FIN 330: Midterm Exam
Fall 2024

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Name: ______________________

Campus ID: __________________

Section: ______________________

Instructions

- This is a closed-book, closed-note exam. You are allowed to use your pens/pencils, your calculator only.
- A formula sheet is attached to the back of the exam. If you remove the formula sheet during the exam, please ensure that the stapling of the exam portion of this document remains intact.
- Show all your work in a well-organized fashion if you wish to get full credit.
- Use legible handwriting. If we can't read what you have written, it will not count.
- When you exit the classroom, please refrain from speaking to other students.

I understand and agree to abide by the exam instructions listed above.

Signature: ______________________

Date: ______________________

1. You receive $6000 in 185 days from now. Assume a 365 day-per-year convention and compute the present value using continuous compounding with a 4.5% discount rate.

$$6000\, e^{-0.045 \cdot \frac{185}{365}} \approx 5864.70$$

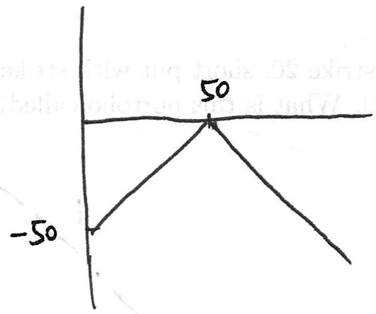
2. You sell (short) a European Put with strike 75. On the expiration date the underlying stock is at 65. What is your payoff?

$$-\max\{75-65, 0\} = -10$$

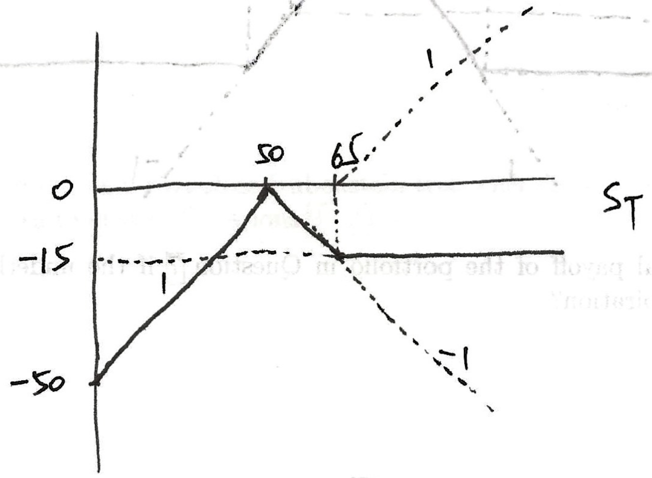
3. You sell a straddle with strike 77. The price of the put is $3 and the call is $4. On the expiration date the underlying stock is at 63. What is your profit?

$$+3+4-|63-77| = -7$$

4. You sell a straddle with strike 50. Graph the payoff on this portfolio.



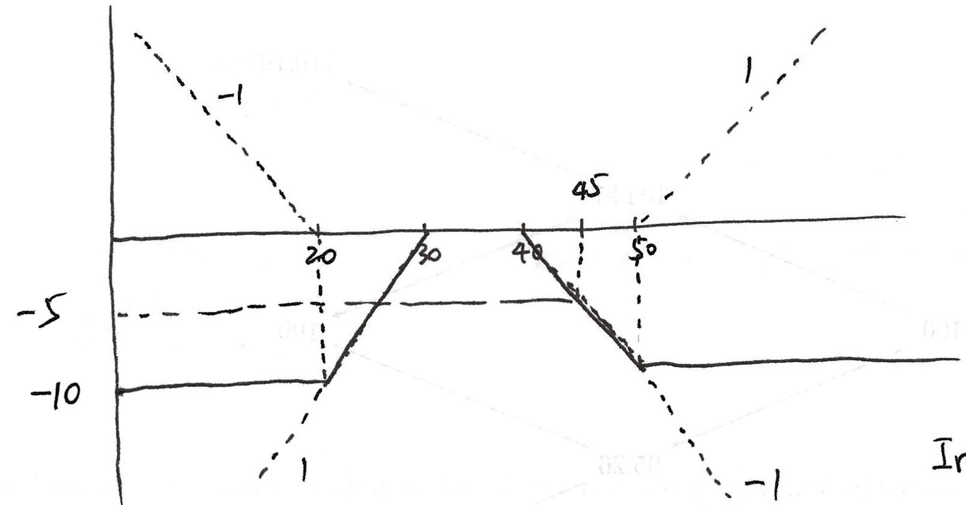
5. Graph the payoff to a short straddle with strike 50 and a long call with strike 65.



6. What is the maximum loss on the trade in the previous question?

-50

7. Graph the payoff on a long put with strike 20, short put with strike 30, short call with strike 40 and a long call with strike 50. What is this portolio called?

