随机相互作用的原子核结构

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我做过什么？

- 配对理论[赵、有马，Physics Reports, Vol. 545, 1 (2014)]

  (跟陈金全老师、有马老师学习技术，后来和学生们一起发展)

  1993.10-1994.2; 1994.12-1995.7; 1998.1-2000.3; 2006.2 - ??

- 随机作用的核结构[赵、有马、吉永，Physics Reports, Vol. 400, 1 (2004)]

  （跟有马朗人老师做博士后，从很低的起点摸索，后来是学生在做）

  2000.3-2003.10；2009 ?? - ??

- 壳模型理论基础性问题 // 空间维数、耦合系数求和规则

  （读书的笔记、个人体会，Lawson 的壳模型书）：我占便宜//就怕动手！

  2002.2 -2005.12；2008；2012；2016；

- 原子核质量描述和预言 // 质量相关量、电荷半径、激发能等

  （个人爱好、穷人的科学）：做事情就怕坚持不懈！

  1990.9-1995.8；1996-1997；1999-2001；2009.6 – 至今

I. 引言

II. 几个问题

- Origin of spin 0 g.s. dominance of even-even nuclei

  Many efforts (examples)： Zhao-Arima, Bijker-Frank, Otsuka et al.,

  Zuker et al., Johnson et al., … ...

- positive parity dominance for even-even nuclei
- average energy
- collective motion of sd bosons (YMZ, Frontiers of Physics, 2018)

IV. Summary

I. Introduction:

Why random interactions ?

The Gaussian orthogonal ensemble of random matrices was first proposed by Wigner, which was a revolutionary thought in understanding the spacings of levels observed in resonances in slow-neutron scattering on heavy nuclei.

The two-body random ensemble (TBRE) was introduced to study statistical properties of spectra of many-body systems, by French and Wong, and by Bohigas and Flores.

I. Introduction

Why the regular structure of atomic nuclei under random interactions is interesting and important?

Themes and Challenges of Modern Science

Taken from [Brad Sherrill and Rick F. Casten, Frontiers of Nuclear structure: Exotic nuclei. Nuclear Physics News, Vol. 15, No.2, Pp. 13 (2005).]

Challenges of modern science reflecting the twin themes of complexity and simplicity in many-body systems

- **Complex systems arising out of basic constituents**

  How the world, with all its apparent complexity and diversity, can be constructed out of a few elementary building blocks and their interactions

- **Simplicity out of complexity [how to understand this ?]**

  How the world of complex systems can display such remarkable regularity and, often, simplicity ***[Franco, Igal and Rick stressed this very loudly in this symposium]***

- **Understanding the nature of the physical universe**

- **Manipulating matter for the benefit of mankind**

In many-body systems such as molecules and atomic nuclei, the interactions by themselves have no trace of symmetry groups for vibrational or rotational modes. However, the low-lying states often exhibit a pattern suggestive of symmetries for these modes. One may ask to what extent the low-lying states acquire order from the basic properties of interactions such as rotational invariance and possibly other symmetries such as isospin invariance. In other words, some properties such as vibration or rotation might dominantly occur in the low-lying states of many-body systems while the others might occur only with small probabilities.

Spin-zero & parity-plus ground states for even-even nuclei, odd-even staggering of binding energies, collective motion, …

One can ask whether these features are robust for general many-body systems. This can be studied by permitting interactions to be more and more arbitrary.

I focus on the features (particularly, orders and correlations) of low-lying levels of many-body systems in the presence of random two-body interactions, explaining both the observations and the present status toward “understanding” these features.

The shell model Hamiltonian that is usually taken includes a one-body term

$$H_1 = \sum_{jmm_t} e_{jm_t} a^{\dagger}_{jm,m_t} a_{jm,m_t} \ ,$$

and a two-body term

$$H_2 = \frac{1}{4} \sum_{j_1 j_2 j_3 j_4, JT} \sqrt{(1+\delta_{j_1 j_2})(1+\delta_{j_3 j_4})} G_{JT}(j_1 j_2, j_3 j_4) \times \sum_{M_J M_T} A^{\dagger}(j_1 j_2)^{(JT)}_{M_J M_T} A(j_3 j_4)^{(JT)}_{M_J M_T} \ ,$$

where

$$A^{\dagger}(j_1 j_2)^{(JT)}_{M_J M_T} = \sum_{m_1 m_2 m_{t_1} m_{t_2}} (j_1 m_1, j_2 m_2 | J M_J) \left(\frac{1}{2} m_{t_1}, \frac{1}{2} m_{t_2} | T M_T \right) a^{\dagger}_{j_1 m_1, m_{t_1}} a^{\dagger}_{j_2 m_2, m_{t_2}} \ ,$$

$$A(j_3 j_4)^{(JT)}_{M_J M_T} = (A^{\dagger}(j_3 j_4)^{(JT)}_{M_J M_T})^{\dagger} \ .$$

II. Two examples

(open questions)

1、 A famous puzzle

spin zero ground state dominance

(still an open problem)

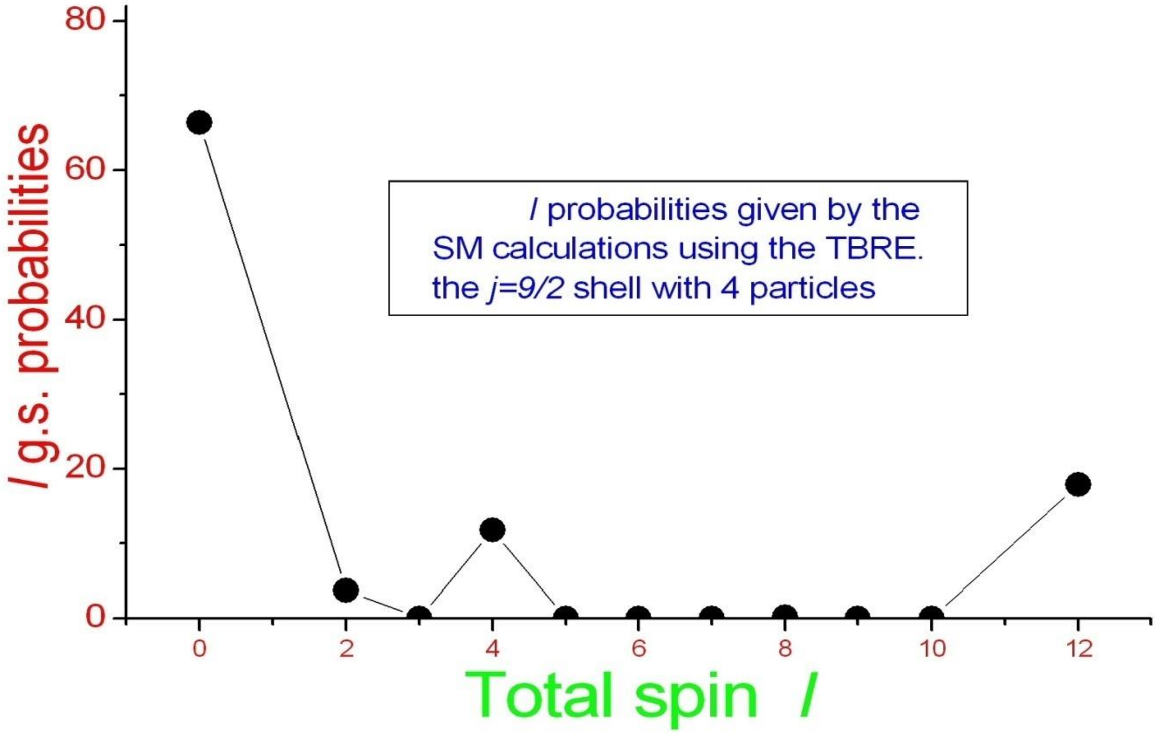
1. What does 0 g.s. dominance mean ?

In 1998, Johnson, Bertsch, and Dean discovered that spin zero ground state dominance can be obtained by using random two-body interactions (Phys. Rev. Lett. 80, 2749).

This result is called the 0 g.s. dominance.

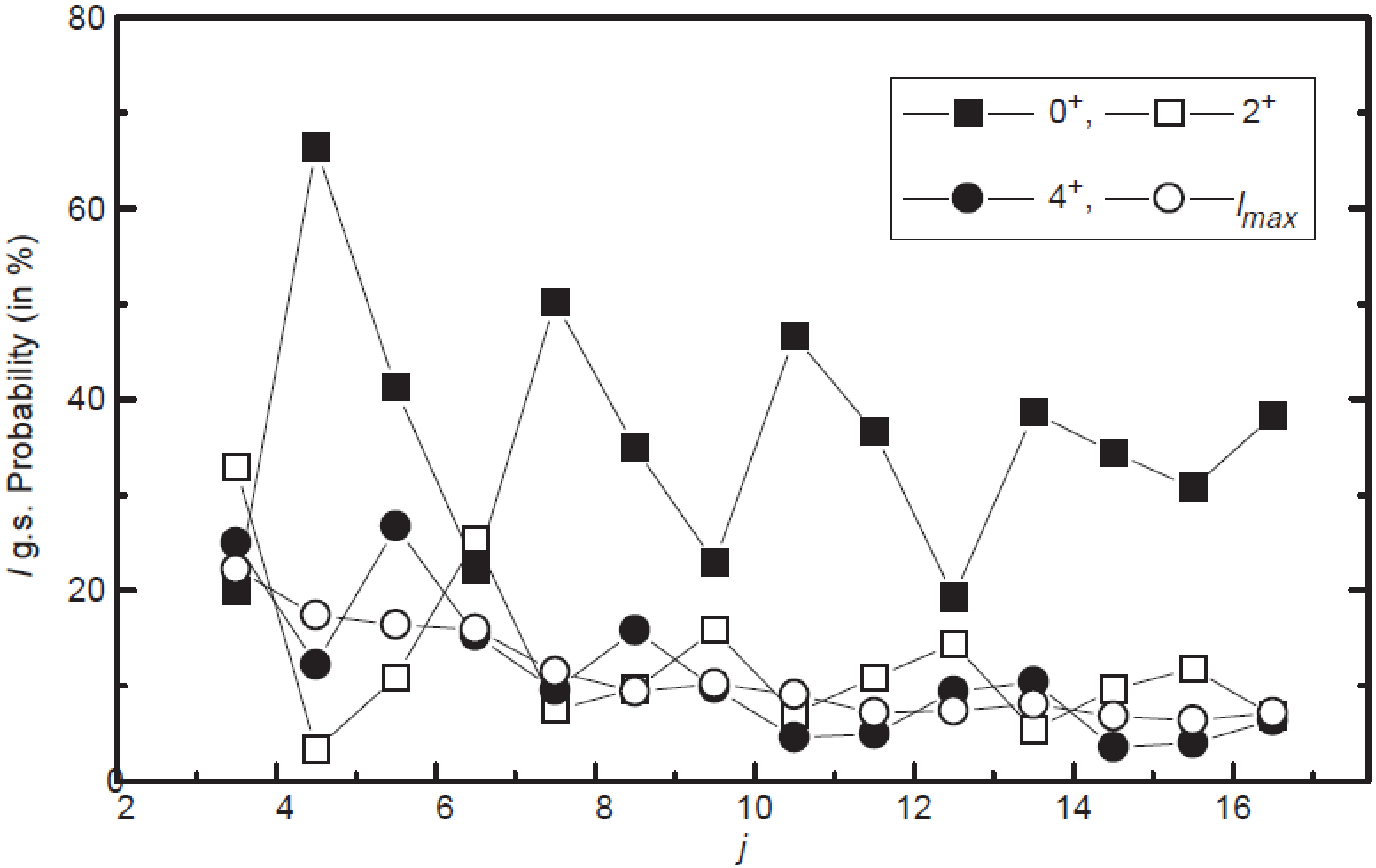
Similar phenomenon was found in other systems, say, sd-boson systems.

