$$\text{Let } f(\boldsymbol{z}) = \boldsymbol{x} - \boldsymbol{z}$$

$$\therefore f'(\boldsymbol{z}) = -1$$

$$\begin{aligned}
\frac{d\left((\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right)}{d\boldsymbol{\mu}} &= \frac{d\left(f(\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}f(\boldsymbol{\mu})\right)}{d\boldsymbol{\mu}} \\
&= \frac{d\left(f(\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}f(\boldsymbol{a})\right)}{d\boldsymbol{\mu}} + \frac{d\left(f(\boldsymbol{a})^T\boldsymbol{\Sigma}^{-1}f(\boldsymbol{\mu})\right)}{d\boldsymbol{\mu}}
\end{aligned}$$

$$(\boldsymbol{a} = \boldsymbol{\mu}, \text{ dummy variable for product rule})$$

$$\begin{aligned}
&= \frac{d\left((\boldsymbol{\Sigma}^{-1}f(\boldsymbol{a}))^T f(\boldsymbol{\mu})\right)}{d\boldsymbol{\mu}} + f(\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1} * -1 \\
&= -(\boldsymbol{\Sigma}^{-1}f(\boldsymbol{\mu}))^T - f(\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1} \\
&= -f(\boldsymbol{\mu})^T((\boldsymbol{\Sigma}^{-1})^T + \boldsymbol{\Sigma}^{-1}) \\
&= -(\boldsymbol{x}-\boldsymbol{\mu})^T((\boldsymbol{\Sigma}^{-1})^T + \boldsymbol{\Sigma}^{-1})
\end{aligned}$$

Is this the chain rule?

$$\frac{\partial}{\partial \boldsymbol{t}} f(g(\boldsymbol{t})) = \nabla f(g(\boldsymbol{t}))^T \frac{\partial g}{\partial \boldsymbol{t}}$$

where g is a vector-valued function and f is a vector-to-scalar function.

Sources: `http://math.stackexchange.com/questions/20694/vector-derivative-w-r-t-its-transpose-fracdaxdxt`
`http://www.cs.rochester.edu/~gildea/2013_Spring/Notes/csc446multivar_review_notes.pdf`