Dispersion and Loss of Complex Structured Plasmonic Surface

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***Abstract*—In this paper, we proposed a new method to analyze the dielectric loss feature of complex structured metallic surface, which is inspired by the idea of the field-network joint method. In order to show the general method of our scheme, we first introduce the behavior of electromagnetic (EM) wave in the upper space using the field view. Meanwhile, we describe the properties of the structured metallic surface based on the view of the circuit topology of the complex structure composed of the transmission line model for continues structure and the lump model for the discontinues structure, which can obtain through the previous research. Hence, by the aid of the mature microwave theory and technology, we can show a general method to analysis the dielectric loss of complex structured metallic surface, which have not been discussed in previous research.**

Keywords—Surface plasmon polaritons, complex structure, dielectric loss.

I. Introduction

Surface plasmon polaritons (SPPs) are electromagnetic (EM) excitations propagating at the interface between a dielectric and conductor, evanescently confined in the perpendicular direction [1]. These EM surface waves arise via the coupling of the EM fields to oscillations of the conductor's electron plasma. Thus, EM fields can be strongly confined to the near vicinity of the interface [2]. In virtue of strong confinement, SPPs provide the possibility of concentrating and channeling light with subwavelength of structures in the optical frequency [3]. The property of SPPs offers the potential for developing new types of photonic devices. This could lead to miniaturized photonic circuits with length scales that are much smaller than those currently achieved. At low frequencies such as microwave or terahertz frequencies, however, the internal fields are identically zero in the limit of a perfect conductor [4]. Perfect metals thus do not support EM surfaces modes, forbidding existence of SPPs.

It would be highly desirable to achieve light localization of subwavelength dimensions at low frequencies. To overcome this difficultly, plenty of efforts have been made to construct plasmonic metamaterials to realize spoof SPPs. Subwavelength corrugated structures have been regarded as the more effective ways to propagate spoof SPPs. Therefore, the concept of spoof SPPs can be applied to very different geometries, such planar [5], cylindrical [6] and so on. Importantly, one of the feature of the spoof SPPs is that their physical properties can be designed at will by tuning the geometrical parameters [7]. The effective medium method and the mode matching technique analyze the surface decorations to provide physical explanations at lower frequencies. However, the mode matching method is so difficult to be implemented technically.

In addition, the slow-wave structures based on Floquet theorem have potential applications on the power electrons, high-power microwave, and CMOS technologies [8] at microwave frequencies. However, it is more difficult to obtain the dispersion relation of the complicated structures that its analysis methods are forced on. Although researchers have introduced the circuit model to analyze SPP-based transmission line [9], the calculated dispersion curves mismatch the simulated result. Recently, the field-network joint solution is proposed to remedy the shortage without the lossy case [10]. In order to accurately obtain the dispersion relation, this paper considers further the dielectric loss of the complex structured surface.

In this paper, the propagation constant and structured field distribution of a metallic surface loaded by lossy dielectric with general subwavelength decorations are investigated in Section II. Secondly, the dispersion relation and loss feature can be calculated by introducing input impedance of the complex subwavelength decorated plasmonic surface using field-network method in Section III.

II. A General Lossy Spoof Surface Plasmon

Like SPPs in the optical frequency, spoof SPPs is a kind of TM surface wave. The inset of Fig. 1 shows a two-demensional (2D) subwavelength corrugated metal structures supporting spoof SPPs, whose plasmonic surface is filled by dielectric material with relative permittivity ε_{r2} and relative permeability μ_{r2} in region II. Whereas, the homogeneous dielectric surrounding with relative permittivity ε_{r1} and relative permeability μ_{r1} in region I. The period of the structural

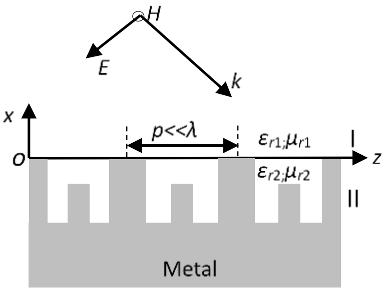

Fig. 1. The schematic diagram of subwavelength decorated metallic plasmonic surface, in which, the *x*-axis is perpendicular the interface between the region I and region II, *z*-axis is along the interface.

