High-spin states of the $N = 82$ isotones ^{136}Xe, ^{137}Cs, and ^{138}Ba: Monopole-driven competition of neutron core-excitations with two-proton excitations to the $h_{11/2}$ high-j orbit

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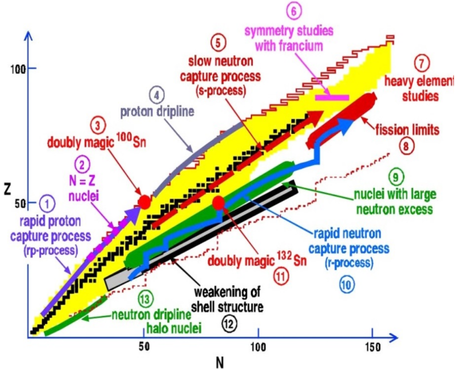
Workshop on nuclear structural theory (Mianyang 2020)

Outline

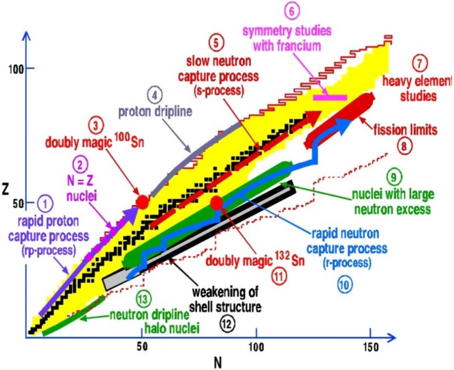
1. **Motivation**
2. **The Model**
3. **Calculated Results**
4. **Conclusion and Discussion**

^{132}Sn mass region



taken from website

^{132}Sn mass region



taken from website

- Magic property of ^{132}Sn[1, 2]

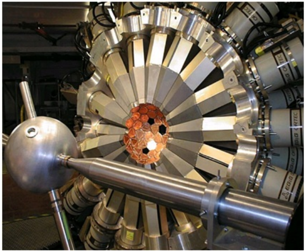
  Nature 465, 454 (2010), Phys. Rev. Lett. 112, 172701 (2014)
- Nucleon-nucleon interactions[3–5]

  Phys. Rev. Lett. 110, 192501 (2013);113, 132502 (2014); 118, 092503 (2017)
- Shell evolutions[6–9] when far away from the stability valley

  Phys. Rev. Lett. 109 , 032501 (2012);111, 152501 (2013);113, 042502(2014); 112, 132501(2014)
- Nucleosynthesis[10–14] relating to the astrophysical *r* process

  Phys. Rev. Lett. 109 , 172501 (2012); 111, 061102 (2013);114, 192501 (2015); 115, 232501 (2015); 118, 202502 (2017)

Neutron core excitations across the $N = 82$ closed shell



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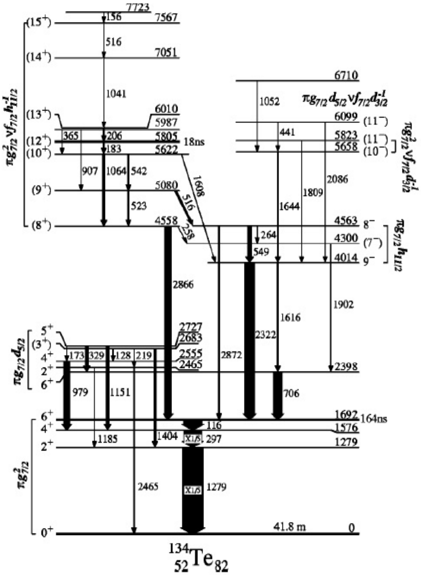
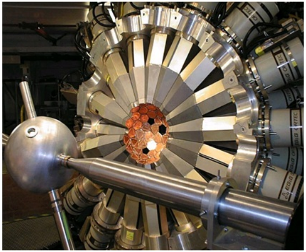


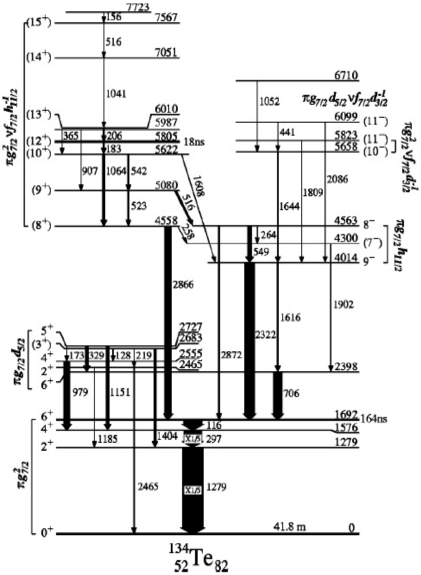
Phys. Rev. C 65, 017302 (2001)

