Lag Length Selection in Vector Autoregressive Models:

Symmetric and Asymmetric Lags

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I. **Introduction**

Vector autoregressive (VAR) models are widely used in forecasting and in analysis of the effects of structural shocks. A critical element in the specification of VAR models is the determination of the lag length of the VAR. The importance of lag length determination is demonstrated by Braun and Mittnik (1993) who show that estimates of a VAR whose lag length differs from the true lag length are inconsistent as are the impulse response functions and variance decompositions derived from the estimated VAR. Lütkepohl (1993) indicates that overfitting (selecting a higher order lag length than the true lag length) causes an increase in the mean-square-forecast errors of the VAR and that underfitting the lag length often generates autocorrelated errors. Hafer and Sheehan (1989) find that the accuracy of forecasts from VAR models varies substantially for alternative lag lengths.

Most VAR models are estimated using symmetric lags, i.e. the same lag length is used for all variables in all equations of the model. This lag length is frequently selected using an explicit statistical criterion such as the AIC or SIC. Symmetric lag VAR models are easily estimated; since the specification of all equations of the model is the same, estimation by ordinary least squares yields efficient parameter estimates. However, there is no compelling reason from economic theory that lag lengths should be the same for all variables in all equations. In fact, Hsaio (1981) suggested estimating VARs in which the lag length on each variable in each equation could differ. The issue of whether VARs should be estimated using symmetric or asymmetric lags has recently been resurrected by Keating (1993; 1995). He suggested estimating asymmetric lag VAR models in which the lag length potentially differs across the variables in the model but is the same for a particular variable in each equation of the model. He demonstrated that this pattern of asymmetry can be derived from a structural representation of the VAR model that has asymmetric lags. In the Keating-type asymmetric lag model, the specification of each equation is the same, so the VAR can be estimated using ordinary least squares. Within the context of a small structural

VAR model, Keating (1995) found that estimates of long-run structural parameters typically had smaller standard errors for his asymmetric lag VAR than for a symmetric lag VAR and that the confidence intervals for impulse response functions and variance decompositions for an asymmetric lag VAR were smaller than for a symmetric lag VAR.

Given the demonstrated importance of lag length selection for VAR models, the aim of this paper is to examine the performance of alternative lag selection criteria for VAR models using Monte Carlo simulations. Although the performance of alternative statistical criterion for lag length selection of symmetric lag VARs has been studied by, among others, Lütkepohl (1993), the performance of statistical lag selection criteria in selecting lag lengths for asymmetric lag VAR models has not been studied. The lag selection criteria considered include Akaike's information criterion (AIC), Schwarz's information criterion (SIC), Phillips' posterior information criterion (PIC), and Keating's (1995)'s application of the AIC and SIC criterion (KAIC and KSIC).[1] Four different bivariate lag models are examined; two are symmetric lag models and two are asymmetric lag models. In the spirit of Kennedy and Simons (1991), the parameters of the lag models are estimated using ten different bivariate models constructed from actual economic data.

The remainder of the study is now outlined. Section II of the paper describes the empirical methodology while the empirical results are reported in Section III. Section IV provides a summary and conclusion.

II. Methodology

Monte Carlo simulations of 1000 draws are used to evaluate the performance of the alternative lag selection criterion in bivariate VAR models. The VARs are simulated using prespecified model parameters and lag length and a random number generator. The alternative lag selection criteria are evaluated by computing the frequency distributions of lag lengths selected by each lag selection criterion. Four different bivariate lag models are considered. The out-of-sample forecasting performance of the models selected by each lag selection criterion are also

examined, as is the ability of each lag selection criterion to generate impulse reponse functions that mimic the true impulse response function.

The lag selection criterion are the AIC, SIC, PIC, and Keating's (1995)'s procedure using, alternatively, the AIC and SIC criterion to select the lag order. The AIC and SIC are well-known and the definition of each is presented without discussion :

$$AIC = \ln \left| \tilde{\Sigma} \right| + \frac{2}{T} \text{(number of freely estimated parameters), and}$$

$$SIC = \ln \left| \tilde{\Sigma} \right| + \frac{\ln T}{T} \text{(number of freely estimated parameters) where } \tilde{\Sigma} = \text{estimated covariance}$$

matrix and T = number of observations.

