

Parabolic Equation Method for Airborne Radar Signal Prediction under the Surface Duct Environment

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Abstract—In this paper, a new parameter, leakage coefficient, is proposed to analyze the trapping effect of the emitting source outside the waveguide layer. In recent years, most of the research on the propagation of atmospheric waveguide is focused on the emission source in the waveguide layer, but few studies have been made on the situation of airborne radar, satellite borne radar and other emission sources outside the waveguide, and the commonly used capture angle can not accurately predict the capture effect of the source in the case of the waveguide. This paper analyzes the trapping effect of the surface waveguide by leakage coefficient, and analyzes the characteristics of airborne radar signal propagation in the surface waveguide by parabolic equation (PE) method, proves the validity of the leakage coefficient.

Keywords—parabolic equation method; leakage coefficient; trapping effect; surface duct;

I. INTRODUCTION

Atmospheric duct is one of the most important factors that influence radio wave propagation in marine environment. Its trapping effect of electromagnetic wave, not only can bend the electromagnetic wave, cause the radar blind area, but also enable the electromagnetic wave to be bound in the waveguide to realize the transmission of the over the horizon, directly affect the combat radius of the airborne radar system. This makes the research on radar signal propagation in the atmospheric duct is of great significance.

The parabolic equation (PE) method was first proposed by Leontovich and Fock in 1946 [1]. Until 1973 Tapper and Hardin put forward the Split-Step Fourier Transform (SSFT) method of parabolic equation (PE) in the field of acoustics, which caused widespread concern and research of parabolic equation (PE) [2]. The parabolic equation (PE) method is an efficient method for the prediction of wave propagation in complex environment. It can itself reflect the refraction and diffraction effect of the wave propagation, and can handle all kinds of complex environmental factors. Especially in the study of the propagation of atmospheric duct, parabolic equation method can not only calculate the wave propagation in the horizon and the range of over the horizon, but also have good stability and accuracy, and more importantly, the parabolic equation(PE) model is very easy to consider the influence of the atmospheric refraction on the wave propagation, so it is widely used.[3]

In the past, most of the research on the propagation of atmospheric waveguide is focused on the source of the source in the waveguide layer, but there is little research on the situation of the airborne radar and the space borne radar, which is located outside the waveguide.[4][5] In this paper, the parabolic equation(PE) method is used to simulate the situation of the emission source outside the waveguide, and the leakage coefficient is proposed to analyze the wave - trapping effect of the emission source outside the waveguide.

II. THEORY

A. Parabolic Equation Model

In a homogeneous region, assuming the time harmonic factor as $e^{-j\omega t}$, the propagation of EM wave satisfied the 2-D Scalar wave equation and given as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} + k_0^2 n^2 \psi = 0 \quad (1)$$

Where ψ denotes the amplitude of electric or magnetic field components and k_0 is the free space wave number.

Assuming $u(x, z) = e^{-ik_0 x} \psi(x, z)$, After several mathematical simplification and Feit-Fleck approximation, a SSFT solution of type wide angle parabolic equations obtained as[6]

$$u(x + \Delta x, z) = e^{ik_0 \Delta x(n-1)} \mathfrak{T}^{-1} \left[e^{i\Delta x \sqrt{k_0^2 - p^2} - k_0} \mathfrak{T}[u(x, z)] \right] \quad (2)$$

\mathfrak{T} and \mathfrak{T}^{-1} denote Fourier positive transformation and inverse transform respectively, $p = k_0 \sin \alpha$ is a frequency domain variable for Fourier transform, α is the elevation angle for each step.

B. Analysis of The Trapping Effect

The trapped effect of atmospheric duct is usually estimated by the capture angle of atmospheric duct. But when the source is located outside the waveguide, the commonly used capture angle can not accurately predict the capture effect of the

source in the case of the waveguide. Therefore, it is necessary to propose a new parameter.

