FIN 330: Midterm Exam
Fall 2022

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Name: ____________________

Campus ID: ________________

Section: ____________________

Instructions

- This is a closed-book, closed-note exam. You are allowed to use your pens/pencils, your calculator only.
- A formula sheet is attached to the back of the exam. If you remove the formula sheet during the exam, please ensure that the stapling of the exam portion of this document remains intact.
- Show all your work in a well-organized fashion if you wish to get full credit.
- Use legible handwriting. If we can't read what you have written, it will not count.
- When you exit the classroom, please refrain from speaking to other students.

I understand and agree to abide by the exam instructions listed above.

Signature: ____________________

Date: ____________________

1. You receive $1900 in 289 days from now. Assume a 365 day-per-year convention and compute the present value using continuous compounding with a 3% discount rate.

$$1900 e^{-0.03 \cdot \frac{289}{365}} \approx 1855.4$$

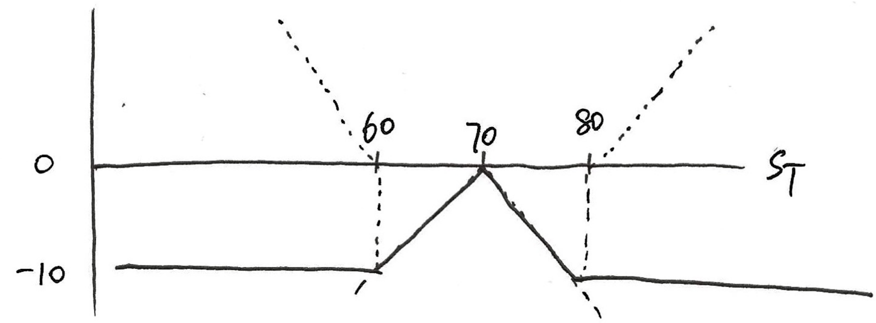
2. You sell (short) a European Put with strike 90. On the expiration date the underlying stock is at 65. What is your payoff?

$$-\operatorname{Max}\{K - S_T, 0\} = -\operatorname{Max}\{90 - 65, 0\} = -25$$

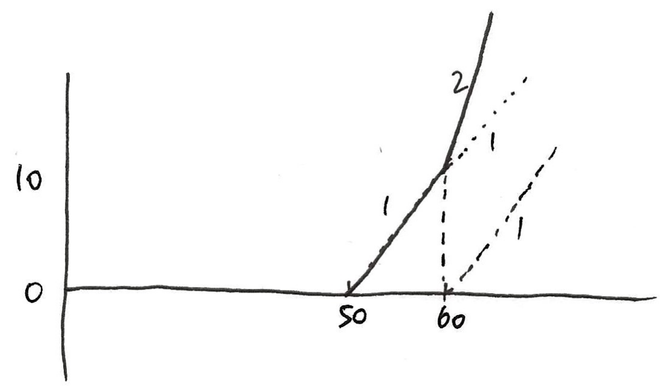
3. You sell a straddle with strike 70. The price of the put is $4 and the call is $6. On the expiration date the underlying stock is at 60. What is your profit?

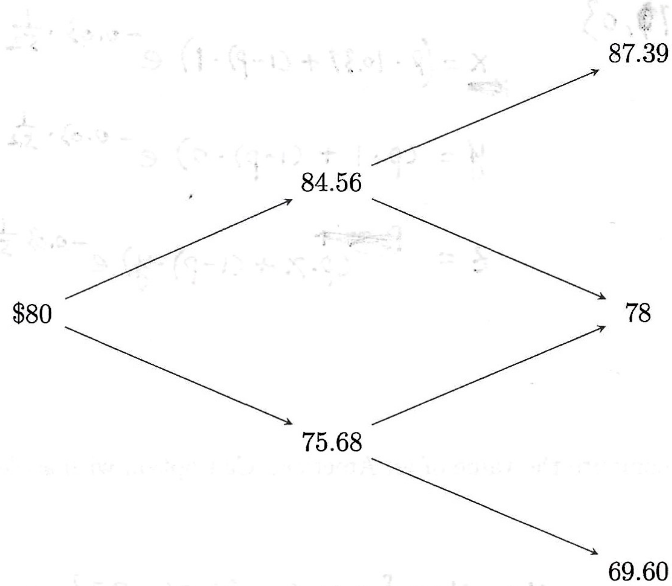
$$-|S_T - K| + c + p = -|60 - 70| + 4 + 6 = 0$$

4. You sell a straddle with strike 70, buy a put with strike 60 and buy a call with strike 80. Graph the payoff on this portfolio.