Phillips (1994) has recently proposed another lag specification criterion called the posterior information criterion (PIC). Phillips derived the PIC from Bayesian analysis in which flat priors were imposed on the parameters to find the posterior distributions. The PIC is defined as:

$$PIC = \ln\left| \tilde{\Sigma} \right| + \frac{1}{T} \left| \tilde{\Sigma}^{-1} \otimes X'X \right|,$$

where $\otimes$ is the kronecker product operator and X = matrix of explanatory variables.

Keating's insight about asymmetric lags can be developed in the following way. Let

(1) $$\Phi(L)y_t = C + \varepsilon_t$$

be a structural model where y_t = N x 1 vector of endogenous variables, C = N x 1 vector of constants, ε_t = N x 1 vector of white-noise error terms which are distributed as multivariate normal with covariance matrix $\sigma^2 I$, $\Phi(L)$ = N x N lag polynomial matrix with the ith row and j^{th} column defined as

$$\Phi_{ij}(L) = \varphi_{0ij} + \varphi_{1ij}L + \varphi_{2ij}L^2 + ... + \varphi_{n_{ij}\,ij}L^{n_{ij}}.$$

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If n_{ij} differs for each element, the structural model is said to have asymmetric lags.

The reduced-form VAR representation of this model can be obtained by premultiplying (1) by Φ_0^{-1} (the inverse of the contemporaneous coefficient matrix). We obtain:

$$(2) \qquad \beta(L)y_t = D + e_t$$

where $\beta(L) = \Phi_0^{-1}\Phi(L)$ with the kth and j^{th} element $\beta_{kj}(L) = \sum_{i=1}^{N}\Phi_0^{ki}\Phi_{ij}(L),\ D = \Phi_0^{-1}C$, and

$$e_t = \Phi_o^{-1}\varepsilon_t.$$

The β_{kj} are thus linear combinations of the structural lag polynomials. The lag length of β_{kj} is the largest value of $n_1, n_2, \ldots, n_{Nj}$. If the lags in the structural model are all the same length, the same will be true in the reduced form model. However, if the structural model is characterized by asymmetric lags, then a particular variable will have the same lag length in each equation of the reduced-form VAR model, although the lag length will vary across variables.[2]   Given this type of asymmetric lag structure in the VAR, Keating (1995) suggests computing AIC and SIC statistics for every possible combination of lag lengths in order to determine the lag structure of the VAR. A systematic search can be set up by varying the lag length of each variable between 1 and M, which requires M^N estimates of the VAR. The lag structure that generates the minimum AIC or SIC is selected as the optimal lag structure.

The bivariate VAR lag models consist of two symmetric lag models and two asymmetric lag models. Lag model one (LM1) has 3 lags on each variable in each equation while lag model two (LM2) has 8 lags on each variable in each equation. Lag model three (LM3) has 3 lags on variable one in each equation and 1 lag of variable two in each equation; lag model four (LM4) has 8 lags on variable one and 5 lags on variable two. The asymmetric lag models are thus of the form

discussed by Keating in which the lags across variables are different, but the lags for a particular variable are the same in each equation.

For each lag model, ten different models were estimated using actual economic data. The stationarity of the data was checked by computing the roots of the characteristic polynomial of the model; the results indicated the series in all models were stationary. The variables in these models and the transformations necessary for stationarity are listed in Appendix A. The data for the Monte-Carlo simulations were generated in the following way. The parameter estimates of these models were taken, in turn, as the "true" parameter values of the lag model.[3] The e_t were selected as a random draw from a normal distribution with mean 0 and variance-covariance matrix equal to the estimated variance-covariance matrix of the model. 212 observations were generated in this fashion, with the first 74 discarded. The lag selection criteria described above were then used to specify the lag length using observations 75-200. The remaining 12 observations were used in an out-of-sample forecasting comparison. In applying the lag selection criterion, a maximum of 8 lags was considered for LM1 and LM3, and a maximum of 16 lags was considered for lag models LM2 and LM4.

