University of Illinois at Urbana-Champaign Department of Physics
Physics 402 Laboratory

Experiment D-2

FRESNEL DIFFRACTION USING A SOLID STATE PHOTODIODE DETECTOR

References:

1. E. Hecht, *Optics*, 2nd ed., Ch. 10.2, Addison-Wesley, Reading, Massachusetts (1987).
2. F. A. Jenkins, H. E. White, *Fundamentals of Optics*, 4th ed., pp. 388-401, 579-599, McGraw-Hill, New York (1976).
3. M. V. Klein, T. E. Furtak, *Optics*, 2nd ed., Ch. 6.1-6.2, 7.1-7.2 John Wiley, New York (1986).
4. Grant R. Fowles, *Introduction to Modern Optics*, 2nd ed, Ch 5.1 – 5.5, Dover Publications, NY (1975).

For Solid State Photodiode Detectors:

1. J. Millman, C. C. Halkias, *Integrated Electronics: Analog and Digital Circuits and Systems*, pp. 79-81, McGraw-Hill, New York (1972).
2. P. Horowitz, W. Hill, *The Art of Electronics*, 2nd ed., pp. 998-1001, Cambridge University Press, Cambridge, Massachusetts (1989).
3. Hamamatsu Silicon Photodiodes catalog or on the web for our detector S1337-1010BQ: http://sales.hamamatsu.com/en/home.php or http://www.hamamatsu.com/us/en/product/category/3100/4001/4103/S1337-1010BQ/index.html

Purpose:

The purpose of this experiment is to measure, using a solid state (silicon) photodiode {Hamamatsu S1337-1010BQ} the intensity variations of both Fraunhofer diffraction patterns from single and multiple slits and the Fresnel patterns made by a narrow obstacle and a straight edge. The resulting intensity patterns are to be compared with calculated curves. Additional instructions may be given to you by the TA with Fourier transform exercise as time permits.

Procedure:

A. Start the LabWindows/CVI DAQ program *Diffn4* (shortcut on the desktop). Turn on the electronics rack power and Initialize the DAQ in that order. Use the *HELP* pull-down menu (at the LHS top of blue taskbar of the program) to familiarize yourself with the features of the program. When you initialize, the detector stage will move to the initial data acquisition position at the end of the stage ($x = 0.0$ cm). Move the photodiode detector to the $x = 2.5$ cm position (middle of its range of travel) using the "*GoTo X = Xuser*" button. Adjust the position of the red He-Ne laser so that the unobstructed laser beam falls precisely on the entrance aperture of the optical fiber head (the far end of which is coupled to the solid-state silicon photodiode and photodiode preamplifier). {n.b. If the computer should crash, *turn the power to the electronics rack off* before hard-rebooting the computer.)

C. Fresnel Diffraction: The Fresnel formalism is valid for any position of the light source and photodetector relative to an obstacle. It is greatly simplified if the source is at infinity; i.e., when the incident rays of light are parallel to each other. Remember that Huygens-Fresnel theory invokes that secondary wavelets are generated at the aperture producing spherical waves. Again use the red He-Ne laser for this portion of the experiment. There are four possible experimental procedures:

1. Ideally, this experiment should be performed with incident plane-waves. This can be achieved by passing the laser beam through a microscope objective in order to focus it to a point, and then expanding the laser beam by placing that point at the focus of a positive lens. In practice, this is difficult (but possible) to achieve, requiring extreme care in the alignment of the optical elements.

2. Place half of a beam expander (basically a lens and collimator) and expand the laser beam to make a spot several centimeters wide at the position of the optical fiber detector, centered on the detector aperture when it is positioned at $x = 2.5$ cm. This will, of course, place the source point at the focus of the diverging lens. Measure the size of the spot at several positions and deduce the effective source position.

3. Use one of the single slits from part A (for example 0.04 mm or the variable slit adjusted to the very small size) to spread the laser beam across the face of the detector. The source is then at the position of the slit. Place the slide holder approximately 1 m from the optical fiber detector, and measure that distance{ = the distance from the source (ℕ, the negative focal point of the diverging lens, or the slit position, depending on the method used)}.

