

Statistical Methods, Exercise 5

Problem 2a

$$\chi^2(a, b) = \sum_{i=1}^n \frac{(y_i - f(x_i))^2}{\sigma_y^2} = \sum_{i=1}^n \frac{1}{\sigma_y^2} (y_i - ax_i - b)^2$$

$$\leadsto \underline{\nabla} \chi^2 = \frac{1}{\sigma_y^2} \sum_{i=1}^n 2(y_i - ax_i - b)(-x_i) \underline{e}_1$$

$$+ \frac{1}{\sigma_y^2} \sum_{i=1}^n 2(y_i - ax_i - b)(-1) \underline{e}_2$$

$$= \frac{2}{\sigma_y^2} \left(a \underbrace{\sum_{i=1}^n x_i^2}_{\equiv S_{xx}} + b \underbrace{\sum_{i=1}^n x_i}_{\equiv S_x} - \underbrace{\sum_{i=1}^n y_i x_i}_{\equiv S_{xy}} \right) \underline{e}_1$$

$$+ \frac{2}{\sigma_y^2} \left(a S_x + nb - \underbrace{\sum_{i=1}^n y_i}_{\equiv S_y} \right) \underline{e}_2$$

$$= \frac{2}{\sigma_y^2} \begin{pmatrix} a S_{xx} + b S_x - S_{xy} \\ a S_x + nb - S_y \end{pmatrix} \stackrel{!}{=} \underline{0}$$

$$\Leftrightarrow \underline{\underline{a = \frac{n S_{xy} - S_x S_y}{n S_{xx} - S_x^2}, \quad b = \frac{S_{xx} S_y - S_x S_{xy}}{n S_{xx} - S_x^2}}}$$