The Monte Carlo simulations employed 1000 draws for each of the ten models estimated for each lag model. The frequency distribution of lags selected by each criterion for each of the ten models estimated for each lag model was computed, and, for each lag model, the results across models were averaged in order to conserve space. For each draw, out-of-sample forecasts at horizons of 1 to 12 periods, and their associated mean-square-errors, were computed. Also, for each draw of the Monte Carlo simulation, the impulse response functions were computed. Since we were working with bivariate models, four impulse response functions were computed; one for the effects of a shock to variable one on itself, one for the effects of a shock to variable one on variable two, and two analogous IRFs for variable two. The impulse response functions were computed by first orthogonalizing the shocks using the Choleski decomposition with variable one

ordered first. This orthogonalization scheme attributes all the contemporaneous correlation between the two variables in the system to the one ordered first. Other orthogonalization schemes use either short-run or long-run restrictions from economic theory to identify structural shocks to the variables in the model.[4] However, since we are not attempting any inference about economic effects of structural shocks or the impact of policy variables, the Choleski decomposition is suitable for our purposes. The "true" impulse response functions were computed for each of the four lag models for each of the ten models from the parameter estimates and estimated variance-covariance matrix from the models estimated using actual data.

III. Results

A. Frequency-Distribution Results

These results are reported in Tables 1-4. For LM1, a symmetric lag model with lag length of 3 for each variable, we see that the AIC criterion selected the true lag approximately 60% of the time; a shorter lag was selected approximately 30% of the time. Keating's method that used the AIC (KAIC) to select each variable's lag length selected the correct lag of 3 approximately 40% of the time, but also selected shorter lags (1 or 2) approximately 40% of the time. The SIC and PIC selected a lag of 1 approximately 70% of the time; the correct lag of 3 was selected only about 15% of the time. The performance of Keating's method that used the SIC (KSIC) to select each variable's lag length was similar to that of the SIC and PIC. A lag of 1 for variable one was selected approximately 70% of the time while a lag of 1 for variable two was selected approximately 60% of the time. Thus, for this symmetric lag model, the AIC-based criteria picked the correct lag more frequently than did the other criteria.

LM2 is a symmetric lag model with a common lag of 8. As in the previous symmetric lag model, the AIC-based criteria selected the correct lag most frequently, although the frequency with which this lag is selected is smaller than for lag model one. The AIC selected a lag of 8 approximately 37% of the time while the KAIC selected a lag of 8 for variable one 26% of the time

and for variable two approximately 20% of the time. The SIC and PIC selected a lag of one about 60% of the time, and the KSIC selected a lag of one for both variables with about the same frequency.

LM3 is an asymmetric lag model with 3 lags on variable one and 1 lag on variable two. The KAIC did best for this lag model, selecting the correct lag on variable one 36% of the time and the correct lag on variable two 76% of the time. KSIC selected a lag of 1 for both variables most frequently; for variable one, a lag of 1 was selected 56% of the time while for variable two a lag of 1 was selected 99% of the time. The standard AIC, SIC, and PIC criteria are at a disadvantage for this model since they select a common lag for all variables in the model.   These three criteria all selected a lag of 1, the shortest lag in the model, most frequently. However, the AIC selected this length about 35% of the time while the SIC and PIC selected this length about 75% of the time.

The last lag model is an asymmetric lag model with a lag of 8 for variable one and a lag of 5 for variable two. The results are similar to LM3. KAIC did best; the correct lag of 8 for variable one was selected about a fourth of the time, as was the correct lag of 5 for variable two. KSIC selected a lag of 1 for both variables; for variable one, this length was selected about 64% of the time while this length was selected for variable two about 53% of the time. The AIC selected the shortest of the true lag lengths, 5, about a fourth of the time, while the SIC and PIC selected a lag of 1 most frequently.

