Neoclassical Growth with Long-Term One-Sided Commitment Contracts

Dirk Krueger and Harald Uhlig, 2022

December 7, 2022

Outline

Introduction

This paper

- Characterizes the stationary general equilibrium of neoclassical production economy with
  - Idiosyncratic income risk
  - Long-term dynamic insurance contract: Endogenous incomplete insurance due to limited commitment friction
    - $\rightarrow$ Different from the Standard incomplete market GE model in Aiyagari (1994) where incomplete asset market is exogenous
  - A continuous-time setup with analytical solutions

Model Setup: Household

Preference:

$$E\left[\int_0^\infty e^{-\rho t}u(c(t))dt\right] \quad \text{with} \quad 0 < -\frac{u''(c)c}{u'(c)} < \bar{\sigma} < \infty$$

- For the analytical characterization, assume log utility
- Infinitely lived agents of unit mass

Labor Productivity and Endowments:

- Labor productivity follows two-state Markov process

$$z_{it} \in Z = \{z_l, z_h\} = \{0, \zeta\}$$

- The transition rate from high to low is ξ and low to high is ν
  - Stationary distribution over productivity follows $(\psi_l, \psi_h) = \left(\frac{\xi}{\xi+\nu}, \frac{\nu}{\xi+v}\right)$
- Newborn draw productivity from the stationary distribution
- Normalize the average productivity to one: $\frac{\nu}{\xi+\nu}\zeta = 1 \quad \rightarrow \quad \nu(\zeta - 1) = \xi$
- Without risk-sharing contract, households consume nontradable endowment $\underline{u} = u(\chi) > -\infty$

Model Setup: Firms

- A competitive sector of production firms with Cobb-Douglas production function

$$AF(K, L) = AK^{\theta} L^{1-\theta}$$

- Capital Accumulation is linear and depreciates at rate δ
- Denote w to be per efficiency unit of labor and r to be rental rate of capital
- Labor is supplied inelastically
- Efficiency unit of labor supplied by high and low productivity households are z_h and z_l respectively, so the aggregate efficiency unit of labor

$$L = \frac{\nu}{\xi + \nu} \zeta = 1$$

Model Setup: Financial Intermediaries and risk-sharing contracts

- A competitive sector of risk-neutral intermediaries
  - Maximize profits and do not have resources on their own
- Households insure against idiosyncratic income risk with intermediaries
- Intermediaries invest the premium payments in capital and so their discount rate is r
- Intermediaries are well-diversified and so not exposed to any risk

One-side Limited Commitment:

1. Intermediaries can fully commit to the contract
2. **Contracting Friction:** Households are free to leave the contract at any time and sign up with a new intermediary

Model Setup: Timing of Events

- At time 0, a newborn household draws labor productivity and signs a contract with intermediaries, delivering lifetime utility $U^{\text{out}}(z)$
- At $t > 0$, first z is realized. Then the hh chooses to commit or leave the contract. In the latter case, hh signs a new contract with another intermediary and receives a lifetime utility $U^{\text{out}}(z)$.

Model Setup: Contract Design Problem (Cost-minimizing Contracts)

Dual Problem:
Intermediaries minimize the net present value of the contract costs $V(z, U)$

$$V(z,U) = \min_{\langle c(\tau)\rangle \geq 0} \mathbf{E}_t \left[\int_t^\infty e^{-r(\tau - t)} [wc(\tau) - wz(\tau)] d\tau \mid z(t) = z \right]$$

subject to

1. Promised Keeping Constraint

$$\mathbf{E}_t \left[\int_t^\infty e^{-\rho(\tau - t)} u(wc(\tau)) d\tau \mid z(t) = z \right] \geq U$$

2. Limited Commitment Constraint

$$\mathbf{E}_s \left[\int_s^\infty e^{-\rho(\tau - s)} u(wc(\tau)) d\tau \mid z(s) \right] \geq U^{\text{out}}\left(z(s)\right)$$

for all $s > t$, for all $\tau \geq t$, for all $z \in Z$ and all $U \in \left[U^{\text{out}}(z), \frac{\bar{u}}{\rho} \right)$.

