Directive Multiple Beam Emissions Lens Formed with Linear Coordinate Transformation Optics

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***Abstract*—Linear coordinate transformation lens that can transform the omnidirectional radiation into reconfigurable number of highly directive beams is presented. In 2-D space, by using our method, the parameters of transformed medium are homogeneous and positive, and the number of beams has no effect on parameters. The proposed structure creates different directive beams under different compression factors. And under high compression factors directivity is increased in electrical field distribution. The antenna lens can be manufactured by common metamaterials and isotropic materials and are realizable.. Full wave simulations are performed to validate our design. The proposed linear coordinate transformation method could promote development towards practicability in this field.**

Keywords—multiple beam emission; transformation optics; metamaterial; antenna.

Since the advent of transformation optics (TO)[1][2], it has become a powerful theoretical tool for scientists and engineers to control the propagation of electromagnetic (EM) waves[3][4]. Development of metamaterial that can realize inhomogeneity and anisotropy of material parameters derived from TO accelerate application of the EM devices in the area of antennas and communication industries [5][6]. A variety of EM devices with novel functions have been fabricated and experimentally validated[7][8][9][10], such as wave concentrator, beam control devices [11], three dimensional flattened Luneburg lens [12], meta-lens designed by smart transformations [13] and multi-beam emission lens [14]. With wide application of satellite communication and MIMO system in last decade, multi-beam emission lens antenna gradually turned to be of vital importance. [1]

Homogeneous, anisotropic material parameters leading to highly directive multi-beam emissions in both near and far fields were deduced from a space transformation [15]. Even after simplified, in the matrix of material parameter, the value in the x direction is less than 1, resulting in that non-diagonal parameter tensor will be educed when rotation angle of unit cell in transformed space is not 90°. Therefore, except for the case that number of beams is four, multi-beam emission lens designed by it is rather difficult to manufacture due to the strong anisotropy of parameters, which requires complicated metamaterials to be designed to satisfy it. Arrays of split-ring resonators (SRRs) were used to achieve the theoretical value of parameters in the case of four beam emissions. The measured far field radiation pattern demonstrated its function. Besides this, in order to eliminate anisotropy of material parameters, quasi-conformal (QC) transformation was adopted. Nearly-isotropic gradient index (GRIN) profile was obtained, which allowed all dielectric implementation [16][17], corresponding structure of metamaterial was also designed and manufactured to verify the function of multi-beam emissions. Nevertheless, for the different number of multi-beam emissions, the parameters are different and unit cell of the related metamaterial must be redesigned.

In this paper we present a linear transformation that multi-beam emissions could be gained through parameters derived from it [18][19][20]. It will be shown how to create such coordinate transformation by compressing space and then expanding it. In 2-D space, when the compression factor is larger than 3 in the compression zone, the radiation intensity in the physical location of radiation source is nearly zero, whereas the radiation intensity of extension zone is rather strong. In other words, the radiation source is virtually moved to the extension zone. Full wave simulations have verified the 3 emissions, 4 emissions, 6 emissions and 8 emissions in 2-D space with the same homogeneous spatially invariant parameter and how the radiation source is virtually moved to extension zone from its real location.

The initial formulation originating from the composite linear optical transformation aimed at designing rotatable illusion media, the transformation proposed by us is to transplant the composite linear transformation from cylindrical coordinate system to Cartesian coordinate system. In 2-D space, the principle of the composite transformation is depicted in Fig. 1. The virtual space is composed of two parts: zone 1 and zone2 and the linear transformation divides the transformed space into two parts: compression zone and extension zone. The zone 1 in the virtual space of which the side length is q_m is compressed into compression zone in the physical space, then zone 2 is extended and transformed into extension zone. The compression factor is q and the side length of compression zone and the whole physical space are m and n respectively. Here, the side length n of the physical space is equal to 75mm and is five times larger than that of m. The value of the compression factor q is adopted as 3 preferentially. Extending it to three dimension, a linear transformation in z direction is added. Then, the inner cube is compression zone and the outside cavity is extension zone. Parameters calculated from it get even simpler, which are isotropic and also positive.

