DATASCI W261, Machine Learning at Scale

Assignement: week #12

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Due: 2016-04-14, 8AM PST



Click-Through Rate Prediction Lab

This lab covers the steps for creating a click-through rate (CTR) prediction pipeline. You will work with the Criteo.com/) dataset that was used for a recent Kaggle competition (https://www.kaggle.com/c/criteo-display-ad-challenge).

This lab will cover:

- ####Part 1: Featurize categorical data using one-hot-encoding (OHE)
- ####Part 2: Construct an OHE dictionary
- ####Part 3: Parse CTR data and generate OHE features
 - #### Visualization 1: Feature frequency
- ####Part 4: CTR prediction and logloss evaluation
 - #### Visualization 2: ROC curve
- ####Part 5: Reduce feature dimension via feature hashing
 - #### Visualization 3: Hyperparameter heat map

Note that, for reference, you can look up the details of the relevant Spark methods in <u>Spark's Python API</u> (https://spark.apache.org/docs/latest/api/python/pyspark.html#pyspark.RDD) and the relevant NumPy methods in the NumPy Reference (https://docs.scipy.org/doc/numpy/reference/index.html)

Start Spark

```
import os
import sys
#Escape L for line numbers
spark_home = os.environ['SPARK_HOME'] = '/Users/leiyang/Downloads/spark-1.6.0-bin-hadoop2.6'
if not spark_home:
    raise ValueError('SPARK_HOME enviroment variable is not set')
sys.path.insert(0,os.path.join(spark_home,'python'))
sys.path.insert(0,os.path.join(spark_home,'python/lib/py4j-0.8.2.1-src.zip'))
execfile(os.path.join(spark_home,'python/pyspark/shell.py'))
```

Welcome to

sampleOHEDictManual[(0,'mouse')] = 2 #<FILL IN>
sampleOHEDictManual[(1,'black')] = 3 #<FILL IN>
sampleOHEDictManual[(1,'tabby')] = 4 #<FILL IN>
sampleOHEDictManual[(2,'mouse')] = 5 #<FILL IN>
sampleOHEDictManual[(2,'salmon')] = 6 #<FILL IN>

Using Python version 2.7.9 (default, Dec 15 2014 10:37:34)
SparkContext available as sc, HiveContext available as sqlContext.

Part 1: Featurize categorical data using one-hot-encoding

(1a) One-hot-encoding

We would like to develop code to convert categorical features to numerical ones, and to build intuition, we will work with a sample unlabeled dataset with three data points, with each data point representing an animal. The first feature indicates the type of animal (bear, cat, mouse); the second feature describes the animal's color (black, tabby); and the third (optional) feature describes what the animal eats (mouse, salmon).

In a one-hot-encoding (OHE) scheme, we want to represent each tuple of (featureID, category) via its own binary feature. We can do this in Python by creating a dictionary that maps each tuple to a distinct integer, where the integer corresponds to a binary feature. To start, manually enter the entries in the OHE dictionary associated with the sample dataset by mapping the tuples to consecutive integers starting from zero, ordering the tuples first by featureID and next by category.

Later in this lab, we'll use OHE dictionaries to transform data points into compact lists of features that can be used in machine learning algorithms.

```
In [2]: # Data for manual OHE
    # Note: the first data point does not include any value for the optional third feature
    sampleOne = [(0, 'mouse'), (1, 'black')]
    sampleTwo = [(0, 'cat'), (1, 'tabby'), (2, 'mouse')]
    sampleThree = [(0, 'bear'), (1, 'black'), (2, 'salmon')]
    sampleDataRDD = sc.parallelize([sampleOne, sampleTwo, sampleThree])

In [3]: # TODO: Replace <FILL IN> with appropriate code
    sampleOHEDictManual = {}
    sampleOHEDictManual[(0, 'bear')] = 0 #<FILL IN>
    sampleOHEDictManual[(0, 'cat')] = 1 #<FILL IN>
```

Test.assertEqualsHashed(sampleOHEDictManual[(0,'bear')], 'b6589fc6ab0dc82cf12099d1c2d40ab994e8410c', "incorrect value for sampleOHEDictManual[(0,'bear')]") Test.assertEqualsHashed(sampleOHEDictManual((0,'cat')), '356a192b7913b04c54574d18c28d46e6395428ab', "incorrect value for sampleOHEDictManual[(0,'cat')]") Test.assertEqualsHashed(sampleOHEDictManual[(0,'mouse')], 'da4b9237bacccdf19c0760cab7aec4a8359010b0', "incorrect value for sampleOHEDictManual[(0,'mouse')]") Test.assertEqualsHashed(sampleOHEDictManual[(1,'black')], '77de68daecd823babbb58edb1c8e14d7106e83bb', "incorrect value for sampleOHEDictManual[(1,'black')]") Test.assertEqualsHashed(sampleOHEDictManual[(1,'tabby')], '1b6453892473a467d07372d45eb05abc2031647a', "incorrect value for sampleOHEDictManual[(1,'tabby')]") Test.assertEqualsHashed(sampleOHEDictManual[(2,'mouse')], ac3478d69a3c81fa62e60f5c3696165a4e5e6ac4', "incorrect value for sampleOHEDictManual[(2,'mouse')]") Test.assertEqualsHashed(sampleOHEDictManual[(2,'salmon')], cldfd96eea8cc2b62785275bca38ac261256e278', "incorrect value for sampleOHEDictManual[(2,'salmon')]") Test.assertEquals(len(sampleOHEDictManual.keys()), 7, 'incorrect number of keys in sampleOHEDictManual')

```
1 test passed.
```

(1b) Sparse vectors

1 test passed. 1 test passed. 1 test passed.

Data points can typically be represented with a small number of non-zero OHE features relative to the total number of features that occur in the dataset. By leveraging this sparsity and using sparse vector representations of OHE data, we can reduce storage and computational burdens. Below are a few sample vectors represented as dense numpy arrays. Use SparseVector (https://spark.apache.org/docs/latest/api/python/pyspark.mllib.linalg.SparseVector) to represent them in a sparse fashion, and verify that both the sparse and dense representations yield the same results when computing dot products (http://en.wikipedia.org/wiki/Dot_product)) (we will later use MLlib to train classifiers via gradient descent, and MLlib will need to compute dot products between SparseVectors and dense parameter vectors) vectors).

Use SparseVector (size, *args) to create a new sparse vector where size is the length of the vector and args is either a dictionary, a list of (index, value) pairs, or two separate arrays of indices and values (sorted by index). You'll need to create a sparse vector representation of each dense vector aDense and bDense.

```
In [5]: import numpy as np
         from pyspark.mllib.linalg import SparseVector
In [6]: # TODO: Replace <FILL IN> with appropriate code
         aDense = np.array([0., 3., 0., 4.])
         aSparse = SparseVector(4,[1,3],[3,4]) #<FILL IN>
         bDense = np.array([0., 0., 0., 1.])
         bSparse = SparseVector(4,[3],[1]) #<FILL IN>
         w = np.array([0.4, 3.1, -1.4, -.5])
         print aDense.dot(w)
         print aSparse.dot(w)
         print bDense.dot(w)
         print bSparse.dot(w)
         7.3
         7.3
         -0.5
         -0.5
In [7]: # TEST Sparse Vectors (1b)
         Test.assertTrue(isinstance(aSparse, SparseVector), 'aSparse needs to be an instance of SparseVector')
Test.assertTrue(isinstance(bSparse, SparseVector), 'aSparse needs to be an instance of SparseVector')
         Test.assertTrue(aDense.dot(w) == aSparse.dot(w),
                            'dot product of aDense and w should equal dot product of aSparse and w')
         Test.assertTrue(bDense.dot(w) == bSparse.dot(w),
                            'dot product of bDense and w should equal dot product of bSparse and w')
         1 test passed.
```

