

随机两体系统综的本征值问题

沈佳杰

jjshen@shmtu.edu.cn

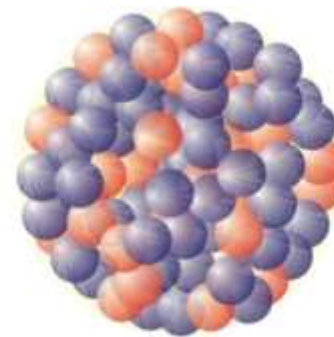
上海海事大学

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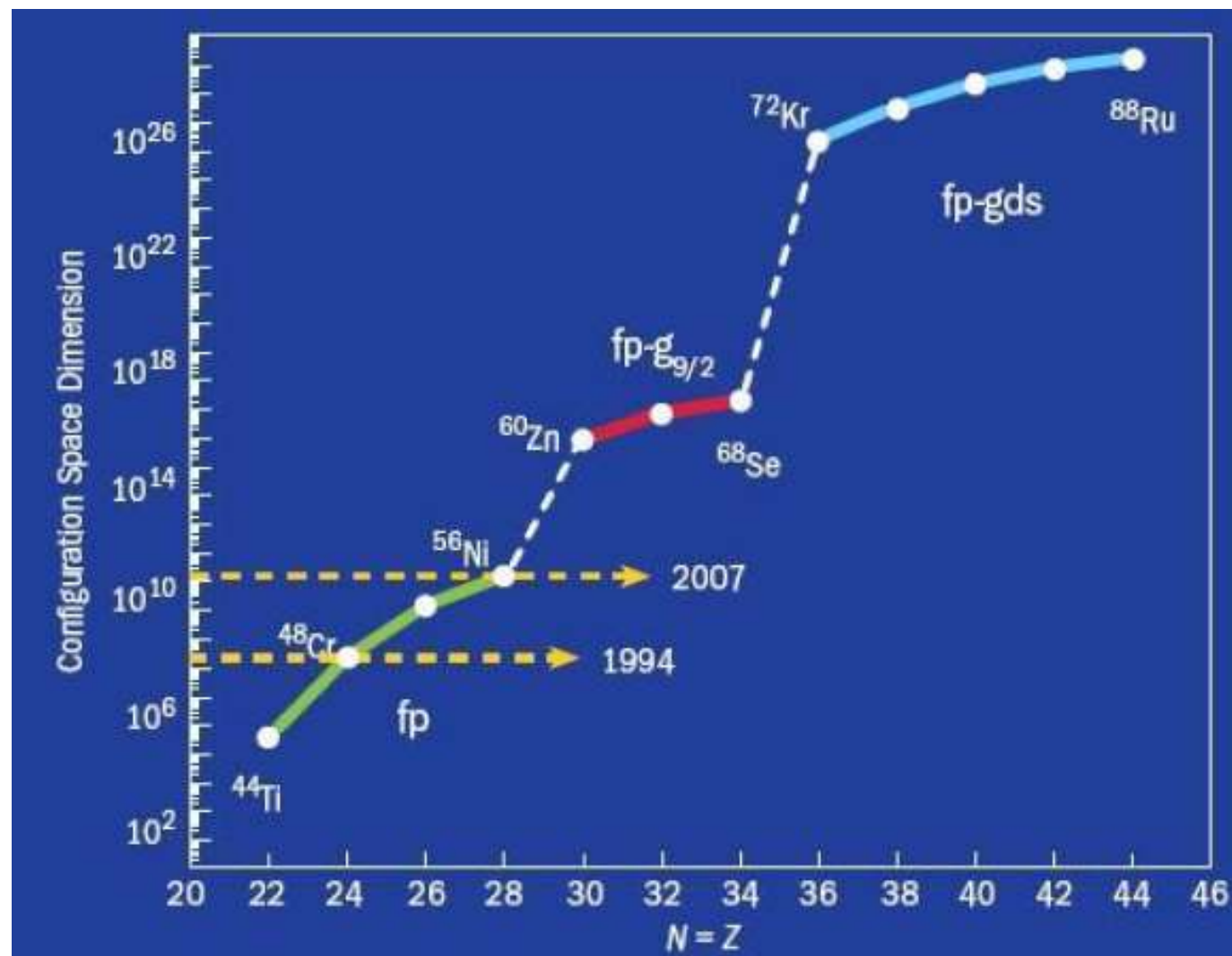
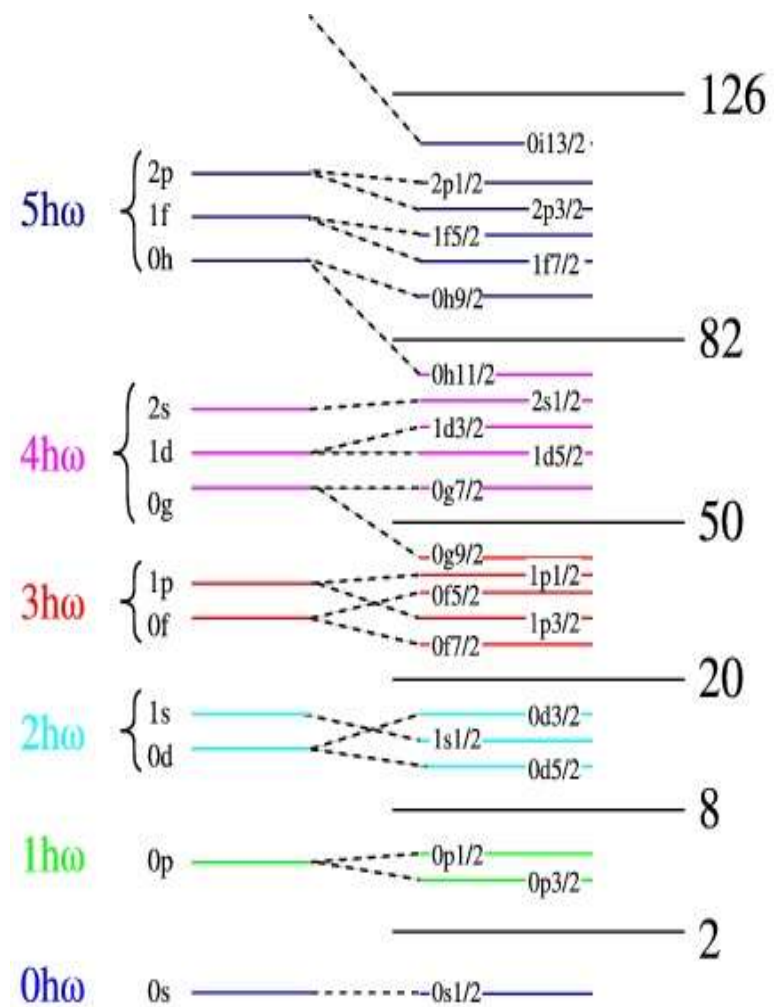
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引入



- “复杂原子核中简单规则的起源问题”（ “**What is the origin of simple patterns in complex nuclei?**” ）是美国核科学顾问委员会撰写的《**核科学前沿：长期计划**》中提出的原子核与核天文学领域五个推动未来发展的首要问题中的第二个问题。
- 偶偶核在随机两体相互作用下**基态零自旋占据主导地位**的现象，在这些相互作用下，偶偶核的低激发态统计性质与真实的原子核非常相似，包括能级结构、结合能的奇偶性以及能谱的非集体转动特性等
- **随机两体系综**等随机矩阵是研究随机相互作用的重要工具之一

壳模型哈密顿量的本征值维数



角动量零自旋占优与最小本征值公式

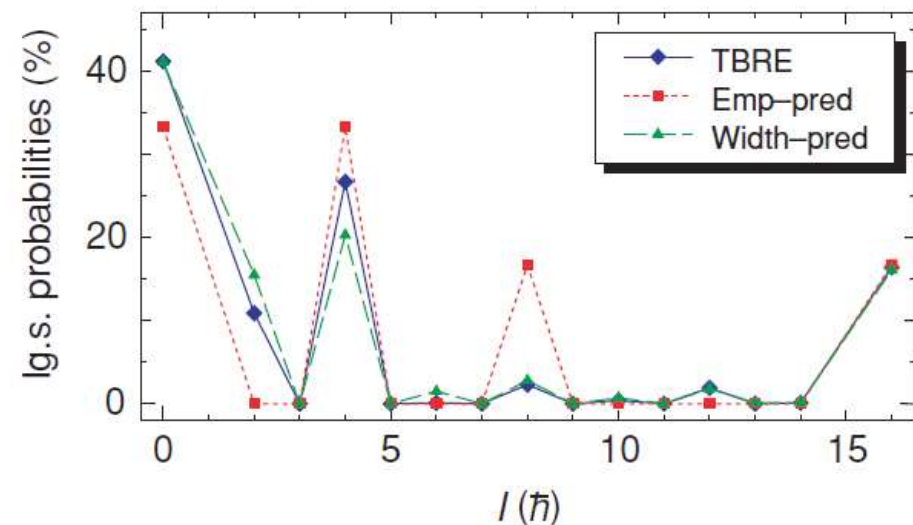
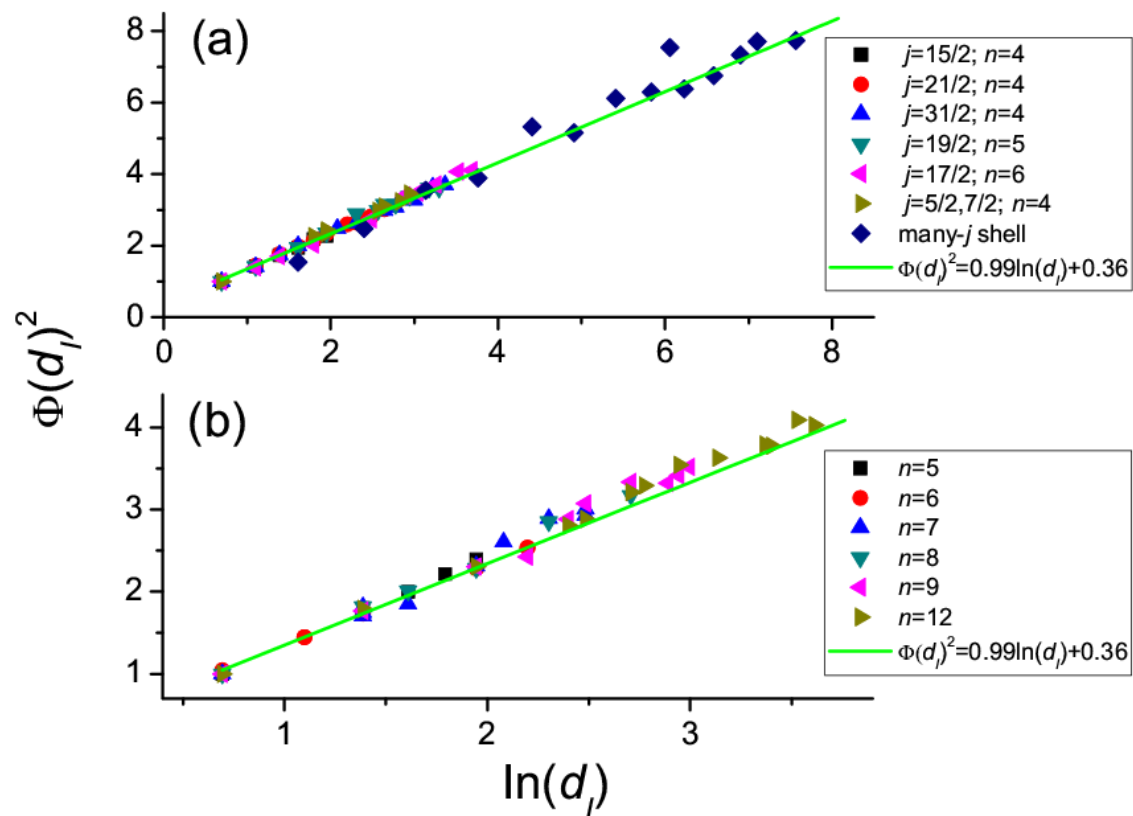


FIG. 2. (Color online) TBRE indicates $P(I)$ for 1000 random interactions with $j = 11/2$ and $n = 4$, Emp-pred indicates the empirical method of Ref. [7], and Width-pred indicates the method proposed in this paper.

$$E_I^{(min)} = \bar{E}_I - \Phi(d_I)\sigma_I\{G_I\}$$

$$\Phi(d_I) = \sqrt{ad_I + b}$$

角动量零自旋占优与最小本征值公式

引入三阶矩表征本征值分布的对称性偏离

$$E_I^{(min)} = \bar{E}_I - \Phi(D_I) \left(1 - \frac{\sqrt{\pi}}{6\sqrt{2}} \left(\frac{\sigma_3}{\sigma_2} \right)^3 \right) \sigma_2$$

j	$\frac{31}{2}$	$\frac{31}{2}$	$\frac{17}{2}$	$\frac{17}{2}$	$\frac{7}{2} \frac{5}{2}$	$\frac{7}{2} \frac{5}{2}$	$s_{\frac{1}{2}} d_{\frac{3}{2}} h_{\frac{11}{2}}$	$s_{\frac{1}{2}} d_{\frac{3}{2}} h_{\frac{11}{2}}$
n	4	4	6	6	4	4	$n_p = n_n = 3$	$n_p = n_n = 3$
d	20	29	21	40	17	19	136	508
I	13	20	5	10	2	12	20	17
ϵ_A	0.130	0.121	0.097	0.121	0.148	0.151	0.210	0.239
ϵ_B	0.087	0.100	0.073	0.088	0.124	0.131	0.141	0.123
ϵ_C	0.061	0.067	0.047	0.057	0.078	0.084	0.099	0.091

