A Hybrid Algorithm Based on FDTD and Spatially filtered FDTD methods for Multiscale Problem

Guoda Xie
Key Laboratory of Intelligent Computing & Signal Processing, Ministry of Education, Anhui University,
Hefei,230039 China

Zhixiang Huang
Key Laboratory of Intelligent Computing & Signal Processing, Ministry of Education, Anhui University,
Hefei ,China zxhuang@ahu.edu.cn

Bo Wu
Key Laboratory of Intelligent Computing & Signal Processing, Ministry of Education, Anhui University,
Hefei ,China xlwu@ahu.edu.cn

Xianliang Wu
Key Laboratory of Intelligent Computing & Signal Processing, Ministry of Education, Anhui University,
Hefei,China startyz@263.net

***Abstract*- A new hybrid technique based on the traditional finite-difference time-domain (FDTD) algorithm and the spatially filtered FDTD (SF-FDTD) method is developed to analyse 2-D multiscale problem. The FDTD method is used to calculate the electromagnetic fields in coarse grid region, and the SF-FDTD method is utilized in the fine grid region. Due to the controllable stability characteristic of SF-FDTD method, a uniform time step can be chosen to the maximum discrete time step corresponding to the coarse grid in the computation domain. Therefore, the temporal interpolation is no long needed at the boundary of the two grids. In addition, by adjusting the field components distribution in the grid and only filtering electric field components, the effect of the filter operation on the interface between the coarse and fine grids is eliminated, ensuring the precision of the proposed method. The accuracy and efficiency of the proposed method was verified by comparing with the standard FDTD method with the fine grids.**

Keywords- hybrid technique; traditional finite-difference time-domain (FDTD); spatially filtered (SF); multiscale problem

I. INTRODUCTION

Duo to the explicit finite difference characteristic, the finite-difference time-domain (FDTD) algorithm need to satisfy the Courant-Friedrich-Levy (CFL) stability condition [1-2]. The maximum time step is limited by the size of the minimum cell grid in calculation area. In order to accurately describe the electromagnetic behavior of the models with multiscale features, the grid size must be very small compared to the operating wavelength. The fine grids decrease the time step size, accordingly, huge memory requirement and CPU time consumption are needed in FDTD simulation. To address this question, subgridding technique has been an alternative way to solve multiscale problems[3-4]. However, in previous research, the classical FDTD method is applied to calculate both the fine and coarse grid regions, both spatial and temporal interpolation are needed at the interface of the two grids. To synchronize the time step in the coarse and fine grid regions, a set of classical implicit unconditionally or weakly conditionally stable FDTD methods, for instance, alternating direction implicit FDTD (ADI-FDTD) algorithm [5], weakly conditionally stable FDTD (WCS-FDTD) method[6], were applied in the fine grid regions[7-8]. As these implicit methods are based on a time split scheme, the derivations of iterative formulas are complex. It is difficult to use these implicit methods in subgridding technology. Recently, a spatially filtered FDTD (SF-FDTD) method is introduced [9]. The principle of the SF-FDTD method is to filter out unstable high frequency harmonics generated in the numerical system due to the selected time step beyond the CFL condition. This algorithm remains the iterative formulas of the traditional FDTD method, and only needs to add a filtering operation in each time-marching loop. Therefore, the SF-FDTD algorithm provides a alternative method for computing the electromagnetic fields in the fine grid area. However, the precision of the numerical results will be destroyed if SF-FDTD method is directly used in subgridding technology. As the fine grid points on the boundary always correspond to high spatial frequency spectrum in frequency domain, If the SF-FDTD method is used to compute the field components in fine grid region, these useful information will be filtered out together with unstable high-frequency components. And finally, the lost information will effect the precision of the whole simulation results.

According to the problem exits in the application of SF-FDTD in multi-scale issue, a simple and effective hybrid spatially filtered subgridding technology is presented. By adjusting the electromagnetic fields distribution in the calculation area and only filtering electric field components, the effect of filter operation on the boundary between the two regions is avoided. The details of the implementation will be given latter.

II. HYBRID SF-FDTD SUBGRIDDING TECHNOLOGY

A. The Theory of Spatially Filtered FDTD Method

The CFL stability condition in the FDTD method requires that all spatial harmonics remain stable in each iteration. However, literature [9] states that only a minority of them have an effect on numerical calculations. The well known numerical dispersion of FDTD method specified that the number of points sampled at a minimum wavelength is no less than 10 ($\Delta<\lambda_{min}/10$), which means the valuable part of the spatial frequency spectrum of a simulated signal in FDTD is within the range of $\boldsymbol{k}\Delta<0.2\pi$. The rest of the wavenumbers is weakly excited. But, the amplitude of these high space harmonics will increase rapidly and cause divergence when the time-step beyond the CFL limit. By bounding the numerical wavenumber $\boldsymbol{k}\Delta<\boldsymbol{k}_{max}$, the new N-D (N=1,2,3) stability limit of the FDTD can be obtained

