Exercise Sheet 4

Andrej Praetzler
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$U = 12 \pm 0{,}3\ V$ with $S = 0{,}2V \Rightarrow \sigma_U = 0{,}1\ V$

$R = 10 \pm 0{,}2\ k\Omega = 10000 \pm 200\ \Omega \Rightarrow \sigma_R = 200\ \Omega$

a) $CoV = \begin{pmatrix} \sigma_U^2 + S^2 & \sigma_U \cdot \sigma_R \\ \sigma_R \cdot \sigma_U & \sigma_R^2 \end{pmatrix} = \begin{pmatrix} 0{,}05\ V^2 & 20\ V\Omega \\ 20\ V\Omega & 40000\ \Omega^2 \end{pmatrix}$

b) $I = \frac{U}{R} = 1{,}2\ mA \qquad \frac{\partial I}{\partial U} = \frac{1}{R}\ ; \quad \frac{\partial I}{\partial R} = -\frac{U}{R^2}$

$$V(I) = \left[\frac{\partial I}{\partial U}\right]^2_{\mu_U, \mu_R} CoV(U,U) + \left[\frac{\partial I}{\partial R}\right]^2_{\mu_R, \mu_U} CoV(R,R) + 2 \frac{\partial I}{\partial U}\Big|_{\mu_U, \mu_R} \frac{\partial I}{\partial R}\Big|_{\mu_R, \mu_U} CoV(U,R)$$

$= 1{,}556 \cdot 10^{-9}\ A^2$

$\Rightarrow \sigma_I = \sqrt{V(I)} \approx 3{,}94 \cdot 10^{-5}\ A \approx$ [illegible] $0{,}039\ mA$

$\Rightarrow I = 1{,}20 \pm$ [illegible] $0{,}039\ mA$

c) $\rho(U,R) = -\frac{1}{2} \qquad \rho(U,R) = \frac{CoV(U,R)}{\sqrt{V(U)\,V(R)}} = -\frac{1}{2}$

$CoV(U,R) =$ [illegible] $= -\frac{1}{2}\sqrt{V(U)\,V(R)}$ with $V(U) = 0{,}05\ V^2$, $V(R) = 40000\ \Omega^2$

$= -\frac{1}{2}\sqrt{0{,}05 \cdot 40000}\ V\Omega$

$\approx -22{,}4\ V\Omega$

$\Rightarrow CoV = \begin{pmatrix} 0{,}05\ V^2 & -22{,}4\ V\Omega \\ -22{,}4\ V\Omega & 40000\ \Omega^2 \end{pmatrix}$

$\Rightarrow V(I) =$ [illegible] $1{,}6316 \cdot 10^{-9}\ A^2$ [illegible]

$\Rightarrow \sigma_I \approx$ [illegible] $0{,}04\ mA$