Transmission of arbitrary electromagnetic beam through uniaxial anisotropic cylinder

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***Abstract*—A semi-analytical solution is presented to the scattering of an arbitrary electromagnetic beam incident on a uniaxial anisotropic cylinder. The scattered fields as well as internal fields are expanded in terms of appropriate cylindrical vector wave functions, and the unknown expansion coefficients are determined by the boundary conditions and the projection method. Numerical results are provided for the normalized internal and near-surface field intensity distributions.**

Keywords—arbitrary electromagnetic beam; cylindrical vector wave function; boundary conditions; projection method

I. Introduction

Recent progress in light scattering has been the theory of interaction between an incident arbitrary electromagnetic (EM) beam and a homogeneous sphere [1-3], later extended to the case in which the scattering particle is a multilayered sphere [4], leading to many applications. Although such a theory allows us to deal with arbitrary-shaped beams, the most extensive effort has been devoted to Gaussian beams, for obvious practical reasons, insofar as laser beams used in the laboratory for experiments are at least approximately Gaussian. Nevertheless, other kinds of beams, such as laser sheets [5-8], and top-hat beams [9, 10] have been considered too. This paper is devoted to an extension in which the uniaxial anisotropic cylinder is illuminated by an arbitrary-shaped beam.

This paper is organized as follows. The descriptions of the scattered fields and internal fields are introduced in Section 2. Section 3 establishes a set of equations derived from boundary conditions at the surface of the cylinder and solves it for the expansion coefficients. In Section 4, the normalized internal and near-surface field intensity distributions are displayed.

II. Description of Scattered and Internal Fields in Cylindrical Coordinates

As shown in Fig.1, an infinite uniaxial anisotropic cylinder of radius r_0 is attached to the Cartesian coordinate system $Oxyz$. An incident arbitrary EM beam propagates in free space and the xOz plane. The angle between the propagation direction and the positive z axis is β. In this paper, the time-dependent part of the EM fields is assumed to be $\exp(-i\omega t)$.

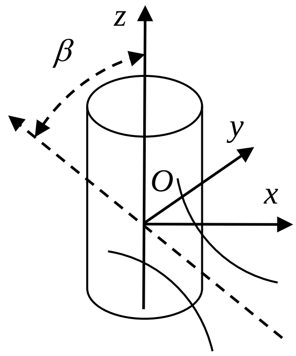


Fig.1. A uniaxial anisotropic cylinder illuminated by an arbitrary EM beam

The scattered fields can be expanded in terms of the cylindrical vector wave functions (CVWFs), as follows:

$$\mathbf{E}^s = E_0 \sum_{m=-\infty}^{\infty} \int_0^{\pi} [\alpha_m(\zeta)\mathbf{m}^{(3)}_{m\lambda} + \beta_m(\zeta)\mathbf{n}^{(3)}_{m\lambda}] e^{ihz} d\zeta \tag{1}$$

$$\mathbf{H}^s = -iE_0 \frac{1}{\eta} \sum_{m=-\infty}^{\infty} \int_0^{\pi} [\alpha_m(\zeta)\mathbf{n}^{(3)}_{m\lambda} + \beta_m(\zeta)\mathbf{m}^{(3)}_{m\lambda}] e^{ihz} d\zeta \tag{2}$$

Where $\lambda = k_0 \sin\zeta$, $h = k_0 \cos\zeta$, k_0 and η are respectively the free space wave number and wave impedance, and $\alpha_m(\zeta)$, $\beta_m(\zeta)$ are the unknown expansion coefficients to be determined.

The uniaxial anisotropic medium is characterized by a permittivity tensor $\bar{\varepsilon} = \hat{x}\hat{x}\varepsilon_t + \hat{y}\hat{y}\varepsilon_t + \hat{z}\hat{z}\varepsilon_z$ and a scalar permeability $\mu = \mu_0$. As described in [11], the EM fields within the uniaxial anisotropic cylinder (internal fields) can correspondingly be expanded as:

$$\mathbf{E}^w = E_0 \sum_{q=1}^{2} \sum_{m=-\infty}^{\infty} \int_0^{\pi} F_{mq}(\zeta)[\alpha_q^e(\zeta)\mathbf{m}^{(1)}_{m\lambda_q} + \beta_q^e(\zeta)\mathbf{n}^{(1)}_{m\lambda_q} + \gamma_q^e(\zeta)\mathbf{l}^{(1)}_{m\lambda_q}] e^{ihz} d\zeta \tag{3}$$