高斯正交系综的最小本征值公式

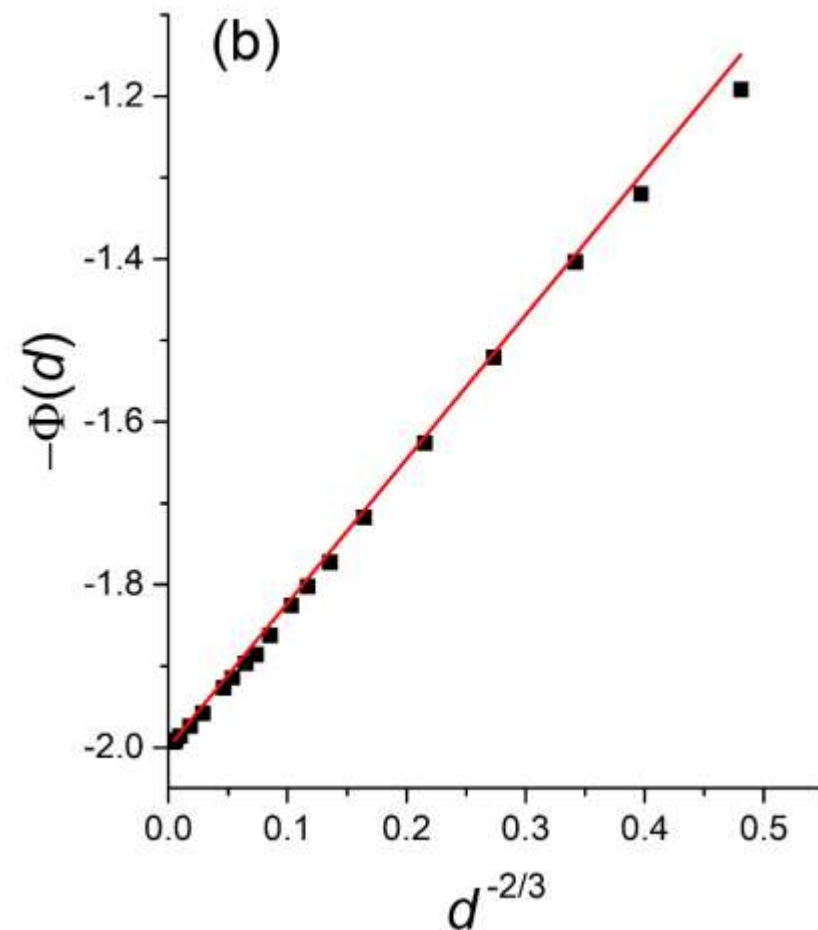
➤ 魏格纳半圆定理:

$$\rho = \frac{1}{2\pi} \sqrt{4 - x^2}, \quad x = \frac{E - \bar{E}}{\sigma}$$

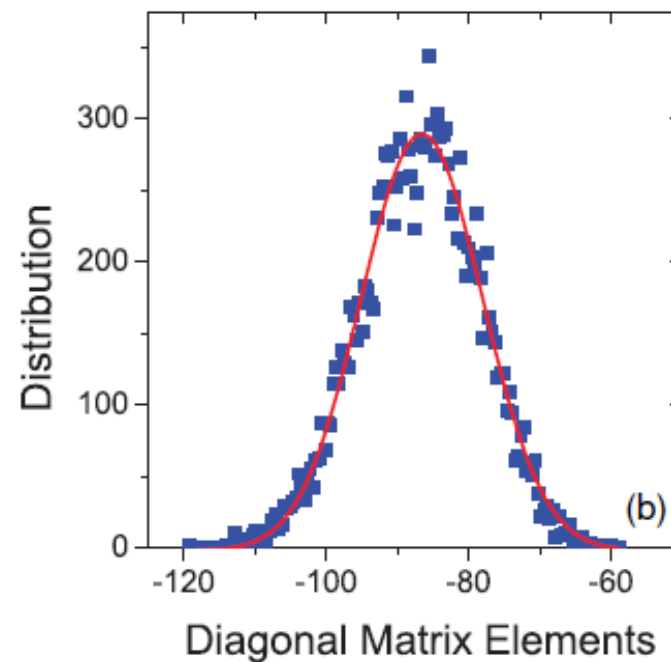
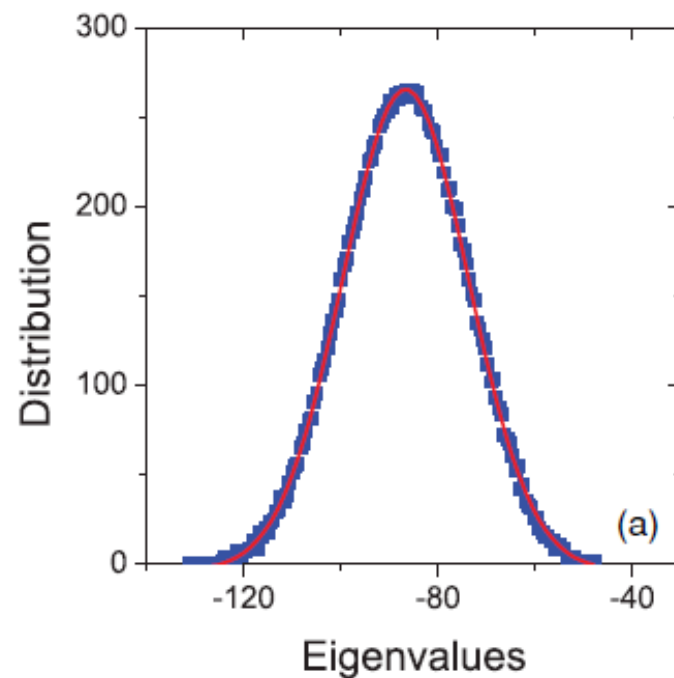


➤ 最小本征值:

$$\Phi(D) = 2 - \left(\frac{9}{16} \pi^2 \right)^{1/3} D^{-2/3}$$

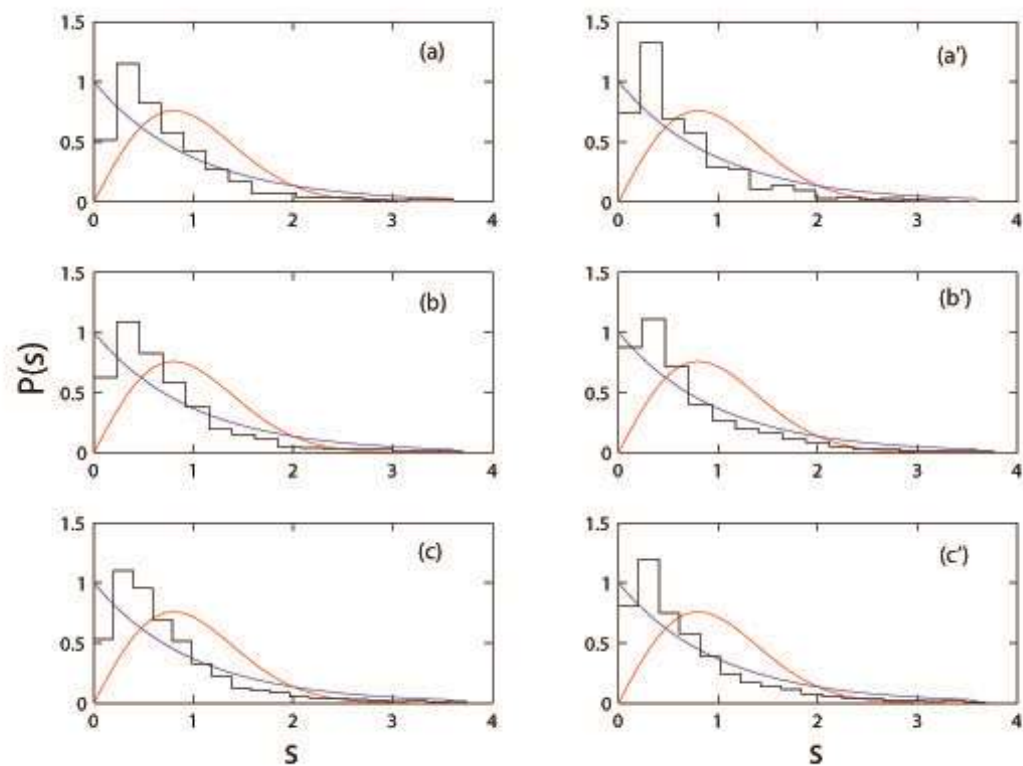
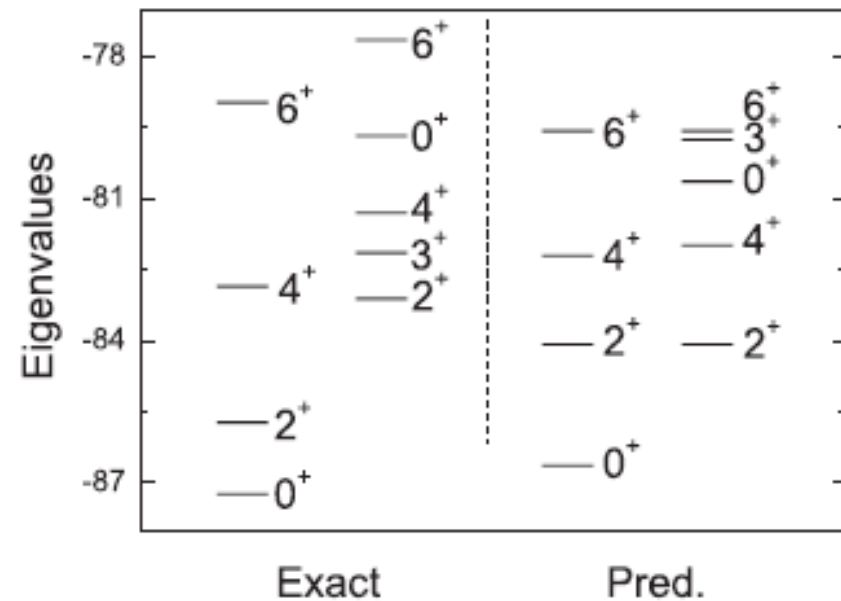
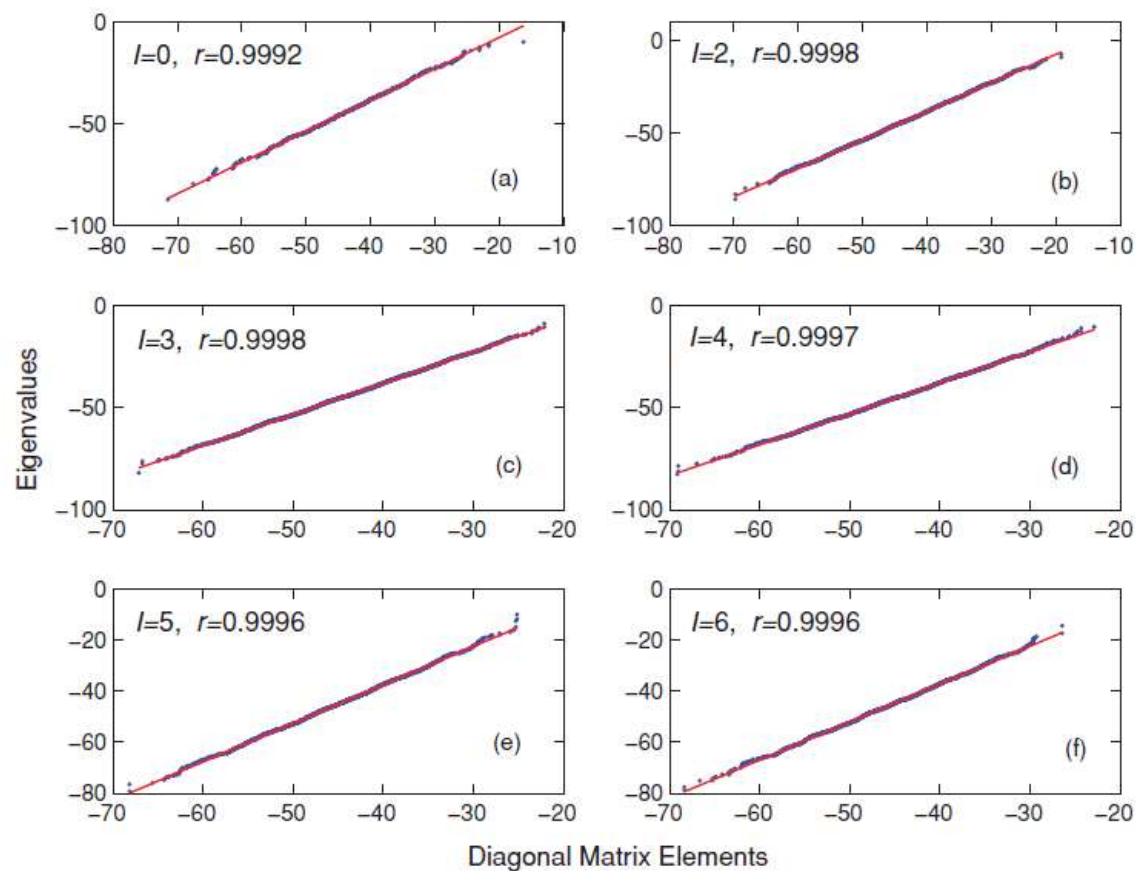