Neutron core excitations across the $N = 82$ closed shell



taken from website

- The medium- or high-spin states with high-excitation energies in some few-valence-nucleon $N = 82$ and $N = 83$ isotones beyond ^{132}Sn have been detected by analyzing different reaction product gamma ray using large gamma-ray detector arrays[15–22].
- Most of them are derived from neutron core excitations across the $N = 82$ closed shell coupling to the valence nucleons. Such core-excited states carry certain structure information.



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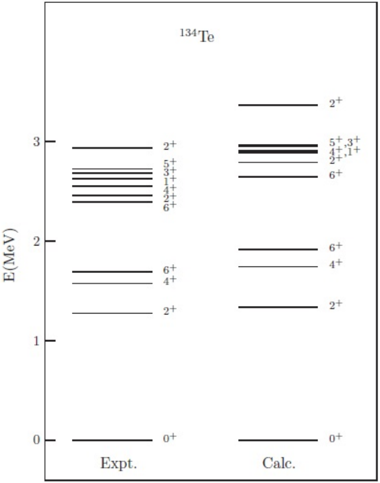
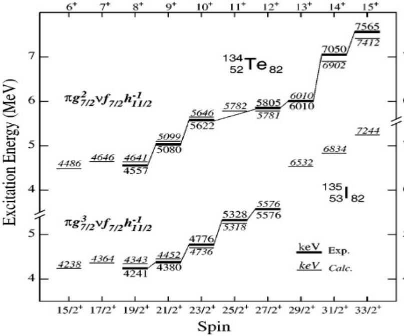
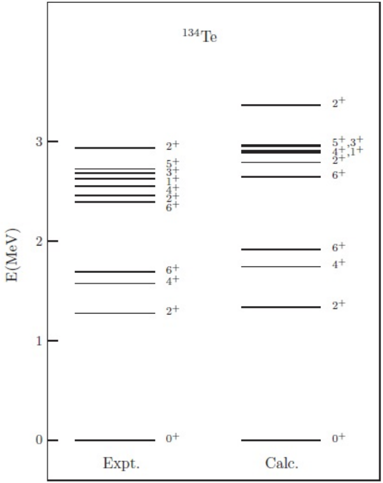


Effective interaction derived from $V_{\text{low-k}}$ approach based on CD-Bonn potential[23])
(Phys. Rev. C 80, 044320 (2009))



Empirical nucleon-nucleon interactions[17]
Phys. Rev. Lett. 77, 3743 (1996)

Shell model description



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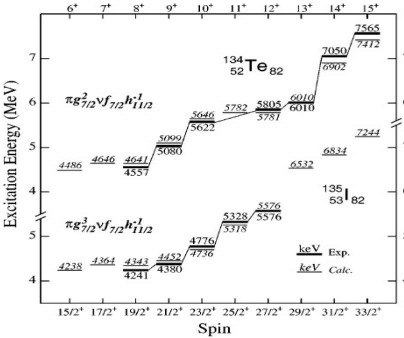

Empirical nucleon-nucleon interactions[17]
Phys. Rev. Lett. 77, 3743 (1996)

Our purpose is to establish a suitable effective shell-model interaction to reproduce the energy levels from low-lying states up to high-spin ones as a whole under a considerably large model space including neutron core excitations.

Outline

proton-neutron (*pn*) representation:

$$
\begin{aligned}
H &= H_{\rm sp} + H_{P_0} + H_{P_2} + H_{QQ} + H_{OO} + H_{HH} + H_{\rm mc} \\
&= \sum_{\alpha,i} \varepsilon_a^i c_{\alpha,i}^\dagger c_{\alpha,i} - \frac{1}{2} \sum_{J=0,2} \sum_{ii'} g_{J,ii'} \sum_M P_{JM,ii'}^\dagger P_{JM,ii'} \\
&- \frac{1}{2} \sum_{\lambda=2,3,4} \sum_{ii'} \frac{\chi_{\lambda,ii'}}{b_0^{2\lambda}} \sum_M : \mathcal{Q}_{\lambda M,ii'}^\dagger \mathcal{Q}_{\lambda M,ii'} : \\
&+ \sum_{j_a \leq j_b, ii'} k_{mc}(ia, i'b) \sum_{JM} A_{JM}^\dagger (ij_a, i'j_b) A_{JM}(ij_a, i'j_b),
\end{aligned} \tag{1}
$$