$$j = \frac{9}{2}, \quad n = 4$$

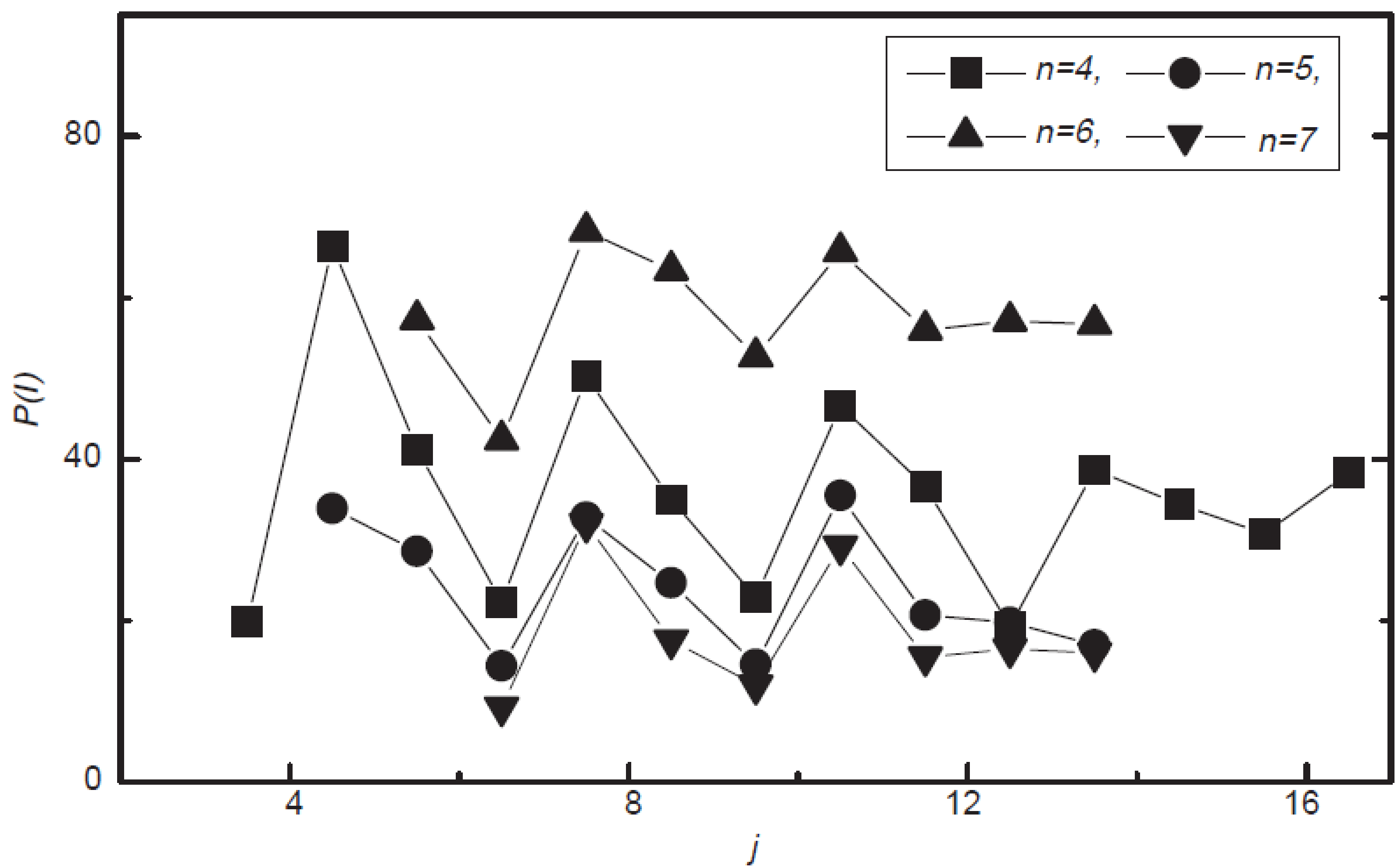


Spin	Dimension
0	2
2	2
3	1
4	3
5	1
6	3
7	1
8	2
9	1
10	1
12	1

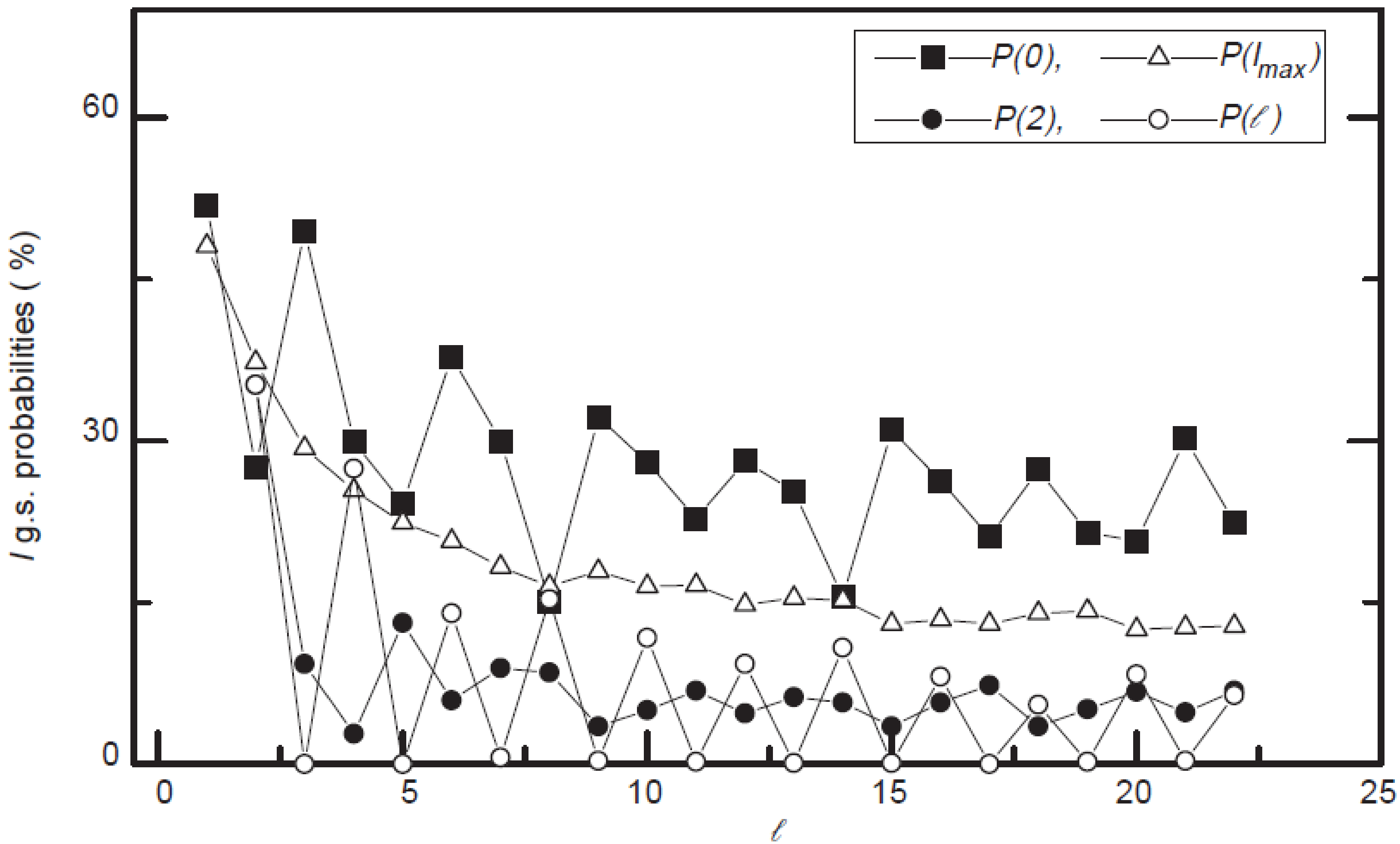
Why is this result interesting?



单轨道情形



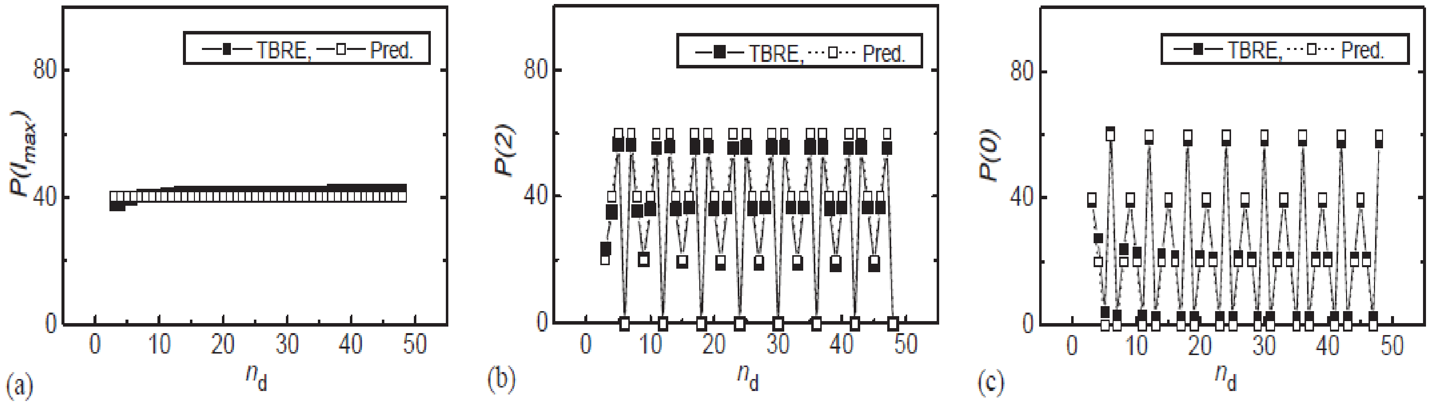
$P(0)$'s of $n = 4, 6$ and $P(j)$'s of $n = 5, 7$ fermions in a single-j shell.