Model Setup: Equilibrium

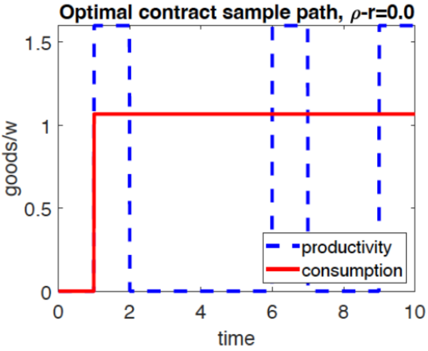
A stationary equilibrium consists of $\{U^{\text{out}}(z)\}_{z \in Z}$, $c(\tau, z, U)$, $V(z, U)$, w, r, $\phi(c)$

1. Given $\{U^{\text{out}}(z)\}_{z \in Z}$ and r, the consumption insurance contract $c(\tau, z, U), V(z, U)$ solves contract design problem
2. The outside options lead to zero profits of intermediaries
$$\forall z \in Z, V\left(z, U^{\text{out}}(z)\right) = 0$$
3. r and w satisfy the firm's optimality conditions: $r = AF_K(K, 1) - \delta \quad w = AF_L(K, 1)$
4. The goods market clears
$$\int wc\phi(c)dc + \delta K = AF(K, 1)$$
5. The capital market clears
$$\underbrace{\frac{w\left[\int c\phi(c)dc - 1\right]}{r}}_{K^s} = K^d$$
   - $w + rK^s = \int wc\phi(c)dc$
6. The stationary consumption pdf is consistent with the dynamics of contract

Characterizing Optimal Contract

Full insurance in the long-run: $\rho = r$

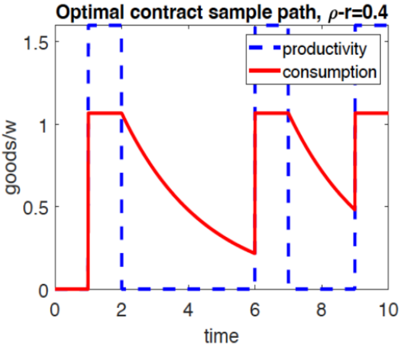
As long as $z_{it} = 0$, hhs consume the nontradable endowment $c_l = \chi$ and signs a consumption contract that has constant consumption $c_h = \left(\frac{\rho+v}{\rho+v+\xi}\right)\zeta$ and remains there forever when the instant labor productivity rises to ξ



Characterizing Optimal Contract

Partial Insurance: $r < \rho$

1. Whenever $z = \zeta$, the hh consumes a constant amount $c_h = \left(\frac{\rho+\nu}{\rho+\nu+\xi}\right)\zeta$
2. When the productivity switches to 0, consumption drift down according to the full-insurance Euler equation $\frac{\dot{c}(t)}{c(t)} = r - \rho < 0$
3. Denote τ the time elapsed since productivity last switched from $z = \zeta$ to 0. Then, $c(\tau) = c_h e^{(r-\rho)\tau}$



Stationary Consumption Distribution

Full-Insurance: $\rho = r$

Consumption distribution places all mass $\phi_h = 1$ on c_h

Hint: Individuals flow out of c_l at rate ν and no inflow to this consumption level

Partial Insurance: $r < \rho$

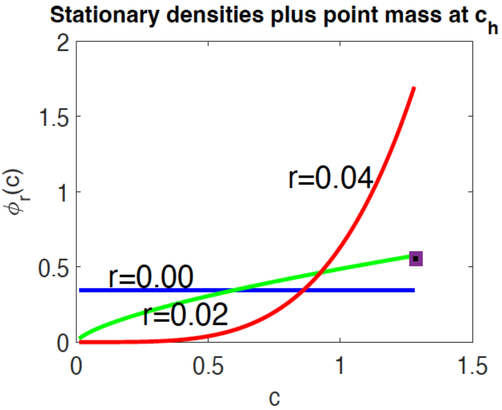
In this case, the stationary distribution is given by a mass point at c_h and a Pareto density below it.

$$\phi_r(c) = \begin{cases} \frac{\xi \nu (c_h)^{-\frac{\nu}{\rho - r}}}{(\rho - r)(\nu + \xi)} c^{\frac{\nu}{\rho - r} - 1} & \text{if} \quad c \in (0, c_h) \\ \frac{\nu}{\nu + \xi} & \text{if} \quad c = c_h \end{cases}$$

Hint: On $(0, c_h)$, the distribution satisfies the Kolmogorov forward equation

$$0 = -\frac{d[(r - \rho)c\phi(c)]}{dc} - v\phi(c)$$

Stationary Consumption Distribution



General Equilibrium and Market Clearing r

Production Side: supply of consumption goods and demand for capital

$$\kappa^d(r) := \frac{K^d(r)}{w(r)} = \frac{\theta}{(1-\theta)(r+\delta)}$$

$$G(r) = \frac{AF\left(K^d(r), 1\right) - \delta K^d(r)}{w(r)} = 1 + \frac{\theta r}{(1-\theta)(r+\delta)}$$