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&- \frac{1}{2}\sum_{\lambda=2,3,4}\sum_{ii'}\frac{\chi_{\lambda,ii'}}{b_0^{2\lambda}}\sum_M :\mathcal{Q}_{\lambda M,ii'}^\dagger \mathcal{Q}_{\lambda M,ii'}: \\
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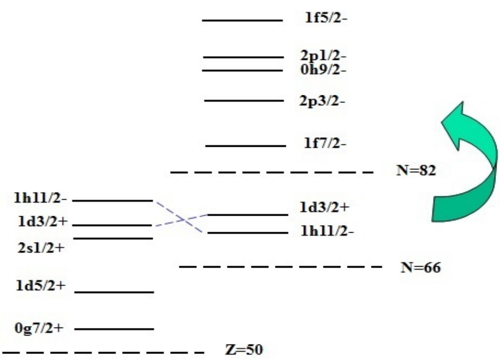
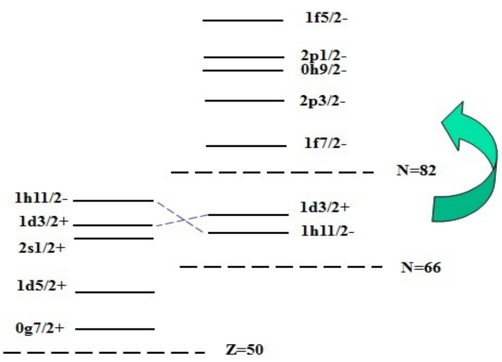
\end{aligned}
\tag{1}
$$

Here, the indices i and i' stand for proton(π) or neutron(ν), and b_0 is the harmonic-oscillator range parameter. The separable forces which include the $J=0$ and $J=2$ pairing (P_0 and P_2) terms, the multipole-multipole terms($\lambda = 2, 3, 4$), and the monopole corrections ($H_{\rm mc}$) are considered.

Model sapce


1f5/2-
2p1/2-
0h9/2-
2p3/2-
1f7/2-
N=82
1h11/2-
1d3/2+
2s1/2+
1d3/2+
1h11/2-
N=66
1d5/2+
0g7/2+
Z=50

Model sapce



$$\varepsilon_a^i = \varepsilon_a'^i - \frac{1}{2j_a + 1} \sum_J \sum_{i',b=hole} (2J+1) \langle i' j_b, i j_a | V | i' j_b, i j_a \rangle_J \qquad (2)$$

- $\varepsilon_a'^i$: single particle or single hole energy from experiments
- ε_a^i: single particle energy for the present model space

Monopole Hamiltonian

$$H_m = \sum_{a,i} \varepsilon_a^i \hat{n}_{ai} + \sum_{ab,ii'} V_{ab}^{ii'} \frac{\hat{n}_{ai}(\hat{n}_{bi'} - \delta_{ab}\delta_{ii'})}{1 + \delta_{ab}\delta_{ii'}}. \tag{3}$$

$V_{ab}^{ii'}$ is the monopole component of the two-body interaction

$$V_{ab}^{ii'} = \frac{\sum_J \langle ij_a, i'j_b | V | ij_a, i'j_b \rangle_J (2J+1)(1+(-1)^J \delta_{ii'}\delta_{ab})}{(2j_a+1)(2j_b+1-\delta_{ii'}\delta_{ab})}, \tag{4}$$

Effective single-particle energies (ESPEs)

Monopole Hamiltonian

$$H_m = \sum_{a,i} \varepsilon_a^i \hat{n}_{ai} + \sum_{ab,ii'} V_{ab}^{ii'} \frac{\hat{n}_{ai}(\hat{n}_{bi'} - \delta_{ab}\delta_{ii'})}{1 + \delta_{ab}\delta_{ii'}}. \quad (3)$$

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ESPE

The ESPE is defined as one-nucleon separation energy for an occupied orbital (or extra binding gained by the addition of a nucleon to an unoccupied orbital) evaluated from the monopole Hamiltonian[24].