I g.s. probabilities vs. *l* of four bosons with spin *l*.

$$
\begin{aligned}
E_{0(0)} &= \tfrac{\mathbf{3}}{\mathbf{2}} G_0 + \tfrac{5}{6} G_2 + \tfrac{3}{2} G_4 + \tfrac{13}{6} G_6 \,, \\
E_{2(2)} &= \tfrac{1}{2} G_0 + \tfrac{11}{6} G_2 + \tfrac{3}{2} G_4 + \tfrac{13}{6} G_6 \,, \\
E_{2(4)} &= G_2 + \tfrac{\mathbf{42}}{\mathbf{11}} G_4 + \tfrac{\mathit{13}}{\mathit{11}} G_6 \,, \\
E_{4(2)} &= \tfrac{1}{2} G_0 + \tfrac{5}{6} G_2 + \tfrac{5}{2} G_4 + \tfrac{13}{6} G_6 \,, \\
E_{4(4)} &= \tfrac{\mathbf{7}}{\mathbf{3}} G_2 + \mathit{1} G_4 + \tfrac{8}{3} G_6 \,, \\
E_{5(4)} &= \tfrac{8}{7} G_2 + \tfrac{192}{77} G_4 + \tfrac{26}{11} G_6 \,, \\
E_{6(2)} &= \tfrac{1}{2} G_0 + \tfrac{5}{6} G_2 + \tfrac{3}{2} G_4 + \tfrac{19}{6} G_6 \,, \\
E_{8(4)} &= \tfrac{\mathit{10}}{\mathit{21}} G_2 + \tfrac{129}{77} G_4 + \tfrac{\mathbf{127}}{\mathbf{33}} G_6 \,.
\end{aligned}
$$

$$
\mathscr{F}(k) = F(k) - F(m) = (\alpha_k^{J'} - \alpha_m^{J'}) G_{J'} + \left(\sum_{J \neq J'} (\alpha_k^J - \alpha_m^J) G_J \right)
$$



n	c_0(min)	c_2(min)	c_2(max)	c_4(min)	c_4(max)
6κ	0	0	$I_{\max}$	$I_{\max}$	0
$6\kappa+1$	2	2	$I_{\max}$	$I_{\max}$	2
$6\kappa+2$	0	2	$I_{\max}$	$I_{\max}$	2
$6\kappa+3$	2	0	$I_{\max}$	$I_{\max}$	0
$6\kappa+4$	0	2	$I_{\max}$	$I_{\max}$	2
$6\kappa+5$	2	2	$I_{\max}$	$I_{\max}$	2

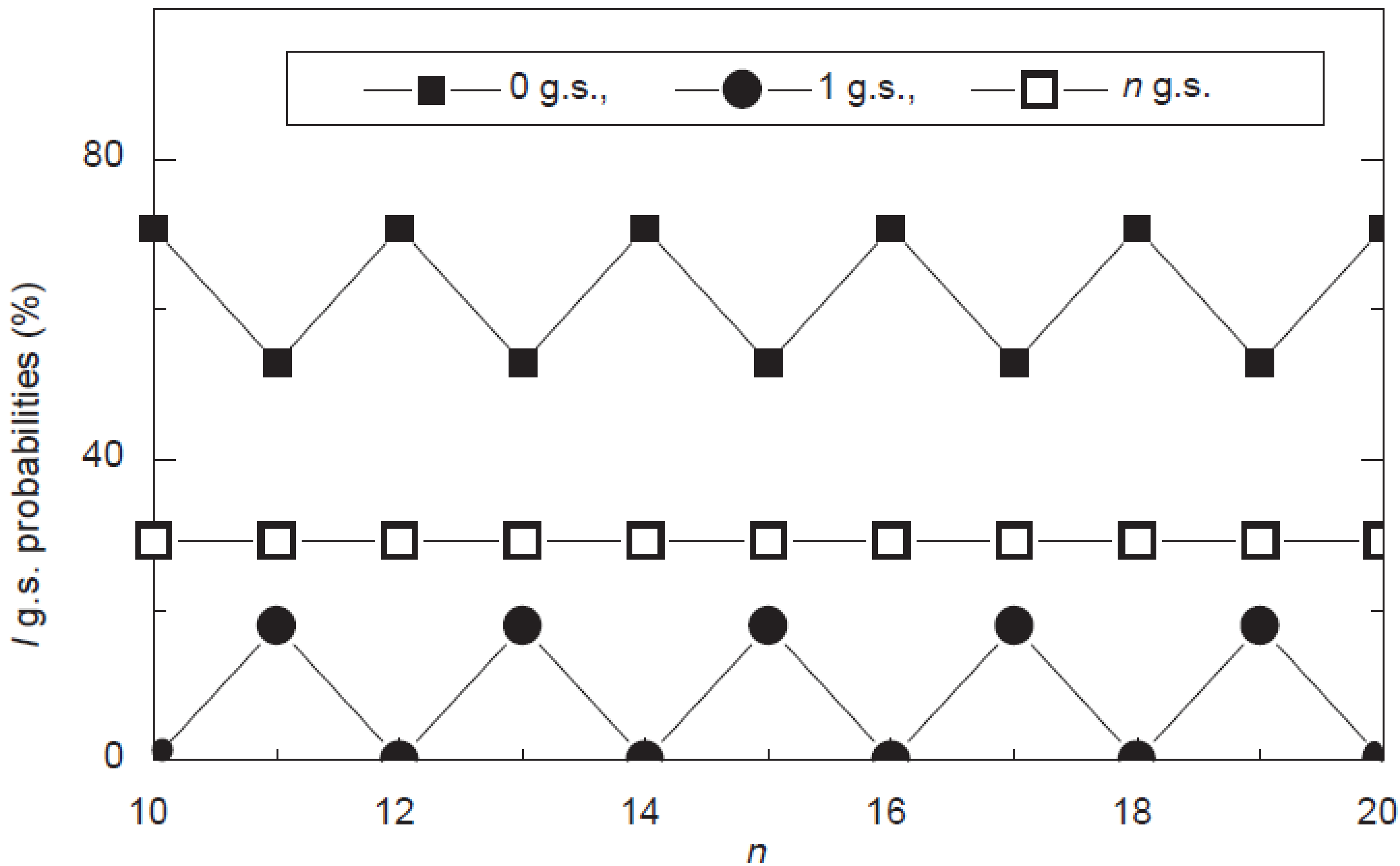
Isacker, 2002 PRC; Y Lu, 2015 PRC.

Mean-field method

(i) $\alpha_0 = 0, \quad -\frac{\pi}{2} < \chi \leqslant \frac{\pi}{2}$;

(ii) $\cos 2\alpha_0 = \cot \chi, \quad \frac{\pi}{4} \leqslant \chi \leqslant \frac{3\pi}{4}$;

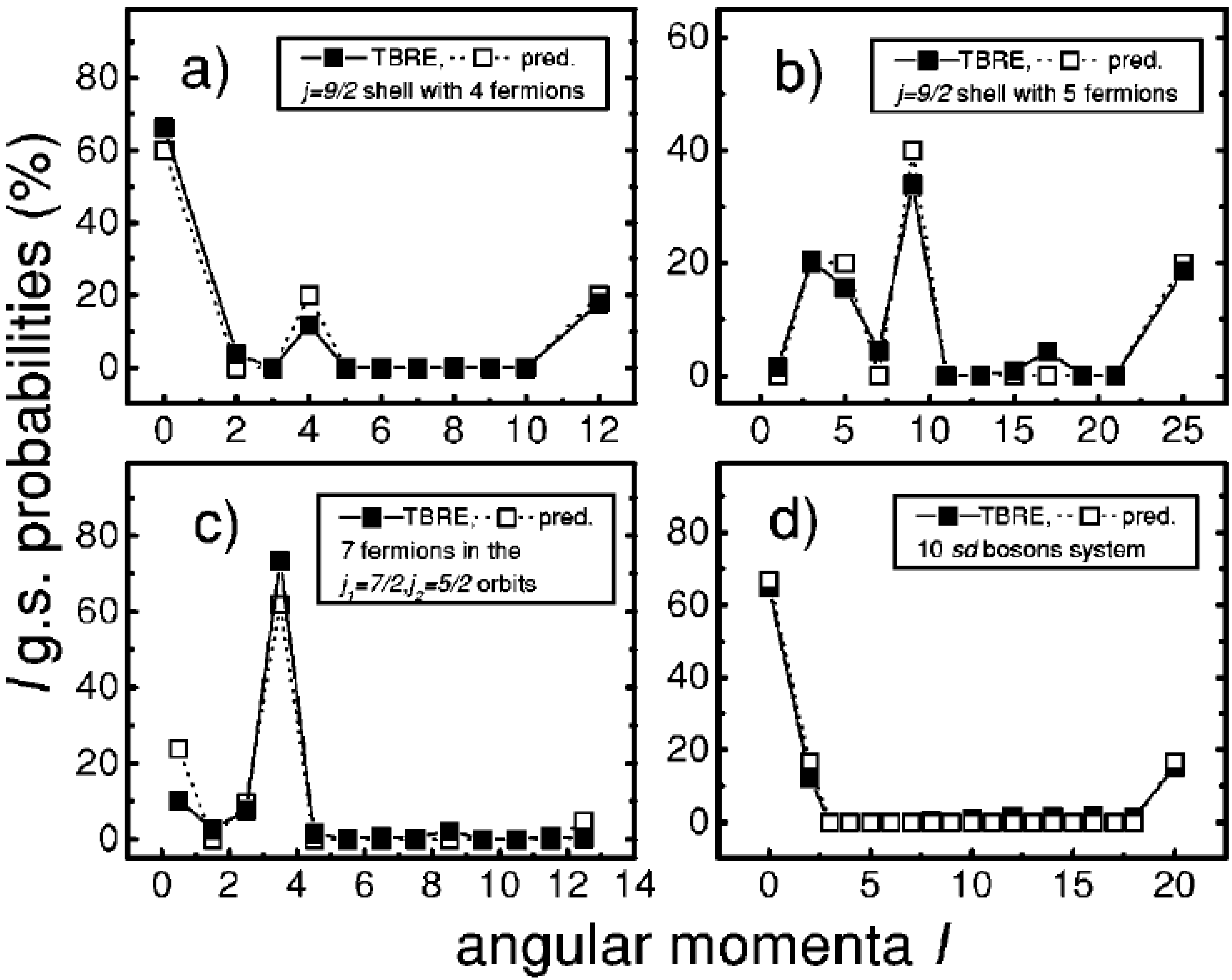
(iii) $\alpha_0 = \frac{\pi}{2}, \quad \frac{3\pi}{4} \leqslant \chi \leqslant \frac{3\pi}{2}$.



经验方法， by Zhao, Arima, and Yoshinaga, PRC 2002

Angular momenta which give the lowest eigenvalues when $G_J = -1$ and all other two-body matrix elements are zero for four fermions in a single-j shell

