Solution

FIN 730: Midterm Exam
Fall 2021

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Name: ____________________

Campus ID: ________________

Section: ___________________

Instructions

- This is a closed-book, closed-note exam. You are allowed to use your pens/pencils and a non-programmable calculator only.
- A formula sheet is attached to the back of the exam. If you remove the formula sheet during the exam, please ensure that the stapling of the exam portion of this document remains intact.
- Show all your work in a well-organized fashion if you wish to get full credit.
- Use legible handwriting. If we can't read what you have written, it will not count.
- When you exit the classroom, please refrain from speaking to other students.

I understand and agree to abide by the exam instructions listed above.

Signature: ____________________

Date: ______________________

1. The S&P 500 index is at 4500. Assuming that the index pays a continuous dividend yield of 1.5%, and that interest rates are 0.2%, find the price of a 3 month S&P 500 futures contract.

$$F_t = 4500 \cdot e^{(0.002 - 0.015)\cdot\left(\frac{3}{12}\right)}$$

$$= \$4485.40$$

2. You enter into the futures contract from the previous question. On the expiration date the S&P 500 index is at 4500. What is your total profit(loss) on the trade?

$$S_T - F_t = 4500 - 4485.40 = \$14.60 \text{ profit}$$

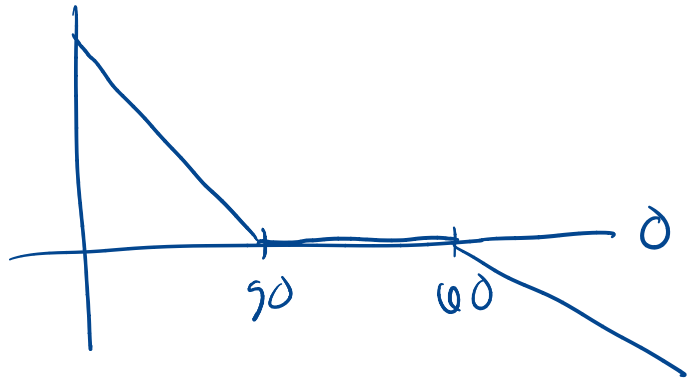
3. You sell short a Straddle with strike 50. On the expiration date the underlying stock is at 69. What is your payoff?

$$\text{short straddle payoff} = -|S_T - K|$$

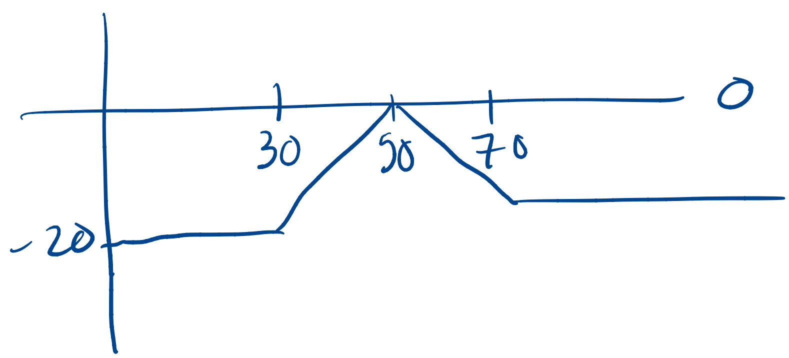
$$= -|50 - 69|$$

$$= -\$19$$

4. You buy a put with strike 50 and sell a call with strike 60. Graph the payoff diagram of this portfolio.



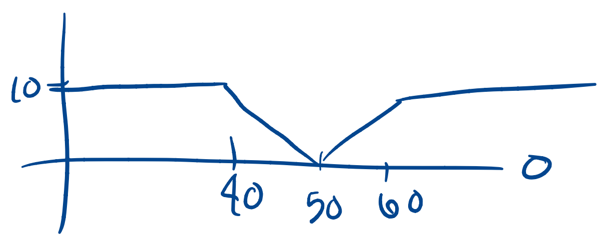
5. You short a straddle with strike 50, long a put with strike 30, and long a call with strike 70. Draw the payoff diagram of this portfolio.



