RISK, UNCERTAINTY AND ENTREPRENEURSHIP

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Abstract. Why are wealthy individuals more likely to become entrepreneurs? With no prior experience, starting a private business is an uncharted territory. As a result, an entrepreneur's investment decision is subject to Knightian uncertainty, which is modelled as wealth-dependent preference for robustness. Agents with a history of good income shocks select themselves to become entrepreneurs, since the buffer of higher wealth allows them to hold relatively optimistic beliefs about investment returns. An empirically plausible increase in uncertainty in the US since 1980s explains most decline in the share of entrepreneurs, and generates a "hollowing out" effect of the middle class.

Keywords: robust control, entrepreneurship, inequality

JEL Classification Numbers: D31, D81, L26

It is this "true" uncertainty, and not risk, as has been argued, which forms the basis of a valid theory of profit and accounts for the divergence between actual and theoretical competition.

—"Risk, uncertainty and profit" (1921, Knight)

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1. INTRODUCTION

It is widely acknowledged that long-term economic growth is propelled by novel non-rivalrous ideas implemented by profit-maximizing entrepreneurs (Romer (1990), Jones (1995)). However, this growth engine has slowed down in recent decades. Figure 1 illustrates the share of entrepreneurs as a proportion of the total labor force from 1985 to 2019 (PSID) using four distinct definitions of entrepreneurs, ranging from the broadest to the most stringent measures. As demonstrated, the share of entrepreneurs has witnessed a sharp decrease within this time period across all measures. For instance, self-employed business owners constituted 10.3% of the total labor force in 1985, whereas they account for a mere 4% in 2020. [1] If this decline solely reflects a decrease in the number of shoe repair shops and Chinese restaurants, macroeconomists need not pay heed. However, if an economy's process of creative destruction (i.e., "out with the old, in with the new") falters, this decline will hinder long-term growth. It is the daring entrepreneurs who invest in and take seriously novel ideas concerning faster computer chips, newer financial services, and more efficient delivery methods, to name a few, who perpetually enhance tomorrow's marketplace compared to that of today. Unfortunately, the slump in the entrepreneur share is accompanied by "the great reversal" of US business dynamism and innovation (Philippon 2019). Patent data from the USPTO reveals that the growth rate of new patent applications declined from 5.4% in 1985 to -3.5% in 2020. Simultaneously, high-tech startup entry rates from Census Bureau BDS data fell from 17.33% in 1985 to 11.41% in 2019. [2] This decline is also widespread across

[1] We define business owners as those who own a business at any point in a year or possess a financial interest in a business enterprise. Active business owners are defined as those who claim to work in their business. Self-employed business owners who declare that their primary occupation is self-employment are classified as entrepreneurs. Lastly, self-employed business owners who have managerial or professional roles in their business are classified as entrepreneurs. For a robustness check, data from the Survey of Consumer Finance reveals a similar declining trend, which is presented in Appendix A.1.

[2] A detailed time-series figure of these trends is provided in Appendix A.2.

age groups and industries. [3] Most strikingly, the US innovation index, measured by the share of RD expenditure in the world, has also experienced a major drop. This begs the question: what factors have contributed to the decline of US business dynamism? The answer to this question can shed light on some of the most significant policy debates, given that governments typically offer incentives and tax breaks for new businesses to promote growth.

Perhaps the most mainstream argument for why individuals do not establish their own business is a lack of capital. Evans and Jovanovic (1989) presents a seminal study demonstrating a positive relationship between initial wealth and entrepreneurship, with a liquidity constraint being a probable explanation. However, further research by Hurst and Lusardi (2004) using data from the PSID and NSSBF points out that a liquidity constraint is not a likely explanation. They find that even in low capital-intensive industries, individuals with higher wealth are more inclined to establish their own businesses. Thus, wealth serves a purpose beyond just providing liquidity for initial capital. Alternative explanations for why wealth encourages entrepreneurship are then needed to address why this force has stagnated over time.

In this paper, we present a novel approach to comprehending the interplay between wealth and entrepreneurship by formalizing and quantifying a concept advanced by Knight (1921). Frank Knight posed the question: what justifies entrepreneurial profit? Profit can be legitimized in an imperfectly competitive market due to a lack of competition. However, Knight notes that profit often persists in perfectly competitive markets. His insight into this apparently perplexing phenomenon is that profits serve as compensation for entrepreneurs who bear *uncertainty*. While measurable uncertainty (i.e., risk) can be mitigated and eliminated by insurance, immeasurable uncertainty cannot. Given that many physical investment decisions lack

[3]See figures in Appendix A.3.

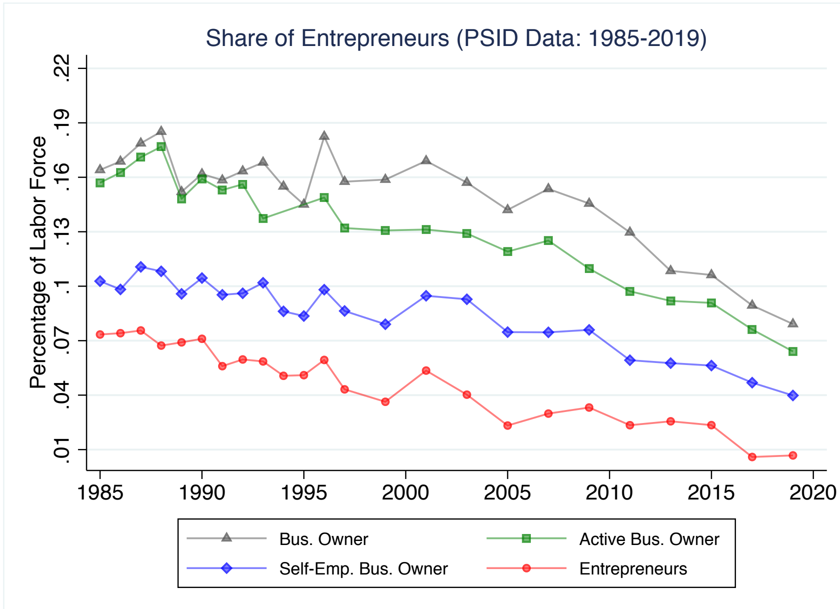

FIGURE 1. Share of entrepreneurs as a fraction of total labor force

a known probability distribution, such uncertainty can only be tackled via an entrepreneur's prudent judgment. Initiating a new private business involves significant uncertainty, with potentially large fluctuations in payoff (Hall and Woodward 2010). Such uncertain outcomes are difficult to characterize by a single probability distribution. While Savage (1972)'s decision theory indicates that, under certain conditions, Knight's distinction between risk and uncertainty can be reconciled and collapsed into a subjective probability measure, numerous recent experimental findings (e.g., the Ellsberg paradox) demonstrate that agents exhibit different behaviors when confronted with known vs. unknown probability measures. Thus, a Bayesian approach is inadequate when it comes to decision making under uncertainty.

To formalize this concept, we assume that individuals face Knightian uncertainty in making occupational and investment choices, which is modeled as a preference for robustness (Hansen and Sargent 2008). In this world, wealth serves a greater purpose than simply financing the initial fixed costs of investment (Greenwood, Han, and Sanchez 2018). Agents who have experienced favorable income shocks are more likely to choose entrepreneurship because their wealth buffer enables them to hold relatively optimistic beliefs about investment returns. For most people, the conflict between their fear of uncertainty and the desire to succeed in business is a significant challenge, but for the wealthy, who are comfortable with Knightian uncertainty, can then make investment decisions even when they lack all the necessary information.

The positive relationship between wealth and optimism is well supported by the data. So far, the best source of micro-level economic expectation data comes from Survey of Consumer Expectations conducted by the New York Fed. Figure 2 displays the monthly data for one-year-ahead expected growth in personal income from June 2013 to July 2021. The blue line illustrates the median estimate for households earning less than $75k$ US dollars annually, while the red line shows the median estimate for households earning over $150k$ annually. The graph indicates that individuals with higher incomes consistently exhibit greater expectations for earnings growth, with an average difference of 0.65%. Furthermore, our results hold up when comparing households with annual income below $50k$ and above $100k$, as well as below $50k$ and above $200k$.

This is not the first paper to study robust investment decisions. For example, Maenhout (2004) used Hansen-Sargent robust control methods to solve a consumption/portfolio problem in a standard Merton-style framework. However, in order to maintain homotheticity, Maenhout (2004) scaled the entropy penalty parameter by a function of wealth. In a recent paper, Kasa and Lei (2018) showed that when the entropy penalty parameter is not scaled, a powerful wealth-dependent preference for robustness is activated. Under increased uncertainty, individuals become more pessimistic, but wealthy individuals can afford to be less so because they have more resources at their disposal. This allows them to allocate a larger portion of their

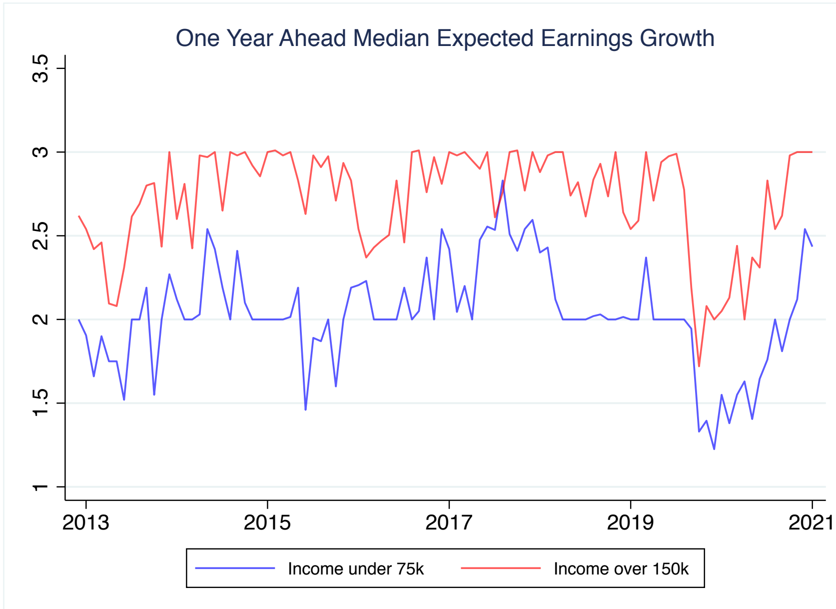

FIGURE 2. One year ahead own earnings growth expectation

wealth into higher-yielding risky assets, leading to greater wealth accumulation. Thus, robust investment decisions provide a micro-foundation for scale-dependent growth, which exacerbates wealth inequality.

We present a generalization of Kasa and Lei (2018) by examining the interplay between occupational and portfolio choice in a continuous time heterogeneous agent model that features idiosyncratic investment risk (Angeletos (2007), Panousi (2010)), robust preferences (Hansen and Sargent 2008), and occupational choice (Quadrini (2000), Cagetti and De Nardi (2006))in a general equilibrium setting. To focus solely on the role of wealth, we eliminate other sources of heterogeneity, such as ability, perseverance, and educational attainment, which could also contribute to the decision to become an entrepreneur. Following Dixit and Pindyck (2012), we adopt a real option model of occupational choice. [4] We assume that all agents

[4]Miao and Wang (2011) solves a real option problem under ambiguity. They argue that while ambiguity about terminal value delays option exercising, the same ambiguity about continuation value accelerates it. Here we consider the ambiguity to be about the terminal value, i.e., the value

start as workers and can switch to an entrepreneur upon paying a fixed cost of entry. Upon becoming an entrepreneur, the agent transitions from a low mean, low volatility wealth growth regime to a higher mean, higher volatility regime. With a fixed cost of starting a business, only the wealthy may find it profitable to do so. We argue that the increase in uncertainty since the 1980s has raised the threshold of wealth required to become an entrepreneur, resulting in a reduced share of entrepreneurs. As Heckman (Ljungqvist and Sargent 2008) noted in a 2003 speech, "a growing body of evidence points to the fact that the world economy is more variable and less predictable today than it was 30 years ago. There is more variability and unpredictability in economic life." Household income risk has also increased over the same period (Gottschalk and Moffitt (2009), Nishimura, Searby, Carmi, Elbedour, Van Maldergem, Fulton, Lam, Powell, Swiderski, Bugge, et al. (2001)). Micro evidence suggests that the world has become more "turbulent" (Ljungqvist and Sargent 1998), so it is reasonable to assume that idiosyncratic uncertainty has also increased.

Although we do not aim to dismiss other alternative mechanisms that contribute to the positive relationship between wealth and entrepreneurship, the wealth-dependent robustness approach offers at least two comparative advantages. First, it features both type-dependence and scale-dependence, providing a micro-foundation for a higher convergence speed of inequality. This reconciles a puzzle identified in Gabaix, Lasry, Lions, and Moll (2016) and is supported by recent empirical evidence (Fagereng, Guiso, Malacrino, and Pistaferri 2020). Those who have experienced a series of bad luck become more pessimistic and are less likely to establish their own businesses, while those who have experienced good luck become entrepreneurs.

of becoming an entrepreneur. We demonstrate that our results on delaying option exercise are consistent with the theoretical result in Miao and Wang (2011).

Moreover, wealthier entrepreneurs grow their wealth faster than less wealthy entrepreneurs because they can afford to take more risks. Wealth, therefore, begets wealth.[5] Second, when income shocks influence beliefs, a powerful force for an endogenous "poverty trap" is ignited. Lower wealth leads to pessimism, which then leads to lower growth, and reinforces pessimism. Increases in uncertainty force more individuals to stay as workers, reducing the size of the middle class and creating a "hollowing out" effect on the wealth distribution, which is consistent with what we have seen in the data (Foster and Wolfson 2010). In other words, the economy overturns the Kuznets curve, leading to lower growth rates and higher wealth inequality (Piketty 2014). Last but not least, the wealth-dependent preferences for robustness approach implies that the new entrepreneurs need to belong to a higher percentile of the wealth distribution than before, which is consistent with empirical findings as will be discussed below.

