Inflation, Risk Premia, and the Business Cycle

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Preview of the paper

Research Focus

- Impulse response of inflation to the risk premium shocks

Empirical Finding (Risk Price Puzzle)

- Risk premium shocks increase inflation (but results are insignificant)

Why it is a puzzle?

- If risk premium shocks are demand shocks [?], NK model would predict a decrease in inflation

How to solve it?

- Model with Fiscal theory of price level can have opposite prediction

Outline

1. Introduction to FTPL
2. Risk Price Puzzle: Empirical Evidence
3. A Two-sector RBC Model
4. Calibration and Estimation
5. Appendix: Pre-1998 and Post-1998

Introduction to FTPL

FTPL (Fiscal Theory of Price Level)

- Government intertemporal budget constraint:

$$B_{t-1} = P_t S_t + Q_t B_t$$

  - Q_t is nominal bond price: $Q_t = \mathbb{E}_t \left[M_{t+1} \frac{P_t}{P_{t+1}} \right]$
- Government debt valuation equation:

$$\frac{B_{t-1}}{P_t} = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t+j} S_{t+j} \right]$$

  - FTPL: Treasury set exogenous S_t rule, then P_t is determined by this equation in equilibrium
- The paper: Effect of risk premium shocks on inflation through FTPL

Risk Price Puzzle: Empirical Evidence

Three Measure of Risk Premium Shocks

1. Volatility Financial Conditions Index (VFCI) [Adrian et al., 2023]

- Def: Conditional Volatility of Output Spanned by Financial Factors
  - Breeden (1979): measure vol of pricing kernel by vol of aggregate consumption for time separable preferences
- Two Steps in Construction:
  1. Pick a few financial variables as "base assets" that span a sizable fraction of variation in the market price of risk

Name	**Description**
ret	Annualized Return on CRSP Value-Weighted Stock Index
vol	Standard Deviation of Daily CRSP Stock Returns
term	10-Year Yield Minus 3-Month Yield on Treasuries
liq	3-Month Treasury Yield Minus Effective Federal Funds Rate
cred	Moody's AAA Corporate Bond Yield Minus 10-Year Treasury Yield
def	Moody's AAA Minus BAA Corporate Bond Yield

Three Measure of Risk Premium Shocks

1. Volatility Financial Conditions Index (VFCI) [Adrian et al., 2023]

- Two Steps in Construction:
  2. Estimate a heteroskedastic linear regression

$$\Delta y_{t+1} = \boldsymbol{\theta} \boldsymbol{PC}_t + \varepsilon_t$$

$$\text{Var}\left(\varepsilon_t^2\right) = \sigma_t^2 = \exp\left(\boldsymbol{\delta PC_t}\right)$$

    - Δy_{t+1} is log real GDP growth
    - $PC_t \equiv [1, PC_1, PC_2, PC_3, PC_4]$

- VFCI: Fitted value of log conditional vol of GDP growth

$$VFCI \equiv \log \sqrt{\hat{\sigma}_t^2} = \hat{\delta} PC_t$$

  - Correlation between VFCI and VIX is 0.78 in post-1990

- Shocks to VFCI: Reiduals from an estimated AR(1)

Three Measure of Risk Premium Shocks

2. News Decomposition [Meeuwis et al., 2023]

- Measure discount rate news term in Campbell (1991) $(\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=1}^{\infty} \rho^j r^s_{t+j}$ using forecasting regression
- Estimate $\Gamma_t = a + b\boldsymbol{\Omega}_{t-1} + \sum_{k=1}^{N} w_k f_{k,t} + \eta_t$
  - $\Gamma_t \equiv \sum_{j=1}^{J} \rho^j r^s_{t+j}$: $\rho = 1/(1 + \overline{DP})$
  - $\boldsymbol{\Omega}_{t-1}$ is a vector of lagged state variables
    - (i) Log real stock return, (ii) Nominal 3-month treasury yield, (iii) Treasury yield spread (nominal 10-year and 3-month) (iv) Log price-dividend ratio
  - Traded factors (f_k): Fama& French (2015) five factors + Momentum
- Assume $\boldsymbol{\Omega}_{t-1}$ captures existing prior belief about Γ_t, then $w_k f_{k,t}$ can be interpreted as discount rate news

