Construction of arbitrarily shaped cloaks using characteristic mode method

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***Abstract*—The paper proposed a robust method to design the invisibility cloaks for arbitrarily shaped object based on the characteristic mode method (CMM). According to discrete dipole approximation (DDA), target with arbitrary geometry can be modelled by a couple of interacting dipoles. By exploiting the characteristic mode analysis, laws of objects with low or nearly identical scattering efficiency are theoretically shown related to the generalized eigenvalue of interacting matrix in DDA method. Hence the relation can be utilized to manipulate the invisibility and illusion images. The accuracy and effectiveness of the developed approach have been demonstrated by numerical arbitrary cloaks in full-wave simulation.**

Keywords—invisibility; arbitrarily-shaped; characteristic mode; discrete-dipole approximation

I. INTRODUCTION

Fast development of defense electronics has triggered the research on electromagnetic invisibility cloak [1-2]. The most powerful approaches, i.e., transformation optics [1, 3] deflects the rays around an object smoothly to achieve perfect invisibility. However, the strongly inhomogeneous, anisotropic and negative material parameters limit the practical realization of TO based cloaks. Afterwards, scattering cancellation (SC) [5-7] was studied to reduce the RCS of target by warping a shell outside. However, it can be only implemented on canonical shapes such as cylinder and sphere due to the restriction of Mie series.

Exploiting the demands on physical realization and freewill geometry, we combine the SC technique with DDA method to achieve stealth or mimicry for objects at will. Therein we apply the DDA method to analyze the electromagnetic scattering from a coated object. Then characteristic mode method is employed to determine the relationship between scattering section of a low observable object and its eigenvalues. Finally, the analytical formulas are derived to achieve the cloaking design.

II. THEORETICAL ANALYSIS

A. Discrete dipole approximation

DDA [8] is an approximation of a continuum target to solve the scattering problems for arbitrarily shaped target. It starts from the volume integral equation (VIE) and then discretizes the target into N cubic elements whose length is d, corresponding volume is $V_j = d^3$ ($j = 1, \cdots, N$). With discretization, the VIE can be written as,

$$\vec{E}_j^{exc} = \vec{E}_j^{inc} + \frac{i}{4\pi\omega\varepsilon_0}\sum_{\substack{k=1\\k\neq j}}^{N}\bar{G}(\vec{r}_j,\vec{r}_k)V_k\vec{J}_k = \vec{E}_j^{inc} + \sum_{\substack{k=1\\k\neq j}}^{N}\bar{G}(\vec{r}_j,\vec{r}_k)\cdot\vec{p}_k \tag{1}$$

Ulteriorly, dipole moment induced by $\vec{E}_j^{exc}$ can be derived

$$\vec{p}_j = \alpha_j\vec{E}_j^{exc} \tag{2}$$

where α_j is dipole polarizability. Combining (1) and (2) a system of $3N$ linear equations can be expressed as

$$\sum_{k=1}^{N} A_{jk}P_j = E_{inc,j} \tag{3}$$

Note that $\bar{A}$ in (3) is a symmetry matrix and the polarization P_j can be got by solving (3). Furthermore, the radar cross section (RCS) can also be derived as

$$\sigma = \lim_{r\to\infty}4\pi r^2\left|\frac{\vec{E}^{sca}}{\vec{E}^{inc}}\right|^2 = 4\pi k^4\left|\sum_{j=1}^{N}e^{-ik\vec{r}_j\cdot\hat{k}^{sca}}\,\vec{p}_j\right|^2 \tag{4}$$

in which, k is the wave number of the incident wave.

B. Characteristic mode method

Characteristic mode is a widely used numerical method to deal with the radiation and scattering problems. According to the matrix feature of $\bar{A}$, we can write the interacting matrix $\bar{A}$ in terms of its real and imaginary parts as

$$\bar{A} = \bar{R} + j\bar{X} \tag{5}$$

where both $\bar{R}$ and $\bar{X}$ are real symmetric matrices. By virtue of CM analysis, we consider the following generalized eigenvalue equation:

$$\bar{A}\cdot\bar{q}_n = (1+i\lambda_n)\bar{R}\cdot\bar{q}_n \tag{6}$$

If the imaginary part $\bar{X}$ is approximately negligible compared with $\bar{R}$, above equation can be reduced to

$$\bar{A}\cdot\bar{q}_n = \bar{R}\cdot\bar{q}_n = \lambda_n\bar{q}_n \tag{7}$$

Considering the normalized orthogonal relationship of the eigenfunctions $\bar{q}_n$, we can expand the dipole moment as

$$\bar{p} = \sum_l \frac{\bar{q}_l^T\cdot\bar{E}^{inc}}{\lambda_l}\bar{q}_l \tag{8}$$

Substituting (8) into (4), the RCS is given by

$$\sigma = 4\pi k^4 \left| \sum_l \frac{V_l^{inc} V_l^{sca}}{\lambda_l} \right|^2 \quad (9)$$

in which, V_l^{sca} can be obtains by the replacement of $\vec{E}^{inc}$ in by $\vec{E}^{sca}$.

