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# Computation of SAR in Layered Prolate Spheroid Head Model Exposed to Thin Wire Antenna Model

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*Abstract-* **The special absorb rate (SAR) calculation in a layered prolate head model exposed a wire model handset antenna model is investigated using null field method (NFM). NFM is used to solve the scattering problem from the layered prolate head model, and analytical tenique is use to obtained current distribution on the thin wire handset antenna. The coupling between the head model and handset antenna is ignored for the thin wire antenna model is used. The accuracy and efficiency of the proposed algorithm are verified by comparing numerical results with other available data.**

## I. INTRODUCTION

The interaction of EM radiation with humans head has raised public concern about potential health effects and the medical use of radio-frequency (RF) and microwave radiation. It is of great importance to seek efficient solutions to electromagnetic (EM) problem from human head combined with mobile phone antennas. The FDTD algorithm [1-3] is widely used for analyzing the interaction between human head and mobile phone antennas.

In this paper, the head of the user of a mobile telephone is modeled by a double layered prolate excited by the near field of a wire model antenna. Null field method (NFM) [4] is used to solve the scattering problem from the head model, and the analytical solution for thin wire model is used to solve the antenna problem [5], in which the delta-function generator is used to model the feed point. As a thin wire model is used to represent the backbone of handset, the coupling between the head model and a wire model can be ignored. The accuracy and efficiency of the proposed algorithm are verified by comparing numerical results with other available data.

## II. THEORY AND METHOD

A wire model of handset antenna operating in front of a prolate head model was the subject of the present study. In the proposed method, the problem is divided into two separated computational regions: the source region and the scatterer region, as shown in Fig. 1. The source region is defined by the thin wire model antenna and is solved by analytical solution, whereas the scatterer region consists of the double layered prolate head model and the NFM is used to solve the near field scattering problem. As the analysis property of vector spherical wave functions (VSWF) used in NFM, the unknowns in NFM are very small, which makes the hybrid NFM and analytical solution of thin wire model be very efficient. Thus, it is easy to offer a real time estimation of the field regarding the mobile terminal's position.

Assume an arbitrary thin wire with radius a and length L is illuminated by an incident field. The boundary condition on wire surface is the total tangential electric field equals to zero. Therefore,

$$\vec{E}^{inc} = -\vec{E}^{sca} = j\omega\vec{A} + j\frac{1}{\omega\mu\varepsilon}\nabla\left(\nabla\cdot\vec{A}\right) \tag{1}$$

$$\vec{A} = \frac{\mu}{4\pi}\int_l I(l')\hat{a}(l')\frac{e^{-jkR}}{R}dl' \tag{2}$$

where $\vec{A}$ is the magnetic potential [6]. If the radius of wire tends to zero and the wire is excited by a delta gap, the current distributions satisfy [5]

$$\frac{d^2I(l)}{dl^2} + k^2I(l) = -j\omega C_o\delta(l-l') \tag{3}$$

where $C_0 = -2\pi\varepsilon/\ln a$ , and $l'$ is the source location. For an arbitrary dipole excited by a delta gap at $l = l'$, the general solution of (2) are written as

$$I(l) = \begin{cases} \dfrac{j\omega C_o}{k}\left(\dfrac{\sin(kl)\sin(k(L-l'))}{\sin(kL)}\right), & 0 < l < l' \\ \dfrac{j\omega C_o}{k}\left(\dfrac{\sin(kl')\sin(k(L-l'))}{\sin(kL)}\right), & l' < l < L \end{cases} \tag{4}$$

For a dual band PIF antenna, a thin wire model is investigated, as shown in Fig. 2. The general solution of the current on these wires is

$$\begin{aligned} &I_1(l) = \begin{cases} C_1\cos kl_1 + D_1\sin kl_1, & 0 < l_1 < l' \\ C_1'\cos kl_1 + D_1'\sin kl_1, & l' < l_1 < L \end{cases} \\ &I_k(l) = C_k\cos kl_k + D_k\sin kl_k, \quad 0 < l_k < L_k, \\ &k = 2,\cdots,7 \end{aligned} \tag{5}$$

where $\{C_i\}_{i=1}^{7}$ and $\{D_i\}_{i=1}^{7}$ are determined by the boundary conditions as

$$
\begin{aligned}
&\left.\frac{dI}{dl}\right|_{l=l'^{+}} - \left.\frac{dI}{dl}\right|_{l=l'^{-}} = -j\omega C_o \\
&I_1(L_1) - I_2(0) - I_3(0) = 0 \\
&I_2(L_2) + I_3(L_3) + I_4(L_4) = 0 \\
&-I_4(0) - I_5(0) + I_6(L_6) = 0 \\
&-I_1(0) + I_5(L_5) + I_7(L_7) = 0
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&I(l'^{+}) = I(l'^{-}) \\
&\phi_1(L_1) = \phi_2(0) = \phi_3(0) \\
&\phi_2(L_2) = \phi_3(L_3) = \phi_4(L_4) \\
&\phi_4(0) = \phi_5(0) = \phi_6(L_6) \\
&\phi_1(0) = \phi_5(L_5) = \phi_7(L_7) \\
&\phi_6(0) = \phi_7(0) = 0
\end{aligned} \tag{7}
$$

