



镜像核质量关系的对称性

Symmetry between masses of mirror nuclei

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Contents



1

Introduction

2

Neutron-proton interaction

3

Separation energies

4

Summary



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1

Introduction

2

Neutron-proton interaction

3

Separation energies

4

Summary



Introduction



Global model

Duflo-Zuker model

J. Duflo and A. P. Zuker, Phys. Rev. C 52, R23 (1995)

Skyrme Hartree-Fock-Bogoliubov theory

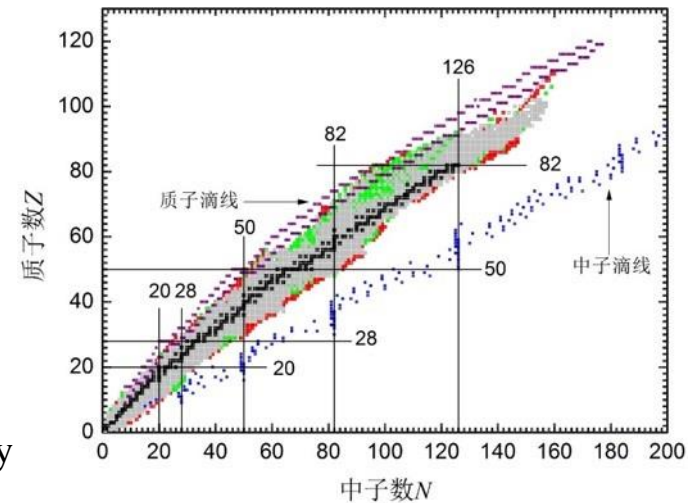
P. Möller, J. R. Nix, At. Data Nucl. Data Tables 59, 185 (1995)

Weizsäcker-Skyrme mass formula

N. Wang, M. Liu and X. Z. Wu, Phys. Rev. C 81, 044322 (2010)

Spherical relativistic continuum Hartree-Bogoliubov theory

X. W. Xia, Y. Lim, et al., At. Data Nucl. Data Tables 121, 1 (2018)



Local mass formulas

Audi-Wapstra method

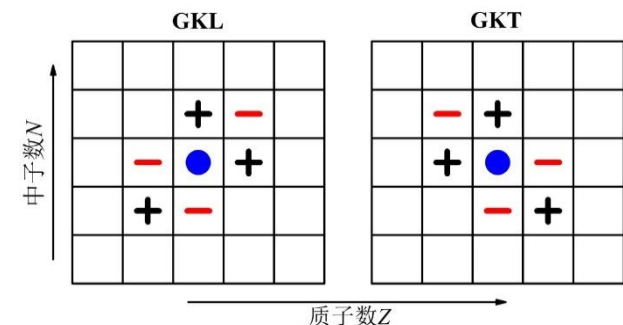
G. Audi, A. H. Wapstra and C. Thibault, Nucl. Phys. A 729, 337 (2003)

Garvey-Kelson mass relations

G. T. Garvey and I. Kelson, Phys. Rev. Lett. 16, 197 (1966)

Mass relation based on neutron-proton interaction

G. J. Fu, et al., Phys. Rev. C 82, 034304 (2010)

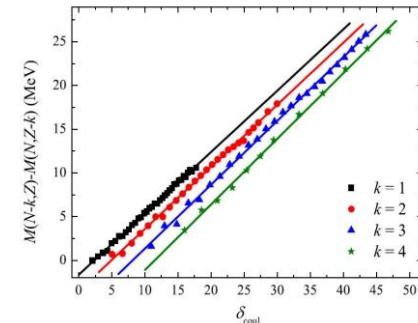


Introduction



- Masses correlation between mirror nuclei

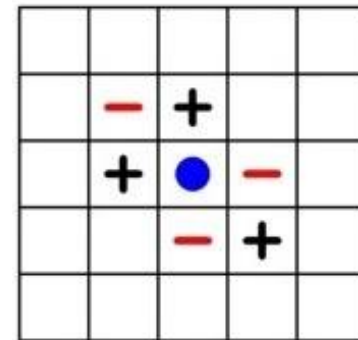
M. Bao, Y. Lu, Y. M. Zhao and A. Arima, Phys. Rev. C 94, 044323 (2016)