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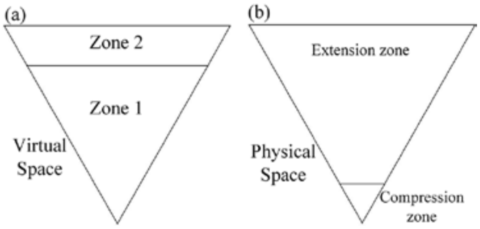

Fig. 1. (a) virtual space (b) physical space

The initial Cartesian space is (x, y, z) and (x', y', z') is the coordinates in the final physical space, the coordinate transformation between the virtual space and the physical space can be expressed as follows. In extension zone (with a =(n-m)/(n-qm)$\frac{n-m}{n-qm}$, 1< q <(n/m) and b =nm(1-q)/(n-qm)):

$$\begin{cases} x' = ax + b \quad qm < x < n \\ y' = ay + b \quad qm < y < n \\ z' = z \end{cases} \tag{1}$$

$\begin{cases} x' = ax + b \quad qm < x < n \\ y' = ay + b \quad qm < y < n \\ z' = z \end{cases}$In compression zone:

$$\begin{cases} x' = \dfrac{x}{q} \quad 0 < x < qm \\ y' = \dfrac{y}{q} \quad 0 < y < qm \\ z' = z \end{cases} \tag{2}$$

The corresponding permittivity and permeability tensors of the transformation media are given as follows. In extension zone:

$$\overline{\varepsilon'} = \overline{\mu'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/a^2 \end{bmatrix} \tag{3}$$

In compression zone:

$$\overline{\varepsilon'} = \overline{\mu'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & q^2 \end{bmatrix} \tag{4}$$

From the equations of transformation, it is obvious that coordinate transformation in x direction and y direction keep the same pace with each other. As depicted in Fig. 2, the unit cell of physical space is isosceles triangle. From the perspective of geometry, the number of edges of regular polygon in the physical space is determined by the apex angle of isosceles triangle and the polygon is formed by rotating isosceles triangle several times with the apex angle of it. Each one of isosceles triangle represents a beam. According to the coordinate rotation transformation in 2-D space, the parameter tensor in isosceles triangle will be altered when it is rotated a certain angle. Whereas, from Eq. 3 and Eq. 4, the only parameter whose value is not 1 is in the z direction of the diagonal tensor. So the parameter tensor in x-y plane is identity matrix which will keep the same form no matter how to rotate it. Therefore, with the same parameter tensor, the number of beams only varies with number of edges of polygon. But if the number of edges of polygon is larger than 6, the function of this concept will gradually perish. With the number of edges of polygon increases, the apex angle of isosceles triangle decreases rapidly, so does the area covered by it. Since each one of isosceles triangle represents a beam, the diminishment of area covered by it directly makes radiation intensity of this beam weaker. In the extreme condition, the number of edges of polygon is infinite, the apex angle of isosceles triangle and the area covered by it reduce to zero. The isosceles triangle turn out to be a line, the expected function of original assumption is totally vanished.

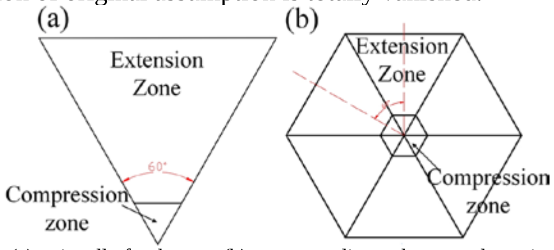

Fig. 2. (a) unit cell of polygon . (b) corresponding polygon and rotation angle of unit cell

To demonstrate the performance of the lens, numerical simulations with COMSOL Multiphysics are made at microwave frequencies. The radiation source is a line source which is located at the center point O. Boundaries of the computational zone are set as impendence match. The theoretical prediction of the lens is to transform the cylindrical wave front emitted by line source into several highly directive beams.

First, the verification of our design is implemented in 2-D space in a transverse electric (TE_z) mode. For the reason analyzed in theoretical design, the number of beams can't exceed 8, the typical multi-beam emissions are 3-beam emissions, 4-beam emissions, 6-beam emissions and 8-beam emissions. Then, each of them is validated respectively. The side length of regular polygon in the extension zone defined by parameter n is 75mm, the counterpart in the compression zone is determined by the value of parameter m. The value of n is set to be five times larger than that of m, so the value of q must be less than 5 in order to guarantee that the derived parameters are positive. On account of previous design in the cylindrical coordinate system, the value of the compression factor q is adopted as 3 preferentially.