$$\mathbf{H}^w = -iE_0 \frac{1}{\eta} \sum_{q=1}^{2} \sum_{m=-\infty}^{\infty} \int_0^{\pi} \frac{k_q}{k_0} F_{mq}(\zeta)[\beta_q^e(\zeta)\mathbf{m}^{(1)}_{m\lambda_q} + \alpha_q^e(\zeta)\mathbf{n}^{(1)}_{m\lambda_q}] e^{ihz} d\zeta \tag{4}$$

where k_q , λ_q , $\alpha_q^e(\zeta)$, $\beta_q^e(\zeta)$ and $\gamma_q^e(\zeta)$ $(q=1,2)$ are described as in [11].

III. DETERMINATION OF EXPANSION COEFFICIETS

The unknown expansion coefficients $\alpha_m(\zeta)$, $\beta_m(\zeta)$ in (1), (2) as well as $F_{m1}(\zeta)$, $F_{m2}(\zeta)$ in (3), (4) can be determined by using the following boundary conditions

$$\left.\begin{aligned}\hat r\times(\mathbf{E}^s+\mathbf{E}^i)=\hat r\times\mathbf{E}^w\\ \hat r\times(\mathbf{H}^s+\mathbf{H}^i)=\hat r\times\mathbf{H}^w\end{aligned}\right\}\quad at \quad r=r_0 \tag{5}$$

where $\mathbf{E}^i$ and $\mathbf{H}^i$ denote the electric and magnetic fields of the incident EM beam, respectively, and $\hat r$ is the unit outward vector of the cylinder surface.

Substituting (1)-(4) into (5), the boundary conditions can be written as

$$\begin{aligned}&\hat r\times E_0\sum_{m=-\infty}^{\infty}\int_0^{\pi}[\alpha_m(\zeta)\mathbf{m}^{(3)}_{m\lambda}+\beta_m(\zeta)\mathbf{n}^{(3)}_{m\lambda}]e^{ihz}d\zeta+\hat r\times\mathbf{E}^i\Big|_{r=r_0}\\ &=\hat r\times E_0\sum_{q=1}^{2}\sum_{m=-\infty}^{\infty}\int_0^{\pi}F_{mq}(\zeta)[\alpha_q^e(\zeta)\mathbf{m}^{(1)}_{m\lambda_q}+\beta_q^e(\zeta)\mathbf{n}^{(1)}_{m\lambda_q}+\gamma_q^e(\zeta)\mathbf{l}^{(1)}_{m\lambda_q}]e^{ihz}d\zeta\end{aligned} \tag{6}$$

$$\begin{aligned}&\hat r\times E_0\sum_{m=-\infty}^{\infty}\int_0^{\pi}[\alpha_m(\zeta)\mathbf{n}^{(3)}_{m\lambda}+\beta_m(\zeta)\mathbf{m}^{(3)}_{m\lambda}]e^{ihz}d\zeta+\hat r\times i\eta\mathbf{H}^i\Big|_{r=r_0}\\ &=\hat r\times E_0\sum_{q=1}^{2}\sum_{m=-\infty}^{\infty}\int_0^{\pi}\frac{k_q}{k_0}F_{mq}(\zeta)[\beta_q^e(\zeta)(\zeta)\mathbf{m}^{(1)}_{m\lambda_q}+\alpha_q^e(\zeta)\mathbf{n}^{(1)}_{m\lambda_q}]e^{ihz}d\zeta\end{aligned} \tag{7}$$

Equations (6) and (7) are multiplied by $\hat z e^{-ih_1 z}e^{-im'\varphi}$ and $\hat\varphi e^{-ih_1 z}e^{-im'\varphi}$ respectively (dot product) and then integrated over the cylindrical surface. By assuming the incident EM fields $\mathbf{E}^i$ and $\mathbf{H}^i$ to be known, the following equations can be obtained.

