Multi-parameter Estimation of Concentred Loop and Dipole Planar Array

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***Abstract*—In this paper the parallel factor model is used to construct a new array signal receiving model, which is more suitable for the vector characteristics of electromagnetic signals. Using the parallel factor model, a novel algorithm for estimating DOA and polarization using the parallel factor model is presented, the proposed algorithm has the advantage of high-precision estimation and parameter automatic matching. Simulation results verified the effectiveness of the proposed algorithm.**

***Keywords*—parallel factor; direction of arrival; array signal processing**

I. INTRODUCTION

The parallel factor (abbreviation as PARAFAC) analysis is three or more way arrays low rank decomposition [1]. The parallel factor analysis concept began in psychological experiment in the field of data analysis [2], later, Caroll and Harshman developed the parallel factor analysis model [3, 4], which was used in the field of spectrophotometric analysis, chemical statistics, psychometric testing and so on. Sidiropoulos and his scientific research partners introduced the PARAFAC analysis concept to the field of signal processing and communication [5-7]. In recent years, PARAFAC-based array signal processing algorithms, especially the PARAFAC-based electromagnetic vector sensor (EMVS) array signal processing algorithm, get more and more attention. In this paper, the DOA parameter estimation algorithm based on trilinear decomposition is proposed, using the planar array. The parallel factor theory is herein introduced to construct the 3-dimensional matrix model of electromagnetic wave receiving array and the corresponding algorithm is also presented. Simulation results show that the proposed algorithm has better DOA estimation performance.

II. RECEIVE SIGNAL MODEL

The receiving array is a planar array which consists of $L_1 \times L_2$ concentred loop and dipole (COLD) pairs in the X-Y plane. The inter-element spacing of the *x*-axis and *y*-axis direction sub-arrays are respectively Δx and Δy, with $\Delta x < \lambda_{\min}/2$ and $\Delta y < \lambda_{\min}/2$, $\lambda_{\min}$ refers to the minimal signals' wavelength of the incident signals, as shown in Fig. 1.

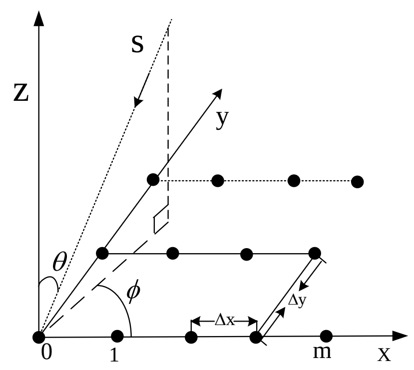

Fig. 1 the geometry structure of planar array

The COLD pairs' steering vector of the *k*th $(1 \le k \le \mathrm{K})$ unit-power electromagnetic source signal is the following 2×1 vector [2]:

$$\mathbf{p}(\theta_k,\gamma_k,\eta_k)=\begin{bmatrix} e_{kz} \\ h_{kz} \end{bmatrix}=\begin{bmatrix} -\sin\theta_k \sin\gamma_k \mathrm{e}^{\mathrm{j}\eta_k} \\ \sin\theta_k \cos\gamma_k \end{bmatrix} \tag{1}$$

where $\theta_k \in [0,\pi/2]$ is the signal's elevation angle measured from the positive *z*-axis, $\phi_k \in [0,2\pi]$ is the signal's azimuth angle measured from the positive *x*-axis, $\gamma_k \in [0,\pi/2]$ represents the auxiliary polarization angle, and $\eta_k \in [-\pi,\pi]$ symbolizes the polarization phase difference.

Let

$$\boldsymbol{\alpha}_1(\theta_k,\phi_k)=\left[1,\mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}\cos\phi_k\sin\theta_k},\ldots,\mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}(\mathrm{L}_1-1)\cos\phi_k\sin\theta_k}\right]^{\mathrm{T}} \tag{2}$$

$$\mathbf{A}_k=\begin{bmatrix} e_{kz}\boldsymbol{\alpha}_1(\theta_k,\phi_k) \\ h_{kz}\boldsymbol{\alpha}_1(\theta_k,\phi_k) \end{bmatrix}=\begin{bmatrix} \mathbf{A}_{ek} \\ \mathbf{A}_{hk} \end{bmatrix} \tag{3}$$

If each column is considered as a uniform linear array, then $\mathbf{A}$ can be expressed as $\mathbf{A}=[\mathbf{A}_1,\mathbf{A}_2,\ldots,\mathbf{A}_K]$.

Hence the received signal on the first column can be expressed as:

$$\mathbf{X}_1 = \mathbf{AS} \tag{4}$$

where $\mathbf{S} \in \boldsymbol{\chi}^{\mathrm{K}\times\mathrm{N}}$ is the matrix of signal source.