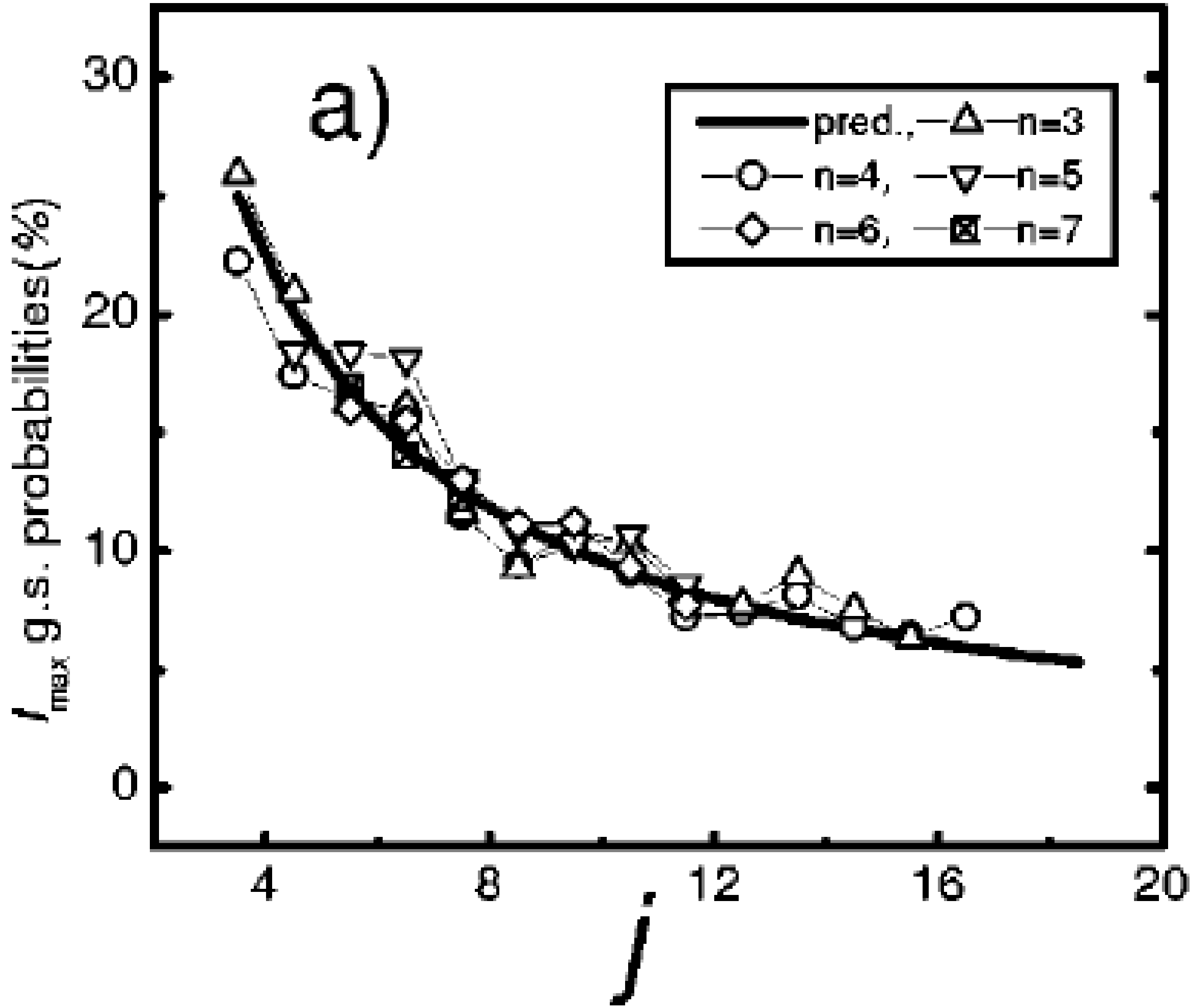
$2j$	G_0	G_2	G_4	G_6	G_8	G_{10}	G_{12}	G_{14}	G_{16}	G_{18}	G_{20}	G_{22}	G_{24}	G_{26}	G_{28}	G_{30}
7	0	4	2	8												
9	0	4	0	0	12											
11	0	4	0	4	8	16										
13	0	4	0	2	2	12	20									
15	0	4	0	2	0	0	16	24								
17	0	4	6	0	4	2	0	20	28							
19	0	4	8	0	2	8	2	16	24	32						
21	0	4	8	0	2	0	0	0	20	28	36					
23	0	4	8	0	2	0	10	2	0	24	32	40				
25	0	4	8	0	2	4	8	10	6	0	28	36	44			
27	0	4	8	0	2	4	2	0	0	4	20	32	40	48		
29	0	4	8	0	0	2	6	8	12	8	0	24	36	44	52	
31	0	4	8	0	0	2	0	8	14	16	6	0	32	40	48	56



a)
TBRE, pred.
j=9/2 shell with 4 fermions
b)
TBRE, pred.
j=9/2 shell with 5 fermions
c)
TBRE, pred.
7 fermions in the
j1=7/2,j2=5/2 orbits
d)
TBRE, pred.
10 sd bosons system
I g.s. probabilities (%)
angular momenta I

Previously, it was noticed that the 0 g.s. dominance is not dependent on having monopole pairing for fermions in the sd shell [1–3]. It was not known which interactions are crucial in order to have 0 g.s. dominance, and it was assumed by many authors that 0 g.s. dominance is thus an intrinsic property of the model space. In this article, however, we can address which interactions (not only monopole pairing interaction) are responsible for 0 g.s. dominance by numerical experiments. For instance, interactions with $J=0$, 6, 8, 12, and 22 give the 0 g.s. dominance for four fermions in a $j=\frac{31}{2}$ shell (refer to Table I).

method one can find which interactions, not only monopole pairing, are important to favor the 0 g.s. dominance. Taking four fermions in a single-j ($j=31/2$) shell as an example, the 0 g.s. probability is $\sim 2.2\%$ if we delete all the two-body interactions which produce $I=0$ g.s. ($J=0$, 6, 8, 12, and 22, refer to the last row of Table I of Ref. [16]). This means that dominance is even independent of monopole pairing [2,3]. It was not known, however, whether a certain two-body matrix element is essential or partly responsible, and how to find which interactions are essential, in producing the 0 g.s. dominance for a given system.


a)
pred., —△—n=3
—○— n=4, —▽—n=5
—◇— n=6, —⊠—n=7
I_{max} g.s. probabilities(%)
30
20
10
0
4
8
12
16
20
j

Progress of Theoretical Physics, Vol. 118, No. 3, September 2007

Ground State Properties with a Random Two-Body Interaction

Noritaka SHIMIZU[1] and Takaharu OTSUKA[1,2,3]

1) E. P. Wigner, Ann. Math **67** (1958), 325.
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4) C. W. Johnson, G. F. Bertsch and D. J. Dean, Phys. Rev. Lett. **80** (1998), 2749.
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17) J. J. Sakurai, *Modern Quantum Mechanics* (Addison-Wesley, 1994).
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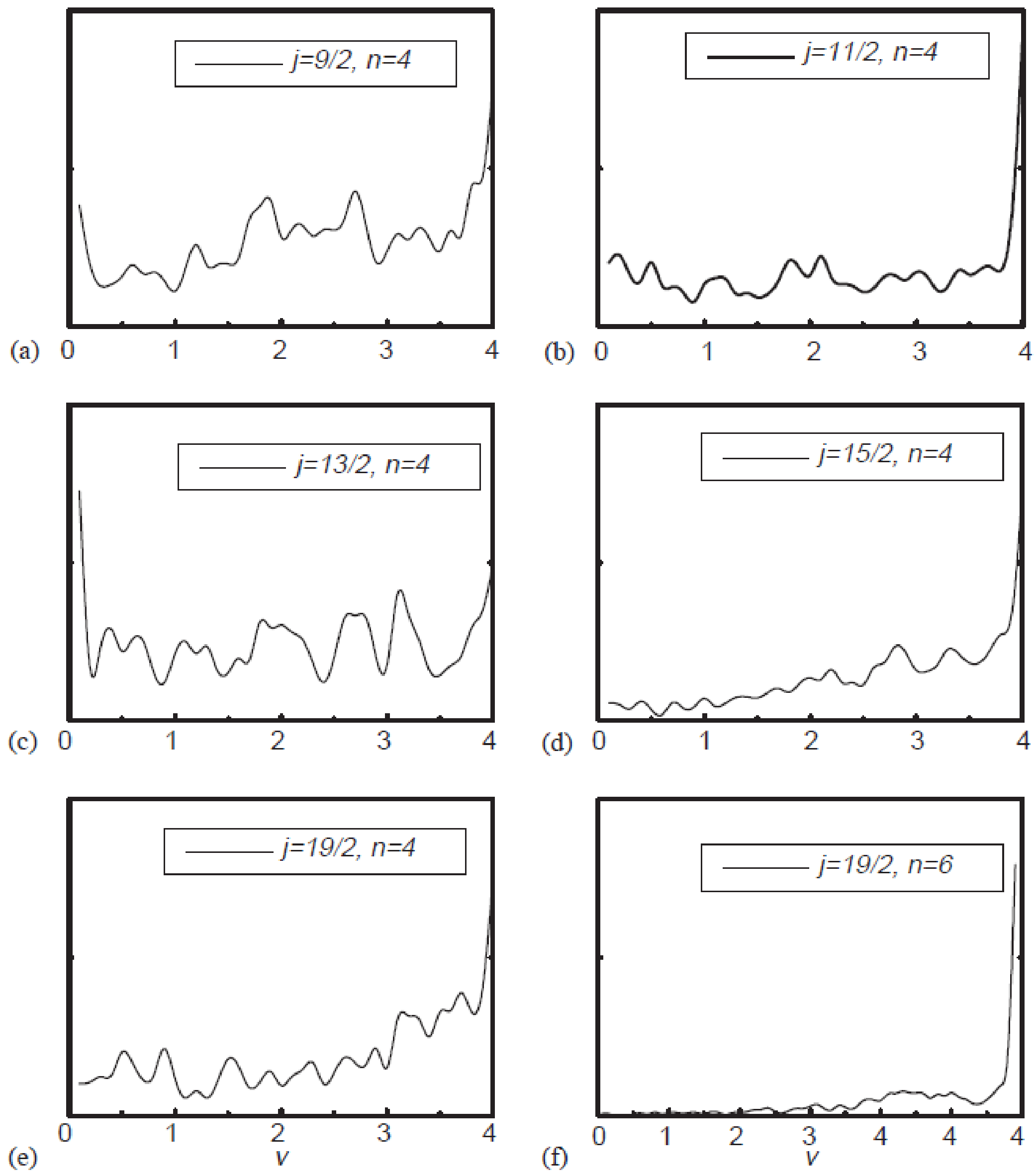
j=9/2, n=4
j=11/2, n=4
j=13/2, n=4
j=15/2, n=4
j=19/2, n=4
j=19/2, n=6
(a)
(b)
(c)
(d)
(e)
(f)
0
1
2
3
4
v

REVIEWS OF MODERN PHYSICS, VOLUME 81, APRIL–JUNE 2009

Random matrices and chaos in nuclear physics: Nuclear structure

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tion of spin-0 states is less than 10%. This result is known as the preponderance of spin-0 ground states. It holds for the random interaction (45). That interaction does not possess a strong pairing force (the agent usually held responsible for the preponderance of spin-0 ground states in actual nuclei). Other regularities were also observed (Johnson *et al.*, 1998, 1999; Zhao, Arima, Shimizu, *et al.*, 2004). We confine ourselves to the preponderance of spin-0 ground states. A large number of theoretical papers is devoted to this phenomenon. We confine ourselves to the two successful explanations that have been offered for the phenomenon and refer the reader to Zelevinsky and Volya (2004) and Zhao, Arima and Yoshinaga (2004) for further references.

The method used by Zhao and Arima (2001) and refined later [Zhao *et al.* (2002) and Zhao, Arima, and Yoshinaga (2004)] is based upon a simple counting procedure. They put one of the $\mathcal{N}$ different matrix elements of the residual interaction equal to (-1) and all others to zero and calculate the spectrum. The procedure is repeated $\mathcal{N}$ times, each time with a different nonzero matrix element. Let $\mathcal{N}_J$ be the number of times the ground state is found to have spin J. We have $\Sigma_J \mathcal{N}_J = \mathcal{N}$. The probability of finding a spin-0 ground state is then estimated as $\mathcal{N}_0/\mathcal{N}$. Comparing the results with an average over many diagonalizations of the TBRE, they found

good agreement for a number of cases (four to six fermions in single j shells and two j shell systems, boson systems).

quantitative agreement with numerical TBRE calculations (Papenbrock *et al.*, 2006).

The method of Papenbrock and Weidenmüller (2004) was improved by Yoshinaga *et al.* (2006). They considered, for instance, a single j shell with identical nucleons. The energy $E(J)$ of the lowest state of spin J was $E(J) = \mathcal{D}(J)^{-1}\,\mathrm{Tr}(\mathcal{H}) - \Phi_J \sigma_J$. This equation differs from that of Papenbrock and Weidenmüller (2004) by inclusion of the trace of $\mathcal{H}$ and by the fact that an analytical form for the function Φ_J was proposed. The trace of $\mathcal{H}$ vanishes upon taking the ensemble average but, for each realization, fluctuates around zero. Inclusion of the trace in the equation for $E(J)$ removes the scatter of the points around the best linear approximation to r_J shown in Fig. 28. The function Φ_J was fitted and given analytically as $\Phi_J = \sqrt{0.99 \ln \mathcal{D}(J) + 0.36}$. Good agreement is obtained for $m = 4$ to 6 fermions in several single j shells and systems with two j shells between TBRE results and the predictions based upon this approach. The method also works for bosons.