$$\begin{aligned}&\xi\frac{d}{d\xi}H_m^{(1)}(\xi)\alpha_m(\zeta)+\frac{hm}{k_0}H_m^{(1)}(\xi)\beta_m(\zeta)\\ &-F_{m1}(\zeta)\xi_1\frac{d}{d\xi_1}J_m(\xi_1)-F_{m2}(\zeta)[\beta_2^e(\zeta)\frac{hm}{k_2}J_m(\xi_2)\\ &-\gamma_2^e(\zeta)imJ_m(\xi_2)]=(\frac{1}{2\pi})^2\frac{1}{E_0}\xi\int_{-\infty}^{\infty}dz\int_0^{2\pi}\hat r\times\mathbf{E}^i\cdot\hat z e^{-im\varphi}e^{-ihz}d\varphi\end{aligned} \tag{8}$$

$$\begin{aligned}&\xi^2H_m^{(1)}(\xi)\beta_m(\zeta)-F_{m2}(\zeta)\frac{k_0}{k_2}\xi_2^2J_m(\xi_2)[\beta_2^e(\zeta)+\gamma_2^e(\zeta)\frac{ihk_2}{\lambda_2^2}]\\ &=(\frac{1}{2\pi})^2\frac{1}{E_0}(k_0r_0)^2\sin\zeta\int_{-\infty}^{\infty}dz\int_0^{2\pi}\hat r\times\mathbf{E}^i\cdot\hat\varphi e^{-im\varphi}e^{-ihz}d\varphi\end{aligned} \tag{9}$$

$$\begin{aligned}&\frac{hm}{k_0}H_m^{(1)}(\xi)\alpha_m(\zeta)+\xi\frac{d}{d\xi}H_m^{(1)}(\xi)\beta_m(\zeta)\\ &-\frac{hm}{k_0}F_{m1}(\zeta)J_m(\xi_1)-\frac{k_2}{k_0}F_{m2}(\zeta)\beta_2^e(\zeta)\xi_2\frac{d}{d\xi_2}J_m(\xi_2)\\ &=i\eta\frac{1}{E_0}(\frac{1}{2\pi})^2\xi\int_{-\infty}^{\infty}dz\int_0^{2\pi}\hat r\times\mathbf{H}^i\cdot\hat z e^{-im\varphi}e^{-ihz}d\varphi\end{aligned} \tag{10}$$

$$\begin{aligned}&\xi^2H_m^{(1)}(\xi)\alpha_m(\zeta)-F_{m1}(\zeta)\xi_1^2J_m(\xi_1)\\ &=i\eta\frac{1}{E_0}(\frac{1}{2\pi})^2(k_0r_0)^2\sin\zeta\int_{-\infty}^{\infty}dz\int_0^{2\pi}\hat r\times\mathbf{H}^i\cdot\hat\varphi e^{-im\varphi}e^{-ihz}d\varphi\end{aligned} \tag{11}$$

where $\xi=\lambda r_0$ and $\xi_q=\lambda_q r_0$ $(q=1,2)$.

Equations (8) - (11) provide a set of simultaneous linear algebraic equations, which can be used to solve for the unknown expansion coefficients. The surface integrals involving $\mathbf{E}^i$ and $\mathbf{H}^i$ are numerically evaluated by Simpson's $1/3$ rule in our calculations.

IV. NUMERICAL RESULTS

In this paper, we will focus our attention on the normalized internal and near-surface field intensity distributions, which are described, respectively, by

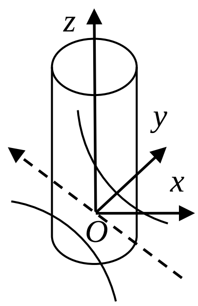
$$\left|\mathbf{E}^w/E_0\right|^2=\left|E_r^w\right|^2+\left|E_\varphi^w\right|^2+\left|E_z^w\right|^2 \tag{12}$$

and

$$\left|(\mathbf{E}^i+\mathbf{E}^s)/E_0\right|^2=\left|E_r^i+E_r^s\right|^2+\left|E_\varphi^i+E_\varphi^s\right|^2+\left|E_z^i+E_z^s\right|^2 \tag{13}$$

A. On-axis Guassian beam

Figure (2) shows the normalized internal and near-surface field intensity distributions $\left|\mathbf{E}^w/E_0\right|^2$ and $\left|(\mathbf{E}^i+\mathbf{E}^s)/E_0\right|^2$ in the xOz plane for a uniaxial anisotropic cylinder, illuminated by a TE polarized Gaussian beam following the Davis first-order approximation in [12]. The radius of the cylinder r_0 and the Gaussian beam waist radius are assumed to be five and two times the wavelength (λ_0) of the incident Gaussian beam. The beam waist middle coincides with origin O , and $\omega\sqrt{\varepsilon_z\mu_0}=\sqrt{2}k_0$, $\omega\sqrt{\varepsilon_t\mu_0}=\sqrt{3}k_0$, $\omega\sqrt{\varepsilon_z\mu_0}=\sqrt{2}k_0$.