5. You buy a call with strike 50 and another call with strike 60. Graph the payoff.





The above tree is a Cox-Ross-Rubenstein tree with $\triangle t = 1/52$, $r = 0.03$, $\sigma = 0.4$ Use these values to answer Questions 6 - 8.

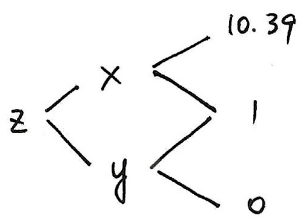
$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.4\sqrt{1/52}}$$

$$d = 1/u$$

$$p = \frac{e^{0.03 \cdot \frac{1}{52}} - d}{u - d} \approx 0.4913$$

6. Use the Binomial tree to compute the value of a European Call option with strike 77.

$\text{Max}\{S_T - 77, 0\}$

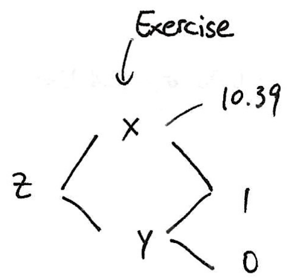


$$x = (p \cdot 10.39 + (1-p) \cdot 1)\, e^{-0.03 \cdot \frac{1}{52}} \approx 5.6104$$

$$y = (p \cdot 1 + (1-p) \cdot 0)\, e^{-0.03 \cdot \frac{1}{52}} \approx 0.4911$$

$$z = (p \cdot x + (1-p) \cdot y)\, e^{-0.03 \frac{1}{52}} \approx 3.0046$$

7. Use the Binomial tree to compute the value of an American Call option with strike 77.



$$x = \text{Max}\{5.6104,\ 84.56 - 77\} = 7.56$$

$$y = \text{Max}\{0.4911,\ 75.68 - 77\} = 0.4911$$

$$z = \text{Max}\{(p \cdot x + (1-p)\, y)\, e^{-0.03 \frac{1}{52}},\ 80 - 77\}$$

$$\doteq \text{Max}\{3.962,\ 3\}$$

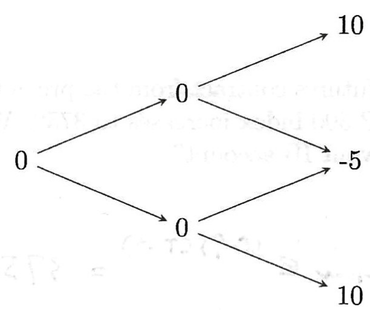
$$= 3.962$$

8. The stock price tree above includes a dividend payment in the final period. What is the dividend payment?

$$u = e^{0.4\sqrt{1/52}} \approx 1.057$$

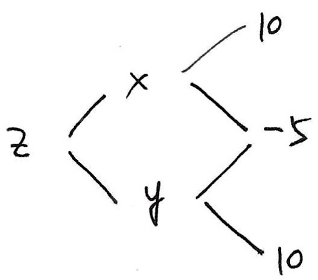
$$S_{uu} = S_0 u^2 \approx 89.39$$

$$89.39 - 87.39 = 2$$



9. The above tree represents the payoff to an exotic derivative security written on the same stock as the preceding questions. In other words, if the stock reaches 87.38, the exotic derivative will pay off $10. And so on.

Find the value of this derivative at time 0.



$$x = (p \cdot 10 + (1-p) \cdot -5)\, e^{-0.03/52} \approx 2.3687$$

$$y = (p \cdot -5 + (1-p) \cdot 10)\, e^{-0.03/52} \approx 2.6285$$

$$z = (p \cdot x + (1-p) \cdot y)\, e^{-0.03/52} \approx 2.50$$

10. The S&P index is at 3700. The continuous compounding interest rate is at 3.3%, and the dividend yield on the S&P 500 is 1.5%. What is the futures price for 6 month maturity S&P 500 futures?

$$F = S e^{(r-q)(T-t)} = 3700\, e^{(0.033 - 0.015)\frac{6}{12}}$$

$$\approx 3733.45$$

11. You short the S&P 500 futures contract from the previous question. Right after you put the position on, the S&P 500 index increases to 3750. What is your approximate P&L? How is this reflected in your IB account?