decoration is p, which is smaller than the wavelength λ of incident EM wave. The inset picture shows the unite structure of the SPPs, which is parallel to the *yoz* plane with periodicity along *z*-direction and perpendicular to the *x*-direction. In TM polarization and incidence in the *xoz* plane, the two components of the EM fields that are parallel to the interface, and thus can be analytically expressed as:

$$E_x = \frac{k_z}{\omega\varepsilon_0\varepsilon_{r1}} A e^{-jk_x x} e^{-jk_z z} \tag{1}$$

$$E_z = \frac{k_x}{\omega\varepsilon_0\varepsilon_{r1}} A e^{-jk_x x} e^{-jk_z z} \tag{2}$$

$$H_y = A e^{-jk_x x} e^{-jk_z z} \tag{3}$$

where ω and ε_0 are the angular frequency and permittivity of free space, k_i ($i = x, z$) the propagation constants in the i-direction, respectively, and A is constant. E_i ($i = x, z$) is the i-component of the electric field, H_y is the y-component of the magnetic field, and the other components (E_y, H_x, H_z) are zero.

According to the dispersion relation for the plane surface, the propagation constans k_i should satisfy the Maxwell's equations only on condition that the following equations also hold :

$$k_z^2 + k_x^2 = k_0^2\varepsilon_{r1}\mu_{r1} = \omega^2\varepsilon_0\mu_0\varepsilon_{r1}\mu_{r1} \tag{4}$$

where k_0 and μ_0 are wavenumber and permeability of free space, respectively. Owing to the impedance boundary conditions and the properties of the plasmonic surface, the input impedance of a corrugated surface is given by

$$Z_{in} = R + jX = \frac{E_z}{H_y} = \frac{k_x}{\omega\varepsilon_0\varepsilon_{r1}} \tag{5}$$

where Z_{in} is the input impedance of the plasmonic surface, R and X are the real part and imaginary part of Z_{in}, respectively. In order to obtain Z_{in}, we need obtain the complex propagation constant k_x.

In a lossy case, we assume that $\varepsilon_{r1} > 0$. ε_{r2} and k_i should be considered in the complex forms, respectively, i.e.

$$\varepsilon_{r2} = \varepsilon'_{r2} - j\varepsilon''_{r2} \tag{6a}$$

$$k_x = k'_x - jk''_x \tag{6b}$$

$$k_z = k'_z - jk''_z \tag{6c}$$

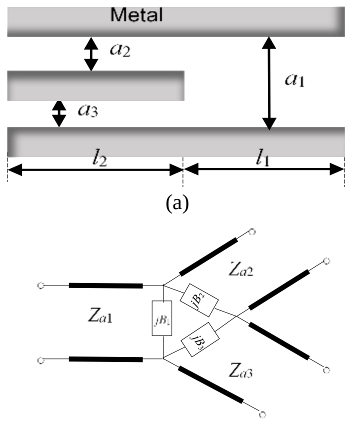

Fig. 2. The schematic diagram of equivalent relation between the transmission lines and circuit topology. (a) the bifurcation of slits and (b) its equivalent circuit.

where Re[k_i] and Im[k_i] are the phase constant and the corresponding attenuation constant in the i-direction, respectively.

Hence, the propagation constant k_z can be also obtained by the input wave impedance as:

$$k_z = \omega\sqrt{\varepsilon_0^2\varepsilon_{r1}^2 Z_{in}^2 - \varepsilon_0\mu_0\varepsilon_{r1}\mu_{r1}} \tag{7}$$

After analysis, $R > 0$ and $X > 0$ must hold, thus the input impedance of the plasmonic surface is inductive. Accordingly, the field in this perpendicular direction is said to be evanescent, reflecting the bound, non-radiative nature of SPs, and prevents power from propagating away from the surface. Parallel to this, the propagation constant k_z of surface plasmon has a non-zero imaginary, which is associated with attenuation of the surface plasmon.

