Approximate Calculation of HPBW for Uniform Circular Array

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***Abstract*—The computation of the half power beam width (HPBW) for uniform circular array (UCA) is usually complicated compared with uniform linear array. In this paper, the calculation of HPBW for UCA in the plane the array placed is investigated, and a simple and effective formula is proposed to compute it approximately. Simulation results are provided to verify the validity of the proposed formula. Also, the accuracy of the proposed formula is evaluated in absolute error and error rate by the numerical simulations.**

Keywords—Uniform circular array (UCA); half power beam width (HPBW); approximate calculation.

I. Introduction

Uniform circular arrays (UCA), sometimes called ring arrays, have some special characteristics compared with uniform linear arrays (ULA). For example, it can offer direction of arrival information in φ and θ, while linear arrays only provide φ or θ. Especially, UCA, illustrated in Fig.1, can provide a full angular scan in *xy* plane almost without distortion. In other words, UCA can scan from 0° to 360° almost without distortion in φ direction in array pattern. Thus, UCA can be applied in many fields [1]-[4].

Although there are a lot of studies focusing on UCA, the researches on the pattern of UCA are not too many, especially about the beam width. In [5], the effects of the radius and element number of UCA on the half power beam width (HPBW) is studied. However, it has not provided the calculation method for the beam width. A method is mentioned in [6], which calculates the HPBW approximately by the series expansion of the power pattern function of UCA at half-power point. But, the calculation is not easy in reality. In [7], an approximate formula is proposed to calculate the HPBW of UCA by using numerical simulation. Although the expression is very simple, its constant result is obviously not suitable for all cases.

Instead of calculating HPBW from the definition, we give out a direct and simple formula to calculate the HPBW in φ plane for UCA under different θ of the main lobe in this paper. The HPBW hereinafter means the half power beam width in φ plane. At last, the proposed formula is verified by numerical simulations in validity and accuracy aspects.

The rest of this paper is organized as follows. Section II gives the general geometry of UCA. Section III presents the analysis of the HPBW calculation. An approximate calculation formula is also given out in this section. The proposed formula is examined in both validity and accuracy via numerical simulations in section IV. Conclusion is drawn in Section V.

II. Geometry of UCA

An UCA with *N* elements in the *xy* plane is shown in Fig.1. The *n*th element is located at the radius of *a* with the phase angle φ_n. Each element has an associated weight I_n and phase α_n, meaning the amplitude and phase of excitation current. Then the array factor (AF) can be written as [8]

$$AF(\varphi,\theta)=\sum_{n=1}^{N} I_n e^{j(ka\sin\theta\cos(\varphi-\varphi_n)+\alpha_n)} \tag{1}$$

$$\alpha_n=-ka\sin\theta_0\cos(\varphi_0-\varphi_n)$$

where k, φ, θ and N are the wavenumber, azimuth angle, elevation angle and the number of elements, respectively, (θ_0,φ_0) is the direction of main beam, φ_n is the angular location and that is,

$$\varphi_n=2\pi\cdot\frac{n-1}{N} \tag{2}$$

It can be seen from (1) that the AF expression of UCA is more complicated than that of ULA as it contains both φ and θ angle information.

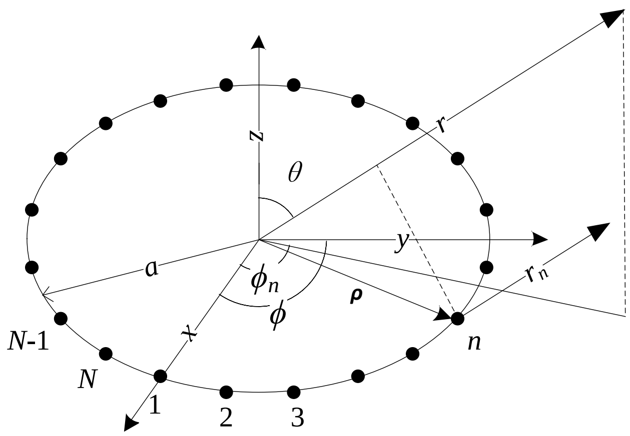

Fig.1 Uniform circular array of *N*-element

Denoting the element spacing by d, the radius of array can be approximately obtained as $a \approx Nd\backslash 2\pi$. In order to avoid high sides lobe and grating lobes, letting $d = \lambda/2$, we have

$$\frac{a}{\lambda} \approx \frac{N}{4\pi} \tag{3}$$

where the left term of (3) is electric length. It reveals the proportional relationship between the electric length of array radius and the number of array element.

