

ASTR404 Homework 9

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1 Question 1

A

Sections A, B, and C will all be answered in a table under the C section header.

B

Sections A, B, and C will all be answered in a table under the C section header.

C

Table 1: The Physical Properties of Cepheid Stars

name	mass (M_{\odot})	luminosity (L_{\odot})	effective temperature (K)
UU Mus	4.94	1126.43	5087.03
FG Vel	2.80	167.44	5332.44
DF Cas	3.44	333.01	5501.84
AE Vel	4.01	557.23	5532.37
YZ Car	5.94	2074.18	5133.26
HW Car	5.05	1207.16	4832.11
XY Car	4.91	1102.86	4976.51
VX Per	4.99	1162.81	5332.45
BK Cen	4.12	610.73	5563.71
GZ Car	3.78	457.40	5563.72
VX Cyg	3.74	443.34	4735.20
GH Car	5.02	1184.96	5699.37
DW Cas	2.93	194.53	5442.94
GX Car	4.72	963.49	5359.26
FM Cas	4.99	1159.70	5332.44
SZ Cyg	4.43	777.74	4852.08
CN Car	3.20	260.88	5254.77
UY Per	2.71	150.31	5563.71

name	mass (M_{\odot})	luminosity (L_{\odot})	effective temperature (K)
SV Vul	7.96	5537.24	4812.34
UZ Car	4.59	877.91	5205.07
BP Cas	2.99	208.78	5442.94
CY Car	4.11	605.61	5563.71
WW Car	4.18	640.82	5817.20
CD Cyg	4.98	1149.89	5042.00
CD Cas	3.08	232.22	5359.26
V459 Cyg	3.18	257.70	5332.44
DR Vel	4.14	621.18	4892.65
VY Per	2.64	137.55	5442.94
SY Cas	3.87	495.08	5663.64
V402 Cyg	3.84	484.34	5472.05
SY Nor	4.25	681.39	5595.96
GH Cyg	3.78	456.71	5414.48
Y Lac	4.70	948.61	5736.64
XY Cas	3.75	445.13	5472.05
SV Vel	5.47	1577.06	5280.22
ST Vel	3.93	520.03	5180.78
SW Cas	3.94	523.84	5472.05
BR Vul	2.99	208.69	5563.71
DL Cas	4.75	980.44	5386.60
XZ Car	5.01	1178.15	4754.20
VZ Cyg	4.69	942.15	5472.05
V Lac	4.74	978.90	5736.64
V496 Cen	3.50	352.19	5442.94
VY Cyg	3.89	505.45	5532.36
BZ Cyg	3.25	276.64	5332.44
Z_{Lac}	5.23	1353.51	5205.07
U_{Car}	8.80	7749.27	4754.20
RZ CMa	3.57	379.44	5629.23
V339 Cen	4.49	815.72	5042.00
X Lac	5.07	1224.92	5629.23
KQ Sco	3.14	246.38	4519.93
BG Lac	4.35	735.06	5414.48
VY Car	6.12	2301.56	4697.71
AS Per	3.42	326.15	5472.05
X Cru	4.82	1034.04	5156.85
V520 Cyg	2.50	113.82	5595.96
V514 Cyg	2.13	67.20	5663.64
XX Cen	5.57	1671.85	5133.26
RX Aur	5.77	1885.61	5109.99
GU Nor	2.75	156.72	5472.05

name	mass (M_{\odot})	luminosity (L_{\odot})	effective temperature (K)
RZ Vel	6.69	3089.88	4998.07
AY Cen	4.17	636.90	5180.78
V600 Aql	3.02	214.82	5472.05
V378 Cen	4.62	896.59	5359.26
UX Car	4.86	1061.14	5736.64
AT Pup	5.26	1382.75	5442.94
RS Pup	6.46	2757.24	4679.22
RS Cas	2.99	209.41	5414.48
RY CMa	4.81	1023.95	5442.94
AG Cru	4.68	934.19	5663.64
CR Cep	2.95	198.97	5332.44
RT Mus	3.56	375.79	5775.77
FN Aql	4.12	610.26	5180.78
MW _{Cyg}	3.04	220.84	5332.44
XX Sgr	3.64	403.34	5532.36
IT Car	4.33	721.09	4955.18
V Car	5.15	1287.03	5180.78
TX Cyg	2.84	175.44	5442.94
RX Cam	4.68	936.77	5386.60
VW Cru	2.74	155.88	5359.26
ER Car	5.75	1863.84	5019.90
X Vul	3.28	284.11	5629.23
T Vel	4.04	572.34	5332.44
U TrA	4.18	642.93	5699.37
V386 Cyg	2.53	119.81	5472.05
X Cyg	5.95	2091.17	4872.26
TU Cas	4.28	693.44	5966.65
V496 Aql	4.04	571.44	5087.03
R Mus	5.89	2025.52	5332.44
BG Vel	4.02	563.63	5109.99
V737 Cen	5.10	1250.21	4998.07
S Cru	5.39	1497.58	5442.94
AW Per	4.20	652.27	5699.37
U Vul	4.56	857.41	5386.60
S Nor	5.46	1572.11	5064.37
S TrA	5.48	1591.91	5306.10
RV Sco	4.48	810.65	5414.48
T Cru	5.00	1167.46	5109.99
SU Cyg	4.69	941.19	5910.48
BB Sgr	4.28	693.06	5156.85
MY Pup	6.13	2316.95	5532.36
V Cen	4.29	699.66	5472.05