谈到这个题目时，我们的工作一直作为
这个题目的重点结果，甚至单独一个section.
我们最后也受邀请写了一篇Physics Reports.

Available Results

Empircal method **Zhao&Arima&Yoshinaga (2002)**

Mean-field method **Bijker-Frank (2003)**

Geometrid method **Chau et al. (2003)**

Spectral Radius **Papenbrock & Weidenmueller (2004)**

Time reversal invariance (TRI) Zuker et al. (2002);

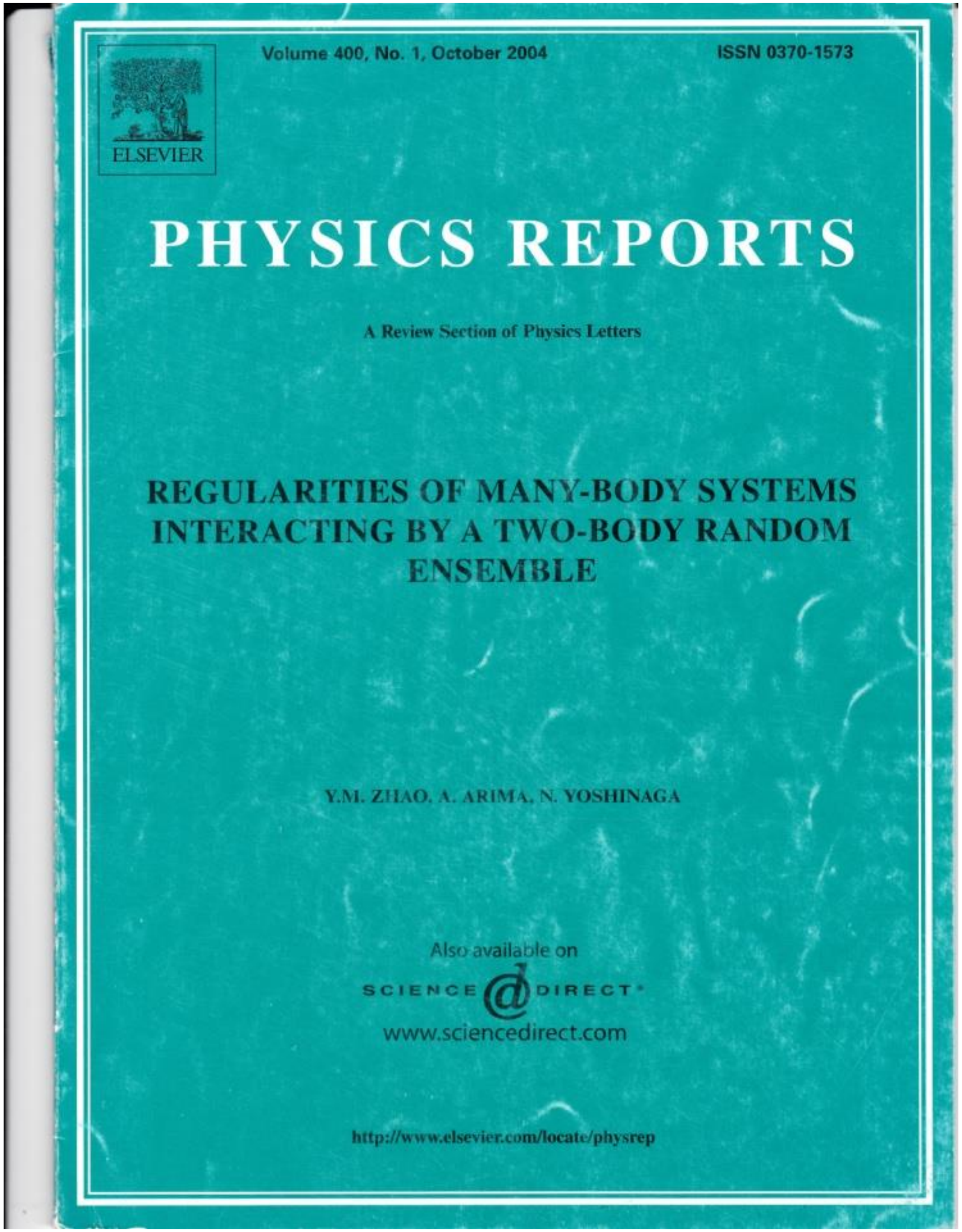
Time reversal invariance? Bijker&Frank&Pittel (1999);

Width ? Bijker&Frank (2000);

off-diagonal matrix elements for I=0 states Drozdz et al. (2001)

Highest symmetry &Time Reversal Otsuka&Shimizu(2004-2007)

Semi-empirical formula Shen, Zhao, Arima, and Yoshinaga(2006-2010)



以中国作为第一
单位的第六篇
Physics Reports
综述论文

2、 A simple and similar pattern

Positive parity dominance

(an open problem)

Let us choose ANY configuration spaces in which we have a few single-particle levels, some of them have positive parity, the other have negative parity.

Then the number of states with positive parity and that with negative parity are very close to each other, roughly 50%-50%.

−−−−−−−−(unit: %)−−−−−−−−−−−−−−−−−−−−

Basis (A)

(0,4)	(0,6)	(2,2)	(2,4)	(2,6)			$\Leftarrow$
86.6	86.2	93.1	81.8	88.8			
(2,3)	(1,4)	(0,5)	(6,1)	(2,1)	(1,3)	(1,5)	
42.8	38.6	45.0	38.4	31.2	77.1	69.8	

Basis (B)

(2,2)	(2,4)	(4,2)						$\Leftarrow$
72.7	80.5	81.0						
(3,4)	(2,3)	(3,2)	(4,1)	(1,4)	(5,0)	(3,3)	(5,1)	
42.5	72.4	39.1	75.1	26.4	44.1	79.4	42.9	

Basis (C)

(2,2)	(2,4)	(4,0)	(6,0)		$\Leftarrow$
92.2	81.1	80.9	82.4		
(2,3)	(5,0)	(4,1)	(1,5)	(1,3)	
52.0	42.6	56.5	64.4	73.0	

Basis (D)

(2,2)	(4,2)	(2,4)	(0,6)	$\Leftarrow$
67.2	76.1	74.6	83.0	
(3,2)	(2,3)	(0,5)	(3,3)	
54.2	54.0	45.9	54.5	

−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−

- The worst case is P(+)=67%，the best case is 99.9%。

  On average P(+)~86%。

- No counter example has been found！

PHYSICAL REVIEW C **78**, 054305 (2008)

Abundance of ground states with positive parity

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(Received 22 August 2008; published 6 November 2008)

Effect of finite number of particles in the systems.

Johnson *et al.* [1] observed that in the two-body ran ensemble (TBRE) of the nuclear shell model, ground states with spin zero occur much more frequently than expected from their statistical weight. That observation caused considerable theoretical activity (see the reviews in Refs. [2,3]). A similar preponderance for states with positive parity was found in Ref. [4]. We wish to explore the reason for that preponderance. We focus attention on parity (rather than spin) because that quantum number is analytically more easily accessible. We

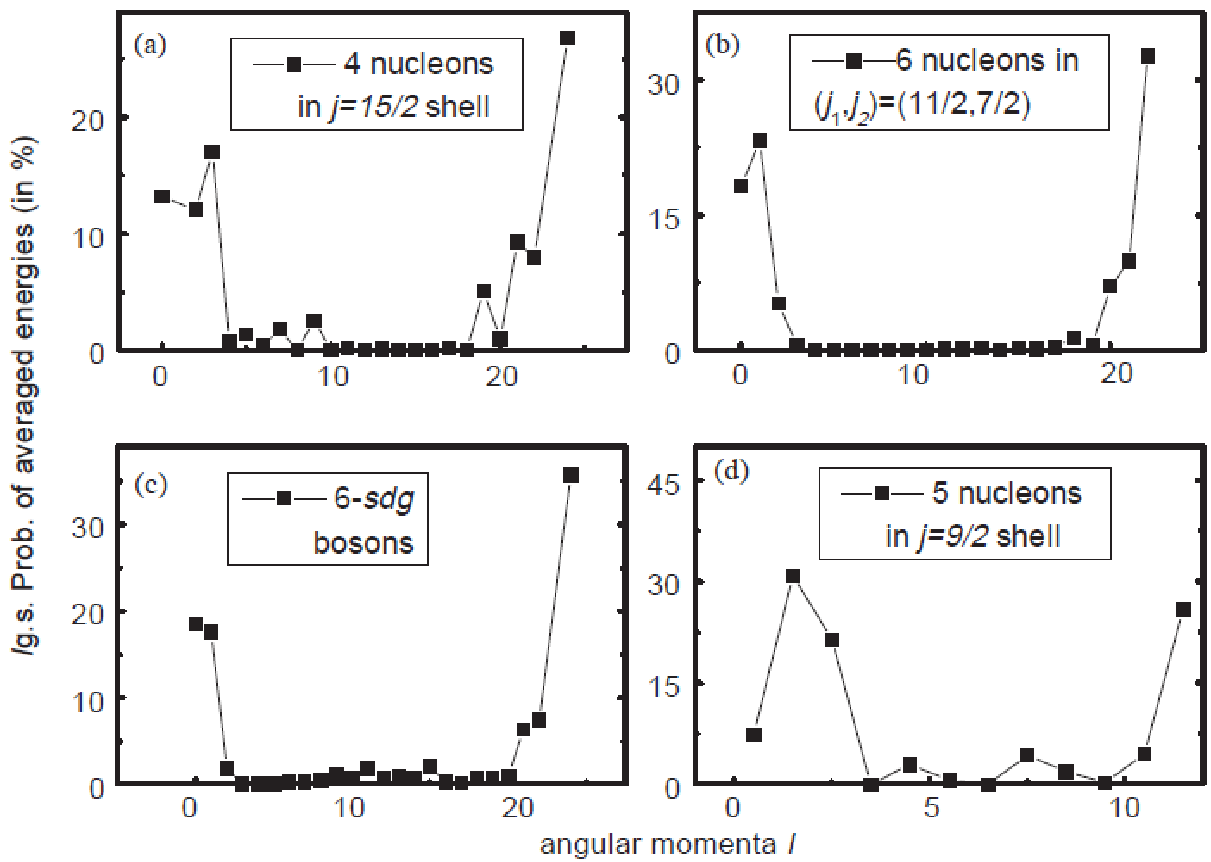
This is the only study of this simple phenomenon so far. This could be part of the full story.

All results applied to the 0 g.s. dominance are applicable to positive parity dominance.