Three Measure of Risk Premium Shocks

2. News Decomposition [Meeuwis et al., 2023]

- Quarterly frequency
- Assume the stock returns after 40 quarters are not predictable
- With estimated loadings $\widehat{w}_k$, we can infer
  - Discount rate news: $N_{DR,t} \equiv \sum_{k=1}^{N} \widehat{w}_k \widetilde{f}_{k,t}$
  - $\widetilde{f}_{k,t}$ are the traded factors orthogonalized w.r.t $\mathbf{\Omega}_{t-1}$

Three Measure of Risk Premium Shocks

3. Structural VAR [Cieslak and Pang, 2021]

- Infer risk premium shocks: SVAR with sign+monotonicity restrictions
- Three types of shocks:
  1. **Growth shocks** ω^g: Shocks to expectations of future cash flows
     - Raises both stock returns and bond yields , with an impact that decreases with maturity
  2. **Risk-free rate shocks** ω^m: Monetary policy shocks
     - Depresses stock returns and raises bond yields, with an impact decreases with maturity
  3. **Risk premium shocks** ω^p:
     - Lowers stock returns and raises bond yields, but with an impact stronger at longer maturity
- Identification: Different impacts on stock and yields across maturities
- Shock Extraction: Estimate a VAR(1) using quarterly innovations in the log pd ratio and nominal Treasury yields of 3-year and 10-year

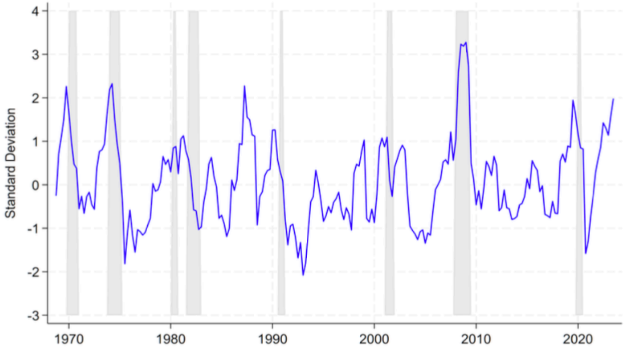
Impulse Responses to Risk Premium Shocks

Local projection impulse responses

$$y_{t+h} = \alpha + \beta^h v_t^{rp} + \Gamma' \mathbf{Z}_t + \varepsilon_{t+h}$$

- y_{t+h} is variable of interest in quarter $t+h$
- $h = \{0, \ldots, 20\}$ quarters
- v_t^{rp}: First principal component of the three risk premium shocks
- $\mathbf{Z}_t$ includes four lags of output, consumption, investment, unemployment, inflation and real stock return

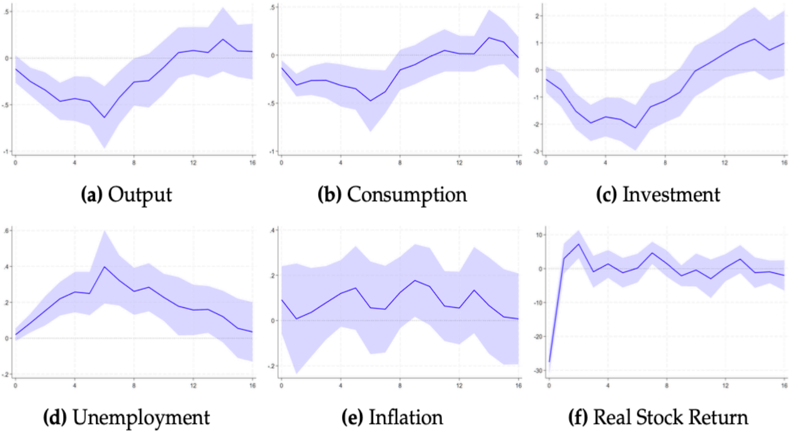
Time Series of Risk Premium Shocks v_t^{rp}



Note: The figure above plots the time series of risk premium shocks, defined as the first principal component of the VFCI shock, discount rate news, and structural risk premium shock. Shaded bars represent NBER recessions.

