Statistical Methods, Exercise 3

Problem 1

Poisson: $p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, \quad k \in \mathbb{N}$

$$\rightsquigarrow \langle k \rangle = \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} k = e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{(k-1)!}$$

$$= e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^{k+1}}{k!} = e^{-\lambda} \lambda e^{\lambda} = \underline{\underline{\lambda}}$$

$$\langle (k - \langle k \rangle)^2 \rangle = \langle k^2 \rangle - \langle k \rangle^2$$

$$= e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{k!} k^2 - \lambda^2$$

$$= \underbrace{e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^{k+1}}{k!} (k+1)}_{\lambda \langle k \rangle + \lambda \cancel{e^{-\lambda}} \cancel{e^{\lambda}}} - \lambda^2 = \underline{\underline{\lambda}}$$

Uniform: $p(x) = \frac{1}{\beta - \alpha}$ if $x \in [\alpha, \beta]$, else 0

$$\rightsquigarrow \langle x \rangle = \int_{-\infty}^{\infty} p(x)\, x \, dx = \int_{\alpha}^{\beta} \frac{1}{\beta - \alpha} x \, dx$$

$$= \frac{1}{\beta - \alpha} \left(\frac{\beta^2}{2} - \frac{\alpha^2}{2} \right) = \underline{\underline{\frac{1}{2} (\alpha + \beta)}}$$

$$\langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

$$= \frac{1}{\beta - \alpha} \left(\frac{\beta^3}{3} - \frac{\alpha^3}{3} \right) - \frac{1}{4} (\alpha + \beta)^2$$

$$= \frac{1}{3} (\alpha^2 + \alpha\beta + \beta^2) - \frac{1}{4} (\alpha + \beta)^2 = \underline{\underline{\frac{1}{12} (\alpha - \beta)^2}}$$