Statistical Methods, Exercise 2

1) a) $P(\text{unfair}) = \frac{1}{10}$

b) $P(\text{tails}) = \frac{9}{10} \cdot \frac{1}{2} + \frac{1}{10} \cdot 1 = \frac{11}{20}$

c) $P(\text{unFair} \mid \text{tails}) = \underbrace{P(\text{tails} \mid \text{unfair})}_{=1} \frac{P(\text{unFair})}{P(\text{tails})} = \frac{2}{11}$

Consistency check:

$P(\text{Fair} \mid \text{tails}) = \underbrace{P(\text{tails} \mid \text{Fair})}_{=1/2} \frac{P(\text{Fair})}{P(\text{tails})} = \frac{9}{20} \frac{20}{11} = \frac{9}{11}$

$\rightsquigarrow P(\text{unFair} \mid \text{tails}) + P(\text{Fair} \mid \text{tails}) = 1$ ✓

d) Python code & output:

```python
import random as rd

N = 100000000

N_tails = 0
N_tails_unfair = 0

for i in range(N):
    if rd.random() <= 0.1:
        N_tails += 1
        N_tails_unfair += 1
    else:
        if rd.random() <= 0.5:
            N_tails += 1

print(N_tails_unfair/N_tails,2/11)
```

✓ 16.9s

```
0.18179510196780543 0.18181818181818182
```

2) a) Poisson process: $P(N(t) = n) = \frac{\lambda^n}{n!} e^{-\lambda}$

Here: $\lambda = 14{,}6/h \cdot 0{,}5\,h = 7{,}3$ (average)

$\rightsquigarrow P(N(t) < 5) = \sum_{n=0}^{4} \frac{\lambda^n}{n!} e^{-\lambda} = e^{-7{,}3} \sum_{n=0}^{4} \frac{7{,}3^n}{n!} \approx 0{,}1473$

b) Gaussian approximation: $P(N(t) = n) \approx \frac{1}{\sqrt{2\pi\lambda}} \exp\left(-\frac{(n-\lambda)^2}{2\lambda}\right)$

$\rightsquigarrow P(N(t) < 5) \approx \sum_{n=0}^{4} \frac{1}{\sqrt{2\pi\lambda}} \exp\left(-\frac{(n-\lambda)^2}{2\lambda}\right) \approx 0{,}1468$