- Risk premia ↑ either immediately before or at the start of recessions

Empirical Impulse Response to Risk Premium Shock



(a) Output **(b)** Consumption **(c)** Investment

(d) Unemployment **(e)** Inflation **(f)** Real Stock Return

- 1968Q4 to 2023Q2; X-axis: quarters Y-axis: percentage points

Risk price puzzle

Risk price puzzle

- Although Risk premium shock decreases output, consumption, investment and employment, but is irrelevant to (or increase) inflation

Why NK model can't rationalize it?

- A positive risk premium shock lower consumption and output gap from Euler equation : $x_t = \mathbb{E}_t x_{t+1} - \psi\left(i_t - \mathbb{E}_t \pi_{t+1}\right) + v_{xt}$
  - v_{xt} shock represents anything that affects output gap at a given risk-free rate, so it includes risk premium shocks from a time-varying price of risk
- Negative output gap lowers inflation along the Phillips curve $\pi_t = \kappa x_t + \rho \mathbb{E}_t \pi_{t+1} + v_{\pi t}; \quad \kappa > 0$
- A positive risk premium shock decreases inflation

A Two-sector RBC Model

Preferences

Two types agents: 1. Shareholders and 2. Hand-to-mouth Workers

Representative Shareholders

- Own firm's equity and government bond
- Time-varying risk aversion x_t and time discount factor β_t

$$U\left(C_t^S\right) = \frac{C_t^{s\,1-x_{t-1}} - 1}{1 - x_{t-1}}; \qquad \beta_t \equiv \frac{\exp(-\delta_t)}{\mathbb{E}_t\left[\frac{C_{t+1}^{s\;-x_t}}{C_t^{s\;-x_{t-1}}}\right]}$$

- SDF

$$M_{t+1} = \beta_t \left(\frac{C_{t+1}^{s\;-x_t}}{C_t^{s\;-x_{t-1}}}\right) = \underbrace{\frac{\exp\left(-\delta_t\right)}{\mathbb{E}_t\left[\left(\frac{C_{t+1}^s}{C_t^s}\right)^{-x_t}\right]}}_{\hat{\beta}_t} \left(\frac{C_{t+1}^s}{C_t^s}\right)^{-x_t} = \hat{\beta}_t \left(\frac{C_{t+1}^s}{C_t^s}\right)^{-x_t}$$

- $x_t = \bar{x} + \phi_x x_{t-1} + \varepsilon_{xt}, \quad \varepsilon_{xt} \sim N\left(0, \sigma_x^2\right)$
- $log(r_{f,t}) = \delta_t = \bar{\delta} + \phi_\delta \delta_{t-1} + \varepsilon_{\delta t}, \quad \varepsilon_{\delta t} \sim N\left(0, \sigma_\delta^2\right)$

Production [Bai and Zhang, 2022]

Representative Firm

- C-D production function: $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$
- $a_{t+1} = log(A_{t+1}) = \bar{a} + \phi_a a_t + \varepsilon_{a,t+1}, \quad \varepsilon_{a,t+1} \sim N\left(0, \sigma_a^2\right)$
- Law of motion of capital
$$K_{t+1} = (1-\xi)K_t + \Phi_t$$
- Installation function
$$\Phi_t \equiv \Phi\left(I_t, K_t\right) = \left[a_1 + \tfrac{a_2}{1-1/v}\left(\tfrac{I_t}{K_t}\right)^{1-1/v}\right] K_t$$
    - $a_1 = \xi/(1-v)$ and $a_2 = \xi^{1/v}$ to ensure no adjustment cost in SS
    - $\partial \Phi_t / \partial I_t = a_2 \left(\frac{I_t}{K_t}\right)^{-1/v} > 0$
    - $\partial^2 \Phi_t / \partial I_t^2 < 0 \quad$ (adjustment cost)

Production [Bai and Zhang, 2022]