Several regularities stand out. Looking across both symmetric lag and asymmetric lag models, the KAIC did best in the sense that the most frequently selected lags are the true lags for both types of lag models. For symmetric lag models, AIC did best, selecting the common lag more frequently than KAIC. For asymmetric lag lag models, AIC selected the shortest lag in the lag model most frequently.   As one would expect from previous studies, the SIC-based criteria typically selected much shorter lags than AIC-based criteria. In fact, for all models studied, the

SIC-based criteria selected a lag of 1 most frequently. The performance of PIC was very similar to that of SIC. Based upon these results, it appears that one should seriously consider KAIC as a lag specification criterion.

B. Forecast Results

A comparison of the out-of-sample forecasting results for the alternative lag selection criteria is presented in Table 5. The results for each lag model are presented separately.   The numbers in this table are the ratio of the average mean-square forecasting error for the criterion listed at the top of the columns to the average mean-square forecasting error for the AIC criterion. Scaling in this fashion generates a unit free measure and enables us to average the mean-square-error over the 10 different models. Three horizons are presented; 3 months, 6 months, and 12 months. Formally, the numbers were calculated for lag model m for horizon 3 as (using the SIC and AIC results as a specific example):

$$\left(\frac{1}{3}\sum_{h=1}^{3}\frac{1}{10}\sum_{c=1}^{10}\text{SICF}_{m,c}^{h} \right) \Big/ \left(\frac{1}{3}\sum_{h=1}^{3}\frac{1}{10}\sum_{c=1}^{10}\text{AICF}_{m,c}^{h} \right) \text{ where}$$

h = horizon length, c = empirical model index = 1, ..., 10, m = lag model = 1, ..., 4,

SICF = mean-square forecasting error for the SIC criterion, averaged over 1000 draws, and

AICF = mean-square forecasting error for the AIC criterion, averaged over 1000 draws. Similar calculations were made for horizons 6 and 12 and for the other lag selection criterion.

We see from Table 5 that the models with lag lengths specified by SIC, PIC, KAIC, and KSIC did slightly better than the model specified using the AIC criterion for the short-lag lag models (LM1 and LM3) as indicated by ratios which are almost always below 1. There is little difference in the results across lag selection criterion, however. For the lag models with longer lags, LM2 and LM4, the AIC-specified lag lengths almost always did better than the other criterion. The difference was more marked for the SIC and PIC results. Somewhat surprisingly in light of the frequency-distribution results, KSIC and KAIC performed about the same in ability to forecast

out-of-sample for these longer-lag models. Furthermore, they generally do only marginally worse than AIC.

C. Impulse Response Function Results

For each draw of our Monte Carlo simulations, impulse response functions were computed for the lags specified by each lag selection criterion. The mean-square-error for the difference between the impulse response functions generated from the true lag length and the impulse response functions generated for the lags specified by a particular lag selection criterion was then computed. These mean-square-errors were then averaged over the 10 empirical models used. Formally, the numbers were calculated for lag model m for horizon 6 as (using the SIC and AIC results as a specific example):

$$\left(\frac{1}{6}\sum_{h=1}^{6}\frac{1}{10}\sum_{c=1}^{10}\text{SICIRF}_{m,c}^{h} \right) \Big/ \left(\frac{1}{6}\sum_{h=1}^{6}\frac{1}{10}\sum_{c=1}^{10}\text{AICIRF}_{m,c}^{h} \right) \text{ where}$$

h = horizon length, c = empirical model index = 1, ..., 10, m = lag model = 1, ..., 4,

SICIRF = mean-square impulse response function error for the SIC criterion, averaged over 1000 draws, and AICIRF = mean-square impulse response function error for the AIC criterion, averaged over 1000 draws. Similar calculations are made for horizons 12 and 24 and for the other lag selection criterion. The results are reported in Tables 6 and 7.

For LM1, the short-lag symmetric-lag model, we see that no one lag selection criterion consistently outperformed the others. The SIC, PIC, KAIC, and KSIC did better than the AIC for the effects of a shock to variable one on itself, but AIC did better for the effects of a shock to variable two on variable one and for the effects of a shock to variable one on variable two. The results for a shock to variable two on itself were mixed. A clearer pattern is present for LM2, the long-lag symmetric-lag model. The AIC generally outperformed the SIC and PIC by a large margin. However, the results for KAIC and KSIC relative to AIC were mixed.