平均能量 (YMZ et al., PRC66, 064322, (2002))：

that $\overline{E_I}$ is a linear combination of G_J's:

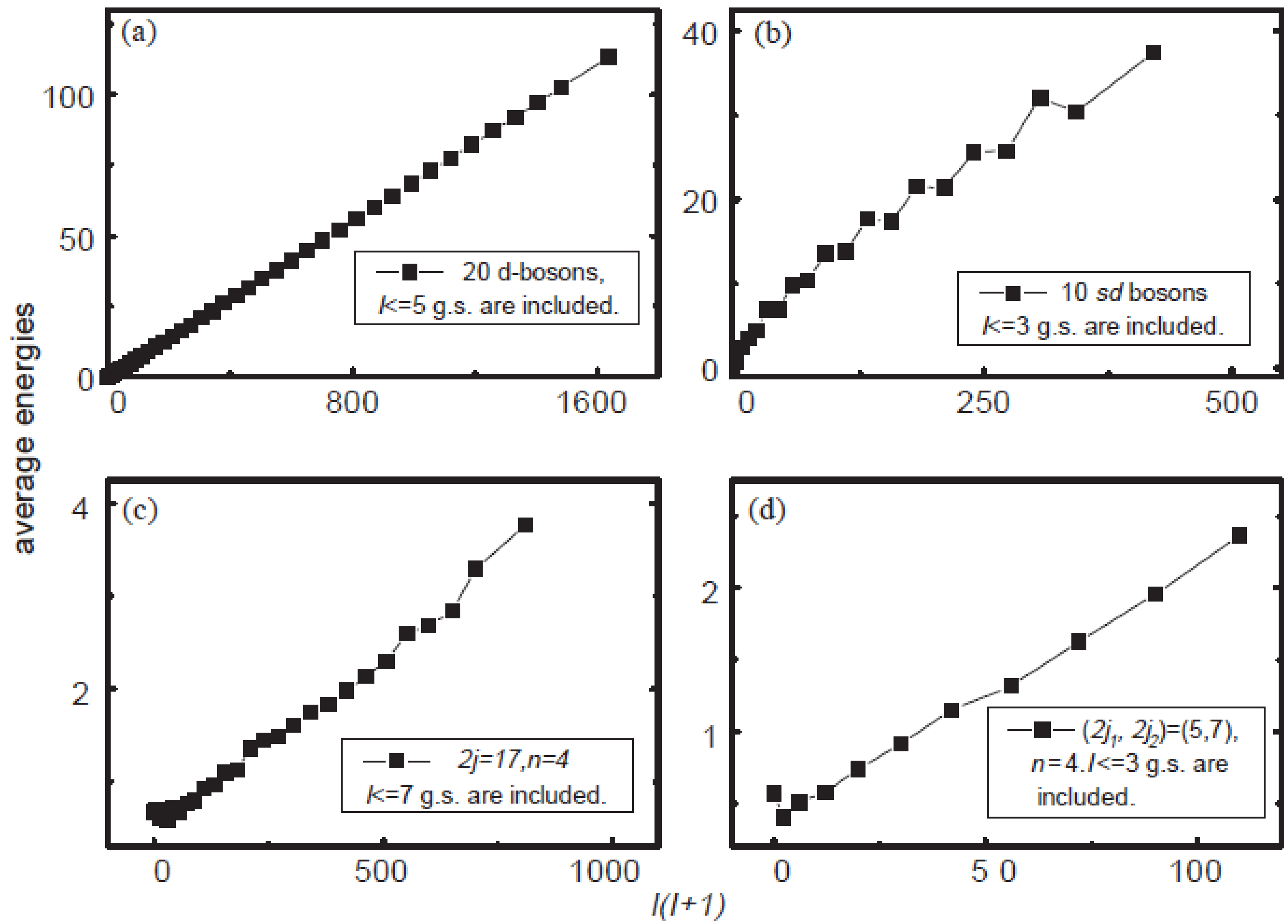
$$\overline{E_I} = \sum_J \overline{\alpha_I^J} G_J \ ,$$



Coefficients $\overline{\alpha_I^J}$ and $\mathscr{P}(I)$ for four fermions in a $j=\frac{9}{2}$ shell

I	G_0	G_2	G_4	G_6	G_8	pred1.(%)	pred2.(%)	TBRE
0	**0.80**	0.35	1.74	2.11	1.01	11.97	11.1	10.2
2	0.30	**1.39**	1.45	*1.29*	1.56	14.51	22.2	15.4
3	0.00	0.36	**2.28**	**2.63**	*0.71*	28.17	33.3	28.9
4	0.20	1.07	1.38	1.91	1.44	1.74	0	1.7
5	0.00	1.00	1.59	1.84	1.57	0.30	06	0.6
6	0.20	0.79	1.50	1.58	1.93	0.22	0	0.3
7	0.00	1.20	1.09	1.40	2.31	3.44	0	3.2
8	0.30	0.48	1.05	1.82	2.36	0.03	0	0
9	0.00	0.17	1.33	2.12	2.38	0.01	0	0
10	0.00	0.70	0.69	1.41	3.21	6.76	0	8.7
12	0.00	*0.00*	*0.52*	1.69	**3.78**	32.64	33.3	31.0

Bold font is used for the largest $\overline{\alpha_I^J}$, and italic for the smallest $\overline{\alpha_I^J}$ for a given J. Probabilities in the column "pred1." are obtained by integrals similar to Eq. (7) in Ref. [47], and those in the column "pred2." are obtained by the empirical formula given in (24). The $\mathscr{P}(I)$'s in the last column "TBRE" (in %) are obtained by diagonalizing the TBRE Hamiltonian for 1000 runs. We take both the smallest and the largest $\overline{\alpha_I^J}$ when counting $\mathscr{N}'_I$.



average energies
(a)
20 d-bosons,
k=5 g.s. are included.
0
50
100
0
800
1600
(b)
10 sd bosons
k=3 g.s. are included.
0
20
40
0
250
500
(c)
2j=17,n=4
k=7 g.s. are included.
2
4
0
500
1000
(d)
(2j1, 2j2)=(5,7),
n=4. I<=3 g.s. are
included.
1
2
0
5 0
100
I(I+1)

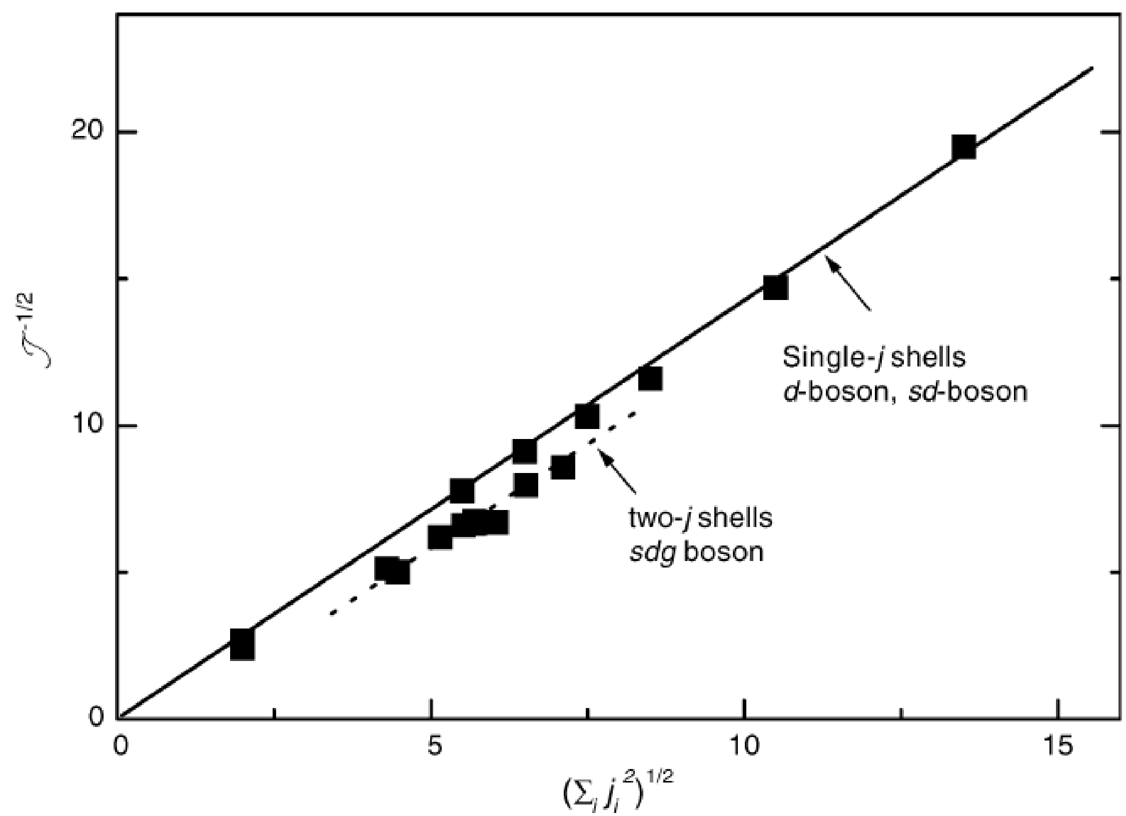


Fig. 17. Correlation between $\sqrt{\mathscr{I}}$ and $j = \sqrt{\sum_i j_i^2}$. $\sqrt{\mathscr{I}} \simeq 1.42\sqrt{\sum_i j_i^2}$ for d, sd bosons, and fermions in a single-j shell. This correlation is shifted very slightly to the right for fermions in many-j shells and sdg bosons.

$$\langle \overline{E_I} \rangle_{\min} = I(I+1)/2\mathscr{I}.$$

$$\overline{E_I} = \sum_J (2J+1) G_J \left(\frac{n}{2j+1} \right)^2 + I(I+1) \sum_J (2J+1) \frac{3(J^2 - 2j(j+1))}{2j^2(j+1)^2(2j+1)^2} G_J + O(I^2(I+1)^2) \ ,$$

VKB Kota 2002, // Mulhall, Volya, Zelevinsky, 2001 PRL

[redacted]

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Regularities with random interactions in energy centroids defined by group symmetries

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III. Two recent results

1、 A regular pattern

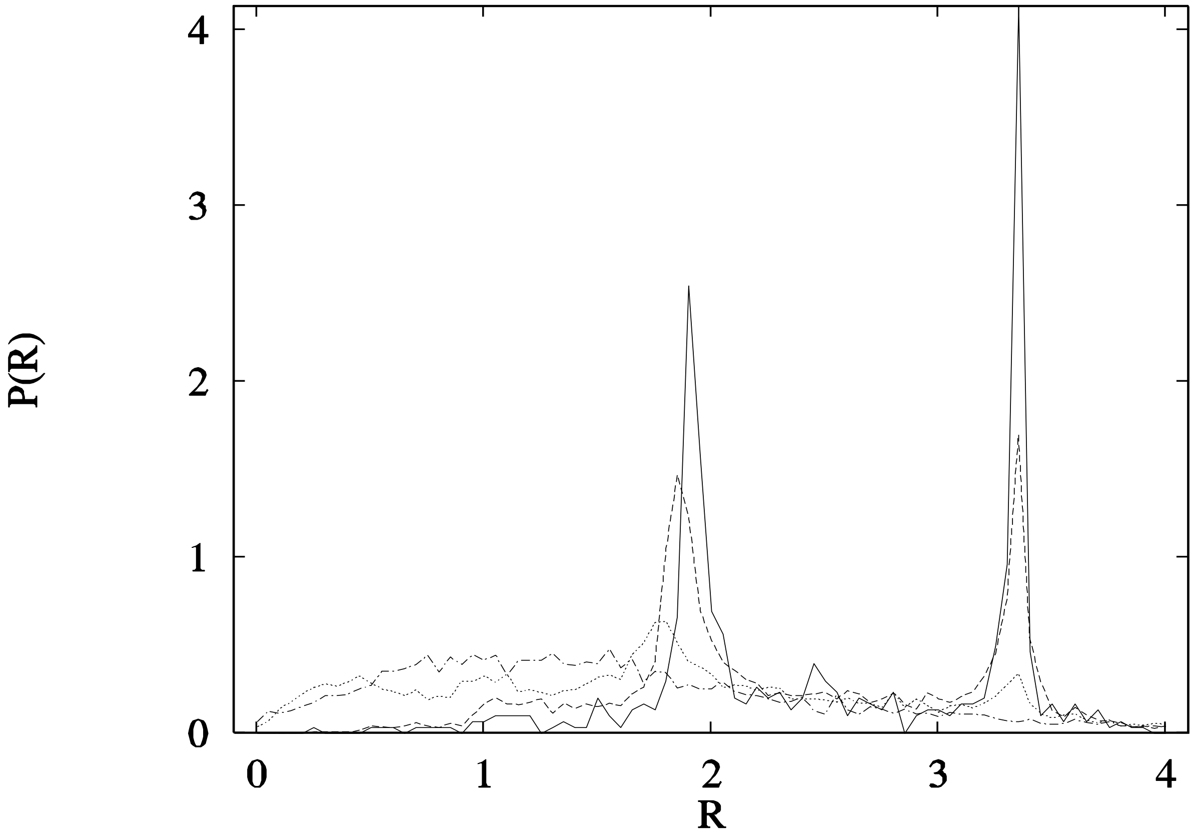
Collective motion in sd-boson systems

(in collaboration with Lei, Yoshida, Pittel, Arima)

The sd interacting boson model (IBM) is a wonderful framework in nuclear structure theory (many thanks to its inventors, Prof. Arima and Prof. Iachello) with full of beauty of symmetry, simplicity, and rich structures exhibited in low-lying states of nuclei. That may be one of the key reasons for its popularity [experimentalists also use it]. It is (possibly) the simplest framework which can be designed in nuclear structure theory.

