Fast analysis of electromagnetic scattering from conducting targets using multilevel characteristic basis function method and recompressed adaptive cross approximation algorithm

Wanqing You, Yufa Sun
Key Lab of Intelligent Computing & Signal Processing, Ministry of Education, Anhui University
Hefei 230601, China
wanqingu@126.com

***Abstract*—The recompressed adaptive cross approximation algorithm (RACA) in conjunction with multilevel characteristic basis function method (MLCBFM) is proposed to solve electromagnetic scattering problems efficiently. The MLCBFM defines a set of higher order characteristic basis functions on each block through the mutual coupling effects of different blocks to obtain high accuracy. Furthermore, the impedance matrices of far field are compressed by RACA algorithm at all levels, which can accelerate the calculation of the reduced matrix and generation of the secondary characteristic basis function (SCBF). Numerical results are given to demonstrate the accuracy and high efficiency of the proposed method.**

Keywords—method of moments; adaptive cross approximation; singular value decomposition; characteristic basis function method

I. INTRODUCTION

The method of moments (MOM) [1] has been commonly studied for solving electromagnetic scattering problems, which transforms the integral equation into a dense matrix equation. To solve this dense matrix equation, the memory requirement and computational time of the conventional MOM are very expensive. Some valid iterative methods such as multilevel fast multipole algorithm (MLFMA) [2], adaptive integral method (AIM) [3], adaptive cross approximation (ACA) algorithm [4] are developed to mitigate this problem. However, these iterative methods suffer from convergence problems of ill-conditional matrices for large targets under analysis. Especially, when the monostatic scattering problems are considered, the iterative process must be repeated for each excitation.

The characteristic basis function method (CBFM) [5] is an iterative-free algorithms, which divides the geometry of scatterer into several blocks, generating a set of characteristic basis functions (CBFs) to express the current distribution of each domain, and the number of CBFs is lower than that of RWG [6] of MOM. For scattering targets, the larger size of the block can lead to the higher compression of unknowns by CBFM. But, with the increase of block size, both the rigorous generation of CBFs of large blocks and calculation of reduced matrix become more and more time and memory consuming. The multilevel characteristic basis function method (MLCBFM) [7] can relive this problem efficiently. In this method, the high-level CBFs defined in large blocks are expressed as a linear combination of the previously generated low-level CBFs defined in the corresponding small blocks, so it can obtain a higher compression rate of unknowns. In [8], ACA algorithm is also employed to accelerate the filling of the impedance matrix of far field.

In this paper, the geometry of large scattering targets is divided with multilevel partitioning approach. In order to obtain high accuracy, the mutual coupling effects of blocks are considered through calculating secondary characteristic basis functions (SCBFs) on each block. Then, the RACA algorithm is applied to compress the impedance matrices of far field at all levels, which further lead to reduce the time of calculation and memory requirement.

II. FORMULATION

A. Review of the MLCBFM

In the conventional CBFM, the geometry of scattering target is firstly divided into M blocks, and the primary characteristic basis function (PCBF) is defined on each block at excitation independent. According to Foldy-Lax multiple scattering equation [9], the SCBFs can be easily obtained through the mutual coupling effects of different blocks. Then, the current distribution of extend block i is expressed by a linear combination of the CBFs, supposing the SCBF is computed to $(K-1)$ th order on each block.

$$\boldsymbol{J}_i = \sum_{k=1}^{K} \alpha_i^k \boldsymbol{J}_i^k \left(i = 1, 2, 3 \cdots M\right) \tag{1}$$

Where K is the number of CBFs including 1 PCBF and $(K-1)$ SCBFs, and α_i^k is weights of CBFs on block i, $\boldsymbol{\alpha}_i = \left[\alpha_i^1 \quad \alpha_i^2 \quad \cdots \quad \alpha_i^K\right]$. In order to solve the unknown

weights $\boldsymbol{\alpha}_i$ for $i=1\sim M$, the reduced matrix equation $\boldsymbol{Z}^R\boldsymbol{\alpha}=\boldsymbol{V}^R$ is constructed and solved as follows

$$\begin{bmatrix} \boldsymbol{R}_{11} & \boldsymbol{R}_{12} & \cdots & \boldsymbol{R}_{1M} \\ \boldsymbol{R}_{21} & \boldsymbol{R}_{22} & \cdots & \boldsymbol{R}_{2M} \\ \vdots & \vdots & \cdots & \vdots \\ \boldsymbol{R}_{M1} & \boldsymbol{R}_{M2} & \cdots & \boldsymbol{R}_{MM} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{\alpha}_2 \\ \vdots \\ \boldsymbol{\alpha}_M \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}_1^R \\ \boldsymbol{V}_2^R \\ \\ \boldsymbol{V}_M^R \end{bmatrix} \tag{2}$$