III. Solving Dispersion Relation and Dielectric Loss Using Input Impedance

As discussed above, the dispersion relation and dielectric loss of the decorated metallic surface solved by the input impedance is so difficult to obtain directly using the common analysis method that a new approach should be applied. Inspired by the analysis method of microwave circuits, the network method with cascading the network topology can apply for getting the input impedance.

In order to introduce spatial dispersion into the model in this paper, the uniform decorated slit using the transmission line model is analyzed. Note that we chose the wave impedance rather than the characteristic impedance to obtain the input impedance of the plasmonic surface in common network. To simplify the calculation and overcome the erroneous result for impedance matching, the average impedance is introduced. Owing to the subwavelength properties of the period structure, the average wave impedance can be analytically expressed as

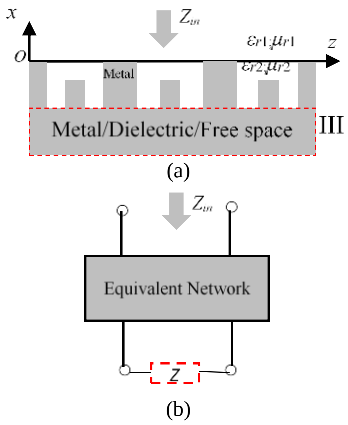

Fig. 3. The schematic diagram of equivalent relation between the terminal structure and circuit topology. (a) the decorated plasmonic surface and (b) its equivalent circuit.

$$Z_a = \frac{a}{p}\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}}\eta = \frac{a}{p}\sqrt{\frac{\mu_{r2}\mu_{r1}}{\varepsilon_{r2}\varepsilon_{r1}}}\eta_0 \tag{8}$$

where Z_a is the average wave impedance, η_0 and η are the intrinsic impedance of free space and lossy dielectric, respectively, and a is the width of 2D slit.

According to the period feature, we can recognize the input impedance as that of a one-port network, which is used to describe the transmission and reflection behaviors of the unit. More generally, the transmission systems with discontinuities lead to simple equivalent circuits with lumped-circuit constants representing the effect of these discontinuities. The discontinuity, Fig. 2, is represented by the three transmission lines in series with a shunting susceptance or reactance placed across each line at the junction. Values for these equivalent susceptance or reactance are obtained. In Fig. 2, Z_{an} (n=1, 2, 3) is the characteristic impedances, and B_n and a_n (n=1, 2, 3) is the susceptance or reactance and width between the transmission lines, respectively. Moreover, l_1 and l_2 the lengths of right slit and left slit, respectively.

Considering practicality instance, the terminal forms of the decorated plasmonic surface is investigated in different cases. We will specialize three special cases, as shown in Fig. 3. Due to the different kinds of medium (metal/dielectric/air), the terminal impedance in Region III is different. The first case of terminal form is Z = 0 because the closed form surface can be regarded as a short circuit terminal. The second case of terminal form is $|Z| \neq 0$ because the introduced dielectric absorb the incidental EM energy. The last case of the terminal form is $|Z| \approx 377\ \Omega$, because the EM wave will be reflected and the other will leak into the air.

As discussed above, the input wave impedance can be solved by obtaining equivalent circuit network topology for the decorated surface. The overall voltage-current matrix of several continuous and discontinuous structure that are cascaded in series can be obtained as

$$\boldsymbol{F}_o = \left[\prod_{i=1}^{n-1} \boldsymbol{T}_{ci}\boldsymbol{T}_{di}\right]\boldsymbol{T}_{cn}\boldsymbol{F}_t \tag{9}$$

where $\boldsymbol{T}_{ci}$ and $\boldsymbol{T}_{di}$ are the transfer matrices of i-th uniformly continuous structure and discontinuous structure, respectively, $\boldsymbol{F}_o$ is the overall voltage-current matrix and $\boldsymbol{F}_t$ is the voltage-current matrix described by the nature of terminal. From (9) the input wave impedance of the plasmonic surface can be written as

$$Z_o = V_o / I_o \tag{10}$$

where $Z_o = Z_{in}$, V_o and I_o are the voltage and current element of the voltage-current matrix $\boldsymbol{F}_o$, respectively. Substituting (10) into (8) gives the dielectric loss and dispersion relation of the decorated surface.

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