- Garvey-Kelson relations for nuclei with $N = Z$

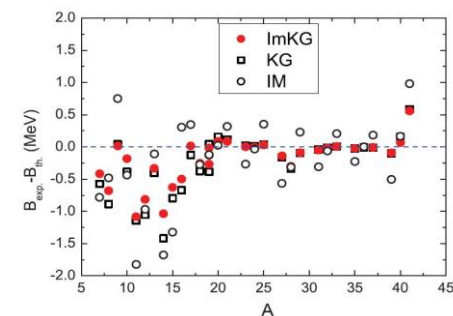
$$M(N-1, Z+1) + M(N, Z-1) + M(N+1, Z)$$

$$-M(N-1, Z) - M(N, Z+1) - M(N+1, Z-1) = 0$$



- Improved Garvey-Kelson mass relations

J. L. Tian, N. Wang, C. Li and J. J. Li, Phys. Rev. C 87, 014313 (2013)



Contents



1

Introduction

2

Neutron-proton interaction

3

Separation energies

4

Summary



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Neutron-proton interaction



- Residual neutron-proton interaction

$$V_{in-jp} = -M(N, Z) + M(N - i, Z) + M(N, Z - j) - M(N - i, Z - j)$$

- For $(N - k, Z)$ and $(N, Z - k)$ pair, we denote

$$\Delta V_{in-jp}(N - k, Z) = V_{in-jp}(N - k, Z) - V_{jn-ip}(N, Z - k) \quad \mathbf{N = Z}$$

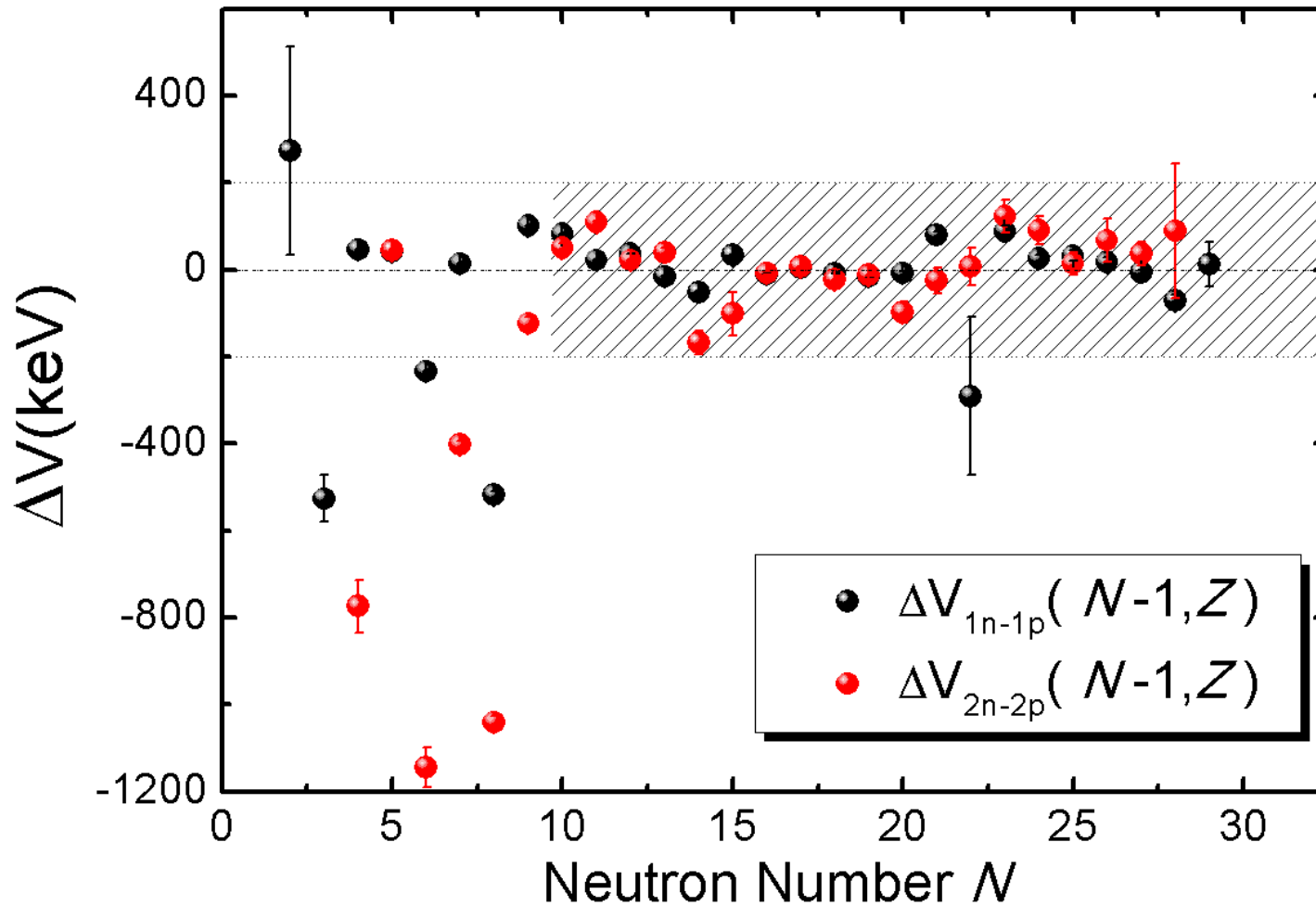
- Suppose that the neutron-proton interactions V_{1n-1p} are equal:

