Planar Beam-Scanning Sparse Array Synthesis with Minimum Spacing Constraint

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***Abstract*—In this paper, a new method is proposed to synthesize a planar beam-scanning sparse array with minimum inter-element spacing constraint. Perturbed compressive sampling (PCS) and convex optimization are two existing sparse array synthesis methods, which have their shortcomings in different indicators. The method of this paper combines the advantages of PCS and convex optimization to maintain the merits of synthesis speed, sparsity and gain while restraining the minimum inter-element spacing. Examples of planar sparse arrays whose minimum spacing is larger than 0.5 wavelength are given to show the performance of this method. The different situations of scanning and not scanning are also shown in the examples.**

Keywords—*minimum spacing, perturbed compressive sampling, convex optimizations, sparse array.*

I. INTRODUCTION

In the design of array antennas, uniform arrays usually need large number of elements and T/R components, which make the cost increase. To reduce the number of elements, lots of techniques have been presented over the last 60 years. Such as the global optimization algorithms, has the huge amount of computation. Some fast algorithms, such as the fast fourier transform (FFT) [1], has fast computational speed, still cannot intelligently determine the number of elements. The matrix pencil method (MPM) [2] can intelligently determine the number of elements, it seems can only be applied to linear arrays.

All these methods are not considering the minimum inter-element spacing. However, in practical applications, if there is no enough space for the elements (usually half wavelength), the mutual coupling will be strong and the elements may even overlap. The method of perturbed compressive sampling (PCS) in [3], has a good effect on the sparsity and speed. The drawback of PCS is that some element positions may be too close to each other. The method of this paper is to deal with the positions of elements to meet the minimum inter-element spacing on the basis of PCS. When a layout of elements is got by PCS, the elements whose spacing is smaller than the minimum spacing will be merged, then a better layout will be got. To make sure the peak sidelobe level (PSLL) and gain meet our requirements, the next work is to update the excitations by convex optimization.

II. SYNTHESIS PROCEDURE

A. Array model

Considering a planar array symmetrical about the x and y axes. The planar is divided into $4N$ grids, and each grid point located an array element. Then the array factor of this array can be written as

$$\begin{aligned} f(u,v) &= \sum_{n=1}^{4N} w_n e^{j\beta x_n u} e^{j\beta y_n v} \\ &= 4\sum_{n=1}^{N} w_n \cos(\beta x_n u)\cos(\beta y_n v) \end{aligned} \tag{1}$$

where $\begin{aligned} u &= \sin\theta\cos\varphi - \sin\theta_0\cos\varphi_0 \\ v &= \sin\theta\sin\varphi - \sin\theta_0\sin\varphi_0 \end{aligned}$

θ_0, φ_0 is the beam pointing direction, $\beta = 2\pi/\lambda$, w_n is the excitation of the nth element.

B. Reducing the Number of Elements

To reduce the number of elements, the following problem needs to be solved.

$$\begin{aligned} &\min_{\mathbf{w},\boldsymbol{\delta}_x,\boldsymbol{\delta}_y} && \|\mathbf{w}\|_p \quad , \quad 0<p<1 \\ &\textit{subject to} && \|(\mathbf{D}+\mathbf{D}_x\boldsymbol{\Lambda}_{\boldsymbol{\delta}_x}+\mathbf{D}_y\boldsymbol{\Lambda}_{\boldsymbol{\delta}_y})\mathbf{w}-\mathbf{f}_{REF}\|_2 \le \varepsilon \end{aligned} \tag{2$$

where $[\mathbf{D}]_{jn} = 4\cos(\beta x_n u_j)\cos(\beta y_n v_j)$, $\mathbf{D}_x$ is the partial derivative of $\mathbf{D}$ with respect to x . $\boldsymbol{\Lambda}_{\boldsymbol{\delta}_x} = diag(\boldsymbol{\delta}_x)$, where $\boldsymbol{\delta}_x$ is a vector of position perturbations $\boldsymbol{\delta}_x = [\delta_{1x}, \delta_{2x}, \cdots \delta_{Nx}]^T$. The definitions of $\mathbf{D}_y$ and $\boldsymbol{\Lambda}_{\boldsymbol{\delta}_y}$ are similar to $\mathbf{D}_x$ and $\boldsymbol{\Lambda}_{\boldsymbol{\delta}_x}$, respectively. Defining a desired pattern function f_r , $\mathbf{f}_{REF}$ is a vector by sampling f_r .

A detailed solution to this problem can be found in [3].

C. Merging Elements and Re-optimizing the Sparse Array

After solving (2), a set of positions can be got, but the minimum spacing of the positions do not meet the minimum array spacing ($d_{\min}$) that we need. Therefore, the distance of two elements smaller than $d_{\min}$ will be merged into one element.

