal. Positions for receiving high-purity vortex wave can be searched by using the threshold of variance.

## IV. Conclusion

This paper describes the definition of the variance of vortex wave order. Simulations show that the purity of the vortex electromagnetic wave in the propagation process will distort with the decrease of propagation distance r as well as the increase of θ in spherical coordinate system. Through utilizing the variance of vortex wave order and corresponding threshold, further studying on the influencing factors of vortex wave property can be reached more effectively.

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