It is useful to temporarily minimize the DAQ program and run the "Fresdiff1.exe", Fresnel Diffraction Simulation program (shortcut on the desktop) for the semi-infinite plane and thin obstacle (see below) using the actual separation distances, the laser wavelength and obstacle dimensions. This will give you a sense of the scan width to use. Note that the program assumes an incident plane wave. It is also necessary to make a geometric correction if the source is not located at ℕ (see below). Make a scan of that range with no obstacle in place. Chose a scan range (selectable from a pull-down menu: "*Change X_Lo, X_Hi Limits*") such that the scan is centered on the profile of laser beam.

Semi-infinite plane: Intercept the light with the edge of a razor blade (epoxied into a slide mount). Position the razor blade such that the edge of its shadow falls on the aperture of the fiber optic detector when positioned at $x = 2.5$ cm. Note that if you position the blade such that the shadow region is scanned last (i.e. $x > 2.5$ cm) then your plots will resemble the calculated curve (see below). Scan over a sufficient range to cover the intensity pattern.

Narrow obstacles: Repeat the above procedure with one or several slide mounts holding wires. Record the distance from each obstacle to detector, and the diameter of the wire, and make copies of your plots

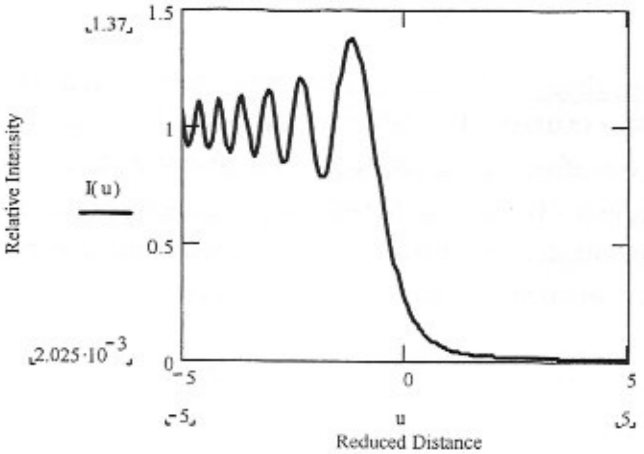


Fig 2. Plot of function *I(u)* vs *u* for semi-infinite plane.

A plot of this function from the old Mathcad program *Fresnel.mcd* is shown above. You may use another program *Fresdiff1.exe* program (shortcut on the desktop) to enter the wavelength of the laser and the blade-to-detector distance for a quantitative comparison with your results. Note that $x = 0$ is defined to be at the edge of the geometrical shadow, identified as the point at which the intensity is ¼ the intensity away from the shadow. You may save your data or print the plot for your report.

The expression for Fresnel diffraction from a thin obstacle is even more complicated, involving the width of the obstacle in the same reduced units. For an actual width w, we define a reduced width $\Delta u = w / \sqrt{2/\lambda r_0}$. The intensity can be written in terms of the Fresnel integrals as:

$$I(u)/I_0 = \frac{1}{2}\left\{\left[1 + C(u-\Delta u) - C(u+\Delta u)\right]^2 + \left[1 + S(u-\Delta u) - S(u+\Delta u)\right]^2\right\}.$$

A plot of this function is shown below for Δu = 0.7.

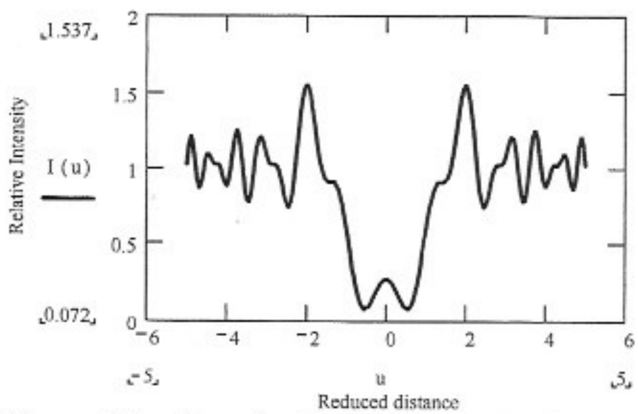


Fig 3. Fresnel function plot for a thin obstacle with Δu = 0.7.

This function is also available on the *Fresdiff1* program, and should be used to make quantitative comparisons with your results.