$$\Delta V_{1n-1p}(N - k, Z) = 0$$

J. Jänecke, Phys. Rev. C **6**, 467 (1972);

Y. H. Zhang, P. Zhang, X. R. Zhang N. Wang, et al., Phys. Rev. C **98**, 014319 (2018)

Neutron-proton interaction



Neutron-proton interaction



- The RMSD (in keV) and number of $\Delta V_{in-jp}(N - k, Z)$ with ^{44}V excluded

$\begin{matrix} k = 1-3 \\ i, j = 1-2 \end{matrix}$	σ_1	N_1	σ_2	N_2	σ_3	N_3
ΔV_{1n-1p}	43	19	64	16	133	5
ΔV_{2n-1p}	55	18	110	6	868	1
ΔV_{1n-2p}	43	19	76	17	136	5
ΔV_{2n-2p}	73	19	152	6	1071	1

M. Wang, G. Audi, F. G. Kondev, W. J. Huang, et al., Chin. Phys. C 41, 030003 (2016)

Neutron-proton interaction



- Mass relations based on this work

$$M(N-2, Z) = M(N, Z-2) + M(N-1, Z) - M(N, Z-1)$$

$$+ M(N-2, Z-1) - M(N-1, Z-2)$$

$$M(N-3, Z) = M(N, Z-3) + M(N-1, Z) - M(N, Z-1) + M(N-2, Z-1)$$

$$- M(N-1, Z-2) + M(N-3, Z-2) - M(N-2, Z-3)$$

$$M(N-3, Z) = M(N, Z-3) + M(N-1, Z) - M(N, Z-1)$$

$$+ M(N-3, Z-1) - M(N-1, Z-3)$$

- Mass extrapolation : AME1995 \longrightarrow AME2016 ($N \geq 10$)

11 proton-rich nuclei

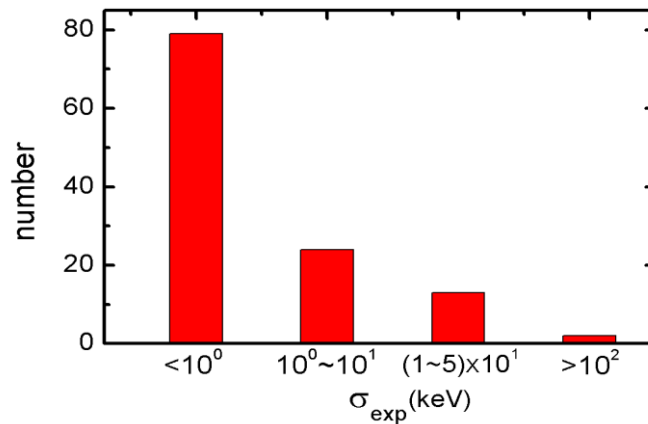
RMSD : **47** keV

Neutron-proton interaction



■ Uncertainty of predictions

$$\sigma = \sqrt{\sigma_{\text{th}}^2 + \sum \sigma_{\text{exp}}^2}$$



■ m_{pred} and σ_{pred}

$$m_{\text{pred}} = F \frac{m_1}{(\sigma_1)^2} + F \frac{m_2}{(\sigma_2)^2} + F \frac{m_3}{(\sigma_3)^2}$$

$$F = \frac{1}{\frac{1}{(\sigma_1)^2} + \frac{1}{(\sigma_2)^2} + \frac{1}{(\sigma_3)^2}}$$

$$\sigma_{\text{pred}} = \sqrt{F}$$

N	Z	A	$mass_{\text{pred}}^{\text{excess}}$	σ_{pred}
27	30	57	-32778	45
28	31	59	-34019	60
29	31	60	-39988	60
29	32	61	-34001	92
30	32	62	-42328	70
30	33	63	-33832	125
31	33	64	-39666	102
31	34	65	-33484	136
32	32	66	-42059	116
32	35	67	-32935	137
33	35	68	-38712	90
33	36	69	-32755	171
34	36	70	-41579	142