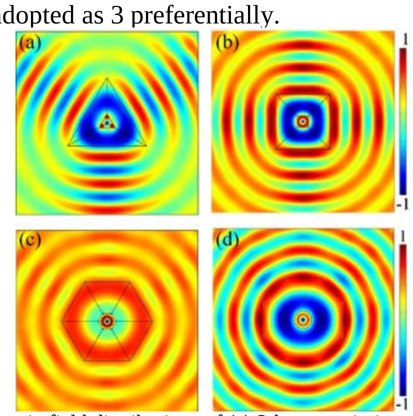

Fig. 3. Electric field distributions of (a) 3-beam emissions, (b) 4-beam emissions, (c) 6-beam emissions, (d) 8-beam emissions.

According to the area of the functional zone and computational zone in our design, the frequency is set as 7GHz. The electric field distribution of those four kind of multi-beam emissions lenses are shown in Fig. 3. Far field patterns of the above four lenses are also calculated and illustrated in Fig. 4. It is obvious that multi-beam emissions in desired directions are highly directive. Far field patterns agree well with the electric field distribution. Both of the results demonstrate our theoretical design.

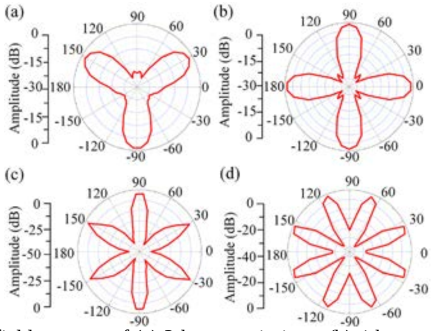

Fig. 4. Far field patterns of (a) 3-beam emissions, (b) 4-beam emissions, (c) 6-beam emissions, (d) 8-beam emissions.

For the 8-beam emissions, although the far field pattern is highly directive to 8 directions, azimuth of far field radiation is not directed at direction that is perpendicular to the sides of the regular octagon as that of the former ones. This phenomenon validates our previous theoretical analysis that when the number of edges of polygon is larger than 6, the function of multi-beam emissions will gradually perish. With regard to extreme condition when the number of edges of polygon is infinite and polygon becomes a circle, the function is totally vanished which can be seen from the electric field distribution.

Since q is the compression factor, when its value increases to 4, the space is further compressed. Then, the radiation energy is squeezed to extension zone from compression zone that is physical location of line source, which indicates that the radiation energy in compression zone is rather weak. The electric field distribution is shown in Fig. 5(a)-(c) and the far field patterns presented in Fig. 5(d)-(f) are still highly directive multi-beam emissions. It is obvious that a virtual source is formed in the extension zone, the radiation source is virtually moved to extension zone from its physical location.

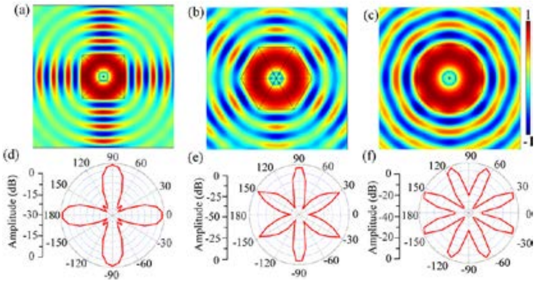

Fig. 5. Electric field distributions of (a) 4-beam emissions, (b) 6-beam emissions, (c) 8-beam emissions. (d)-(f) show the corresponding far field patterns.

In conclusion, highly directive N-beam emissions lenses in 2-D space have been designed using TO method. This manipulation of electromagnetic waves is enabled by linear transformation which is utilized to construct the mapping between virtual space and physical space. The final physical space is composed of two parts: compression zone and extension zone. In 2-D space, four typical kinds of multi-beam emissions lenses are generated with only one set of homogenous parameters, and when the compression factor increases, the radiation source is virtually moved to the extension zone from its original location in compression zone, which provides the possibility to fool detectors. As common metamaterials and isotropic materials can satisfy the theoretical value of designed parameters. The performance of the lenses was verified by numerical simulations. Our design proves to be a feasible way to promote the application of multi-beam emissions lens.

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