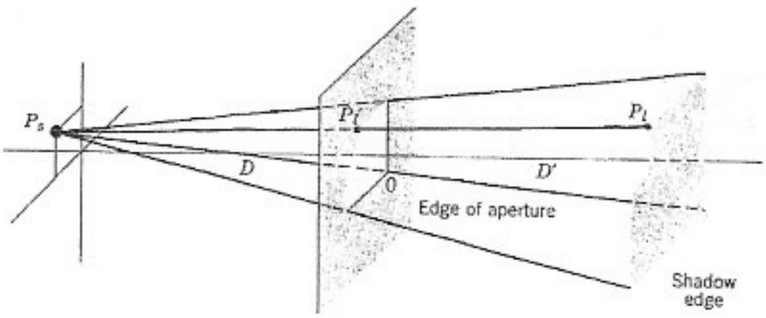


Fig. 8.6

Here P_l is at the edge of the geometrical shadow of the aperture produced by the light from the point source P_s. We may then write

$$\mathbf{r}_l' - \mathbf{r}_m = \frac{D}{D+D'}(\mathbf{r}_l - \mathbf{r})$$

and then rewrite (8.22) as

$$\boldsymbol{\eta}_l = \left(\frac{2}{\lambda}\right)^{1/2}\left[\frac{D+D'}{DD'}\,\frac{D^2}{(D+D')^2}\right]^{1/2}(\mathbf{r}_l - \mathbf{r}) = \left[\frac{2D}{\lambda D'(D+D')}\right]^{1/2}(\mathbf{r}_l - \mathbf{r})$$

$$= \frac{(\mathbf{r}_l - \mathbf{r})}{S_F} \tag{8.24}$$

where

$$S_F = \left[\frac{\lambda D'(D+D')}{2D}\right]^{1/2} \tag{8.25}$$

is a scale factor having the dimension of length.

When the limits on the η_x and η_y integrations in Eq. (8.21) are mutually independent, the integral factors giving

$$\tilde{E}(\mathbf{r}) = \tfrac{1}{2}i\tilde{E}_{na}(\mathbf{r})\,I_x I_y$$

$$I_x = \int e^{-(\pi i/2)\eta_x^2}\,d\eta_x, \qquad I_y = \int e^{-(\pi i/2)\eta_y^2}\,d\eta_y \tag{8.26}$$

If $E_{e,na}(\mathbf{r})$ represents the flux density with no aperture, then the flux density is

$$E_e(\mathbf{r}) = E_{e,na}(\mathbf{r})\,\frac{|I_x|^2\,|I_y|^2}{4} \tag{8.26$'$}$$

B. APPLICATION TO A RECTANGULAR APERTURE AND ITS LIMITING CASES

In this section we discuss Fresnel diffraction by a rectangular aperture and the limiting cases obtained when one or more of the edges go to infinity. The aperture and its shadow in the observation plane are illustrated in Fig. 8.7 for two dimensions. Because where

$$I(\eta) = \int_0^{\eta} e^{-i(\pi/2)u^2}\,du \tag{8.28c}$$

The integral in Eq. (8.28*c*) is called the *complex Fresnel integral*. It can be split into real and imaginary parts as follows:

$$I(\eta) = C(\eta) + iS(\eta) \tag{8.29a}$$

where

$$C(\eta) = \int_0^{\eta} \cos\left(\frac{\pi}{2}u^2\right) du \tag{8.29b}$$

and

$$S(\eta) = -\int_0^{\eta} \sin\left(\frac{\pi}{2}u^2\right) du \tag{8.29c}$$

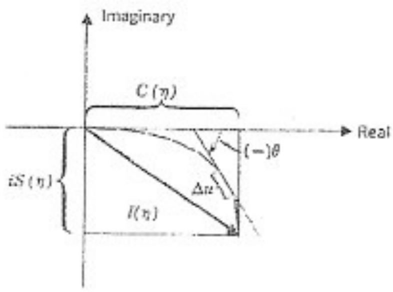


Fig. 8.8. Polygon approximation to vibration curve.

