Asset Pricing with Endogenously Uninsurable Tail Risk

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Outline

Introduction

Challenge for macro asset pricing theory:

- Large magnitude of equity premia
  - $\rightarrow$ requires large variation in pricing kernel
- Large variations of equity premium over time
  - $\rightarrow$ requires large variation in the volatility of pricing kernel
- Large exposure of firms' cash flow to aggregate risk

This paper:

- Resolve these puzzles under a framework of dynamic contract where uninsurable idiosyncratic labor income risk feedback into pricing kernel
- Meanwhile, discount rate variations feedback into employment decisions for labor market dynamics, which further amplify the discount rate variations

Introduction

Optimal contracting in GE framework

- Diversified owners of firms insure workers using long-term compensation contracts
- Contracts restricted by limited commitment of both parties
- Moral Hazard in firm's retention effort

Key results

- Firm side limited commitment generates tail risks in earnings
- Consumption share of capital owners is procyclical
- Cash flow exposure due to endogenous operating leverage
- Moral hazard amplifies primitive shocks

Model Setup

- Discrete time with infinite horizon
- Two groups of agents: a unit measure of firm owners and a unit measure of workers
  - $\rightarrow$ In each period, workers die with prob $1-\kappa$. Same measure of new workers are born. Each with one unit of human capital.
- Both groups have common EZ preferences with RRA γ and IES ψ
- N firms. Perfectly competitive.
- If employed in period t, worker i with human capital $h_{i,t}$ produces output

$$y_{i,t} = Y_t h_{i,t}$$

  where Y_t is the aggregate productivity

$$\ln Y_{t+1} = \ln Y_t + g_{t+1}$$

  where g_t is a finite state Markov process with transition matrix $\pi(g' \mid g)$
- In each period, unemployed workers receive unemployment benefit by_{it}

Model Setup

Law of motion for the human capital:

- For the worker i who remains employed with firm j in $t+1$:

$$h_{i,t+1} = h_{i,t} e^{\eta_{j,t+1} + \varepsilon_{i,t+1}}$$

  Conditional on g_{t+1}, the firm component $\eta_{j,t+1}$ is i.i.d. across firms but common to all workers in a firm; the work-specific shock $\varepsilon_{i,t+1}$ is i.i.d. across workers; they are mutually independent.

  - Normalize (η, ε) so that $\mathbb{E}\left[e^{\varepsilon_{i,t}} \mid g_t\right] = \mathbb{E}\left[e^{\eta_{j,t}} \mid g_t\right] = 1$
  - Use $z_{i,j,t} = \left(\eta_{j,t}, \varepsilon_{i,t}\right)$ for match-specific shocks

- For an unemployed worker in $t+1$:

$$h_{i,t+1} = \lambda h_{i,t}$$

Model Setup

Matching and separation

- The probability of retaining the worker $\theta(g)$ with $\theta_H > \theta_L$
- Besides, both groups can voluntarily initiate a separation.
  - $\rightarrow$ In equilibrium, no voluntary separation.
  - $\rightarrow$ Reason: separations leads to human capital losses, therefore lower worker utility without benefiting firms. Optimal contract avoids such inefficiency.
- In each period, an unemployed worker receives an employment opportunity with prob $\chi \in (0, 1)$. Every newborn workers also have an employment opportunity.
- A worker with an employment opportunity can choose to establish a match with the firm that offers the most favorable contract. Firm compete with no cost for posting vacancies.

Model Setup

A contract to a newly employed worker at time τ specifies

$$\mathscr{C}_{i,j,\tau} \equiv \left\{ C_{i,j,t}\left(h_{i,\tau}, z_{i,j}^{\tau \to t}, g^{t} \right) \right\}_{t=\tau}^{\infty}$$

Denote $\Lambda_t\left(g^t\right)$ as state prices, then the firm value from a worker i follows,

$$V_t\left(\mathscr{C}_{i,j,\tau}\right) = y_{i,t} - C_{i,j,t} + \kappa\theta_t \mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}\left(\mathscr{C}_{i,j,\tau}\right)\right]$$

Let $U^*\left(h, g^t\right)$ be highest utility to worker from a new match, then the utility for an unemployed worker $\overline{U}\left(h_{i,t}, g^t\right)$

$$\overline{U}\left(h_{i,t}, g^t\right) = \left[(1-\beta)\left(by_{i,t}\right)^{1-\frac{1}{\psi}} + \beta\overline{\mathbb{M}}\left(h_{i,t}, g^t\right)^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}$$

$$\overline{\mathbb{M}}\left(h_{i,t}, g^t\right) = \left(\kappa\mathbb{E}_t\left[(1-\chi)\overline{U}\left(h_{i,t+1}, g^{t+1}\right)^{1-\gamma} + \chi U^*\left(h_{i,t+1}, g^{t+1}\right)^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}$$