In general, the merger of the elements will change the radiation pattern. Hence, a new set of excitations for this sparse array will be solved. Suppose there are p elements of the sparse array after merging elements, the array factor of this array can be written as

$$f_c(u,v)=\sum_{n=1}^{p} w_c(n)e^{j\beta x_n u}e^{j\beta y_n v} \tag{3}$$

where $w_c(n)$ is the finial excitation the nth element. Now the problem becomes: finding an optimal $\mathbf{w}_c$ to achieve the targets we need. It can be written as

$$\begin{aligned} &f_c(u_0,v_0)=1 \\ &\left|f_c(u_{sll},v_{sll})\right|_{(u_{sll},v_{sll})\in Sidelobe} \le Expcet\_SLL \end{aligned} \tag{4}$$

where $u_0=\sin\theta_0\cos\varphi_0, v_0=\sin\theta_0\sin\varphi_0$, $Sidelobe$ is the sidelobe region, and $Expcet\_SLL$ is the PSLL that we want.

III. NUMERICAL RESULTS

Considering a 40×40 uniform array with a −35 dB Taylor pattern as the reference pattern. Our goal is to achieve the same performance (gain, PSLL) with as little as possible array elements. Setting $d_{\min}>0.5\lambda$, the final results that not scanning are shown in Fig.1. The performance are summarized in Table I. As it can be seen that the number of elements of the sparse array is 40% lower than the uniform Taylor array, and the performance is the same as the uniform array with the minimum spacing greater than $d_{\min}$. The same problem has been solved in [4], it used 1024 elements to get the same performance, and there is no spacing constraint in [4]. What's more, paper [4] did not consider the situation of scanning. When the array scans to 30°, the results are shown in Fig .2.

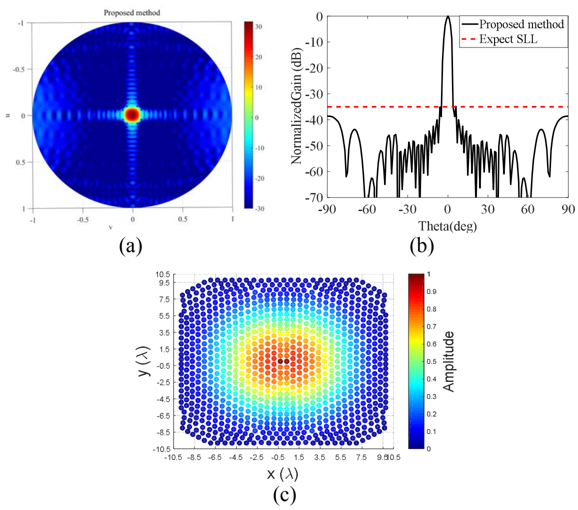

Fig. 1. Results of not scanning. (a) 3D view of the power pattern synthesized by proposed method. (b) 2D normalized power pattern ($\varphi=0°$). (c) Resulting array layout relative to (a).

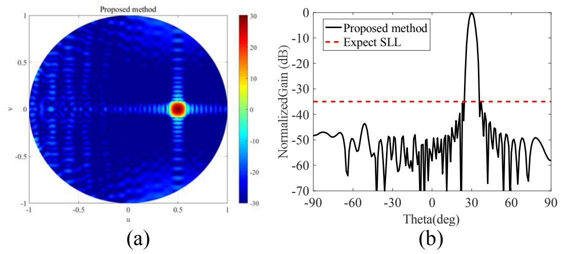

Fig. 2. Results of scanning to 30°. (a) 3D view of the power pattern synthesized by proposed method. (b) 2D normalized power pattern ($\varphi=0°$).

TABLE I THE PERFORMANCE OF ARRAYS

Array	Gain(dB)	PSLL(dB)	Number of elements(p)	$d_{\min}$ (λ)
40×40 Taylor	31.67	-35	1600	0.5
Proposed method	31.65	-35.1	960	0.501
Paper [4]	/	-35	1024	<0.5
Scanning to 30°				
40×40 Taylor	30.36	-35	1600	0.5
Proposed method	30.34	-34.8	960	0.501

The performance of the sparse array also maintains a similar level as the uniform array when it scans to 30°.

IV. CONCLUSION

A new method to synthesize a beam-scanning sparse planar array with the minimum spacing constraint is presented. Numerical examples are provided to validate the effectiveness of the proposed method. The results show the method can maintain the merits of sparsity, PSLL and gain under the premise of ensuring the minimum array spacing.

ACKNOWLEDGMENT

This work is supported by the Sichuan Provincial Science and technology support program (No. 2017HH0041), and National Key Research and Development Plan (2016YFC0303501). International Science and technology cooperation project of Dongguan (No.2016508102009).

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