Because of its simplicity, it is an ideal tool to study the behavior of the random hamiltonian by using the IBM.

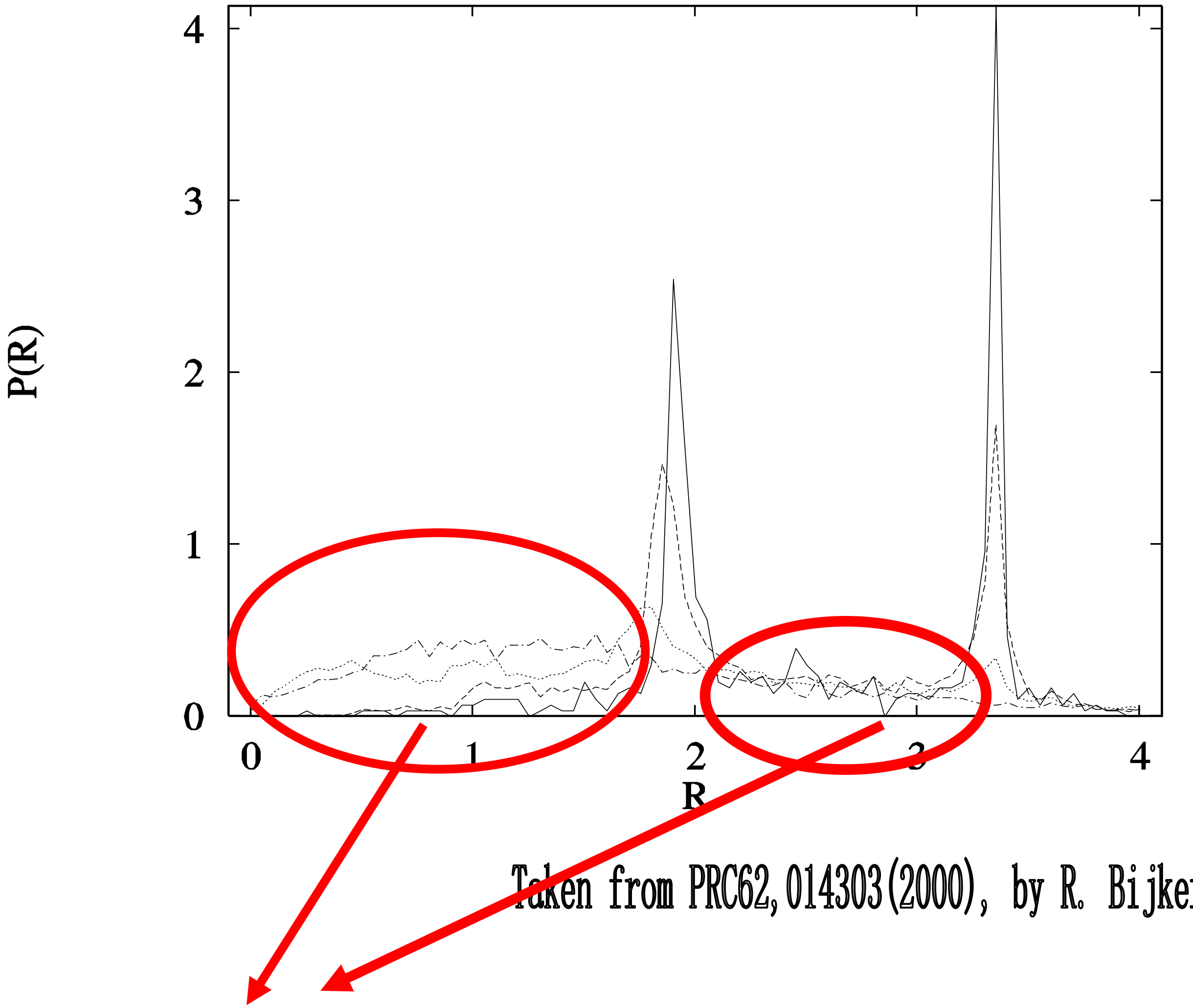
Collectivity in the IBM under random interactions



Taken from PRC62,014303(2000), by R. Bijker and A. Frank

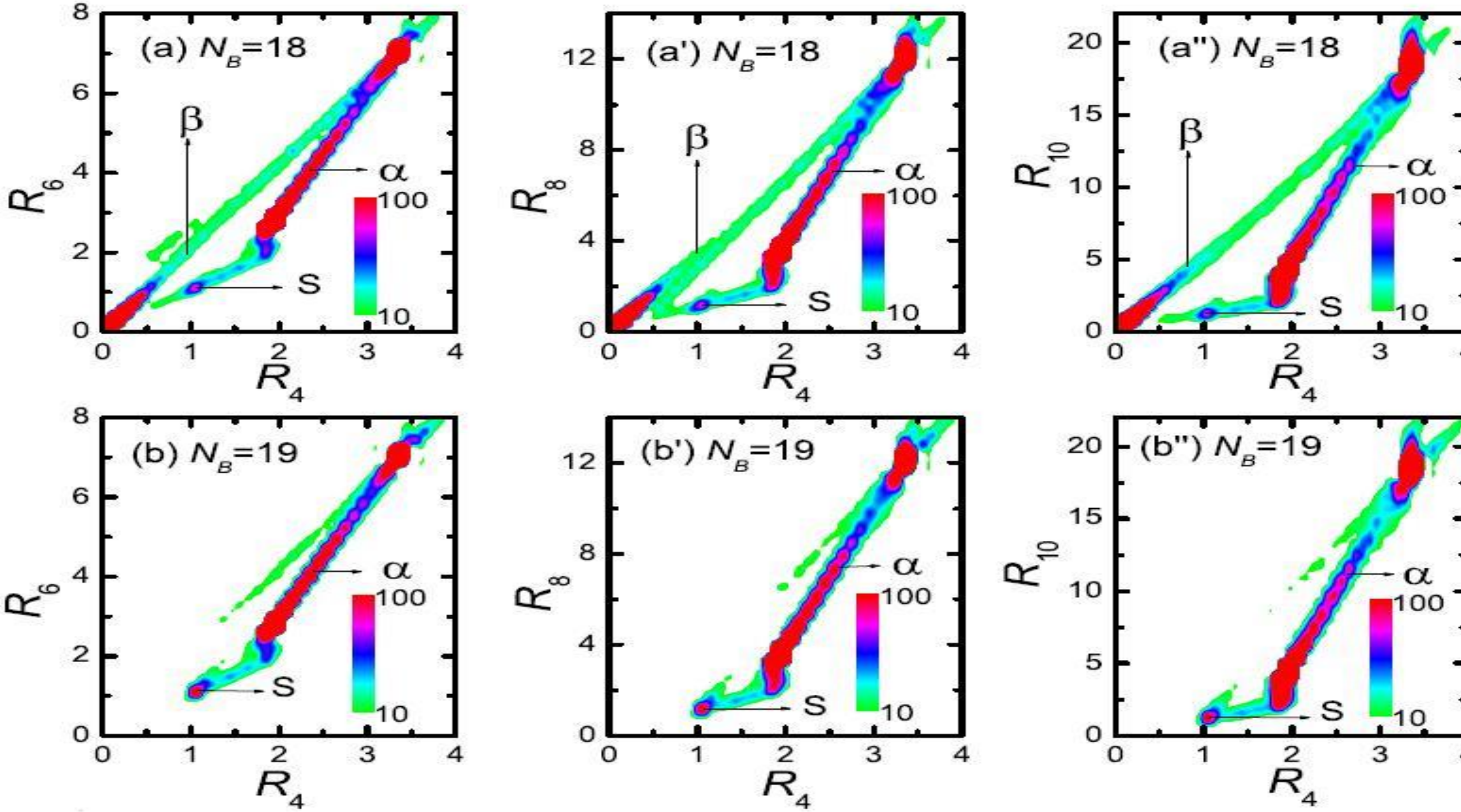
R=E(4)/E(2), the ratio between 4+ energy and 2+ energy

Collectivity in the IBM under random interactions



Taken from PRC62,014303(2000), by R. Bijker and A. Frank

What are they ? Are they noisy (random) backgrounds ?



R_4=E4(+)/E(2+), R6=E6(+)/E(2+), R8=E8(+)/E(2+), R10=E4(+)/E(2+).

We found the origin of the above patterns. Yrast states are highly regular with random interactions.

Lei, Jiang, Zhao, Yoshida, Pittel, and Arima, PRC (2011).