Now let's see how we can represent the OHE features for points in our sample dataset. Using the mapping defined by the OHE dictionary from Part (1a), manually define OHE features for the three sample data points using SparseVector format. Any feature that occurs in a point should have the value 1.0. For example, the DenseVector for a point with features 2 and 4 would be [0.0, 0.0, 1.0, 0.0, 1.0, 0.0, 0.0].

```
In [ ]: # Reminder of the sample features
         # sampleOne = [(0, 'mouse'), (1, 'black')]
# sampleTwo = [(0, 'cat'), (1, 'tabby'), (2, 'mouse')]
         # sampleThree = [(0, 'bear'), (1, 'black'), (2, 'salmon')]
         # sampleOHEDictManual[(0,'bear')] = 0 #<FILL IN>
         # sampleOHEDictManual[(0, 'cat')] = 1 #<FILL IN>
# sampleOHEDictManual[(0, 'mouse')] = 2 #<FILL IN>
# sampleOHEDictManual[(1, 'black')] = 3 #<FILL IN>
         # sampleOHEDictManual[(1,'tabby')] = 4 #<FILL IN>
         # sampleOHEDictManual[(2,'mouse')] = 5 #<FILL IN>
# sampleOHEDictManual[(2,'salmon')] = 6 #<FILL IN>
In [8]: # TODO: Replace <FILL IN> with appropriate code
         \verb|sampleOneOHEFeatManual| = SparseVector(7, [2,3],[1,1]) \# < FILL \ IN>
         sampleTwoOHEFeatManual = SparseVector(7, [1,4,5], [1,1,1]) #<FILL IN>
         sampleThreeOHEFeatManual = SparseVector(7, [0,3,6], [1,1,1]) #<FILL IN>
In [9]:
         # TEST OHE Features as sparse vectors (1c)
         Test.assertTrue(isinstance(sampleOneOHEFeatManual, SparseVector),
                            'sampleOneOHEFeatManual needs to be a SparseVector')
         Test.assertTrue(isinstance(sampleTwoOHEFeatManual, SparseVector),
                            'sampleTwoOHEFeatManual needs to be a SparseVector')
         Test.assertTrue(isinstance(sampleThreeOHEFeatManual, SparseVector),
                            'sampleThreeOHEFeatManual needs to be a SparseVector')
         Test.assertEqualsHashed(sampleOneOHEFeatManual,
                                      ecc00223d141b7bd0913d52377cee2cf5783abd6',
                                     'incorrect value for sampleOneOHEFeatManual')
         Test.assertEqualsHashed(sampleTwoOHEFeatManual,
                                      '26b023f4109e3b8ab32241938e2e9b9e9d62720a',
                                      'incorrect value for sampleTwoOHEFeatManual')
         Test.assertEqualsHashed(sampleThreeOHEFeatManual,
                                      c04134fd603ae115395b29dcabe9d0c66fbdc8a7',
                                     'incorrect value for sampleThreeOHEFeatManual')
         1 test passed.
         1 test passed.
         1 test passed.
```

(1d) Define a OHE function

1 test passed.
1 test passed.
1 test passed.

Next we will use the OHE dictionary from Part (1a) to programatically generate OHE features from the original categorical data. First write a function called oneHotEncoding that creates OHE feature vectors in SparseVector format. Then use this function to create OHE features for the first sample data point and verify that the result matches the result from Part (1c).

```
"""Produce a one-hot-encoding from a list of features and an OHE dictionary.
                 You should ensure that the indices used to create a SparseVector are sorted.
             Args:
                 rawFeats (list of (int, str)): The features corresponding to a single observation. Each
                     feature consists of a tuple of featureID and the feature's value. (e.g. sampleOne)
                 OHEDict (dict): A mapping of (featureID, value) to unique integer.
                 numOHEFeats (int): The total number of unique OHE features (combinations of featureID and
                     value).
                 SparseVector: A SparseVector of length numOHEFeats with indicies equal to the unique
                     identifiers for the (featureID, value) combinations that occur in the observation and
                     with values equal to 1.0.
             #<FILL IN>
             # build index array
             ids = [OHEDict[x] for x in rawFeats]
             ones = [1]*len(ids)
             return SparseVector(numOHEFeats, np.sort(ids), ones)
         # Calculate the number of features in sampleOHEDictManual
         numSampleOHEFeats = len(sampleOHEDictManual) #<FILL IN>
         # Run oneHotEnoding on sampleOne
         sampleOneOHEFeat = oneHotEncoding(sampleOne, sampleOHEDictManual, numSampleOHEFeats) #<FILL IN>
         print sampleOneOHEFeat
         (7,[2,3],[1.0,1.0])
In [11]: # TEST Define an OHE Function (1d)
         Test.assertTrue(sampleOneOHEFeat == sampleOneOHEFeatManual,
                          sampleOneOHEFeat should equal sampleOneOHEFeatManual')
         Test.assertEquals(sampleOneOHEFeat, SparseVector(7, [2,3], [1.0,1.0]),
                           'incorrect value for sampleOneOHEFeat')
         Test.assertEquals(oneHotEncoding([(1, 'black'), (0, 'mouse')], sampleOHEDictManual,
                                          numSampleOHEFeats), SparseVector(7, [2,3], [1.0,1.0]),
                           'incorrect definition for oneHotEncoding')
         1 test passed.
         1 test passed.
         1 test passed.
         (1e) Apply OHE to a dataset
         Finally, use the function from Part (1d) to create OHE features for all 3 data points in the sample dataset.
In [12]: # TODO: Replace <FILL IN> with appropriate code
         sampleOHEData = sampleOataRDD.map(lambda p: oneHotEncoding(p, sampleOHEDictManual, numSampleOHEFeats)) #<FILL IN>
         print sampleOHEData.collect()
         [SparseVector(7, {2: 1.0, 3: 1.0}), SparseVector(7, {1: 1.0, 4: 1.0, 5: 1.0}), SparseVector(7, {0: 1.0, 3: 1.0, 6:
          1.0})]
In [13]: # TEST Apply OHE to a dataset (1e)
         sampleOHEDataValues = sampleOHEData.collect()
         Test.assertTrue(len(sampleOHEDataValues) == 3, 'sampleOHEData should have three elements')
         Test.assertEquals(sampleOHEDataValues[0], SparseVector(7, {2: 1.0, 3: 1.0}),
                           'incorrect OHE for first sample')
         Test.assertEquals(sampleOHEDataValues[1], SparseVector(7, {1: 1.0, 4: 1.0, 5: 1.0}),
                           'incorrect OHE for second sample')
         1 test passed.
         1 test passed.
         1 test passed.
         1 test passed.
         Part 2: Construct an OHE dictionary
         (2a) Pair RDD of (featureID, category)
```

To start, create an RDD of distinct (featureID, category) tuples. In our sample dataset, the 7 items in the resulting RDD are (0, 'bear'), (0, 'cat'), (0, 'mouse'), (1, 'black'), (1, 'tabby'), (2, 'mouse'), (2, 'salmon'). Notably 'black' appears twice in the dataset but only contributes one item to the RDD: (1, 'black'), while 'mouse' also appears twice and contributes two items: (0, 'mouse') and (2, 'mouse'). Use

1 test passed.

(2b) OHE Dictionary from distinct features

Next, create an RDD of key-value tuples, where each (featureID, category) tuple in sampleDistinctFeats is a key and the values are distinct integers ranging from 0 to (number of keys - 1). Then convert this RDD into a dictionary, which can be done using the collectAsMap action. Note that there is no unique mapping from keys to values, as all we require is that each (featureID, category) key be mapped to a unique integer between 0 and the number of keys. In this exercise, any valid mapping is acceptable. Use zipWithIndex (https://spark.apache.org/docs/latest/api/python/pyspark.html#pyspark.RDD.zipWithIndex) followed by collectAsMap (https://spark.apache.org/docs/latest/api/python/pyspark.html#pyspark.RDD.collectAsMap).

In our sample dataset, one valid list of key-value tuples is: [((0, 'bear'), 0), ((2, 'salmon'), 1), ((1, 'tabby'), 2), ((2, 'mouse'), 3), ((0, 'mouse'), 4), ((0, 'cat'), 5), ((1, 'black'), 6)]. The dictionary defined in Part (1a) illustrates another valid mapping between keys and integers.