Finally, we show that the mechanism described above is exacerbated in a general equilibrium setting that allows for borrowing and lending. This is because of a reduction in the equilibrium interest rate, which occurs in response to an increase in uncertainty. [6] Intuitively, increases in uncertainty lead to more precautionary saving and fewer business entries. As a result, aggregate precautionary saving increases, which puts downward pressure on the risk-free rate. This amplifies inequality in two ways: first, for any given portfolio, entrepreneurs can now enjoy a higher excess mean return on risky investment; and second, cheaper borrowing costs allow entrepreneurs to take on more leverage, which further increases their average return to wealth.

[5]One implication of wealth-dependent entrepreneurial entry is that early entrepreneurs can take advantage of experiential learning and scale up the game. For example, Félix, Karmakar, Sedlácek, et al. (2021) shows that firms of serial entrepreneurs are larger, more productive, and grow faster.

[6]This aligns with recent findings from Greenwald, Leombroni, Lustig, and Van Nieuwerburgh (2021) and İmrohoroğlu and Zhao (2022).

The rest of the paper is organized as follows. Section 2 presents the theoretical framework of robust entrepreneurial decisions. Section 3 discusses the model's implication on individual wealth dynamics. Section 4 compares the stationary tail index in the robust vs. the non-robust economies. Section 5 simulates the model economy and presents policy functions in both the partial equilibrium and general equilibrium. Section 6 illustrates the main calibration results and various decomposition exercises. We review relevant literature in section 7, and provide additional empirical evidence from Survey of Economic Expectation data in section 8. Finally, we conclude and discuss future research direction in section 9.

2. The model

The model blends the workhorse Blanchard (1985) overlapping generation model and a real option model of occupational choice. Time is continuous, and the economy consists of a measure 1 continuum of finitely lived agents. We start by describing a single agent's problem, for both the entrepreneur and the worker. Then we move on to the discussion of the general equilibrium.

2.1. **The setup.**

In this economy, birth and death occurs at an identical Poisson rate ν. When an agent dies, he/she is instantly replaced by a newborn. Uncertainty is represented by a filtered probability space (Ω, $\mathcal{P}$,$\mathcal{F}_{t\geq 0}$, $\mathcal{F}$) induced by an (unobserved) one-dimensional standard Brownian motion $B = \{B_t : t \in [0, \infty]\}$. Each agent is born as a worker and is endowed with a positive initial financial wealth w_0, which is normalized to 1. He/she receives a continuous stream of labor income ω_t which follows the process of

$$\omega = \sigma_l w dB^l \tag{2.1}$$

where w denotes the financial wealth.[7] The labor income shock B^l is idiosyncratic, thus is $i.i.d$ across all workers. Workers do not have access to the capital market, and can only save in risk free asset (i.e.: a bond). The bond pays an instantaneous rate of return $\tilde{R}$, which will be determined in equilibrium. To focus on entrepreneurial entry and investment channel, we fix the consumption/savings channel by assuming a linear consumption rule where agents consume a constant fraction of their financial wealth,[8] i.e.:

$$c = \psi w \tag{2.2}$$

We assume that agents do not have bequest motive. Instead, an annuity contract is signed so that upon death, the unintended bequests are gathered by an annuity company. In return, the annuity product makes coupon payments while agents are alive. Under free entry and zero profit condition, the annuity product pays a rate of return $R = \tilde{R} + \nu$. Agents would then find it optimal to pour all the risk-free investment into the annuity market. Mathematically, the wealth evolution for a worker is given by

$$dw = (Rw + \omega - c)dt = (R - \psi)wdt + \sigma_l w dB^l \tag{2.3}$$

Each worker has the opportunity to make an irreversible decision of becoming an entrepreneur, which would require him/her to pay a fixed entry cost K. Let k denote the value of risky capital, The return process of capital follows

$$dk = \mu_k k dt + \sigma_k k dB^k \tag{2.4}$$

[7]The notion that the income shock is proportional to wealth is only for analytical convenience. We adopt a reduced form way of characterizing labor income without explicitly modeling the labor market. We will also simplify notation here by dropping index of the agent and calendar time.

[8]Knightian uncertainty can potentially affect both savings rate and portfolio reallocation. Under CRRA preferences (as will be assumed later), savings rate decreases with wealth due to uncertainty (See Kasa and Lei (2018)). This is inconsistent with empirical evidence (Dynan, Skinner, and Zeldes (2004), Fagereng, Holm, Moll, and Natvik (2019)). Since our focus is on investment decisions instead of savings decisions, here we fix the savings rate across all agents.

where dB^k captures the idiosyncratic investment risk that is $i.i.d$ across entrepreneurs. Let x denote the wealth of an entrepreneur. It follows that the entrepreneur's wealth evolution can be expressed as

$$dx = (R + \alpha(\mu_k - R) - \psi)xdt + \alpha\sigma_k x dB^k \tag{2.5}$$

where α is the fraction of wealth allocated to the risky capital.

2.2. **An entrepreneur's problem.** We assume that each agent has a time-additive constant relative risk aversion (CRRA) preference. Each entrepreneur needs to make continuous decisions on the fraction of wealth invested in the risky capital, α, in order to maximize his/her expected life time utility from consumption. The key feature of this paper is to assume that agents have doubts about the return process of the risky capital, which can be interpreted as their own ability to generate profit in a business. Following Gilboa and Schmeidler (2004), those doubts cannot be captured by a unique Bayesian prior. To formalize this, we allow agents to consider all unstructured perturbation of the benchmark model that can plausibly be generated from historical data as in Hansen and Sargent (2008). We use conditional relative entropy to measure the distance between two models. Let q_t^0 denote the probability distribution of the capital return process in the benchmark model, and q_t be a distorted probability distribution that is absolutely continuous with respect to q_t^0, the (discounted) relative entropy between q_t and q^0 then becomes

$$\mathcal{R}(q) = \int_0^\infty e^{-\rho t} \left[\int \log \left(\frac{dq_t}{dq_t^0} \right) dq_t \right] \tag{2.6}$$

where ρ is the rate of time preference, and that $\frac{dq_t}{dq_t^0}$ is a Radon-Nikodym derivative. [9]Let $h = \{h_t : t \in [0, \infty]\}$ be a square-integrable process that is progressively

[9]Discounting modifies the law of large numbers and delivers stationary decision rules. See Hansen, Sargent, Turmuhambetova, and Williams (2006) for more details.

measurable with respect to the filtration generated by q_t^0, i.e.:

$$\mathbb{E}^{q_t^0}\left[\frac{1}{2}\exp\left(\int_0^t h_s^2 ds\right)\right] < \infty \tag{2.7}$$

By Cameron-Martin-Girsanov Theorem, there then exists an equivalent measure q_t s.t.:

$$\frac{dq_t}{dq_t^0} = \exp\left[-\int_0^t h_s dB_s^k - \frac{1}{2}\int_0^t h_s^2 ds\right] \tag{2.8}$$

which is a positive martingale. Further, define the Brownian motion $\tilde{B}^k = \{\tilde{B}_t^k : t \in [0, \infty]\}$ as

$$\tilde{B}_t^k = B_t^k + \int_0^t h_s ds \tag{2.9}$$

under measure q_t. In other words, the change of measure can be reduced to a change of drift s.t.:

$$d\tilde{B}^k = \frac{1}{\sigma_k}\left[\frac{dk}{k} - \tilde{\mu}_k dt\right] = dB^k + hdt \tag{2.10}$$

where $h = \frac{\mu_k - \tilde{\mu}_k}{\sigma_k}$ and $\tilde{\mu}_k$ reflect the subject belief of the mean return. As in Hansen and Sargent (2008), we want our agents to be prudent, but not paranoid, so the worst case distortion is then subject to an entropy cost. Specifically, agents view themselves as being engaged in a dynamic zero-sum max-min game where an evil agent is trying to minimize their utility while they themselves attempt to maximize against it.

$$V^E(x_0) = \max_{\alpha} \min_{h} E \int_0^{\infty} \left(\frac{(\psi x_t)^{1-\gamma}}{1-\gamma} + \frac{1}{2\epsilon}h^2\right)e^{-\rho t} dt \tag{2.11}$$

subject to

$$dx = (R + \alpha(\mu_k - R) - \psi + \alpha\sigma_k h)xdt + \alpha\sigma_k x dB^k \tag{2.12}$$

Here, $V^E(x_0)$ is the value function for entrepreneurs with wealth x_0. Notice that $\rho = \tilde{\rho} + \nu$ is the effective discount rate, where $\tilde{\rho}$ denotes agents' subjective time discount rate. The cost parameter ϵ disciplines the level of belief distortion. This is related to the degree of uncertainty in the environment. We will discuss in detail the choice of ϵ in the later section. When $\epsilon = 0$, this reduces to the a standard rational expectation dynamic portfolio choice problem. However, when ϵ gradually deviates from zero, agents guard against their unknown knowledge of the world and attempt to form robust policies. This robustness game can be solved using dynamic programming. The (stationary) Hamilton–Jacobi–Bellman (HJB) equation associated with this problem reads

$$\rho V^E = \max_{\alpha} \min_{h} \frac{(\psi x)^{1-\gamma}}{1-\gamma} + V_x^E(R + \alpha(\mu_k - R) - \psi + \alpha\sigma_k h)x + \frac{1}{2\epsilon}h^2 + \frac{1}{2}\sigma_k^2 V_{xx}^E x^2 \alpha^2 \tag{2.13}$$

The first order conditions w.r.t. h and α can be expressed as

$$h^* = -\epsilon\alpha\sigma_k x V_x^E \tag{2.14}$$

$$\alpha^* = -\frac{(\mu_k - R)V_x^E}{[V_{xx}^E - \epsilon(V_x^E)^2]x\sigma_k^2} \tag{2.15}$$

Studying these two expressions yields several intriguing insights even before delving into their solution. Firstly, the positive marginal value of wealth implies that the drift distortion is invariably negative, which means that Knightian uncertainty inherently generates pessimism. Secondly, when fixing the values of wealth x and risky portfolio share α, increasing values of either ϵ or σ_k lead to greater pessimism. Heightened volatility conceals the mean return, thereby legitimizing the pessimistic inner voice of the agent. This interplay between perceived risk ϵ and actual risk σ_k intensifies pessimism. Thirdly, wealth affects pessimism in two ways: via risky portfolio holding αx, and through the marginal utility of wealth V_x^E. On one hand, prosperous entrepreneurs possess more stake in risky capital, thereby leading to more conservative return estimates. On the other hand, having additional funds in their bank accounts allows them to relax, given the decreasing marginal value of each extra dollar. Having said that, it is still essential to fully solve the value function to determine which effect dominates, a topic we will revisit later. For now, we turn to the portfolio choice expression, where we observe that the solution for α is akin to Merton's portfolio choice problem, with the exception of non-homotheticity.

The uncertainty term $\epsilon(V_x^E)^2$ discourages risk-taking, with this effect tapering off as wealth increases. For the ultra-rich, uncertainty no longer influences their risk-taking behavior. Finally, the interdependence between α and h embodies the conflict between aspiration and apprehension in the agent's head.

As mentioned before, to examine the wealth effect on the pessimistic drift distortion, we still need to solve for the value function $V^E(x)$. This involves substituting eqn. 2.14 and 2.15 into the HJB eqn. 2.13. The resulting maximized HJB equation is a second order nonlinear ODE , which does not permit an exact analytical solution. However, we will exploit the fact that when $\epsilon = 0$, the value function does have an exact solution, and we will look for a perturbation solution around $\epsilon = 0$ which takes the form of $V^E(x) \approx A_0 V_0^E(x) + \epsilon A_1 V_1^E(x) + \mathcal{O}(\epsilon^2)$.

Proposition 1. *The perturbation solution of the value function for entrepreneurs reads*

$$V^E(x) = A_0 \frac{x^{1-\gamma}}{1-\gamma} + \epsilon A_1 \frac{x^{2(1-\gamma)}}{2(1-\gamma)} \tag{2.16}$$

where

$$A_0 = \frac{\psi^{1-\gamma}}{\rho - (1-\gamma)(R + \frac{(\mu_k - R)^2}{\gamma\sigma_k^2} - \psi) + (1-\gamma)\frac{(\mu_k - R)^2}{2\gamma\sigma_k^2}} \tag{2.17}$$

$$A_1 = \frac{\sigma_k^2 A_0^2}{2[R + \frac{(\mu_k - R)^2}{\gamma\sigma_k^2} - \psi] - \frac{\rho}{1-\gamma} + (1-2\gamma)\frac{(\mu_k - R)^2}{\gamma^2\sigma_k^2}} \tag{2.18}$$

Proof. See Appendix A.4. □

This allows us to derive the solutions to the two policy functions.

