DOA Estimation based on Sparse Representation of Covariance Matrix for 4-D Antenna Arrays

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***Abstract*—In this paper, Direction Of Arrival (DOA) estimation algorithm based on sparse representation of covariance matrix for four dimensional (4-D) antenna arrays with unknown source number is proposed to solve defect that DOA estimation needs to know source number in existing DOA estimation algorithms for 4-D antenna arrays. The algorithm represents sparse representation of covariance matrix for 4-D antenna arrays directly. The whole sparse reconstruction process does not need to know source number. Simulation results show that the algorithm can achieve accurate DOA estimation under unknown source number, low SNR, closely spaced signals and coherent signals.**

Keywords—4-D antenna arrays, covariance matrix, sparse representation, unknown source number, DOA estimation

I. INTRODUCTION

In the 50s and 60s last century, researchers introduced time modulation technology into antenna arrays which were 4-D antenna arrays or Time Modulated Antenna arrays (TMAA) [1-2]. Compared with phased arrays, the 4-D antenna arrays can have larger design freedom, simplify feed network and reduce cost of antenna arrays [3-4]. In recent years, focus of research on 4-D antenna arrays are formation of ultralow sidelobe, pattern synthesis, adaptive beamforming, direction finding, beam control and so on [5-7].

DOA estimation is a major research direction in array signal processing. In recent years, study of DOA estimation for 4-D antenna arrays has attracted more attention [9]. Existing DOA estimation algorithm for 4-D antenna arrays has high DOA estimation precision. But it is necessary to know source number before DOA estimation. For the insufficiency, because DOA estimation algorithm based on sparse representation of covariance matrix can carry out accurate DOA estimation without knowing source number [10], this paper applies the algorithm to 4-D antenna arrays to realize DOA estimation algorithm for 4-D antenna arrays with unknown source number.

II. THEOREM

A. Sparse Representation Model

M far field signals are incident to N antenna element linear 4-D antenna arrays with equal distance. The harmonic signal of $2Q+1$ pathes (Q is harmonic orde) includes fundamental and harmonic components. So data matrix [8] of 4-D antenna arrays of $2Q+1$harmonic signals is:

$$\mathbf{Y}(l) = \mathbf{BX}(l) = \mathbf{B}\left[\mathbf{AS}(l) + \mathbf{N_0}(l)\right] = \mathbf{BAS}(l) + \mathbf{N}_1(l) \tag{1}$$

Sparse representation of array covariance matrix is an important step of L1-SRACV algorithm [9]. The sparse reconstruction optimization model obtained is:

$$\min_{\mathbf{C}} \left\|\overline{\mathbf{C}}\right\|_1 \quad s.t.\, \mathbf{R} = \mathbf{B}\overline{\mathbf{A}}\mathbf{C} + \sigma^2\mathbf{I}_{2Q+1} \tag{2}$$

The over complete basis $\overline{\mathbf{A}}$ determines angle interval $\left[\theta_1 \;\; \theta_2 \;\; \cdots \;\; \theta_P\right]$ of space meshing. When knowing covariance matrix $\mathbf{R}$ of 4-D antenna arrays received data and over complete $\overline{\mathbf{A}}$, reconstruction of sparse signal vector is $\overline{\mathbf{C}}$. Angle positions corresponding to former maximum reconfiguration value is angles of targets.

B. Principle of DOA Estimation

Because number of snapshots is limited, estimated covariance matrix is obtained by using sample covariance matrix $\mathbf{R}$ of multiple snapshots.

$$\mathbf{R} = E\left\{\frac{1}{L}\sum_{l=1}^{L}\mathbf{Y}(l)\mathbf{Y}(l)^H\right\} = \mathbf{R} + \Delta\mathbf{R} \tag{3}$$

Suppose $\mathbf{J}^{-1/2} = \sqrt{L}\mathbf{R}^{-T/2} \otimes \mathbf{R}^{-1/2}$, $\mathbf{R}^{-1/2}$ is inverse Hermitianmean square root:

$$\left\|\mathbf{J}^{-1/2} vec\left(\mathbf{R} - \mathbf{B}\overline{\mathbf{A}}\mathbf{C} - \sigma^2\mathbf{I}_{2Q+1}\right)\right\|_2^2 \sim As\chi_\rho^2\left(\left(2Q+1\right)^2\right) \tag{4}$$

