Likelyhood function:

a) $\mathcal{L}(\theta|x) = f_\theta(x)$

$$f(t, T_{1/2}) = \frac{\ln 2}{T_{1/2}} \cdot e^{-t \cdot \frac{\ln 2}{T_{1/2}}}$$

$$\mathcal{L}(t_i, t_{1/2}) = \prod_{i=1}^{n} f(t_i, T_{1/2}) = \left(\frac{\ln 2}{T_{1/2}}\right)^n e^{-\frac{\ln 2}{T_{1/2}}\left(\sum_{i=1}^{n} t_i\right)}$$

$$\ln(\mathcal{L}(t_i, t_{1/2})) = n \cdot \ln\left(\frac{\ln 2}{T_{1/2}}\right) - \frac{\ln 2}{T_{1/2}} \underbrace{\sum_{i=1}^{n} t_i}_{n \cdot \langle t \rangle}$$

b) $\hat{T}$: $\left.\frac{\ln \mathcal{L}}{d T_{1/2}}\right|_{T_{1/2} = \hat{T}}$

$$= \left(-\frac{n}{T_{1/2}} + \frac{\ln 2}{T_{1/2}^2} \sum_{i=1}^{n} t_i\right)_{T_{1/2} = \hat{T}}$$

$$= \left(\frac{\ln(2)}{\hat{T}^2} \sum_{i=1}^{n} t_i\right) - \frac{n}{\hat{T}} = 0$$

$$0 = n \frac{\ln 2}{\hat{T}^2} \langle t \rangle - n \frac{1}{\hat{T}}$$

$$\Rightarrow \frac{1}{\hat{T}} = \frac{\ln 2}{\hat{T}^2} \langle t \rangle$$

$$\Rightarrow \underline{\underline{\hat{T} = \ln 2 \langle t \rangle}}$$

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$$b = \langle \hat{T}_{1/2} \rangle - T_{1/2}$$

So: $\langle \hat{T}_{1/2} \rangle = \int_0^\infty d^n t_i \; \hat{T}_{1/2} \, L(\{t_i\}, T_{1/2})$

$$= \int_0 d^n t_i \, (\ln 2 \langle t \rangle) \left(\frac{\ln 2}{T_{1/2}}\right)^n \exp\left(-\frac{\ln 2}{T_{1/2}} \sum_{j=1}^n t_j\right)$$

$$= \int_0^\infty d^n t_i \; \ln 2 \langle t \rangle \left(\frac{\ln 2}{T_{1/2}}\right)^n \exp\left(-\frac{\ln 2}{T_{1/2}} n \langle t_j \rangle\right)$$

$$= \left(\frac{\ln 2}{n}\right)\left(\frac{\ln 2}{T_{1/2}}\right)^n \int_0^\infty d^n t_i \sum_{i=1}^n t_i \prod_{j=1}^n \exp\left(-\frac{\ln 2}{T_{1/2}} t_j\right)$$

$$= \left(\frac{\ln 2}{n}\right)\left(\frac{\ln 2}{T_{1/2}}\right)^n \sum_{i=1}^n \left(\int_0^\infty t_i \exp\left(-\frac{\ln 2}{T_{1/2}} t\right) dt \right. \text{ \& are theile}$$

$$\left. \cdot \left(\int_0^\infty \exp\left(-\frac{\ln 2}{T_{1/2}} t_{\neq i}\right) dt_{\neq i} \right)^{n-1} \right)$$

$$= \left(\frac{\ln 2}{n}\right)\left(\frac{\ln 2}{T_{1/2}}\right)^n n \left(\frac{T_{1/2}}{\ln 2}\right)^2 \left(\frac{T_{1/2}}{\ln 2}\right)^{n-1}$$

$$= \frac{\cancel{(\ln 2)^{n+1}}}{T_{1/2}^n} \frac{T_{1/2}^{n+1}}{\cancel{(\ln 2)^{n+1}}} = \underline{\underline{T_{1/2}}}$$

Also $\underline{\underline{b = \langle \hat{T}_{1/2} \rangle - T_{1/2} = 0}}$

$\Rightarrow$ <u>Unbiased</u>