CSC336 A2 Report

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Q1

Assume $n \in \mathbb{N}$

Assume $x \in \mathbb{R}^n$

Then $\|x\|_2$

$= (|x_1|^2+|x_2|^2+\cdots+|x_n|^2)^{\frac{1}{2}}$

$\leq [(|x_1|^2+\cdots+|x_n|^2)+(2|x_1||x_2|+2|x_1||x_3|+\cdots+2|x_{n-1}||x_n|)]^{\frac{1}{2}}$

$= [(|x_1|+|x_2|+\cdots+|x_n|)^2]^{\frac{1}{2}}$

$= |x_1|+|x_2|+\cdots+|x_n|$

$= \|x\|_1$

Then $\forall n \in \mathbb{N}$ $\forall x \in \mathbb{R}^n$ $\|x\|_2 \leq \|x\|_1$

Q2

Possible :

Let $x=(3,4)$, $y=(1,5)$.

Then $\|x\|_1 = 7$, $\|x\|_2 = 5$

$\|y\|_1 = 6$, $\|y\|_2 = \sqrt{26}$

$\|x\|_1 > \|y\|_1$

$\|x\|_2 < \|y\|_2$

Q3

Assume A is invertible

Let $Ax=b$; $A\hat{x}=\hat{b}$

Then $A^{-1}b=x$; $A^{-1}\hat{b}=\hat{x}$

Since $Ax=b$, $A\hat{x}=\hat{b}$ $\Rightarrow$ $\frac{\|x-\hat{x}\|}{\|x\|} \leq cond(A)\frac{\|b-\hat{b}\|}{\|b\|}$

Then $\frac{\|b-\hat{b}\|}{\|b\|} \leq cond(A^{-1})\frac{\|x-\hat{x}\|}{\|x\|}$

Since $cond(A) = \|A^{-1}\|\,\|(A^{-1})^{-1}\| = \|A^{-1}\|\,\|A\| = cond(A)$

Then $\frac{1}{cond(A)}\frac{\|b-\hat{b}\|}{\|b\|} \leq \frac{\|x-\hat{x}\|}{\|x\|}$

Q4

code:

```
% a): compute f_hat
a = sqrt(2)/2;
A = zeros(13, 13);
b = [0 10 0 0 0 0 0 15 0 20 0 0 0]';
A(1,2)=1; A(1,6)=-1;
A(2,3)=1;
A(3,1)=a; A(3,4)=-1; A(3,5)=-a;
A(4,1)=a; A(4,3)=1; A(4,5)=a;
A(5,4)=1; A(5,8)=-1;
A(6,7)=1;
A(7,5)=a; A(7,6)=1; A(7,9)=-a; A(7,10)=-1;
A(8,5)=a; A(8,7)=1; A(8,9)=a;
A(9,10)=1; A(9,13)=-1;
A(10,11)=1;
A(11,8)=1; A(11,9)=a; A(11,12)=-a;
A(12,9)=a; A(12,11)=1; A(12,12)=a;
A(13,12)=a; A(13,13)=1;
f_hat = A\b;
%b compute bound using 2 norm
condition = cond(A);
relative_residual = norm(b-(A*f_hat))/norm(b);
upper_bound = condition*relative_residual;
lower_bound = relative_residual/condition;
```

a):

```
f_hat =
  -28.2843
   20.0000
   10.0000
  -30.0000
   14.1421
   20.0000
         0
  -30.0000
    7.0711
   25.0000
   20.0000
  -35.3553

   25.0000
```

b):

Since $A f = b$, $A\hat{f} = \hat{b}$

Since $\frac{1}{\text{cond}(A)} \frac{\|b-\hat{b}\|}{\|b\|} \leq \frac{\|f-\hat{f}\|}{\|f\|} \leq \text{cond}(A) \frac{\|b-\hat{b}\|}{\|b\|}$

Then $\frac{1}{\text{cond}(A)} \frac{\|r\|}{\|b\|} = \frac{1}{\text{cond}(A)} \frac{\|b-A\hat{f}\|}{\|b\|} \leq \frac{\|f-\hat{f}\|}{\|f\|} \leq \text{cond}(A) \frac{\|b-A\hat{f}\|}{\|b\|} = \text{cond}(A) \frac{\|r\|}{\|b\|}$

upper_bound = 2.0952e-15

lower_bound = 1.9326e-17

Q5

code:

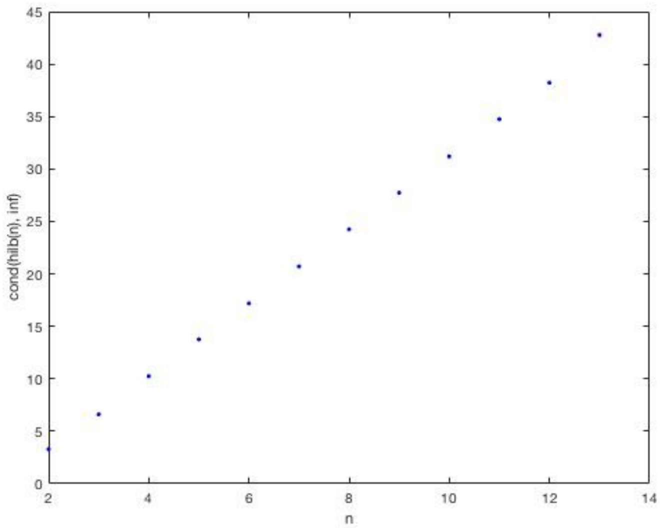
```
C = zeros(12, 1);
logC = zeros(12, 1);
D = zeros(12, 1);
R = zeros(12, 1);
N = (2:13)';
for n = 2:13
    H = hilb(n);
    x = ones(n,1);
    b = H*x;
    x_hat = H\b;
    relative_error = norm(x-x_hat, inf)/norm(x, inf);
    R(n-1) = relative_error;
    if relative_error >= 1
        max_n = n-1;
    end
    condition = cond(H, inf);
    C(n-1) = condition;
    logC(n-1) = log(condition);
    % use -log10(r_e) to approximate number of correct digits
    num_correct_digits = floor(-log10(relative_error));
    D(n-1) = num_correct_digits;
end

% part1: find the largest n with relative error < 1
fprintf('The largest n with relative error smaller than one: %d\n\n', max_n);

% part2: plot n versus log(cond(hilb(n)))
figure;
plot(N, logC, 'b.');
xlabel('n');
ylabel('cond(hilb(n), inf)');

% part3: print table of number of matched digits versus condition
Table = table(N, R, C, D, 'VariableNames', ...
        {'n', 'relative_error', 'condition_number', 'number_correct_digits'});
disp(Table);
```

result:



Condition number of hill(n) is positively proportional to exp(n)

n	relative_error	condition_number	number_correct_digits
2	7.7716e-16	27	15
3	7.4385e-15	748	14
4	4.6785e-13	28375	12
5	3.4417e-13	9.4366e+05	12
6	3.9883e-10	2.907e+07	9
7	1.5394e-08	9.8519e+08	7
8	5.7771e-07	3.3873e+10	6
9	2.1096e-05	1.0996e+12	4
10	0.00027738	3.5354e+13	3
11	0.021296	1.2311e+15	1
12	0.1144	3.9473e+16	0
13	9.4529	3.7285e+18	-1

The largest n with relative error smaller than 1 is 12.
As n gets larger , condition number gets larger, number of correct digits gets fewer.