The scattered fields from the induced currents on the surface of wire model antenna are treated as the source in NFM domain. Considering the EM scattering problem form two layered dielectric head model with permittivity and permeability $(\varepsilon_1, \mu_1)$ and $(\varepsilon_2, \mu_2)$, respectively, which is in homogeneous space which is characterized by the parameters $(\varepsilon_0, \mu_0)$, as shown in Fig. 3. According to the equivalence principle, the problem can be solved by considering two simpler equivalent problems, an internal equivalent problem and an external equivalent problem, respectively. Then, we can obtain the general null field equations

$$
\begin{aligned}
&\nabla\times\int_{S_1}(-\bar{J}_{1m})\cdot\bar{\bar{G}}(k_0,\bar{r},\bar{r}')ds' \\
&+\frac{i}{\omega\varepsilon_0}\nabla\times\nabla\times\int_{S_1}(-\bar{J}_{1e})\cdot\bar{\bar{G}}(k_0,\bar{r},\bar{r}')ds' = -\bar{E}^{inc} \\
&\qquad \bar{r}\in D_{in} = D_1\cup D_2\cup S_2
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
&-\nabla\times\int_{S_1}(-\bar{J}_{1m})\cdot\bar{\bar{G}}(k_1,\bar{r},\bar{r}')ds' - \frac{i}{\omega\varepsilon_1}\nabla\times\nabla\times\int_{S_1}\bar{J}_{1e} \\
&\cdot\bar{\bar{G}}(k_1,\bar{r},\bar{r}')ds' + \nabla\times\int_{S_2}(-\bar{J}_{2m})\cdot\bar{\bar{G}}(k_1,\bar{r},\bar{r}')ds' \\
&+\frac{i}{\omega\varepsilon_1}\nabla\times\nabla\times\int_{S_2}\bar{J}_{2e}\cdot\bar{\bar{G}}(k_1,\bar{r},\bar{r}')ds' = 0,\ \bar{r}\in D_0\cup D_2
\end{aligned} \tag{9}
$$

where $\bar{J}_{1m}, \bar{J}_{1e}$, and $\bar{J}_{2m}, \bar{J}_{2e}$ are the equivalent electric current and magnetic current on surface $S_1$ and $S_2$, respectively. $\bar{\bar{G}}(\bar{r},\bar{r}') = \frac{e^{ik_0|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|}\bar{\bar{I}}$ are the dyadic green function of free space. Considering the general null field equations (8) and (9), one can expand the incident field $\bar{E}^{inc}(\bar{r})$ and the dyad green function $\bar{\bar{G}}(\bar{r},\bar{r}')$ with the vector spherical wave functions (VSWF's) [7], approximate the equivalent surface currents $\bar{J}_{1m}, \bar{J}_{1e}$, and $\bar{J}_{2m}, \bar{J}_{2e}$ by the complete set of regular VSWF for interior domain, and use the orthogonality of the VSWF on spherical surface to obtain [4]

$$\bar{Q}_1^{21}(k_0,k_1)\bar{i}_1 + \bar{Q}_1^{22}(k_0,k_1)\tilde{i}_1 = -\bar{e} \tag{10}$$

$$-\tilde{i}_1 + \bar{Q}_2^{11}(k_1,k_2)\bar{i}_2 = 0 \tag{11}$$

$$\bar{i}_1 + \bar{Q}_2^{21}(k_1,k_2)\bar{i}_2 = 0 \tag{12}$$

where $\bar{i}_1 = [c_{1\mu}^N, d_{1\mu}^N]^T$, $\tilde{i}_1 = [\tilde{c}_{1\mu}^N, \tilde{d}_{1\mu}^N]^T$, $\bar{i}_2 = [c_{2\mu}^N, d_{2\mu}^N]^T$, and as before, $\bar{e} = [a_\nu, b_\nu]^T$ is the vector containing the expansion coefficients of incident field. Solving the system of matrix equations, the unknown coefficients $\bar{i}_1$, $\tilde{i}_1$, and $\bar{i}_2$ can be obtained. Then, the electromagnetic field inside and outside of the head model can be computed.