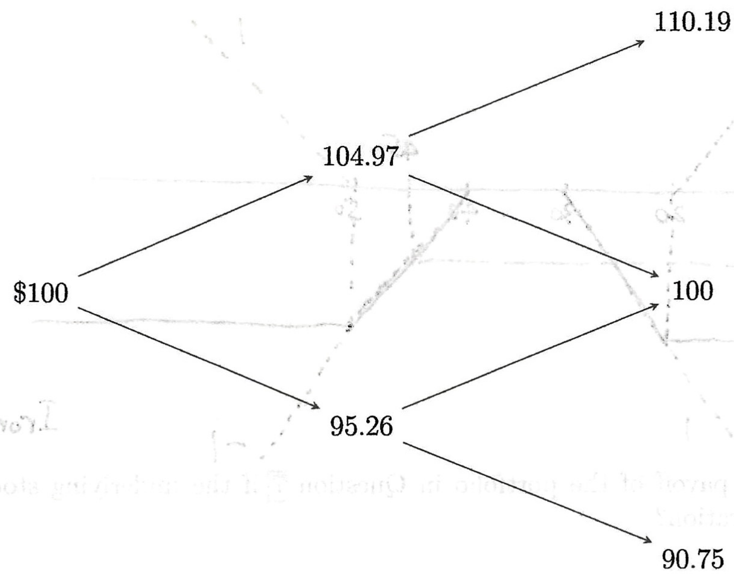


Iron Condor

8. What is the terminal payoff of the portfolio in Question 7 if the underlying stock is trading at \$45 at expiration?

$\text{Max}\{20-45,0\} - \text{Max}\{30-45,0\} - \text{Max}\{45-40\} + \text{Max}\{45-50,0\}$

$= 0 - 0 - 5 + 0 = -5$



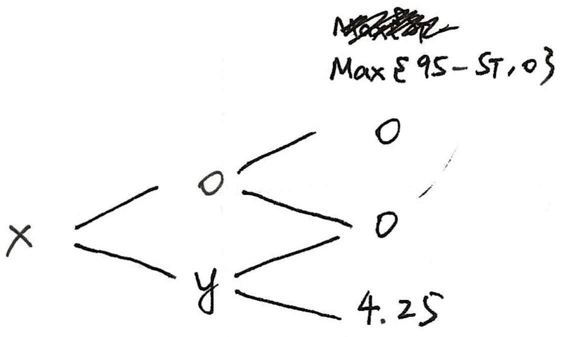
The above tree is a Cox-Ross-Rubenstein tree with $\Delta t = 1/52$, $r = 0.05$, $\sigma = 0.35$ Use these values to answer Questions 9 - 11.

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.35 \cdot \sqrt{1/52}}$$

$$d = 1/u$$

$$p = \frac{e^{r\Delta t} - d}{u-d} = \frac{e^{0.05 \cdot \frac{1}{52}} - d}{u-d} \approx 0.49777$$

9. Use the Binomial tree to compute the value of a European Put option with strike 95.



$$y = (p\cdot 0 + (1-p)\cdot 4.25)\cdot e^{-0.05\cdot\frac{1}{52}} \approx 2.133$$

$$x = (p\cdot 0 + (1-p)\cdot 2.133)\cdot e^{-0.05\cdot\frac{1}{52}} \approx 1.07$$

10. Find the value of a European ~~Put~~ Call with strike 95 using the easiest method you can think of.

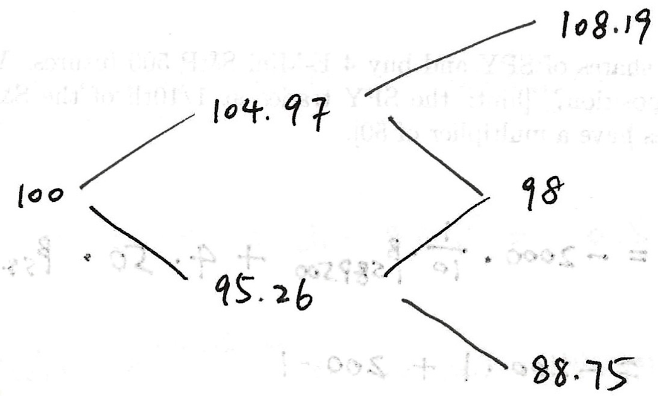
put call parity:

$$c = P + S - Ke^{-rT}$$

$$= 1.07 + 100 - 95e^{-0.05\cdot\frac{2}{52}}$$

$$\approx 6.253$$

11. Assume that the stock above pays a \$2 dividend in the final period. Adjust the tree to account for this.

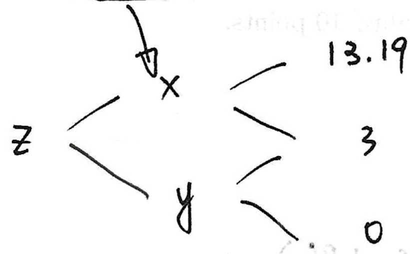


12. Given the dividend-adjusted tree above in Question 11, is it ever optimal to exercise the American call early?

$\Delta\ x = Max\{(p\cdot 13.19 + (1-p)\cdot 3)\,e^{-0.05\cdot\frac{1}{52}},\ 104.97-95\}$