For the lag model with short asymmetric lags (LM3), the SIC, PIC, and KSIC generally outperformed both the AIC and KAIC, often by large margins. This is perhaps not too surprising since SIC, PIC, and KSIC tended to select short lags frequently. However, the results are dramatically different when the lag model with long asymmetric lags (LM4) is considered. In this case, the AIC outperformed the SIC, PIC, and KSIC in all cases, generally by a large margin. Additionally, the AIC outperformed the KAIC about half the time.

III. Summary and Conclusion

This study used Monte Carlo simulations to study the performance of alternative lag selection criterion for symmetric lag and asymmetric lag models. Lag models with short lags and with long lags were considered. The alternative criteria considered were the AIC, SIC, Phillips' (1994) Posterior Information Criterion (PIC), and Keating's modification of the AIC and SIC (KAIC and KSIC, respectively). The alternative criteria were evaluated by computing the frequency distribution of lags selected by each criterion, by computing the out-of-sample forecasting performance of models with lags selected using each criterion, and by comparing the ability of models with lags selected using each criterion to mimic the "true" impulse response functions for the lag model.

In terms of frequency distribution results, the AIC criterion dominated for the symmetric lag models in the sense that the AIC selected the true lags more frequently than the other criteria. The KAIC performed similarly for the asymmetric lag models. Furthermore, considering both symmetric and asymmetric lag models, the KAIC did best in that the most frequently selected lags are the true lags for both types of lag models. This is an important result since one does not generally know whether the true model has symmetric or asymmetric lags or short or long lags.

The results are more ambiguous in terms of forecasting performance. For the short-lag models, both symmetric and asymmetric, the SIC, PIC, KAIC, and KSIC slightly outperform the AIC, but the difference is small. For the longer-lag models with both symmetric and asymmetric

lags, the AIC-specified lag length models almost always forecast better than the other criterion, especially the SIC and PIC. The KAIC and KSIC generally do only marginally worse for these longer-lag models than the AIC. Given the inherent uncertainty about the true lag length and whether the lags are symmetric or asymmetric, these results suggest a preference for the AIC, KAIC, and KSIC over the SIC and PIC. The KAIC and KSIC do slightly better than AIC in the short-lag models but slightly worse in the long-lag models. The AIC does slightly worse than KAIC, KSIC, SIC, and PIC for the short-lag models, but much better than SIC and PIC for the longer-lag models, and slightly better than KAIC and KSIC for the long-lag models.

The most ambiguity about the preferred lag selection criterion arises for the impulse response functions. For the long-lag models, the AIC results are generally much closer to the true impulse response functions than are the SIC and PIC results. The AIC outperforms the KSIC for the long-lag asymmetric-lag model by a large margin, but the results are mixed for the long-lag symmetric-lag case. The abilities of models with lags selected by AIC and KAIC to mimic the true impulse response functions are very similar. The results are difficult to generalize for the short-lag models. For the symmetric-lag model, no one criterion dominates. For the asymmetric-lag model, the SIC, PIC, and KSIC do better than the AIC and KAIC. It is thus difficult to generalize about the preferred lag selection technique when there is uncertainty about the true lag length and whether the lags are symmetric or asymmetric.

ENDNOTES

[1]Although Hsiao's technique of lag length selection has been used in the literature (see, for example, Caines, Keng, and Sethi (1981) and McMillin and Fackler (1984)), the implementation of this criterion is problematic in the sense that lag length determination is often sensitive to the order in which variables are considered. Caines, Keng, and Sethi (1981) attempted to deal with this problem by suggesting a criterion--the specific-gravity criterion--for selecting the order in which the lag length of the model's variables is determined. We also note that, since the specification of each equation of the model is different, estimation using ordinary least squares is not appropriate. Because an extensive iterative procedure is required to appropriately specify a Hsiao-type model even when the specific gravity criterion is applied (see McMillin and Fackler (1984)), this type of model is not considered in our Monte Carlo study.

