

# *A two-step method for the synthesis of Massively-thinned large planar arrays*

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**Abstract**—a two-step method is proposed to address the low sidelobe synthesis of the massively-thinned large planar array (MTLPA). In the first step, an MTLPA is obtained with well-behaved sidelobe performance through performing the iterative Fourier transform (IFT). Thanks to the high convergence of the IFT, this operation just require a few seconds CPU-time. Furthermore, in the second step, a total of 100 elements within the MTLPA that spaced greater than half wavelength are selected, and their locations are further optimized by the algorithm of differential evolution (DE) to decrease the sidelobe level. Some numerical results indicate that the proposed method can reduce the sidelobe level of the thinned array by at least 1.3dB, as is compared to the published reports.

**Keywords**—massively-thinned; planar array; differential evolution

## I. INTRODUCTION

Thinned array (TA) means to remove some elements from a periodic, fully populated antenna array, and thereby the power consumption, the antenna weight and cost are decreased. The thinned array can make it possible to build a highly directive array with reduced gain, and the cost will be further reduced by exciting the array with a uniform illumination [1]. For this reason, the use of TA had found the applications in the area such as the satellite receiving antennas, ground-based high frequency radars, remote sensing, radio astronomy, and so on [1, 2]. The advantageous of TA become more notable when we consider achieving the equivalent array performance by using the massively thinned array, which denotes the array has fewer than half the elements of their filled counterparts.

Over the past few decades, the synthesis of thinned array had been extensively addressed. A number of tools, such as the genetic algorithm (GA) [3, 4], the simulated annealing (SA) [5], the ant colony optimization (ACO) [6], and the particle swarm optimization (PSO) [7, 8], had been employed. However, the use of stochastic approaches is only limited to small or mid-sized array, they will face great computational burden when considering for large arrays. Fortunately, some other synthesis methods, including the stochastic approach [9], the deterministic approach [10], the difference sets-based methods (DSs) [11-12], and the iterative Fourier transform (IFT) [13-14], had been proposed. Among all the above related methods, the IFT had been confirmed very effective and efficient for synthesizing the array when the aperture ranges

from small to very large size. Recently, a modified form of the IFT (MIFT) is proposed in [15], where the method shows great advantage in sidelobe suppression for the large thinned planar array with filling factor greater than 40%, as is compared to its initial form. However, the MIFT can not be used for the MTLPA. As a kind of deterministic approach, although the IFT could deal with the synthesis of MTLPA, the approach is apt to stick into local minima. For this reason, a two-step method is proposed to address the issue. Firstly, a MTLPA with improved sidelobe performance is obtained by the IFT. Then, in the second step, a total of 100 elements within the MTLPA whose inter element spacing is greater than half wavelength, are selected, and their locations are further optimized by the DE. The method is actually based on the consensus that more degree of freedom for element locations is benefit for the sidelobe suppression. Due to the fast convergence of the IFT, and just a limited number of element locations need to be optimized, thus the computer burden is considerably alleviated.

The paper is organized as follows. Section I gives the introduction, while section II describes the formulation of the proposed method. Section III provides some numerical examples to validate the effectiveness of the proposed method. Finally, the conclusion is summarized in Section IV.

## II. FORMULATION OF THE METHOD

Consider a planar array with isotropic elements arranged in a  $M \times N$  square lattices spaced at  $0.5\lambda$  ( $\lambda$  denotes the wavelength), the array factor can be described as below [15].

$$AF(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{mn} e^{jkd(mu+nv)} \quad (1)$$

where  $u = \sin(\theta)\cos(\phi)$ ,  $v = \sin(\theta)\sin(\phi)$ .  $\theta$ , and  $\phi$  respectively denotes the elevation and the azimuth angle.  $A_{mn}$  is the element excitation in location  $(m, n)$ . According to (1), the collection of element excitations  $\{A_{mn}\}$  is related to the array factor  $AF(u, v)$  through a 2D Fourier transform.

Suppose that the synthesis refers to a circular planar array with filling factor equal to  $f_0$ , and the total number of lattices is represented by  $M_{tot}$ , therefore the two steps of the proposed method can be described as follows.

**Step 1: perform the IFT to get an initial MTLPA with reduced sidelobe level [14-15].**

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- 1) Randomly initialize the element excitations equal to one or zero with equal probability.
- 2) Compute  $AF(u)$  from  $\{A_{mn}\}$  through a  $K \times K$  points 2D-IFFT.
- 3) Adjust the values of  $AF(u)$  to match the prescribed sidelobe value.
- 4) Compute  $\{A_{mn}\}$  from  $AF(u)$  by a  $K \times K$  points 2D-FFT.
- 5) Truncate  $\{A_{mn}\}$  from the  $K \times K$  samples to  $M \times N$  samples that coincide with the columns and rows of the square lattices.
- 6) Set the samples located outside the circular aperture equal to zero.
- 7) Force the  $M_{tot} \cdot f_0$  samples of element excitations with higher amplitudes equal to one while the rest equal to zero.
- 8) Repeat 2) to 7) until the same element distribution between two adjacent iterations is obtained, or the iteration arrives its maxima.

**Step 2: for the elements of MTLPA spaced greater than  $0.5\lambda$ , randomly select 100 of them as the candidates with their locations being further optimized by the algorithm of DE [16].**

Once the current trial is finished, the optimum element distribution is retained. Then the procedure will proceed to the next numerical trial.