$$\Delta t \le \frac{\Delta}{\sqrt{N}c} \times \left(\sin \frac{\boldsymbol{k}_{max}\Delta}{2\sqrt{N}} \right)^{-1} = \Delta t_{CFL-ND} \times CE \tag{1}$$

$$CE = 1 \Big/ \sin \frac{\boldsymbol{k}_{max}\Delta}{2\sqrt{N}} \tag{2}$$

where $\Delta t_{CFL\text{-}ND}$ is the standard N dimensional (N=1,2,3) CFL condition (assuming Δx=Δy=Δ) and CE denotes a CFL extension factor. Once CE is determined, $\boldsymbol{k}_{max}$ can be obtained from equation (2). Taking a example of two dimensional case , a circular low-pass spatial filter is then defined as

$$F(\boldsymbol{k}) = \begin{cases} 1, & \sqrt{\boldsymbol{k}_x^2 + \boldsymbol{k}_y^2} \le \boldsymbol{k}_{max} \\ 0, & otherwise \end{cases} \tag{3}$$

The number 0 implies that all the unstable space harmonics should be completely discarded, because even a small portion of them will grow exponentially and cause late-time instability. As the filter worked in spatial frequency domain, the field components should be transformed to spatial frequency domain, then applying this circular low-pass filter to filter out unstable high frequency components, and finally transforming all the fields from the frequency to the spatial domain.

B. Implementation of subgridding technology

In subgrid region, the electromagnetic fields distribution are the same with that of the coarse grid region. Fig.1 shows the arrangement of the field components for the 2D TMz mode. The electric fields (**E** and **e**) was positioned at the center of the grid, and the magnetic fields (**H** and **h**) located at the edges of the grids. The coarse grid magnetic field components are marked as black arrow, and the red arrows represent the fine grid magnetic fields. The circles and black dots denote the coarse and fine grid electric fields respectively, and the collocated electric field point was marked as a circle with a dot. For simplicity, the ratio of coarse to fine grid R=3 is chosen. The odd ratio can simplify the interpolation operation, since more collocated fields could be acquired in the computation domain, reduces the number of points need to be interpolated on the boundary of the two grids.

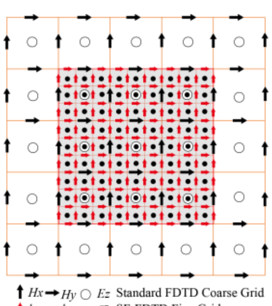

Fig.1. The field components in the fine and coarse grids

The exchange of electromagnetic fields value on the boundary or nearby points between the main coarse grids and the fine grids is very important in subgridding technology. In this paper. The exchange method is shown in Fig.2, the magnetic fields value H_x, H_y (black arrow) at the center of the coarse grid sides are transmitted to the fine grid magnetic field points which located at the same location. The non-collocated fine grid magnetic fields h_x, h_y (red arrow) required for fine grid computation can be obtained using a first order linear interpolation scheme. In order to fed back the fine grids information to the coarse grids, the collocated fine grids ez fields which are in one and a half of a coarse grid into the fine grid are passed to the coarse grid. The detailed interpolation process can be seen in Fig.3. Take the x direction magnetic field as an example, according to the ratio of coarse to fine grids in this letter, we can get

$$h_1 = 2/3 \times H_2 + 1/3 \times H_5 \tag{7}$$

$$h_2 = 1/3 \times H_2 + 2/3 \times H_5 \tag{8}$$

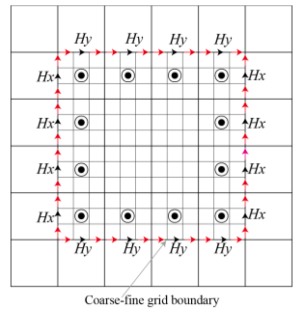

Fig.2. The transfer mode of fields value between on coarse and fine grid

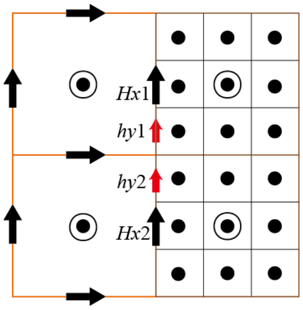

Fig.3. Sketch map of line interpolation

To synchronize the time-step of SF-FDTD method with that of the maximum temporal increment used in traditional FDTD, CE is selected to 3, correspond to the ratio of coarse to fine grid (R=3). According to equation (2), the wavenumber $\boldsymbol{k}_{max}$ can be determined, then a circular low-pass spatial filter is then defined. From the previous introduction we know that all the fine grid magnetic points located on the interface between coarse and fine grids are obtained by interpolation and assignment operation. These magnetic points often correspond to high spatial frequency spectrum in frequency domain. If filter operation is applied on fine grid magnetic field components, these useful information will be filter out together with some unstable high frequency component by the high frequency filter. Since all the electric field values in the fine grid region can be obtained by the iterative computations without interpolation processes. Therefore, all the tangential electric field components at the interface between coarse and fine grids are left intact if filter operations are applied only to the electric field components e_z. Finally, the spatial interpolation manipulation in the subgridding technology will not be disturbed by filter operations when the SF-FDTD algorithm is used in the fine grid region. In summary, the whole calculation process is given as follow:

1) Update the electric fields E_z in the entire coarse gid region.
2) Apply the interpolation technique, obtain the noncollocated fine magnetic field (h_x,h_y) values on the interface between the

coarse and fine grids, then assign the collocated coarse magnetic field (H_x,H_y) value on the interface to the corresponding fine magnetic field components (h_x,h_y).
3) Update the electric field components (e_z) and magnetic field components (h_x,h_y) in fine grid region with SF-FDTD method.
4) Transfer the collocated fine electric fields (e_z) which are in one and a half of a coarse grid into the fine grid to the corresponding collocated coarse magnetic fields (H_x,H_y).
5) Update the magnetic field components (Hx,H_y) of the entire area of the coarse grid.

III. NUMERICAL EXAMPLES

To verify the accuracy and efficiency of the proposed method, a 2-D metal waveguide model is being simulated on the *x-y* plane, as show in Fig. 4. The size of the entire area has been marked in this model. The solid line in the model represents the PEC which has a 0.9m slit. A small area contains slit is finely divided, the rest part is divided by coarse grid. The size of subgrid region is 1.2m×1.2m, the fine grid size is 0.01m, and the ratio of coarse to fine grid is chosen to be R=3, so the slit can be divided into 30×R fine grids. The ends of the waveguide are truncated by CPML, and the periodic boundary conditions are set along x directions. A plane wave Gaussian pulse function is adopted as the excitation. Its time domain is $\exp(-(t-t_0)^2/Ts^2)$, where Ts=0.64ns, t_0=3Ts. Hence λ_{min}=0.3 being ten times of the coarse grid size. The observation point e_z(62,62) located at the point of the fine grid region which corresponds to the coarse grid point E_z(90,230).

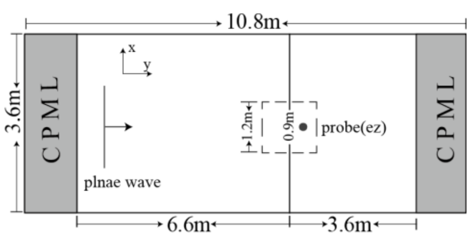

Fig. 4 A 2-D waveguide separated into two parts by a PEC with a slit

Fig.5 shows the time domain waveform of the observation point obtained by using three different methods. It can be seen that the result of the proposed method is closer to the standard FDTD method with fine grid when compared with the standard FDTD method with coarse grid.

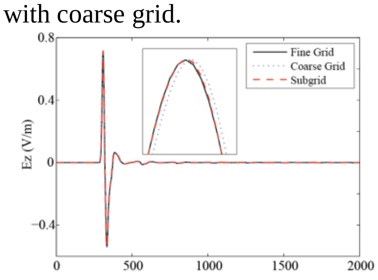

Fig.5 The waveform versus iterations of the probe point calculated by using uniform fine grid, uniform coarse grid and subgrid respectively.

At last, the execution time and memory cost for these three different methods are calculated and listed on Table I. It is shown that the proposed method saves a large amount of CPU time and memory cost for long-time simulation within the allowable range of precision.

Table I

Test Case	CPU time (s)	Memory cost(M)	grid size (m)	Time step (ps)
coarse grid	5.02	1.8637	0.03	70.063
fine grid	229.28	7.361	0.01	23.354
subgrid	8.47	1.321	0.03 and 0.01	70.063

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IV. CONCLUSION

An hybrid 2D algorithm based on the spatially filtered FDTD and traditional FDTD is proposed. This technique adopts the SF-FDTD method in fine grid area, and uses the traditional FDTD method in the coarse region. Making the time step used in the whole simulation is only determined by the coarse grid, the temporal interpolation is avoided in this method. In addition, the effect of the filter operation on the interpolated boundary is eliminated by only filtering electric fields components and adjusting the electromagnetic distribution in the calculation area. Guaranteed the integrity of spatial interpolation information. Numerical example shows that the hybrid method has obvious advantages in computational efficiency and memory occupation within the range of accuracy. This hybrid method is simple and easy to program, providing a more simple and effective technical means for simulating more complex multiscale problems.

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