Corollary 2.1. *The perturbation solution to portfolio choice is*

$$\alpha^*(x) \approx \alpha_0 + \epsilon\alpha_1(x) \tag{2.19}$$

where $\alpha_0 = \frac{\mu_k - R}{\gamma\sigma_k^2}$ *and* $\alpha_1(x) = -\frac{\mu_k - R}{\sigma_k^2}\left[\frac{A_0^2+(\gamma-1)A_1}{A_0\gamma^2}\right]x^{1-\gamma}$*. The drift distortion can be expressed as*

$$h^* \approx -\epsilon\sigma_k\alpha_0 A_0 x^{1-\gamma} \tag{2.20}$$

Proof. See Appendix A.4. □

As we can observe, the optimal portfolio share generally deviates from homotheticity, except when either $\epsilon = 0$ or $\gamma = 1$. In the former case, agents have rational expectations, and the solution for portfolio choice aligns with Merton's. In the latter case, agents possess log utility. Although log agents are equally averse to uncertainty, the degree of aversion remains constant across the wealth distribution. Wealthier entrepreneurs invest more in risky capital but exhibit less sensitivity to marginal wealth variations. Log agents strike a perfect balance between these two factors, leading to the two effects of increasing uncertainty offsetting one another. However, once $\gamma > 1$ (as is typically the case in portfolio choice), pessimistic drift distortion decreases with wealth. This enables prosperous entrepreneurs to undertake greater risks in private businesses, which is the primary driving force behind scale dependence. Additionally, it is essential to note that homotheticity renders the unit of parameters irrelevant. In contrast, scale dependence could potentially cause the magnitude of belief distortion to be influenced by the units of x. Nonetheless, what matters for pessimistic drift distortion is $\epsilon x^{1-\gamma}$, implying that the exact unit of x still holds no significance, provided that the penalty cost parameter is appropriately adjusted. [10]

2.3. **A worker's problem.** Unlike the entrepreneur, a worker does not have access to the risky capital market. Given a linear consumption rule, the only decision he/she needs to make is when and whether to switch to an entrepreneur. Since he/she can switch at any time from now on, it is essentially a real option problem.

[10]We will calibrate ϵ and delve into its empirical counterpart in a later section.

With a fixed cost K, the worker chooses an optimal time to switch. [11] The only relevant state variable for the worker is his wealth w. Denote w_0 as the worker's initial wealth, and $G(w_0)$ be the value function. Formally, his/her optimal stopping time problem can be written as

$$G(w_0) = \max_{\tau} E \int_0^{\tau} \frac{(\psi w_t)^{1-\gamma}}{1-\gamma} e^{-\rho t} dt + V^E(w_\tau - K) e^{-\rho\tau} \tag{2.21}$$

subject to:

$$dw = (R - \psi) w dt + \sigma_l w dB^l \tag{2.22}$$

where eqn. 2.22 describes the wealth evolution of the worker in the inaction region (i.e., before switching). Define $V^W(w_\tau) = E \int_\tau^\infty \frac{(\psi w_t)^{1-\gamma}}{1-\gamma} e^{-\rho t} dt$ as the present value of staying as a permanent worker with current wealth w_τ. We can then rewrite the value function $G(w_0)$ as

$$G(w_0) = \max_{\tau} E \int_0^{\infty} \frac{(\psi w_t)^{1-\gamma}}{1-\gamma} e^{-\rho t} dt + (V^E(w_\tau - K) - V^W(w_\tau)) e^{-\rho\tau} \tag{2.23}$$

The first term in the value function denotes the present value of staying as a worker throughout one's lifetime, which equals $V^W(w_0)$. The second term represents the option value of waiting to become an entrepreneur given that the switch happens at time τ, and it is non-negative. In the limit when $\tau \to \infty$, the worker never switch, and the option value goes to zero. However, as long as $\tau < \infty$, there is positive value of waiting.

Proposition 2. *The solution to the worker's value function $G(w)$ is characterized by the following five conditions:*

[11] Recent evidence shows that both entry and exit rates of businesses have declined in the US. However, the decline of entry is stronger than the decline of exit. (Decker, Haltiwanger, Jarmin, and Miranda (2020)) Here, we focus on the entry decision and abstract away from exit decision.

(i). $G(w)$ has the limiting property that

$$\lim_{w\to 0}\left(G(w)-V^W(w)\right)=0 \tag{2.24}$$

(ii). $G(w)$ satisfies

$$\rho G(w)=\frac{(\psi w)^{1-\gamma}}{1-\gamma}+G_w(R-\psi)w+\frac{1}{2}G_{ww}\sigma_l^2 w^2, w<w_\tau \tag{2.25}$$

$$G(w)=V^E(w-K), w\geq w_\tau \tag{2.26}$$

(iii). At the exercise boundary, we have

$$\lim_{w\to w_\tau} G(w)=V^E(w-K) \tag{2.27}$$

(iv). The first derivative of the value function needs to satisfy

$$\lim_{w\to w_\tau} G_w(w)=V_w^E(w-K) \tag{2.28}$$

(v). At the exercise boundary, the worker's wealth and (soon to be) entrepreneur's wealth is linked by

$$x_\tau=w_\tau-K \tag{2.29}$$

The first condition stipulates that when an individual's wealth is at its lowest (bound by zero due to the geometric Brownian motion process of wealth growth), their value function is equivalent to that of a permanent worker. Absolute risk aversion is so intense among individuals with limited wealth that engaging in entrepreneurial endeavors becomes unwarranted. The second condition states that the necessary Hamilton-Jacobi-Bellman equation must be fulfilled in the inaction region, and the value function must match that of the entrepreneur's after exercise. The third and fourth conditions correspond to the usual value matching and smooth pasting conditions, respectively. Finally, the last condition takes into account the accounting of wealth during the switching process.

Corollary 2.2. *The value function for workers reads*

$$G(w) = B_0 \frac{w^{1-\gamma}}{1-\gamma} + B_1 w^{\beta} \tag{2.30}$$

where

$$B_0 = \frac{\psi^{1-\gamma}}{\rho - (1-\gamma)(R - \psi - \gamma\sigma_l^2/2)} \tag{2.31}$$

and that β is the positive root of

$$\frac{1}{2}\sigma_l^2\beta^2 + (R - \psi - \frac{1}{2}\sigma_l^2)\beta - \rho = 0 \tag{2.32}$$

Proof. See Appendix A.5. □

The first component of the value function corresponds to the present value of the agent's life if they remain a worker indefinitely, denoted as $V^W(w)$ and previously defined. The second component reflects the inherent value of the option to switch to entrepreneurship. As the option value is always non-negative and diminishes to zero when wealth approaches zero, we must have $B_1 > 0$ and $\beta > 0$.

Corollary 2.3. *The optimal stopping rule is characterized by a threshold wealth w_τ that solves the algebraic equation:*

$$F(w_\tau - K, \epsilon) = F(x_\tau, \epsilon) = 0 \tag{2.33}$$

Let x_τ^0 be the solution to $F(x, 0) = 0$. We can approximate the solution to $F(x, \epsilon) = 0$ as:

$$x_\tau = x_\tau^0 + \epsilon \frac{\partial x_\tau}{\partial \epsilon} \tag{2.34}$$

where $\frac{\partial x_\tau}{\partial \epsilon} > 0$.

Proof. See Appendix A.6 for the expression of $F(x_\tau, \epsilon)$. □

Although we do not have an explicit formula for x_τ, we will show in the numerical section that it increases in ϵ monotonically. That is, uncertainty leads to greater option value of waiting, thus driving up the wealth threshold.

2.4. **Market clearing.** We allow entrepreneurs and workers to borrow and lend through the bond and the annuity market. Since the annuity return strictly dominates the bond, the de facto borrowing and lending occurs in the annuity market. The return on bond acts as a shadow rate instead. Workers deposit their savings in the annuity company, who then lends to the entrepreneurs to invest. The entrepreneur could either self-finance (when $\alpha^* < 1$) or borrow from the saving of workers and other entrepreneurs. At any instant t, the equilibrium annuity market clearing condition reads

$$\int_0^{w_\tau} w f_t^W(w)dw + \int_0^{\infty}(1-\alpha^*(x))x f_t^E(x)dx = 0 \tag{2.35}$$

where $f_t^W(w)$ and $f_t^E(x)$ represent the marginal densities of wealth for the workers and the entrepreneurs at time t, respectively. Let s be the cohort born at time s where $s < t$, we have $f_t^W(w) = \int_{-\infty}^t f_{s,t}^W(w)ds$ and $f_t^E(w) = \int_{-\infty}^t f_{s,t}^E(w)ds$. The market clearing condition can then be usef to pin down the endogenous equilibrium interest rate.

2.5. **Discussion of micro-foundation.** It is worth noting that the seemingly simple model presented above is actually quite rich, as it captures the mechanism behind a robust entrepreneurial investment decision. For those who are interested, we provide a more comprehensive model in Appendix A.9, which includes monopolistic competition and endogenous wage and rental rates, and show that the mechanism holds in the full model as well.

3. Implication on individual wealth dynamics

In this section, we study the wealth dynamics of the economy where each agent solves the optimal occupational choice problem and the risky investment problem as described before.

3.1. **Individual wealth dynamics.**

Proposition 3. *Beyond the exercise threshold, we have*

$$dx = (\overline{a}_0 + \epsilon \overline{a}_1 x^{1-\gamma})xdt + (b_0 + \epsilon b_1 x^{1-\gamma})xdB^k \tag{3.36}$$

where

$$\begin{aligned}
\overline{a}_0 &= R + \alpha_0(\mu_k - R) - \psi && (3.37)\\
\overline{a}_1 &= -\sigma_k^2 \alpha_0 \left[\frac{A_0^2 + (\gamma - 1)A_1}{A_0}\right] < 0 && (3.38)\\
b_0 &= \sigma_k \alpha_0 && (3.39)\\
b_1 &= -\sigma_k \alpha_0 \left[\frac{A_0^2 + (\gamma - 1)A_1}{A_0 \gamma}\right] < 0 && (3.40)
\end{aligned}$$

In the inaction region, we have the following log of wealth dynamics of the worker where

$$dw = (R - \psi)wdt + \sigma_l w dB^l \tag{3.41}$$

Proof. See Appendix A.7. □

The non-homothetic portfolio share delivers "scale-dependence" of wealth. Two special cases are worth discussing. First, when $\epsilon = 0$, wealth evolution degenerates to the usual "random growth" where all agents experience the same constant drift and volatility of exponential growth. Second, the case of $\gamma = 1$ delivers the "random growth" feature as well, only with slightly reduced growth and volatility parameters. This is the knife-edge case where the conflict between income and substitution channels is well resolved. Finally, notice that in the exercise region, as long as $\gamma > 1$, wealthier entrepreneurs accumulate wealth faster, while in the inaction region, agents have a much lower growth rate. In other words, the endogenous "scale dependence" and "type dependence" are ignited in this economy. We will show in the numerical analysis that the entrepreneurs' average growth rate is much higher than the workers'.

4. The Kolmogorov-Fokker-Plank Equation

In this section, we delve into the examination of the stationary distribution of wealth, given the individual wealth dynamics discussed in the previous section. For this purpose, we employ the Kolmogorov-Fokker-Plank equations, which describe the transition dynamics of the wealth distribution for both types. It transpires that the equations are more manageable to solve when formulated in terms of the logarithm of wealth.

Proposition 4. *Let $y = \log w$, (or in the case of entrepreneur, $y = \log x$), and let f_t^E, f_t^W denote the density of log of wealth for the entrepreneur and the worker, respectively. The evolution of the log wealth distribution $f_t(x) = f^E(x) + f^W(x)$ can be characterized by the following (coupled) linear PDEs.*

$$f_t^E = -\frac{\partial}{\partial y}((a_0 + \epsilon a_1 e^{(1-\gamma)y}) f_t^E) + \frac{1}{2}\frac{\partial^2}{\partial y^2}((b_0 + \epsilon b_1 e^{(1-\gamma)y})^2 f_t^E) - \nu f_t^E + q_t \zeta_0(\log(x_\tau)) \tag{4.42}$$

$$f_t^W = -\frac{\partial}{\partial y}((R - \psi - \frac{1}{2}\sigma_l^2) f_t^W) + \frac{1}{2}\frac{\partial^2}{\partial y^2}(\sigma_l^2 f_t^W) - \nu f_t^W - q_t \zeta_0(\log(w_\tau)) + \nu \zeta_0(\log(w_0)) \tag{4.43}$$

where q_t is the inflow of new entrepreneurs, and $\zeta_0(.)$ is the Dirac delta function.

In general, there is no exact solution to this system due to changes in the entry rate q_t. However, in the stationary distribution, we have $q_t = q$, which is equal to the birth and death rate ν multiplied by the fraction of entrepreneurs.

Proposition 5. *The non-robust stationary distribution of log of wealth is double exponential for entrepreneurs, and truncated double exponential for workers. Using f_∞^W and f_∞^E to describe the stationary log wealth distribution of workers and entrepreneurs respectively, we have*

$$f_\infty^E = \begin{cases} c_1 e^{\phi_1^E y} & \phi_1^E > 0,\ -\infty < y < \log x_\tau \\ c_2 e^{\phi_2^E y} & \phi_2^E < 0,\ y > \log x_\tau \end{cases} \tag{4.44}$$

$$f_\infty^W = \begin{cases} d_1 e^{\phi_1^W y} & \phi_1^W > 0, \ -\infty < y < 0 \\ d_2 e^{\phi_2^W y} & \phi_2^W < 0, \ 0 < y < \log w_\tau \end{cases} \tag{4.45}$$

where $c_{1,2}$ *and* $d_{1,2}$ *are constants to ensure that the integrals of two distributions sum to one (i.e., total population is normalized to one), and* f_∞^E *and* f_∞^W *are continuous at* $\log x_\tau$ *and* 0*, respectively. The solutions also guarantee that the following boundary conditions hold:* $f_\infty^E(-\infty) = 0, f_\infty^E(\infty) = 0, f_\infty^W(-\infty) = 0$.