III. Analysis of the HPBW Calculation

According to [7], it is known that the HPBW of UCA with half-wavelength spacing of elements can be approximately calculated as

$$\mathrm{HPBW} = \Theta \approx 0.3637 \cdot \frac{\lambda}{a} = 21^\circ \cdot \frac{\lambda}{a} \tag{4}$$

Obviously, the result of formula (4) is determined when λ and a are given. Unfortunately, it is found from TABLE I that the HPBW varies with θ. And the formula (4) is only applicable for $\theta = 90°$.

The calculation expression of the HPBW must contain more parameters which are included in AF, such as φ, θ and so on. Meanwhile, according to [5], the HPBW of UCA decreases as array radius grows. Also, the number of element has almost no influence on the HPBW for a given radius, which can also be seen from (4). This indicates that the HPBW is only affected by the array radius and the direction of the main lobe. It is known from array antennas theory that the HPBW is not changing with the φ. Therefore, only array radius and θ can affect the HPBW. Based on the AF in (1), it is easy and reasonable to deduce that the HPBW has a sine or cosine relation with θ. In order to study the relation further, the HPBWs of UCA are computed from AF under different θ. The results are recorded in TABLE I.

In TABLE I, the values of θ and HPBW corresponding to different θ, namely Θ_θ, are shown in the first and second columns in degree respectively, and the ratios of Θ_θ to Θ_{90} are presented in the third column. The data in the first and second columns indicate a trend that the HPBW decreases with the increasing of θ until $\theta = 90°$. This trend is coincident with the variation of $\cos\theta$ or $1/\sin\theta$. In addition, the values in third column show that $\Theta_\theta/\Theta_{90}$ reaches its minimum of 1 at $\theta = 90°$. This property is just well-matched with that of $1/\sin\theta$. Thus, the HPBW of UCA can be expressed as

TABLE I. HPBW under samples of θ

θ (deg.)	Θ_θ (deg.)	$\Theta_\theta/\Theta_{90}$
10	29.82	5.78
30	10.33	2.00
50	6.74	1.31
70	5.49	1.06
90	5.16	1.00

$$\Theta_\theta \approx \Theta_{90} \times \frac{1}{\sin\theta} \tag{5}$$

where Θ_{90} is given by (4). Substituting (4) into (5), an approximate calculation formula for the HPBW of UCA is obtained as

$$\Theta_\theta \approx 21^\circ \cdot \frac{\lambda}{a} \cdot \frac{1}{\sin\theta} \tag{6}$$

where $\theta \in (0°, 180°)$. Note that the absolute value of Θ_θ in (6) is approaching to infinity when θ is closing to 0° and 180°. This absolutely does not happen in reality. It is known that the HPBW of main lobe is only measured in elevation direction when $\theta = 0°$ and $\theta = 180°$. In other words, $\theta = 0°$ and $\theta = 180°$ are singularities for the HPBW in φ direction, which means that it is meaningless to compute the azimuthal HPBW at $\theta = 0°$ and $\theta = 180°$. Consequently, to avoid the singularities, θ in (6) is limited at $(10°, 170°)$.

IV. Simulation Results

Due to the symmetry of UCA, the simulation results in this section are symmetrical about $\theta = 90°$. For the sake of simplicity, only the properties under $\theta \in [10°, 90°]$ are presented in this paper.

A. Validity Verification

It is known from (6) that the HPBW of UCA is only affected by the ratio of wavelength to array radius and θ of the main lobe. In order to examine the validity of (6), the HPBW is calculated via formula (6) under different values of λ/a and θ respectively.

Letting $d = \lambda/2$, the HPBWs for different θ are calculated via (6) under several different array radii which are 3.18λ, 3.98λ and 4.77λ. The results compared with the ones computed from AF in (1) are shown in Fig.2. It is clear that the results computed from the proposed approximate formula and