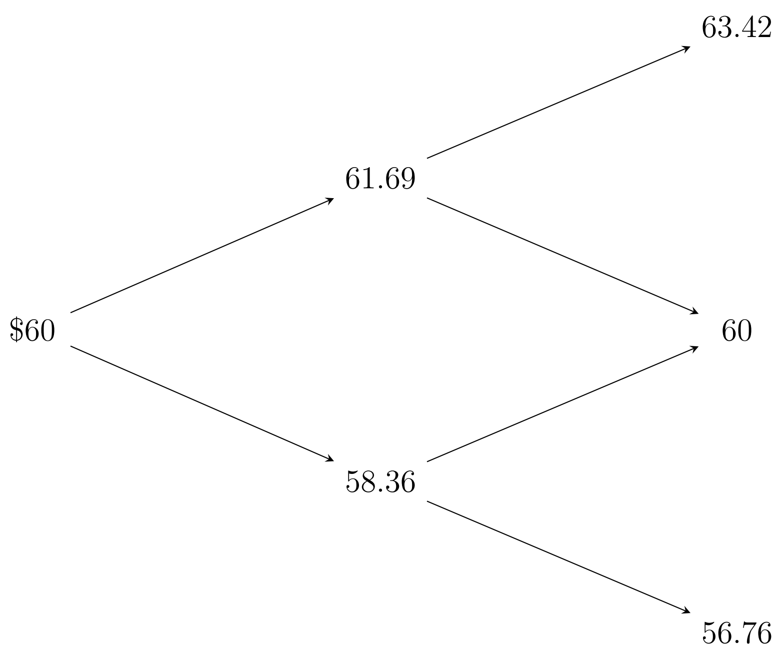
6. What could be the motive of someone buying the option portfolio from the previous question?

Shorting a straddle is selling volatility. It is a bet that the volatility of the underlying asset will not move that much. when you long call & long put w/ different strike prices you are minimizing your future losses. you get positive cash flow from shorting straddle & are betting the portfolio is ATM @ expiry.

7. You buy a straddle with strike 50, sell a call with strike 60 and sell a put with strike 40. Draw the payoff diagram of this portfolio.



8. Suppose you buy the portfolio in question 7. On the expiration date the underlying stock is at 65. What is your total payoff?



The above tree is a Cox-Ross-Rubenstein tree with $\triangle t = 1/52$, $r = 0.01$. Use these values to answer Questions 9 - 13.

9. Find the volatility, σ, used to construct the tree.

$(60)(u) = 61.69 \quad \& \quad u = e^{\sigma\sqrt{\Delta t}}$

$\Rightarrow \dfrac{61.69}{60} = e^{\sigma\sqrt{\frac{1}{52}}} \quad \Rightarrow \sigma = 0.2003$

10. Find the value of a European call option with strike 59 that matures in two weeks.

$$
\begin{tikzcd}[row sep=small]
 & & 4.42 \\
 & C_u \arrow[ur, no head] \arrow[dr, no head] & \\
C_0 \arrow[ur, no head] \arrow[dr, no head] & & 1 \\
 & C_d \arrow[ur, no head] \arrow[dr, no head] & \\
 & & 0
\end{tikzcd}
$$

$p = 0.4965, \ u = 1.028, \ d = 0.973$

$$C_u = (0.4965(4.42) + (1-0.4965)(1))e^{-0.01/52} = 2.6975$$

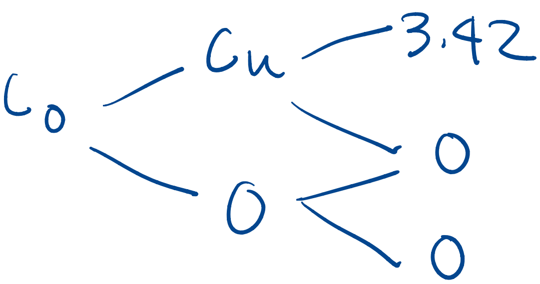
$$C_d = (0.4965(1) + (1-0.4965)(0))e^{-0.01/52} = 0.4964$$

$$c_0 = (0.4965(2.6975) + (1-0.4965)(0.4964))e^{-0.01/52} = \$1.5889$$

11. Assume that the tree above represents the stock price under the assumption that the stock does not pay dividend. Modify the tree so as to represent the price of the stock assuming that the stock pays a $1 dividend in week 2 such that the stock price in week 2 (terminal period) represents the ex-dividend price.

$$
\begin{tikzcd}[row sep=small]
 & & 62.42 \\
 & 61.69 \arrow[ur, no head] \arrow[dr, no head] & \\
60 \arrow[ur, no head] \arrow[dr, no head] & & 59 \\
 & 58.36 \arrow[ur, no head] \arrow[dr, no head] & \\
 & & 55.76
\end{tikzcd}
$$

12. Find the time 0 value of an American Call option with 2 weeks to maturity and a strike of 59 on the dividend paying stock using the tree in the previous question.



exercise C_u

$\max(2.69, 0) = 2.69$

dont exercise C_u

$((3.42)(0.4965) + (0)(1-.4965))\, e^{-\frac{0.01}{52}}$

$= 1.697$

exercise C_0

$\max(1, 0) = 1$

dont exercise C_0

$[(2.69)(0.4965) + 0] = e^{-0.01/52} = 1.335$

$\Rightarrow C_0 = \$1.335$

13. Find the time 0 price of a 3 week maturity futures on the dividend paying stock (again, information given above Question 9 applies).