[2]This will be true as long as none of the elements in Φ_0^{-1} are zero. If Φ_0^{-1} has a zero element(s), then it is possible for a particular variable to have different lag lengths in different equations of the model.

[3]The coefficient estimates are available upon request, as are all the RATS programs used in the analysis.

[4]See, for example, Bernanke (1986) and Blanchard and Quah (1989).

DATA APPENDIX

The following series were used in the bivariate VAR models. All series were taken from Citibase, and the Citibase name is provided in parentheses. The transformations indicated were required to render the models stationary. Quarterly data from 1959:3-1996:1 were used in all models except model 4 where data availability dictated using the 1967:1-1994:4 period.

1. the change in the log of nonborrowed reserves (FMRNBA) and the change in the log of real GDP (GDPQ).

2. the change in the log of M2 (FM2) and the change in the rate of CPI (PUNEW) inflation.

3. the unemployment rate (LHUR) and the change in the log of the GDP deflator (GDPDFC).

4. the change in the log of plant and equipment investment expenditures (IXIQ) and the change in the log of the capacity utilization rate (IPX).

5. the spread between the 10-year Treasury note (FYGT10) rate and the 3-month T-bill rate (FYGM3) and the change in the log of real GDP.

6. the change in the difference between the log of the GDP deflator and the log of the producer price index for crude oil (PW561) and the change in the log of employment (LHNAG).

7. the change in the log of the bilateral U.S.-Japanese exchange rate (EXRJAN) and the rate of change in the GDP deflator.

8. the change in the log of real consumption (GCQ) and the change in the log of real stock prices (FSPCOM/(GDP Deflator/100)).

9. the change in the log of real private domestic investment (GPIQ) and the change in the log of real GDP.

10. the change in the log of the money multiplier (FM2/FMRNBA) and the change in the log of real GDP.

ACKNOWLEDGEMENTS

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Lag	AIC	SIC	PIC	KAIC Variable		KSIC Variable	
				1	2	1	2
1	12.3	70.2	73.2	28.6	21.6	70.2	60.4
2	17.8	13.5	12.9	15.4	18.3	17.6	14.3
3	57.3	16.3	14.0	39.5	43.1	12.0	24.7
4	8.1	0.0	0.0	8.6	8.2	0.2	0.6
5	2.6	0.0	0.0	3.7	3.9	0.0	0.0
6	1.0	0.0	0.0	1.9	2.3	0.0	0.0
7	0.6	0.0	0.0	1.4	1.4	0.0	0.0
8	0.4	0.0	0.0	1.0	1.3	0.0	0.0

Table 1
Frequency Distribution
Model 1 (3,3)

Lag	AIC	SIC	PIC	KAIC Variable		KSIC Variable	
				1	2	1	2
1	3.5	58.4	61.4	9.5	9.5	57.6	56.2
2	5.1	10.8	11.0	6.0	7.1	9.9	13.7
3	8.0	13.7	13.5	9.2	13.9	10.3	21.6
4	7.4	6.4	6.3	9.1	9.4	5.8	4.5
5	9.5	5.3	4.6	10.5	8.8	4.9	2.1
6	9.8	1.1	1.0	8.1	8.7	1.1	0.7
7	8.1	0.1	0.1	5.0	10.3	0.1	0.6
8	37.3	4.3	2.2	26.3	19.6	9.7	0.7
9	6.0	0.0	0.0	6.5	4.9	0.5	0.0
10	2.3	0.0	0.0	3.6	2.5	0.1	0.0
11	1.3	0.0	0.0	2.1	1.9	0.0	0.0
12	0.6	0.0	0.0	1.7	1.2	0.0	0.0
13	0.5	0.0	0.0	1.0	0.9	0.0	0.0
14	0.3	0.0	0.0	0.6	0.5	0.0	0.0
15	0.2	0.0	0.0	0.5	0.5	0.0	0.0
16	0.2	0.0	0.0	0.4	0.5	0.0	0.0

Table 2
Frequency Distribution
Model 2 (8,8)

