Novel Strategies Based on Domain Decomposition Method for Electromagnetic Problem Analysis*

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***Abstract*—We present a high-performance program for parallel analysis of electromagnetic problems. This program is intended to perform large-scale numerical simulation of electromagnetic radiation, propagation and coupling based on Finite Element Method (FEM) and its advanced acceleration techniques. A double-level parallel scheme based on an advanced infrastructure is used to enable the parallelization scale to tens of thousands of CPU cores. In this paper, we demonstrate the architecture and main features of the program. The Domain Decomposition Method (DDM), *p*-adaptive strategy and fast frequency sweep technique are discussed as well. Several numerical examples are presented to demonstrate its capability in Electromagnetic applications.**

Keywords—parallel computing; domain decomposition method (DDM); finite element method (FEM); supercomputer

I. Introduction

During the last decade, the emergence of parallel computing architectures resulted in a great boost of the electromagnetic simulation performance [1]-[3]. Therefore, we have the opportunity to address various electromagnetic environmental effects (E3) of more complex geometrically and larger electrically with high accuracy and efficient time. This has put finite element method (FEM) to its limits because it always faces itself with a highly ill-conditioning, and possibly indefinite stiff matrix under such circumstance.

Fortunately, the introduction of Domain Decomposition Method (DDM) provides an alternative way to overcome this bottleneck [4], [5]. Moreover, extra *p*-refinement becomes essential to counter numerical dispersions, where *p*-type multiplicative Schwartz (pMUS) method can be adopted to exploit the multilevel feature of the reduced systems [6], even with mixed–order meshes. However, when large-scale parallel computing gets involved, *p*-refinement will always bring up a new challenge that workload balances can be deteriorated, along with obstacles due to repeated solving of the problem, where multilevel method can also degrade when dealing with hundreds of millions of unknowns at the coarsest level. Thus, it is intuitive to combine DDM and *p*-refinement when performing large-scale parallel computing, In which every *p*-adaptive step can be viewed as a coarser grid compared with the next refined system and used to perform dynamic coarse-level correction.

In this paper, we present the main features of a high-performance program JEMS-FD (J ElectroMagnetic Solver - Frequency Domain) and its advanced technologies. It is based on an advanced infrastructure JAUMIN (J parallel Adaptive Unstructured Mesh applications Infrastructure) [7]. We also demonstrate that with a careful redesign of the algorithms, *p*-adaption results can be achieved quickly and efficiently based on DDM and multilevel method integrated in JEMS-FD, even confronting hundreds of millions of unknowns.

II. Mathematical Implementation

We start by considering the time-harmonic propagation of an electrical wave in an open region Ω. Proper boundary conditions are set on $\partial\Omega$ in order to solve this problem with FEM, where a Silver-Müller radiation condition is used to truncate the infinite domain. This leads to the following problem:

$$\begin{cases} \nabla\times\left(\mu_r^{-1}\nabla\times\mathbf{E}\right)-k_0^2\varepsilon_r\mathbf{E}=-j\omega\mu_0\mathbf{J} & \text{in } \Omega \\ \gamma_{\mathrm{t}}\left(\mathbf{E}\right)=0 & \text{on } \partial\Omega\cap\Gamma_{\mathrm{PEC}} \\ \gamma_{\mathrm{t}}\left(\mu_r^{-1}\nabla\times\mathbf{E}\right)+jk\gamma_{\mathrm{T}}\left(\mathbf{E}\right)=0 & \text{on } \partial\Omega\cap\Gamma_{\infty} \end{cases} \tag{1}$$

where k is the wavenumber, j is the imaginary unit, and the tangential trace and the tangential component trace operators for any vector $\mathbf{v}$ are given by $\gamma_{\mathrm{T}}\left(\mathbf{v}\right):\mathbf{v}\mapsto\mathbf{n}\times\mathbf{v}\times\mathbf{n}$ and $\gamma_{\mathrm{t}}\left(\mathbf{v}\right):\mathbf{v}\mapsto\mathbf{n}\times\mathbf{v}$, where $\mathbf{n}$ is the unit vector outwardly oriented normal to Ω. For the implementation of DDM, a general Schwarz algorithm for two subdomains $\Omega=\Omega_1\cup\Omega_2$ then solves iteratively for subdomain problems. We consider Robin transmission boundary condition between subdomains to make the subdomain problems well-posed, as