When electromagnetic waves pass through the air duct, we regard the waveguide as a n layer medium. we assume that P_i is the energy of incident wave, and P_t is the energy of the outgoing wave. So

$$P_t = P_i \prod_{i=1}^n (1 - |\Gamma_n(\theta)|^2) \quad (3)$$

Define the $L_{out}(\theta)$ as the leakage coefficient, so

$$L_{out}(\theta) = \prod_{i=1}^n (1 - |\Gamma_n(\theta)|) \quad (4)$$

Where

$$\Gamma_n(\theta) = \begin{cases} \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} & (\text{Vertical polarization}) \\ \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_t \cos \theta_i + n_i \cos \theta_t} & (\text{Parallel polarization}) \end{cases} \quad (5)$$

θ_i is the angle of incidence and θ_t is the angle of refraction, n_i is the atmospheric refractive index of the n layer medium., and n_t is the atmospheric refractive index of the $n+1$ layer medium.

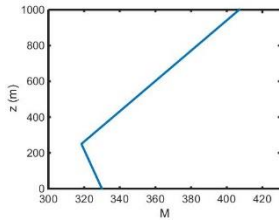


Fig. 1. Refractive index profile curve of atmospheric duct

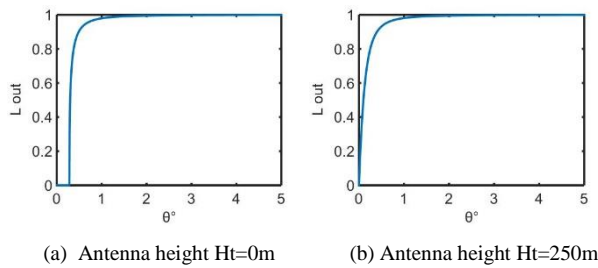


Fig. 2. Curve of leakage coefficient with angle of incidence

When $\theta < \theta_c$, $L_{out}(\theta) = 0$, the total reflection occurs in the waveguide, which can be called complete trapping, as shown in Fig. 2 (a), $\theta_c = 0.28^\circ$.

When $\theta_c < \theta < \theta_{0.9}$, $0 < L_{out}(\theta) \leq 0.9$, the electromagnetic wave partially reflects in the waveguide,

which can be called partial trapping, as shown in Fig. 2(a) and (b). When $\theta > \theta_{0.9}$, $0.9 < L_{out}(\theta) \leq 1$, we can think that electromagnetic waves do not trap in the atmospheric duct.

As can be seen from Fig. 2(a), when the transmitting antenna is located in the waveguide, both the complete trap and partial trapping are observed in the waveguide. From Fig. 2(b), we know that only partial trapping occurs in the waveguide when the transmitting antenna is located outside the waveguide.

III. NUMBER RESULTS

In order to verify the effectiveness of the leakage coefficient $L_{out}(\theta)$, the propagation characteristics of airborne signal in the surface waveguide were simulated. Assuming frequency $f=1\text{GHz}$, the transmitting antenna is a horizontal polarization Gauss antenna, the height of the antenna is $H_t=10\text{Km}$, the width of the 3dB lobe $\theta_0=1^\circ$, the grazing angle $\theta=2.995^\circ$ (Corresponding incidence angle $\theta_i=0.02^\circ$, $L_{out}(\theta_i)=0.15$), Therefore, there is a partial trapping effect in the waveguide), the maximum propagation distance 1200Km, the waveguide thickness $H_d=250\text{m}$, the waveguide strength $\Delta M = 11.5$, and the receiving antenna height $Z=10\text{m}$. The waveguide refractive index profile, as shown in Fig. 1.

