Exercise Sheet 3

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Poisson

$$f(x,\mu) = \frac{\mu^x e^{-\mu}}{x!}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x) = \sum_{x\geq 0} x \frac{\mu^x e^{-\mu}}{\cancel{x}(x-1)!} = e^{-\mu}\cdot\mu \underbrace{\sum_{x\geq 1} \frac{\mu^{(x-1)}}{(x-1)!}}_{x'=x-1} = \cancel{e^{-\mu}}\cdot\mu \underbrace{\sum_{x'\geq 0} \frac{\mu^{x'}}{x'!}}_{\cancel{e^{\mu}}} = \underline{\underline{\mu}}$$

$$E(x^2) = \sum_{x\geq 0} x^2 \frac{\mu^x e^{-\mu}}{x!} = e^{-\mu}\cdot\mu \sum_{x\geq 1} x \frac{\mu^{(x-1)}}{(x-1)!}$$

$$= e^{-\mu}\cdot\mu\left[\sum_{x\geq 1} \frac{(x-1)\,\mu^{(x-1)}}{(x-1)!} + \underbrace{\sum_{x\geq 1} \frac{\mu^{(x-1)}}{(x-1)!}}_{e^{\mu}}\right]$$

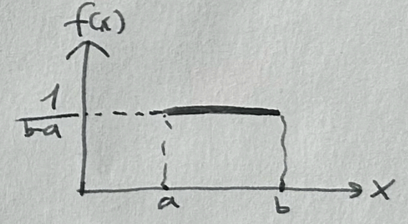
$$= e^{-\mu}\cdot\mu\left[\mu \underbrace{\sum_{x\geq 2} \frac{\mu^{(x-2)}}{(x-2)!}}_{e^{\mu}} + e^{\mu}\right] = \cancel{e^{-\mu}}\mu(\mu \cancel{e^{\mu}} + \cancel{e^{\mu}})$$

$$E(x^2) = \underline{\mu^2 + \mu}$$

$$V(x) = \mu^2 + \mu - \mu^2 = \underline{\underline{\mu}}$$

Uniform

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & x < a \text{ and } x > b \end{cases}$$



$$E(x) = \int_a^b x \frac{1}{(b-a)} dx = \frac{1}{(b-a)} \frac{x^2}{2}\Big|_a^b = \frac{1}{(b-a)} \frac{b^2-a^2}{2} = \underline{\frac{b+a}{2}}$$

$$E(x^2) = \int_a^b x^2 \frac{1}{(b-a)} dx = \frac{1}{(b-a)} \frac{x^3}{3}\Big|_a^b = \frac{1}{(b-a)} \frac{b^3-a^3}{3} = \frac{\cancel{(b-a)}(b^2+ab+a^2)}{3\cancel{(b-a)}}$$

$$= \underline{\underline{\frac{b^2+ab+a^2}{3}}}$$

$$V(x) = E(x^2) - (E(x))^2 = \frac{b^2+ab+a^2}{3} - \frac{\overbrace{(b+a)^2}^{b^2+2ab+a^2}}{4} = \frac{4b^2+4ab+4a^2-3b^2-6ab-3a^2}{12}$$

$$= \frac{b^2-2ab+a^2}{12} = \underline{\underline{\frac{(b-a)^2}{12}}}$$