$$\begin{aligned} \mathrm{T}_{\mathbf{n}}\left(\mathbf{E}\right)&=\left(\mathbf{I}+\left(\delta_1 S_{\mathrm{TM}}+\delta_2 S_{\mathrm{TE}}\right)\right)\left(\gamma_{\mathrm{t}}\left(\nabla\times\mathbf{E}\right)\right) \\ &\quad -j\omega\sqrt{\varepsilon\mu}\left(\mathbf{I}-\left(\delta_3 S_{\mathrm{TM}}+\delta_4 S_{\mathrm{TE}}\right)\right)\left(\gamma_{\mathrm{T}}\left(\mathbf{E}\right)\right) \text{ on } \Gamma \end{aligned} \tag{2}$$

where $S_{TM}=\nabla_\tau\nabla_\tau\cdot$, $S_{TE}=\nabla_\tau\times\nabla_\tau\times$, τ denotes the tangential direction, and δ_l ($l=1$, 2, 3, and 4) are the parameters for different transmission conditions. Using the FEM discretization, the matrix equation resulted from above problem can be written in a compact form, for the case of two subdomains, as

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{A}_2 \end{bmatrix}\begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix}=\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} \tag{3}$$

where the matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ are subdomain matrices and $\mathbf{C}_{12}$ and $\mathbf{C}_{21}$ are interface coupling matrices.

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III. Massively Parallel Infrastructure And Advanced Technologies

As illustrated in Fig. 1, to make sure that parallel processes in JEMS-FD do equal amounts of work, spatial locality is addressed with dynamic scheduling, by defining a minimum number of the successive subdomains assigned to each process. To do this, JAUMIN uses the MPI parallel programming scheme. And the dynamic scheduling of patches is updated at runtime after every iteration. During this procedure, every process spawns a series of threads mapped into the CPU cores to execute sub-linear system formation and then solve it. To perform this task, JAUMIN uses the OpenMP API. The multi-processes and multi-threads parallel strategy of the proposed algoritm integrated into the JAUMIN can be seen in the shaded blocks in Fig. 1.

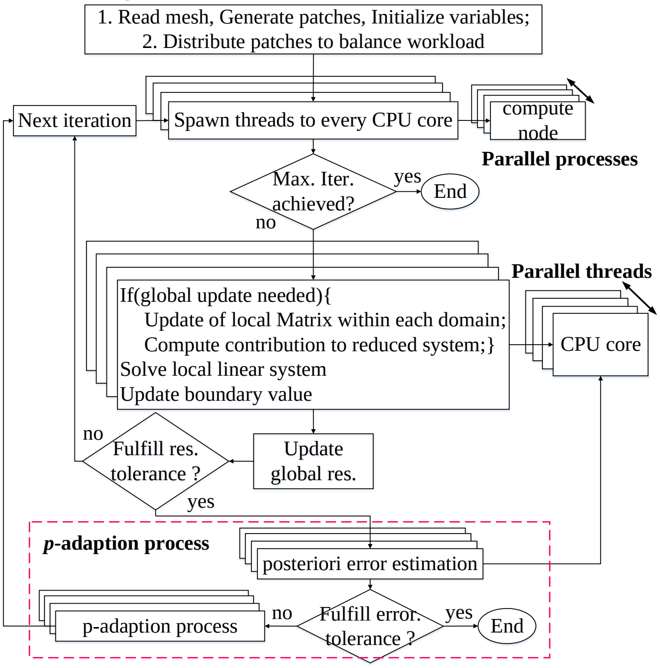


Fig. 1 Proposed parallel algorithm of DDM integrated with JAUMIN

P-adaption refinement in JEMS-FD is proceeded once DDM iteration criterion is achieved. It was suggested in ref. [6] that the solution of the lowest level can be used as a preconditioner to accelerate the solving at subsequent adaptive steps. What is different here is that every adaptive result is used to define the coarse-level correction, since each adaptive step can be viewed as *p*-type coarse-level compared with the next more refined system. We advance this step by updating the boundary value defined on subdomain boundary, then solving the inner values using DDM method.