Model Setup

The utility of a matched worker $U_t\left(h_{i,\tau}, z_{i,j}^{\tau\to t}, g^t \mid \mathscr{C}_{i,j,\tau}\right)$ follows

$$U_t\left(\mathscr{C}_{i,j,\tau}\right) = \left[(1-\beta)\left(C_{i,j,t}\right)^{1-\frac{1}{\psi}} + \beta \mathbb{M}_t\left(\mathscr{C}_{i,j,\tau}\right)^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}$$

$$\mathbb{M}_t\left(\mathscr{C}_{i,j,\tau}\right) = \left(\kappa \mathbb{E}_t\left[\theta_t U_{t+1}\left(\mathscr{C}_{i,j,\tau}\right)^{1-\gamma} + (1-\theta_t)\,\overline{U}\left(h_{i,t+1}, g^{t+1}\right)^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}$$

Frictions:

- The firm-side limited commitment requires

$$V_t\left(h_{i,\tau}, z_{i,j}^{\tau\to t}, g^t \mid \mathscr{C}_{i,j,\tau}\right) \geq 0$$

- The worker-side limited commitment requires

$$U_t\left(\mathscr{C}_{i,j,\tau}\right) \geq \overline{U}\left(h_{i,t}, g^t\right)$$

Model Setup

Let $X_t(g^t)$ be firm-owner consumption, the utility of firm owner $W_t(g^t)$:

$$W_t(g^t) = \left\{(1-\beta)X\left(g^t\right)^{1-\frac{1}{\psi}} + \beta\mathbb{N}_t\left(g^t\right)^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}$$

$$\mathbb{N}_t\left(g^t\right) = \left(\mathbb{E}_t W_{t+1}\left(g^{t+1}\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$$

The state prices $\Lambda_t(g^t)$ respects firm owners' equilibrium consumption

$$\frac{\Lambda_{t+1}\left(g^{t+1}\right)}{\Lambda_t\left(g^t\right)} = \beta\left[\frac{X_{t+1}\left(g^{t+1}\right)}{X_t\left(g^t\right)}\right]^{-\frac{1}{\psi}}\left[\frac{W_{t+1}\left(g^{t+1}\right)}{\mathbb{N}_t\left(g^t\right)}\right]^{\frac{1}{\psi}-\gamma}$$

Resource Constraint and State Variables

Use normalized promised utility $u \equiv \frac{U}{y}$ as a state variable
The resource constraint:

$$Y \int bh\Phi_0(dh) + Y \sum_{j=1}^{N} \iint c(u,S)h\Phi_j(du,dh) + Yx(S) = Y \sum_{j=1}^{N} \iint h\Phi_j(du,dh)$$

$$\Downarrow$$

$$B + \int c(u,S)\phi(du) + x(S) = \int \phi(du)$$

- where $\Phi_j(du,dh)$ is the distribution of (u,h) for workers in firm j and $\Phi_0(dh)$ is the distribution of h of unemployed workers.
- $\phi(du)$ is the average human capital of employed workers of type u

$$\phi(du) = \sum_{j=1}^{N} \int h\Phi_j(dh \mid u)$$

- B is the total compensation to all unemployed workers normalized by Y_t
- It reduces the $N+1$ two-dimensional distributions $\{\Phi_j\}_{j=0}^{N}$ into a one-dimensional measure ϕ and a scalar B
- The aggregate history can be summarized by $S \equiv (g, \phi, B)$

Normalization

Homotheticity in preferences and technology $\Rightarrow$ normalize variables by y

$$u \equiv \frac{U}{y}, \quad v(u,S) = \frac{V(y,U,S)}{y}, \quad u^*(S) = \frac{U^*(y,g^t)}{y}, \quad \overline{u}(S) = \frac{\overline{U}(y,g^t)}{y},$$
$$c(u,S) = \frac{C(y,U,S)}{y}$$

Let $x_t(g^t) = \frac{X_t(g^t)}{Y_t(g^t)}$ be the normalized consumption of the firm owners

$$\Lambda(S',S) = \beta \left[\frac{x(S')\, e^{g'}}{x(S)}\right]^{-\frac{1}{\psi}} \left[\frac{w(S')\, e^{g'}}{n(S)}\right]^{\frac{1}{\psi}-\gamma}$$