$= Max\{8.065,\ 9.97\} = 9.97$ (E)

Exercise $Max\{S_T - 95, 0\}$



$\Delta\ y = Max\{(p\cdot 3 + (1-p)\cdot 0)\cdot e^{-0.05\frac{1}{52}},\ 95.26-95\}$

$= Max\{1.492,\ 0.26\} = 1.492$ (NE)

$\Delta\ z = Max\{(p\cdot x + (1-p)\cdot y)\cdot e^{-0.05\cdot\frac{1}{52}},\ 100-95\}$

$= Max\{5.707,\ 5\} = 5.707$ (NE)

13. The S&P index is at 4400. The continuous compounding interest rate is at 4.3%, and the dividend yield on the S&P 500 is 1.7%. What is the futures price for 6 month maturity S&P 500 futures?

$F_t = S_t e^{(r-q)(T-t)} = 4400\, e^{(0.043-0.017)\cdot\frac{6}{12}} \approx 4457.57$

14. You sell 2000 shares of SPY and buy 4 E-Mini S&P 500 futures. What is the market beta on this position? [hint: the SPY trades at 1/10th of the S&P 500 index. The E-mini Futures have a multiplier of 50].

$$\beta_{portfolio} = -2000 \cdot \frac{1}{10} \cdot \beta_{S\&P500} + 4 \cdot 50 \cdot \beta_{S\&P500}$$

$$= -200 \cdot 1 + 200 \cdot 1$$

$$= 0$$

15. Consider the SPY and E-mini position from the previous question again. Assume that interest rates go down 100BPS (1%). Will you make or lose money? Assume that Futures-spot-parity holds. Verbal answer: 5 points: Quantitative answer by plugging into parity formula: 8 points. Analytical answer using calculus: 10 points.

$$200 \cdot \frac{\partial (F_t - S_t)}{\partial r} \cdot \Delta r$$

$$= 200 \cdot \frac{\partial (S_t e^{r(T-t)} - S_t)}{\partial r} \cdot (-1\%)$$

$$= -2 \cdot S_t (T-t) e^{r(T-t)} < 0$$

lose money

16. Find the price of a European Call option with initial stock price $S_0 = 300$, strike $K = 300$, interest rate $r = 0.05$, volatility $\sigma = 0.4$ and time to maturity of four years $T - t = 4$ using the Black-Scholes model.

$$d_1 = \frac{\ln(\frac{S_T}{K}) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln(\frac{300}{300}) + (0.05 + \frac{1}{2} \cdot 0.4^2) \cdot 4}{0.4\sqrt{4}} = 0.65$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = d_1 - 0.4\sqrt{4} = -0.15$$

$$C = 300 \cdot N(d_1) - e^{-0.05 \cdot 4} \cdot 300\, N(d_2) = 114.48$$

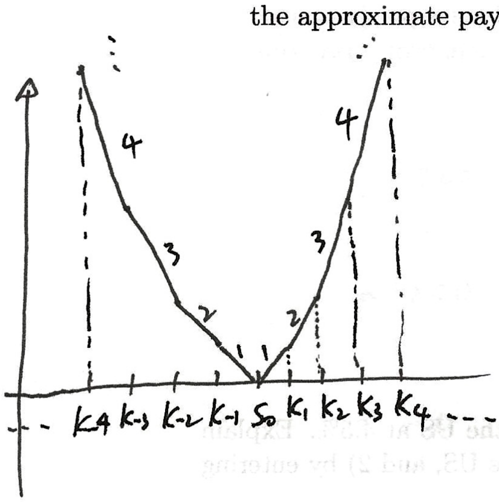
17. Without doing any math, would the European option in Question $\boxed{16}$ be worth more or less if the underlying stock pays a $10 dividend each year before expiration?

Less

18. Suppose you can borrow in Yen (JPY) at 0.5% and invest in the US at 4.5%. Explain how to exploit this 1) by borrowing in Yen and investing in the US, and 2) by entering a futures contract on JPY. Are the two equivalent?

19. Suppose the Japanese central bank (Bank of Japan) increases rates unexpectedly to 1%. This leads to an increase of 1.5% in the JPY/USD spot exchange rate. Name two ways in which the futures trade from the previous question makes or loses money.

20. Graph the payoff to a portfolio that buys every out-of-the-money PUT and CALL option with strikes $K_1, K_2, ..., K_n$ and state what the slopes are between each strike. What is the approximate payoff? What is the interpretation of the portfolio?



Approximate payoff: $a(S_T - S_0)^2$ for some a

invest $1/a$

$\rightarrow E(\text{payoff}) \approx E[(S_T - S_0)^2] = Var(S_T)$

Variance Payoff: payoff proportional to variance of the underlying stock