(2c) Automated creation of an OHE dictionary

Now use the code from Parts (2a) and (2b) to write a function that takes an input dataset and outputs an OHE dictionary. Then use this function to create an OHE dictionary for the sample dataset, and verify that it matches the dictionary from Part (2b).

```
In [18]: # TODO: Replace <FILL IN> with appropriate code
def createOneHotDict(inputData):
    """Creates a one-hot-encoder dictionary based on the input data.

Args:
    inputData (RDD of lists of (int, str)): An RDD of observations where each observation is
    made up of a list of (featureID, value) tuples.

Returns:
    dict: A dictionary where the keys are (featureID, value) tuples and map to values that are
    unique integers.

"""
#<FILL IN>
    return inputData.flatMap(lambda p: p).distinct().sortBy(lambda p: p).zipWithIndex().collectAsMap()
sampleOHEDictAuto = createOneHotDict(sampleDataRDD) #<FILL IN>
print sampleOHEDictAuto
```

```
{(2, 'mouse'): 5, (0, 'cat'): 1, (0, 'bear'): 0, (2, 'salmon'): 6, (1, 'tabby'): 4, (1, 'black'): 3, (0, 'mouse'): 2}
```

- 1 test passed.
- 1 test passed.

Part 3: Parse CTR data and generate OHE features

Before we can proceed, you'll first need to obtain the data from Criteo. If you have already completed this step in the setup lab, just run the cells below and the data will be loaded into the rawData variable.

Below is Criteo's data sharing agreement. After you accept the agreement, you can obtain the download URL by right-clicking on the "Download Sample" button and clicking "Copy link address" or "Copy Link Location", depending on your browser. Paste the URL into the # TODO cell below. The file is 8.4 MB compressed. The script below will download the file to the virtual machine (VM) and then extract the data.

If running the cell below does not render a webpage, open the <u>Criteo agreement (http://labs.criteo.com/downloads/2014-kaggle-display-advertising-challenge-dataset/)</u> in a separate browser tab. After you accept the agreement, you can obtain the download URL by right-clicking on the "Download Sample" button and clicking "Copy link address" or "Copy Link Location", depending on your browser. Paste the URL into the # __TODO cell below.

Note that the download could take a few minutes, depending upon your connection speed.

Out[87]:





Download Kaggle Display Advertising Challenge Dataset

CRITEO LABS DATA TERMS OF USE

```
import glob
import os.path
import tarfile
import urllib
import urlparse
# Paste url, url should end with: dac sample.tar.gz
url = 'http://labs.criteo.com/wp-content/uploads/2015/04/dac_sample.tar.gz' #'<FILL IN>'
url = url.strip()
baseDir = os.path.join('data')
inputPath = os.path.join('cs190', 'dac_sample.txt')
fileName = os.path.join(baseDir, inputPath)
inputDir = os.path.split(fileName)[0]
def extractTar(check = False):
    # Find the zipped archive and extract the dataset
    tars = glob.glob('dac sample*.tar.gz*')
    if check and len(tars) == 0:
     return False
    if len(tars) > 0:
        try:
           tarFile = tarfile.open(tars[0])
        except tarfile.ReadError:
            if not check:
               print 'Unable to open tar.gz file. Check your URL.'
           return False
        tarFile.extract('dac_sample.txt', path=inputDir)
        print 'Successfully extracted: dac_sample.txt'
       return True
    else:
       print 'You need to retry the download with the correct url.'
       print ('Alternatively, you can upload the dac_sample.tar.gz file to your Jupyter root ' +
              'directory')
        return False
if os.path.isfile(fileName):
   print 'File is already available. Nothing to do.'
elif extractTar(check = True):
   print 'tar.gz file was already available.'
elif not url.endswith('dac_sample.tar.gz'):
   print 'Check your download url. Are you downloading the Sample dataset?'
else:
    # Download the file and store it in the same directory as this notebook
       urllib.urlretrieve(url, os.path.basename(urlparse.urlsplit(url).path))
    except IOError:
       print 'Unable to download and store: {0}'.format(url)
   extractTar()
```

File is already available. Nothing to do.

[u'0,1,1,5,0,1382,4,15,2,181,1,2,,2,68fdle64,80e26c9b,fb936136,7b4723c4,25c83c98,7e0ccccf,de7995b8,1f89b562,a73ee5 10,a8cd5504,b2cb9c98,37c9c164,2824a5f6,ladce6ef,8ba8b39a,891b62e7,e5ba7672,f54016b9,21ddcdc9,b1252a9d,07b5194c,,3a 171ecb,c5c50484,e8b83407,9727dd16']

(3a) Loading and splitting the data

We are now ready to start working with the actual CTR data, and our first task involves splitting it into training, validation, and test sets. Use the randomSplit method (https://spark.apache.org/docs/latest/api/python/pyspark.html#pyspark.RDD.randomSplit) with the specified weights and seed to create RDDs storing each of these datasets, and then thttps://spark.apache.org/docs/latest/api/python/pyspark.html#pyspark.RDD.cache) each of these RDDs, as we will be accessing them multiple times in the remainder of this lab. Finally, compute the size of each dataset.

```
seed = 42
         # Use randomSplit with weights and seed
         rawTrainData, rawValidationData, rawTestData = rawData.randomSplit(weights, seed) #<FILL IN>
         # Cache the data
         #<FILL IN>
         rawTrainData.cache()
         rawValidationData.cache()
         rawTestData.cache()
         # count the data
         nTrain = rawTrainData.count()
         nVal = rawValidationData.count()
         nTest = rawTestData.count() #<FILL IN>
         print nTrain, nVal, nTest, nTrain + nVal + nTest
         print rawData.take(1)
         79911 10075 10014 100000
         [u'0,1,1,5,0,1382,4,15,2,181,1,2,,2,68fdle64,80e26c9b,fb936136,7b4723c4,25c83c98,7e0cccf,de7995b8,1f89b562,a73ee5]
         171ecb,c5c50484,e8b83407,9727dd16']
In [23]: # TEST Loading and splitting the data (3a)
         Test.assertTrue(all([rawTrainData.is_cached, rawValidationData.is_cached, rawTestData.is_cached]),
                         'you must cache the split data')
         Test.assertEquals(nTrain, 79911, 'incorrect value for nTrain')
Test.assertEquals(nVal, 10075, 'incorrect value for nVal')
Test.assertEquals(nTest, 10014, 'incorrect value for nTest')
         1 test passed.
         1 test passed.
         1 test passed.
         1 test passed.
```

(3b) Extract features

We will now parse the raw training data to create an RDD that we can subsequently use to create an OHE dictionary. Note from the take() command in Part (3a) that each raw data point is a string containing several fields separated by some delimiter. For now, we will ignore the first field (which is the 0-1 label), and parse the remaining fields (or raw features). To do this, complete the implemention of the parsePoint function.

```
In [24]: # TODO: Replace <FILL IN> with appropriate code
         def parsePoint(point):
               ""Converts a comma separated string into a list of (featureID, value) tuples.
                 featureIDs should start at 0 and increase to the number of features - 1.
             Aras:
                 point (str): A comma separated string where the first value is the label and the rest
                     are features.
             Returns:
                 list: A list of (featureID, value) tuples.
             #<FILL IN>
             fea = point.strip().split(',')[1:]
             return [(i, fea[i]) for i in range(len(fea))]
         parsedTrainFeat = rawTrainData.map(parsePoint)
         numCategories = (parsedTrainFeat
                           .flatMap(lambda x: x)
                          .distinct()
                          .map(lambda x: (x[0], 1))
                          .reduceByKey(lambda x, y: x + y)
                          .sortByKey()
                           .collect())
         print numCategories[2][1]
         #print parsedTrainFeat.take(2)
```

```
In [25]: # TEST Extract features (3b)
Test.assertEquals(numCategories[2][1], 855, 'incorrect implementation of parsePoint')
Test.assertEquals(numCategories[32][1], 4, 'incorrect implementation of parsePoint')
```

