ASTR 404 Fall 2020
Homework #9

Due online **Sunday, November 8 at 11:55 pm**. Please remember to show all work on quantitative questions. No credit will be given without it, **even if the numerical result is correct**. Point values for each question are indicated in parenthesis after the question number.

1. (60) Cepheid variable stars experience dynamical-timescale pulsations whose period has a well-known relationship with their intrinsic luminosity. In this problem you will examine the characteristics of a sample of Galactic Cepheids.

   Download the file "cepheids_proc.txt" from the course web site. This file contains a sample of Galactic classical Cepheids (which pulsate at their fundamental oscillation frequency) with pulse-averaged V-band (visual) apparent magnitudes and $B - V$ colors from the David Dunlap Observatory database of Galactic Cepheids[1] and distances and pulsation periods from the Gaia Data Release 2 (Ripepi et al. 2019, AA, 625, A14). The data columns in this file are

ID	A common name used in the literature.
GAIA_ID	The corresponding Gaia DR2 source identification number.
PERIOD	The pulsation period in days (Π).
PARALX	The parallax measured by Gaia in **milli**arcseconds.
VMAG	The pulse-averaged V-band apparent magnitude (V).
BV	The pulse-averaged $B - V$ color (ie., the difference between the B and V apparent magnitudes).
EBV	The color excess $E(B - V)$ due to interstellar reddening.

   You should be able to work with the data in this file either using a spreadsheet or by writing a program (e.g. in Python).

   a) Compute the pulse-averaged effective temperature T_{eff} of each star. To do this, first compute the intrinsic $B - V$ color by using the color excess to account for interstellar reddening:

   $$(B - V)_{\text{intrinsic}} = (B - V)_{\text{observed}} - E(B - V)$$

   (the value in the table is the observed quantity). The intrinsic color can be used to estimate the effective temperature using results from spectroscopic observations of red giants. A simple fitting formula based on data in Buzzoni et al. (2010, MNRAS, 403, 1592) is

   $$T_{\text{eff}} = (6050\ \text{K}) \exp\left[-\left(\frac{(B - V)_{\text{intrinsic}} - 0.45}{3.0}\right)^{0.75}\right].$$

   b) Now compute the pulse-averaged bolometric luminosity L of each star. To do this, first compute the star's apparent bolometric magnitude (ie. the apparent magnitude corresponding to the star's entire output, not just the light in the V band) using a bolometric correction

1 https://www.astro.utoronto.ca/DDO/research/cepheids/cepheids.html

determined by Buzzoni et al.:

$$m_{\text{bol}} = V + \frac{1}{1000}\exp\left(\frac{27500\ \text{K}}{T_{\text{eff}}}\right).$$

Then use the apparent bolometric magnitude and the parallax to compute the absolute bolometric magnitude M_{bol}. Finally, using the fact that the Sun's absolute bolometric magnitude is $M_{\text{bol},\odot} = 4.74$, compute L in units of the Sun's luminosity.

c) So far we have used observational data (or fits to data). Now use the theoretical mass-luminosity relation for classical Cepheids

$$\log(L/L_\odot) = 0.90 + 3.35\log(M/M_\odot) + 1.36\log Y - 0.34\log Z$$

from Bono et al. (2000, ApJ, 543, 955) to compute the mass of each star, assuming solar abundance values ($Y = 0.28$, $Z = 0.02$). Produce a table listing each star's name, mass (in units of $M_\odot$), luminosity (in units of $L_\odot$), and effective temperature (in units of K).

d) The pulsation period Π is roughly equal to the star's dynamical time: $\Pi \sim \sqrt{\frac{R^3}{GM}}$. Using the spherical blackbody relation, write an expression for the pulsation period Π in terms of the mass and the unperturbed luminosity and effective temperature. Plot this expected period against the observed period (use a log-log scale). Be sure to use appropriate axis labels. Do you see the expected scaling? What might explain any observed difference in amplitude? (See note in (g) below for the lowest-luminosity stars.)

e) Since Cepheids are horizontal branch stars, they slowly move along (very roughly) horizontal paths across part of the HR diagram as they evolve. They can pass through the Cepheid instability strip multiple times as they go back and forth. Suppose the evolutionary track of a star is horizontal, so that there is no evolution in the star's unperturbed luminosity, and neglect mass loss. Show that the rate of change of the star's pulsation period (ie. $\frac{d\Pi}{dt}$) is given by

$$\frac{d\Pi}{dt} = -3\frac{\Pi}{T_{\text{eff}}}\frac{dT_{\text{eff}}}{dt},$$

where $\frac{dT_{\text{eff}}}{dt}$ is the rate of change of the star's unperturbed effective temperature due to its evolution. Integrate this equation for stars moving rightward (redward) on the HR diagram to determine the ratio of the period at the red edge (RE) of the instability strip to the period at the blue edge (BE), $\Pi_{\text{RE}}/\Pi_{\text{BE}}$, in terms of the ratio of the unperturbed effective temperatures where the star's evolutionary track crosses those boundaries ($T_{\text{eff,RE}}/T_{\text{eff,BE}}$).

f) Theoretical models by De Somma et al. (2020, ApJS, 247, 30) place the locations of the blue and red edges of the Cepheid instability strip at

$$\log\left(\frac{T_{\text{eff,BE}}}{\text{K}}\right) = 3.91 - 0.05\log\left(\frac{L}{L_\odot}\right)$$
$$\log\left(\frac{T_{\text{eff,RE}}}{\text{K}}\right) = 3.97 - 0.08\log\left(\frac{L}{L_\odot}\right)$$

given the Bono et al. mass-luminosity relation we have used. Using your result from (e), determine $\log\frac{\Pi_{\text{RE}}}{\Pi_{\text{BE}}}$ as a function of $\log(L/L_\odot)$. Plot the result versus $\log(L/L_\odot)$ between 1000

and $30,000 L_\odot$. Be sure to use appropriate axis labels. Which horizontal branch stars (lower mass or higher mass) exhibit the largest period changes as they cross the Cepheid instability strip?

g) Plot the luminosities and temperatures of the Cepheids from the data file as points on the Hertzsprung-Russell diagram. Use the blue and red edge temperature relations in (f) to plot curves showing the blue and red edges of the Cepheid luminosity strip. Label your plot appropriately. You should find that most of the Cepheids fall within the theoretical bounds of the instability strip. (Note: At the moment there seem to be some lower-luminosity stars in the sample that don't fall within the classical Cepheid instability strip and don't follow the Cepheid period-luminosity relation. There may be some contamination with W Virginis (Type II Cepheid) variable stars. However, if you have done everything correctly up to this point you should find a substantial population of stars inside the instability strip.)

h) Identify two stars of similar luminosity in your plot from (d), one that is close to the red edge and one that is close to the blue edge of the instability strip. Find the periods of these two stars in the table and compute their ratio. How does it compare with the theoretical expectation in (f), given the average of their luminosities?