Monopole Hamiltonian

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ESPE

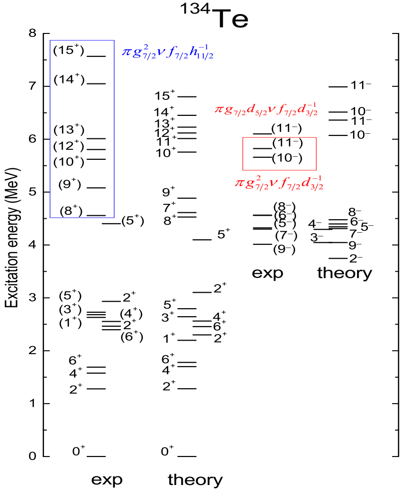
The ESPE is defined as one-nucleon separation energy for an occupied orbital (or extra binding gained by the addition of a nucleon to an unoccupied orbital) evaluated from the monopole Hamiltonian[24].

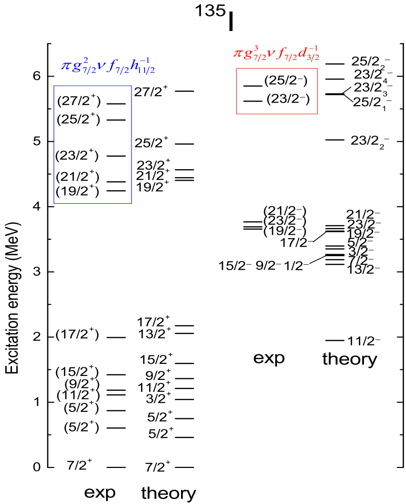
Monopole corrections :

$$\begin{aligned}
&k_{mc}(\nu h_{11/2}, \nu f_{7/2}) = 0.04,\\
&k_{mc}(\nu d_{3/2}, \nu f_{7/2}) = 0.15,\\
&k_{mc}(\pi g_{7/2}, \pi h_{11/2}) = -0.15,\\
&k_{mc}(\pi g_{7/2}, \nu h_{9/2}) = -0.6,\\
&k_{mc}(\pi h_{11/2}, \nu f_{7/2}) = -1.0,\\
&k_{mc}(\pi h_{11/2}, \nu h_{9/2}) = -0.8. \quad \text{(in MeV)}
\end{aligned}$$

N = 82 isotones ^{134}Te, ^{135}I

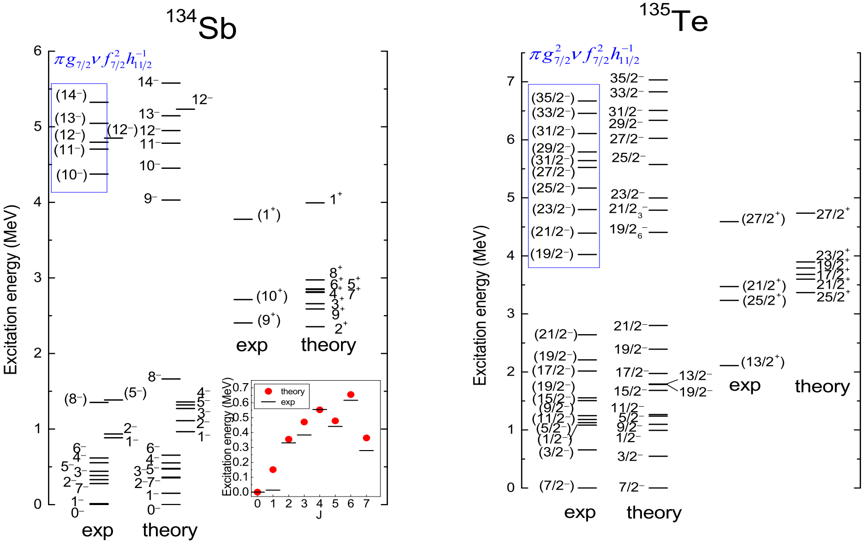
Jin,Hasegawa,Tazaki,Kaneko,and Sun, Phys. Rev. C 84, 044324 (2011)

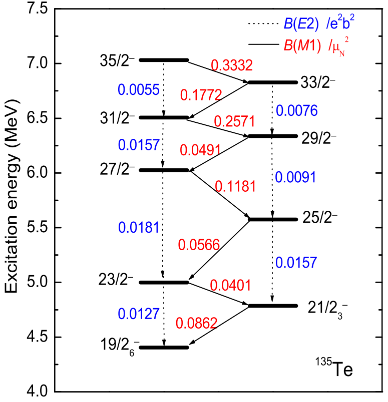




$$k_{mc}(\nu h_{11/2}, \nu f_{7/2}) = 0.04,$$
$$k_{mc}(\nu d_{3/2}, \nu f_{7/2}) = 0.15. \quad \text{(in MeV)}$$