Where $\boldsymbol{Z}^R$ is a $KM\times KM$ reduced matrix, $\boldsymbol{\alpha}$ and $\boldsymbol{V}^R$ are $KM\times 1$ vectors, $\boldsymbol{R}_{ij}$ is an $K\times K$ sub-matrix of $\boldsymbol{Z}^R$. However, with the increase of the size of targets, the generation of CBFs of blocks and calculation of reduced matrix $\boldsymbol{Z}^R$ need more and more time consumption. The MLCBFM is adopted to mitigate this problem, which divides the geometry of scattering targets under analysis into 1 to L level. Supposing the reduced matrix $\left(\boldsymbol{Z}_i^l\right)^R$ belongs to the lth level block i ($l\le L$), the SCBF is computed to $(K-1)$ th order on each block at all levels, then

$$\left(\boldsymbol{Z}_i^l\right)^R = \begin{bmatrix} \boldsymbol{R}_{11}^{(l-1)} & \boldsymbol{R}_{12}^{(l-1)} & \cdots & \boldsymbol{R}_{1B}^{(l-1)} \\ \boldsymbol{R}_{21}^{(l-1)} & \boldsymbol{R}_{22}^{(l-1)} & \cdots & \boldsymbol{R}_{2B}^{(l-1)} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{R}_{B1}^{(l-1)} & \boldsymbol{R}_{B2}^{(l-1)} & \cdots & \boldsymbol{R}_{BB}^{(l-1)} \end{bmatrix} \quad (l>1) \tag{3}$$

Where $\left(\boldsymbol{Z}_i^l\right)^R$ is a $KB\times KB$ reduced matrix, B is the sum of son blocks on lth level extended block i. The $\left(\boldsymbol{Z}_i^l\right)^R$ is much smaller than $\boldsymbol{Z}^R$, which can be easily solved. By constructing a reduced matrix equation of B son blocks like (2) and directly solving it, the current coefficients which belongs to the extend region of block i are discard, we get the real CBF $\boldsymbol{J}_i^l$ of lth level block i

Similarly, the CBF of each block on $(l+1)$ th level is expressed as a linear combination of the CBFs defined in the corresponding small block on the lth level. Until the highest-level (Lth level) CBFs are calculated, we can get the induced current on the scattering target through solving the reduced matrix equation at last. Therefore, $\left(\boldsymbol{Z}^{L+1}\right)^R$ is given by

$$\left(\boldsymbol{Z}^{L+1}\right)^R = \begin{bmatrix} \boldsymbol{R}_{11}^{(L)} & \boldsymbol{R}_{12}^{(L)} & \cdots & \boldsymbol{R}_{1N}^{(L)} \\ \boldsymbol{R}_{21}^{(L)} & \boldsymbol{R}_{22}^{(L)} & \cdots & \boldsymbol{R}_{2N}^{(L)} \\ \vdots & \vdots & \cdots & \vdots \\ \boldsymbol{R}_{N1}^{(L)} & \boldsymbol{R}_{N2}^{(L)} & \cdots & \boldsymbol{R}_{NN}^{(L)} \end{bmatrix} \tag{4}$$

Where N is the number of blocks on the highest-level. $\left(\boldsymbol{Z}^{L+1}\right)^R$ is a $KN\times KN$ reduced matrix. The sub-matrix $\boldsymbol{R}_{ij}^{(l)}$ for $l=1-L$ is computed by

$$\boldsymbol{R}_{ij}^{(l)} = \left(\boldsymbol{J}_i^l\right)^H \boldsymbol{Z}_{ij}^l \boldsymbol{J}_j^l \tag{5}$$

Where $\boldsymbol{R}_{ij}^{(l)}$ is a $K\times K$ matrix, $\boldsymbol{Z}_{ij}^l$ is an $N_i\times N_j$ impedance matrix, N_i and N_j are the unknowns of block i and block j on the lth level. If block i and block j are parting overlapping, $\boldsymbol{Z}_{ij}^l\in N_i\times\left(N_j-N_{ij}^{overlapping}\right)$, the $N_{ij}^{overlapping}$ are overlapping part.