Proof. Guess and verify the (truncated) double exponential function, and solve for the ϕ^W and ϕ^E. Using the integration condition and continuity conditions, we can get $c_{1,2}$ and $d_{1,2}$. □

For entrepreneurs, random growth, along with constant birth and death, produces the standard recipe of power law (Gabaix (2009)).[12] Since workers' wealth is upper truncated at w_τ, the right tail of the wealth distribution is exclusively populated by entrepreneurs. Therefore, the (right tail) Pareto exponent of the wealth distribution is given by $\phi_2^E = \frac{a_0 - \sqrt{a_0^2 + 2\nu b_0^2}}{b_0^2}$. The two left tail indexes ϕ_1^E and ϕ_1^W can be expressed as

$$\phi_1^E = \frac{a_0 + \sqrt{a_0^2 + 2\nu b_0^2}}{b_0^2} \tag{4.46}$$

$$\phi_1^W = \frac{R - \psi - \frac{1}{2}\sigma_l^2 + \sqrt{(R - \psi - \frac{1}{2}\sigma_l^2)^2 + 2\nu\sigma_l^2}}{\sigma_l^2} \tag{4.47}$$

Since we are evaluating the density where $x < 0$, $\phi_1^W < \phi_1^E$ would then imply that the left tail is dominated by the entrepreneurs' wealth distribution. This then requires that $\frac{R-\psi-\frac{1}{2}\sigma_l^2+\sqrt{(R-\psi-\frac{1}{2}\sigma_l^2)+2\nu\sigma_l^2}}{\sigma_l^2} > \frac{a_0+\sqrt{a_0^2+2\nu b_0^2}}{b_0^2}$. That is, the differences in growth rates and volatility matter for which group dominates the Pareto left tail.

Proposition 6. *The robust stationary distribution of log of wealth is double exponential for entrepreneurs, and truncated double exponential for workers. Using* $\tilde{f}_\infty^W$ *and*

[12] When log wealth follows a double exponential distribution, it follows that wealth follows a double Pareto distribution, where the Pareto tail index $\phi_{1,2}^E$ equals the exponential decay parameters.

$\tilde{f}^E_\infty$ to describe the stationary log wealth distribution of workers and entrepreneurs respectively, we have

$$\tilde{f}^E_\infty = \begin{cases} \tilde{c}_1 e^{\tilde{\phi}^E_1 y} & \tilde{\phi}^E_1 > 0,\ -\infty < y < \log \tilde{x}_\tau \\ \tilde{c}_2 e^{\tilde{\phi}^E_2 y} & \tilde{\phi}^E_2 < 0,\ y > \log \tilde{x}_\tau \end{cases} \tag{4.48}$$

$$\tilde{f}^W_\infty = \begin{cases} \tilde{d}_1 e^{\tilde{\phi}^W_1 y} & \tilde{\phi}^W_1 > 0,\ -\infty < y < 0 \\ \tilde{d}_2 e^{\tilde{\phi}^W_2 y} & \tilde{\phi}^W_2 < 0,\ 0 < y < \log \tilde{w}_\tau \end{cases} \tag{4.49}$$

where $\tilde{c}_{1,2}$ and $\tilde{d}_{1,2}$ are constants to ensure that the integrals of two distributions sum to one (i.e., total population is normalized to one), and $\tilde{f}^E_\infty$ and $\tilde{f}^W_\infty$ are continuous at $\log \tilde{x}_\tau$ and 0, respectively. The solutions also guarantee that the following boundary conditions hold: $\tilde{f}^E_\infty(-\infty) = 0, \tilde{f}^E_\infty(\infty) = 0, \tilde{f}^W_\infty(-\infty) = 0$. Further, the exponents $-\phi^E_{1,2}$ are solutions of

$$\epsilon + H(s) + \beta\Phi'(s - \beta) = 0 \tag{4.50}$$

where $\beta, H(.), \Phi(.)$ are defined in the Appendix.

Proof. See Appendix A.8. □

5. Simulation

In this section, we simulate and plot the stationary distributions for the two economies in 1985 and 2019. To do this, we discretize our continuous time economy into an annual frequency discrete time economy. In 1985, we assume that $\epsilon = 0$, hence agents only face measurable uncertainty. We then assume that there is a one-time turbulence "MIT shock" where the uncertainty parameter ϵ suddenly increased in value, and the economy gradually transitions into a new stationary distribution, which we assume is the 2019 economy. We increase ϵ to 0.0353 to match the increase of top 10% wealth share between 1985 and 2019. Since ϵ disciplines the degree of ambiguity in the economy, we will discuss its validity of it in detail using likelihood ratio statistics in the later section. The values and sources for the rest parameters are shown in Table 1.

TABLE 1. Benchmark Parameter Values

Parameters	Value	Data Source
w_0	1	Initial wealth is normalized to 1 for all new born.
$\tilde{\rho}$	2%	Upper bound of empirical estimate is 2%.
ν	2.5%	Average working life of 40 years.
γ	1.25	Small deviation from log utility satisfying parameter constraints.
K	0.39	Calibrated to match 1985 fraction of entrepreneur.
μ_k	5.9%	1985 Median private equity return by Moskowitz Jorgensen (2002).
σ_k	9.5%	Lower bound of private equity s.d from BBZ (2016).
σ_l	4.5%	Roughly half of entrepreneurial income risk (Heaton Lucas 2000).
ψ	4.1%	Falling in the range of estimated wealth to consumption ratio.
ϵ	0.0353	Small deviation from zero such that DEP is large.

First, we normalize initial wealth, i.e., $w_0 = 1$. Next, we set the discount rate $\tilde{\rho}$ to be 2% to match the upper bound of empirical estimate. The birth and death rate ν equals 2.5% annually, as average working life in the U.S. is approximately 40 years. Since our analytical approximation assumes that γ is close to 1, we set $\gamma = 1.25$ to ensure a real value of the entry threshold. The fixed cost K is calibrated to match the fraction of entrepreneurs in 1985, which is roughly 10% defined as "self-employed business owner" in PSID. [13]It is reported by Moskowitz and Vissing-Jørgensen (2002) that the median private equity return is 6.9%. To be conservative, we set μ_k to be 5.9% to satisfy parameter constraints. [14] Next, σ_k is calibrated to the lower bound of private equity standard deviation as in Benhabib, Bisin, and Zhu (2016). Finally, as suggested by Heaton and Lucas (2000), the standard deviation of

[13]According to the US small business administration, the initial startup cost is around 3000 USD in 2022, and the first year operation cost could be anywhere between 30k to 40k USD. Given an average household net worth of 135332 USD (St. Louis Fed data), this fixed cost amounts to 2.2% to 28% of average wealth, depending on whether one counts the first year cost as fixed cost or not. If $K = 0.39$ and a mean wealth of 2.32 in the 1985 economy (as will be shown later in the calibration), we get a reasonably close cost share of 16.8%, which falls in the range in the data.
[14]The value of implied threshold x_τ needs to be real.

the labor income σ_l is roughly half of entrepreneurial income risk. To be conservative, we set $\sigma_l = 4.5\%$. Finally, even though we do not explicitly model consumption choice, the marginal propensity of consumption $\psi = 4.1\%$ is roughly equal to the MPC of an economy where the entrepreneurs optimally make joint consumption and portfolio decision in 1985. [15] For a back of envelope check, the implied wealth to consumption when $\psi = 4.1\%$ also falls in the range of the empirical estimates in Lustig, Van Nieuwerburgh, and Verdelhan (2013). Finally, we calibrate the uncertainty parameter ϵ to match the empirically observed increase in the top 10% wealth share, and exmaine the model's performance in other dimensions of the data. We shall discuss the choice of ϵ in more detail later.

Figure 3 shows the threshold of entry x_τ as a function of the uncertainty parameter ϵ given the above parameters using the interest rate as in 1985 (which will be computed in the calibration that follows). As one can see, x_τ increases monotonically with an increase in uncertainty.

Figure 4 and Figure 5 present the entrepreneurs' policy functions of belief distortions and risky portfolio share, respectively. [16] As one can see, with no uncertainty in 1985, the optimal drift distortion is zero, and that all agents have rational expectations. In contrast, with an increase in uncertainty, in 2019, wealth-dependent pessimism emerges. Wealthier entrepreneurs tend to be less pessimistic about their investment returns. More interestingly, the general equilibrium effect deepens the *level* as well as the *slope* of such pessimism. This is because as pessimism grows, aggregate precautionary savings motive increases, which leads to the reduction of equilibrium interest rate. This raises the equity premium and potentially induces the entrepreneurs to invest more in risky portfolio share. To hedge against the volatility

[15] In that world, the entrepreneurs' consumption function is given by $c = \tilde{A}_0^{-1/\gamma}$ where $\tilde{A}_0 = \left[\frac{1}{\gamma}\left(\rho - (1-\gamma)(R + \frac{(\mu_k - R)^2}{2\gamma\sigma_k^2})\right)\right]^{-\gamma}$. This gives us $\psi = 4.7\%$. We set ψ slightly lower to satisfy parameter constraints.

[16] Since the relevant range of log wealth starts from x_τ, we start our plot from x_τ.

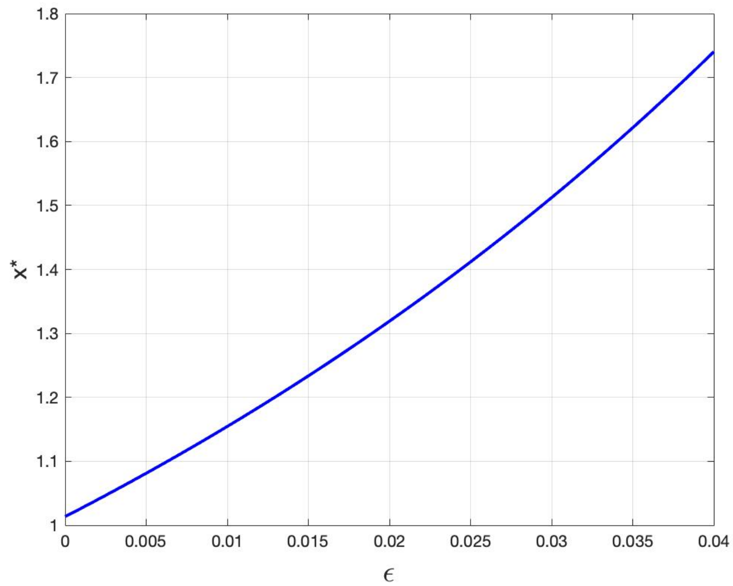

Figure 3. robust wealth threshold

of risky returns, the "evil agent" in the entrepreneur's head exercises extra caution. This effect is stronger for the poor entrepreneurs, because the volatility generated by the same (potential) increase in portfolio share leads to more utility reduction. Likewise, everyone invests a constant proportion of their wealth into risky portfolios back in 1985. However, in the year 2019, the allocation of risky shares changed with respect to wealth. Wealthy entrepreneurs are now channeling a greater portion of their wealth into higher-yielding and more daring investments. Notably, in the partial equilibrium, risky shares remain lower than the 1985 benchmark. This is attributable to the prevailing uncertainty, which has led to increased pessimism, causing people to take fewer risks in the business arena. This is the same observation made in Kasa and Lei (2018). Yet, what is novel here is the magnitude of this wealth-dependent behavior that is amplified in the general equilibrium. For the super-rich, their portfolio share may even exceed the 1985 level. Put simply, affluent entrepreneurs can now leverage and profit from the augmented precautionary savings of workers owing to the decrease in borrowing costs. This is due to the

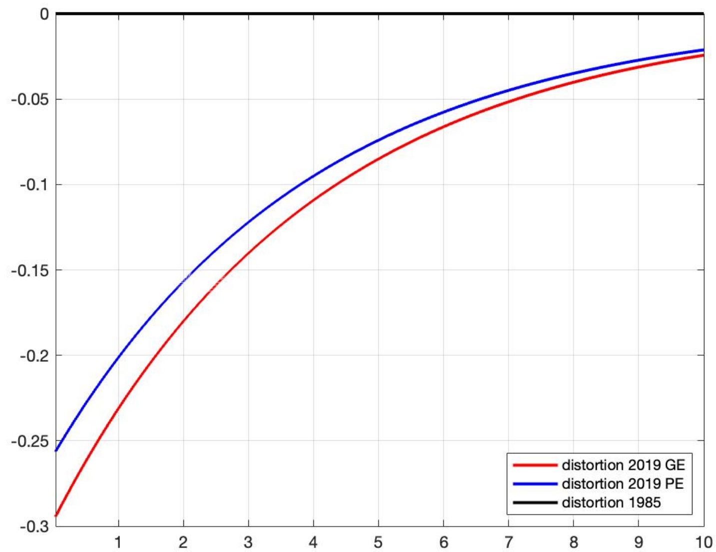

Figure 4. implied drift distortion

amplified slope of drift distortion. In this new paradigm, the despair of the destitute is facilitating the profit of the affluent, as they take on more risks and earn even greater rewards.