The integrals in Eqs. (8.28*a*, *b*, *c*) and (8.27*a*) have a graphical interpretation in terms of a *vibration curve* in the complex plane. We break the interval from 0 to η into N equally spaced segments of length $\Delta u = \eta/N$. Then we write

$$I(\eta) = \lim_{N\to\infty} \sum_{j=0}^{N-1} e^{-i(\pi/2)u_j^2}\Delta u$$

where $u_j = j\Delta u$. Before the limit is taken, the vector sum can be represented as the chord of a polygon in the complex plane. As $N \to \infty$ the polygon in Fig. 8.8 becomes a vibration curve, and $I(\eta)$ represents the complex vector lying along the chord of this curve with its tail at the origin and its head at the head of the last infinitesimal vector $\exp(-\pi i u^2/2)\,du$.

The total arc length S of the curve is given by

$$S = \int_0^{\eta} du = \eta$$

The inclination angle θ of Fig. 8.8 is the phase angle of the complex vector

$$du\, e^{-(1/2)\pi i u^2}$$

Hence

$$\theta(u) = -\frac{\pi u^2}{2}$$

This quadratic dependence of θ on arc length, which is characteristic of Fresnel diffraction, comes originally from the quadratic dependence of the optical path difference $[\widehat{P_sP'P}] - [\widehat{P_sP_mP}]$ on

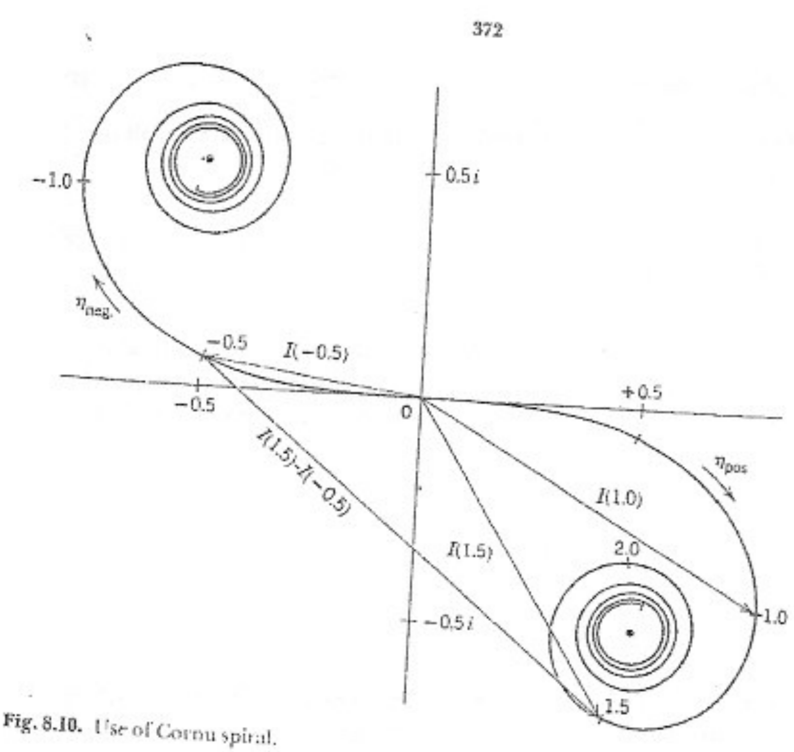


Fig. 8.10. Use of Cornu spiral.

The limiting values of the integrals as $\eta \to \pm \infty$ can be shown to be

$$C(+\infty) = -C(-\infty) = \tfrac{1}{2}$$

$$S(+\infty) = -S(-\infty) = -\tfrac{1}{2} \tag{8.30a}$$

Thus the "eye" of the spiral in the fourth quadrant is at

$$I(+\infty) = C(+\infty) + iS(+\infty) = \tfrac{1}{2}(1-i)$$

and the eye in the second quadrant is at

$$I(-\infty) = C(-\infty) + iS(-\infty) = -\tfrac{1}{2}(1-i) \tag{8.30b}$$

We now turn to some limiting cases of the rectangular aperture.

3. Open Aperture. Here there is no obstruction and hence $X_1 =$ $Y_1 = X_2 = Y_2 = +\infty$. Then $\eta_{x2} = \eta_{y2} = +\infty$ and $\eta_{x1} = \eta_{y1} = -\infty$, and from Eq. (8.30) we obtain

$$I_x = I_y = I(+\infty) - I(-\infty) = (1-i)$$

Equation (8.26) then gives

$$\tilde{E}(\mathbf{r}) = \tfrac{1}{2}i(1-i)^2 \tilde{E}_{na}(\mathbf{r}) = \tfrac{1}{2}i(1-1-2i)\,\tilde{E}_{na}(\mathbf{r}) = \tilde{E}_{na}(\mathbf{r})$$

This hardly unexpected result justifies the choice of $C = i/\lambda$ used in Eq. (7.6). The vector representing $I_x = I_y$ goes from eye to eye of the Cornu spiral, as shown in Fig. 8.11.