N = 83 isotones ^{134}Sb, ^{135}Te

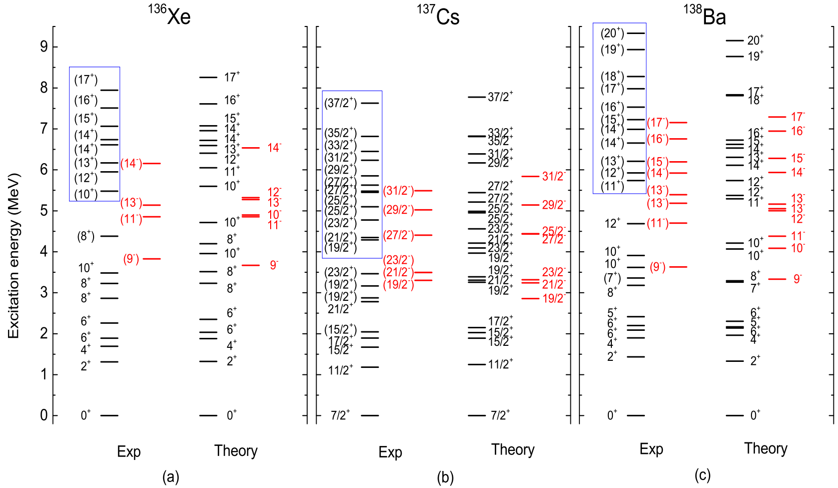




Calculated $B(E2)$ and $B(M1)$ values for the core-excitation states in ^{135}Te

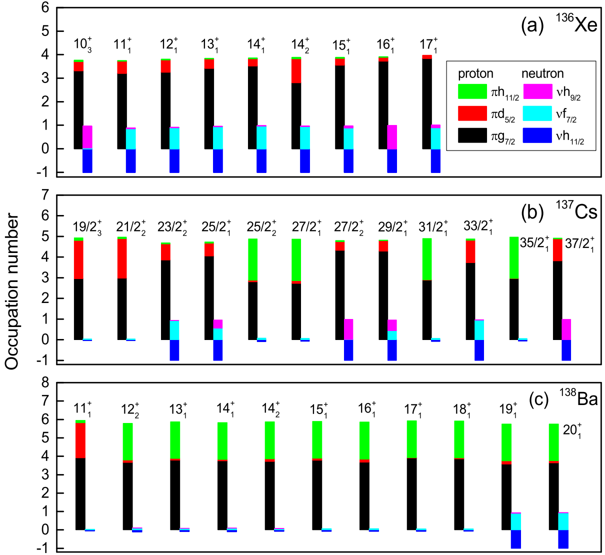
Heavier $N = 82$ isotones ^{136}Xe, ^{137}Cs, ^{138}Ba

Jin,Tazaki,Kaneko,Wang and Sun, Phys. Rev. C 100, 064316 (2019)

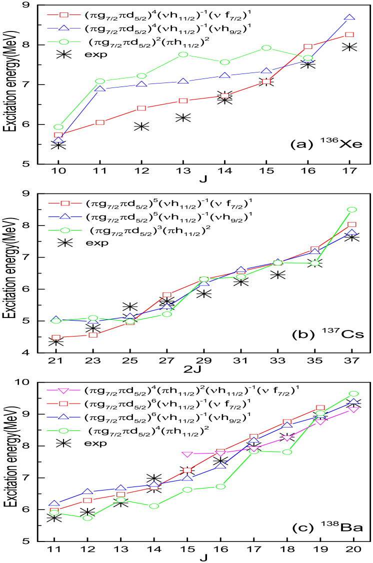


Most of the high-spin states with negative-parity have the predominant configuration of $(\pi g_{7/2}\pi d_{5/2})^{n-1}(\pi h_{11/2})^1$ except the highest 16_1^- and 17_1^- states in ^{138}Ba.

Occupation numbers

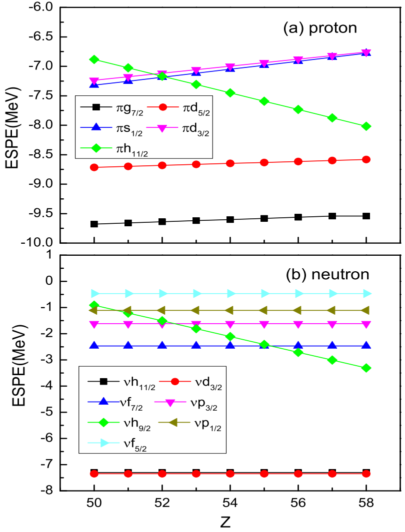


Configurations



Competitions between the configurations of two-proton excitation to the $\pi h_{11/2}$ orbit and those of neutron core excitation across the $N = 82$ closed shell.

