Gaussian beam scattering from a sphere on or near a plane surface

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***Abstract*—A theoretical procedure is presented for calculating the scattered fields from a sphere on or near a plane surface. The incident Gaussian beam is expanded in series of cylindrical vector wave functions, and the scattered as well as internal fields in series of spherical ones. By using the boundary conditions and the method of moments technique, the unknown expansion coefficients are determined. Numerical results of the normalized differential scattering cross section are given, and the scattering characteristics are discussed concisely.**

Keywords—Scattering; Gaussian beam; sphere above a plane surface

I. INTRODUCTION

The study of light scattering from a spherical particle has a long and complex history[1]. More recently, systems containing a single particle and a plane surface have been examined[2-5]. However, a Gaussian beam (focused TEM00 mode laser beam) is often used, which is of great importance in some practical situations. In the previous papers, we have studied the Gaussian beam transmission through a uniaxial anisotropic slab by expanding the Gaussian beam in terms of the cylindrical vector wave functions (CVWFs) [6], and analyzed the arbitrarily shaped beam scattering from an anisotropic particle by following the method of moments (MoM) procedure [7] . In this paper, a combination of such an expansion and the MoM scheme enables us to investigate the Gaussian beam scattering by a sphere above a plane surface.

II. FORMULATION

The geometry of the scattering system is shown in Fig.1. A spherical particle is situated near a plane surface, with its symmetry axis coinciding with the normal to the plane surface. Let us introduce a Cartesian coordinate system $Oxyz$ with the z axis along the symmetry axis of the sphere. Origin O is situated at a distance d from the plane surface. The incident radiation is a Gaussian beam traveling in the xOz plane, oriented at an angle ζ with respect to the z axis. The middle of Gaussian beam waist O' has the Cartesian coordinates $(x_0, 0, z_0)$ in $Oxyz$. The wave number in free space is denoted by k .

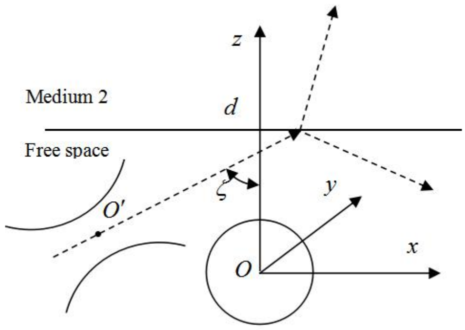

Fig.1.Geometry of the scattering system.

The incident Gaussian beam strikes the sphere either directly or after interacting with the surface. An expansion has been obtained of the electromagnetic (EM) fields of an incident Gaussian beam in terms of the CVWFs, which, for the sake of subsequent applications of boundary conditions, can be written as:

$$\mathbf{E}^i = E_0 \sum_{m=-\infty}^{\infty} \int_0^{\pi} [I_{m,TE} \mathbf{m}_{m\lambda}^{(1)} + I_{m,TM} \mathbf{n}_{m\lambda}^{(1)}] e^{ihz} d\alpha \tag{1}$$

where $\lambda = k \sin\zeta$, $h = k\cos\zeta$, and $I_{n,TE}^m$, $I_{n,TM}^m$ are the Gaussian beam shape coefficients for a TE polarized Gaussian beam.

$$I_{m,TE} = \frac{i^{m+1}}{2k} \sum_{n=|m|}^{\infty} g_n \frac{2n+1}{n(n+1)} \frac{(n-m)!}{(n+m)!} [\frac{dP_n^m(\cos\zeta)}{d\zeta} \frac{dP_n^m(\cos\alpha)}{d\alpha} + m^2 \frac{P_n^m(\cos\zeta)}{\sin\zeta} \frac{P_n^m(\cos\alpha)}{\sin\alpha}] \tag{2}$$

$$I_{m,TM} = \frac{i^{m+1}}{2k} m \sum_{n=|m|}^{\infty} g_n \frac{2n+1}{n(n+1)} \frac{(n-m)!}{(n+m)!} [\frac{dP_n^m(\cos\zeta)}{d\zeta} \frac{P_n^m(\cos\alpha)}{\sin\alpha} + \frac{P_n^m(\cos\zeta)}{\sin\zeta} \frac{dP_n^m(\cos\alpha)}{d\alpha}] \tag{3}$$

where

$$g_n = \frac{\exp(ikl_0)}{1+2isl_0/w_0} \exp[ih(x_0\cot\zeta - z_0)] \exp\left[\frac{-s^2(n+1/2)^2}{1+2isl_0/w_0}\right] \tag{4}$$

and $s = 1/(kw_0)$, $l_0 = -x_0/\sin\zeta$, and w_0 is the beam waist radius.

