Exercise Sheet 1

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Exercise 1

Data:

sepal length [cm]: [5.1, 4.9, 4.7, 4.6, 5.0, 5.4, 4.6] = w

sepal width [cm]: [3.5, 3.0, 3.2, 3.1, 3.6, 3.9, 3.4] = x

petal length [cm]: [1.4, 1.4, 1.3, 1.5, 1.4, 1.7, 1.4] = y

petal width [cm]: [0.2, 0.2, 0.2, 0.2, 0.2, 0.4, 0.3] = z

a) $\text{Median}_w = 4.6\,\text{cm}$

$\text{Median}_x = 3.1\,\text{cm}$

$\text{Median}_y = 1.5\,\text{cm}$

$\text{Median}_z = 0.2\,\text{cm}$

Arithmetic Mean: $\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$

$N = 7$

$$\bar{w} = \frac{1}{7}(5.1 + 4.9 + 4.7 + 4.6 + 5.0 + 5.4 + 4.6)_{cm} = 4.89\,\text{cm}$$

$$\bar{x} = \frac{1}{7}(3.5 + 3.0 + 3.2 + 3.1 + 3.6 + 3.9 + 3.4)_{cm} = 3.38\,\text{cm}$$

$$\bar{y} = \frac{1}{7}(1.4 + 1.4 + 1.3 + 1.5 + 1.4 + 1.7 + 1.4)_{cm} = 1.44\,\text{cm}$$

$$\bar{z} = \frac{1}{7}(0.2 + 0.2 + 0.2 + 0.2 + 0.2 + 0.4 + 0.3)_{cm} = 0.24\,\text{cm}$$

Variance: $V(x) = \bar{x^2} - (\bar{x})^2$

$$V(w) = \left(\frac{1}{7}(5.1^2 + 4.9^2 + 4.7^2 + 4.6^2 + 5.0^2 + 5.4^2 + 4.6^2) - 4.89\right)_{cm} = 0.074\,\text{cm}^2$$

$$V(x) = \left(\frac{1}{7}(3.5^2 + 3.0^2 + 3.2^2 + 3.1^2 + 3.6^2 + 3.9^2 + 3.4^2) - 3.38\right)_{cm} = 0.084\,\text{cm}^2$$

$$V(y) = \left(\frac{1}{7}(1.4^2 + 1.4^2 + 1.3^2 + 1.5^2 + 1.4^2 + 1.7^2 + 1.4^2) - 1.44\right)_{cm} = 0.014\,\text{cm}^2$$

$$V(z) = \left(\frac{1}{7}(0.2^2 + 0.2^2 + 0.2^2 + 0.2^2 + 0.2^2 + 0.4^2 + 0.3^2) - 0.24\right)_{cm} = 0.005\,\text{cm}^2$$

Standard deviation: $\sigma = \sqrt{V(x)}$

$\sigma_w = 0{,}27\ cm$

$\sigma_x = 0{,}29\ cm$

$\sigma_y = 0{,}12\ cm$

$\sigma_z = 0{,}07\ cm$

b) $CoV(x,y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$

Covariance matrix:

$$CoV = \begin{pmatrix} V(w) & CoV(w,x) & CoV(w,y) & CoV(w,z) \\ CoV(x,w) & V(x) & CoV(x,y) & CoV(x,z) \\ CoV(y,w) & CoV(y,x) & V(y) & CoV(y,z) \\ CoV(z,w) & CoV(z,x) & CoV(z,y) & V(z) \end{pmatrix}$$

$CoV(w,x) = CoV(x,w) = 0{,}060\ cm^2$

$CoV(w,y) = CoV(y,w) = 0{,}020\ cm^2$

$CoV(w,z) = CoV(z,w) = 0{,}010\ cm^2$

$CoV(x,y) = CoV(y,x) = 0{,}021\ cm^2$

$CoV(x,z) = CoV(z,x) = 0{,}015\ cm^2$

$CoV(y,z) = CoV(z,y) = 0{,}007\ cm^2$

$$CoV = \begin{pmatrix} 0{,}074 & 0{,}060 & 0{,}020 & 0{,}010 \\ 0{,}060 & 0{,}084 & 0{,}021 & 0{,}015 \\ 0{,}020 & 0{,}021 & 0{,}014 & 0{,}007 \\ 0{,}010 & 0{,}015 & 0{,}007 & 0{,}005 \end{pmatrix} cm^2$$

Correlation coefficient $\rho = \frac{CoV(x,y)}{\sqrt{V(x)\,V(y)}}$

$$\Rightarrow \rho = \begin{pmatrix} \frac{V(w)}{V(w)} & \frac{CoV(w,x)}{\sqrt{V(w)\,V(x)}} & \frac{CoV(w,y)}{\sqrt{V(w)\,V(y)}} & \frac{CoV(w,z)}{\sqrt{V(w)\,V(z)}} \\ \frac{CoV(x,w)}{\sqrt{V(x)\,V(w)}} & \frac{V(x)}{V(x)} & \frac{CoV(x,y)}{\sqrt{V(x)\,V(y)}} & \frac{CoV(x,z)}{\sqrt{V(x)\,V(z)}} \\ \frac{CoV(y,w)}{\sqrt{V(y)\,V(w)}} & \frac{CoV(y,x)}{\sqrt{V(y)\,V(x)}} & \frac{V(y)}{V(y)} & \frac{CoV(y,z)}{\sqrt{V(y)\,V(z)}} \\ \frac{CoV(z,w)}{\sqrt{V(z)\,V(w)}} & \frac{CoV(z,x)}{\sqrt{V(z)\,V(x)}} & \frac{CoV(z,y)}{\sqrt{V(z)\,V(y)}} & \frac{V(z)}{V(z)} \end{pmatrix} = \begin{pmatrix} 1 & 0{,}76 & 0{,}62 & 0{,}50 \\ 0{,}76 & 1 & 0{,}60 & 0{,}70 \\ 0{,}62 & 0{,}60 & 1 & 0{,}78 \\ 0{,}50 & 0{,}70 & 0{,}78 & 1 \end{pmatrix} cm$$