6. Calibration

In this section, we turn to the the main calibration results listed in Table 2. With an increase in the immeasurable uncertainty, the threshold wealth for becoming an entrepreneur increases. Fewer individuals can afford to start their business. Hence we see a reduction in fraction of entrepreneurs from 10.69% in 1985 to 4.40% in 2019, which is a 6.29% reduction. Compared with a decrease of 6.30% in the data during the same time period, the model can explain almost all of the declining entrepreneur share. Second, the equilibrium interest rate is reduced from 3.99% to 3.68%, which features a 7.8% reduction in percentage terms, and explains about one tenth of the data. Next, we can compare these two economies and examine the "hollowing out" of the middle class in wealth distribution. The bottom 50%

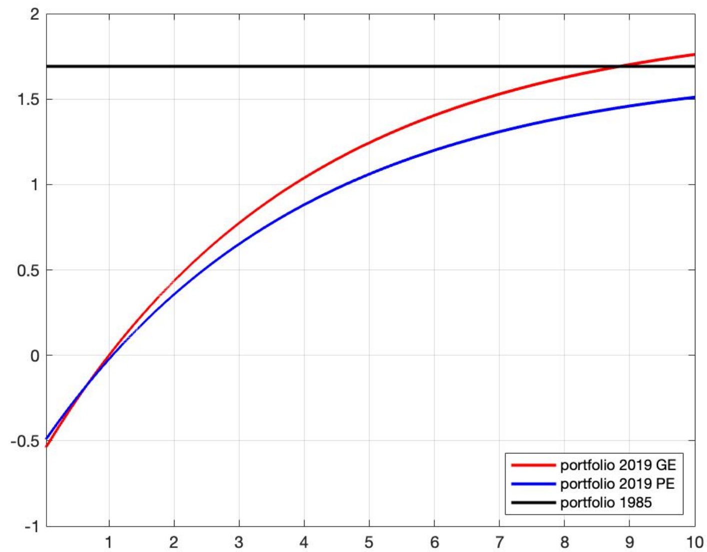

FIGURE 5. Robust risky portfolio share

wealth share has reduced from 17.10% in 1985 to 13.44% in 2019, while the top share has increased. Since we calibrate ϵ to match the increase in top 10% share, we should therefore determine the model's performance by examining other parts of the distribution. Interestingly, the further out in right tail of the wealth distribution, the more the increase of wealth share of top percentile. For example, the top 0.1% wealth share has increased by 35.58%, which accounts for 41.65% of the increase in the data. The last two rows in Table 2 report the turnover statistics. The rate of new entrepreneurs in the United States, as predicted by the model, has decreased significantly from 0.267% to 0.11% between 1985 and 2019, resulting in a decline of approximately 58.5%. This reduction is higher than the actual decline of 31.71% observed in the BDS data. This disparity can be explained by the fact that the model does not account for the entrepreneurial exit, whereas in reality, the exit rate has also decreased along with the entry rate, although at a slightly slower pace. Secondly, the mobility index for the top 1%, as predicted by the model, has shown a consistent decline from 7% in 1985 to 2.33% in 2019. This index measures the annual percentage of individuals who were not in the top 1% wealth share in the previous year but have entered this bracket in the current year. This trend indicates that the American dream of upward social mobility is moving in the opposite direction, making society more immobile, which is consistent with recent empirical evidence (Chetty, Grusky, Hell, Hendren, Manduca, and Narang 2017). This result is perhaps more concerning than simply having a higher level of inequality. Finally, the model suggests that in 2019, new entrepreneurs were, on average, wealthier than those in 1985. Prior to paying the initial cost of K, the model implies that the wealth threshold was approximately 94.51% of the wealth distribution, which increased to 96.67% in 2019. To determine the empirical equivalent, we analyzed the changes in the average wealth of new entrepreneurs during this period. Using PSID data, we calculated the average wealth of households that were not active business owners in 1985 but became active business owners in 1989. In 1989, their average wealth (prior to subtracting debt) was approximate $\$122,608$, placing them in the top 9% of the wealth distribution. However, when we computed the same statistics for households that were not active business owners in 2017 but became active business owners in 2019, we found that an average wealth of $\$363,143$ was required, which places them in the top 7.5% of the wealth distribution in 2019. Therefore, the model produces a reasonable, albeit slightly higher increase in the wealth percentile of new entrepreneurs.

Table 2. Main Results

	1985	2019	Δ Model	Δ Data
Threshold Wealth x_τ	1.0135	1.0352	0.0217	--
Fraction of Entrepreneurs	10.69%	4.40%	−6.29%	−6.30%
			Δ% Model	Δ% Data
Equilibrium R^*	3.9922%	3.6797%	−7.83%	−73.3%
Bottom 50%	17.10%	13.44%	−21.40%	−44.44%
Top 10%	63.92%	70.59%	10.43%	10.82%
Top 1%	54.47%	64.29%	18.03%	40.73%
Top 0.1%	41.18%	55.83%	35.58%	85.42%
Rate of new entrepreneurs	0.267%	0.11%	−58.80%	−31.71%
Top 1% Mobility	7.00%	2.33%	−66.71%	--
Wealth percentile of new entrepreneurs	94.51%	96.67%	2.16%	1.50%

Although this is not a paper about growth per se, the model does produce the interesting "type dependence" as well as the "death of Kuznets curve" dynamics with a lower growth and higher inequality. This can be seen from Table 3. Two measures of average wealth are computed: a population average (median), and a wealth distribution weighted average. As one can see, workers are on average getting poorer while the entrepreneurs are becoming richer in both measure. At the aggregate level, the median growth rate has decreased (lower growth) while the wealth-distribution weighted average growth rate has increased (higher inequality) (Jones and Kim (2018)).

To investigate the factors driving the rise of wealth inequality and the decline of entrepreneurship, we employ a decomposition exercise. While the strong economy has led to a general increase in inequality, it is important to recognize that various channels impact inequality trends during times of heightened uncertainty, and

TABLE 3. Average growth rate of wealth (%)

population average	worker	entrepreneur	total
1985	−0.099	2.34	0.16
2019	−0.41	3.55	−0.24
wealth-weighted average			
1985	−0.099	9.72	6.11
2019	−0.41	12.15	8.10

not all of these channels have positive effects. Therefore, we pose the question of which specific forces are at play. Table 4 provides a decomposition analysis of these channels. To fix ideas, we first focus on the intensive margin effect, defined as one where the wealth threshold to become an entrepreneur is fixed at its 1985 level. [17] This intensive margin effect can be further decomposed into a partial equilibrium (PE) effect and a general equilibrium (GE) effect. In the partial equilibrium, increased uncertainty first makes everyone more pessimistic, which reduces the average risky investment of the economy. On the other hand, with scale dependent portfolio choice, wealthier entrepreneurs also invest more in risky projects, which makes them even richer. The effect of this on wealth inequality is ambiguous, and in general depends on the parameters. Quantitatively, we witness a reduction of inequality in the partial equilibrium. [18] In general equilibrium, we have the opposite result. The reduction of equilibrium interest rate leads to both increased equity premium and increased risky portfolio share. Both serve to increase wealth inequality. Numerically, the GE effect significantly dominates the PE effect. Finally, changes in the threshold wealth in 2019 has a dampening (but almost negligible) effect on wealth inequality.

[17] Notice that this is not to say that the fraction of entrepreneurs are fixed. Given changes in return differences, the percentage of population who desire to be entrepreneurs could differ even if the threshold wealth is the same

[18] Note: the unit in $f(E)$ in Table 4 is the absolute percentage change, while those in other columns features the percentage change with respect to the 1985 level, normalized by the overall percentage change so that all effects sum up to the total effect.

Table 4. Decomposing: how does robustness shape wealth distribution(Δ%)

	f(E)	Top 0.1%	Top 1%	Top 10%	Bottom 50%
Data	-6.30	85.42	40.73	10.82	-44.44
Model					
Total	-6.29	35.58	18.03	10.43	-21.40
1985 x_τ					
PE ($\epsilon\uparrow$)	0	-43.25	-33.12	-19.71	35.32
GE ($R\downarrow$)	-5.74	78.19	51.85	31.16	-58.48
2019 x_τ	-0.55	0.63	-0.7	-1.02	1.75

Table 5. Wealth Inequality Within Entrepreneurs

	1985	2019	Δ% Model	Δ% Data
Bottom 50%	3.18%	1.86%	-41.51%	-97.43%
Top 10%	87.39%	91.77%	5.01%	10.88%
Top 1%	68.77%	78.87%	14.69%	-22.95%

6.1. **Comments on "irreversibility".** In the proposed theoretical model, it was assumed that the choice to become an entrepreneur is a permanent decision. However, it may be argued that this assumption may not necessarily be in the best interest of an entrepreneur if there are no costs associated with returning to the role of a worker. In order to examine the likelihood of an entrepreneur returning to the role of a worker, we performed numerical analysis and discussed the implications of this decision. Our model was calibrated to match the 1985 fraction of entrepreneurs, so what is of particular importance is the probability that it would be optimal for an entrepreneur to return to being a worker in 2019. There are three main factors that influence this decision: the cost of re-entry into the workforce, the cost of exiting the entrepreneurial role, and pessimism about future returns. In order for an entrepreneur to return to being a worker, they must be sufficiently pessimistic about future returns such that, even after considering the costs of exit and potential re-entry into the workforce, it is still financially beneficial to return to the role of a worker. This occurs when an entrepreneur's wealth falls below a certain threshold that is much lower than x_τ. Therefore, the probability that one's wealth falls below x_τ gives us an upper bound of which how likely one wants to exit. In our calibration, this probability is only around 6.7%. That is, the "irreversibility" assumption can be viewed as an outcome rather than a constraint.

6.2. **Detection error probability.** The key mechanism that generates declining entrepreneurship and rising inequality is an increase in Knightian uncertainty, characterized by an increase in ϵ in our model. We have chosen this parameter to match the increase of the top 10% wealth share. Here, we examine this choice in detail. In other words, is the worst case model that the agents consider reasonable? Why they cannot update belief out of it from past data? As in Hansen and Sargent (2008), we want our agents to be prudent, but not paranoid. To implement this, we consider our agents as statisticians that attempt to discriminate models based on likelihood ratio statistics. If the worst case model is too far apart from the benchmark model, the chance of making either a type I or type II error is small. Formally, we define the detection error probability (DEP) as the average of type I and type II error, i.e.;

$$DEP = \frac{1}{2}prob(H_1|H_0) + \frac{1}{2}prob(H_0|H_1) \tag{6.51}$$

where the null and the alternative hypothesis are treated equally. Thus, we want our agents to consider models whose DEP is large, i.e,: cannot be distinguished easily using historical data. Our model features continuous time stochastic processes, so the likelihood ratio is the integrals of the those wealth-dependent drift distortions $h(x)$.

Following Anderson, Hansen, and Sargent (2012), an upper bound of the DEP is given by

$$ave(DEP) \leq \frac{1}{2}\mathbb{E}\exp\left(-\frac{1}{8}\int_0^T h^2(x)dx\right) \tag{6.52}$$

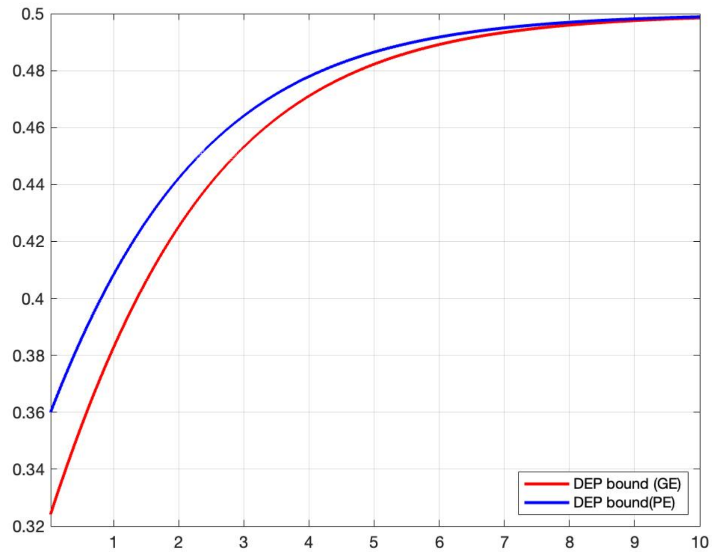

FIGURE 6. Detection error probability bound

where T is the sample length. In our economy, changes in x is stochastic, which makes it hard to evaluate this integral. Instead, we approximate the bound using a series of wealth-dependent DEP, i.e.: $ave(DEP) \leq \frac{1}{2}\exp\left(-\frac{1}{8}Th^2(x)\right)$. Figure 6 plots such state-dependent bound using a sample length that equals to the average life expectancy in the model. This assumes that agents learn from their own life time experience Malmendier and Wachter (2021). As one can see, the value of DEP is fairly large across all wealth level, indicating that the wealthier entrepreneurs have a higher DEP. [19]This makes sense because they suffer less from pessimistic drift distortion, so the worst case scenario is harder to be distinguished from the true data generating process. The slope of this bound is again increased in the general equilibrium when the less wealthy becomes dis-proportionally more cautious in response to a potential increase in risky portfolio share.

[19]Following Hansen and Sargent (2008), a reasonable calibration of ϵ requires a DEP bound to be higher than 20%.

7. Literature review

This paper is broadly related to two strands of literature: entrepreneurship and wealth. In this section, we will examine the relation of our work to them in detail. First, the paper contributes to the entrepreneurship literature by providing an alternative mechanism of entrepreneurial entry. Arguably the dominant explanation for why individuals do not start a business is due to a liquidity constraint (Evans and Jovanovic (1989), Quadrini (2000), Cagetti and De Nardi (2006), Boháček (2006)). However, Hurst and Lusardi (2004) find no evidence that wealth matters more for business that requires higher initial capital. Several other papers examine alternative forces that encourages entrepreneurship (Poschke (2013),Walter and Heinrichs (2015),Levine and Rubinstein (2017) among the few). Recent work by Davis and Haltiwanger (2014) and Haltiwanger, Hyatt, and McEntarfer (2018) argue that changes in demographics are mostly responsible for the declining business dynamism, as documented in Decker, Haltiwanger, Jarmin, and Miranda (2014), Salgado (2020) and Akcigit and Ates (2019). Slowdown in labor supply growth also contributes to the declining startup rates (Pugsley and Ṣahin (2018), Engbom (2019), Karahan, Pugsley, and Şahin (2019) and Hopenhayn, Neira, and Singhania (2022)). Jiang and Sohail (2023) argue that the observed decline in entrepreneurship can be primarily attributed to skill-neutral technological change and an escalating share of college graduates. While all those attempts are plausible and insightful, they rely on the assumption of rational expectations —the notion that agents perfectly grasp the data generating process of return to entrepreneurship. In this project, we depart from rational expectation, and argues that the wealth-dependent preference for robustness not only rationalizes the positive relationship between wealth and entrepreneurship, but also explains the secular decline of entrepreneurship.