The received signal on the second column can be expressed as

$$\mathbf{X}_2 = \mathbf{A}_1 \mathrm{diag}\left(\mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}\sin\phi_1 \sin\theta_1}, \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}\mathrm{in}\phi_2 \sin\theta_2}, \cdots, \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}\sin\phi_{\mathrm{K}} \sin\theta_{\mathrm{K}}} \right) \tag{5}$$

Similarity, the received signal on the L_2 column can be expressed as

$$\mathbf{X}_{\mathrm{L}_2} = \mathbf{A}_1 \mathrm{diag}\left(\mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}(\mathrm{L}_2-1)\sin\phi_1 \sin\theta_1}, \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}(\mathrm{L}_2-1)\sin\phi_2 \sin\theta_2}, \cdots, \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}(\mathrm{L}_2-1)\sin\phi_{\mathrm{K}} \sin\theta_{\mathrm{K}}} \right) \tag{6}$$

Let

$$\mathbf{\Phi}_2 = [\boldsymbol{\alpha}_2(\theta_1,\phi_1), \boldsymbol{\alpha}_2(\theta_2,\phi_2), \ldots, \boldsymbol{\alpha}_2(\theta_{\mathrm{K}},\phi_{\mathrm{K}})] \in \boldsymbol{\chi}^{\mathrm{L}_2\times\mathrm{K}} \tag{7}$$

$$\boldsymbol{\alpha}_2(\theta_k,\phi_k) = \left[1, \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}\sin\phi_k \sin\theta_k}, \ldots, \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}(\mathrm{L}_2-1)\sin\phi_k \sin\theta_k} \right]^{\mathrm{T}} \tag{8}$$

The array received signal is herein expressed as:

$$\mathbf{X}_{l_2} = \mathbf{A}_1 \mathrm{diag}(\mathbf{\Phi}_2(l_2,:))\mathbf{S} + \mathbf{E}_{l_2} \quad l_2 = 1, \cdots, \mathrm{L}_2 \tag{9}$$

where $\mathbf{E}$ is noise matrix. Equation (9) is the tri-linear model, so the parallel factor method can be used to analysis the data.

III. ESTIMATION OF DOA

The steering vector matrix $\mathbf{A}$ is divided into two parts, i.e., the electric dipole array steering vector $\hat{\mathbf{A}}_e$ and the magnetic dipole array steering vector $\hat{\mathbf{A}}_h$.

Let

$$\hat{\mathbf{q}}_{kb} = \frac{\hat{\mathbf{A}}_{ek}(2:\mathbf{M})}{\hat{\mathbf{A}}_{ek}(1:\mathbf{M}-1)} \qquad \begin{bmatrix} \hat{\alpha}_k \\ \hat{\beta}_k \end{bmatrix} = \begin{bmatrix} \sin\hat{\theta}_k \sin\hat{\phi}_k \\ \sin\hat{\theta}_k \cos\hat{\phi}_k \end{bmatrix} \tag{10}$$

$$\mathbf{W} = \frac{2\pi\mathrm{R}}{\lambda} \begin{bmatrix} \sin\varphi_2 - \sin\varphi_1 & \cos\varphi_2 - \cos\varphi_1 \\ \sin\varphi_3 - \sin\varphi_2 & \cos\varphi_3 - \cos\varphi_2 \\ \vdots & \vdots \\ \sin\varphi_{\mathrm{M}} - \sin\varphi_{\mathrm{M}-1} & \cos\varphi_{\mathrm{M}} - \cos\varphi_{\mathrm{M}-1} \end{bmatrix} \tag{11}$$

Then

$$\arg\left(\hat{\mathbf{q}}_{kb}\right) = \mathbf{W} \begin{bmatrix} \sin\theta_k \sin\phi_k \\ \sin\theta_k \cos\phi_k \end{bmatrix} \tag{12}$$

The following relationship can be obtained:

$$\mathbf{C} = \arg\left(\hat{\mathbf{q}}_{kb}\right) \qquad \begin{bmatrix} \hat{\alpha}_k \\ \hat{\beta}_k \end{bmatrix} = \mathbf{W}^{\#}\mathbf{C} \tag{13}$$

From equation (13), the DOA estimation is obtained.

$$\begin{cases} \hat{\phi}_k = \begin{cases} \arctan\left(\dfrac{\beta_1}{\beta_2}\right) & \beta_2 > 0 \\ \pi + \arctan\left(\dfrac{\beta_1}{\beta_2}\right) & \beta_2 < 0 \end{cases} \\ \hat{\theta}_k = \arcsin\left(\sqrt{\hat{\alpha}_k^2 + \hat{\beta}_k^2}\right) \end{cases} \tag{14}$$

The relationship between the electric dipole array steering vector and the magnetic dipole array steerin vector can be expressed as:

$$\hat{\mathbf{A}}_e = \hat{\mathbf{A}}_h \mathbf{\Omega} \tag{15}$$

where

$$\hat{\mathbf{\Omega}} = \mathrm{diag}\left(-\mathrm{tg}\hat{\gamma}_1 \mathrm{e}^{\mathrm{j}\hat{\eta}_1}, \cdots, -\mathrm{tg}\hat{\gamma}_k \mathrm{e}^{\mathrm{j}\hat{\eta}_k} \right) \tag{16}$$