1 test passed.
1 test passed.

```
In [28]: # TODO: Replace <FILL IN> with appropriate code
           ctrOHEDict = createOneHotDict(parsedTrainFeat) #<FILL IN>
           numCtrOHEFeats = len(ctrOHEDict.keys())
           print numCtrOHEFeats
           print ctrOHEDict[(0, '')]
           233286
In [29]: # TEST Create an OHE dictionary from the dataset (3c)
           Test.assertEquals(numCtrOHEFeats, 233286, 'incorrect number of features in ctrOHEDict')
           Test.assertTrue((0, '') in ctrOHEDict, 'incorrect features in ctrOHEDict')
           1 test passed.
           1 test passed.
           (3d) Apply OHE to the dataset
           Now let's use this OHE dictionary by starting with the raw training data and creating an RDD of <a href="LabeledPoint"><u>LabeledPoint</u></a> (<a href="http://spark.apache.org/docs/1.3.1/api/python/pyspark.mllib.html#pyspark.mllib.regression.LabeledPoint">LabeledPoint</a>) objects using OHE features. To do this, complete the implementation of the <a href="parseOHEPoint">parseOHEPoint</a> is an extension of the <a href="parsePoint">parsePoint</a> function from Part (3b) and it uses the oneHotEncoding function from Part (1d).
In [30]: from pyspark.mllib.regression import LabeledPoint
In [31]: # TODO: Replace <FILL IN> with appropriate code
           def parseOHEPoint(point, OHEDict, numOHEFeats):
                  ""Obtain the label and feature vector for this raw observation.
                Note:
                     You must use the function `oneHotEncoding` in this implementation or later portions
                     of this lab may not function as expected.
                     point (str): A comma separated string where the first value is the label and the rest
                          are features.
                     OHEDict (dict of (int, str) to int): Mapping of (featureID, value) to unique integer.
                     numOHEFeats (int): The number of unique features in the training dataset.
                Returns:
                     LabeledPoint: Contains the label for the observation and the one-hot-encoding of the
                          raw features based on the provided OHE dictionary.
                #<FILL IN>
                label, fea = point.strip().split(',', 1)
                return LabeledPoint(label, oneHotEncoding(parsePoint(point), OHEDict, numOHEFeats))
           OHETrainData = rawTrainData.map(lambda point: parseOHEPoint(point, ctrOHEDict, numCtrOHEFeats))
           OHETrainData.cache()
           print OHETrainData.take(1)
           # Check that oneHotEncoding function was used in parseOHEPoint
           backupOneHot = oneHotEncoding
           oneHotEncoding = None
           withOneHot = False
           try: parseOHEPoint(rawTrainData.take(1)[0], ctrOHEDict, numCtrOHEFeats)
           except TypeError: withOneHot = True
           oneHotEncoding = backupOneHot
           [Labeled Point (0.0, (233286, [2,147,3206,3467,6199,25073,25873,26398,27079,28303,28323,28390,28554,28906,29422,65104] \\
```

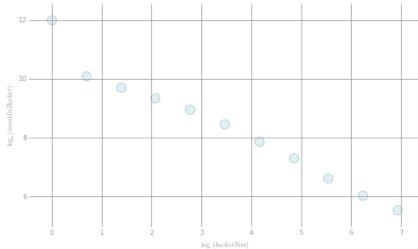
In [32]: # TEST Apply OHE to the dataset (3d)
 numNZ = sum(parsedTrainFeat.map(lambda x: len(x)).take(5))
 numNZAlt = sum(OHETrainData.map(lambda lp: len(lp.features.indices)).take(5))
 Test.assertEquals(numNZ, numNZAlt, 'incorrect implementation of parseOHEPoint')
 Test.assertTrue(withOneHot, 'oneHotEncoding not present in parseOHEPoint')

1 test passed.
1 test passed.

Visualization 1: Feature frequency

We will now visualize the number of times each of the 233,286 OHE features appears in the training data. We first compute the number of times each feature appears, then bucket the features by these counts. The buckets are sized by powers of 2, so the first bucket corresponds to features that appear exactly once (2^0), the second to features that appear twice (2^1), the third to features that occur between three and four (2^2) times, the fifth bucket is five to eight (2^3) times and so on. The scatter plot below shows the logarithm of the bucket thresholds versus the logarithm of the number of features that

```
In [33]: | def bucketFeatByCount(featCount):
                                 ""Bucket the counts by powers of two."""
                              for i in range(11):
                                      size = 2 ** i
                                      if featCount <= size:</pre>
                                               return size
                             return -1
                    featCounts = (OHETrainData
                                                   .flatMap(lambda lp: lp.features.indices)
                                                   .map(lambda x: (x, 1))
                                                    .reduceByKey(lambda x, y: x + y))
                    featCountsBuckets = (featCounts
                                                                   .map(lambda x: (bucketFeatByCount(x[1]), 1))
                                                                   .filter(lambda (k, v): k != -1)
                                                                   .reduceByKey(lambda x, y: x + y)
                                                                   .collect())
                    print featCountsBuckets
                    [(512,\ 414),\ (1024,\ 255),\ (2,\ 24076),\ (4,\ 16639),\ (32,\ 4755),\ (64,\ 2627),\ (128,\ 1476),\ (256,\ 748),\ (16,\ 7752),\ (8,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),\ (1024,\ 1476),
                       11440), (1, 162813)]
In [34]: import matplotlib.pyplot as plt
                    %matplotlib inline
                    x, y = zip(*featCountsBuckets)
                    x, y = np.log(x), np.log(y)
                    def preparePlot(xticks, yticks, figsize=(10.5, 6), hideLabels=False, gridColor='#999999',
                                                        gridWidth=1.0):
                              """Template for generating the plot layout."""
                             plt.close()
                              fig, ax = plt.subplots(figsize=figsize, facecolor='white', edgecolor='white')
                             ax.axes.tick_params(labelcolor='#999999', labelsize='10')
                             for axis, ticks in [(ax.get_xaxis(), xticks), (ax.get_yaxis(), yticks)]:
                                      axis.set_ticks_position('none')
                                      axis.set_ticks(ticks)
                                      axis.label.set color('#999999')
                                      if hideLabels: axis.set ticklabels([])
                             plt.grid(color=gridColor, linewidth=gridWidth, linestyle='-')
                             map(lambda position: ax.spines[position].set_visible(False), ['bottom', 'top', 'left', 'right'])
                             return fig, ax
                    # generate layout and plot data
                    fig, ax = preparePlot(np.arange(0, 10, 1), np.arange(4, 14, 2))
                    ax.set\_xlabel(r'\$\lceil (bucketSize)\$'), \ ax.set\_ylabel(r'\$\lceil (countInBucket)\$')
                    plt.scatter(x, y, s=14**2, c='#d6ebf2', edgecolors='#8cbfd0', alpha=0.75)
                     #pass
                    plt.show()
```



(3e) Handling unseen features

We naturally would like to repeat the process from Part (3d), e.g., to compute OHE features for the validation and test datasets. However, we must be careful, as some categorical values will likely appear in new data that did not exist in the training data. To deal with this situation, update the oneHotEncoding() function from Part (1d) to ignore previously unseen categories, and then compute OHE features for the validation data.

```
If a (featureID, value) tuple doesn't have a corresponding key in OHEDict it should be
                  ignored.
        Args:
                  rawFeats (list of (int, str)): The features corresponding to a single observation. Each
                         feature consists of a tuple of featureID and the feature's value. (e.g. sampleOne)
                  OHEDict (dict): A mapping of (featureID, value) to unique integer.
                  numOHEFeats (int): The total number of unique OHE features (combinations of featureID and
                           value).
        Returns:
                  SparseVector: A SparseVector of length numOHEFeats with indicies equal to the unique
                           identifiers for the (featureID, value) combinations that occur in the observation and
                           with values equal to 1.0.
         #<FILL IN>
        ids = [OHEDict[x] for x in rawFeats if x in OHEDict]
         ones = [1]*len(ids)
         return SparseVector(numOHEFeats, np.sort(ids), ones)
OHEValidationData = rawValidationData.map(lambda point: parseOHEPoint(point, ctrOHEDict, numCtrOHEFeats))
OHEValidationData.cache()
print OHEValidationData.take(1)
[LabeledPoint(0.0, (233286,[0,1970,3117,3577,9095,24758,26026,26425,26863,28301,28311,28390,28681,28709,29599,5898
3,82008,87062,87178,93196,94413,94641,96215,104657,137971,142919,144970,149229,164814,178594,179212,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,182197,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,180974,1809740,180974,180974,180974,1809740,180974,180974,180974,180974,180974,180974,180974,180974,180
```

"""Produce a one-hot-encoding from a list of features and an OHE dictionary.

1 test passed.