Second, our work resonates with the larger literature on wealth inequality, particularly those based on entrepreneurial risks models. Jones and Kim (2018) propose a Schumpeterian model of top income inequality where entrepreneurs exert effort to improve productivity, which raises top wealth concentration. Panousi (2010) studies the effect of fiscal policy on wealth distribution in an entrepreneurial risk model. There are also some recent attempts that examine the relationship between declining interest rate and wealth inequality. Favilukis (2013) jointly explains rising inequality and a declining interest rate with a loosening of borrowing constraints, an increase in the volatility of labor income shocks and a fall in participation costs. İmrohoroğlu and Zhao (2022) study the role played by the declining world interest rates in accounting for top wealth inequality. In their model, entrepreneurs benefit from lower financing costs while workers are harmed by lower returns. Asriyan, Laeven, Martin, der Ghote, and Vanasco (2022) examine the effect of declining interest rates on the resource allocation efficiency among the entrepreneurs and aggregate output. The key difference in our paper is that both the declining interest rate and the rising inequality are endogenously generated. The increase in uncertainty produces powerful "scale dependence" and "type dependence", which provides a micro-foundation for drastically rising inequality found in Gabaix, Lasry, Lions, and Moll (2016).

8. Empirical evidence

In this section, we use micro data to examine the relationship between wealth, optimism and entrepreneurship. To do this, we turn to the New York Federal Reserve Survey of Economic Expectation data.

The SCE is a national survey representing a rotating panel of approximately 1,300 U.S. household heads from 2013 to present. It consists of seven unique surveys designed to extract information on households and their expectations towards future economic and financial uncertainties. We have chosen to merge three out of their seven sub-survey panels to link belief variables with demographic and wealth variables. The panel data acquired from the survey allows us to observe patterns in behaviors of the same individuals, relative to other respondents, over a (max) twelve month period. We restrict our sample to those observations who are in the labor force aged between 22 and 60 years old, and who define themselves as either working or being self-employed, and who have realistic beliefs about future income growth. [20]

The three datasets that we merged together are: Survey of Consumer Expectations, the SCE Household Finance Survey, and the SCE Housing Survey. First, we want to construct a measure to elicit household income growth expectations and optimism. The Survey of Consumer Expectations provides us with the question, "[Over the next twelve months] by about what percent do you expect your total household income to increase or decrease?", which we define as expected future income growth. For robustness checks, we use two additional measures for optimism, which include questions asking respondents about macroeconomic expectations. The two questions are namely, "What do you think is the percent chance that 12 months from now the unemployment rate in the U.S. will be higher than it is now?", and "What do you think is the percent chance that 12 months from now, on average, stock prices in the U.S. stock market will be higher than they are now?". Second, we extract income growth and wealth information from the Household Finance Survey. We collect answers to the question, "By what percent did your [total] household income increase or decrease [in the last twelve months vs. the previous year]?". We use this percentage change in income growth as a variable in our regression to control for last year income growth. We also use another question from the Finance survey as our first proxy for wealth. The question is, "[What is your] current value of savings and investments, excluding retirement accounts [?]". The last survey, SCE Housing

[20]We winsorize the data removing the top 2.5 and bottom 2.5 percent of answers for expected future income growth.

Survey, includes information on the respondent's current home value, which we use as a our second proxy for wealth. The last question is, "[What is the] value of [your] own home today[?]".

To examine the relationship between wealth and beliefs, we begin by selecting the month containing the most information across all surveys for each variable used in our analysis. Since past year income growth information is recorded only in August, we choose August as our Survey month and assume other annual variables' values as the August values. [21]. After dropping observations with missing values, we are left with a total of 956 observations. Appendix A.10 reports a summary statistics table of all used variables with adjusted weights.

To see whether an increase in individual wealth level increases one's optimism about future economic and financial situations we run a fixed effect panel regression that takes the following functional form

$$\begin{aligned} expinc_{i,t} &= \beta_0 + \beta_1 \log w_{i,t} + \beta_2 worker_{i,t} + \beta_3 incomegrowth_{i,t-1} + \beta_4 edu_{i,t} \\ &\quad + \beta_5 age_{i,t} + \beta_6 race_i + \beta_7 year_t, \end{aligned} \tag{8.53}$$

where $expinc_{i,t}$ is the percentage of expected income growth of next calendar year for respondent i in year t, which is then used as a proxy for optimism. The explanatory variables are (log of) wealth, $\log w_{i,t}$, [22] worker status, $worker_{i,t}$, percentage of last year income growth, $incomegrowth_{i,t-1}$, control variables, $edu_{i,t}$, $age_{i,t}$, and $race_i$, and year fixed effect, $year_t$. To rule out the possibility that higher expected income growth can be driven by high real income growth rate, we control for last year income growth in the RHS of the regression. Robust standard error is used in the final regression.

[21] The Survey of Consumer Expectations is conducted each month from June 2013 to July 2021, the SCE Household Finance Survey is conducted every August from 2014 to 2019, and the SCE Housing Survey is conducted every February from 2014 to 2019. This implies that the February value in the housing survey is assumed to be the value of August

[22] Wealth is defined as either (a) the value of savings and investments or (b) the value of savings and investments plus home value, as will be clear in the results table.

Table 1 presents the empirical results. Two things are notable. First, one can see there is a positive and statistically significant correlation between wealth and expected percentage income growth. In particular, a percentage increase in wealth increases expected income growth rate by 0.278%. Second, non-workers has a higher expected future income growth of about 0.389% compared with workers, which is consistent with our model. Since housing value occupies a large fraction of households asset, we add home value into the definition of wealth in part (b), and the results are consistently robust. To further check the robustness of our results, we use expected unemployment rate and expected stock price growth as proxies for level of optimism in Column (2)-(3). As one can see, wealthier respondents expect lower probability that next year unemployment rate will go up, and higher probability that stock price will go up, which is consistent with the theoretical prediction that they are more optimistic about the future.

9. Conclusion

This paper presents a pioneering investigation of the impact of Knightian uncertainty on entrepreneurship. Drawing on the insights of Knight (1921), we contend that the rise of such uncertainty since the 1980s has discouraged entrepreneurial entry, exacerbating the already substantial wealth gap. Wealthier entrepreneurs are more comfortable with uncertainty, allowing them to undertake more investment projects that are rewarded with faster wealth accumulation on average. Conversely, the middle class is being gradually squeezed out of entrepreneurship, creating a "hollowing out" effect. This mechanism demonstrates both "type-dependence" and "scale-dependence," surpassing the conventional random growth model in generating a significant increase in wealth concentration. In general equilibrium, wealth inequality is magnified as the uncertainty-averse workers optimally supply more credit to the wealthier entrepreneurs.

Table 2: Panel regression on belief variables vs. (log) wealth

	(1) Expected Income Growth	(2) Unemployment Rate†	(3) Stock Prices†
(a) Household wealth proxy: savings/investment			
Log wealth	0.278***	-0.351***	0.803***
	(0.014)	(0.037)	(0.040)
Non-worker	0.389***	2.937***	2.068***
	(0.092)	(0.278)	(0.293)
Past year income growth	0.036***	0.067***	0.072***
	(0.003)	(0.005)	(0.005)
Age	-0.109***	-0.024***	-0.239***
	(0.003)	(0.008)	(0.009)
Education	0.164***	2.451***	2.182***
	(0.021)	(0.052)	(0.055)
Non-white	-0.820***	-6.584***	-3.239***
	(0.067)	(0.264)	(0.294)
Constant	7.172***	32.570***	35.966***
	(0.225)	(0.496)	(0.553)
Observations	763	763	763
R-squared	0.0685	0.0591	0.0681
(b) Household wealth proxy: savings/investment + home value			
Log wealth	0.229***	-1.736***	0.459***
	(0.012)	(0.034)	(0.036)
Non-worker	0.468***	0.391	1.793***
	(0.079)	(0.240)	(0.268)
Past year income growth	0.045***	0.050***	0.032***
	(0.002)	(0.005)	(0.005)
Age	-0.092***	0.060***	-0.225***
	(0.003)	(0.008)	(0.008)
Education	0.226***	2.800***	2.473***
	(0.018)	(0.047)	(0.050)
Non-white	-0.941***	-8.276***	-4.757***
	(0.058)	(0.228)	(0.262)
Constant	5.883***	43.104***	36.116***
	(0.206)	(0.506)	(0.543)
Observations	880	880	880
R-squared	0.0617	0.0879	0.0623

Note: *** p<0.01, ** p<0.05, * p<0.10

Standard errors in parentheses

† Likelihood of higher unemployment rate and higher stock prices 12 months ahead

Data source: (FRBNY) Survey of Consumer Expectations, SCE Household Finance Survey, SCE Housing Survey

There are various potential directions for further research. Firstly, since the model used in this study assumes AK technology and finite lives, it does not consider long-term growth. Expanding the framework to include an RD-based endogenous growth model could provide valuable insights into long-term growth. As uncertainty is known to hinder innovation and result in stagnation, research in this area could be particularly informative. Secondly, it could be beneficial to explicitly distinguish between capitalists and entrepreneurs to analyze the increase in inequality from capital gains versus business profits. Such a distinction could also facilitate the examination of the potential interplay between uncertainty and financial frictions and explore how they affect declining business dynamism. Thirdly, the post-pandemic entrepreneurial boom, which saw a decrease in uncertainty in certain industries, such as online-based businesses, presents a natural application of our model.

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Appendix A.

A.1. Figure: Share of entrepreneurs (Survey of Consumer Finance). Figure 7 plots the declining share of self-employed business owners fron 1989 to 2019 using SCF data. We define self-employed business owners as households whose head has an active role in at least one privately owned business (variable X3104 in SCF) and who are self-employed in their current job (variable X4106 in SCF).

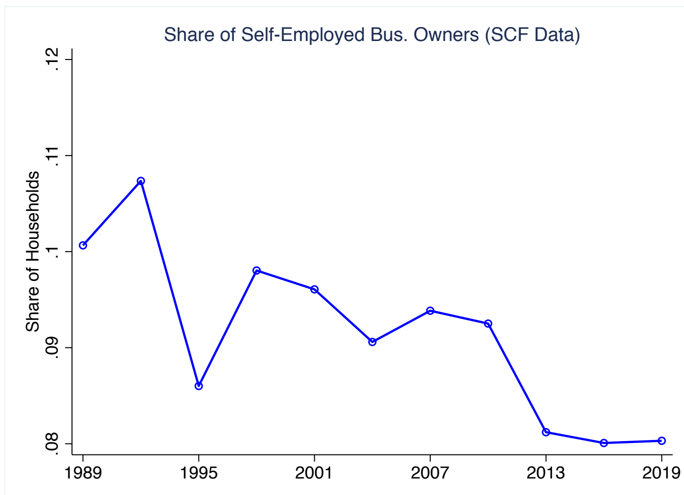

Figure 7. Share of self-employed business owners (1985-2019)

A.2. Figure: startup entry rate vs. US R&D expenditure share. Figure 8 plots the total establishment entry rate and the establishment entry rate of high technology firms in US using Census Bureau BDS data. Establishment entry rate is defined as the count of establishment entrants in year t divided by the average count of employment active establishments in year t and year $t-1$. We plot on the same figure the US share of worldwide R&D expenditure using data from OECD Main Science and Technology Indicators (MSTI) database.

A.3. Age and industry distribution. Figure 9 and 10 show the decline of the share of entrepreneurs (defined as "self-employed business owner") across the age and industry distribution.