本征值与对角元的相关性

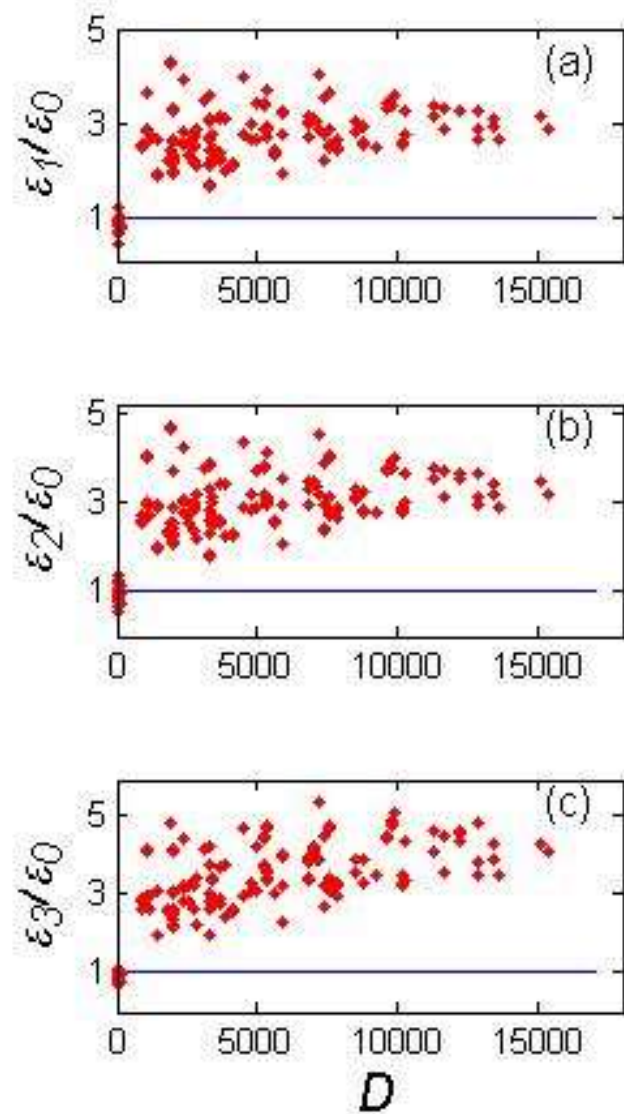
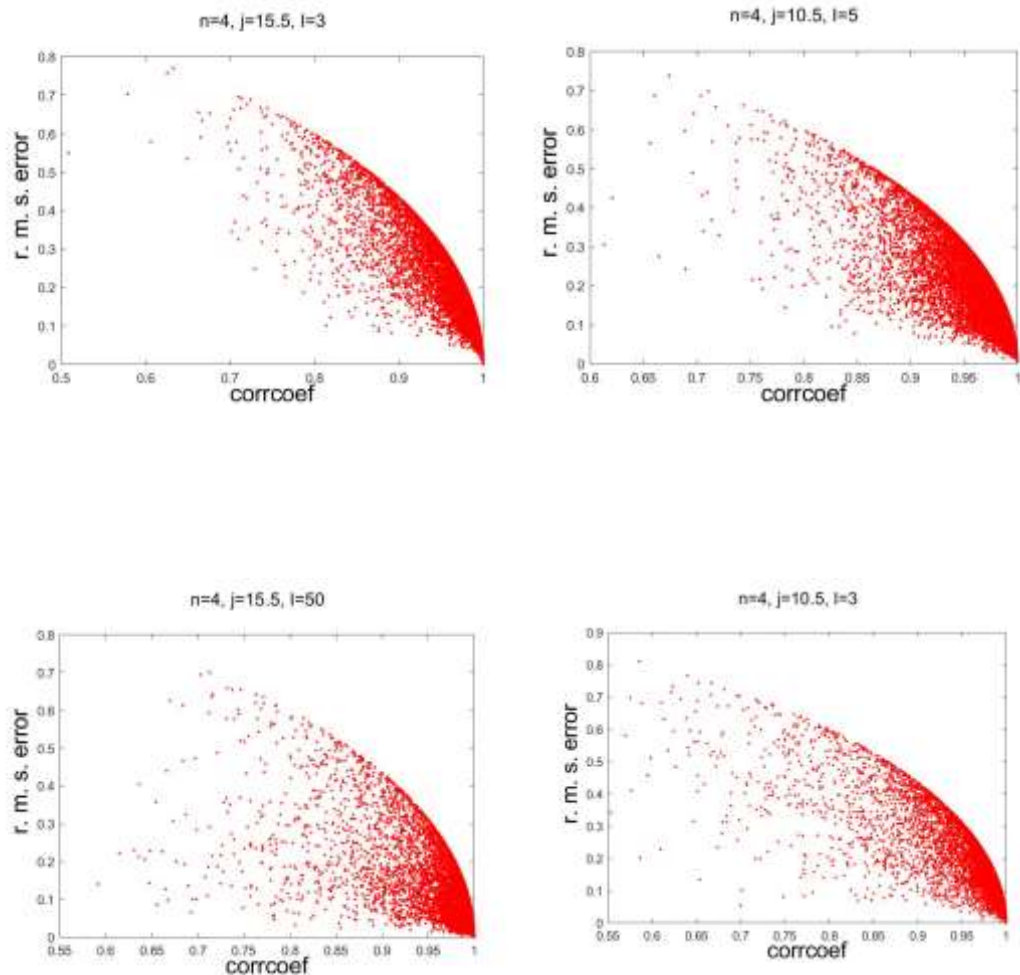


$$E_i^{Pred.} = H_{ii} \sqrt{\frac{d \sum_i \sum_j H_{ij}^2 - (\sum_i H_{ii})^2}{d \sum_i H_{ii}^2 - (\sum_i H_{ii})^2}} + \frac{1}{d} \sum_i H_{ii} \left(1 - \sqrt{\frac{d \sum_i \sum_j H_{ij}^2 - (\sum_i H_{ii})^2}{d \sum_i H_{ii}^2 - (\sum_i H_{ii})^2}} \right)$$

本征值与对角元的相关性



本征值与对角元的相关性



$$\text{r. m. s. error} \leq \sqrt{2(1 - r)}$$

高斯正交系综的本征值与对角元关系

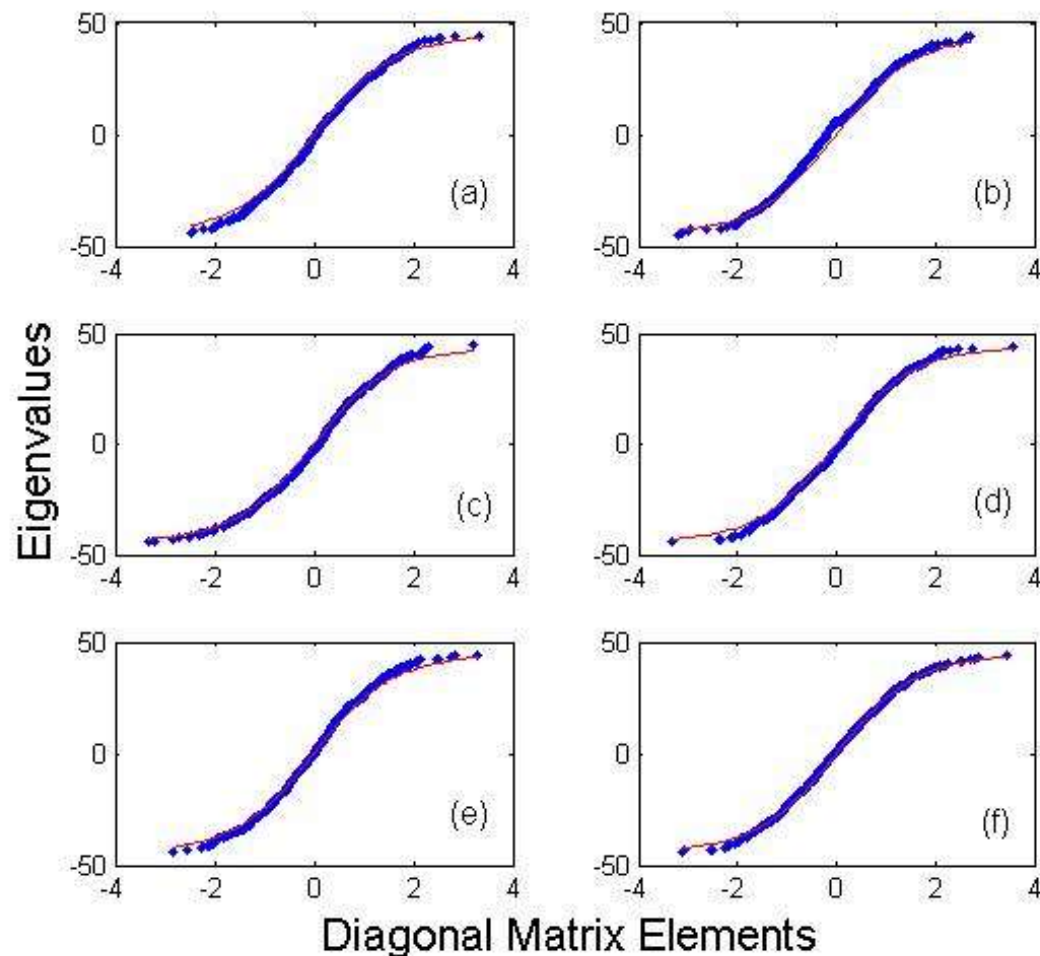
➤ 魏格纳半圆定理:

$$\rho = \frac{1}{2\pi} \sqrt{4 - x^2}, \quad x = \frac{E - \bar{E}}{\sigma}$$



➤ 本征值与对角元关系:

$$E_i = 2\sqrt{D}\sigma \tanh\left(-\frac{\sqrt{\pi}}{4\sigma} H_{ii}\right)$$



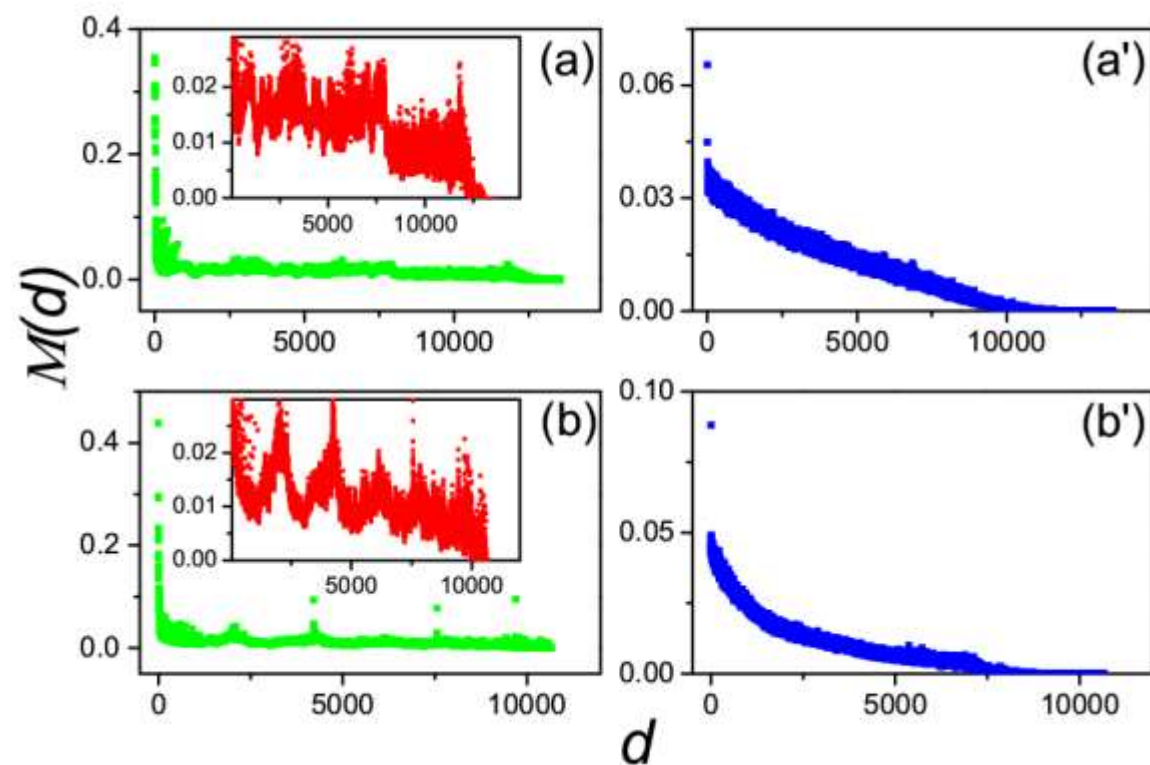
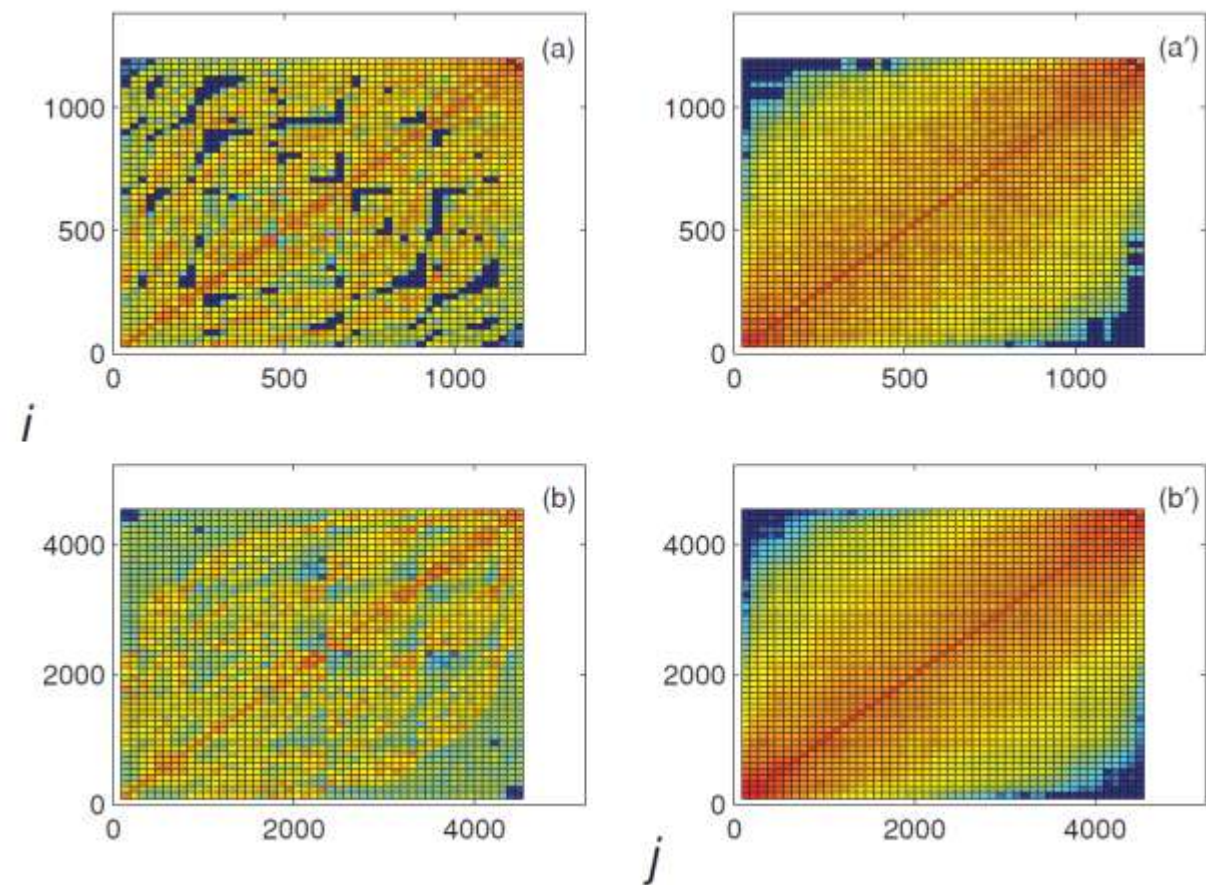
壳模型空间截断与外推近似

壳模型基矢根据哈密顿量对角元排序

$$H_{11} \leq H_{22} \leq H_{33} \leq \cdots H_{DD}$$

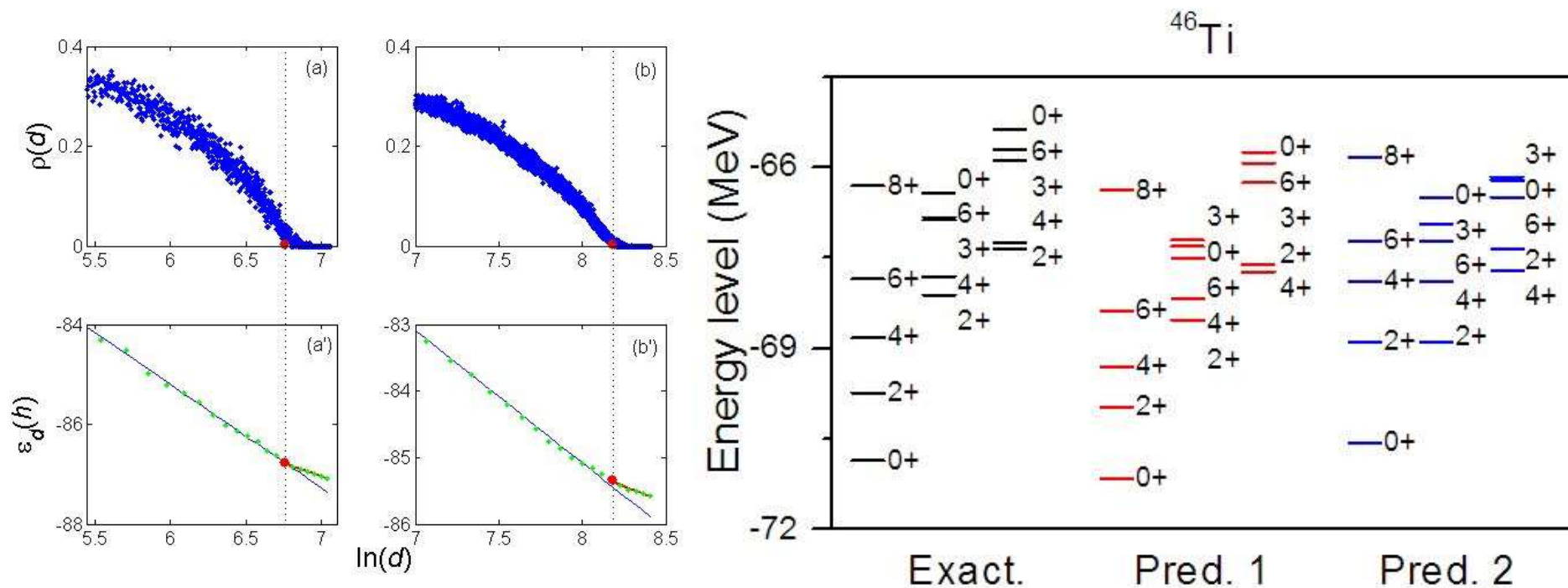
$$M(d) = \overline{|H(i, i + d)|}$$

$$d = |i - j|$$



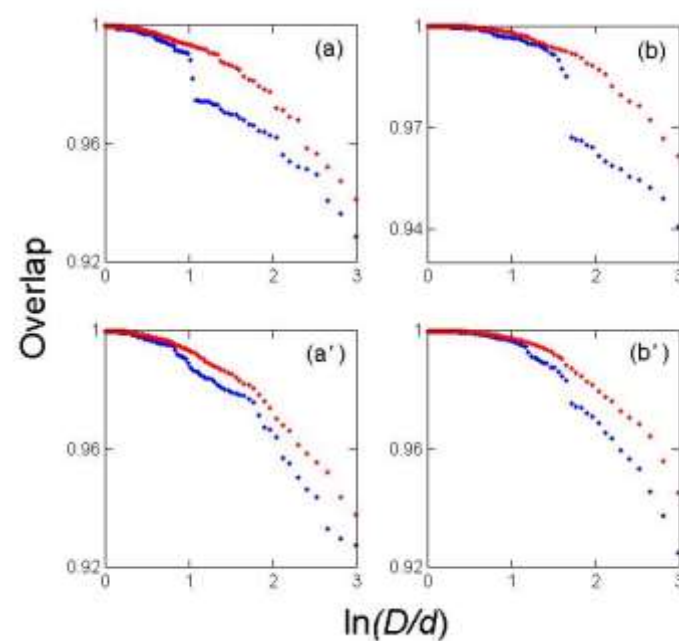
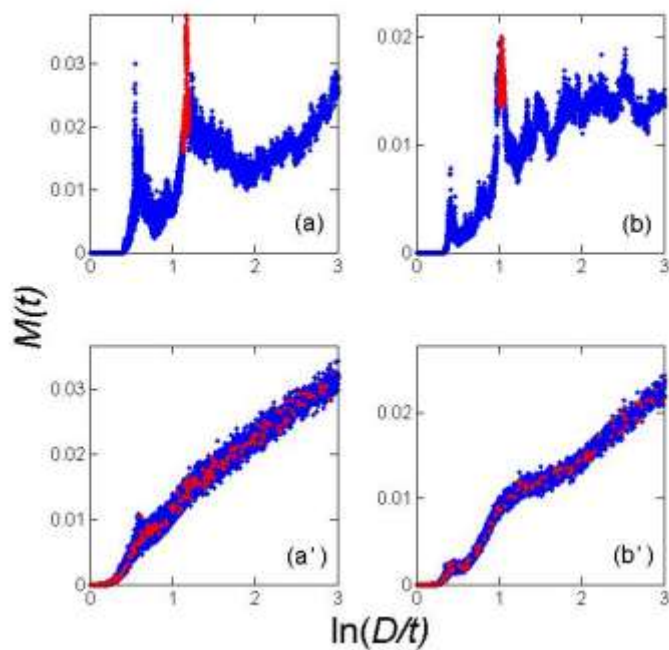
壳模型空间截断与外推近似