Part 4: CTR prediction and logloss evaluation

sortedWeights = sorted(model0.weights)
print sortedWeights[:5], model0.intercept

(4a) Logistic regression

We are now ready to train our first CTR classifier. A natural classifier to use in this setting is logistic regression, since it models the probability of a click-through event rather than returning a binary response, and when working with rare events, probabilistic predictions are useful. First use LogisticRegressionWithSGD (https://spark.apache.org/docs/latest/api/python/pyspark.mllib.html#pyspark.mllib.classification.LogisticRegressionWithSGD) to train a model using

(https://spark.apache.org/docs/latest/api/python/pyspark.mllib.html#pyspark.mllib.classification.LogisticRegressionWithSGD) to train a model using OHETrainData with the given hyperparameter configuration. LogisticRegressionWithSGD returns a LogisticRegressionModel (https://spark.apache.org/docs/latest/api/python/pyspark.mllib.html#pyspark.mllib.regression.LogisticRegressionModel). Next, use the LogisticRegressionModel .weights and LogisticRegressionModel .intercept attributes to print out the model's parameters. Note that these are the names of the object's attributes and should be called using a syntax like model .weights for a given model.

 $[-0.45899236853575626,\ -0.37973707648623972,\ -0.36996558266753299,\ -0.36934962879928268,\ -0.32697945415010637]\ 0.56455084025$

1 test passed.
1 test passed.

(4b) Log loss

Throughout this lab, we will use log loss to evaluate the quality of models. Log loss is defined as:

$$\ell_{log}(p,y) = \begin{cases} -\log(p) & \text{if } y = 1\\ -\log(1-p) & \text{if } y = 0 \end{cases}$$
 (1)

where p is a probability between 0 and 1 and y is a label of either 0 or 1. Log loss is a standard evaluation criterion when predicting rare-events such as click-through rate prediction (it is also the criterion used in the <u>Criteo Kaggle competition (https://www.kaggle.com/c/criteo-display-ad-challenge)</u>). Write a function to compute log loss, and evaluate it on some sample inputs.

```
In [40]: # TODO: Replace <FILL IN> with appropriate code
         from math import log
         def computeLogLoss(p, y):
              """Calculates the value of log loss for a given probabilty and label.
                 log(0) is undefined, so when p is 0 we need to add a small value (epsilon) to it
                 and when p is 1 we need to subtract a small value (epsilon) from it.
             Aras:
                 p (float): A probabilty between 0 and 1.
                 y (int): A label. Takes on the values 0 and 1.
             Returns:
                 float: The log loss value.
             epsilon = 10e-12
             #<FILL IN>
             return -log(p+epsilon) if y==1 else -log(1-p+epsilon)
         print computeLogLoss(.5, 1)
         print computeLogLoss(.5, 0)
         print computeLogLoss(.99, 1)
         print computeLogLoss(.99, 0)
         print computeLogLoss(.01, 1)
         print computeLogLoss(.01, 0)
         print computeLogLoss(0, 1)
         print computeLogLoss(1, 1)
         print computeLogLoss(1, 0)
         0.69314718054
         0.69314718054
         0.0100503358434
         4.60517018499
         4.60517018499
         0.0100503358434
         25.3284360229
         -1.00000008274e-11
         25.3284360229
In [41]: # TEST Log loss (4b)
         Test.assertTrue(np.allclose([computeLogLoss(.5, 1), computeLogLoss(.01, 0), computeLogLoss(.01, 1)],
                                      [0.69314718056, 0.0100503358535, 4.60517018599]),
                          'computeLogLoss is not correct')
         {\tt Test.assertTrue(np.allclose([computeLogLoss(0,\ 1),\ computeLogLoss(1,\ 1),\ computeLogLoss(1,\ 0)],}
                                      [25.3284360229, 1.00000008275e-11, 25.3284360229]),
                          'computeLogLoss needs to bound p away from 0 and 1 by epsilon')
         1 test passed.
```

(4c) Baseline log loss

1 test passed.

Next we will use the function we wrote in Part (4b) to compute the baseline log loss on the training data. A very simple yet natural baseline model is one where we always make the same prediction independent of the given datapoint, setting the predicted value equal to the fraction of training points that correspond to click-through events (i.e., where the label is one). Compute this value (which is simply the mean of the training labels), and then use it to compute the training log loss for the baseline model. The log loss for multiple observations is the mean of the individual log loss values.

```
# In practice click-through rate can be one to two orders of magnitude lower classOneFracTrain = OHETrainData.map(lambda p: p.label).mean() #<FILL IN> print classOneFracTrain

logLossTrBase = OHETrainData.map(lambda p: computeLogLoss(classOneFracTrain, p.label)).mean() #<FILL IN> print 'Baseline Train Logloss = {0:.3f}\n'.format(logLossTrBase)

0.22717773523

Baseline Train Logloss = 0.536

In [45]: # TEST Baseline log loss (4c)
Test.assertTrue(np.allclose(classOneFracTrain, 0.22717773523), 'incorrect value for classOneFracTrain')
Test.assertTrue(np.allclose(logLossTrBase, 0.535844), 'incorrect value for logLossTrBase')
```

(4d) Predicted probability

1 test passed.
1 test passed.

In order to compute the log loss for the model we trained in Part (4a), we need to write code to generate predictions from this model. Write a function that computes the raw linear prediction from this logistic regression model and then passes it through a sigmoid function (http://en.wikipedia.org/wiki/Sigmoid_function) $\sigma(t) = (1 + e^{-t})^{-1}$ to return the model's probabilistic prediction. Then compute probabilistic predictions on the training data.

Note that when incorporating an intercept into our predictions, we simply add the intercept to the value of the prediction obtained from the weights and features. Alternatively, if the intercept was included as the first weight, we would need to add a corresponding feature to our data where the feature has the value one. This is not the case here.

```
In [46]: # TODO: Replace <FILL IN> with appropriate code
         from math import \exp \# exp(-t) = e^-t
         def getP(x, w, intercept):
             """Calculate the probability for an observation given a set of weights and intercept.
             Note:
                 We'll bound our raw prediction between 20 and -20 for numerical purposes.
             Args:
                 x (SparseVector): A vector with values of 1.0 for features that exist in this
                     observation and 0.0 otherwise.
                 w (DenseVector): A vector of weights (betas) for the model.
                 intercept (float): The model's intercept.
             Returns:
             float: A probability between 0 and 1.
             rawPrediction = x.dot(w) + intercept #<FILL IN>
             # Bound the raw prediction value
             rawPrediction = min(rawPrediction, 20)
             rawPrediction = max(rawPrediction, -20)
             return 1/(1+exp(-rawPrediction)) #<FILL IN>
         trainingPredictions = OHETrainData.map(lambda p: getP(p.features, model0.weights, model0.intercept)) #<FILL IN>
         print trainingPredictions.take(5)
         [0.3026288202391112,\ 0.10362661997434088,\ 0.2836342478387561,\ 0.1784610205788012,\ 0.5389775379218853]
```

1 test passed.

(4e) Evaluate the model

We are now ready to evaluate the quality of the model we trained in Part (4a). To do this, first write a general function that takes as input a model and data, and outputs the log loss. Then run this function on the OHE training data, and compare the result with the baseline log loss.

```
"""Calculates the log loss for the data given the model.
        model (LogisticRegressionModel): A trained logistic regression model.
        data (RDD of LabeledPoint): Labels and features for each observation.
   Returns:
    float: Log loss for the data.
   #<FILL IN>
   return data.map(lambda p: computeLogLoss(getP(p.features, model.weights, model.intercept), p.label)).mean()
logLossTrLR0 = evaluateResults(model0, OHETrainData)
print ('OHE Features Train Logloss:\n\tBaseline = {0:.3f}\n\tLogReg = {1:.6f}'
       .format(logLossTrBase, logLossTrLR0))
OHE Features Train Logloss:
       Baseline = 0.536
```

```
LogReg = 0.456903
```

```
In [52]: # TEST Evaluate the model (4e)
         Test.assertTrue(np.allclose(logLossTrLR0, 0.456903), 'incorrect value for logLossTrLR0')
```

1 test passed.

(4f) Validation log loss

Next, following the same logic as in Parts (4c) and 4(e), compute the validation log loss for both the baseline and logistic regression models. Notably, the baseline model for the validation data should still be based on the label fraction from the training dataset.

