

# Statistical Methods, Exercise 3

## Problem 1

Poisson:  $p(k) = e^{-\lambda} \frac{\lambda^k}{k!}$ ,  $k \in \mathbb{N}$

$$\begin{aligned} \Rightarrow \langle k \rangle &= \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} k = e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{(k-1)!} \\ &= e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^{k+1}}{k!} = e^{-\lambda} \lambda e^{\lambda} = \underline{\underline{\lambda}} \end{aligned}$$

$$\begin{aligned} \langle (k - \langle k \rangle)^2 \rangle &= \langle k^2 \rangle - \langle k \rangle^2 \\ &= e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{k!} k^2 - \lambda^2 \\ &= e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^{k+1}}{k!} (k+1) - \lambda^2 = \underline{\underline{\lambda}} \\ &\quad \lambda \langle k \rangle + \lambda e^{-\lambda} e^{\lambda} \end{aligned}$$

Uniform:  $p(x) = \frac{1}{\beta - \alpha}$  if  $x \in [\alpha, \beta]$ , else 0

$$\begin{aligned} \Rightarrow \langle x \rangle &= \int_{-\infty}^{\infty} p(x) x dx = \int_{\alpha}^{\beta} \frac{1}{\beta - \alpha} x dx \\ &= \frac{1}{\beta - \alpha} \left( \frac{\beta^2}{2} - \frac{\alpha^2}{2} \right) = \underline{\underline{\frac{1}{2}(\alpha + \beta)}} \end{aligned}$$

$$\begin{aligned} \langle (x - \langle x \rangle)^2 \rangle &= \langle x^2 \rangle - \langle x \rangle^2 \\ &= \frac{1}{\beta - \alpha} \left( \frac{\beta^3}{3} - \frac{\alpha^3}{3} \right) - \frac{1}{4} (\alpha + \beta)^2 \\ &= \frac{1}{3} (\alpha^2 + \alpha\beta + \beta^2) - \frac{1}{4} (\alpha + \beta)^2 = \underline{\underline{\frac{1}{12}(\alpha - \beta)^2}} \end{aligned}$$