A Block Vectorized-Filling Method with Unified Quadrature

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***Abstract*—In the present paper, a novel block vectorized-filling method with unified quadrature is proposed for fast generating the impedance matrix. It is deduced from the unified quadrature on triangle cells in groups, in which basis- and testing- functions and Green's functions on triangle cells in groups are sampled and vectorized, hence the optimized efficiency of floating multiplies. As a result of grouping and vectorization, the impedance matrix is decomposed into several sub-matrices corresponding to the interactions between two groups, in which every sub-matrix of a group-pair is decomposed into some localized matrices including shared basis- and testing-matrices and independent Green's matrices. As a result of an optimization of both the numbers and the manners of floating multiplies, redundancies in matrix filling can be reduced, and the efficiency of the matrix generation can be improved.**

Keywords—MoM; RWG function; block vectorized-filling; floating multiply

I. INTRODUCTION

Although primary theories, kinds of modifications [1], and wide applications [2] have been studied, method of moments (MoM) and its matrix generations for most problems are still very time consuming. Their matrix may be deduced from methods like electric field integral equation (EFIE), magnetic field integral equation or current and charge integral equation which are discretized by RWG basis functions [3], pyramid-shaped functions [4] or higher order basis functions [5]. Among of the above solving methods, the method to solve the discretized EFIE with RWG functions is preferred [6]. However, this method and its primary matrix-filling are usually very time consuming.

To accelerate matrix fillings, many researchers contribute a lot. Some researchers accelerated it by using parallel technologies [5]; some replaced the RWG-RWG loop in matrix filling by the triangle-triangle loop [5]; some resolved every matrix element into several matrices for optimizing floating multiplies [7]; some utilized kinds of interpolation schemes [8]. Among them, two methods are very interesting. One is the triangle-triangle loop method. It is deduced from the linear basis- and testing- functions, in which the effects of triangles are extracted and shared. The other is the method of resolving elements into several matrices. It is deduced from the quadrature on triangle cells, in which elements are rebuilt by several matrix-vector multiplies. Although MoM can be improved, its matrix-filling is still very time consuming.

In the matrix-filling, there still are severe redundancies between different matrix elements. In order to reduce redundancy, it is also important to optimize quadrature operations between different triangle-pairs (a triangle-pair is with two triangle cells). However, after a depth investigation, we find that it is still lack of the quadrature optimization between different triangle-pairs. In order to reduce redundancies between different matrix elements, we propose a fast matrix filling method based on a quadrature optimization. The proposed method is a novel block vectorized-filling method with unified quadrature (BVFM-UQ). It is deduced from the unified quadrature operations on triangle cells in groups. By virtual of grouping and unified quadrature, the BVFM-UQ can globally improve the efficiency of matrix filling.

II. THE BLOCK VECTORIZED-FILLING METHOD

Before the proposed method, a common vectorized-filling method is provided as a base. In the common vectorized-filling method, both RWG and half-RWG functions are applied when the EFIE is discretized, hence the following matrix element

$$Z_{mn} = \iint \left[\mathbf{f_m} \cdot \mathbf{g_n} - \frac{\nabla \cdot \mathbf{f_m} \nabla' \cdot \mathbf{g_n}}{k^2} \right] g(\mathbf{r}, \mathbf{r}') \mathrm{d}s' \mathrm{d}s \qquad (1)$$

where k is the wavenumber and $g(\mathbf{r}, \mathbf{r}')$ is the Green's function. $\mathbf{g_n}$ and $\mathbf{f_m}$ are the n_{th} basis functions and the m_{th} testing functions, respectively. Both them can be either RWG functions or half-RWG functions. Since a RWG function consists of two half-RWG functions, the matrix element of Z deduced from RWG functions consists of four matrix elements of Z^{HR} deduced from half-RWG functions.

After the common vectorized filling method, the block vectorized-filling method with unified quadrature. The proposed BVFM-UQ is an approach to reduce waste, in which the above redundancy can be alleviated, and the above scattered multiplies can also be integrated. In the proposed method, it groups triangle cells and shares basis and testing vectors in the groups and integrates several matrix-vector multiplies into sparse matrix-matrix multiplies. We call its manner as group-group loop manner.

Firstly, some preparations are necessary, which include grouping and unifying. Grouping denotes the preparation including grouping triangle cell and identifying pairs of groups. All triangle cells are usually grouped by several congruent cubes. Unifying denotes the preparation that unifies quadrature rule for triangle cells in a group.