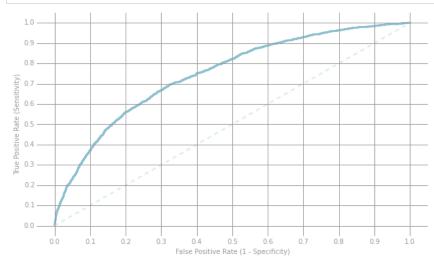
```
In [54]: # TODO: Replace <FILL IN> with appropriate code
         logLossValBase = OHEValidationData.map(lambda p: computeLogLoss(classOneFracTrain, p.label)).mean() #<FILL IN>
         logLossValLR0 = evaluateResults(model0, OHEValidationData) #<FILL IN>
         .format(logLossValBase, logLossValLR0))
         OHE Features Validation Logloss:
                 Baseline = 0.528
                 LogReg = 0.457
In [55]: # TEST Validation log loss (4f)
         Test.assertTrue(np.allclose(logLossValBase, 0.527603), 'incorrect value for logLossValBase')
Test.assertTrue(np.allclose(logLossValLR0, 0.456957), 'incorrect value for logLossValLR0')
```

Visualization 2: ROC curve

1 test passed. 1 test passed.

We will now visualize how well the model predicts our target. To do this we generate a plot of the ROC curve. The ROC curve shows us the trade-off between the false positive rate and true positive rate, as we liberalize the threshold required to predict a positive outcome. A random model is represented by the dashed line.

```
labelsAndWeights = labelsAndScores.collect()
labelsAndWeights.sort(key=lambda (k, v): v, reverse=True)
labelsByWeight = np.array([k for (k, v) in labelsAndWeights])
length = labelsByWeight.size
truePositives = labelsByWeight.cumsum()
numPositive = truePositives[-1]
falsePositives = np.arange(1.0, length + 1, 1.) - truePositives
truePositiveRate = truePositives / numPositive
falsePositiveRate = falsePositives / (length - numPositive)
# Generate layout and plot data
fig, ax = preparePlot(np.arange(0., 1.1, 0.1), np.arange(0., 1.1, 0.1))
ax.set_xlim(-.05, 1.05), ax.set_ylim(-.05, 1.05)
ax.set_ylabel('True Positive Rate (Sensitivity)')
ax.set_xlabel('False Positive Rate (1 - Specificity)')
plt.plot(falsePositiveRate, truePositiveRate, color='#8cbfd0', linestyle='-', linewidth=3.)
plt.plot((0., 1.), (0., 1.), linestyle='--', color='#d6ebf2', linewidth=2.)  # Baseline model
pass
```



Part 5: Reduce feature dimension via feature hashing

(5a) Hash function

As we just saw, using a one-hot-encoding featurization can yield a model with good statistical accuracy. However, the number of distinct categories across all features is quite large -- recall that we observed 233K categories in the training data in Part (3c). Moreover, the full Kaggle training dataset includes more than 33M distinct categories, and the Kaggle dataset itself is just a small subset of Criteo's labeled data. Hence, featuring via a one-hot-encoding representation would lead to a very large feature vector. To reduce the dimensionality of the feature space, we will use feature hashing.

Below is the hash function that we will use for this part of the lab. We will first use this hash function with the three sample data points from Part (1a) to gain some intuition. Specifically, run code to hash the three sample points using two different values for numBuckets and observe the resulting hashed feature dictionaries.

```
def hashFunction(numBuckets, rawFeats, printMapping=False):
               ""Calculate a feature dictionary for an observation's features based on hashing.
                  Use printMapping=True for debug purposes and to better understand how the hashing works.
                  numBuckets (int): Number of buckets to use as features.
                  rawFeats (list of (int, str)): A list of features for an observation. Represented as
                      (featureID, value) tuples.
                  printMapping (bool, optional): If true, the mappings of featureString to index will be
                      printed.
              Returns:
                  dict of int to float: The keys will be integers which represent the buckets that the
                      features have been hashed to. The value for a given key will contain the count of the
                      (featureID, value) tuples that have hashed to that key.
              .....
              mapping = {}
              for ind, category in rawFeats:
                  featureString = category + str(ind)
                  mapping[featureString] = int(int(hashlib.md5(featureString).hexdigest(), 16) % numBuckets)
              if(printMapping): print mapping
              sparseFeatures = defaultdict(float)
              for bucket in mapping.values():
                  sparseFeatures[bucket] += 1.0
              return dict(sparseFeatures)
         # Reminder of the sample values:
         # sampleOne = [(0, 'mouse'), (1, 'black')]
# sampleTwo = [(0, 'cat'), (1, 'tabby'), (2, 'mouse')]
          # sampleThree = [(0, 'bear'), (1, 'black'), (2, 'salmon')]
In [58]: # TODO: Replace <FILL IN> with appropriate code
          # Use four buckets
         sampOneFourBuckets = hashFunction(4, sampleOne, True)
          sampTwoFourBuckets = hashFunction(4, sampleTwo, True)
         sampThreeFourBuckets = hashFunction(4, sampleThree, True)
         \# Use one hundred buckets
          sampOneHundredBuckets = hashFunction(100, sampleOne, True)
         sampTwoHundredBuckets = hashFunction(100, sampleTwo, True)
         sampThreeHundredBuckets = hashFunction(100, sampleThree, True)
         print '\t\t 4 Buckets \t\t\t 100 Buckets'
         print 'SampleOne:\t {0}\t\t {1}'.format(sampOneFourBuckets, sampOneHundredBuckets)
         print 'SampleTwo:\t {0}\t\t {1}'.format(sampTwoFourBuckets, sampTwoHundredBuckets)
         print 'SampleThree:\t {0}\t {1}'.format(sampThreeFourBuckets, sampThreeHundredBuckets)
         {'black1': 2, 'mouse0': 3}
{'cat0': 0, 'tabby1': 0, 'mouse2': 2}
         {'bear0': 0, 'black1': 2, 'salmon2': 1}
         {'black1': 14, 'mouse0': 31}
{'cat0': 40, 'tabby1': 16, 'mouse2': 62}
         {'bear0': 72, 'black1': 14, 'salmon2': 5}
                           4 Buckets
                                                            100 Buckets
         SampleOne:
                           {2: 1.0, 3: 1.0}
                                                            {14: 1.0, 31: 1.0}
         SampleTwo:
                           {0: 2.0, 2: 1.0}
                                                             {40: 1.0, 16: 1.0, 62: 1.0}
         SampleThree:
                           {0: 1.0, 1: 1.0, 2: 1.0}
                                                             {72: 1.0, 5: 1.0, 14: 1.0}
In [59]: # TEST Hash function (5a)
         Test.assertEquals(sampOneFourBuckets, {2: 1.0, 3: 1.0}, 'incorrect value for sampOneFourBuckets')
         Test.assertEquals(sampThreeHundredBuckets, {72: 1.0, 5: 1.0, 14: 1.0},
                             'incorrect value for sampThreeHundredBuckets')
         1 test passed.
         1 test passed.
```