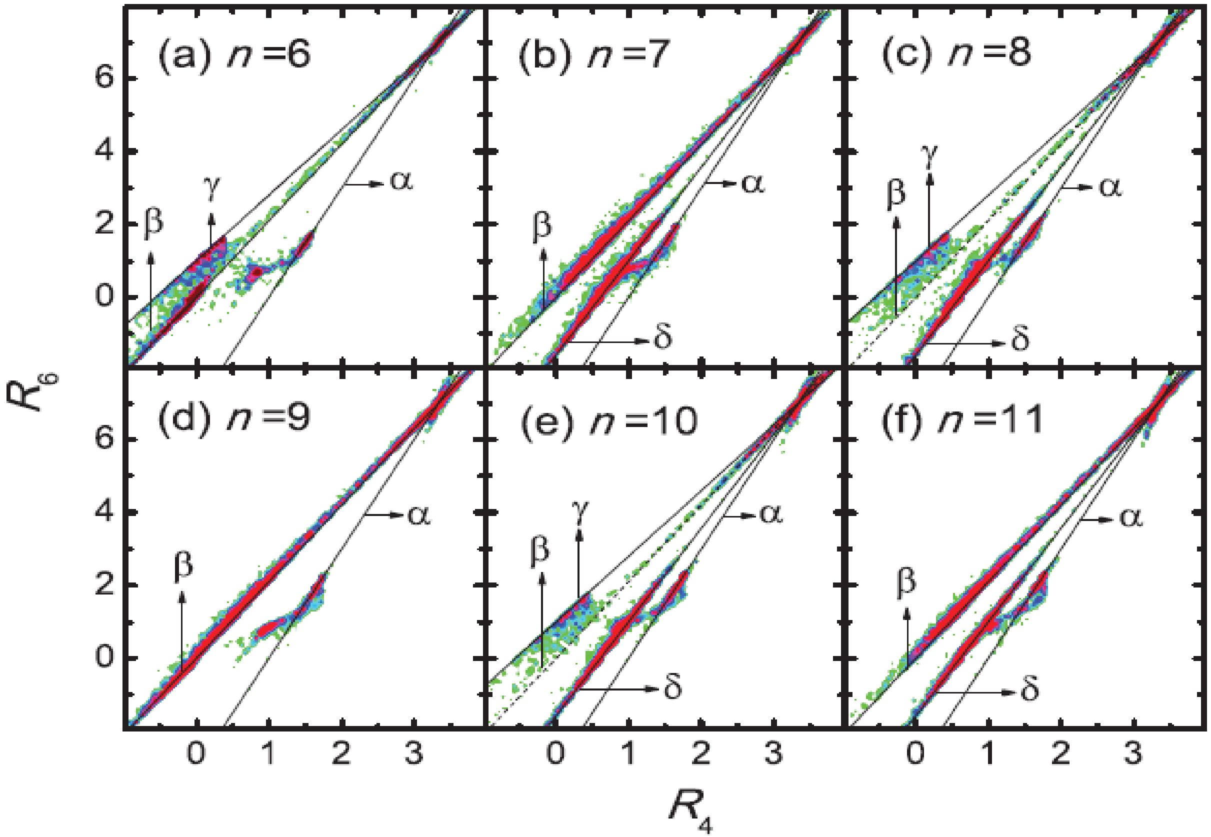


FIG. 1. (Color online) Correlation of R_6 versus R_4 of sd bosons with two-body random interactions. n is the boson number. Only random samples with spin-*nonzero* ground states are considered here. In addition to the peaks at $(R_4, R_6) = (3.3, 7.0)$, which corresponds to rotational motion, and $(R_4, R_6) = (1.0, 1.0)$, which corresponds to the seniority-type correlation, strong correlations (denoted by α, β, γ, δ, and ξ) between R_4 and R_6 are seen. Correlations β and ξ are described by the same formula in the R_6-R_4 plot, and thus are "mixed" in the case of odd n when $n = 3k$. See the text for more details.

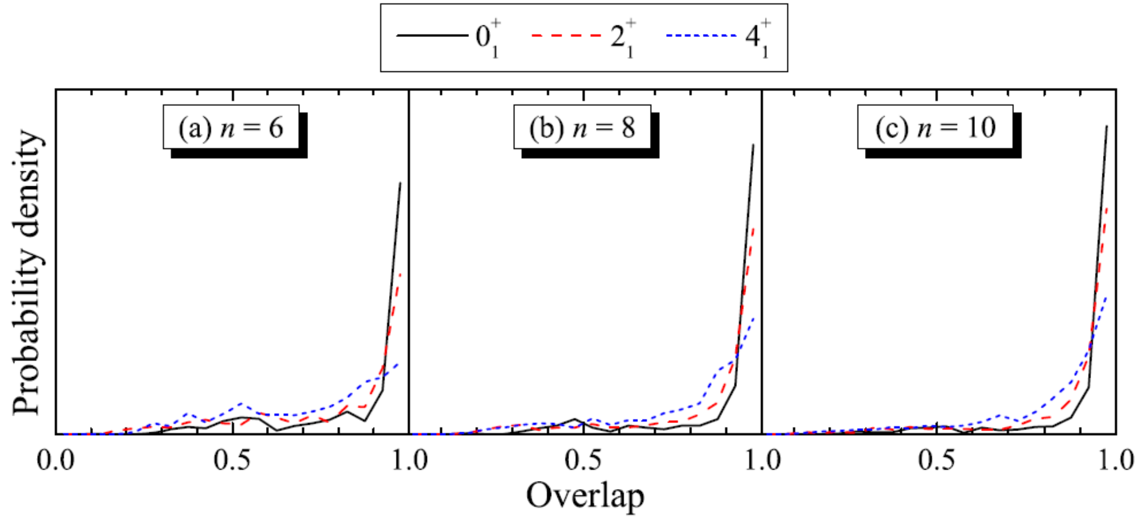


FIG. 1. Distribution of the largest overlap between the wave function obtained under the TBRE and U(5)-limit states for the random samplings of the vibrational peak ($R_4 \approx 2$). The solid curve in black, the dashed curve in red, and the dotted curve in blue are for the 0_1^+, 2_1^+, 4_1^+ states, respectively.

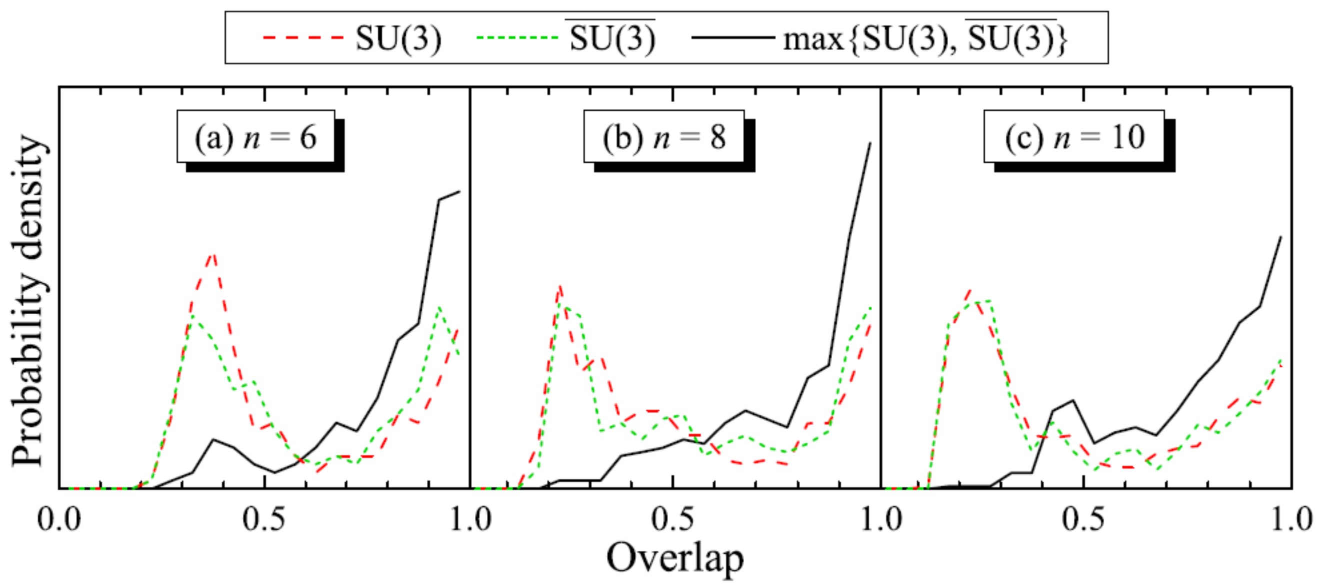
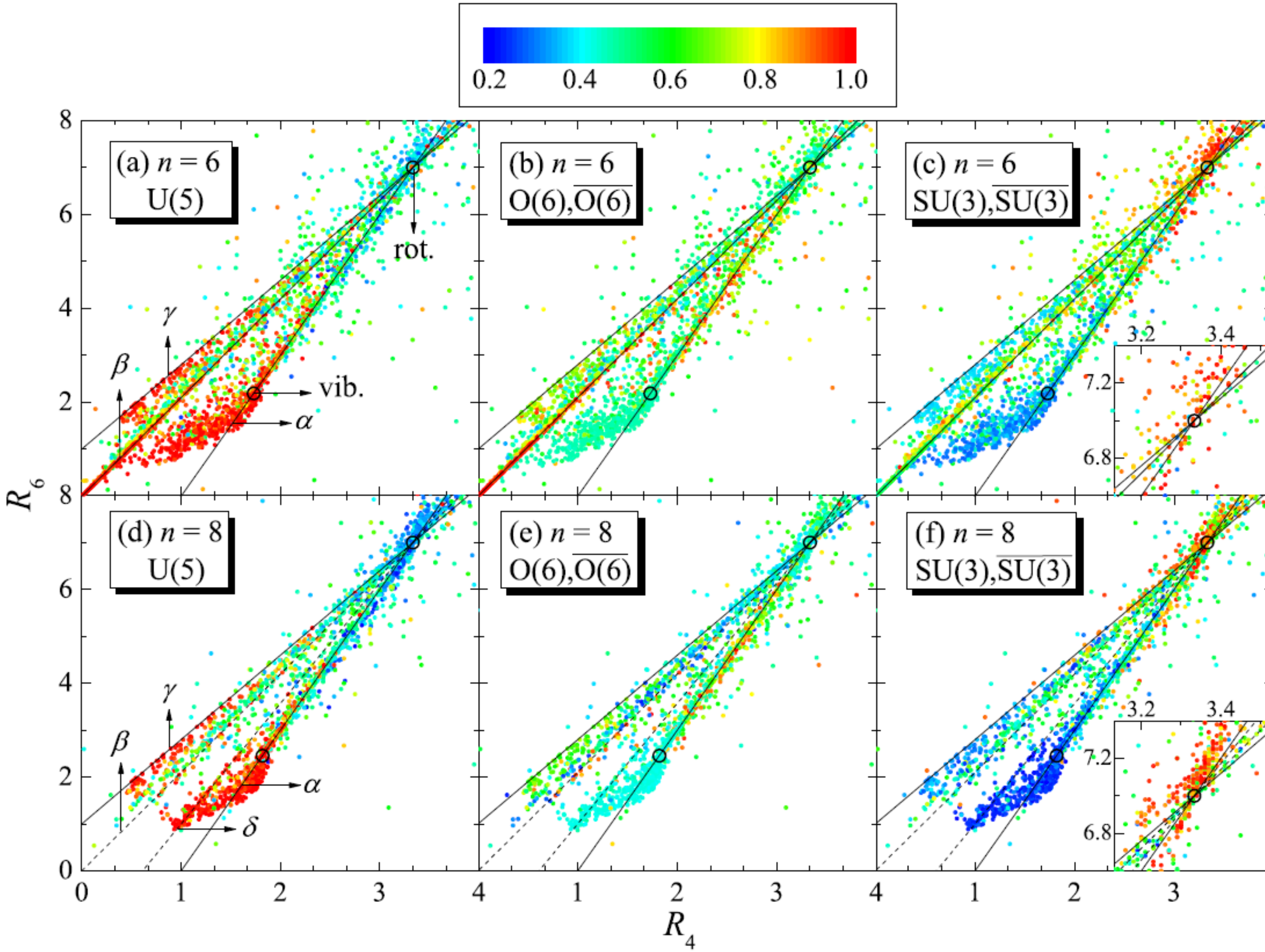


FIG. 2. Distribution of the largest overlap between the wave function obtained under the TBRE and SU(3)/$\overline{\mathrm{SU(3)}}$-limit states for the 0_1^+ state of the random samplings of the rotational peak ($R_4 \approx 3.33$).



0.2 0.4 0.6 0.8 1.0
(a) $n = 6$
U(5)
(b) $n = 6$
O(6),$\overline{\text{O(6)}}$
(c) $n = 6$
SU(3),$\overline{\text{SU(3)}}$
(d) $n = 8$
U(5)
(e) $n = 8$
O(6),$\overline{\text{O(6)}}$
(f) $n = 8$
SU(3),$\overline{\text{SU(3)}}$
rot.
vib.
α
β
γ
δ
R_6
R_4

Summary

Two examples:

- Spin zero ground state dominance
- Positive parity dominance
- Everage energies

Two recent results in collaboration with Prof. Arima:

- Collective motion in the *sd*-boson systems

"Computers understand the problem, but I wish to understand it, too"

---E. P. Wigner

[This statement was already cited by Franco Iachello in 2010 in Okinawa, and by another feature article in nuclear physics news]

未来的世界是你们的，要靠各位。

希望各位能相互多讨论、多合作

前程似锦！