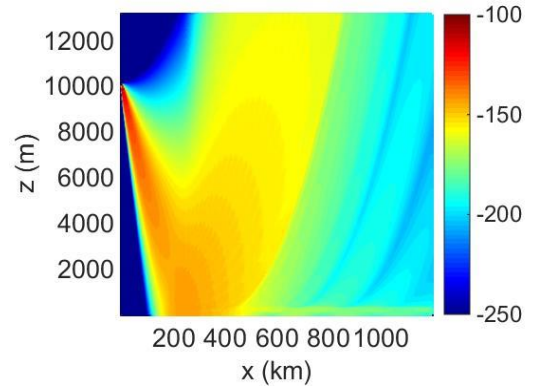


Fig. 3. The distribution of pseudo color propagation factor

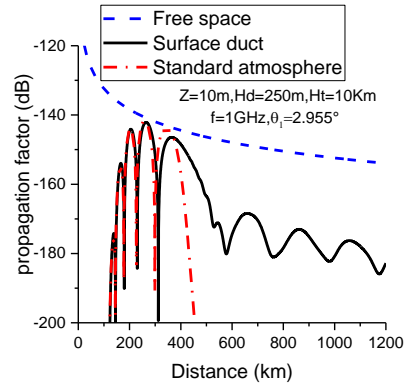
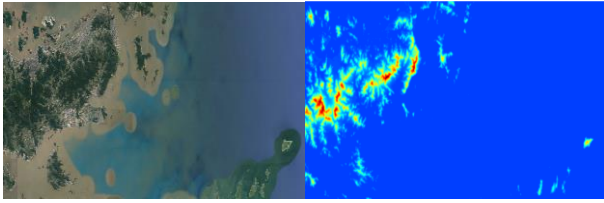
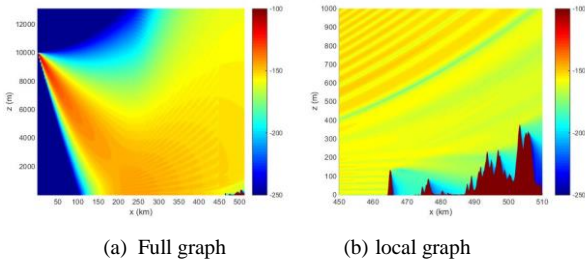


Fig. 4. The variation curve of propagation factor with the propagation distance

Fig. 3 gives a pseudo color map of the distribution of the wave propagation factor in the range of high 14km propagation distance 1200km. It can be seen from the diagram that the electromagnetic wave has an obvious capture effect in the waveguide. This is consistent with the prediction result of leakage coefficient, which proves the validity of leakage coefficient. Fig. 4 gives the curve of propagation factor with propagation distance at high $z=10\text{m}$ of receiving antenna. It can be seen from the diagram that after the electromagnetic wave is incident to the waveguide at propagation distance $x=300\text{km}$, because only a small part of the energy is trapped by the surface waveguide, the propagation factor decreases sharply. When the propagation distance is greater than 600km, the propagation factor will slow down. The loss at this stage mainly comes from the leakage of signal energy in the waveguide.



(a) Image map (b) elevation map
Fig. 5. Zhoushan coastal image and elevation map



(a) Full graph (b) local graph
Fig. 6. Zhoushan coastal propagation factor distribution of pseudo color

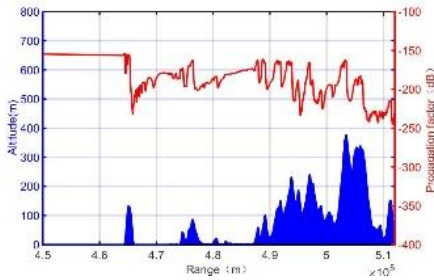


Fig. 7. Variation curves of propagation factors with propagation distance near Zhoushan coast

On the basis of the above, the Zhoushan terrain data as shown in Fig. 5 is introduced. The terrain began to appear at the propagation distance $x=450\text{km}$. Fig. 6 shows the

distribution of Zhoushan coastal wave propagation factor pseudo color, which (a) is the full graph, (b) is the local map. It can be seen from the Fig. 6 that the electric wave signal is launched from ten thousand meters high, and enters the waveguide at the distance of 200 kilometers. It travels to the 450km near Zhoushan in the waveguide, and passes through the occlusion and diffraction of the terrain near Zhoushan, and there is a clear shadow area after the terrain.

Figure 7 is the variation curve of Zhoushan coastal propagation factor with the propagation distance, and the height of the receiving antenna is 10m. It can be seen from the graph that the attenuation trend of radio waves slows down after long distance propagation before encountering terrain. The first time the terrain was encountered at 465km, the radio wave propagation factor sharply decreased, which is due to the shadow effect of the terrain. And there is a shadow area behind it. After 465km, the radio propagation factor will be concussion due to the occlusion, reflection and diffraction of the terrain.

IV. CONCLUSION

In this paper, a new parameter, leakage coefficient, is proposed to analyze the trapping effect of the emitting source outside the waveguide layer, and the radio wave propagation characteristics of airborne radar signal in marine atmospheric duct are analyzed by parabolic equation method. The validity of the leakage coefficient is verified. Besides, the simulation of airborne radar signal propagation in the surface waveguide environment is carried out in the vicinity of Zhoushan island.

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