The scattered fields from the induced currents on the surface of the wire model antenna are treated as the source in the NFM domain. Using the expansion of dyadic green function, the expansion coefficients of incident field for NFM region (induced by current I) can be obtain [8]

$$\begin{pmatrix} a_\nu \\ b_\nu \end{pmatrix} = -\eta_0 k_0^2 \int_{l_1}^{l_2} \begin{pmatrix} \bar{M}_{\bar{\nu}}^{1}(k_0\bar{r}') \\ \bar{N}_{\bar{\nu}}^{1}(k_0\bar{r}') \end{pmatrix} \cdot \bar{u}\, I(l)dl \tag{13}$$

where is the unit direction vector of current I.

In the calculation of specific absorption rate (SAR) distribution inside the head model, the following definition is used:

$$SAR = \frac{\sigma}{\rho}\left|\vec{E}\right|^2 \tag{14}$$

where $\sigma$, $\rho$, $|\vec{E}|$ are the conductivity, density and rms electric field intensity, respectively.

## III. Numerical Results

To illustrate the accuracy and efficiency of the NFM, we consider the EM scattering from a double layered prolate head model under the illumination of plane wave. The model is charactered by major semi-axial lengths of outer and inner layer are 100 mm and 98 mm, respectively. The ellipticity of the prolate spheroid is 0.8. The permittivity, conductivity and mass density parameters of the dielectric layer and the core are $\varepsilon_{r1} = 30$, $\sigma_1 = 0.5\,\text{S/m}$, and $\varepsilon_{r2} = 50$, $\sigma_2 = 1.0\,\text{S/m}$, respectively. The incident wave is polarized in the x-direction and propagating in the z-direction. From Fig. 4, we clearly observe a good agreement of the near scattered on the line x=0,

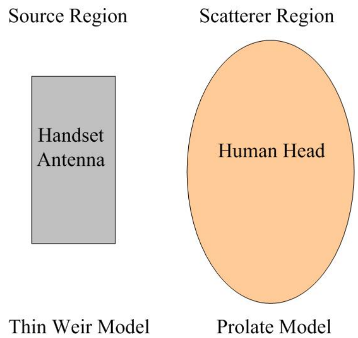


Fig.1 Geometry model of head model exposed to a mobile antenna

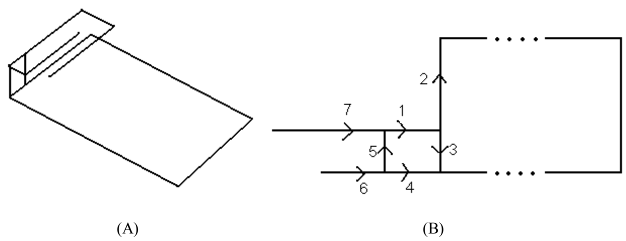


Fig.2 (A) Dual band PIFA wire model. (B) PIFA antenna segmented into 7 wires.

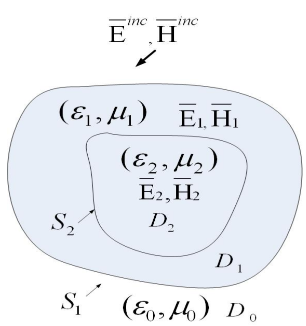


Fig.3 Scattering problem form two layered dielectric object.

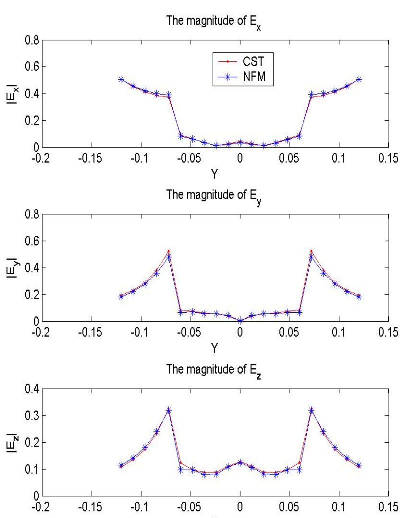


Fig.4 Electric field distribution on the line of x=0,z=50mm. Double layered prolate head model with plane incident wave.

z=50mm between the NFM and CST results. Then, a half-wavelength wire dipole directed along the z-axis is used to test the accuracy of analytical resolution for wire model. As shown in Fig. 5, the field on the plane y =10mm are calculated by analytical solution, which is in good agreement with the result observed by and MoM method [8].