(5b) Creating hashed features

Next we will use this hash function to create hashed features for our CTR datasets. First write a function that uses the hash function from Part (5a) with numBuckets = $2^{15} \approx 33K$ to create a LabeledPoint with hashed features stored as a SparseVector. Then use this function to create new training, validation and test datasets with hashed features. Hint: parsedHashPoint is similar to parseOHEPoint from Part (3d).

```
"""Create a LabeledPoint for this observation using hashing.
                                             point (str): A comma separated string where the first value is the label and the rest are
                                             numBuckets: The number of buckets to hash to.
                                             LabeledPoint: A LabeledPoint with a label (0.0 or 1.0) and a SparseVector of hashed
                                                        features.
                                    # <FILL IN>
                                   elem = point.strip().split(',')
                                   rawFea = [(i, elem[i+1]) for i in range(len(elem) - 1)]
                                   index = np.sort(hashFunction(numBuckets, rawFea, False).keys())
                                   return LabeledPoint(elem[0], SparseVector(numBuckets, index, [1]*len(index)))
                        numBucketsCTR = 2 ** 15
                        hashTrainData = rawTrainData.map(lambda p: parseHashPoint(p, numBucketsCTR)) #<FILL IN>
                        hashTrainData.cache()
                        hashValidationData = rawValidationData.map(lambda p: parseHashPoint(p, numBucketsCTR)) #<FILL IN>
                        hashValidationData.cache()
                        hashTestData = rawTestData.map(lambda p: parseHashPoint(p, numBucketsCTR)) #<FILL IN>
                        hashTestData.cache()
                        print hashTrainData.take(1)
                        [Labeled Point (0.0, (32768, [1305, 2883, 3807, 4814, 4866, 4913, 6952, 7117, 9985, 10316, 11512, 11722, 12365, 13893, 14735, 15816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 13816, 
                        16198, 17761, 19274, 21604, 22256, 22563, 22785, 24855, 25202, 25533, 25721, 26487, 26656, 27668, 28211, 29152, 29402, 29873, 30039, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 2016, 201
                        In [62]: # TEST Creating hashed features (5b)
                        hashTrainDataFeatureSum = sum(hashTrainData
                                                                                                .map(lambda lp: len(lp.features.indices))
                                                                                                .take(20))
                        hashTrainDataLabelSum = sum(hashTrainData
                                                                                          .map(lambda lp: lp.label)
                                                                                          .take(100))
                        hashValidationDataFeatureSum = sum(hashValidationData
                                                                                                             .map(lambda lp: len(lp.features.indices))
                                                                                                             .take(20))
                        hashValidationDataLabelSum = sum(hashValidationData
                                                                                                       .map(lambda lp: lp.label)
                                                                                                        .take(100))
                        hashTestDataFeatureSum = sum(hashTestData
                                                                                           .map(lambda lp: len(lp.features.indices))
                                                                                             .take(20))
                        hashTestDataLabelSum = sum(hashTestData
                                                                                       .map(lambda lp: lp.label)
                                                                                        .take(100))
                        Test.assertEquals(hashTrainDataFeatureSum, 772, 'incorrect number of features in hashTrainData')
Test.assertEquals(hashTrainDataLabelSum, 24.0, 'incorrect labels in hashTrainData')
                        Test.assertEquals(hashValidationDataFeatureSum, 776,
                                                                         'incorrect number of features in hashValidationData')
                        Test.assertEquals(hashValidationDataLabelSum, 16.0, 'incorrect labels in hashValidationData')
                        Test.assertEquals(hashTestDataFeatureSum, 774, 'incorrect number of features in hashTestData')
Test.assertEquals(hashTestDataLabelSum, 23.0, 'incorrect labels in hashTestData')
                        1 test passed.
                        1 test passed.
```

(5c) Sparsity

Since we have 33K hashed features versus 233K OHE features, we should expect OHE features to be sparser. Verify this hypothesis by computing the average sparsity of the OHE and the hashed training datasets.

Note that if you have a SparseVector named sparse, calling len(sparse) returns the total number of features, not the number features with entries. SparseVector objects have the attributes indices and values that contain information about which features are nonzero. Continuing with our example, these can be accessed using sparse.indices and sparse.values, respectively.

```
"""Calculates the average sparsity for the features in an RDD of LabeledPoints.
                   data (RDD of LabeledPoint): The LabeledPoints to use in the sparsity calculation.
                   d (int): The total number of features.
                  n (int): The number of observations in the RDD.
              float: The average of the ratio of features in a point to total features. """
              #<FTT.T. TN>
              return data.map(lambda p: 1.0*len(p.features.indices)/d).mean()
          averageSparsityHash = computeSparsity(hashTrainData, numBucketsCTR, nTrain)
          averageSparsityOHE = computeSparsity(OHETrainData, numCtrOHEFeats, nTrain)
          print 'Average OHE Sparsity: {0:.7e}'.format(averageSparsityOHE)
          print 'Average Hash Sparsity: {0:.7e}'.format(averageSparsityHash)
          Average OHE Sparsity: 1.6717677e-04
          Average Hash Sparsity: 1.1805561e-03
In [64]: # TEST Sparsity (5c)
          Test.assertTrue(np.allclose(averageSparsityOHE, 1.6717677e-04),
                            'incorrect value for averageSparsityOHE')
          Test.assertTrue(np.allclose(averageSparsityHash, 1.1805561e-03),
                            'incorrect value for averageSparsityHash')
          1 test passed.
          1 test passed.
          (5d) Logistic model with hashed features
          Now let's train a logistic regression model using the hashed features. Run a grid search to find suitable hyperparameters for the hashed features, evaluating via log loss on the validation data. Note: This may take a few minutes to run. Use 1 and 10 for stepSizes and 1e-6 and 1e-3 for
          regParams.
In [65]: numIters = 500
          regType = '12'
          includeIntercept = True
          # Initialize variables using values from initial model training
          bestModel = None
          bestLogLoss = 1e10
In [79]: # TODO: Replace <FILL IN> with appropriate code
          stepSizes = [1, 10] #<FILL IN>
          regParams = [1e-6, 1e-3] #<FILL IN>
          for stepSize in stepSizes:
              for regParam in regParams:
                   model = (LogisticRegressionWithSGD
                             .train(hashTrainData, numIters, stepSize, regParam=regParam, regType=regType,
                                    intercept=includeIntercept))
                  logLossVa = evaluateResults(model, hashValidationData)
                  print ('\tstepSize = {0:.1f}, regParam = {1:.0e}: logloss = {2:.6f}'
                          .format(stepSize, regParam, logLossVa))
                   if (logLossVa < bestLogLoss):</pre>
                       bestModel = model
                       bestLogLoss = logLossVa
          print ('Hashed Features Validation Logloss:\n\tBaseline = {0:.6f}\n\tLogReg = {1:.6f}'
                  .format(logLossValBase, bestLogLoss))
                   stepSize = 1.0, regParam = 1e-06: logloss = 0.474694
                  stepSize = 1.0, regParam = 1e-03: logloss = 0.474999
                  stepSize = 10.0, regParam = 1e-06: logloss = 0.449679
                  stepSize = 10.0, regParam = 1e-03: logloss = 0.451841
          Hashed Features Validation Logloss:
                  Baseline = 0.527603
                  LogReg = 0.449679
```

Visualization 3: Hyperparameter heat map

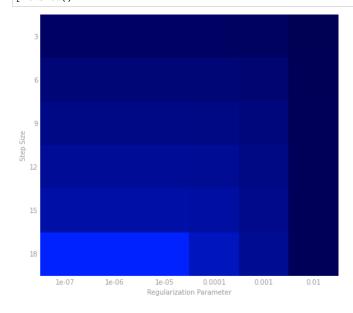
In [80]: # TEST Logistic model with hashed features (5d)

1 test failed. incorrect value for bestLogLoss

We will now perform a visualization of an extensive hyperparameter search. Specifically, we will create a heat map where the brighter colors correspond to lower values of logLoss.

Test.assertTrue(np.allclose(bestLogLoss, 0.4481683608), 'incorrect value for bestLogLoss')

```
In [68]: from matplotlib.colors import LinearSegmentedColormap
            # Saved parameters and results. Eliminate the time required to run 36 models
            stepSizes = [3, 6, 9, 12, 15, 18]
            regParams = [1e-7, 1e-6, 1e-5, 1e-4, 1e-3, 1e-2]
            {\tt logLoss = np.array([[~0.45808431,~~0.45808493,~~0.45809113,~~0.45815333,~~0.45879221,~~0.46556321]),}
                                      [ 0.45188196, 0.45188306, 0.4518941, 0.4520051, 0.45316284, 0.46396068], [ 0.44886478, 0.44886613, 0.44887974, 0.44902096, 0.4505614, 0.46371153],
                                     [ 0.44706645, 0.4470698, 0.44708102, 0.44724251, 0.44905525, 0.46366507], [ 0.44588848, 0.44589365, 0.44590568, 0.44606631, 0.44807106, 0.46365589], [ 0.44508948, 0.44509474, 0.44510274, 0.44525007, 0.44738317, 0.46365405]])
            numRows, numCols = len(stepSizes), len(regParams)
            logLoss = np.array(logLoss)
            logLoss.shape = (numRows, numCols)
            fig, ax = preparePlot(np.arange(0, numCols, 1), np.arange(0, numRows, 1), figsize=(8, 7),
                                        hideLabels=True, gridWidth=0.)
            ax.set_xticklabels(regParams), ax.set_yticklabels(stepSizes)
            ax.set xlabel('Regularization Parameter'), ax.set ylabel('Step Size')
            colors = LinearSegmentedColormap.from_list('blue', ['#0022ff', '#000055'], gamma=.2)
image = plt.imshow(logLoss,interpolation='nearest', aspect='auto',
                                     cmap = colors)
            #pass
            plt.show()
```



(5e) Evaluate on the test set

Finally, evaluate the best model from Part (5d) on the test set. Compare the resulting log loss with the baseline log loss on the test set, which can be computed in the same way that the validation log loss was computed in Part (4f).