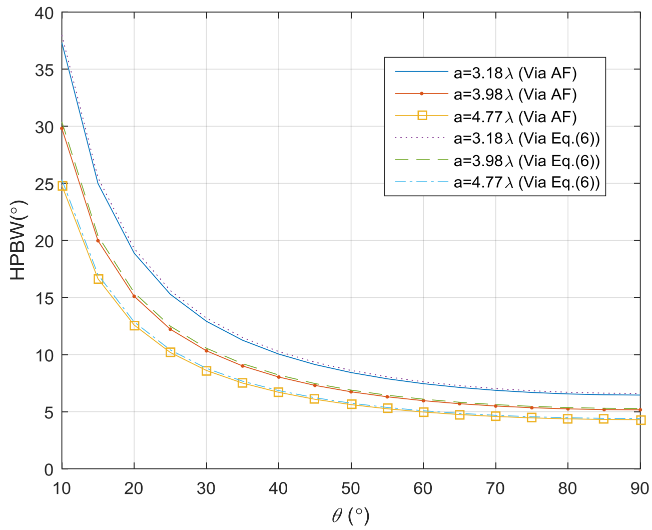

Fig.2 HPBW of UCA under different θ

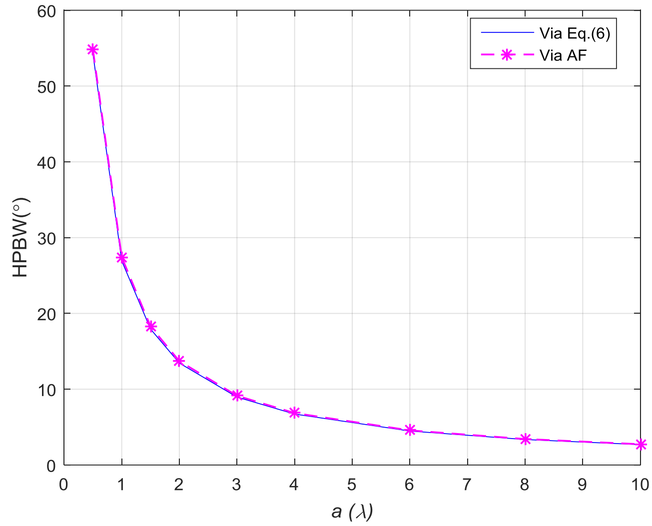

Fig.3 HPBW under different radii

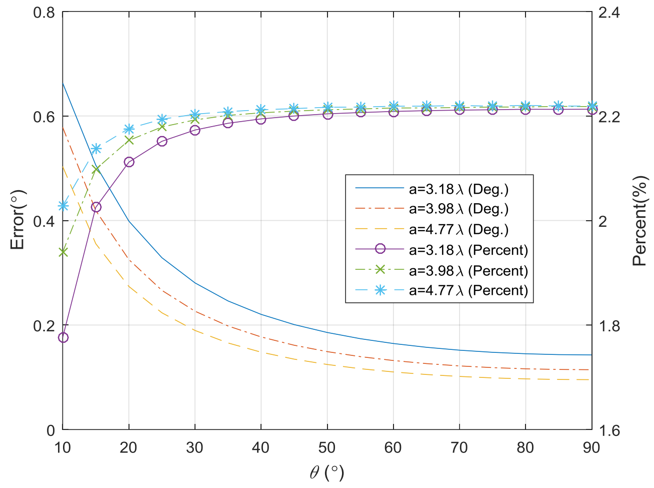

Fig.4 Errors under different θ

AF are almost the same for the same array radii. It means that the proposed approximate formula is available for different θ and different array radii. This simulation demonstrates the validity of (6) under different situations. Its good accuracy is also seen in this simulation.

In order to examine that the proposed formula can calculate the HPBW correctly while array radius changing, an example is calculated under $\theta = 50°$ and shown in Fig.3. It depicts that the result calculated by the proposed formula is almost same as the one computed by AF. Thus, the validity of (6) for different array radii is also proved.

Therefore, the numerical simulations demonstrates that the proposed approximate formula (6) can calculate the HPBW correctly under different array radii and different θ of the main lobe for UCA.

B. Calculation Errors

It is well known that the calculation error is very important for an approximate formula and affects the accuracy of the calculation result. Although the validity of the proposed formula is proved in the last subsection, its accuracy still needs to be further evaluated.

Taking the result of AF as a reference, the calculation error of the proposed formula is evaluated and the results are shown in Fig.4. The error results in this simulation are corresponding to the simulation results in Fig.2, and are presented in both absolute error and error rate, i.e. "Deg." and "Percent".

From the results in Fig.4, it can be figured out that the differences between the results computed by (6) and AF are very small. The absolute errors decline with the increasing of θ, and reach their minimums at $\theta = 90°$. But, the error rate curves present a contrary trend that the error rates go up as θ increases, and their maximums occur at $\theta = 90°$. Moreover, the results in Fig.4 reveal that there is an upper limit of error rate for all radii, around 2.2% at $\theta = 90°$, and it is almost not affected by array radius. Consequently, the good accuracy of the proposed approximate formula in this paper is demonstrated.

V. Conclusion

A simple formula is provided to calculate the HPBW of UCA approximately in φ plane in this paper. Both validity and accuracy of the proposed approximate formula are verified by numerical simulations. Based on the simulation results, a conclusion can be drawn that the proposed approximate formula is valid for different θ and different array radii. In addition, it has a good calculation accuracy that the maximum error rate is only around 2.2%.

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