Representative Firm

- Post job vacancies V_t to attract unemployed workers U_t
- Vacancies filled via a CRS matching function:
$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^{\iota} + V_t^{\iota})^{\frac{1}{\iota}}}$$
- Employment evolves as
$$N_{t+1} = (1 - s)N_t + G(U_t, V_t)$$
  s: Constant separation rate
- Define labor market tightness $\theta_t = \frac{V_t}{U_t}$, then the Prob that a vacancy is filled (job filling rate) is
$$q(\theta_t) = \frac{G(U_t, V_t)}{V_t} = (1 + \theta_t^{\iota})^{-1/\iota}$$
- Aggregate investment in hiring is κV_t

Production

Representative Firm

Firm maximize cum-dividend market value

$$E_t \equiv \max_{\{V_{t+\tau}, N_{t+\tau+1}, I_{t+\tau}, K_{t+\tau+1}\}_{\tau=0}^{\infty}} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau} \right]$$

- Dividend: $D_t = Y_t - W_t N_t - \kappa V_t - I_t$
- Subject to Law of motion of capital and labor

Firm Euler Equations

Investment Euler Equation:

$$1 \equiv \mathbb{E}_t \left[M_{t+1} R^k_{t+1} \right] = \mathbb{E}_t \left[M_{t+1} \left(\frac{\alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{1}{a_2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^{1/\nu} (1 - \xi + a_1) + \frac{1}{\nu - 1} \frac{I_{t+1}}{K_{t+1}}}{\frac{1}{a_2} \left(\frac{I_t}{K_t} \right)^{1/\nu}} \right) \right]$$

- Marginal cost of investment is $1 / \left(\partial \Phi_t / \partial I_t \right) = \frac{1}{a_2} \left(\frac{I_t}{K_t} \right)^{1/\nu}$
- Marginal benefit of investment:
    - Marginal product of capital: $\partial Y_{t+1} / \partial K_{t+1} = \alpha Y_{t+1} / K_{t+1}$
    - Marginal continuation value net of depreciation $(1 - \xi) / \left(\partial \Phi_{t+1} / \partial I_{t+1} \right) = \frac{1}{a_2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^{1/\nu} (1 - \xi)$
    - Marginal impact of an extra unit of capital on installation technology $\left(\partial \Phi_{t+1} / \partial K_{t+1} \right) / \left(\partial \Phi_{t+1} / \partial I_{t+1} \right)$

Firm Euler Equations

Hiring Euler Equation:

$$1 \equiv \mathbb{E}_t \left[M_{t+1} R^n_{t+1} \right] = \mathbb{E}_t \left[M_{t+1} \left(\frac{(1-\alpha)\frac{Y_{t+1}}{N_{t+1}} - W_{t+1} + (1-s)\frac{\kappa}{q(\theta_{t+1})}}{\frac{\kappa}{q(\theta_t)}} \right) \right]$$

- Marginal cost of hiring: $\frac{\kappa}{q(\theta_t)}$
- Marginal benefit of hiring:
  - Marginal product of labor $\partial Y_{t+1}/\partial N_{t+1} = (1-\alpha) Y_{t+1}/N_{t+1}$
  - Net of Wage $-W_{t+1}$
  - Marginal cost of hiring next period net of separation $(1-s)\frac{\kappa}{q(\theta_{t+1})}$

Firm Euler Equations

- Due to CRS production function, gross stock return R^S_{t+1} is a weighted average of R^k_{t+1} and R^n_{t+1}:

$$R^s_{t+1} = w^k_t R^k_{t+1} + \left(1 - w^k_t\right) R^n_{t+1}$$

- Value weights $\quad w^k_t \equiv \mu^k_t K_{t+1} / \left(\mu^k_t K_{t+1} + \mu^n_t N_{t+1}\right)$
- Shadow value of capital $\mu^k_t = (1/a_2)\,(I_t/K_t)^{(1/v)}$
- Shadow value of labor $\mu^n_t = \kappa / q\,(\theta_t)$

Model Primitives: Wages and Worker's consumption

Real wage determined by Nash bargaining

$$W_t = \frac{1}{1-\tau+\tau\eta}\left[\eta\left((1-\alpha)\frac{Y_t}{N_t}+(1-s)\frac{\kappa}{q(\theta_t)}\right)+(1-\eta)b\right]$$