The state of the art in multilevel solving strategy is exemplified by hybrid domain decomposition – spectral – algebraic multigrid method. As shown in Fig.2, it exploits the hierarchical feature of the sub-domain system through spectral coarsening, and at last employs the AMG solver BoomerAMG from the parallel High Performance Preconditioners (Hypre) library to solve the coarsest level system. This multilevel method would make the optimal efficiency in terms of solving time in parallel computing and is verified in subsequently reported performance results.

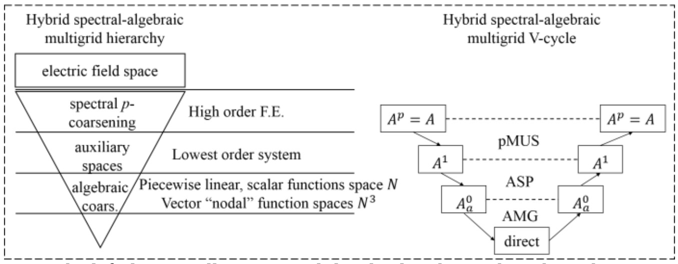


Fig.2 The left diagram illustrates multilevel solver hierarchy. The right diagram shows the multigrid V-cycle consisting of smoothing at each level of the hierarchy (square) and inter-grid transfer operators (arrows downward for restriction and arrows upward for prolongation).

IV. Numerical Experiments

A. *Surface Current Distribution of Large Platform*

We first simulate the surface current distribution of a large ship platform in the presence of an incident plane wave. The simulation is performed at 0.1GHz, with the plane wave imposed at the incident angle of 45 degree and the azimuthal angel of 0 degree. The size of the large-scale perfectly electric conductor (PEC) ship is $130.8\text{m}\times 20\text{m}\times 23.1\text{m}$, leading to the highly non-uniform 216,074,000 tetrahedral grids. The visualization of surface current distribution is plotted in Fig. 3, and the parallel information gathered from TianHe-II supercomputer is shown in Fig. 4.

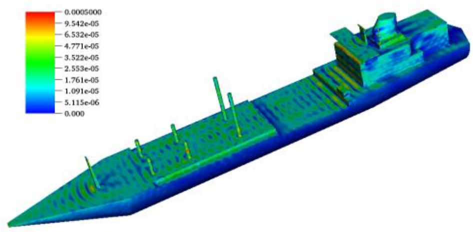


Fig. 3 Visualization of the surface current (A/m) over the ship surface

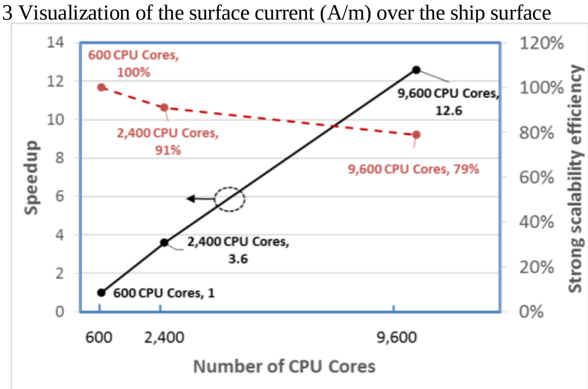


Fig. 4 Scalability experiments on the TianHe-II supercomputer.

B. *Fast Frequency Sweep based on DDM*

Our practically numerical experiment is exemplified by shielding effectiveness analysis of a metallic rectangular cavity, with geometrical details such as many apertures and slots on its wall. This test was done on platform 1 by initiating 96 threads on 96 CPU cores. The simulation was performed from 0.3GHz to 2.5GHz with a z-direction, x-polarization incident plane wave. The number of tetrahedral grids increases from 2 million to 6 million with the frequency sweeping. The fast frequency sweep technique is acquired with the aid of DDM in JEMS-FD instead of traditional asymptotic waveform evaluation (AWE)[8]. Specifically, we can calculate the field at a particular frequency point by expanding the boundary coupling value of DDM instead of the solution itself. And iteration will converge quickly using DDM iterations. The results of simulations and measurement are plotted in Fig. 5 for comparison. The current distribution over the surface is also captured in Fig. 6 for visualization.