name	mass (M_{\odot})	luminosity (L_{\odot})	effective temperature (K)
Y Oph	5.03	1189.68	5133.26
U Sgr	4.19	643.40	5205.07
R TrA	4.26	681.57	5472.05
T Vul	5.08	1228.07	5501.83
BG Cru	5.27	1393.40	5563.71
X Sgr	4.61	895.84	5629.23
AZ Cen	5.24	1374.11	5817.20
QZ Nor	4.55	848.69	5359.26
BY Cas	3.02	216.47	5736.64
VZ CMa	3.82	475.09	5861.67
V532 Cyg	3.92	517.34	5736.64
EU Tau	4.17	635.64	5817.20
V950 Sco	5.04	1199.89	5736.64
V397 Car	3.78	456.12	5910.48
TT Aql	4.91	1097.25	4955.18
V482 Sco	3.94	525.03	5414.48
FM Aql	3.59	383.12	5359.26
CO Aur	4.20	649.37	6050.00
V350 Sgr	4.32	713.98	5442.94
BP Cir	4.26	682.97	5910.48
AV Cir	4.33	720.96	5736.64
IR Cep	3.72	431.91	5966.65
AH Vel	6.27	2495.27	5736.64
SZ Tau	4.51	831.87	5595.96
DT Cyg	5.03	1194.77	5775.77
FF Aql	5.40	1512.57	5629.23
AX Vel	4.76	988.30	5910.48
UZ Cen	4.05	577.65	5966.65
V381 Cen	4.54	842.38	5472.05

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D

So the starting equation is:

$$\Pi = \sqrt{\frac{R^3}{GM}} \quad (1)$$

The equation for luminosity, solved for the radius, is:

$$R = \frac{1}{T_{eff}^2} \sqrt{\frac{L}{4\pi\sigma}} \quad (2)$$

Plugging this back into the equation gives us:

$$\Pi = \frac{1}{T_{eff}^3} \sqrt{\frac{1}{GM} \left(\frac{L}{4\pi\sigma}\right)^{3/2}} \quad (3)$$

The plot of this is:

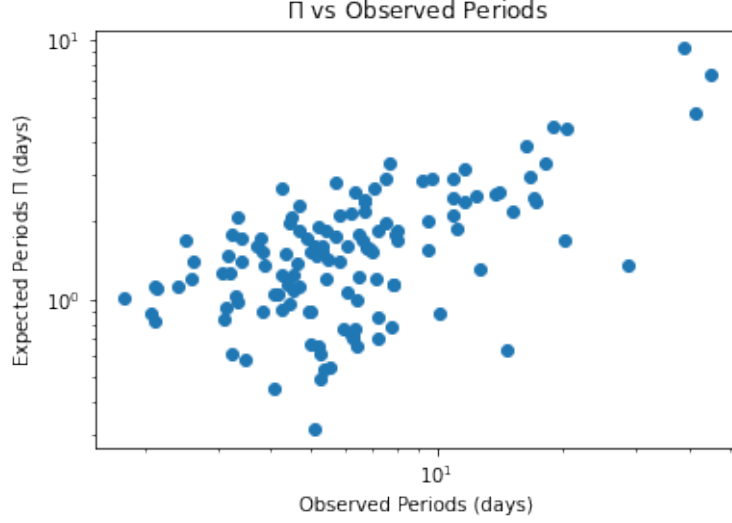


Figure 1: The expected periods vs the observed periods.

This is a pretty clear linear increase, although there are some amplitudes that are outliers near the lower end of the expected periods. This could be that these stars aren't classically cepheids, and so their expected periods end up smaller than their observed periods.

