

Topic: Linear Algebra

Subtopic: Row echelon form

Q: Determine which of the following matrices are in row echelon form: a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \\ 0 & 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

A: c and d

B: None of them

C: All of them

D: a and b

Q: Reduce the following matrix to ordinary row echelon form (1), determine the rank (2), and identify the basic columns (3): $\begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

A: (1) $\begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ (2) rank=3 (3) A^*1 , A^*2 and A^*4

B: (1) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (2) rank=2 (3) A^*1 , A^*2

C: (1) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (2) rank=2 (3) A^*1 and A^*4

D: (1) $\begin{bmatrix} 0 & 2 & 3 & 3 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ (2) rank=0 (3) A^*2

Q: Determine the reduced row echelon form EA of the following matrix (1) and then express each nonbasic column in terms of the basic columns (2): $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

A: (1) $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (2) $A^*3 = 2A^*1 + 2A^*2$

B: (1) $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (2) $A^*3 = 2A^*1 + 1/2A^*2$

C: (1) $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix}$ (2) $A^*3 = 2A^*1 + 1/2A^*2$

D: (1) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ (2) $A^*3 = A^*1$

Q: Determine which of the following systems are consistent: (a) $\{x + 2y + z = 2, 2x + 4y = 2, 3x + 6y + z = 4\}$. (b) $\{2x + 2y + 4z = 2, x + y + z = 1, 3x + 6y + z = 4\}$.

A: a and c are consistent, while b is inconsistent

B: a and b are consistent, while c is inconsistent

C: All are consistent

D: None of them are consistent

Q: Which of these rules gives a correct definition of the rank of A ? By EA we denote the reduced row echelon form of a matrix

A: All true

B: (a) True (b) False (c) True (d) False

C: (a) False (b) True (c) False (d) True

D: All false

Q: Suppose that $[A|b]$ is reduced to a matrix $[E|c]$. (a) Is $[E|c]$ in ordinary row echelon form, if E is in ordinary row echelon form

A: (a) No (b) No

B: (a) Yes (b) No

C: (a) No (b) Yes

D: (a) Yes (b) Yes

Subtopic: Matrices

Q: Is this matrix symmetric, skew symmetric or neither? $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \end{bmatrix}$

A: Skew symmetric

B: Neither

C: Symmetric

Q: Let matrices $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -5 & 4 \\ 4 & -3 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 7 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Compute the product $AB = \begin{bmatrix} 10 & 15 \\ 1 \end{bmatrix}$

A: It exists and it's correct

B: It doesn't exist

C: It exists but it's not correct

Q: What is the 2 by 2 identity matrix I such that I times (x, y) equals (x, y) ?

A: $I = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

B: $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C: It doesn't exist

D: $I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Q: Is this matrix symmetric, skew symmetric or neither? $\begin{bmatrix} 1 & -3 & 3 \\ -3 & 4 & -3 \\ 3 & 3 & 0 \end{bmatrix}$

A: Symmetric

B: Skew symmetric

C: Nonsensical

D: Neither

Q: What is the 2 by 2 exchange matrix P such that P times (x, y) equals (y, x)?

A: $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

B: $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

C: It doesn't exist

D: $P = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Q: Let matrix $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ and $x = (1, 1, 1, 2)$. Compute the product Ax by dot products of the rows

A: $Ax = (4, 3, 5, 5)$

B: $Ax = (5, 5, 4, 3)$

C: $Ax = (3, 4, 5, 5)$

Q: Is this matrix symmetric, skew symmetric or neither? $\begin{bmatrix} 0 & -3 & -3 \\ 3 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$

A: Neither

B: Symmetric

C: Skew symmetric

Q: Let matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 1 \\ -4 & 1 & 2 \end{bmatrix}$ and $x = (2, 2, 3)$. Compute the product Ax by dot products of the rows with the columns

A: $Ax = (-18, -5, 0)$

B: $Ax = (-18, 5, 0)$

C: $Ax = (18, -5, 0)$

D: $Ax = (18, 5, 0)$

Q: Consider the following linear system $\begin{cases} x + y + z = 2 \\ x + 2y + z = 3 \\ 2x + 3y + 2z = 5 \end{cases}$ Determine how many solutions has the system