- 能级的线性外推



壳模型空间截断与外推近似

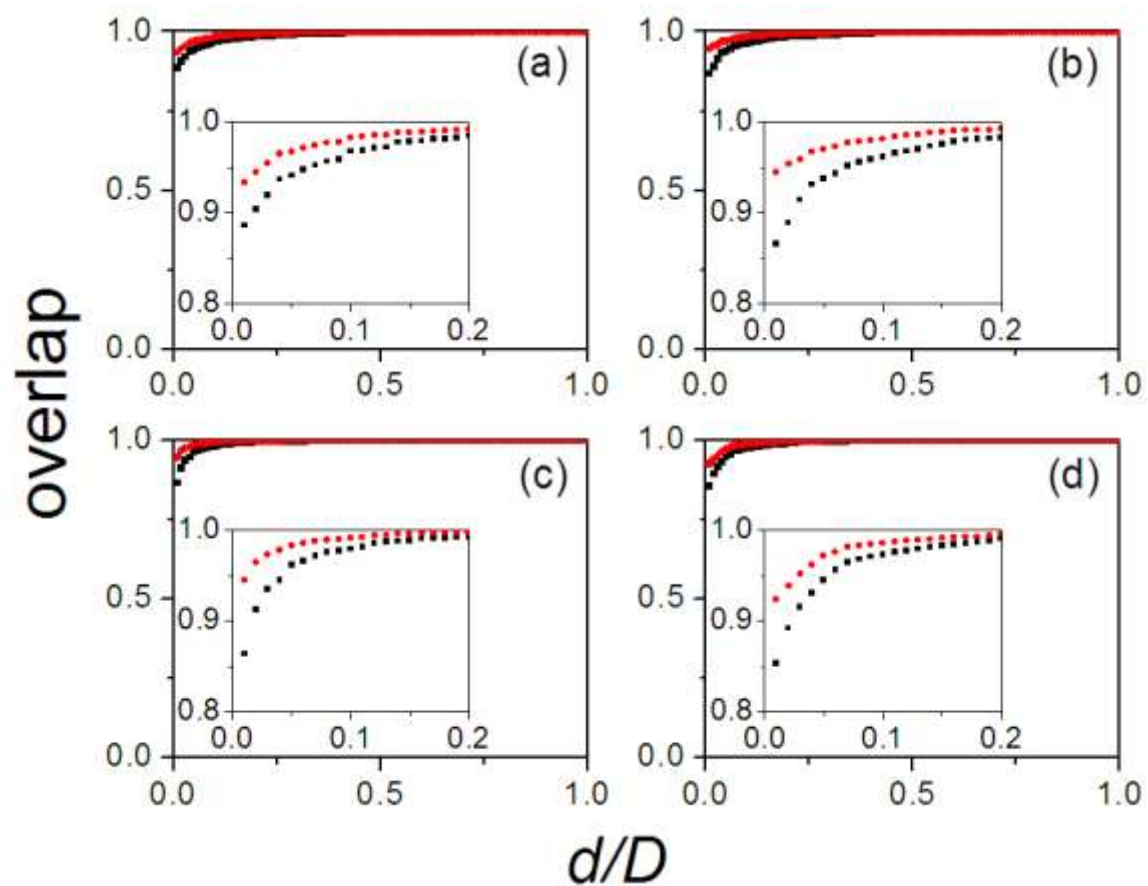
- 对角元截断与单粒子能级截断比较



- Converge faster
- More smooth

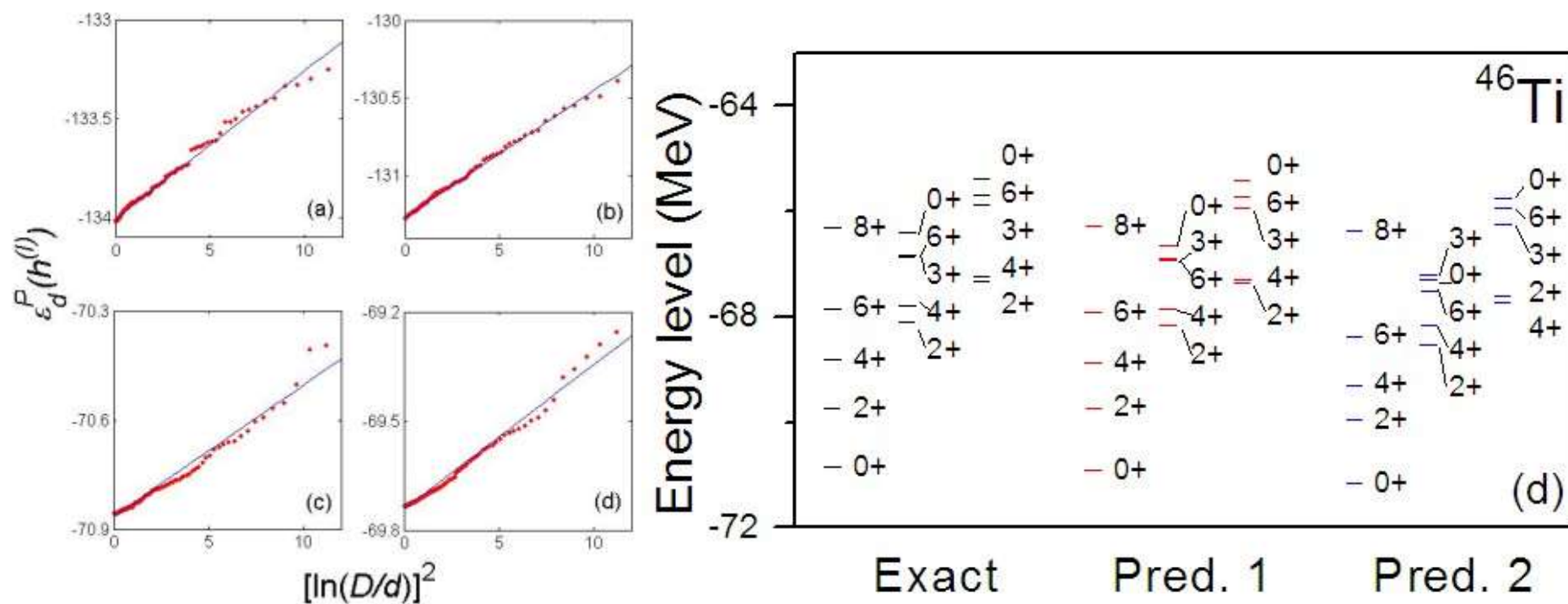
壳模型空间截断与外推近似

- 对角元截断与单粒子能级截断比较



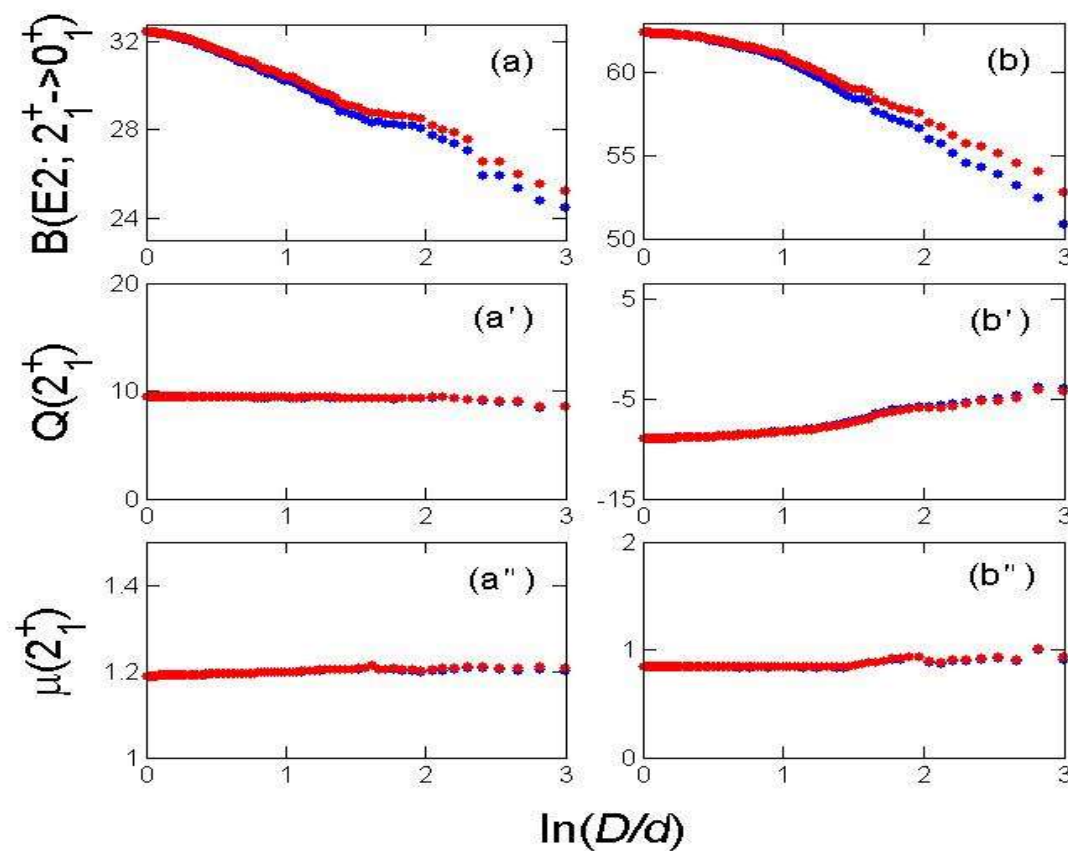
壳模型空间截断与外推近似

- Quadratic extrapolation method with perturbation



壳模型空间截断与外推近似

- extrapolation of electromagnetic properties



壳模型的微扰近似求解

有效哈密顿量

$$H^{\text{eff}} = h + \sum_{n=2}^{\infty} V^T A B^{n-2} C V$$

式中

$$\begin{aligned} A_{i,j} &= H_{i,d+j}, \\ B_{i,j} &= (1 - \delta_{ij}) \frac{H_{d+i,d+j}}{\lambda_w - H_{d+i,d+i}}, \\ C_{i,j} &= \frac{H_{d+i,j}}{\lambda_w - H_{d+i,d+i}} \end{aligned}$$

壳模型的微扰近似求解

- The approximation of the effective

$$H^{\text{eff}}(\lambda_w) = h + \sum_{k=2}^{\infty} V^T A B^{k-2} C V$$

$$\approx \mathbf{h} + \sum_{k=2}^n \sum_{i=1}^m V^T a_i b_{ii}^{k-2} c_i V$$

$$\begin{pmatrix} \mathbf{h} & a_1 & \cdots & a_2 \\ c_1 & \mathbf{b}_{11} & \cdots & b_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ c_1 & b_{m1} & \cdots & \mathbf{b}_{mm} \end{pmatrix}$$

推导过程

- 壳模型空间基矢按对角元排序后哈密顿量定义为 H

$$H_{i,j} = \langle \phi_i | H | \phi_j \rangle$$

- 截断空间哈密顿量为 h ，其中

$$h_{i,j} = H_{i,j}$$

$$i, j = 1, 2, 3, \dots, d$$

- 定义截断空间下哈密顿量对角化：

$$V^T h V = \begin{pmatrix} \varepsilon_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \varepsilon_d \end{pmatrix}$$

• 全空间形式

$$\begin{pmatrix} V^T & 0 \\ 0 & I \end{pmatrix} H \begin{pmatrix} V & 0 \\ 0 & I \end{pmatrix} = H_0 + H_1$$

式中

$$H_0 = \begin{pmatrix} \varepsilon_1 & \cdots & 0 & & & \\ \vdots & \ddots & \vdots & & & 0 \\ 0 & \cdots & \varepsilon_d & & & \\ & & & H_{d+1,d+1} & \cdots & 0 \\ & & & \vdots & \ddots & \vdots \\ & 0 & & 0 & \cdots & H_{D,D} \end{pmatrix}$$

$$H_1 = \begin{pmatrix} 0 & V^T G_1 \\ G_1 V^T & G_2 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} H_{1,d+1} & \dots & H_{1,D} \\ \vdots & \ddots & \vdots \\ H_{d,d+1} & \dots & H_{d,D} \end{pmatrix}$$

$$G_2 = \begin{pmatrix} 0 & H_{d+1,d+2} & \dots & H_{d+1,D} \\ H_{d+2,d+1} & 0 & \ddots & H_{d+2,D} \\ \vdots & \ddots & \ddots & \vdots \\ H_{D,d+1} & H_{D,d+2} & \dots & 0 \end{pmatrix}$$

- 对于对角矩阵 H_0 定义

$$H_0|\gamma\rangle = \varepsilon_\gamma|\gamma\rangle$$

$$H_0|d+i\rangle = H_{d+i,d+i}|d+i\rangle$$

式中

$$\gamma = 1, \dots, d$$

$$i = 1, \dots, D-d$$

定义全空间哈密顿量本征值（已排序）为 λ_ω ，本征函数为 $|\Psi_\omega\rangle$

$$H_0|\Psi_\omega\rangle + H_1|\Psi_\omega\rangle = \lambda_\omega|\Psi_\omega\rangle$$

左乘 H_0 的本征函数

$$(\varepsilon_\gamma - \lambda_\omega)\langle\gamma|H_1|\Psi_\omega\rangle + \langle\gamma|H_1|\Psi_\omega\rangle = 0$$

$$(H_{d+i,d+i} - \lambda_\omega)\langle d+i|H_1|\Psi_\omega\rangle + \langle d+i|H_1|\Psi_\omega\rangle = 0$$

• 代入 $H_1 = \begin{pmatrix} 0 & V^T G_1 \\ G_1 V^T & G_2 \end{pmatrix}$ 可得,

$$(\varepsilon_\gamma - \lambda_\omega) \langle \gamma | \Psi_\omega \rangle + \sum_i \tilde{H}_{\gamma, d+i} \langle d + i | \Psi_\omega \rangle = 0$$

$$\langle d + i | \Psi_\omega \rangle = \frac{1}{\lambda_\omega - H_{d+i, d+i}} \left[\sum_i \tilde{H}_{d+i, \gamma'} \langle \gamma' | \Psi_\omega \rangle + \sum_{i' \neq i} H_{d+i, d+i'} \langle d + i' | \Psi_\omega \rangle \right]$$

式中

$$\tilde{H}_{i, d+j} \equiv \sum_{l=1}^d V_{l, i} H_{l, d+j}$$

- 上面两式合并以后可得

$$H^{\text{eff}}(\lambda_w)|\psi_\omega\rangle = \lambda_\omega|\psi_\omega\rangle$$

式中 $|\psi_\omega\rangle$ 为 $|\Psi_\omega\rangle$ 的前 d 个分量

$$H^{\text{eff}}(\lambda_w) = h + \sum_{n=2}^{\infty} V^T A B^{k-2} C V$$

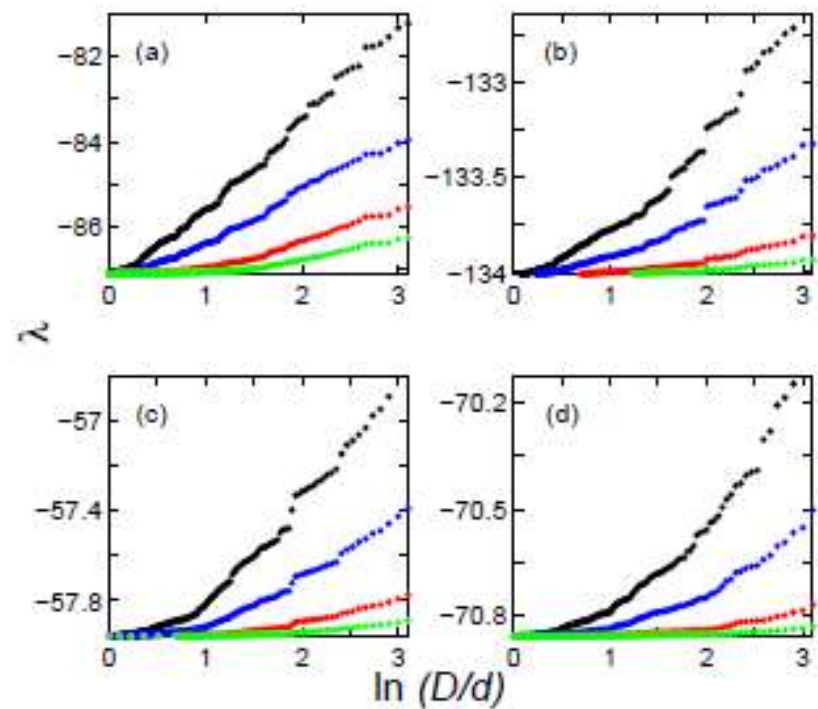
$$|\Psi_\omega\rangle = \begin{pmatrix} I_d \\ \sum_{k=1}^{\infty} (B^{k-1}) C \end{pmatrix} V |\psi_\omega\rangle$$