For a TM polarized Gaussian beam, the corresponding expansions can be obtained only by replacing $I^m_{n,TE}$ in (1) by $iI^m_{n,TM}$, and $I^m_{n,TM}$ by $iI^m_{n,TE}$.

By following (1), the reflected beam is expanded as

$$\mathbf{E}^r = E_0 \sum_{m=-\infty}^{\infty} \int_0^{\frac{\pi}{2}} [I_{m,TE} R_{TE}(\alpha)\mathbf{m}^{(1)}_{m\lambda}(-h) + I_{m,TM} R_{TM}(\alpha)\mathbf{n}^{(1)}_{m\lambda}(-h)] e^{-ihz} d\alpha \quad (5)$$

where $R_{TE}(\alpha)$, $R_{TM}(\alpha)$ respectively denote the Fresnel reflection coefficients.

The electric fields scattered by and existing within the sphere can be respectively expanded in terms of radiating and regular spherical vector wave functions (SVWFs)

$$\mathbf{E}^s = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [\alpha_{mn}\mathbf{M}^{(3)}_{mn}(k) + \beta_{mn}\mathbf{N}^{(3)}_{mn}(k)] \quad (6)$$

$$\mathbf{E}^w = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [\delta_{mn}\mathbf{M}^{(1)}_{mn}(k') + \gamma_{mn}\mathbf{N}^{(1)}_{mn}(k')] \quad (7)$$

where α_{mn}, β_{mn}, δ_{mn} and γ_{mn} are the expansion coefficients, $k' = k\tilde{n}'$, and $\tilde{n}'$ is the refractive index of the material of the sphere relative to that of free space.

To investigate the reflection of the scattered EM field by the interface, we use the integral representations of the radiating SVWFs in terms of the CVWFs [9, 13]

$$(\mathbf{M} \quad \mathbf{N})^{(3)}_{mn} = \frac{i^{m-n-1}}{k} \int_0^{\frac{\pi}{2}-i\infty} [\tau_{mn}(\alpha)(\mathbf{m} \quad \mathbf{n})^{(1)}_{m\lambda} + \pi_{mn}(\alpha)(\mathbf{n} \quad \mathbf{m})^{(1)}_{m\lambda}] e^{ihz} d\alpha \quad (8)$$

where $\tau_{mn}(\alpha) = \frac{d}{d\alpha} P^m_n(\cos\alpha)$, $\pi_{mn}(\alpha) = m\frac{P^m_n(\cos\alpha)}{\sin\alpha}$.

Substituting (8) into (6), we can obtain the series expansion of the scattered electric field in terms of the CVWFs

$$\mathbf{E}^s = E_0 \sum_{m=-\infty}^{\infty} \int_0^{\frac{\pi}{2}-i\infty} [A_m(\alpha)\mathbf{m}^{(1)}_{m\lambda}(h) + B_m(\alpha)\mathbf{n}^{(1)}_{m\lambda}(h)] e^{ihz} d\alpha \quad (9)$$

where the expansion coefficients $A_m(\alpha)$ and $B_m(\alpha)$ are given by

$$A_m(\alpha) = \frac{i^m}{k} \sum_{n=|m|}^{\infty} i^{-n-1} [\alpha_{mn} \frac{d}{d\alpha} P^m_n(\cos\alpha) + \beta_{mn} m \frac{P^m_n(\cos\alpha)}{\sin\alpha}] \quad (10)$$

$$B_m(\alpha) = \frac{i^m}{k} \sum_{n=|m|}^{\infty} i^{-n-1} [\alpha_{mn} m \frac{P^m_n(\cos\alpha)}{\sin\alpha} + \beta_{mn} \frac{d}{d\alpha} P^m_n(\cos\alpha)] \quad (11)$$