B. Compressed high-level impedance matrix of far field using RACA algorithm

It can be found that there are many $\boldsymbol{R}_{ij}^{(l)}$ in (5) need to be computed for each level. When $l=1$, the $\boldsymbol{Z}_{ij}^l$, which belongs to the mutual impedance matrix of non-adjacent block i and block j, is rank deficient and can be compressed by ACA algorithm. Moreover, some studies [10, 11] show that the high-level impedance matrices between nonoverlapping high-level blocks also have low-rank property and can be compressed efficiently. Therefore, in order further to accelerate generation of SCBFs and the calculation of the reduced matrix, the ACA algorithm is employed at all levels.

$$\boldsymbol{Z}_{ij}^l \approx \tilde{\boldsymbol{Z}}^{N_i\times N_j} = \boldsymbol{Z}_U^{N_i\times r}\left(\boldsymbol{Z}_V^{N_j\times r}\right)^T \tag{6}$$

Where r is the rank of $\boldsymbol{Z}_{ij}^l$. The two low-rank matrices $\boldsymbol{Z}_U^{N_i\times r}$ and $\boldsymbol{Z}_V^{N_j\times r}$ obtained by ACA algorithm are not orthogonal with some redundancy. Therefore, the QR decomposition [12, 13] is applied to the $\boldsymbol{Z}_U^{N_i\times r}$ and $\boldsymbol{Z}_V^{N_j\times r}$

$$\boldsymbol{Z}_U^{N_i\times r} = \boldsymbol{Q}_U^{N_i\times r}\boldsymbol{R}_U^{r\times r} \tag{7}$$

$$\boldsymbol{Z}_V^{N_j\times r} = \boldsymbol{Q}_V^{N_j\times r}\boldsymbol{R}_V^{r\times r} \tag{8}$$

Where $\boldsymbol{Q}_U^{N_i\times r}$ and $\boldsymbol{Q}_V^{N_j\times r}$ are the orthonormal matrices, $\boldsymbol{R}_U^{r\times r}$ and $\boldsymbol{R}_V^{r\times r}$ are the upper-triangular matrices. Then, $\boldsymbol{R}_U^{r\times r}\left(\boldsymbol{R}_V^{r\times r}\right)^T$ is recompressed by SVD [14, 15]

$$\boldsymbol{R}_U^{r\times r}\left(\boldsymbol{R}_V^{r\times r}\right)^T = \boldsymbol{U}_U^{r\times r'}\left(\boldsymbol{S}^{r'\times r'}\right)^l\boldsymbol{V}_V^{r'\times r} \tag{9}$$

Finally, the impedance matrix $\boldsymbol{Z}_{ij}^l$ is expressed as

$$\begin{aligned} \boldsymbol{Z}_{ij}^l &\approx \tilde{\boldsymbol{Z}}^{N_i\times N_j} \\ &= \left(\boldsymbol{Q}_U^{N_i\times r}\boldsymbol{U}_U^{r\times r'}\right)\left(\boldsymbol{S}^{r'\times r'}\right)^l\left(\boldsymbol{Q}_V^{N_j\times r}\boldsymbol{V}_V^{r\times r'}\right)^T \\ &= \tilde{\boldsymbol{Z}}_U^{N_i\times r'}\left(\tilde{\boldsymbol{Z}}_V^{N_j\times r'}\right)^T \end{aligned} \tag{10}$$

Where r' is the valid rank of $\boldsymbol{Z}_{ij}^l$ by ACA-SVD, $r'<r$. It can be seen that the impedance matrix of far field obtained by ACA algorithm is recompressed by SVD, and the calculated amount of the interactions between block i and block j is reduced from $O\left((N_i+1)\times N_j\right)$ to $O\left(r'\left(N_i+N_j+1\right)\right)$. Therefore, the RACA algorithm can further reduce the computational complexity and memory requirement.

III. Numerical Results And Discussions

In this section, the proposed method is applied in several different test examples for calculating the bistatic radar cross-section (RCS). All the results are computed on the Intel (R) Xeon (R) CPU E5-2630 V4@2.2GHz, 256 RAM PC. The compiler is used Code Blocks. The SCBF is computed to 5th order for the proposed method, the ACA and SVD threshold are chosen be 10^{-3}, the incident wave is illuminated along the negative direction of z-axis in these numerical examples.

A. A Group of PEC Targets

Firstly, we choose 16 discrete PEC targets with random shapes at the incident frequency of 300 MHz, which include the sphere with the radius 0.5m, the cone and cylinder with the radius and height 0.5m and 1m, the cube with the length 1m. The distance between two adjacent targets is 1m. The geometry is divided into 11440 triangular patches and the number of unknowns is 17160, which divided into 16 first-level blocks and 4 second-level blocks. The bistatic RCS in HH and VV polarization calculated by conventional ACA-MLCBFM and proposed method are shown in Fig.1 and Fig.2.

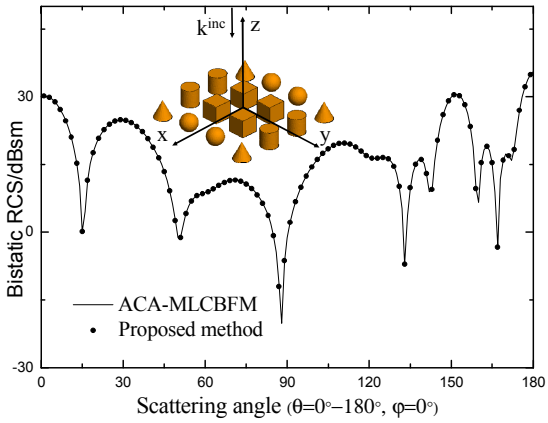

Fig.1. Bistatic RCS of 16 discrete PEC targets in HH polarization.