A: Infinitely many solutions

B: One solution

C: No solution

Q: Let matrices $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -5 & 4 \\ 4 & -3 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 7 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$. Compute the product $CB = \begin{bmatrix} 8 & 15 & 12 \end{bmatrix}$

A: It doesn't exist

B: It exists but it's not correct

C: It exists and it's correct

Q: Let matrices $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -5 & 4 \\ 4 & -3 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 7 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$. Compute the product $BA = \begin{bmatrix} 10 & 15 & 8 \end{bmatrix}$

A: It doesn't exist

B: It exists and it's correct

C: It exists but it's not correct

Q: Consider the following linear system $\begin{cases} x + y + z = 2 \\ x + 2y + z = 3 \\ 2x + 3y + 2z = 9 \end{cases}$ Determine how many solutions has the system

A: No solution

B: Infinitely many solutions

C: One solution

Q: Is this matrix symmetric, skew symmetric or neither? $\begin{bmatrix} 0 & -3 & -3 \\ -3 & 0 & 3 \\ -3 & 3 & 1 \end{bmatrix}$

A: Symmetric

B: Skew symmetric

C: Neither

Subtopic: Gaussian elimination

Q: Consider the following linear system: $\begin{cases} ax + 3y = 3 \\ 4x + 6y = 6 \end{cases}$. For which numbers a does elimination break down (1) permanently

A: (1) $a=2$ (2) $a=0$

B: Any value of a breaks down elimination permanently

C: (1) $a=0$ (2) $a=3$

D: (1) $a=2$ (2) $a=4$

Q: What multiple L2 of equation 1 should be subtracted from equation 2 to eliminate unknown x in equation 2 ? $\begin{cases} 2x + 3y = 1 \\ 10x + 15y = 5 \end{cases}$

A: L2.1=-5

B: L2.1=10

C: L2.1=2

D: L2.1 = $10/2=5$

Q: Consider the following linear system: $\begin{cases} 2x + by = 16 \\ 4x + 8y = g \end{cases}$ Choose a coefficient b that makes this system singular, the

A: $b=4$ and $g=32$

B: $b=4$ and $g=4$

C: $b=0$ and $g=32$

D: $b=8$ and $g=4$

Q: Use the Gauss-Jordan method to solve the following system: $\begin{cases} 0x+4y-3z = 3 \\ x+7y-5z = 4 \\ x+8y-6z = 5 \end{cases}$ Give the solution

A: (1, 0, 1)

B: (0, 1, 1)

C: (1, 1, -1)

D: (1, 0, 1)

Q: What multiple of equation 1 should be subtracted from equation 2 below, to eliminate unknown x in equation 2 ? $\begin{cases} 2x - 4y = 6 \\ 10x - 20y = 30 \end{cases}$

A: -1/2

B: 2

C: 1

D: 1/2

Q: Consider the following linear system $\begin{cases} 3x + 2y = 10 \\ 6x + 4y = ? \end{cases}$ Choose a right side b1 which gives no solution and another r

A: There is no solution to such system

B: $b_1=(10,20)$ and $b_2=(20, 10)$

C: There is no solution unless $b_1=(10,20)$

D: $b_1=(20,10)$ and b_2 can be any other two dimensional vector

Q: Consider the following linear system: $\{kx + 3y = 6, \{3x + ky = 6$. For which three numbers k does elimination break down ?(1

A: The only solutions are $k=3$ and $k=-3$, and both have only one solution

B: The system has no solution, regardless of the value of k .

C: (1) $k=3$, $k=-3$ and $k=0$ (2) $k=0$ (3) no solution, infinite solutions and one solution

D: (1) $k=3$, $k=-3$ and $k=0$ (2) $k=0$ (3) no solution, no solution and infinitely many solutions

Subtopic: Vectors

Q: Let vectors $u = (-0.6, 0.8)$, $v = (3, 4)$ and $w = (8, 6)$. Calculate $u \cdot v$, $u \cdot w$, $u \cdot (v + w)$ and $w \cdot v$.