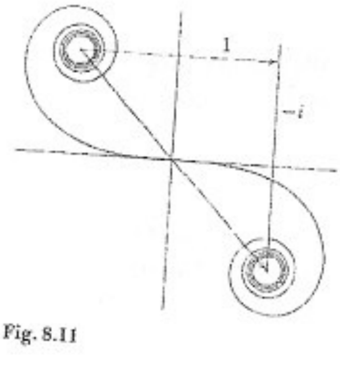
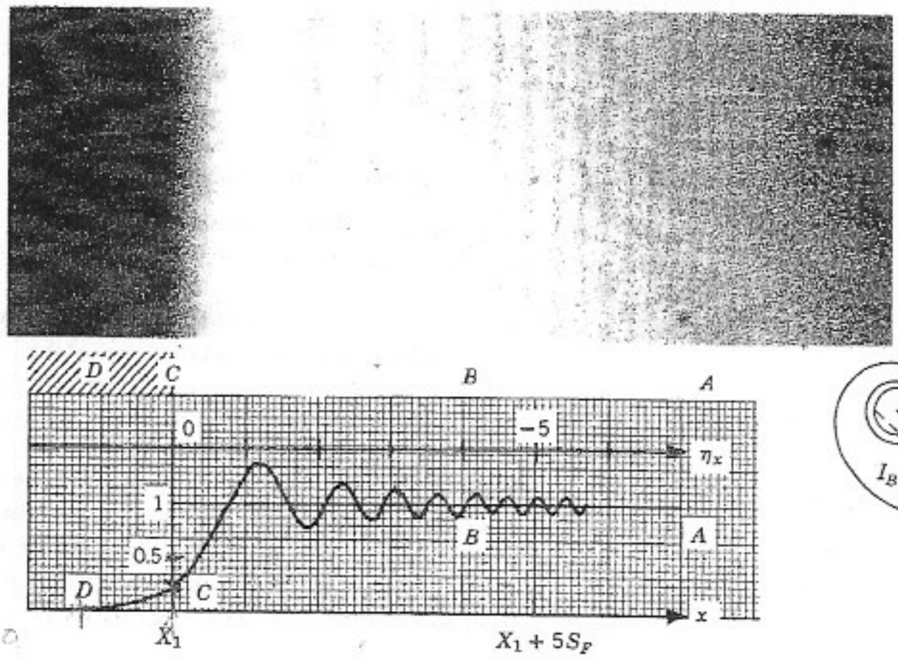


Fig. 8.11



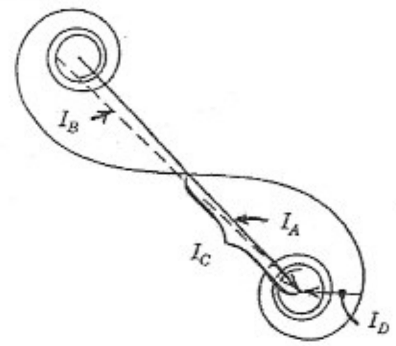


Fig. 8.13. Fresnel diffraction by a straight edge. At the top is an actual photograph. At the bottom the intensity is plotted. The vectors on the Cornu spiral at the right correspond to the labeled points.

For $x < X_1$, η_{x1} is positive, and the tail of the vector for I_x is on the lower right half of the spiral. As x continues to decrease η_{x1} grows, and the tail spirals in toward the eye. This is region D. The magnitude of I_x decreases monotonically and for $\eta_{x1} > +5$, say, I_x is quite small. This is the region of the geometrical shadow.

5. Wide Slit. Let the edges of the slit be at $x' = \pm x_0$. Then the shadow edges in the observation plane are at $x = X_1$ and $x = X_2$, where

$$X_1 = -\frac{(D+D')}{D}x_0 - \frac{D'}{D}x_s, \qquad X_2 = \frac{(D+D')}{D}x_0 - \frac{D'}{D}x_s$$

and the shadow region has width

$$\Delta X = X_2 - X_1 = \frac{(D+D')}{D}2x_0 \tag{8.31}$$

The integral for I_y still has infinite limits and gives $I_y = (1-i)$.