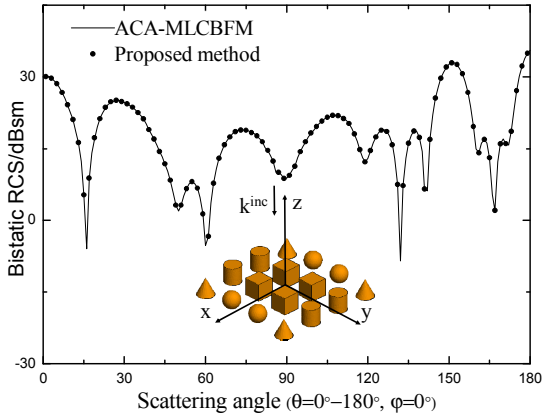

Fig.2. Bistatic RCS of 16 discrete PEC targets in VV polarization.

Table I shows the computational time and memory requirement of 16 discrete PEC targets in HH polarization. It can be found that proposed method outperforms the conventional ACA-MLCBFM.

TABLE I. Comparison of CPU time and memory

	Two Methods	
	ACA-MLCBFM	***Proposed method***
CBFs generation	59s	43s
Reduced matrix solving	356s	323s
Total time	428s	379s
Memory(GB)	1.85	1.64

B. Almond

Finally, the bistatic RCS of a 252.3744-mm PEC almond is calculated at the incident frequency of 10 GHz, The geometry is divided into 10616 triangular patches and the number of unknowns is 23906, which is divided into 16 first-level blocks and 4 second-level blocks, and each block is extended 0.05λ on the first-level and 0.15λ on the second-level in all directions. The bistatic RCS in HH and VV polarization calculated by conventional ACA-MLCBFM and proposed method are shown in Fig.3 and Fig.4. Table II shows the computational time and memory requirement of the PEC almond in HH polarization. It can be found from the figure that proposed method outperforms the conventional ACA-MLCBFM.

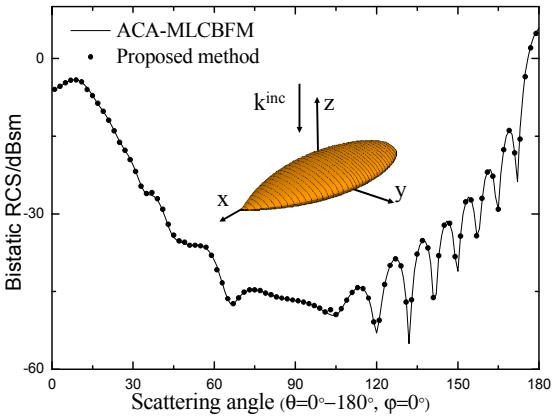

Fig.3. Bistatic RCS of the PEC almond in HH polarization.

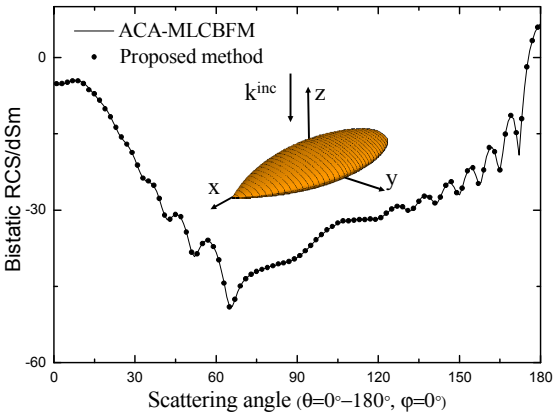

Fig.4. Bistatic RCS of the PEC almond in VV polarization.

TABLE II. COMPARISON OF CPU TIME AND MEMORY

	Two Methods	
	ACA-MLCBFM	***Proposed method***
CBFs generation	157s	121s
Reduced matrix solving	924s	768s
Total time	1101s	909s
Memory(GB)	3.33	2.94

IV. CONCLUSION

In this paper, the RACA-MLCBFM is proposed to solve electromagnetic scattering problem efficiently. It was found that the proposed method can keep high accuracy through calculating the higher order CBFs on each block. Moreover, the RACA algorithm is applied to further compress the impedance matrices of far filed, which requires less matrix equation solutions and decreases the time of SCBFs generation. The numerical results show more advantage in the computational time and memory requirement for large scattering targets.

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