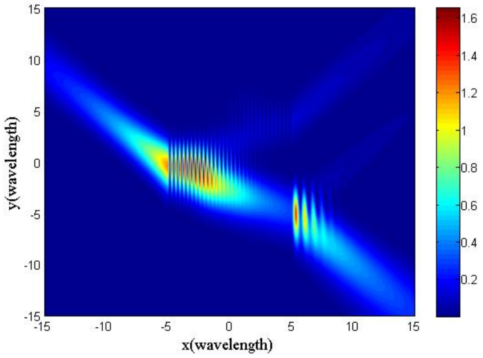


Fig.2. $\left|\mathbf{E}^w/E_0\right|^2$ and $\left|(\mathbf{E}^i+\mathbf{E}^s)/E_0\right|^2$ for a uniaxial anisotropic cylinder illuminated by a TE polarized Gaussian beam

By comparing figure (2) and the results in [11], an excellent agreement is achieved, which to a great extent verifies our solutions and results.

B. Hertzian electric dipole

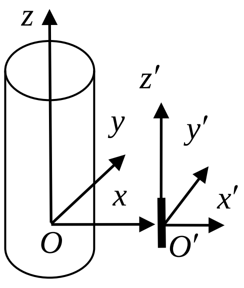
The radiated EM fields due to a Hertzian electric dipole (HED) pointing in the z' direction and located at origin O' can be written as [13]

$$\mathbf{E}^i(\mathbf{r}') = E_0[\hat{z}' + \frac{1}{k_0^2}\nabla'\frac{\partial}{\partial z'}]\frac{e^{ik_0 r'}}{k_0 r'} \tag{14}$$

$$\mathbf{H}^i(\mathbf{r}') = -i\frac{E_0}{\eta_0}\frac{1}{k_0}\nabla'\frac{e^{ik_0 r'}}{k_0 r'}\times\hat{z}' \tag{15}$$

where $r' = \sqrt{x'^2+y'^2+z'^2}$, $E_0 = i\omega\mu Il\frac{k_0}{4\pi}$, and the prime indicates that the EM fields are described in the shaped beam coordinate system $O'x'y'z'$.

Figures (3) show the normalized internal and near-surface field intensity distributions $\left|\mathbf{E}^w/E_0\right|^2$ and $\left|(\mathbf{E}^i+\mathbf{E}^s)/E_0\right|^2$ in the xOz plane for a uniaxial anisotropic cylinder, illuminated by the (HED) radiation. The radius of the cylinder r_0 is assumed to be 1.2 times the incident wavelength (λ_0), and origin O has the coordinates $(-2\lambda_0, 0, 0)$ in $O'x'y'z'$, $\beta = 0$, $\omega\sqrt{\varepsilon_t\mu_0} = \sqrt{3}k_0$, $\omega\sqrt{\varepsilon_z\mu_0} = \sqrt{2}k_0$.



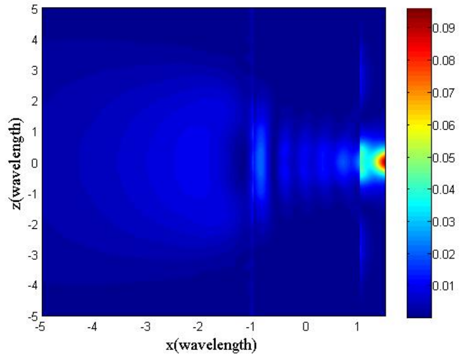


Fig.3. $\left|\mathbf{E}^w/E_0\right|^2$ and $\left|(\mathbf{E}^i+\mathbf{E}^s)/E_0\right|^2$ for a uniaxial anisotropic cylinder illuminated by the HED radiation.

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