Finally, the calculation of SAR distribution in a double layered prolate head model caused by handset antenna is investigated. The head model used is a layered dielectric prolate, whose major semi-axial lengths of outer and inner layer are 100 mm and 98 mm, respectively. The ellipticity of the prolate spheroid is 0.8. The permittivity, conductivity and mass density parameters of the dielectric layer and the core are $\varepsilon_{r1} = 5$ , $\sigma_1 = 0.05\,\text{S/m}$ , $\rho_1 = 1000\,\text{kg/m}^3$ , and $\varepsilon_{r2} = 42$ , $\sigma_2 = 0.99\,\text{S/m}$ , $\rho_2 = 1000\,\text{kg/m}^3$ , respectively. The handset antenna is modeled by a dual band PIFA antenna using a wire model and a patch model, as shown in Fig. 2. The dual-band antenna works at GSM band (900 MHz) and DCS band (1800 MHz). Fig. 6 shows the calculated return loss of the wire model and that of the patch model for the dual band PIFA antenna. The distance between the antenna and the head model is set to 10mm. Fig. 7 and Fig.8 show the SAR distribution with the wire model and the patch model at operating frequencies of 0.9GHz and 1.8 GHz, respectively. The proposed quick NFM solution is used to the wire model case, while the rigorous hybrid NFM/MoM method [8] is used for the patch model case. As shown in Figs. 7 and 8, similar SAR distribution can be observed by the two models .

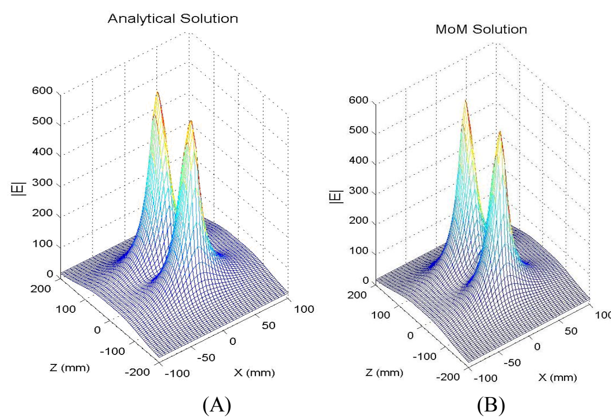


Fig.5 Electric field distributions on plane y = 10mm.
(A) Analytical solution. (B) MoM solution.

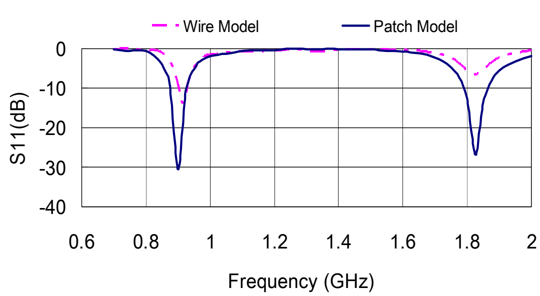
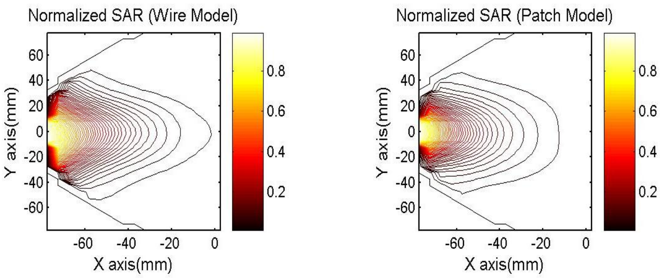
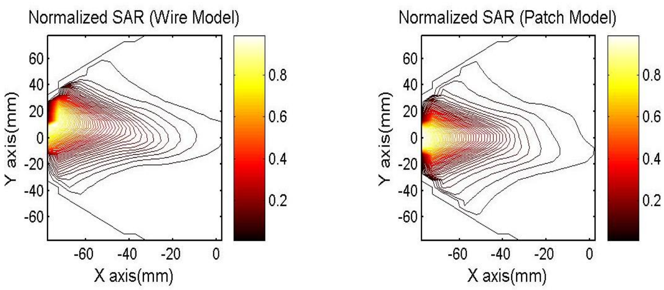


Fig. 6 Return loss of dual ban PIFA antenna.




(A) (B)

Fig. 7 Normalized SAR distribution on plane z=0 with operating frequency 0.9 GHz. (A) Wire model for PIFA; (B) Patch Model for PIFA.




(A) (B)

Fig. 8 Normalized SAR distribution on plane z=0 with operating frequency 1.8 GHz. (A) Wire model for PIFA; (B) Patch Model for PIFA.

## IV. CONCLUSION

The electromagnetic radiation of a handset antenna operating in the vicinity of a prolate head model is investigated by a novel hybrid NFM and analytical solution approach. The analytical solution and NFM were used to solve the source region and scatter region, respectively, which can offer a real time estimation of the field regarding the handset antenna's position. Numerical results show the accuracy and efficiency of the proposed hybrid method.

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