In addition to the three fields described by (1), (5) and (6), a fourth field exists in free space. This field, which for convenience is called the interacting field, is a result of the scattered field reflecting off the surface and striking the sphere. The interacting field can be expressed in the following form

$$\mathbf{E}^{sr} = E_0 \sum_{m=-\infty}^{\infty} \int_0^{\frac{\pi}{2}-i\infty} [R_{TE}(\alpha) A_m(\alpha)\mathbf{m}^{(1)}_{m\lambda}(-h) + R_{TM}(\alpha) B_m(\alpha)\mathbf{n}^{(1)}_{m\lambda}(-h)] e^{-ihz} d\alpha \quad (12)$$

By inserting (10) and (11) into (12), we can get a representation of the interacting field, as follows:

$$\mathbf{E}^{sr} = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [\alpha_{mn}\mathbf{X}^e_{mn}(k) + \beta_{mn}\mathbf{Y}^e_{mn}(k)] \quad (13)$$

where the basis vector wave functions $\mathbf{X}^e_{mn}$ and $\mathbf{Y}^e_{mn}$ in terms of the CVWFs can be readily derived in obtaining (13).

As shown above, only the series expansions of the electric fields are provided, and the corresponding expansions of the magnetic fields of the incident Gaussian beam $\mathbf{H}^i$, reflected beam $\mathbf{H}^r$, scattered field $\mathbf{H}^s$, internal field $\mathbf{H}^w$, and interacting field $\mathbf{H}^{sr}$ can be obtained with the following relations

$$\mathbf{H} = \frac{1}{i\omega\mu} \nabla \times \mathbf{E}, \quad (\mathbf{m} \quad \mathbf{n})_{m\lambda} e^{ihz} = \frac{1}{k} \nabla \times [(\mathbf{n} \quad \mathbf{m})_{m\lambda} e^{ihz}] \quad (14)$$

The boundary conditions require that the tangential components of the EM fields at the sphere's surface S be continuous

$$\hat{r} \times (\mathbf{E}^i + \mathbf{E}^r + \mathbf{E}^s + \mathbf{E}^{sr}) = \hat{r} \times \mathbf{E}^w \quad (15)$$

$$\hat{r} \times (\mathbf{H}^i + \mathbf{H}^r + \mathbf{H}^s + \mathbf{H}^{sr}) = \hat{r} \times \mathbf{H}^w \quad (16)$$

Substituting (6), (7) and (13) into (15) and (16), we can have

$$\hat{r} \times E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \{\alpha_{mn}[\mathbf{M}^{(3)}_{mn}(k) + \mathbf{X}^e_{mn}(k)] + \beta_{mn}[\mathbf{N}^{(3)}_{mn}(k) + \mathbf{Y}^e_{mn}(k)]\} - \hat{r} \times E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [\delta_{mn}\mathbf{M}^{(1)}_{mn}(k') + \gamma_{mn}\mathbf{N}^{(1)}_{mn}(k')] = -\hat{r} \times (\mathbf{E}^i + \mathbf{E}^r) \quad (17)$$

$$\hat{r} \times E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \{\alpha_{mn}[\mathbf{N}^{(3)}_{mn}(k) + \mathbf{X}^h_{mn}(k)] + \beta_{mn}[\mathbf{M}^{(3)}_{mn}(k) + \mathbf{Y}^h_{mn}(k)]\} - \hat{r} \times \tilde{n}' E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [\delta_{mn}\mathbf{N}^{(1)}_{mn}(k') + \gamma_{mn}\mathbf{M}^{(1)}_{mn}(k')] = -\hat{r} \times i\eta(\mathbf{H}^i + \mathbf{H}^r) \quad (18)$$

where η is the free space wave impedance.

Equations (17) and (18) are multiplied (dot product) by the vector spherical harmonics $\mathbf{m}_{-mn} = \nabla \times (\mathbf{r}Y_{-mn})$ and $\mathbf{n}_{-mn} = r\nabla Y_{-mn}$ respectively, where $Y_{mn} = P^m_n(\cos\theta)e^{im\phi}$, and then integrated over the sphere surface. By such a simple mathematical performance, a system of equations can be obtained, which generates an adequate number of relations among the expansion coefficients and then can be used to solve for them.