A: $u \cdot v = 1.4$; $u \cdot w = 0$; $u \cdot (v + w) = 1.4$; $w \cdot v = 48$

B: $u \cdot v = -1.4$; $u \cdot w = 0$; $u \cdot (v + w) = 1.4$; $w \cdot v = 48$

C: $u \cdot v = 1.4$; $u \cdot w = 0$; $u \cdot (v + w) = -1.4$; $w \cdot v = 48$

D: $u \cdot v = -1.4$; $u \cdot w = 0$; $u \cdot (v + w) = -1.4$; $w \cdot v = 48$

Q: Let vectors u , v and w in three dimensions. Let u be perpendicular to v and w . v and w are parallel.

A: TRUE

B: False, v and w can be any vectors in the plane perpendicular to u

Q: Let vectors $v = (2, 1)$ and $w = (1, 2)$. Find $3v + w$ and $cv + dw$.

A: $3v + w = (7, 5)$ and $cv + dw = (2c + d, c + 2d)$

B: $3v + w = (7, 5)$ and $cv + dw = (2c + d, -c + 2d)$

C: $3v + w = (7, 5)$ and $cv + dw = (c + 2d, c + 2d)$

D: $3v + w = (7, -5)$ and $cv + dw = (2c + d, -c + 2d)$

Q: Let vectors u , v and w . Let u be perpendicular to v and w . u is perpendicular to v and $2w$.

A: FALSE

B: TRUE

Q: Let vectors $\mathbf{v} = (1, -2, 1)$ and $\mathbf{w} = (0, 1, -1)$. Find $c\mathbf{v} + d\mathbf{w}$. Then, find c and d so that $c\mathbf{v} + d\mathbf{w} = (3, 3, -6)$

A: $c\mathbf{v} + d\mathbf{w} = (c, -2c + d, c - d)$; $c = 3$ and $d = 9$

B: $c\mathbf{v} + d\mathbf{w} = (c, 2c - d, c - d)$; $c = 3$ and $d = 9$

C: $c\mathbf{v} + d\mathbf{w} = (c, -2c + d, c - d)$; $c = -3$ and $d = -9$

D: $c\mathbf{v} + d\mathbf{w} = (c, -2c + d, c - 2d)$; $c = -3$ and $d = 9$

Q: Let vectors $\mathbf{v} = (3, 1)$ and $\mathbf{w} = (-1, -2)$. Calculate the angle between these two vectors.

A: $3/4$ radians

B: $2/3$ radians

C: $/4$ radians

D: $-3/4$ radians

Q: Let vectors $\mathbf{u} = (-0.6, 0.8)$, $\mathbf{v} = (3, 4)$ and $\mathbf{w} = (8, 6)$. Compute the lengths of these vectors.

A: u 's length = -1 , v 's length = 5 , w 's length = 10

B: u 's length = 1 , v 's length = 5 , w 's length = 11

C: u 's length = 1 , v 's length = 5 , w 's length = 10

D: u 's length = 1 , v 's length = 6 , w 's length = 10

Q: Let vectors $\mathbf{v} = (2, 2, -1)$ and $\mathbf{w} = (2, -1, 2)$. Is the angle between these two vectors 90° (perpendiculars)?

A: TRUE

B: FALSE

Q: Let vectors \mathbf{v} and \mathbf{w} . $\mathbf{v} + \mathbf{w} = (5, 1)$ and $\mathbf{v} - \mathbf{w} = (1, 5)$. Compute \mathbf{v} and \mathbf{w} .

A: $\mathbf{v} = (3, 3)$ and $\mathbf{w} = (2, -2)$

B: $\mathbf{v} = (5, 2)$ and $\mathbf{w} = (4, -3)$

C: $\mathbf{v} = (3, -3)$ and $\mathbf{w} = (2, -2)$

D: $v = (-3, 3)$ and $w = (-2, -2)$

Q: Let vectors u and v be perpendicular unit vectors. $u-v$'s length = 2

A: FALSE

B: TRUE