Contents



1

Introduction

2

Neutron-proton interaction

3

Separation energies

4

Summary



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Separation energies



- Weizsäcker mass formula

$$B(N, Z) = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a (N - Z)^2 A^{-1} + a_p \delta_{\text{pair}} A^{-1/2}$$

- The relation between nuclear masses and the Coulomb energy for $N = Z$

$$M(N - 1, Z) - M(N, Z - 1) = a_c (A - 1)^{2/3} + M_p - M_n$$

Namely $\Delta_m(N - k, Z) = M(N - k, Z) - M(N, Z - k)$

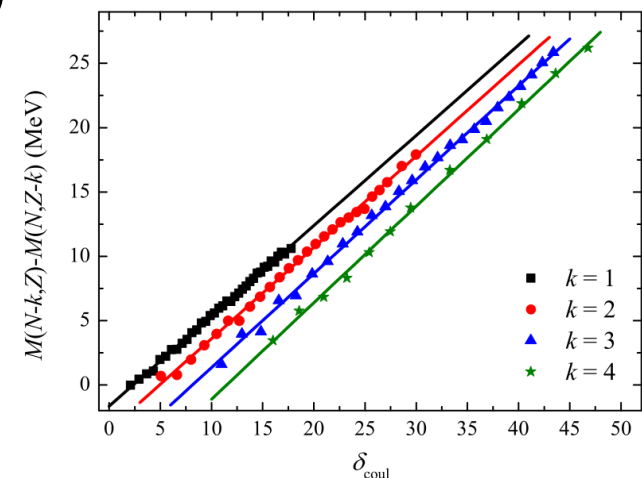
$$= a_c \delta_{\text{coul}} + k(M_p - M_n)$$

where $\delta_{\text{coul}} = k(A - k)^{2/3}$, $k = 1 - 4$

- RMSD values :134, 239, 309, 275 (in keV)

for $k = 1, 2, 3$, and 4, respectively.

M. Bao, Y. Lu, Y. M. Zhao and A. Arima, Phys. Rev. C 94, 044323 (2016)





Separation energies



- One-neutron (proton) separation energy $S_n(N, Z)$, $S_p(N, Z)$:

$$S_n(N, Z) = M(N - 1, Z) - M(N, Z) + M_n;$$

$$S_p(N, Z) = M(N, Z - 1) - M(N, Z) + M_p$$

- We next define two quantities, Δ_n and Δ_p :

$$\Delta_n(N - k, Z) = S_n(N - k, Z) - S_p(N, Z - k) + (M_p - M_n);$$

$$\Delta_p(N - k, Z) = S_p(N - k, Z) - S_n(N, Z - k) + (M_n - M_p)$$

- Finally, we obtain

$$\Delta_n(N - k, Z) = M(N - 1 - k, Z) - M(N - k, Z) - M(N, Z - 1 - k) + M(N, Z - k) = a_c \delta_c^n + (M_p - M_n);$$

$$\Delta_p(N - k, Z) = M(N - k, Z - 1) - M(N - k, Z) - M(N - 1, Z - k) + M(N, Z - k) = a_c \delta_c^p + (M_n - M_p)$$

where $\delta_c^n = (k + 1)(A - k - 1)^{\frac{2}{3}} - k(A - k)^{\frac{2}{3}}$; $\delta_c^p = (k - 1)(A - k - 1)^{\frac{2}{3}} - k(A - k)^{\frac{2}{3}}$