- $\eta \in (0,1)$: workers' relative bargaining weight
- τ: income tax rate; b: workers' flow value of unemployment
- W_t is increasing in
  - Marginal product of labor $(1-\alpha)Y_t/N_t$
  - Marginal cost of hiring net of separation $(1-s)\kappa/q(\theta_t)$
- Wage inertia: lower η reduces wage elasticity to labor productivity
- Worker's consumption:

$$C_t^w = N_tW_t - T_t + bU_t = (1-\tau)N_tW_t + bU_t$$

- Population nominalize to one: $U_t = 1 - N_t$

Model Primitives: Government

- Government's real primary surplus S_t:
$$S_t = T_t - G_t = \tau W_t N_t - bU_t - \chi Y_t$$
  - bU_t is unemployment benefit
  - Government spending χY_t is proportional to output
- Nominal intertemporal budget constraint
$$B_{t-1} = P_t S_t + Q_t B_t; \quad Q_t = \mathbb{E}_t \left[M_{t+1} \frac{P_t}{P_{t+1}} \right]$$
- Government debt valuation equation
$$\frac{B_{t-1}}{P_t} = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t+j} S_{t+j} \right]$$
  - FTPL: Price level P_t adjusts so that this equation holds

Discount rate shocks and Inflation

A positive risk premium shock

- <u>Discount rate channel:</u> Surplus claim is risky, so discount rate on government debt $\uparrow \Rightarrow$ inflation $\uparrow$
- <u>Cash flow channel:</u> Surpluses $\downarrow \Rightarrow$ inflation $\uparrow$

A positive risk-free shock

- Discount rates $\uparrow \Rightarrow$ inflation $\uparrow$

Calibration and Estimation

Estimation: Impluse response matching

- Solve model using a third-order perturbation around non-stochastic steady state
- $\hat{\Psi}$: vector that stacks point estimates of $\underline{\text{empirical impulse responses to a risk premium shock across all horizons}}$ + $\underline{\text{unconditional average equity premium}}$
- $\Psi(\Pi)$ denotes corresponding model-implied counterparty of $\hat{\Psi}$

$$\mathcal{L}(\Pi) \equiv (\hat{\boldsymbol{\Psi}} - \boldsymbol{\Psi}(\Pi))' \mathbf{W} (\hat{\boldsymbol{\Psi}} - \boldsymbol{\Psi}(\Pi))$$

Calibrated Parameters

Table 3: Calibrated Parameters

Name	Description	Value
α	Labor Share	1/3
ξ	Depreciation Rate	3.8%
s	Separation Rate	8.73%
$\bar{\delta}$	Risk-Free Rate Mean	0.3%
σ_δ	Risk-Free Rate Volatility	0.2%
ρ_δ	Risk-Free Rate Persistence	0.88
χ	Discretionary Spending-to-Output Ratio	0.16

- Mean, persistence, and vol of real rate are set to match the data counterparts

Estimation

Table 4: Estimated Parameters

Name	Description	Value
ρ_a	TFP Persistence	0.99
σ_a	TFP Volatility	0.005
$\bar{x}$	Risk Price Mean	20
ρ_x	Risk Price Persistence	0.85
σ_x	Risk Price Volatility	4.5
ν	Supply Elasticity of Capital	0.145
η	Worker Bargaining Weight	0.79
ι	Curvature of Matching Function	0.65
κ	Vacancy Cost Parameter	0.106
b	Flow Value of Unemployment	0.05
τ	Income Tax Rate	0.33

- Average risk price of 20 is low relative to the literature
- Low supply elasticity of capital

Estimation

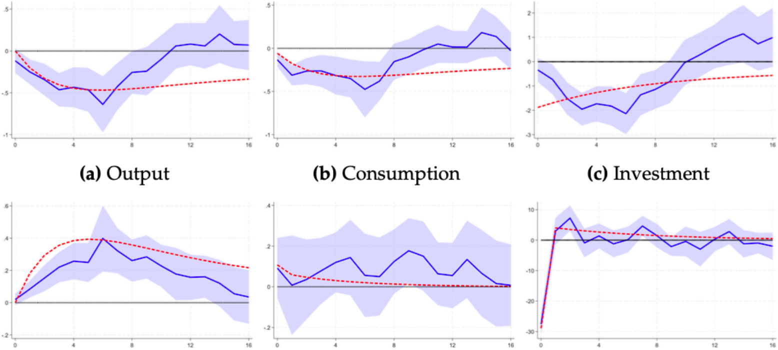
Table 5: Unconditional Macroeconomic and Asset Pricing Moments