Bellman Equation

$$v(u,S) = \max_{c,\{u'(\zeta')\}_{\zeta'}} 1 - c + \kappa\theta \int \Lambda\left(S', S\right) e^{g'+\eta'+\varepsilon'} v\left(u'\left(\zeta'\right), S'\right) \Omega\left(d\zeta' \mid g\right)$$

subject to:

$$u = \left[(1-\beta)c^{1-\frac{1}{\psi}} + \beta m^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}$$

$$v\left(u'\left(\zeta'\right), S'\right) \geq 0, \text{ for all } \zeta'$$

$$\left[u'\left(\zeta'\right) - \lambda\bar{u}\left(S'\right)\right] \geq 0, \text{ for all } \zeta'$$

where

$$m = \left\{\kappa \int e^{(1-\gamma)\left(g'+\eta'+\varepsilon'\right)} \left[\theta u'\left(\zeta'\right)^{1-\gamma} + (1-\theta)\lambda\bar{u}\left(S'\right)^{1-\gamma}\right] \Omega\left(d\zeta' \mid g\right)\right\}^{\frac{1}{1-\gamma}}$$

Recursive Competitive Equilibrium

A RCE consists of

- SDF: $\Lambda(S', S)$
- Value of unemployed workers: $\overline{u}(S)$, Value from a new match: $u^*(S)$
- Firm value function: $v(u, S)$
- Policy Functions: $\left(c(u, S), \{u'(u, S, \zeta')\}_{\zeta'}\right)$
- Consumption share of firm owners $x(S)$
- Law of motions Γ_ϕ and Γ_B

such that

1. The firm value function and the policy functions solve the contracting problem
2. SDF respects $x(S)$
3. $u^*(S)$ and $\overline{u}(S)$ satisfy

$$u^*(S) = \max\{u : v(u, S) \geq 0\} \quad \bar{u}(S) = \left[(1-\beta)b^{1-\frac{1}{\psi}} + \beta\lambda\bar{m}(S)^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}$$

4. Law of motions Γ_ϕ and Γ_B satisfy ...
5. The resource constraint holds

Proposition

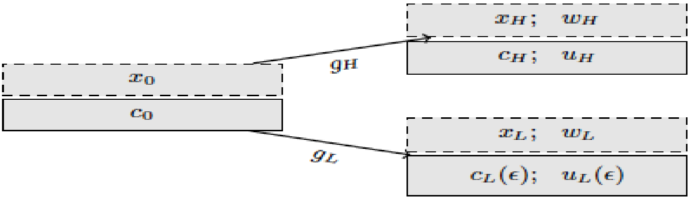
There exist threshold levels $\underline{\varepsilon}(u, S, g')$ and $\bar{\varepsilon}(u, S, g')$ such that

1. $\forall \varepsilon' + \eta' > \bar{\varepsilon}(u, S, g')$,
$$u'(u, S, \zeta') = \lambda \bar{u}(S')$$
2. $\forall \varepsilon' + \eta' < \underline{\varepsilon}(u, S, g')$,
$$u'(u, S, \zeta') = u^*(S')$$
3. $\forall \varepsilon' + \eta' \in [\underline{\varepsilon}(u, S, g'), \bar{\varepsilon}(u, S, g')]$
$$\left[\frac{x(S')}{x(S)}\right]^{-\frac{1}{\psi}} \left[\frac{w(S')}{n(S)}\right]^{\frac{1}{\psi}-\gamma} = e^{-\gamma(\eta'+\varepsilon')} \left[\frac{c(u'(u, S, \zeta'), S')}{c(u, S)}\right]^{-\frac{1}{\psi}} \left[\frac{u'(u, S, \zeta')}{m(u, S)}\right]^{\frac{1}{\psi}-\gamma}$$

- For a large $\eta' + \varepsilon'$ shock, u' is bounded below by a constant. So the level of continuation utility $ye^{\varepsilon'+\eta'}u'$ must increase with positive productivity shocks
- For a negative enough realizations of $\eta' + \varepsilon'$, the level of continuation utility need to decrease with a more negative productivity shock to provide incentives for the firm to continue the match $\Rightarrow$ **Endogenous uninsurable tail risk in labor earning**
- In the interior, the intertemporal MRS of all agents has to equalize

A Two Period Model

- $g_t \in \{g_L, g_H\}$. $g_t = g_1$ for all $t \geq 1$
- Each firm has a single worker and $\eta = 0$
- $\varepsilon \mid g_L \sim$ negative exponential (ξ) $\quad \varepsilon \mid g_H \sim$ degenerate
- Preferences satisfy $\gamma \geq \psi = 1$
- Agency friction: firm-side limited commitment only
- For t = 2,3,..., both employed and unemployed workers produce output and consume a fraction of their output $C_t = \alpha y_t$