Separation energies

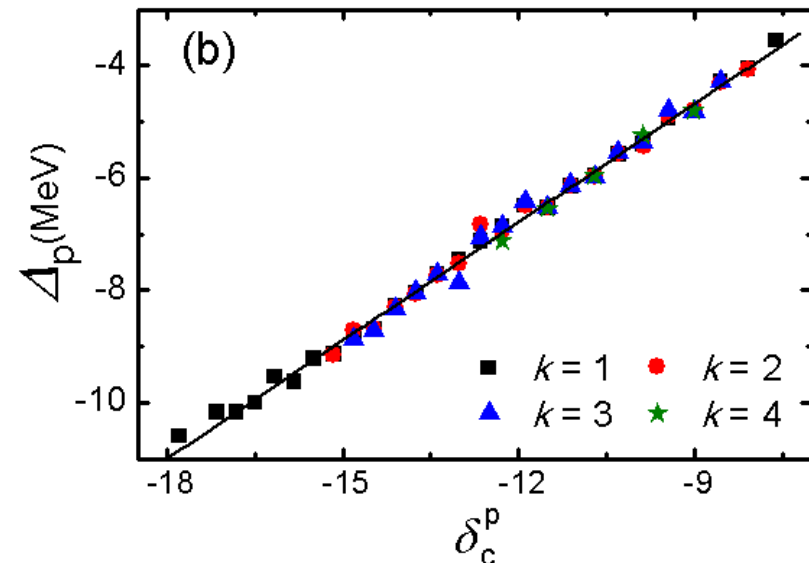
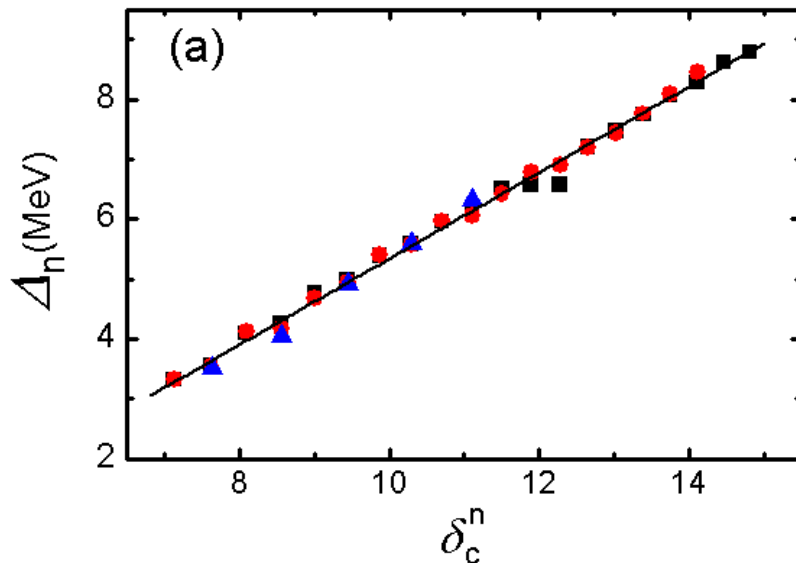


- We rewrite the simple relations in short as

$$\Delta = a_c \delta_c + C$$

Strong linear correlations between Δ and δ_c , independent of k :

$\Delta_{(N \geq 10)}$	RMSD	Number	a_c	C
Δ_n	113	46	718	-1833
Δ_p	132	68	702	+1637



Separation energies



- Mass relations based on this work

$$\begin{aligned} M(N - k, Z) &= M(N - 1 - k, Z) - M(N, Z - 1 - k) \\ &\quad + M(N, Z - k) - a_c \delta_c^n - C \end{aligned}$$

$$\begin{aligned} M(N - k, Z) &= M(N - k, Z - 1) - M(N - 1, Z - k) \\ &\quad + M(N, Z - k) - a_c \delta_c^p - C \end{aligned}$$

- Mass extrapolation : AME2003 \longrightarrow AME2016 ($N \geq 10$)

13 proton-rich nuclei RMSD : **102** keV (346 keV with Δ_m)

- We predict 58 nuclear masses for proton-rich nuclei with $N=[10,50]$,
and $(Z-N) \leq 4$

Contents



1

Introduction

2

Neutron-proton interaction

3

Separation energies

4

Summary



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Summary



- We report two simple relations of masses between corresponding mirror nuclei
for $N \geq 10$.
- The neutron-proton interactions of two mirror nuclei are very close to each other,
namely $\Delta V_{in-jp}(N - 1, Z) \sim 0$.
- The difference between on-nucleon separation energies of two mirror nuclei,
namely $\Delta_n(\Delta_p)$, exhibits strong linear correlations with coulomb correction term.
- These correlations provide a remarkably accurate approach to predict 58 proton-rich nuclei.

谢谢!



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