A "wide" slit is one that has a total arc length on the Cornu spiral

$$\Delta\eta = \eta_{x2} - \eta_{x1}$$

that is very large compared with unity. This condition can be rewritten using Eq. (8.27*b*):

$$1 \ll (\eta_{x2} - \eta_{x1}) = \frac{X_2 - X_1}{S_F} = 2x_0\left(\frac{2}{\lambda\rho}\right)^{1/2} \tag{8.32}$$

$$S_F = \left[\frac{\lambda D'(D+D')}{2D}\right]^{1/2}, \qquad \frac{1}{\rho} = \frac{1}{D} + \frac{1}{D'}$$

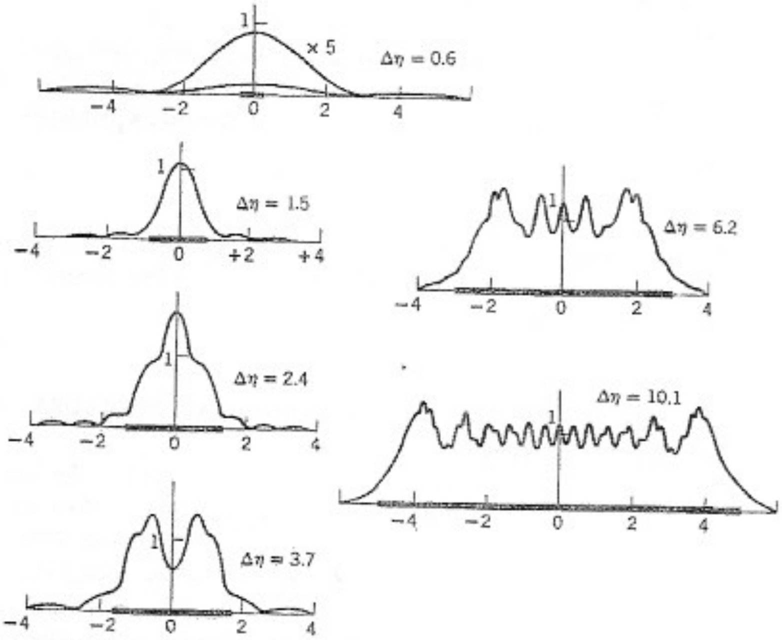


Fig. 8.15. Fresnel diffraction by slits of varying width. We plot the ratio of irradiance or flux density $E_r/E_{r,na}$ versus x/S_F and assume the pattern to be symmetrical about $x = 0$. Heavy bars denote the regions of geometrical brightness.

diffraction pattern much greater than that of the shadow. We can show that in the limit $\Delta\eta \to 0$ the Fresnel integrals do give the Fraunhofer diffraction expression.

*7. **The "Limit" of Geometrical Optics.*** Consider a slit that is very wide. The requirement is (Eq. [8.32])

$$1 \lll \Delta\eta = \eta_{x2} - \eta_{x1} = 2x_0 \left(\frac{2}{\lambda\rho}\right)^{1/2}$$

No matter how large $\Delta\eta$ is, there are always departures from the predictions of geometrical optics near the edge of the geometrical shadow. This occurs for $|\eta_{x1}|$ or $|\eta_{x2}|$ of the order of 10, for then the oscillations in region B and the nonzero intensity in region D of Fig. 8.14 occur. These diffraction effects take place when x is within a distance Δx of the shadow edge given by

$$\Delta x \sim 10 S_F = 10 \left[\frac{\lambda D'(D + D')}{2D}\right]^{1/2}$$

Hence, for fixed D and D', Δx can be made arbitrarily small by decreasing the wavelength sufficiently, although the size of the oscillations in E_r will not diminish. In this sense geometrical optics can be obtained from physical, or wave optics in the limit as $\lambda \to 0$.

*8. **The Two-Dimensional Pattern for Apertures of Rectangular Symmetry.*** An expression for the irradiance from (8.26) is

$$E_r(\mathbf{r}) = E_{r,na}(\mathbf{r}) \; \frac{|I_x|^2 |I_y|^2}{4}$$