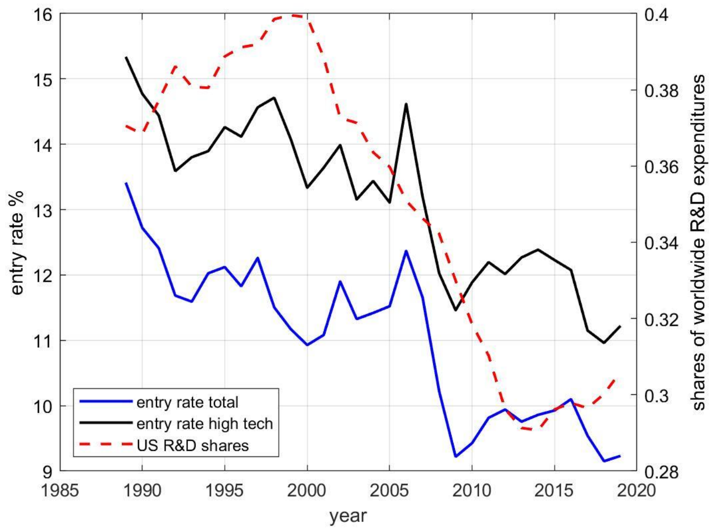

FIGURE 8. Startup entry rate vs. US R&D expenditure share

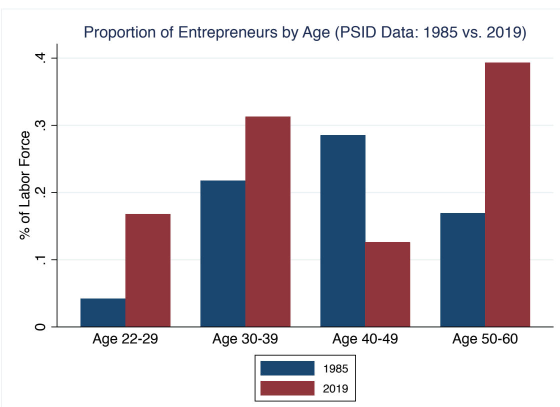

FIGURE 9. Share of entrepreneurs vs. age

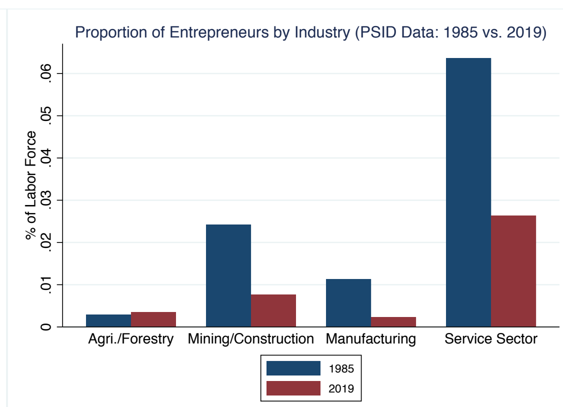

FIGURE 10. Share of entrepreneurs vs. industry

A.4. **Derivation of entrepreneur's value function.** To get perturbation solution of entrepreneur's value function, we first guess the functional form

$$V^E(x) = A_0 \frac{x^{1-\gamma}}{1-\gamma} + \epsilon A_1 \frac{x^{2(1-\gamma)}}{2(1-\gamma)} \tag{A.54}$$

It follows that $V_x^E = A_0 x^{-\gamma} + \epsilon A_1 x^{1-2\gamma}$ and $V_{xx}^E = -A_0\gamma x^{-\gamma-1} + \epsilon A_1(1-2\gamma)x^{-2\gamma}$. Recall that the first order condition for portfolio choice,

$$\alpha^* = -\frac{(\mu_k - R)V_x^E}{[V_{xx}^E - \epsilon(V_x^E)^2]x\sigma_k^2}, \tag{A.55}$$

By substituting the guess of the value function into α^* and dropping $\mathcal{O}(\epsilon^2)$ terms, we obtain:

$$\begin{aligned}
\alpha^* &= \frac{\mu_k - R}{\sigma_k^2}\left(\frac{1}{\gamma} + \frac{A_1 - \frac{1}{\gamma}(A_0^2 - A_1(1-2\gamma))}{A_0\gamma + \epsilon(A_0^2 - A_1(1-2\gamma))}\epsilon x^{1-\gamma}\right) && \text{(A.56)}\\
&\approx \frac{\mu_k - R}{\sigma_k^2}\left(\frac{1}{\gamma} + \frac{A_1 - \frac{1}{\gamma}(A_0^2 - A_1(1-2\gamma))}{A_0\gamma}\epsilon x^{1-\gamma}\right) && \text{(A.57)}\\
&= \frac{\mu_k - R}{\sigma_k^2}\left(\frac{1}{\gamma} - \frac{A_1(\gamma-1) + A_0^2}{A_0\gamma^2}\epsilon x^{1-\gamma}\right) && \text{(A.58)}\\
&= \alpha_0 + \epsilon\alpha_1(x) && \text{(A.59)}
\end{aligned}$$

where α_0, α_1 are defined in Corollary 2.1. Similarly, we substitute the guess of the value function into the first order condition for drift distortion: $h^* = -\epsilon\alpha\sigma_k x V_x^E$, and drop $\mathcal{O}(\epsilon^2)$ terms, we get:

$$h^* \approx -\epsilon\sigma_k\alpha_0 A_0 x^{1-\gamma} \tag{A.60}$$

Substituting the simplified expressions of portfolio choice and distortion into the eqn. 2.13 and matching coefficients in front of ϵ and constant, we get the solutions for A_0 and A_1.

A.5. **Derivation of worker's value function.** We will take a guess and verify approach to solve for the value function of the worker. Guess that

$$G(w) = B_0\frac{w^{1-\gamma}}{1-\gamma} + B_1 w^\beta \tag{A.61}$$

where B_0, B_1, β are unknown constants. It follows that $G_w = B_0 w^{-\gamma} + B_1\beta w^{\beta-1}$ and $G_{ww} = -B_0\gamma w^{-\gamma-1} + B_1\beta(\beta-1)w^{\beta-2}$. Plugging the guess into the HJB equation, we have

$$\rho G = \frac{(\psi w)^{1-\gamma}}{1-\gamma} + G_w (R-\psi) w + \frac{1}{2} G_{ww} \sigma_l^2 w^2 \tag{A.62}$$

Solving for B_0, we have

$$B_0 = \frac{\psi^{1-\gamma}}{\rho - (1-\gamma)(R - \psi - \gamma \sigma_l^2/2)} \tag{A.63}$$

Further, the characteristic equation of β is given by

$$\frac{1}{2}\sigma_l^2 \beta^2 + (R - \psi - \frac{1}{2}\sigma_l^2)\beta - \rho = 0 \tag{A.64}$$

To ensure that the option value $B_1 w^\beta$ goes to zero when w goes to zero, we choose the positive root where $\beta > 0$.

A.6. **Derivation of optimal stopping rule.** Combining (iii). and (iv). in Proposition 2 together gives equation $F(x, \epsilon) = 0$:

$$A_0 x^{-\gamma} \left(\frac{x}{1-\gamma} - \frac{w}{\beta} \right) + \epsilon A_1 (x)^{(1-2\gamma)} \left(\frac{x}{2(1-\gamma)} - \frac{w}{\beta} \right) - B_0 \left(\frac{1}{1-\gamma} - \frac{1}{\beta} \right) (w)^{(1-\gamma)} = 0 \tag{A.65}$$

where $w = x + K$. Solving this equation gives x_τ, and hence w_τ. Plugging x_τ into either (iii) or (iv) gives the value of B_1. Although x_τ does not have an analytical solution, we show in Figure 3 in the numerical section that it increases in ϵ monotonically, conditional on a fixed risk-free rate R, i.e., in a partial equilibrium setting.

A.7. **Derivation of entrepreneur's wealth dynamics.** In the exercise region, we have

$$\begin{aligned} dx &= (R + \alpha(\mu_k - R) - \psi) x dt + \sigma_k \alpha x dB^k && \text{(A.66)} \\ &= a(x, \epsilon) x dt + b(x, \epsilon) x dB^k && \text{(A.67)} \end{aligned}$$

Hence

$$a(x,\epsilon) = R + \alpha_0(\mu_k - R) - \psi + \epsilon\alpha_1(x)(\mu_k - R) \tag{A.68}$$

$$= \overline{a}_0 + \epsilon\overline{a}_1 x^{1-\gamma} \tag{A.69}$$

where $\overline{a}_0 = R + \alpha_0(\mu_k - R) - \psi$, $\overline{a}_1 = -\sigma_k^2\alpha_0\left[\frac{A_0^2+(\gamma-1)A_1}{A_0}\right]$.

Similarly, we obtain

$$b(x,\epsilon) = \sigma_k\alpha_0 + \epsilon\sigma_k\alpha_1(x) \tag{A.70}$$

$$= b_0 + \epsilon b_1 x^{1-\gamma} \tag{A.71}$$

where $b_0 = \sigma_k\alpha_0$, $b_1 = -\sigma_k\alpha_0\left[\frac{A_0^2+(\gamma-1)A_1}{A_0\gamma}\right]$.

Since $\gamma > 0$ and we adopt a small ψ to ensure $A_0, A_1 > 0$, we have $\overline{a}_1, b_1 < 0$.

Let $y = \log(x)$. From Ito's lemma, we have:

$$dy = \frac{dx}{x} - \frac{1}{2}b^2(x,\epsilon)dt \tag{A.72}$$

$$= (a(x,\epsilon) - \frac{1}{2}b^2(x,\epsilon))dt + b(x,\epsilon)dB^k \tag{A.73}$$

Substituting $e^{(1-\gamma)y} = x^{1-\gamma}$ into the equation and dropping $\mathcal{O}(\epsilon^2)$ terms gives:

$$dy = (a_0 + \epsilon a_1 e^{(1-\gamma)y})dt + (b_0 + \epsilon b_1 e^{(1-\gamma)y})dB^k \tag{A.74}$$

where

$$a_0 = \overline{a}_0 - \frac{1}{2}b_0^2 \tag{A.75}$$

$$a_1 = \overline{a}_1 - b_0 b_1 \tag{A.76}$$

A.8. **Solution of robust stationary distribution.** In this section, we lay out the derivation of the Pareto tail in the robust economy in the stationary distribution. Define $\tilde{y} = y - \log x_\tau$, we can then rewrite the stationary KFP equation into

$$0 = -\frac{\partial}{\partial\tilde{y}}((a_0 + \epsilon\tilde{a}_1 e^{(1-\gamma)\tilde{y}})f^E) + \frac{1}{2}\frac{\partial^2}{\partial\tilde{y}^2}((b_0 + \epsilon\tilde{b}_1 e^{(1-\gamma)\tilde{y}})^2 f^E) - \nu f^E + q\zeta_0(0) \tag{A.77}$$

where $\tilde{a}_1 = a_1 x_\tau^{1-\gamma}$, $\tilde{b}_1 = b_1 x_\tau^{1-\gamma}$. Notice that that last term $\zeta_0(0)$ is the Dirac delta function evaluated at 0, which makes it convenient to Laplace transform. Use the (double-sided) Laplace transform where

$$\mathcal{L}(f(x)) \equiv F(s) \equiv \int_{-\infty}^{\infty} f(x) e^{-sx} dx \tag{A.78}$$

where $-s$ has the interpretation as the $-s$ moments of the stationary distribution. We then have

$$\mathcal{L}(f_{\tilde{y}}) = sF(s); \mathcal{L}(f_{\tilde{y}\tilde{y}}) = s^2 F(s); \mathcal{L}(e^{\beta x} f) = F(s-\beta); \mathcal{L}(\zeta_0(0)) = 1 \tag{A.79}$$

where we used the first shift theorem of the Laplace transformation. We then use the additive property to rewrite eqn. A.77 into

$$0 = H(s)F(s) + \epsilon \Phi(s-\beta) F(s-\beta) + q \tag{A.80}$$

where $\beta = 1 - \gamma$ and that

$$H(s) = \frac{1}{2} b_0^2 s^2 - a_0 s - \nu \tag{A.81}$$

$$\Phi(s) = b_0 \tilde{b}_1 s^2 + (2 b_0 \tilde{b}_1 \beta - \tilde{a}_1) s + \beta (b_0 \tilde{b}_1 \beta - \tilde{a}_1) \tag{A.82}$$

In the special case where $\epsilon = 0$, we simply have

$$F(s) = -\frac{\nu}{H(s)} \tag{A.83}$$

Since $H(s)$ is linear quadratic in s and that any moments higher than $-s$ is infinity, applying the inverse Laplace transformation gives us

$$\mathcal{L}^{-1}(F(s)) = f_\infty^E(\tilde{y}) = \nVdash(x \geq \log x_\tau) \hat{c}_1 e^{\phi_2^E \tilde{y}} + \nVdash(x < \log x_\tau) \hat{c}_2 e^{\phi_1^E \tilde{y}} \tag{A.84}$$

where $\nVdash$ is the indicator function, the exponential rate parameters $-\phi_{1,2}^E$ are the positive and negative roots of $H(s) = 0$, and $\hat{c}_{1,2}$ are again constants of integration.

Rewriting $f^E_\infty(.)$ in terms of y, we will get back the non-robust stationary distribution.

When $\epsilon > 0$, we do not have an easy algebraic equation anymore due to the term from the first shift theorem. Instead, we will pursue a solution where $F(s-\beta) \approx F(s) - \beta\frac{\partial F}{\partial s}$ so that $F(s)$ solves

$$\epsilon\beta F_s = \left[H(s) + \epsilon\Phi(s-\beta)\right] F(s) + \nu \tag{A.85}$$

This solution when $\epsilon = 0$ provides a natural perturbation point. However, since ϵ is multiplied by F_s, this poses a "singular perturbation" problem, where a naive regular n^{th} order perturbation provides divergent instead of a convergent series. As a result, we transform $\hat{s} = s/\epsilon$ so that eqn. A.85 can be written into

$$F_{\hat{s}} = \frac{1}{\beta}\left(\epsilon + \frac{H(\hat{s})}{\Phi(\hat{s})}\right) F(\hat{s}) + \frac{\nu}{\beta\Phi} \tag{A.86}$$

which has the solution

$$F(\hat{s}) = -\frac{\nu}{\epsilon + H(\hat{s}) + \beta\Phi'(\hat{s}-\beta)} \tag{A.87}$$

As $\beta \to 0$ and $\epsilon \to 0$, we get back the solution in the non-robust economy. However, in general, the exponential rate parameters $\tilde{\phi}^E_{1,2}$ is the solutions of

$$\epsilon + H(\hat{s}) + +\beta\Phi'(\hat{s}-\beta) = 0 \tag{A.88}$$

A.9. **The case of monopolistic competition and endogenous wages.** In this section, we illustrate how the model in the main text can be thought of as an abstraction of a full model where entrepreneurs face monopolistic competition and optimally choose labor and capital input to maximize profits. In this world, both wage and rental rate of capital are endogenous. As one shall see, the intuition of the model in the main text carries through. However, the full model can provide some micro foundations for parameter calibration.