Hashed Features Test Log Loss:
 Baseline = 0.537438
 LogReg = 0.453568

```
'incorrect value for logLossTestBaseline')
Test.assertTrue(np.allclose(logLossTest, 0.455616931), 'incorrect value for logLossTest')
```

- 1 test passed.
- 1 test failed. incorrect value for logLossTest

HW12.2 OPTIONAL Homework (please include your solution in your HW12.1 notebook)

- Implement a decision tree algorithm for regression for two continuous input variables and one categorical input variable on a single core computer using Python.
- Use the IRIS dataset to evaluate your code, where the input variables are: Petal.Length Petal.Width Species and the target or output variable is Sepal.Length.
- Use the same dataset to train and test your implementation.
- Stop expanding nodes once you have less than ten (10) examples (along with the usual stopping criteria).
- Report the mean squared error for your implementation and contrast that with the MSE from scikit-learn's implementation on this dataset (http://scikit-learn.org/stable/modules/generated/sklearn.tree.DecisionTreeRegressor.html)

Regression Tree:

We seek the splitting variable j and split point s that solve:

$$\min_{j,s} \left[\min_{c_1} \sum_{x_i \in R_1(j,s)} (y_i - c_1)^2 + \min_{c_2} \sum_{x_i \in R_2(j,s)} (y_i - c_2)^2 \right]$$

• For any choice j and s, the inner minimization is solved by

$$\hat{c}_1 = \operatorname{ave}(y_i \mid x_i \in R_1(j, s))$$
 and $\hat{c}_2 = \operatorname{ave}(y_i \mid x_i \in R_2(j, s))$

Load Iris Data

```
Iris Plants Database
```

Notes

Data Set Characteristics:

:Number of Instances: 150 (50 in each of three classes)
:Number of Attributes: 4 numeric, predictive attributes and the class
:Attribute Information:

- sepal length in cm
- sepal width in cm
- petal length in cm
- petal width in cm
- class:
 - Iris-Setosa
 - Iris-Versicolour
- Iris-Virginica

:Summary Statistics:

	==========	====	====	======	=====	=======	======	
		Min	Max	Mean	SD	Class Cor	rrelation	
	==========	====	====	======	=====	========		
	sepal length:	4.3	7.9	5.84	0.83	0.7826		
	sepal width:	2.0	4.4	3.05	0.43	-0.4194		
	petal length:	1.0	6.9	3.76	1.76	0.9490	(high!)	
	petal width:	0.1	2.5	1.20	0.76	0.9565	(high!)	
			====		=====			
:Missing Attribute Values: None :Class Distribution: 33.3% for each of 3 classes.								
	:Creator: R.A. Fisher :Donor: Michael Marshall (MARSHALL%PLU@io.arc.nasa.gov) :Date: July, 1988							

This is a copy of UCI ML iris datasets. http://archive.ics.uci.edu/ml/datasets/Iris

The famous Iris database, first used by Sir R.A Fisher

This is perhaps the best known database to be found in the pattern recognition literature. Fisher's paper is a classic in the field and is referenced frequently to this day. (See Duda & Hart, for example.) The data set contains 3 classes of 50 instances each, where each class refers to a type of iris plant. One class is linearly separable from the other 2; the latter are NOT linearly separable from each other.

References

- Fisher,R.A. "The use of multiple measurements in taxonomic problems" Annual Eugenics, 7, Part II, 179-188 (1936); also in "Contributions to Mathematical Statistics" (John Wiley, NY, 1950).
- Duda, R.O., & Hart, P.E. (1973) Pattern Classification and Scene Analysis.
- (Q327.D83) John Wiley & Sons. ISBN 0-471-22361-1. See page 218.

 Dasarathy, B.V. (1980) "Nosing Around the Neighborhood: A New System Structure and Classification Rule for Recognition in Partially Exposed Environments". IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. PAMI-2, No. 1, 67-71.
- Gates, G.W. (1972) "The Reduced Nearest Neighbor Rule". IEEE Transactions on Information Theory, May 1972, 431-433.
- See also: 1988 MLC Proceedings, 54-64. Cheeseman et al"s AUTOCLASS II conceptual clustering system finds 3 classes in the data.
- Many, many more ...

Data Preparation

```
In [20]: import numpy as np
# Petal.Length Petal.Width Species
feature_names = ['petal length (cm)', 'petal width (cm)']
target_name = 'sepal length (cm)'
fea_index = [iris.feature_names.index(f) for f in feature_names]
n_data, n_feature = len(iris.target), len(iris.target_names)
# one-hot-encoding for species
OHE_species = np.zeros([n_data, n_feature])
for i in range(n_data):
    OHE_species[i][iris.target[i]] = 1
features = np.append(iris.data[:, fea_index], OHE_species, axis=1)
target = iris.data[:, iris.feature_names.index(target_name)]
# first column is target, rest are predictors
trainingData = np.hstack(([[x] for x in target], features))
```

```
left = None
right = None
def __init__(self, data, verbose=False, depth=0):
    self.data = data
    self.depth = depth
    self.verbose = verbose
    self.sample_size = data.shape[0]
    if self.verbose:
        print 'new node depth: %d, point: %d' %(self.depth, self.sample_size)
def grow(self):
    if self.sample_size < 10 or np.var(self.data[:,0]) < 0.01:</pre>
        return
    # find split variable and split point
    self.split_id, self.split_point = -1, -1
    cur_var = float("inf")
    for i in range(1, self.data.shape[1]):
        pt, var = self.findSplitPoint(i)
        if var < cur var:</pre>
            cur var = var
            self.split_id = i
            self.split_point = pt
    if self.verbose:
        print 'split on variable %d at %.4f' %(self.split_id, self.split_point)
    # if the point is not splitting, don't do anything
    left = self.data[:, self.split_id] <= self.split_point</pre>
    if sum(left) == self.sample_size: return
    # otherwise split the data, and add children
    l_node = TreeNode(self.data[left, :], self.verbose, self.depth+1)
    r_node = TreeNode(self.data[~left, :], self.verbose, self.depth+1)
    self.left = l_node
    self.right = r_node
    # grow the children
    self.left.grow()
    self.right.grow()
def findSplitPoint(self, var_id):
    # return split point and variance reduction
    points = np.unique(self.data[:, var_id])
    cur_var, cur_s = np.var(self.data[:, 0]), points[-1]
    # not checking the last (largest point), as it doesn't split the data
    for s in points[:-1]:
       left = self.data[:, var_id] <= s</pre>
        n_{eft} = sum(left)
        n_right = self.sample_size - n_left
        var = (n_left*np.var(self.data[left,0]) + n_right*np.var(self.data[~left,0]))/self.sample_size
        if var < cur_var:</pre>
            cur_var = var
            cur_s = s
    return (cur_s, cur_var)
def predict(self, point):
    if (not self.left) and (not self.right):
        return np.mean(self.data[:,0])
    elif point[self.split_id] <= self.split_point:</pre>
       return self.left.predict(point)
    else:
        return self.right.predict(point)
```

Grow the tree

```
In [91]: root = TreeNode(trainingData)
root.grow()
```

Evaluate MSE - tied with sklearn tree

```
print 'MSE of the homegrown tree: %.8f' %MSE

# sklearn MSE
from sklearn.tree import DecisionTreeRegressor
tree = DecisionTreeRegressor(min_samples_split=10)
tree.fit(trainingData[:,1:], trainingData[:,0])
print 'MSE of the sklearn tree: %.8f' %np.mean((tree.predict(trainingData[:,1:])-trainingData[:,0])**2)
```

MSE of the homegrown tree: 0.06410849 MSE of the sklearn tree: 0.06410849