Table 3 Frequency Distribution Model 3 (3,1)							
Lag	AIC	SIC	PIC	KAIC Variable		KSIC Variable	
				1	2	1	2
1	34.4	73.9	75.1	23.8	76.4	55.8	98.6
2	31.8	25.0	24.1	25.8	11.4	33.4	1.2
3	26.2	1.1	0.8	35.6	5.3	10.8	0.2
4	4.7	0.0	0.0	7.4	3.0	0.1	0.0
5	1.6	0.0	0.0	3.5	1.8	0.0	0.0
6	0.8	0.0	0.0	1.8	0.9	0.0	0.0
7	0.4	0.0	0.0	1.2	0.6	0.0	0.0
8	0.2	0.0	0.0	0.9	0.6	0.0	0.0

Table 4 Frequency Distribution Model 4 (8,5)							
Lag	AIC	SIC	PIC	KAIC Variable		KSIC Variable	
				1	2	1	2
1	3.8	64.5	68.4	8.8	11.3	64.0	52.9
2	5.3	15.5	14.9	4.5	10.7	16.9	15.6
3	14.1	15.2	13.5	8.5	21.5	10.0	22.8
4	12.9	3.4	2.6	10.9	15.3	4.3	4.0
5	22.8	1.3	0.6	11.2	23.6	1.8	4.4
6	14.8	0.2	0.1	11.9	6.7	2.2	0.2
7	6.9	0.0	0.0	8.0	3.6	0.4	0.0
8	14.0	0.0	0.0	23.7	2.8	0.5	0.0
9	2.6	0.0	0.0	5.3	1.4	0.0	0.0
10	1.2	0.0	0.0	2.8	1.1	0.0	0.0
11	0.7	0.0	0.0	1.7	0.7	0.0	0.0
12	0.3	0.0	0.0	1.0	0.5	0.0	0.0
13	0.2	0.0	0.0	0.7	0.3	0.0	0.0
14	0.2	0.0	0.0	0.5	0.2	0.0	0.0
15	0.1	0.0	0.0	0.4	0.2	0.0	0.0
16	0.1	0.0	0.0	0.4	0,2	0.0	0.0

Table 5 Forecast Mean-Square Errors [*]								

Model 1 (3,3)

Horizon	SIC Variable		PIC Variable		KAIC Variable		KSIC Variable	
	1	2	1	2	1	2	1	2
3	1.003	0.997	1.005	0.994	1.002	1.011	1.020	1.006
6	0.991	0.996	0.990	0.995	0.999	1.008	0.996	1.000
12	0.989	0.998	0.988	0.997	1.002	1.005	0.989	0.999

Model 2 (8,8)

Horizon	SIC Variable		PIC Variable		KAIC Variable		KSIC Variable	
	1	2	1	2	1	2	1	2
3	1.023	1.021	1.025	1.024	1.001	1.024	0.987	1.010
6	1.016	1.012	1.021	1.014	1.002	1.019	0.989	1.008
12	1.008	1.001	1.009	1.002	1.009	1.010	0.997	0.998

Model 3 (3,1)

Horizon	SIC Variable		PIC Variable		KAIC Variable		KSIC Variable	
	1	2	1	2	1	2	1	2
3	0.987	0.993	0.983	0.993	0.995	0.994	0.980	0.989
6	0.986	0.992	0.984	0.992	0.997	0.996	0.989	0.990
12	0.987	0.996	0.987	0.996	0.999	0.998	0.992	0.995

Model 4 (8,5)

Horizon	SIC Variable		PIC Variable		KAIC Variable		KSIC Variable	
	1	2	1	2	1	2	1	2
3	1.071	1.031	1.076	1.031	1.018	1.003	1.073	1.023
6	1.047	1.012	1.052	1.012	1.021	1.001	1.042	1.004
12	1.026	1.004	1.031	1.004	1.014	1.003	1.018	1.000

[*]The numbers in the table are ratios of the forecast mean-square errors for the lag selection technique indicated to the forecast mean-square errors for the AIC.