In this economy, final goods market is competitive. A representative final goods firm produces output Y by combining differentiated intermediate goods i of price p_i with a CES production function

$$Y = (\int_0^1 Y_i^\theta di)^{\frac{1}{\theta}}, 0 < \theta < 1 \tag{A.89}$$

where $\frac{1}{1-\theta}$ denotes elasticity of substitution among input goods. The final goods firm then solves an optimal input demand problem,

$$\max_{Y_i}(\int_0^1 Y_i^\theta di)^{\frac{1}{\theta}} - \int_0^1 p_i Y_i di \tag{A.90}$$

The first order condition gives

$$(\frac{Y}{Y_i})^{1-\theta} = p_i \tag{A.91}$$

The intermediate goods firm i is owned by entrepreneur i. He/she combines labor and capital to produce output. Formally, its production function is given by

$$F(K_i, L_i) = AK_i^\alpha L_i \tag{A.92}$$

where A reflects the level of technology. This parameter is the same across all entrepreneurs, but they just don't know it. Instead, each of them believes $A = A_i$. The entrepreneur solves

$$\max_{Y_i} p_i Y_i - wL_i = Y^{1-\theta} Y_i^\theta - w\frac{Y_i}{A_i K_i^\alpha} \tag{A.93}$$

The first order condition gives

$$Y_i = (\frac{w}{\theta A_i K_i^\alpha})^{\frac{1}{\theta-1}} Y \tag{A.94}$$

where w denotes the base wage. Substituting eqn. A.94 into final good production function gives us the equilibrium base wage rate

$$w = \theta(\int_0^1 (A_i K_i^\alpha)^{\frac{\theta}{1-\theta}} di)^{\frac{1-\theta}{\theta}} = \theta(\int_0^1 A_i^{\frac{\theta}{1-\theta}} K_i di)^{\frac{1-\theta}{\theta}} = \theta(\int_0^1 A_i^{\frac{1}{\alpha}} K_i di)^\alpha \tag{A.95}$$

where we assume that $\alpha\frac{\theta}{1-\theta} = 1$ for analytical convenience. It follows that the labor demand equation is given by

$$L_i = \frac{(\frac{w}{\theta A_i K_i^{\alpha}})^{\frac{1}{\theta-1}} Y}{AK_i^{\alpha}} \tag{A.96}$$

The labor market clearing condition reads $\int_0^1 L_i di = L$ where labor supply is inelastic. We can then derive the aggregate output

$$\begin{aligned} Y &= [\int_0^1 (\frac{w}{\theta})^{\frac{1}{\theta-1}} (A_i K_i^{\alpha})^{\frac{1}{1-\theta}} K_i^{-\alpha} di]^{-1} AL && \text{(A.97)} \\ &= [\int_0^1 (\frac{\theta}{w} A_i K_i^{\alpha\theta})^{\frac{1}{1-\theta}} di]^{-1} AL && \text{(A.98)} \\ &= \{\int_0^1 [(\int_0^1 A_i^{\frac{1}{\alpha}} K_i di)^{-\alpha} A_i K_i^{\alpha\theta}]^{\frac{1}{1-\theta}} di\}^{-1} AL && \text{(A.99)} \\ &= g(\int A_i, \int K_i) AL && \text{(A.100)} \end{aligned}$$

The entrepreneur's perceived profit becomes

$$\begin{aligned} \pi_i &= Y^{1-\theta} Y_i^{\theta} - w\frac{Y_i}{AK_i^{\alpha}} && \text{(A.101)} \\ &= (\frac{w}{\theta A_i})^{\frac{1}{\theta-1}} (\frac{w}{\theta A_i} - \frac{w}{A}) Y K_i && \text{(A.102)} \\ &= w^{\frac{\theta}{\theta-1}} (\theta A_i)^{\frac{1}{1-\theta}} (\frac{1}{\theta A_i} - \frac{1}{A}) Y K_i && \text{(A.103)} \end{aligned}$$

Following the spirit in the main text, we assume that entrepreneur's profit π_i is subject to multiplicative idiosyncratic risks. Formally,

$$d\Pi_i = \pi dt + \sigma K_i dB_{i,t}^k \tag{A.104}$$

where π denotes the profit under the true data generating process. Express profit profit rate as return to capital, we have

$$\frac{d\Pi_{it}}{K_{it}} = w^{\frac{\theta}{\theta-1}}(\theta A)^{\frac{1}{1-\theta}}(\frac{1}{\theta A}-\frac{1}{A})Ydt+\sigma dB^k_{i,t} \tag{A.105}$$
$$= r_t dt + \sigma dB_{i,t} \tag{A.106}$$

Profits are observable, but the return parameter A is not. Again, Girsanov's theorem implies that the entrepreneur's perceived profit process becomes

$$\frac{d\Pi_{it}}{K_{it}} = w^{\frac{\theta}{\theta-1}}(\theta A_i)^{\frac{1}{1-\theta}}(\frac{1}{\theta A_i}-\frac{1}{A})Ydt+\sigma d\widetilde{B}^k_{i,t} \tag{A.107}$$
$$= r_{it}dt + \sigma d\widetilde{B}^k_{i,t} \tag{A.108}$$
$$= (r_t + \sigma h_{it})dt + \sigma(dB^k_{i,t} - h_{it}dt) \tag{A.109}$$

where h_{it} is the distortion:

$$h_{it} = \frac{r_{it}-r_t}{\sigma} \tag{A.110}$$
$$= \frac{1}{\sigma}w^{\frac{\theta}{\theta-1}}Y[(\theta A_i)^{\frac{1}{1-\theta}}(\frac{1}{\theta A_i}-\frac{1}{A})-(\theta A)^{\frac{\theta}{1-\theta}}(1-\theta)] \tag{A.111}$$

Since $h_{i,t} < 0$ and $Ai < A$ (as will be proved later), we have $\frac{\partial h}{\partial A_i} > 0$. That is, entrepreneurs who are more optimistic about their own TFP have less pessimistic drift distortion with respect to their capital return process.

At each instant, workers are compensated with a base wage determined by the competitive labor market, plus a Brownian motion type of labor income shock with volatility σ_l. Entrepreneurs are compensated with the base wage as well, but are not subject to such uncertainty. Let w denote the base wage. Then at any time t, a worker with wealth $x_{j,t}$ gets a wage

$$w_{i,t} = w + \sigma_l x_{j,t} dB^l_{j,t}; \tag{A.112}$$

A bank is introduced to lend the total base wage for each new born, which equals their initial wealth. Entrepreneurs and workers pay back the bank the base wage each period. However, workers have to bear the idiosyncratic labor income risk themselves.

Next, define total effective wealth of the entrepreneur i, $x_{i,t}$ as the sum of financial wealth $(K_{it} + b_{i,t})$ and human wealth $(H_{i,t})$:

$$x_{i,t} = K_{it} + b_t + H_{i,t} \tag{A.113}$$

where $H_{i,t} = \mathbb{E}\int_t^{\infty} e^{-\int_t^s R_j dj} ds$. The evolution of total wealth under the true data generating process can be written as

$$\begin{aligned} dx_{i,t} &= d\Pi_{i,t} + [R_t b_{i,t} + R_t H_{i,t} - w + w + c_{i,t}]dt && \text{(A.114)} \\ &= r_t K_{it} dt + \sigma K_{it} dB_t^k + [R_t b_t + R_t H_{i,t} + c_{i,t}]dt && \text{(A.115)} \\ &= [r_t \alpha_{i,t} x_{i,t} + R_t (1 - \alpha_{i,t}) x_{i,t} - c_{i,t}] dt + \sigma \alpha_{i,t} x_{i,t} dB_t, && \text{(A.116)} \end{aligned}$$

where $\alpha_t \geq 0$ is the fraction of total wealth invested in physical capital, r_t is the risky return, and R_t is the risk-free rate, which is the same as the return to annuity in the main text. From the entrepreneur's perspective, the perceived wealth dynamics however, is

$$dx_{i,t} = [r_{i,t} \alpha_{i,t} x_t + R_t (1 - \alpha_{i,t}) x_{i,t} - c_{i,t}] dt + \sigma \alpha_t x_{i,t} d\widetilde{B}_{i,t}, \tag{A.117}$$

To fix consumption to wealth ratio, we adopt recursive preferences and assume the intertemporal elasticity of substitution equals to one. Formally, the entrepreneur's value function can be written as

$$
\begin{aligned}
0 = &\sup_{\{c_t,\alpha_t\}} \inf_{\{h_t\}} \rho(1-\gamma)V[\log(c_t) - \frac{1}{1-\gamma}\log((1-\gamma)V)] + \frac{1}{2\varepsilon}h_t^2 \\
&+ V_x[r_t\alpha_t x_t + R_t\left(1-\alpha_t\right)x_t - c_t + \alpha_t x_t \sigma h_t] + \frac{1}{2}V_{xx}\alpha_t^2 x_t^2\sigma^2 + V_t
\end{aligned}
\tag{A.118}
$$

where index i is suppressed and $\rho = \widetilde{\rho} + \nu$ is the sum of rate of time preference and birth/death rate, $\gamma > 0$ is the coefficient of relative risk aversion. The first order conditions for portfolio share and consumption are then [23]

$$
\alpha_{i,t} = -\frac{V_x(r_{i,t}-R_t)}{V_{xx}x_t\sigma^2} = \frac{r_{i,t}-R_t}{\gamma\sigma^2} \tag{A.119}
$$

$$
c_{i,t} = \frac{\rho(1-\gamma)V}{V_x} = \rho x_{i,t} \tag{A.120}
$$

For simplicity, we assume that workers use the same consumption rule as entrepreneurs. Solving the min-max problem of optimal drift distortion gives

$$
\begin{aligned}
h_t &= -\varepsilon\frac{r_t-R_t}{\gamma\sigma}\exp\{(1-\gamma)[\log\rho + \frac{R_t-\rho}{\rho} - \frac{1}{2}\frac{(r_t-R_t)^2}{\rho\gamma\sigma^2}]\}x_t^{1-\gamma} && \text{(A.121)}\\
&= \frac{1}{\sigma}\theta^{\frac{1}{1-\theta}}w^{\frac{\theta}{\theta-1}}Y[A_i^{\frac{1}{1-\theta}}(\frac{1}{\theta A_i}-\frac{1}{A}) - A^{\frac{1}{1-\theta}}(\frac{1}{\theta A}-\frac{1}{A})] && \text{(A.122)}\\
&- \varepsilon\sigma\frac{r-R}{\gamma\sigma}\exp\{(1-\gamma)[\log\rho + \frac{R-\rho}{\rho} - \frac{1}{2}\frac{(r-R)^2}{\rho\gamma\sigma^2}]\}x_t^{1-\gamma} && \text{(A.123)}\\
&= \theta^{\frac{1}{1-\theta}}w^{\frac{\theta}{\theta-1}}Y[A_i^{\frac{1}{1-\theta}}(\frac{1}{\theta A_i}-\frac{1}{A}) - A^{\frac{1}{1-\theta}}(\frac{1}{\theta A}-\frac{1}{A})] && \text{(A.124)}
\end{aligned}
$$

This gives us a one-to-one mapping between drift distortion and distortion with respect to TFP. Since the base wage rate $w\left(K\right) = \theta(\int_0^1 A_i^{\frac{1}{\alpha}}K_i di)^{\alpha}$, the share invested

[23]Note that agents with A_j low enough (such that the risk-adjusted capital return is always lower than the risk-free rate) will not invest in risky asset. Then we have $\alpha_{j,t} = 0$ and $c_{j,t} = \rho x_{j,t}$.

in the risky business for the entrepreneurs and workers can be written as

$$\alpha_i\left(K,R\right) = \frac{w^{\frac{\theta}{\theta-1}}(\theta A_i)^{\frac{1}{1-\theta}}(\frac{1}{\theta A_i}-\frac{1}{A})Y-R}{\gamma\sigma^2} \tag{A.125}$$

$$\alpha_j\left(K,R\right) = 0 \tag{A.126}$$

Hence the dynamics of wealth of entrepreneurs and workers are simplified as

$$\frac{dx_{i,t}}{x_{i,t}} = \left[r_t\alpha_{i,t}+R_t\left(1-\alpha_{i,t}\right)-\rho\right]dt+\sigma\alpha_{i,t}dB_{i,t}^k \tag{A.127}$$

$$\frac{dx_{j,t}}{x_{j,t}} = (R_t-\rho)dt+\sigma_l dB_{j,t}^l \tag{A.128}$$

From here, the worker solves a real option problem of occupational choice as presented in the main text.

A.10. **Summary statistics: Survey of Consumer Expectations.** The following table provides summary statistics for the variables from NY Fed Survey of Consumer Expectations used in our empirical analysis.

Summary statistics

	Mean	Median	SD	Min	Max	N	Units
Demographic variables							
Age	42.89	44	10.51	22	60	956	years
Education	3.77	4	1.64	1	9	956	categorical (asc.)
Race	1.54	1	1.55	1	6	956	categorical
Survey of Consumer Expectations							
Expected income growth	4.12	3	6.51	-15	40	956	%
Unemployment rate†	35.83	35	22.18	0	100	956	%
Stock prices†	39.47	45	22.71	0	100	956	%
SCE Household Finance Survey							
Past year income growth	3.26	2	16.60	-100	100	956	%
Savings/investments	105,536.20	13,000	621,979.64	0	11,000,000	763	dollars
SCE Housing Survey							
Home value	294,537.87	200,000	1,754,636.80	14,000	60,000,000	706	dollars

†Note: Likelihood of higher unemployment rate and higher stock prices 12 months ahead