The Effective Multiplication Constant and Moderator Density

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Introduction

The effective multiplication factor is a highly important number in nuclear engineering. This ratio of neutrons from one generation in a reactor to the one before it seems innocuous, but it defines the reactivity of the reactor and is the number that determines the criticality of a reactor [1]. The k constant can also be difficult to predict, as some minor changes could have massive effects to the ratio while some major changes could leave the ratio unchanged. While a good nuclear engineer can quickly estimate how k changes under different reactor conditions, to actually calculate the ratio is more time intensive.

Description

The problem given is to calculate the maximum value of the reactivity of a given reactor pin with different moderator to fuel ratios. The reactivity, as mentioned, is a number that arrives from the effective multiplication factor k [1]:

$$\mathrm{p} = \frac{\mathrm{k} - 1}{\mathrm{k}} \tag{1}$$

This then requires that k be calculated. The effective multiplication factor is subject to six other factors in the six-factor formula:

$$k = pf\epsilon\eta P_{NL}^{F} P_{NL}^{th} \tag{2}$$

These are the fast fission factor (ϵ), thermal utilization factor (f), thermal fission factor (η), thermal and fast non-leakage probability (P^{th} & P^{F}), and resonance escape probability (p, separate from the reactivity) [1]. These factors have the following equations:

$$p = \exp[\frac{-a}{\xi}(\frac{N_A/N_M}{\sigma_{sM}10^{24}})^{1-c} \tag{3}$$

$$f = \frac{\Sigma_a^F}{\Sigma_a^F + \Sigma_a^{NF}} \tag{4}$$

$$\epsilon = a + b\sqrt{0.6974\frac{N_{238}}{N_W}} - c * exp[-0.6974\frac{N_{238}}{N_W}] \tag{5}$$

$$\eta = \vartheta\frac{\Sigma_f^F}{\Sigma_a^F} \tag{6}$$

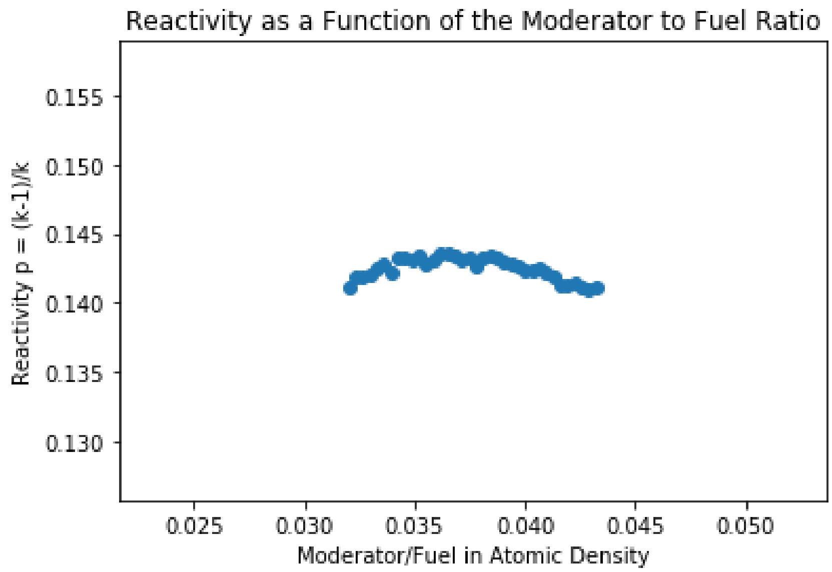
$$P_{NL}^{th} = \frac{1}{1 + L_T^2 B_c^2} \tag{7}$$

$$P_{NL}^{f} = e^{-B_c^2 \tau_T} \tag{8}$$

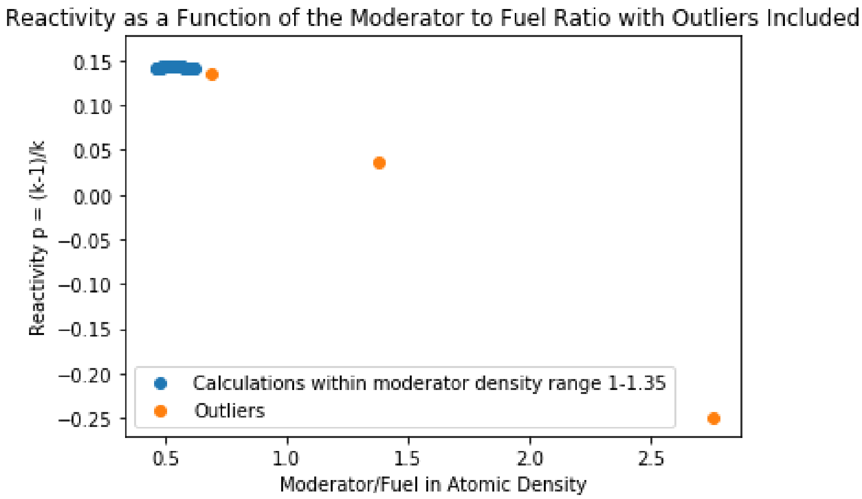
All of these equations have a reliance on the moderator (in the fast and thermal non-leakage probabilities, the reliance is within the L_T and B_c terms) except for η. η requires only the ratio between ^{238}U and ^{235}U, which do not change so η should remain constant. As an estimate, the reactivity should increase to a maximum, then decrease as the moderator density increases.

<u>Results</u>

The effective multiplication factor was calculated using the Monte Carlo machine Serpent and varying the moderator density. This started at 6 g/cm^3, then decreased by 2 until the returned k_{eff} reached a stable number around 1.25 g/cm^3. The moderator density was then varied from 1 g/cm^3 to 1.35 g/cm^3 by .01 g/cm^3, calculating the k_{eff} for each value. These values were then used to calculate the reactivity and the mass densities were converted into atomic densities and multiplied by the volume to calculate the ratio between the moderator and the fuel, returning:



This graph does not include the outliers that were calculated to understand the range of calculation. This next graph does include these outliers, to show the behavior at much higher densities than those calculated before.



The maximum reactivity was 0.14363 which occurred at the ratio 0.036499, or the k_{eff} 1.16772 which occurred at the density 1.14 g/cm^3.

Discussion

The trend hypothesized in the description section was correct. The reactivity increased to a maximum then decreased from there. This occurs because of neutron scattering and neutron absorption. As the density of the moderator was increased to the maximum, the moderator scattered more neutrons back into the fuel pin than it absorbed. However, as the moderator density was increased past the maximum, the moderator absorbed more neutrons than it scattered back into the fuel pin. This trend can be starkly seen in the second graph, where the reactivity drops to a negative value because the moderator is absorbing many more neutrons than the fuel pin can produce. This would create a horrible reactor that makes no power in contrast to the reactor that would be made with the maximum ratio, which should work fine.

Bibliography

[1] Shultis, J. Kenneth, Richard E. Faw. 2017. “Chapter 10: Principles of Nuclear Reactors” *Fundamentals of Nuclear Science and Engineering Third Edition* : 321-369.