其中

$$A_{i,j} = H_{i,d+j}, \quad B_{i,j} = (1 - \delta_{ij}) \frac{H_{d+i,d+j}}{\lambda_w - H_{d+i,d+i}}, \quad C_{i,j} = \frac{H_{d+i,j}}{\lambda_w - H_{d+i,d+i}}$$

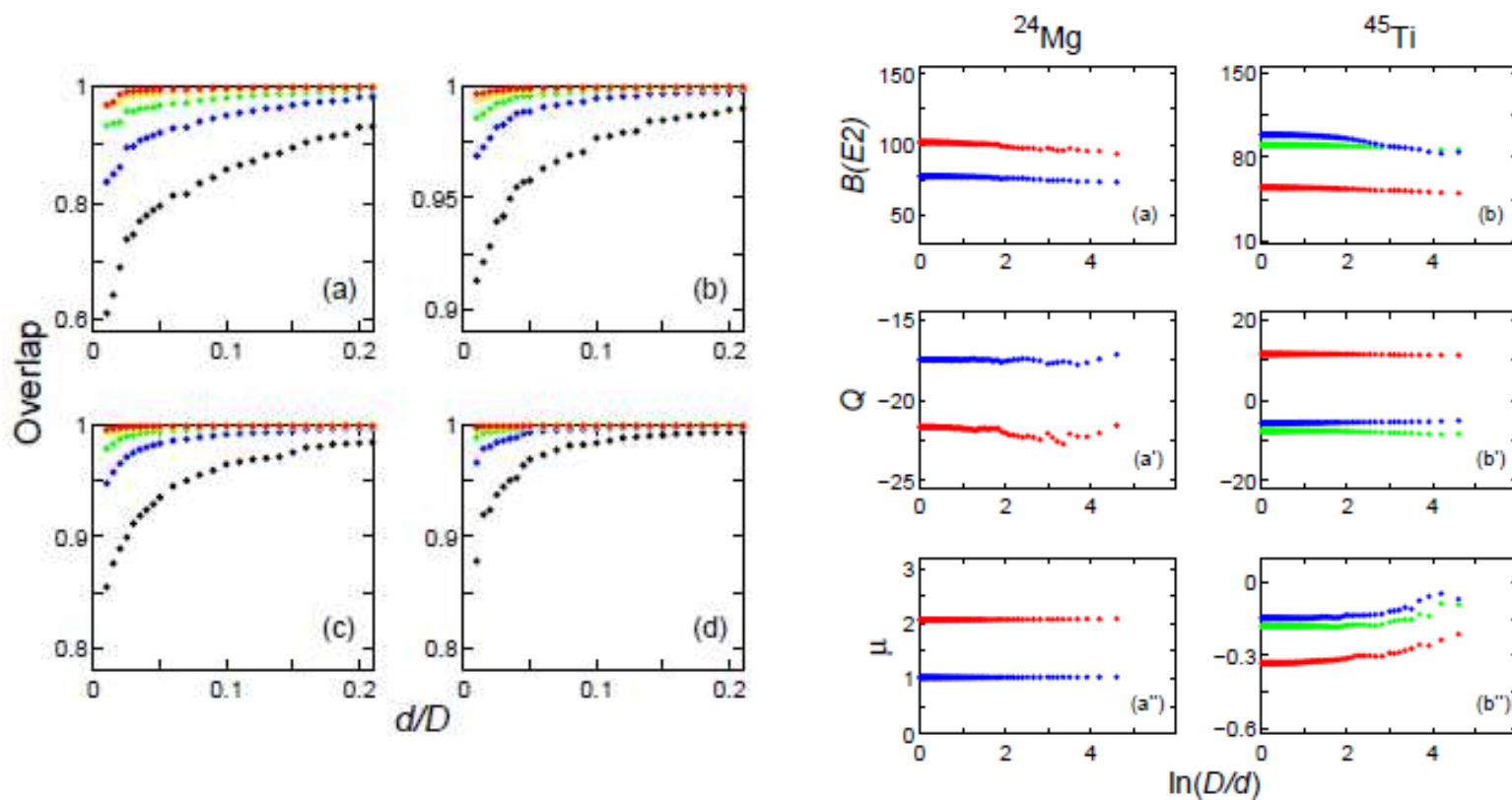
壳模型的微扰近似求解

- The extrapolation based on the effective Hamiltonian:
energy levels

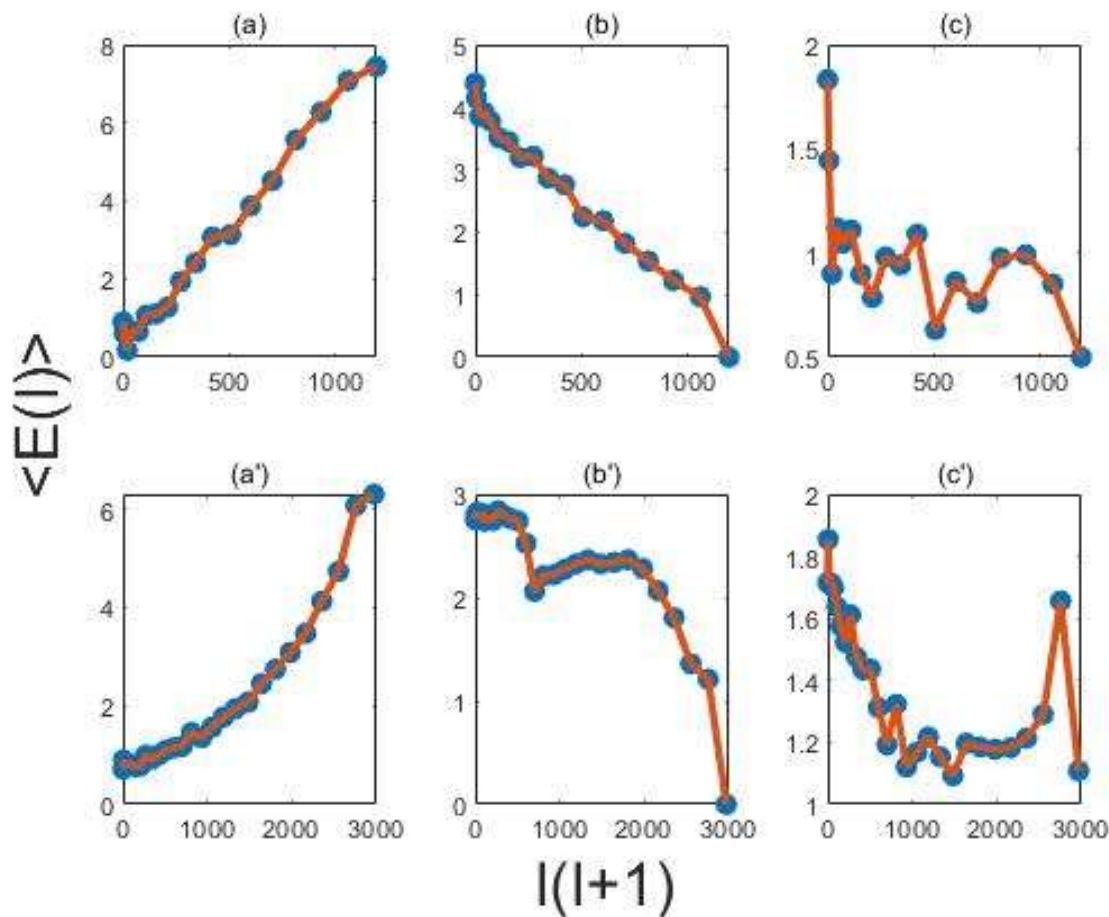


壳模型的微扰近似求解

- ④ The extrapolation based on the effective Hamiltonian: wave function and electromagnetic properties



不规则能谱的随机两体系统综（单 j 核）



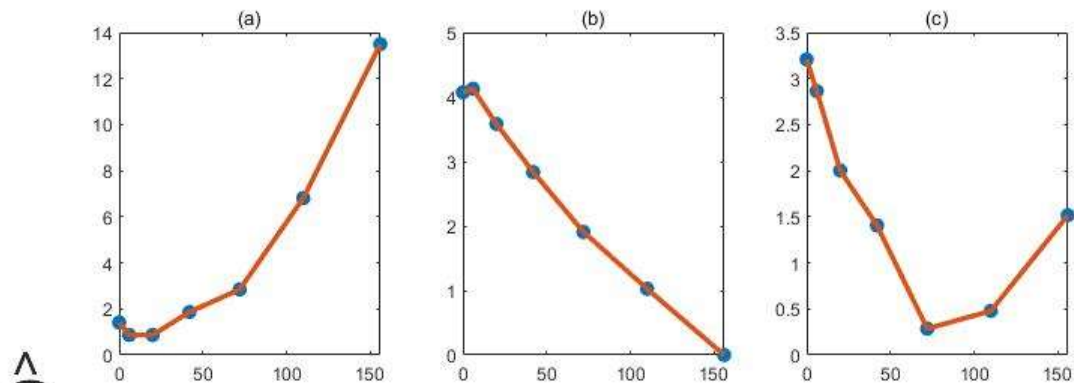
$$j = \frac{17}{2}, n = 6$$

$$\begin{cases} (a) & E(I=0) < E(I_{max}) \\ (b) & E(I=0) > E(I_{max}) \text{ 且 } E(I_{max}) = 0 \\ (c) & E(I=0) > E(I_{max}) \text{ 且 } E(I_{max}) \neq 0 \end{cases}$$

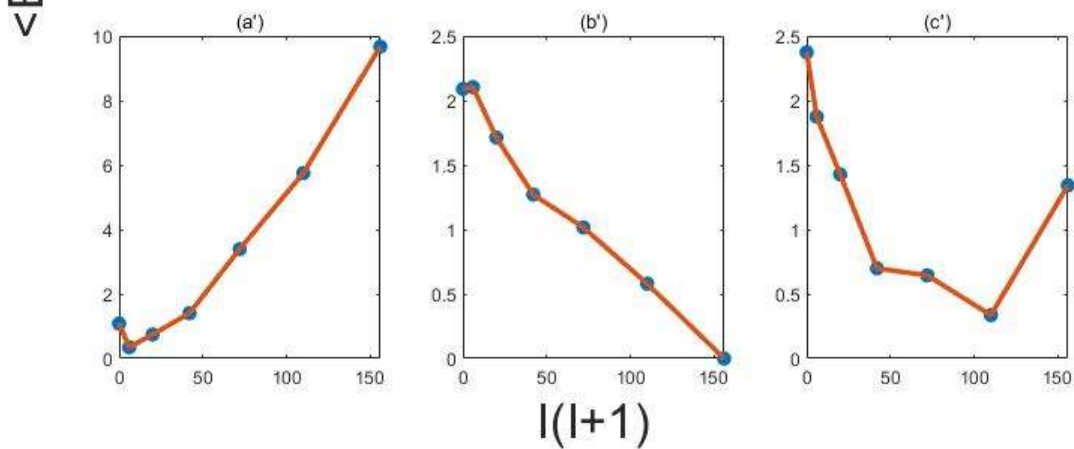
$$j = \frac{31}{2}, n = 4$$

不规则能谱的随机两体系综（偶偶核）

sd-shell



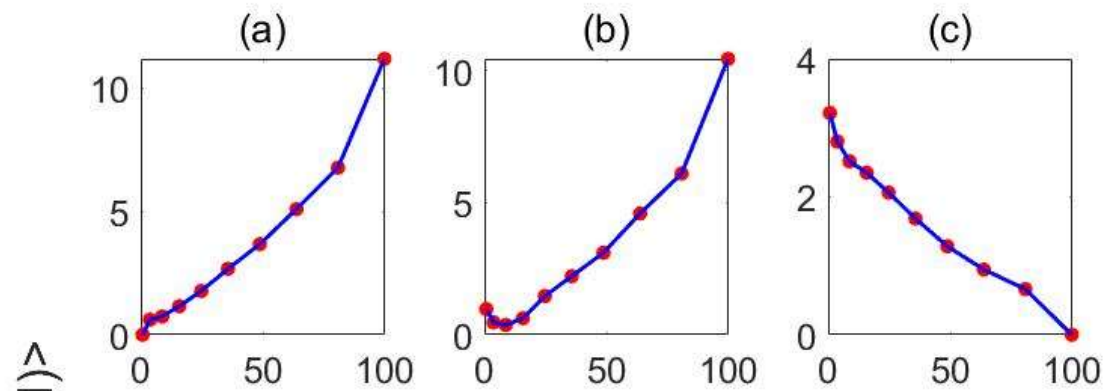
pf-shell



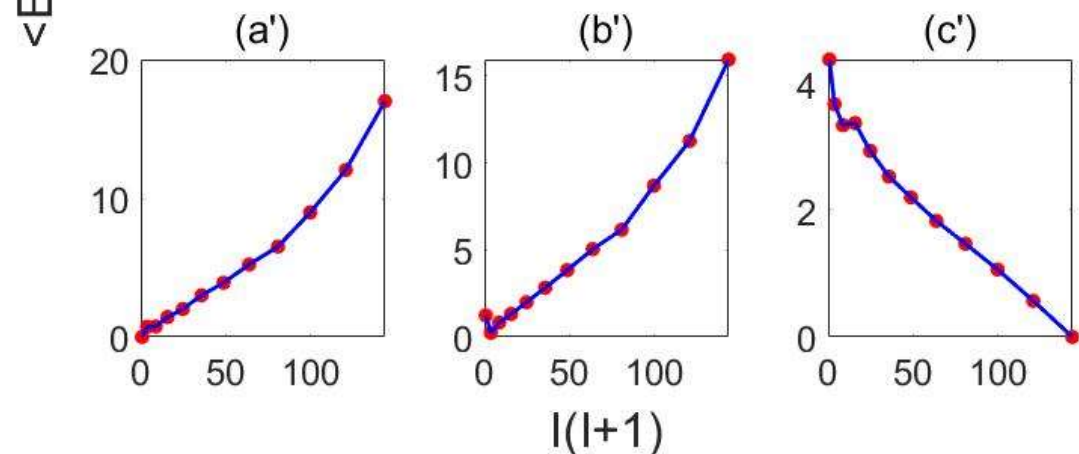
$$\left\{ \begin{array}{ll} (a) & E(I=0) < E(I_{max}) \\ (b) & E(I=0) > E(I_{max}) \text{ 且 } E(I_{max}) = 0 \\ (c) & E(I=0) > E(I_{max}) \text{ 且 } E(I_{max}) \neq 0 \end{array} \right.$$