Consumption distribution $\to$ Demand of consumption $\to$ supply of capital

$$C(r) = \int c\phi_r(c)dc \quad \to \quad \kappa^s(r) = K^s(r)/w(r) = \frac{C(r)-1}{r}$$

Supply of Capital:

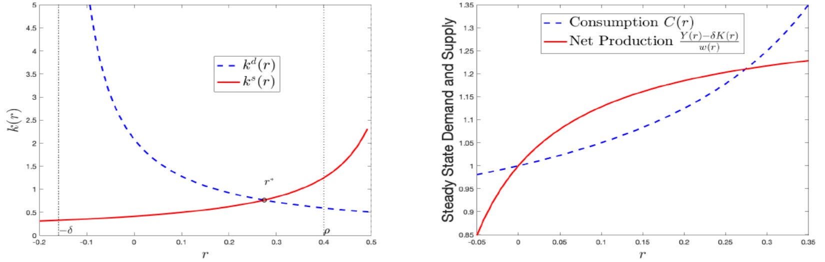
1. Full insurance ($r = \rho$): $\kappa^s(r) := \kappa^{FI} = \frac{\xi}{\nu(\nu+\rho+\xi)}$
   - Unique $\rho^{FI} = r^{FI}$: $\kappa^d(r^{FI}) = \kappa^{FI}$
2. Partial insurance ($r < \rho$): $\kappa^s(r) = \frac{\xi}{(\nu+\rho-r)(\nu+\rho+\xi)}$

General Equilibrium and Market Clearing r

Unique equilibrium features partial insurance

Suppose $\frac{\theta}{(1-\theta)(\rho+\delta)} < \frac{\xi}{\nu(\nu+\rho+\xi)}$, then there exists a unique stationary equilibrium with interest rate $r^* \in (-\delta, \rho)$ follows

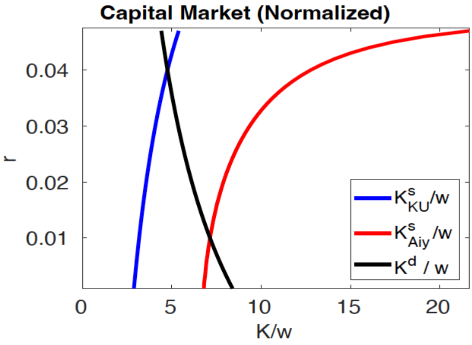
$$r^* = \frac{\theta(\nu+\rho+\xi)(\nu+\rho) - \xi\delta(1-\theta)}{\xi + \theta(\nu+\rho)}$$



(a) Capital Demand $\kappa^d(r)$ and Supply $\kappa^s(r)$ as a Function of the Interest Rate r

(b) Goods Demand $w(r)C(r)$ and Net Supply $Y(r) - \delta K(r)$ as a Function of r

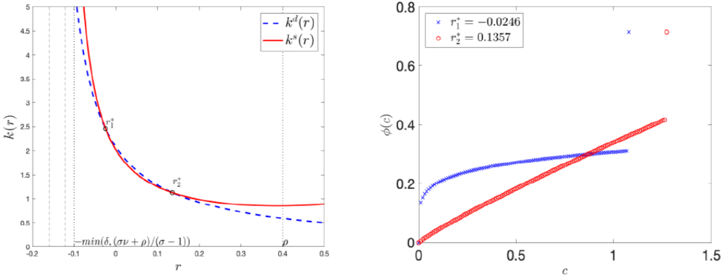
Comparsion to Aiyagari (1994)



- $\forall r$, the supply of assets is less in KU economy $\rightarrow$ a lower r^*
  - Intuition: In the presence of explicit income insurance, the need to accumulate capital for precautionary reasons is reduced

Multiple Equilibria

Figure 5: Two equilibria with partial insurance when $\sigma > 2$.



(a) Capital Market Clearing

(b) Equilibrium Consumption Distributions

This figure plots an example of two equilibria, both with partial insurance, under parameter values $\sigma = 10, \theta = 0.25, \delta = 0.16, \nu = 0.05, \xi = 0.02, \rho = 0.4$. The two equilibrium interest rates are given by $r_1^* = -0.0246, r_2^* = 0.1357$. Left panel: solid line represents the capital supply curve $k^s(r)$, dashed line represents the capital demand curve $k^d(r)$. The right panel displays the two equilibrium consumption distributions.