$F_0 = S_0 e^{r(T-t)} - D e^{r(T-S)}$

$= \$60\, e^{(0.01)(3/52)} - \$1^{(0.01)(\frac{1}{52})}$

$= \$59.03$

	LAST	CHANGE	
EUR.USD	1.1593^4	$+32^4$	0.28%
EUR Nov15'21 @GLOBEX	1.15960	+0.00...	0.29%
EUR Dec13'21 @GLOBEX	1.16030	+0.00...	0.30%
EUR Jan14'22 @GLOBEX	1.16125	+0.00...	0.26%
EUR Mar14'22 @GLOBEX	1.16260	+0.00...	0.26%
EUR Jun13'22 @GLOBEX	1.16500	+0.00...	0.26%
EUR Dec19'22 @GLOBEX	1.16990	+0.00...	0.15%

For Question 14 and 15, use the above USD-EUR futures prices. The spot EUR-USD exchange rate is given in the first entry.

14. A corporation has a 130 Million EUR accounts payable come due January 14, 2022. Detail what to do to hedge the currency risk associated with this cash flow. Show the cash-flows at initiation (today) and January 14, 2022.

long futures contract

CF0 : 0

CF Jan14 : (-130 mil EURO)(1.1625 $\frac{USD}{EURO}$) + 130 mil Euro -130 mil Euro

15. Find the interest rate differential $r_{US} - r_{EU}$ for 4-month interest rates (hint: the January 14, 2022 maturity contract has approximately 5 months to maturity).

$$F_t = S_t e^{(r_{US} - r_{EU})(T-t)}$$

$$\ln\left(\frac{F_t}{S_t}\right) = (r_{US} - r_{EU})(T-t)$$

$$\Rightarrow r_{US} - r_{EU} = \frac{\ln(F_t) - \ln(S_t)}{(T-t)} = \frac{\ln(1.16125 - \ln(1.1593)}{4/12}$$

$$= 0.005042$$

$$= .5042\%$$

16. The S&P 500 index is at 4500. Use the Black-Scholes formula to find the price of a Call with 1 year to maturity, strike of 4500. Assume that the continuously compounding interest rate is 0%, and that the index has an annualized volatility of $\sigma = 0.16$ (16%).

$C_t = \$287.1$

17. Suppose the market price of the option in the previous question equals the price you found in that question. What is the implied volatility?

implied vol = 0.16 or 16%

18. Suppose the implied volatility of the option from the previous two questions increases. What is the impact on the market price of the option? Assume everything else remain as in Question 17.

iy impeied vol $\uparrow$, mkt prices of options $\uparrow$

19. Given your answer to the previous question, do you think the VIX index is likely to have increased, decreased or remained unchanged? Justify the response.

unm impeied vol $\uparrow$, VIX $\uparrow$

Revisit your answer from Question 10, including all the information pertaining to the stock price tree in Questions 9 (i.e, the non-dividend tree) through 10 in answering the following question.

20. (1.5x credit) A compound option is an option on an option. For example, a call on a call has payoff $\max(C_\tau - K, 0)$ where C_τ is the price of a Call option at time τ and K is the strike of the compound option. Note here that the expiration date for the underlying call, T, typically is after the maturity of the compound call ($0 < \tau < T$). Also, the strike of the compound option is different from the strike of the underlying option.

    Use the binomial tree to find the price of a one-week ($\tau = 1$) maturity compound call with $K = \$1$ strike on an underlying European call with two week maturity ($T = 2$) and strike 59 (hint: use information about the price of the underlying call from Question 10).

C_0

$\max(C_1 - K, 0) = \max(2.6975 - 1, 0) = 1.6975$

$\max(C_1 - K, 0) = \max(0.4964 - 1, 0) = 0$

$$\Rightarrow C_0 = ((0.4965)(1.6975) + 0)e^{\frac{-0.01}{52}} = \$0.84$$

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