Secondly, decomposing matrix based on gathering is its kernel, which is different from the method of resolving matrix elements into matrices. The matrix decomposition is followed by gathered matrices, which integrates several small and scattered matrix-vector multiplies of a group-pair into sparse matrix-matrix multiplies as follows

$$\tilde{Z}^{\mathrm{HR}}_{[M,N]} = \begin{cases} \sum_{i=1}^{4} F_i^{[M]} K_{[M,N]} G_i^{[N]} \\ \sum_{i=1}^{4} F_i^{[M]} \left[K^{\mathrm{RT}}_{[M,N]} G_i^{[N]} + \left(KG\right)^{\mathrm{ST}}_{i,[M,N]} \right] \end{cases} \tag{2}$$

The proposed group-group loop manner also benefits from the simplification of shared matrices. In the simplification, $F_i^{[M]}$ and $G_i^{[N]}$ can be further decomposed as

$$F_i^{[M]} = R_F^{\mathrm{T}} Q_F^{\mathrm{T}} D_F^{\mathrm{T}} \tag{3}$$

and

$$G_i^{[N]} = D_G Q_G R_G \tag{4}$$

where $R_{F/G}$ and $Q_{F/G}$ are also highly sparse matrices, and $D_{F/G}$ are diagonal matrices. The mark `T' denotes the matrix transpose operator. The above decomposition can be demonstrated by the following example. For a group with 2 triangle cells ($\#1$ and $\#2$) supporting 6 half-RWG functions ($\{\#1, r_+^1\}$, $\{\#1, r_+^2\}$, $\{\#1, r_+^3\}$, $\{\#2, r_+^2\}$, $\{\#2, r_+^3\}$ and $\{\#2, r_+^4\}$) and 2-point quadrature rule used ($\{r_1, r_2\} \in \#1$, $\{r_3, r_4\} \in \#2$, $w_1 = w_3$, $w_2 = w_4$), those sparse matrices in (3) are

$$D_F = \mathrm{diag}\left(\begin{bmatrix} w_1 & w_2 & | & w_1 & w_2 \end{bmatrix}\right)$$

and

$$Q_F = \begin{bmatrix} r_1 & -1 & 0 & 0 \\ r_2 & -1 & 0 & 0 \\ 0 & 0 & r_3 & -1 \\ 0 & 0 & r_4 & -1 \end{bmatrix}$$

and

$$R_F = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ r_+^1 & r_+^2 & r_+^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & r_+^2 & r_+^3 & r_+^4 \end{bmatrix}$$

where the index i (=1,2,3,4) is omitted for a concise description. Generally speaking, the number of floating multiplies can be reduced by 1/3, which seems the effect of the triangle-triangle loop.

Finally, accumulating localized matrices of group-pairs is its end. We get the completed matrix of RWG functions by

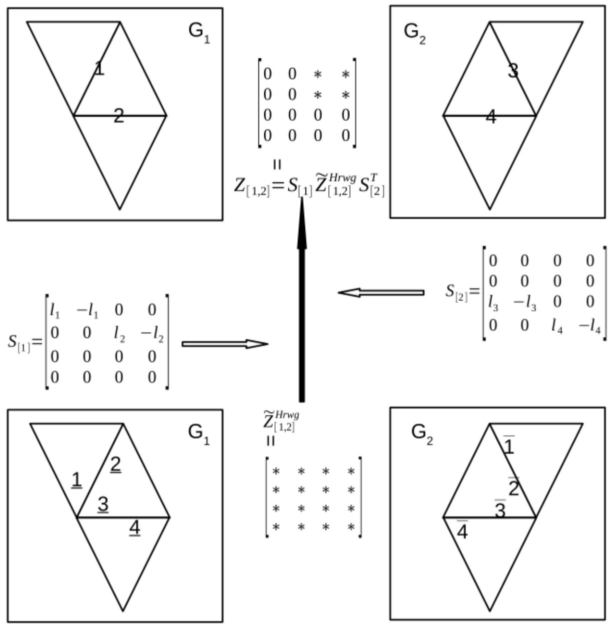

Fig. 1 Transforming localized matrix of group-pair (G_1 - G_2) to global matrix. Indexes 1,...,4 denote the global RWG functions, and $\underline{1},...,\underline{4}$ and $\bar{1},...,\bar{4}$ denote the localized Half-RWG functions in G_1 and G_2, respectively

accumulating all sub-matrices of half-RWG functions as

$$Z = \sum_{M,N} Z_{[M,N]} = \sum_{M,N} S_{[M]} \, \tilde{Z}^{\mathrm{HR}}_{[M,N]} S^{\mathrm{T}}_{[N]} \tag{5}$$

Sparse matrices $S_{[M]}$ and $S_{[N]}$ are sparse sub-matrices which map local indexes of half-RWG functions in each group to global indexes of RWG functions as shown in Fig. 1.