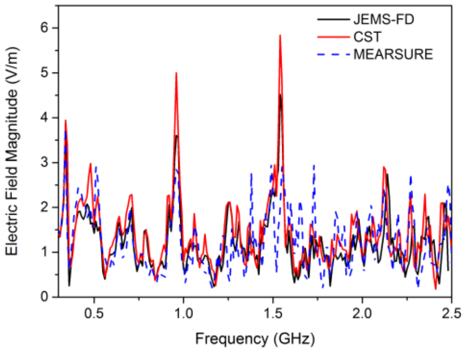
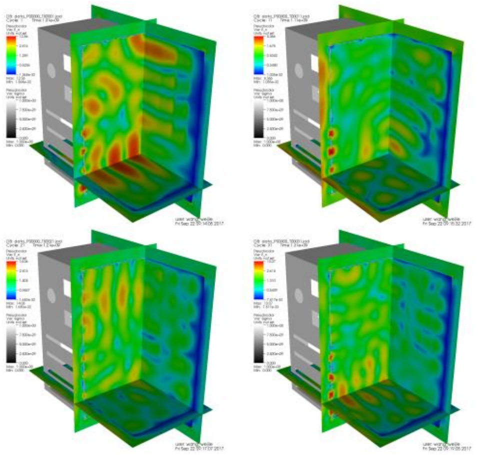


Fig.5 Calculated results of missile for different simulations.



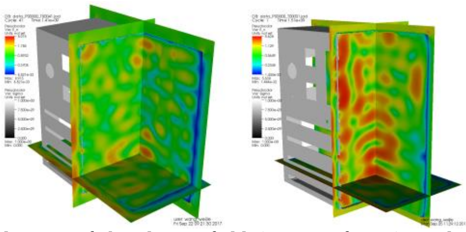


Fig.6 Visualization of the electric field (in unit of V/m) inside the metallic rectangular cavity at frequency of 1.0GHz, 1.1GHz, 1.2GHz, 1.3GHz, 1.4GHz, 1.5GHz, respectively.

C. *Adaptive Numerical Analysis of Large Platform*

Our last numerical example is to test the scalability of our program for the above ship platform with hundreds of millions of unknowns. The simulation was done on TianHe-II. We initiated 450 processes on 450 compute node and 10,800 threads on 10,800 CPU cores for simulation frequency 0.5GHz, with its size approximated by $218\lambda\times33.5\lambda\times37.5\lambda$. The computational statistics are summarized in Table I. An iteration number of 43 for DDM iteration and 81 for *p*-adaptive refinement were obtained as shown in Fig. 7, which indicates the robustness of the proposed algorithms for large-scale electromagnetic problems simulations. The quick convergence after each adaptive step indicates the efficiency of the *p*-type coarse-level correction.

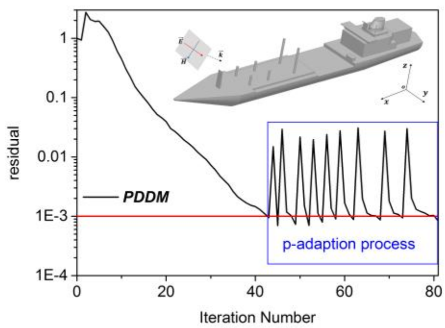


Fig.7 Iteration process for large-scale adaptive simulation.

TABLE I

SIMULATION SUMMARIZATIONS FOR THE CASE OF A LARGE-SCALE PEC SHIP

Frequency (GHz)	Number of Grids	Number of Compute Nodes/ CPU Cores	Iterations of DDM for Initial/Last Adaptive System	Number of Unknowns in the Initial System	Number of Unknowns in the Last Adaptive System	Adaptive Simulation Time(Sec.)
0.5	259,152,000	450/10,800	43/81 (*res.* = 1.0e-3)	328,745,000	521,655,000	27,052.7

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