Results of the simple model

Optimal risk sharing

$$\underbrace{\left(\frac{x_L}{x_H}\right)^{-\frac{1}{\psi}}\left(\frac{w_L}{w_H}\right)^{\frac{1}{\psi}-\gamma}}_{\text{MU of capital owner}} = \underbrace{\left(\frac{e^{\underline{\varepsilon}}c_L(\underline{\varepsilon})}{c_H}\right)^{-\frac{1}{\psi}}\left(\frac{e^{\underline{\varepsilon}}u_L(\underline{\varepsilon})}{u_H}\right)^{\frac{1}{\psi}-\gamma}}_{\text{MU of marginal worker}}$$

Market clearing

$$x_H + c_H = 1$$

$$x_L + \int e^{\varepsilon} c_L(\varepsilon) f\left(\varepsilon \mid g_L\right) = 1$$

Proposition

(i) If $\gamma = \frac{1}{\psi}$, firm owners' consumption share is countercyclical, $x_H < x_L$

(ii) $\exists \hat{\gamma} \in [1, 1+\xi)$ such that if $\gamma > \hat{\gamma}$, then $x_H > x_L$

Intuition of the simple model

Optimal risk sharing with expected utility

$$\frac{x_L}{x_H} = \frac{e^{\underline{\varepsilon}} c_L(\underline{\varepsilon})}{c_H}$$

If without the friction

$$e^{\varepsilon} c_L(\varepsilon) = c_H = c; x_H = x_L$$

- A larger fraction of worker-firm pairs are constrained in recessions
- Constrained firms cut compensation, so a higher fraction of resources available for firm owners during a recession
- These resources are allocated between firm owners and the unconstrained workers by equating their intertemporal MRS.
- The consumption share for firm owners increases in a recession: $x_L > x_H$

Intuition of the simple model

Optimal risk sharing with EZ preferences

$$\underbrace{\left(\frac{x_L}{x_H}\right)^{-\frac{1}{\psi}}\left(\frac{w_L}{w_H}\right)^{\frac{1}{\psi}-\gamma}}_{\text{MU of capital owner}} = \underbrace{\left(\frac{e^{\underline{\varepsilon}}c_L(\underline{\varepsilon})}{c_H}\right)^{-\frac{1}{\psi}}\left(\frac{e^{\underline{\varepsilon}}u_L(\underline{\varepsilon})}{u_H}\right)^{\frac{1}{\psi}-\gamma}}_{\text{MU of marginal worker}}$$

$\star$ If $\gamma > \frac{1}{\psi}$, then marginal utility decreases with continuation utility

New force: tail risks in the future affect current marginal utilities

$\Rightarrow$ Optimal risk sharing requires transferring resources away from the firm owners to unconstrained workers in recessions
$\Rightarrow$ If risk aversion is high enough, this effect dominates
$\Rightarrow$ procyclical consumption share of capital owner $x_H > x_L$

Optimal contract generates operating leverage

Define the valuation risk exposure or beta of a firm as

$$\mathcal{B}(u_0) = \left(\frac{v_H(u_0)}{\mathbb{E}\left[e^{\varepsilon} v_L(u_0, \varepsilon)\right]} \right)$$

Proposition

For $\gamma > \gamma^* \in (\frac{1}{\psi}, 1 + \xi)$, we have $\frac{\partial}{\partial u_0}\mathcal{B}(u_0) > 0$

- Individual firm's risk exposure (hence, expected return) increases in promised utility
- In the model, valuation ratio decreases with promised utility

$\Rightarrow$ Expected return decreases with valuation ratio
$\Rightarrow$ Resolve Value premium puzzle

A Brief Description of Algorithm

- Use an algorithm similar to Krusell and Smith (1998): Replace the distribution ϕ with suitable summary statistics x_t
- Assume a forecasting rule Γ_x for x_t

$$\log x' = a\left(g, g'\right) + b\left(g, g'\right) \log x$$

- The forecasting function Γ_x pins down SDF $\Lambda\left(g', x, g\right)$
- Inner Loop: Given $\Gamma_x\left(x, g, x'\right)$ and $\Lambda\left(g', x, g\right)$, solve value function and policy functions in contracting problem
- Outer loop: Given the policy functions, simulate a panel of agents and use the simulated data to update the law of motion Γ_x
- Iterate until the function Γ_x converge: the unconditional R^2 approaches 99.9%