The proposed method has two important advantages. One is the optimizing floating multiplies. From (2)-(5), the BVFM-UQ optimizes the number of floating multiplies by shared matrices, and optimizes the manner of floating multiplies by matrix decompositions. The other is the matrix decomposition. The matrix decomposition extracts the effects of Green's function by cleaning effects of basis and testing functions, which can be combined with other fast methods. The latter of the above advantages has been demonstrated in the above, and the former will be discussed by the following example.

III. Numerical Examples

In order to demonstrate the above first advantage, the proposed BVFM-UQ is realized and validated by a benchmark. The BVFM-UQ is realized in GC++, developed based on GSL. Since it is natural parallelizable, the BVFM-UQ supporting the multi-thread is also developed. It is validated on a Lenovo W530 laptop with i7-3720QM CPU supporting 8 threads. The benchmark is simulated by the software named Feko.

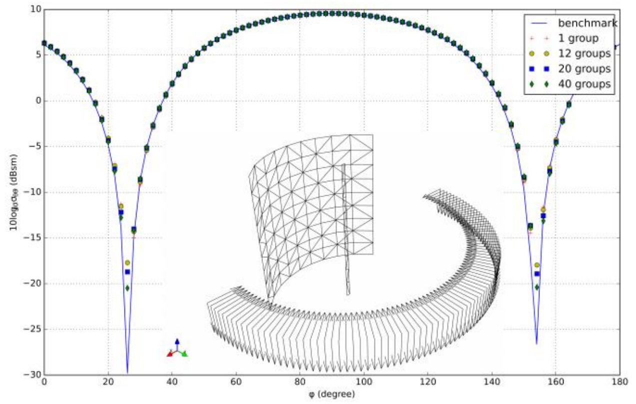

Fig. 2 Comparisons between results for a cylinder-backed dipole from the MoM with the BVFM-UQ of different grouping and the benchmark from the MoM solver in Feko. Azimuth angle varies from 0° to 180°, and pitching angle is 90°

For the benchmark, the accuracy of the proposed method can be validated as shown in Fig. 2, and the efficiencies can be demonstrated as shown in Table I. The benchmark is a typical scattering problem [2], which is a dipole backed with a cylinder, where the dipole is 2m length, the cylinder is with the radius of 1.25m and the height of 1m, and their distance is 1m. It is meshed by 120 nodes and 180 triangles. In Fig. 2, results are of mono-static radar-cross-section (RCS) related to incident waves scanned along the azimuth plane, which can be used for two comparisons. One comparison is between the results from the MoM with BVFM-UQ and from the traditional MoM (Feko), which suggests that the accuracy of the BVFM-UQ is acceptable. The other is among the results from the MoM with BVFM-UQ of different grouping, which suggests that the BVFM-UQ of different grouping is also acceptable. These different groupings are uniform divisions with element sizes of 2.3m, 0.9m, 0.6m and 0.4m, respectively. In Table I, consumed time (six threads available) including some real-time (running time) and system-time (CPU time) is listed, which can be used for determining the best grouping. Table I suggests that the efficiencies increase with grouping refinement, and the best grouping may be with element size of 0.6m.

Table I
CPU TIME OF THE BVFM-UQ WITH DIFFERENT GROUPING

size(m)/groups	2.3/1	0.9/12	0.6/20	0.4/40
real-time(sec)	8.6	1.7	0.8	0.7
system-time(sec)	8.6	5.6	3.6	3.6

IV. CONCLUSION

In this letter, a novel block vectorized-filling method with unified quadrature in groups is proposed for fast generating matrix in MoM. It is deduced from the unified quadrature operations on triangle cells in groups, in which basis and testing functions and Green's functions on triangle cells in groups are unified sampled and shared, and the efficiency of floating multiplies is optimized. The proposed method is not only an optimized filling method, but also a natural paralleling method for concurrently generating different sub-matrices of different group-pairs.

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