III. NUMERICAL RESULTS

In this section, we present some numerical results for the differential scattering cross section (DSCS) in the sphere system $Oxyz$ [8].

In the following calculations, all the formulations are programmed with MATLAB using double precision complex arithmetic. The material of the sphere and medium 2 (silicon) are assumed to be nonmagnetic, and the wavelength of the incident Gaussian beam $\lambda_0 = 0.6328\mu m$ is chosen.

To verify our solutions and results, a comparison is made in Fig.2 between the normalized DSCS of a polystyrene sphere situated on a silicon substrate evaluated by the present method and that by the EBCM in [8]. As expected, an excellent agreement is achieved.

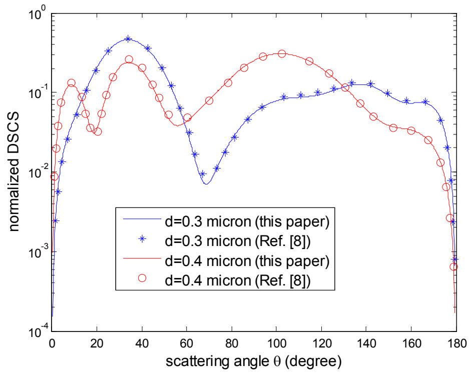

Fig. 2. Nomalized DSCS by πa^2 in the azimuthal plane $\phi = 0$ for a sphere (radius $a = 0.3\mu m$, $\tilde{n}' = 1.59$) deposited on ($d = 0.3\mu m$) and located at a distance $d = 0.4\mu m$ from a silicon substrate ($\tilde{n} = 3.88 + i0.02$), illuminated by a p-polarized plane wave (wavelength $\lambda_0 = 0.6328\mu m$ and $\zeta = \pi/3$).

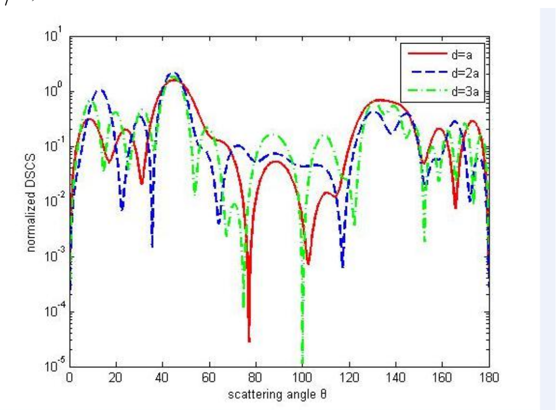

Fig. 3. Same model as in Fig.2 ($a = \lambda_0$, and $d = a$, $2a$ and $3a$ respectively), but illuminated by a TE polarized Gaussian beam ($\zeta = \pi/4$, $w_0 = 5\lambda$, $x_0 = z_0 = -0.2\lambda_0$).

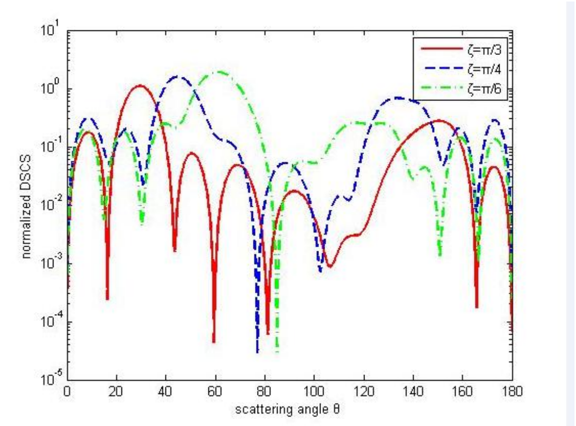

Fig. 4. Same model as in Fig.3 ($a = d = \lambda_0$, and $\zeta = \pi/3$, $\pi/4$ and $\pi/6$ respectively).

Figures 3 and 4 respectively show the normalized DSCSs for a spherical particle situated on a silicon substrate, illuminated by a TE polarized Gaussian beam. As shown above, there is a peak in the direction around angle ζ.

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