The polarization parameters are herein estimated:

$$\begin{cases} \hat{\gamma}_k = \arctan\left(\left|\hat{\Phi}_{kk}\right|\right) \\ \hat{\eta}_k = \arg(-\hat{\Phi}_{kk}) \end{cases} \tag{17}$$

IV. COMPUTER SIMULATION AND ANALYSIS

The following assumptions are made, the noise is additive Gauss white noise, the number of the planar array is 6, the column number is 4. The source number is 2, the azimuth and elevation angles are respective $\left(60^{\circ},120^{\circ}\right)$ and $\left(65^{\circ},80^{\circ}\right)$, the auxiliary polarization angle (APA) and polarization phase difference (PPD) are respective $\left(30^{\circ},68^{\circ}\right)$ and $\left(58^{\circ},80^{\circ}\right)$. The snapshot number is 500. The performance of the algorithm is verified by the standard deviation.

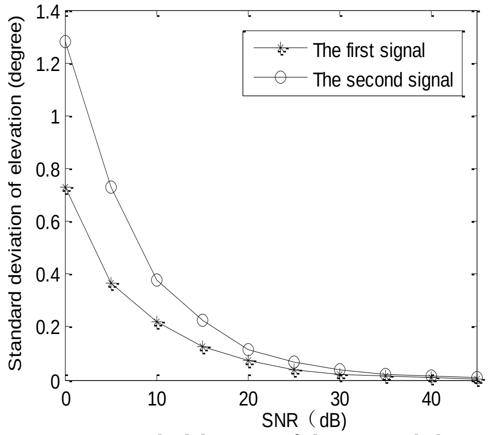

Fig. 2. standard deviation of elevation with the SNR

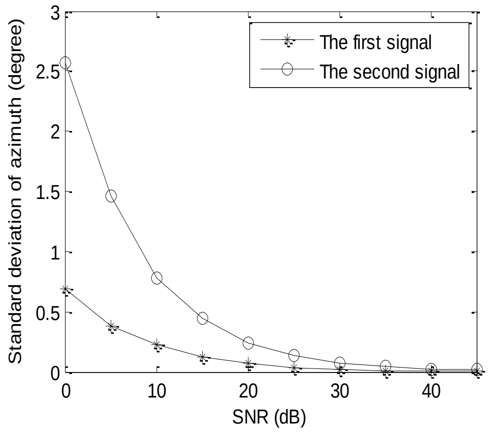

Fig. 3. standard deviation of azimuth with the SNR.

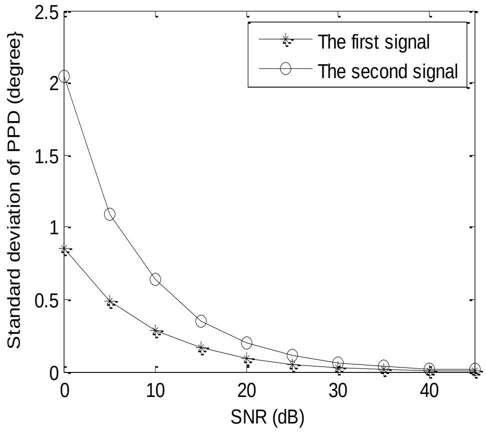

Fig. 4. standard deviation of PPD with the SNR.

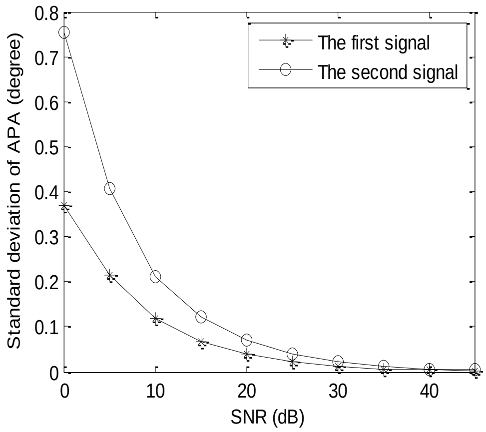

Fig. 5. standard deviation of APA with the SNR.

From Figures 2-5, it can be shown that the proposed algorithm has better elevation angle, azimuth angle, APA and PPD parameter estimation performance, the standard deviation decrease monotonically with the signal-to-noise ratio increase. That is to say, with the increase of the signal to noise ratio, the standard deviation of elevation, azimuth, APA and PPD are getting smaller and smaller. In addition, in the case of low SNR, these DOA and polarization estimated parameters also have better accuracy. For example, the elevation angle is estimated to be less than 0.6 degrees in the signal to noise ratio of 10dB.

V. CONCLUSION

In this paper, a 2-D directional angle estimation algorithm based on three linear decomposition is proposed in this paper. The algorithm firstly algorithm COMFAC to estimate the direction matrix, and then take advantage of Vandermonde characteristics of the direction matrix, the 2-D angle estimation based on least square method, the simulation results show that, the algorithm has better DOA estimation performance and faster convergence speed.

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