Exercise Sheet 2

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Exercise 1

10 coins, 9 fair (head/tail: 0,5/0,5)

1 unfair (tail: 1)

a) $p(\text{unfair}) = \frac{n_{\text{unfair coins}}}{n_{\text{coins}}} = \frac{1}{10} = \underline{0,1} \Rightarrow p(\text{fair}) = 0,9$

b) $p = p(\text{tail}|\text{unfair}) \cdot p(\text{unfair}) + p(\text{tail}|\text{fair}) \cdot p(\text{fair})$

$= 1 \cdot 0,1 + 0,5 \cdot 0,9 = \underline{0,55}$

c) $p(\text{unfair}|\text{tail}) = \frac{p(\text{tail}|\text{unfair}) \cdot p(\text{unfair})}{p(\text{tail})} = \frac{2}{11} \approx \underline{0,18}$

d) view Jupyter Notebook

Exercise 2

a) rate: $14,6 \frac{1}{h} \hat{=} 7,3 \frac{1}{30\,\text{min}}$

less than 5 events in 30 min?

$\lambda = 7,3 \frac{1}{30\,\text{min}}$

Poisson: $P(r, \lambda) = e^{-\lambda} \frac{\lambda^r}{r!}$, r: number of events

$\Rightarrow P(r<5, \lambda) = \sum_{r=0}^{4} e^{-\lambda} \frac{\lambda^r}{r!}$

$= e^{-7,3} \cdot \left(\frac{7,3^0}{0!} + \frac{7,3^1}{1!} + \frac{7,3^2}{2!} + \frac{7,3^3}{3!} + \frac{7,3^4}{4!} \right)$

$\approx \underline{0,147}$

b) With Gaussian approximation: $\mu = \lambda$, $\sigma = \sqrt{\lambda}$

$\Rightarrow P(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \Rightarrow P(x; \lambda, \sqrt{\lambda}) = \frac{1}{\sqrt{2\pi\lambda}} e^{-\frac{(x-\lambda)^2}{2\lambda}}$

$P(x<5; \lambda, \sqrt{\lambda}) = \int_0^5 \frac{1}{\sqrt{2\pi \cdot 7,3}} \cdot e^{-\frac{(x-7,3)^2}{2 \cdot 7,3}} dx \approx \underline{0,193}$

↑ Wolfram Alpha