Moment	Model	Empirical
Equity Risk Premium	4.87%	4.97%
Sharpe Ratio	0.43	0.35
Debt Risk Premium	9.54%	—
Inflation Volatility	4.2%	2.5%
Consumption Growth Volatility	2.68%	2.0%
Unemployment Volatility	0.5%	1.65%
Output Growth Volatility	2.7%	2.18%
Investment Growth Volatility	10.4%	11.1%
Transfer Spending-Output Ratio	2.36%	3.45%
Income Tax Revenue-Output Ratio	19.7%	16.4%

- In this table, only average equity premium was targeted

Impulse Responses

Figure 8: Model-Implied Impulse Responses to Risk Premium Shock

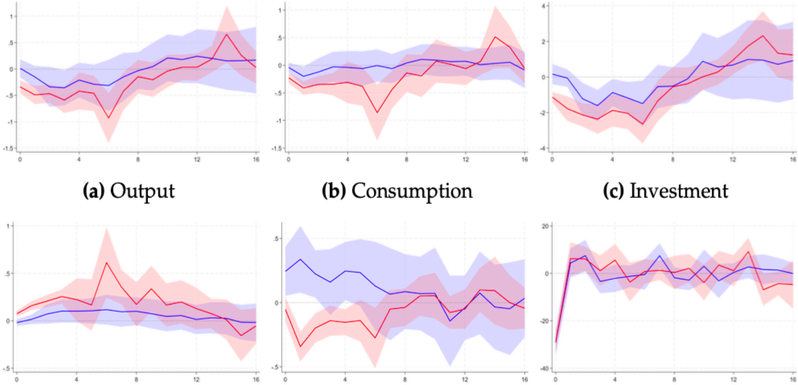


(a) Output **(b)** Consumption **(c)** Investment

(d) Unemployment **(e)** Inflation **(f)** Real Stock Return

Appendix: Pre-1998 and Post-1998

Empirical Impulse Response to Risk Premium Shock



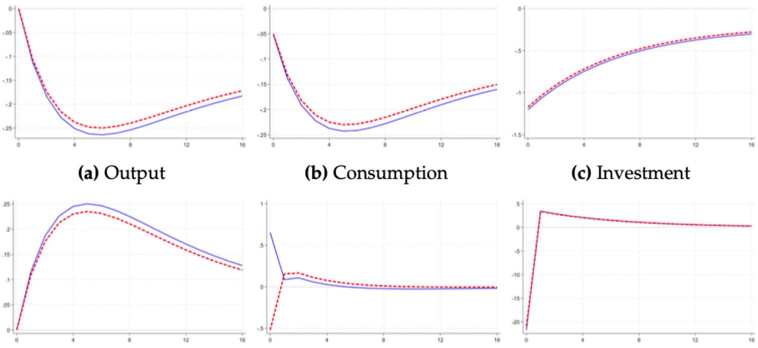
(a) Output **(b)** Consumption **(c)** Investment

(d) Unemployment **(e)** Inflation **(f)** Real Stock Return

- Blue: Pre-1998 v.s. Red: Post-1998: Inflation respond differently
- Paper rationalizes it by the changing correlation between shocks to risk premium and real risk-free rate

Impulse Responses

Figure 9: Model-Implied Impulse Responses to Risk Premium and Risk-Free Rate Shocks



(a) Output **(b)** Consumption **(c)** Investment

(d) Unemployment **(e)** Inflation **(f)** Real Stock Return

- Blue: a shock that increases both risk premium and real risk-free rate
  - Positive stock bond correlation
- Red: a shock that increases risk premium and lowers the risk free rate
  - Negative stock bond correlation

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