不规则能谱的随机两体系统综（奇A核）

sd-shell



pf-shell



$$\begin{cases} (a) & E\left(I = \frac{1}{2}\right) = 0 \\ (b) & E\left(I = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\right) = 0 \\ (c) & E(I_{max}) = 0 \end{cases}$$

转动与振动峰

$$R_I = \frac{E_{I1} - E_{01}}{E_{21} - E_{01}}$$

$$R_4 = \frac{E_{41} - E_{01}}{E_{21} - E_{01}}$$

$$R_6 = \frac{E_{61} - E_{01}}{E_{21} - E_{01}}$$

$$R_8 = \frac{E_{81} - E_{01}}{E_{21} - E_{01}}$$

vibrational mode:

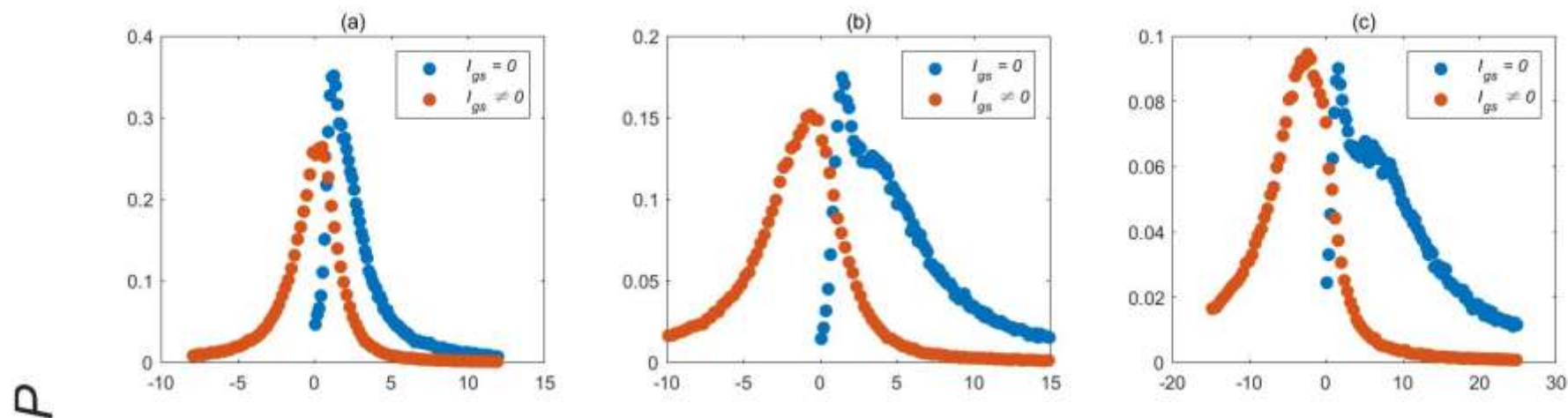
$$E_I \sim I, R_4 \sim 2$$

rotational mode:

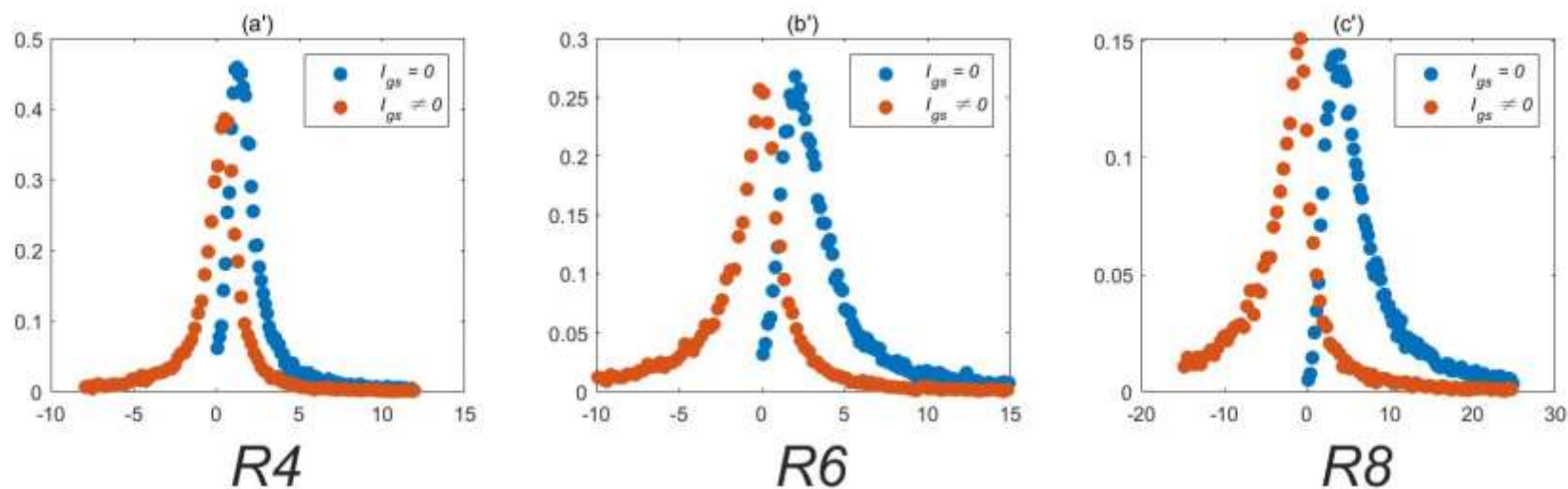
$$E_I \sim I(I + 1), R_4 \sim 3.33$$

转动与振动峰

sd-shell



pf-shell



转动与振动峰

$$R'_I = \frac{E_{I1} - E_{01}}{E_{41} - E_{01}}$$

$$R'_6 = \frac{E_{61} - E_{01}}{E_{41} - E_{01}}$$

$$R'_8 = \frac{E_{81} - E_{01}}{E_{41} - E_{01}}$$

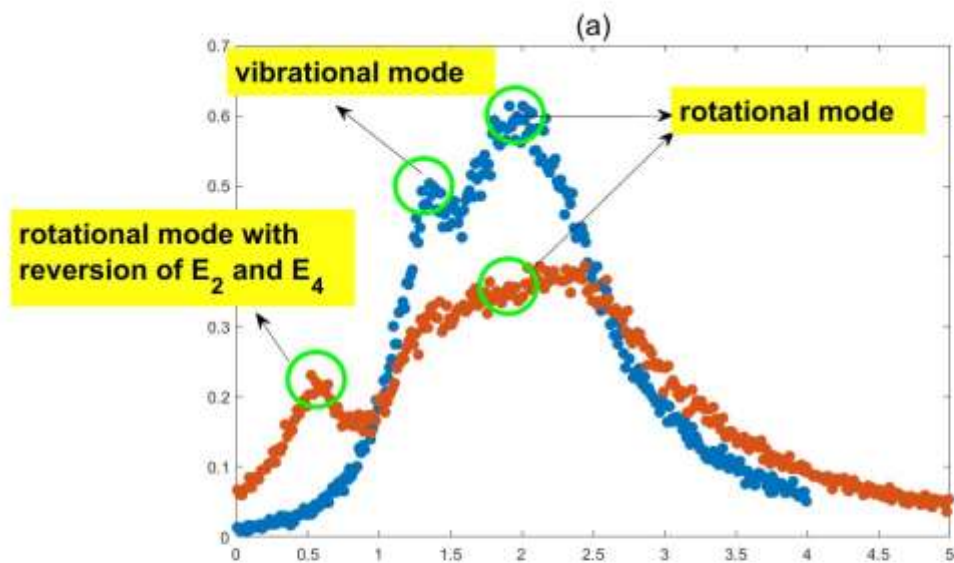
vibrational mode:

$$E_I \sim I, R'_6 \sim 1.5, R'_8 \sim 2$$

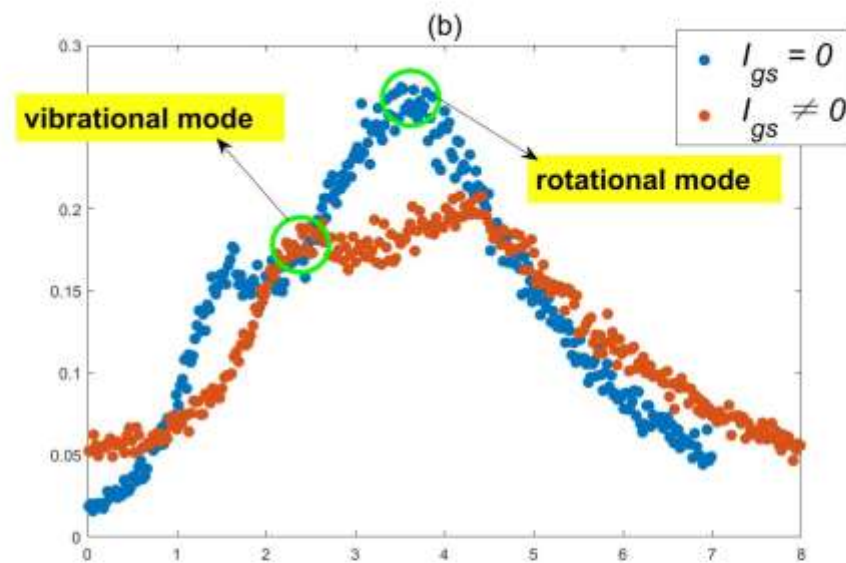
rotational mode:

$$E_I \sim I(I + 1), R'_6 \sim 2.1, R'_8 \sim 3.6$$

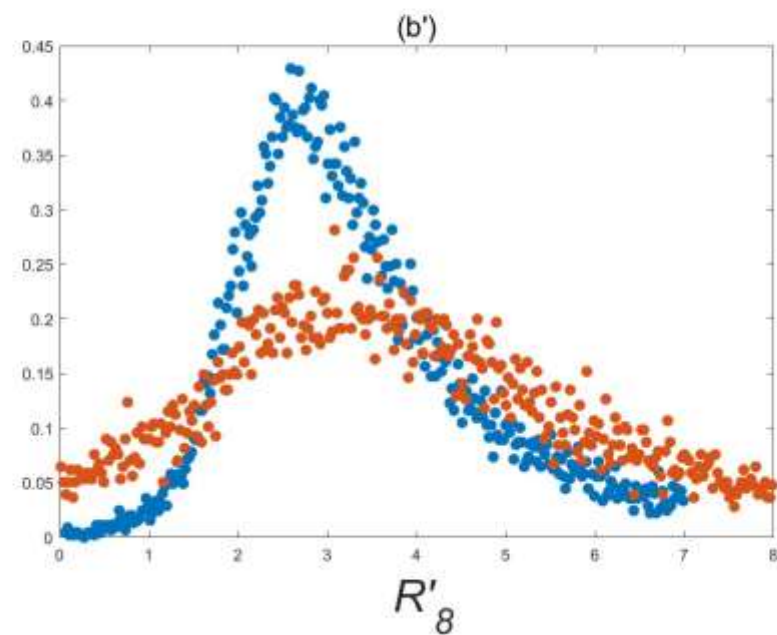
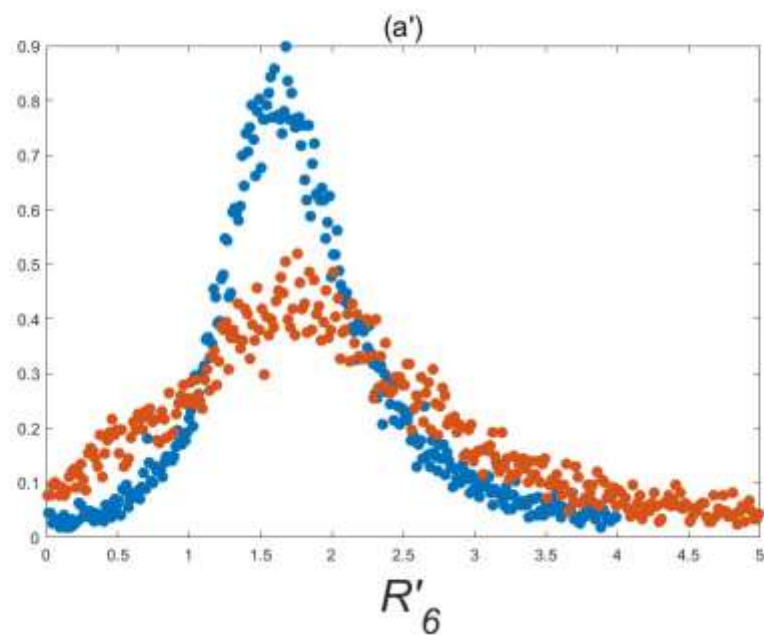
sd-shell