E

Since the only time dependent value from the answer to (d) is the effective temperature, the derivative is:

$$\frac{d\Pi}{dt} = \frac{-3}{T_{eff}^4} \sqrt{\frac{1}{GM} \left(\frac{L}{4\pi\sigma}\right)^{3/2}} \frac{dT_{eff}}{dt} \quad (4)$$

This can be separated out into:

$$\frac{d\Pi}{dt} = \frac{-3}{T_{eff}} \frac{1}{T_{eff}^3} \sqrt{\frac{1}{GM} \left(\frac{L}{4\pi\sigma}\right)^{3/2}} \frac{dT_{eff}}{dt} = \frac{-3\Pi}{T_{eff}} \frac{dT_{eff}}{dt} \quad (5)$$

Which is the form we want. From here, we cancel the dt's and move like terms together:

$$\frac{d\Pi}{\Pi} = -3 \frac{dT_{eff}}{T_{eff}} \quad (6)$$

Integrating this from their blue edge values to their red edge values gives:

$$\ln(\Pi_{RE}) - \ln(\Pi_{BE}) = -3(\ln(T_{effRE}) - \ln(T_{effBE})) \quad (7)$$

This is equal to:

$$\ln\left(\frac{\Pi_{RE}}{\Pi_{BE}}\right) = \ln\left(\left(\frac{T_{effRE}}{T_{effBE}}\right)^{-3}\right) \quad (8)$$

Which gives the final value of the ratio as:

$$\frac{\Pi_{RE}}{\Pi_{BE}} = \left(\frac{T_{effRE}}{T_{effBE}}\right)^{-3} \quad (9)$$

F

The log of the ratio relates to the log of the luminosity through:

$$\log\left(\frac{\Pi_{RE}}{\Pi_{BE}}\right) = \log(10^{-0.18}\left(\frac{L}{L_{\odot}}\right)^{0.09}) \quad (10)$$

The plot of this is:

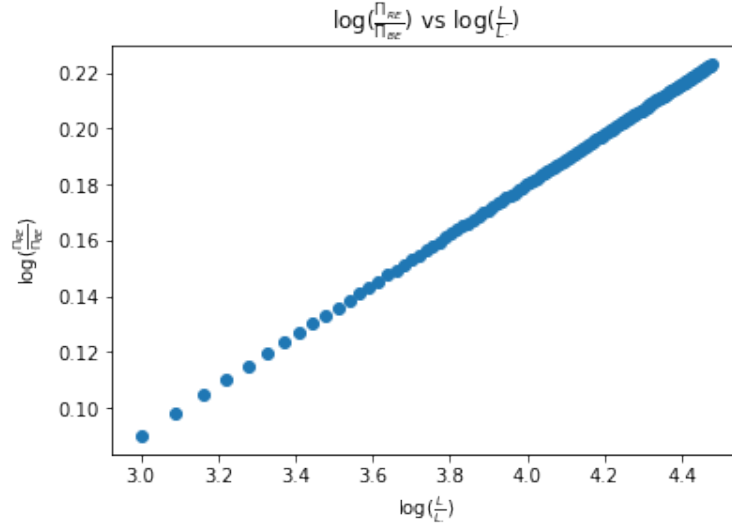


Figure 2: Graph of the log of the ratio periods vs the log of the luminosity, from $1000 L_{\odot}$ to $30000 L_{\odot}$

Higher mass stars experience a greater period change as they cross the instability strip.

G

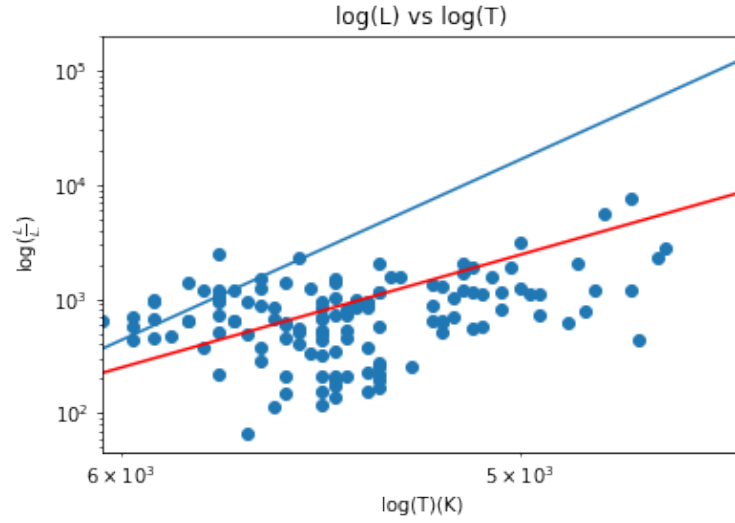


Figure 3: An HR diagram constructed of just stars in our data set.

H

My blue edge star is IR Cep, at a period of 2.11410 days with a luminosity of 5966.65, and my red edge star is V532 Cyg, at a period of 3.2836 days with a luminosity of 5736.64. The ratio of periods is then 1.55, slightly higher than the 1 we want it to be. The average luminosity is 5851.645, which gives a ratio of 1.44, which is still slightly off, but closer to the 1 we want.