$$F_{new} = S_{new}\, e^{(r-q)(T-t)} = 3750\, e^{(0.033 - 0.015)\frac{6}{12}}$$

$$\approx 3783.90$$

$$F - F_{new} = -50.45$$

contract size

$$P\&L = -50.45 \times 250 = -12613$$

Cash balance $\downarrow$

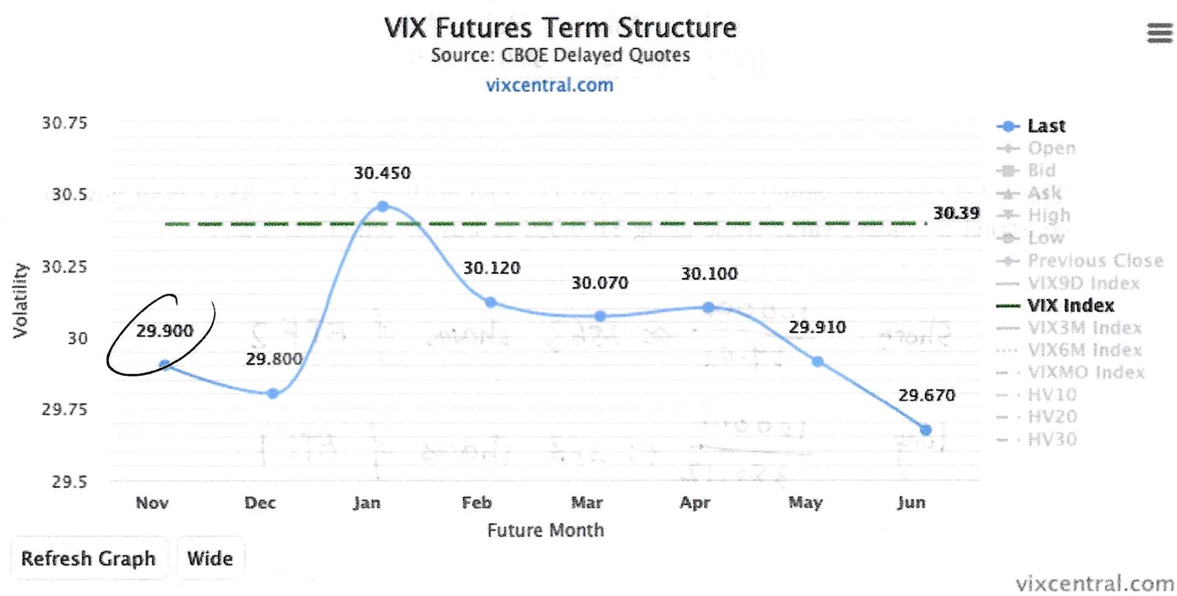


Figure 1: VIX term structure on Oct. 24, 2022

12. Suppose you buy 1 contract of the November maturity VIX futures. If the VIX index, shown in the stapled line at 30.39, is unchanged at the expiration of the futures contract. What is your P&L?

$$30.39 - 29.9 = 0.49$$

$$P\&L = 0.49 \times \underbrace{1000}_{\text{contract size}} = 4900$$

13. Based on the above VIX term structure, suggest a pairs trade.

E.g. Short one contract of Jan

long one contract of Dec

14. Two ETF's that track the S&P 500 index with leverage 1 are trading at

ETF1		ETF2	
Bid	Ask	Bid	Ask
380.1	380.12	39.01	39.02

The NAV (intrinsic value) of ETF1 is 380.11. The NAV of ETF2 is 38. Detail how to construct a relative value trade using $100,000 to buy or sell either ETF.

Short $\frac{100000}{39.01} \approx 2563$ shares of ETF 2

long $\frac{100000}{380.12} \approx 263$ shares of ETF 1

15. Describe how the previous question relates to a trade we did in class.

VXX & VIXY pair trade

The following prices are relevant for Questions 16 to 20

CALL		Strike	PUT	
Bid	Ask		Bid	Ask
13.7	13.8	60	3.7	3.8
8.3	8.4	70	8.3	8.4
4.75	4.85	80	14.75	14.85

16. Suppose you sell the 70 strike straddle, buy the 60 strike put, and buy the 80 strike call (same as Question 4). What is your cash flow when initiating the trade? Assume that you cross the market with limit orders or use market orders.

$$+8.3 + 8.3 - 3.8 - 4.85 = 7.95$$

17. Suppose the stock price is at 86 at the expiration of the options. Find your profit.

$$\text{Payoff} = |86-70| + \text{Max}\{86-80, 0\} + \text{Max}\{60-86, 0\}$$

$$= -16 + 6 + 0 = -10$$

$$\text{Profit} = -10 + 7.95 = -2.05$$

18. Use midpoints to find out what the stock price is. Assume a zero interest rate.

Using Put-Call Parity

$$S = C - P + Ke^{-rT}$$

$$= 8.35 - 8.35 + 70$$

$$= 70$$

19. What is the lowest payoff you can receive on the position in Question 16.

See the plot in Q4

-10

20. With the answers to Questions 16 and 19 in mind, derive a no-arbitrage condition for price of the portfolio. Extra points if you derive the condition algebraically. Hint: it will be a function of strike prices.

Denote straddle strike to be K_S, call strike to be K_c and put strike to be K_p

$$0 \leq P \leq \text{Max}\{K_c - K_s,\ K_s - K_p\} = 10$$