P



pf-shell



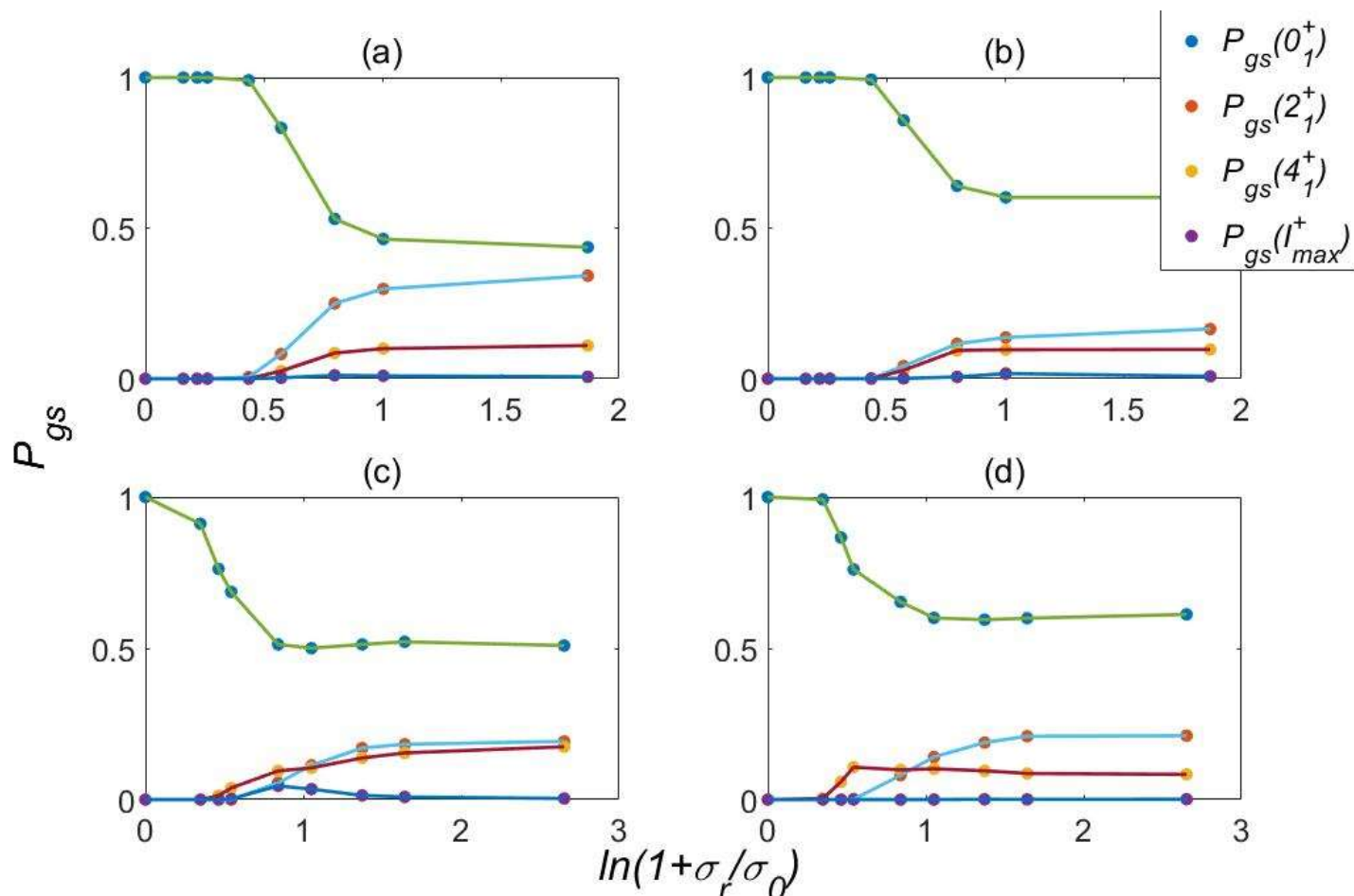
随机相互作用

$$G_{JT}(j_1 j_2; j_3 j_4) = G_{JT}^{real}(j_1 j_2; j_3 j_4) + G_{JT}^{TBRE}(j_1 j_2; j_3 j_4)$$

$$\sigma^2 = \frac{1}{2} \sigma_r^2 (1 + \delta_{j_1 j_2; j_3 j_4})$$

真实相互作用的随机扰动----基态0自旋几率

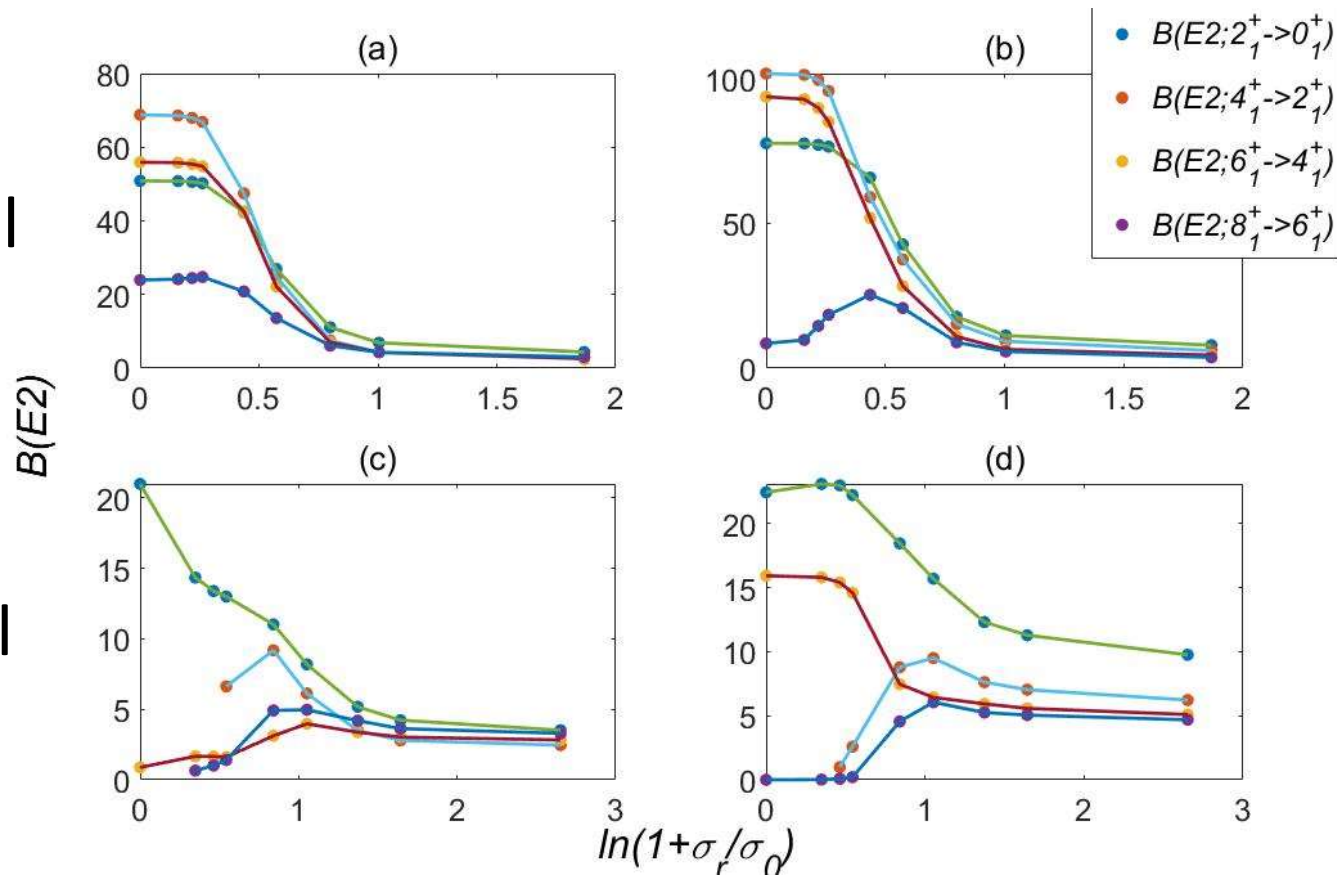
sd-shell



pf-shell

真实相互作用的随机扰动---BE2

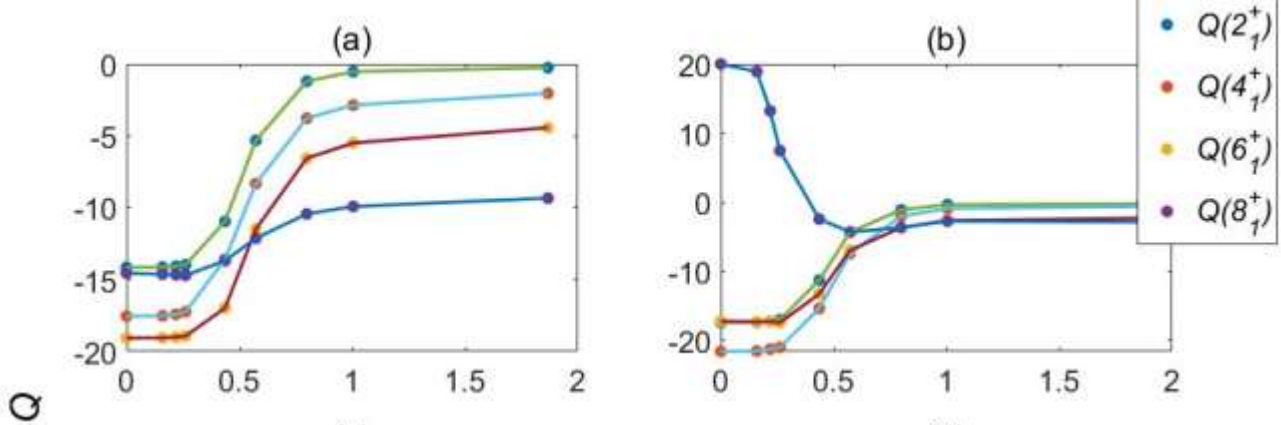
sd-shell



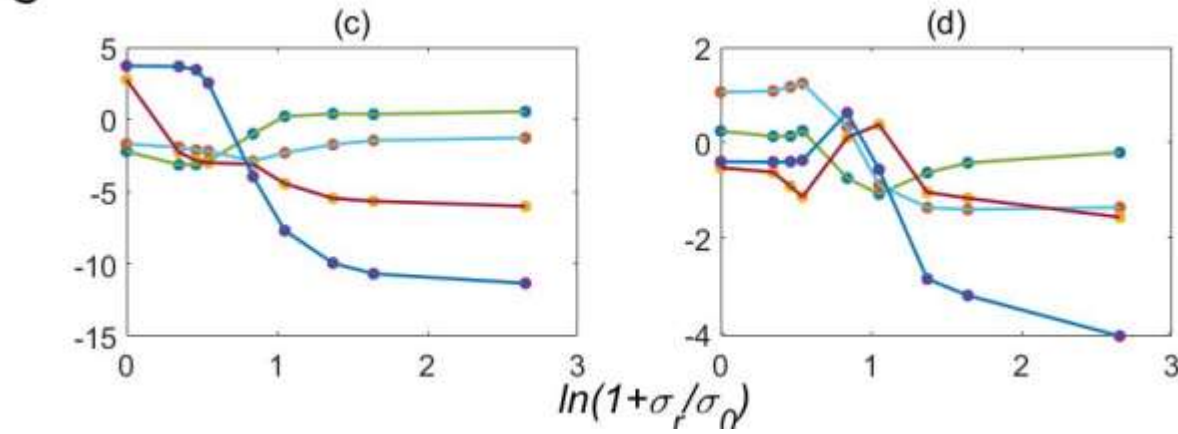
pf-shell

真实相互作用的随机扰动-----电四极矩

sd-shell



pf-shell



总结

- 我们研究并获得了随机两体系综的本征值经验公式，包括最小本征值用多阶矩展开以及本征值与对角矩阵元的相关性。
- 我们研究了并获得一种新的组态截断方法以及相应的本征值和波函数外推及微扰方法，通过此方法本征值和波函数会快速并光滑地收敛，用外推方法可以比较精确的近似求解哈密顿量。
- 我们研究了原子核在随机相互作用下基态非0自旋下的非转动集体行为，发现了yrast带能谱的能级期望值与 $I(I + 1)$ 的正相关性，与基态0自旋的结果一致
- 我们研究了原子核在真实相互作用下增加随机相互作用的扰动，通过研究基态0自旋几率和电磁跃迁的相变点，寻找真实相互作用和随机相互作用的临界点

谢谢！