Table 6								
Impulse Response Function Mean-Square Errors[*]								

Model 1 (3,3)

Shock to Variable 1/Effects on Variables 1 and 2

Horizon	SIC Variable		PIC Variable		KAIC Variable		KSIC Variable	
	1	2	1	2	1	2	1	2
6	0.917	1.050	0.920	1.051	0.997	1.047	1.016	1.133
12	0.874	1.012	0.856	1.013	0.968	1.087	0.894	1.096
24	0.872	1.005	0.856	1.006	0.994	1.089	0.880	1.088

Shock to Variable 2/Effects on Variables 1 and 2

Horizon	SIC Variable		PIC Variable		KAIC Variable		KSIC Variable	
	1	2	1	2	1	2	1	2
6	1.347	1.002	1.467	1.006	1.125	1.039	2.506	0.983
12	1.193	0.967	1.270	0.971	1.103	1.071	1.997	0.939
24	1.142	0.998	1.210	1.004	1.101	1.073	1.882	0.948

Model 2 (8,8)

Shock to Variable 1/Effects on Variables 1 and 2

Horizon	SIC Variable		PIC Variable		KAIC Variable		KSIC Variable	
	1	2	1	2	1	2	1	2
6	1.432	1.253	1.571	1.263	0.971	0.957	0.819	1.304
12	1.214	1.150	1.282	1.155	0.988	0.989	0.888	1.172
24	1.121	1.105	1.175	1.112	1.007	0.980	0.929	1.128

Shock to Variable 2/Effects on Variables 1 and 2

Horizon	SIC Variable		PIC Variable		KAIC Variable		KSIC Variable	
	1	2	1	2	1	2	1	2
6	1.190	1.366	1.280	1.393	0.957	1.099	0.942	1.299
12	1.010	1.305	1.038	1.320	0.989	1.074	0.957	1.244
24	1.000	1.200	1.024	1.213	0.980	1.073	0.887	1.145

[*]The numbers in the tables are the ratios of the impulse response function mean-square errors for the lag selection technique indicated to the impulse response function mean-square errors for the AIC.

Table 7
Impulse Response Function Mean-Square Errors[*]

Model 3 (3,1)

Shock to Variable 1/Effects on Variables 1 and 2

Horizon	SIC Variable		PIC Variable		KAIC Variable		KSIC Variable	
	1	2	1	2	1	2	1	2
6	0.993	1.038	1.002	1.031	0.982	0.952	0.990	1.027
12	0.921	0.982	0.920	0.975	0.974	0.996	0.994	0.984
24	0.898	0.977	0.897	0.970	0.996	1.001	0.975	0.980

Shock to Variable 2/Effects on Variables 1 and 2

Horizon	SIC Variable		PIC Variable		KAIC Variable		KSIC Variable	
	1	2	1	2	1	2	1	2
6	0.559	0.661	0.535	0.650	0.751	0.799	0.542	0.596
12	0.520	0.640	0.497	0.630	0.768	0.826	0.522	0.586
24	0.514	0.642	0.493	0.632	0.775	0.835	0.524	0.590

Model 4 (8,5)

Shock to Variable 1/Effects on Variables 1 and 2

Horizon	SIC Variable		PIC Variable		KAIC Variable		KSIC Variable	
	1	2	1	2	1	2	1	2
6	2.032	1.349	2.008	1.366	0.861	0.969	1.663	1.304
12	4.670	1.258	5.224	1.268	0.925	0.996	2.243	1.250
24	3.677	1.221	4.083	1.231	0.965	1.024	1.867	1.216

Shock to Variable 2/Effects on Variables 1 and 2

Horizon	SIC Variable		PIC Variable		KAIC Variable		KSIC Variable	
	1	2	1	2	1	2	1	2
6	1.149	1.542	1.230	1.585	0.959	1.110	3.533	1.414
12	1.340	1.301	1.433	1.332	1.186	1.037	3.461	1.163
24	1.232	1.207	1.304	1.234	1.124	1.033	2.849	1.096

[*]The numbers in the table are the ratios of the impulse response function mean-square errors for the lag selection technique indicated to the impulse response function mean-square errors for the AIC.