### III. NUMERICAL EXAMPLES

The synthesis by this method refers to obtaining a low sidelobe circular thinned planar array with aperture size respectively equal to  $25\lambda$  and  $33.33\lambda$ , and the filling factor is equal to 40%. The parameters related to DE, including the maxima of evolutionary generation, the population size, the scale factor, and the crossover probability, is equal to 20, 20, 0.7, and 0.9, respectively.

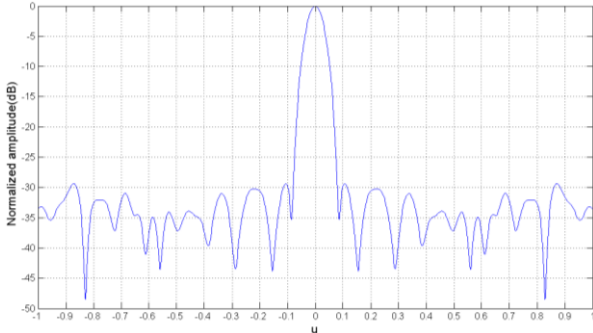


Fig.1 the u-cut far field pattern of the MTLPA with aperture size of  $25\lambda$  and filling factor of 40%.

The first case we consider a circular array with aperture size of  $25\lambda$ . The best u-cut far field pattern among 15 trials of this method, as is described in Fig. 1, has the sidelobe level of  $-27.84\text{dB}$ , decreased about  $1.4\text{dB}$  comparing with the published report in [14]. Furthermore, Fig. 2 shows the element distribution of this array, where the symbols of “+” describe the elements whose locations is retained after performing the IFT, and the symbols of “solid square” indicate the elements whose locations were optimized by the DE. We can see that the location-optimized elements are no longer

confined in the square lattices, but are randomly located around them.

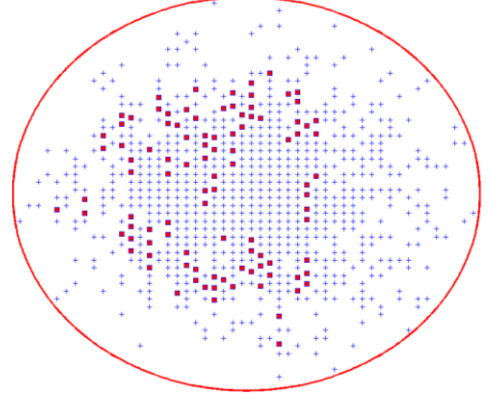


Fig.2 the element distribution of the MTLPA with aperture size of  $25\lambda$  and filling factor of 40%.

In the second case, we suppose that the array aperture size increased to  $33.33\lambda$ . The obtained MTLPA has the sidelobe level of  $-29.82\text{dB}$ , about  $1.3\text{dB}$  lower than the value presented in [14]. Fig. 3 and Fig. 4 respectively give the u-cut far field pattern as well as the element distribution of this array.

All the above results are obtained based on a PC equipped with an 8-GB RAM as well as an Intel I7-6700 Processor that operates at 3.4 GHz. The calculation cost of the first numerical case is about 11 hours, while the second case needs about 20 hours. Although it seems very time consuming, the computational burden is still considerably alleviated comparing with the situation when all the element locations need to be optimized.

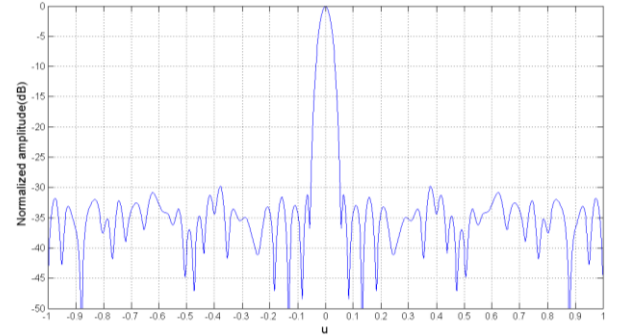


Fig.3 the u-cut far field pattern of the MTLPA with aperture size of  $33.33\lambda$  and filling factor of 40%.

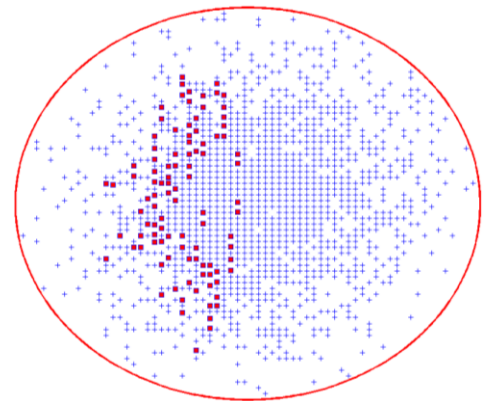


Fig.4 the element distribution of the MTLPA with aperture size of  $33.33\lambda$  and filling factor of 40%.

#### IV. CONCLUSION

To achieve good sidelobe suppression for MTLPA, the IFT is performed firstly to determine the initial element distribution. Then, among the elements that spaced greater than half wavelength, we randomly select 100 of them as the candidates whose locations will be further optimized by the algorithm of DE. For the fast convergence of the IFT, the main CPU time of this method is occupied by the DE. Furthermore, the limited parameters for each individual of DE make the computational cost considerably reduced. The numerical results confirmed the effectiveness of the proposed method in synthesizing the massively thinned large planar arrays.

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