Calibration

Table 1: PARAMETERS

Parameters	Values	Targeted moments	Values
Aggregate Risk			
g_H, g_L	0.35%, -0.15%	Mean, std of consumption growth	1.08%, 2.14%
$\pi(g_H \| g_H)$	0.99	Duration of booms	12 yrs
$\pi(g_L \| g_L)$	0.95	Duration of recessions	4 yrs
$\sigma_{\mathcal{E}}$	1.2%	Autocorr of consumption growth	0.44
Labor Market			
$a_{1,H}, a_{1,L}$	.995,.9925	Annualized separations rates	2%, 3%
χ	8%	Long-term unemployment duration	3 years
λ	96%	PV of earning losses on separation	30%
b	1	Flow value of unemployment	40-95%
κ	0.99	Duration of working life	25 years
Idiosyncratic Risk			
α	82%	Across firm wage variation	40%
σ_L, σ_H	7.0%, 8.0%	Std. of labor earnings change in booms and recessions	32%, 31%
τ, ρ	4.155, 2%	Kelly skewness of labor earnings change in booms and recession	-3.2%, -8.9%
Other parameters			
β,ψ,γ	0.989, 2, 5	Discount factor, IES, risk aversion	

- Labor share of consumption in the data has mean 75%, s.d. 2.94% and autocorrelation 0.88. Model: mean 70%, s.d. 3% and autocorrelation 0.58.

Aggregate Asset Pricing Implications

Table 2: AGGREGATE ASSET PRICING IMPLICATIONS

Moments	Model		Data
	Baseline	No Frictions	
Excess return on consumption			
mean	3.59%	0.62%	-
std.	7.40%	2.86%	-
Excess return on dividends			
mean	3.67%	0.62%	6.06%
std.	7.61%	2.86%	19.8%
Std of log SDF			
booms	19.15%	17.83%	38.00%
recessions	35.70%	27.80%	66.00%
Risk free rate			
mean	2.81%	5%	0.56%
std.	2.86%	0.85%	2.89%

- Use 50% D/E ratio, the model implies a equity premium of 5.5%

Cross-section: value premium

- Value premium: Stock with low price to earning per share ratio (value stocks) earn higher average returns
- Sort stocks into three portfolios ranked by earnings-to-price ratios.

	Mean High-minus-low Return
Data	6.27%
Model	4.66%

- Mechanism: firms with high-u workers have a high operating leverage and a low valuation ratio. Such firms should have a higher expected return.

Summary

- A unified theory of asset pricing and labor market dynamics
- Uninsured idiosyncratic tail risk in labor earnings arises as an outcome of optimal risk-sharing arrangements with frictions
- Tail risk generate equity premia and its time variation drives the variation in equity premia
- A recent paper by Tong and Ying (2020) adds capital into the model.

Appendix

Moral Hazard and Endogenous θ

- Firm can exert effort to change the probability of retaining the worker θ_t (also the effort level) with cost function $A(\theta)$.
- Retention effort choice should incentive compatible from the firm's perspective

$$A'(\theta) = \kappa \int \Lambda \left(S', S\right) e^{g' + \eta' + \varepsilon'} v \left(u' \left(\zeta'\right), S'\right) \Omega \left(d\zeta' \mid g\right)$$

- Higher discount rate in recession decreases the present value of profits that a worker can create, hence lowering the retention effect and increasing unemployment rate
- Higher separations in recessions magnify the downside risk in labor earnings and hence the need for insurance, leading to more procyclical consumption for marginal agents
- GE: Reinforce each other

Aggregate Return Predictability

Horizon (quarters)	Model				Data	
	Baseline		No Frictions			
	β	R^2	β	R^2	β	R^2
2	-0.356	0.157	-0.381	0.001	-0.062	0.042
4	-0.580	0.251	-0.739	0.001	-0.113	0.075
8	-0.788	0.329	-1.409	0.002	-0.190	0.119
12	-0.860	0.345	-2.029	0.003	-0.236	0.142
16	-0.871	0.328	-2.600	0.003	-0.277	0.166

- Regressions: $\sum_{j=1}^{J}\left(r_{t+j}-r_{f,t+j}\right)=\alpha+\beta\left(pd_t\right)+\epsilon_{t+j}$

Cross-section: labor share and excess return

Coefficients	Using LS	Using ELS
Labor share	1.38	1.25
	(0.41)	(0.19)
Time fixed effects	Yes	Yes
no. of obs.	15170	83611
no. of entities	1645	9591

- Regressions: Excess Return $_{f,t+1} = \alpha_r + \beta_r \times$ LaborShare $_{f,t} + \lambda_{rt}$
- Labor share predicts expected returns
- Robust to including controls such as leverage and total assets