The introduction of the parameter β makes the follow formula at a high probability:

$$\left\|\mathbf{J}^{-1/2} vec\left(\mathbf{R} - \mathbf{B}\overline{\mathbf{A}}\mathbf{C} - \sigma^2\mathbf{I}_{2Q+1}\right)\right\|_2^2 \le \beta \tag{5}$$

Therefore, (2) can be transformed to sparse reconstruction optimization model of DOA estimation based on sparse representation of covariance matrix with unknown source number of 4-D antenna arrays (TMAA-L1-SRACV algorithm):

$$\min_{\mathbf{C}} \left\|\bar{\mathbf{C}}\right\|_1 \quad s.t. \left\|\mathbf{Z}-\Psi vec\left(\mathbf{C}\right)\right\|_2^2 \le \beta \tag{6}$$

Where, $\mathbf{Z}=\mathbf{J}^{-1/2}vec\left(\mathbf{R}-\sigma^2\mathbf{I}_{2Q+1}\right)$. $\mathbf{\Psi}=\mathbf{J}^{-1/2}\left(\mathbf{I}_{2Q+1}\otimes\mathbf{B}\bar{\mathbf{A}}\right)$ indicates a new redundant dictionary. $\beta=\sqrt{As\chi_\rho^2\left(\left(2Q+1\right)^2\right)}$ is parameter determining noise.

III. Numerical Results

Four incoherent signals are from $-30°$, $-10°$, $15°$ and $25°$, SNR is 5dB, snapshots number is 150. TMAA-L1-SRACV algorithm can carry out DOA estimation without knowing source number in advance, but TMAA-MUSIC algorithm needs to know source number. For TMAA-MUSIC algorithm, 3, 4 sources are assumed respectively. Two DOA estimation algorithms is as follows:

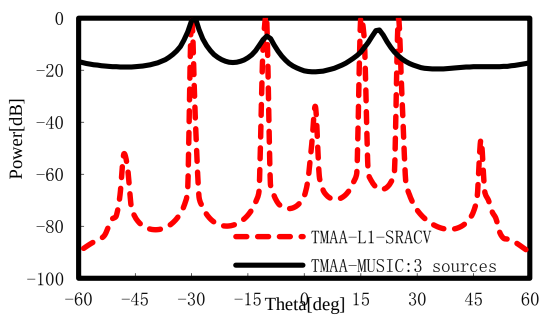

Fig. 2(a). 3 sources

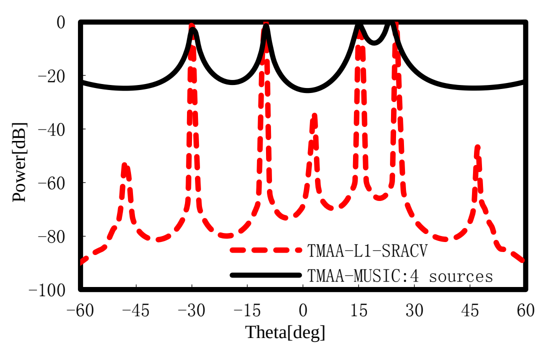

Fig. 2(b). 4 sources

Fig. 2. Spectrum of two DOA estimation algorithms for 4-D antenna arrays with different source number

From Figure 2, it shown that TMAA-MUSIC algorithm can estimate error of number of sources and error of sources angles when source number is inaccurate. But TMAA-L1-SRACV algorithm can still accurately estimate source number of sources and angle of sources when knowing wrong sources number.

IV. Conclusion

DOA estimation algorithm based on sparse representation of covariance matrix for 4-D antenna arrays with unknown source number is studied in this paper. The algorithm applies DOA estimation algorithm based on sparse representation of covariance matrix to 4-D dimensional antenna arrays, and solves the shortage of direction finding error when existing DOA estimation algorithms for 4-D antenna arrays do not know source number. The algorithm realizes accurate DOA estimation for 4-D antenna arrays under unknown source number, low SNR, closely spaced signals and signal coherence.

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