Note that the eigenvalue λ_n which ranges from $-\infty$ to $+\infty$ is of great importance to the scattering nature. There will be significant scattering energy when λ_n is close to zero. On the other hand, it leads to no electromagnetic scattering when λ_n tends to infinity. The phenomenon provides us a valuable and feasible way to achieve invisible cloaking.

C. *Eigenvalue analysis of Invisibility and Illusion Conditions*

According to the above discussion, we can manipulate the scattering of an object by regulating its generalized eigenvalue. Given an arbitrary target with relative permittivity ε_1, we cover it with a passive layer with ε_2 to make it show semblable behavior to another illusion object ε_e. According to (9), the coated target has a similar behavior as illusion object if they have same eigenfunctions and eigenvalues. To approximately accomplish this condition, the same matrix traces are used, i.e.,

$$tr\left[\overline{X}^{Illusion}(\overline{R}^{Illusion})^{-1}\right] = tr\left[\overline{X}^{coated}(\overline{R}^{coated})^{-1}\right] \quad (10)$$

For an electrically moderate target, non-diagonal elements can be reasonably omitted because the diagonal elements of $\overline{A}$ are dominant. Therefore, matrices in (10) can be reduced to

$$\overline{X}^{Illusion} = \overline{X}^{coated} = diag\left[-\frac{2}{3}k^3 \quad \cdots \quad -\frac{2}{3}k^3\right] \quad (11)$$

$$\overline{R}^{Illusion} = diag\left[\beta_e \quad \cdots \quad \beta_e\right] \quad (12)$$

$$\overline{R}^{coated} = diag\left[\beta_2 \quad \cdots \quad \beta_2 \quad \beta_1 \quad \cdots \quad \beta_1 \quad \beta_2 \quad \cdots \quad \beta_2\right] (13)$$

where $\beta_s = \frac{4\pi}{3d^3}\frac{\varepsilon_s+2}{\varepsilon_s-1} + \frac{k^2}{d}(b_1+b_2\varepsilon_s)$ $(s=1,2,e)$.

Then substituting (11)-(13) into (10), the illusion condition is determined by

$$\frac{N_1}{\beta_1} + \frac{N_2}{\beta_2} = \frac{N}{\beta_e} \quad (14)$$

in which, N represents the total number of the dipoles for the coated and illusion objects. N_1 and N_2 are the number of the dipoles for the original target and the coating respectively. Particularly, the result also satisfies for invisibility condition if $\varepsilon_e = 1$.

III. Numerical Result

Based on our above theoretical analysis, a numerical design for an arbitrary object is given to validate the invisibility performance. An incident plane wave is used in the following simulation.

For the cloak design, we characterize how well a monolayer cloak cancels the scattering of a goblet-shaped target. The relative permittivity of the concealed goblet is 2.5 and the proposed formula indicates that a shell with $\varepsilon_2 = 0.76$ can achieve the invisibility condition. With this initial solution we further optimize the coating for minimal scattering fields. The optimized coating has $\varepsilon_2 = 0.45$ [9]. Fig. 1 shows the RCSs in XOY and XOZ planes when the initial and optimized coatings are used respectively. There is 5.8dB RCS reduction for the initial covering. With the optimized coating, nearly 15dB RCS reduction is achieved. Besides, comparison of the near field distribution plotted in Fig.2 also demonstrates the good invisible performance of the proposed coating.

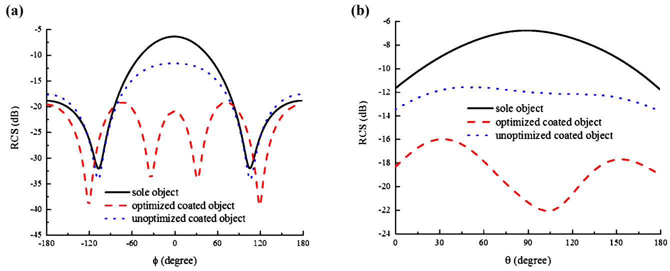

Fig.1. RCS comparison between the coated and sole object. (a). XOY plane. (b). XOZ plane.

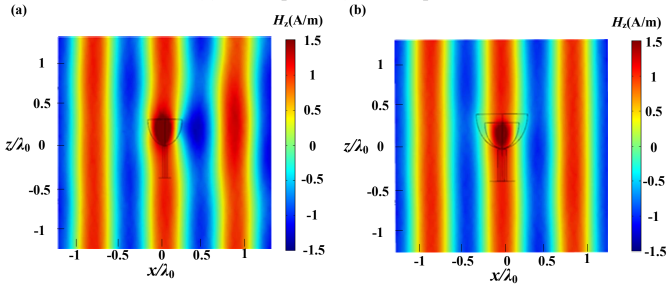

Fig.2. Comparison of near field distribution. (a). Sole object. (b). Coated object.

IV. Conclusion

The proposed approach creates a new way to drastically change the scattering efficiency of an arbitrary target based on characteristic mode in the text of scattering cancellation. We have explored the relation between the scattering field and generalized eigenvalues. With the developed method, cloaks for arbitrarily shaped target demonstrate a perfect performance.

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