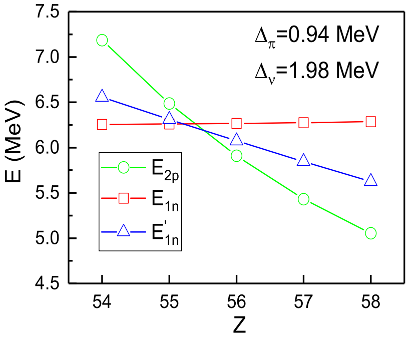
Analysis of ESPEs



$k_{mc}(\pi g_{7/2}, \pi h_{11/2}) = -0.15,$

$k_{mc}(\pi g_{7/2}, \nu h_{9/2}) = -0.6,$ (in MeV)

Quasiparticles' excitation based on the BCS theory



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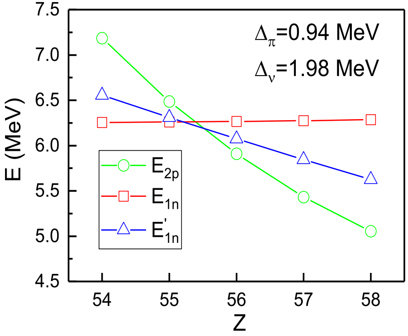
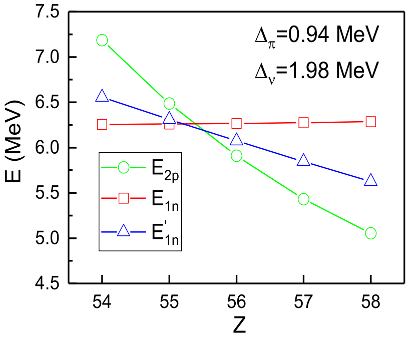


$(\pi g_{7/2})^{n-2}(\pi h_{11/2})^2 \rightarrow \mathrm{E}_{2p}$ $(2p2h)$

$(\pi g_{7/2})^{n}(\nu h_{11/2})^{-1}(\nu f_{7/2})^1 \rightarrow \mathrm{E}_{1n}(1p1h)$

$(\pi g_{7/2})^{n}(\nu h_{11/2})^{-1}(\nu h_{9/2})^1 \rightarrow \mathrm{E}'_{1n}(1p1h)$

Quasiparticles' excitation based on the BCS theory



$(\pi g_{7/2})^{n-2}(\pi h_{11/2})^2 \rightarrow \mathrm{E_{2p}}\ (2p2h)$
$(\pi g_{7/2})^{n}(\nu h_{11/2})^{-1}(\nu f_{7/2})^1 \rightarrow \mathrm{E_{1n}}(1p1h)$
$(\pi g_{7/2})^{n}(\nu h_{11/2})^{-1}(\nu h_{9/2})^1 \rightarrow \mathrm{E'_{1n}}(1p1h)$

- Quasiparticle energy
  $E_{ik} = \sqrt{(\varepsilon_{ik} - \lambda_i)^2 + \Delta_i^2}, \quad i = \pi \text{ or } \nu$
- Valence-particle number
  $N_i = \sum_{k>0}\{1-(\varepsilon_{ik}-\lambda_i)/[(\varepsilon_{ik}-\lambda_i)^2+\Delta_i^2]^{1/2}\}$

Conclusion

1. The effective interaction derived from the EPQQM model has been used to carry out the shell-model calculation to describe the high-spin states in the heavier N = 82 isotones beyond ^{132}Sn.
2. The competition between neutron core excitation and two-proton excitation to the $\pi h_{11/2}$ orbit has been found in the high-spin states for the heavier N =82 isotones.
3. Such competition has been analyzed by the theoretical ESPEs.The adopted monopole corrections contribute to the variation of ESPEs and lead to the structural evolution.

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3. Such competition has been analyzed by the theoretical ESPEs.The adopted monopole corrections contribute to the variation of ESPEs and lead to the structural evolution.

Discussion

1. Spurious center-of-mass motion
2. To construct the monopole terms starting from the monopole-based universal force V_{MU}[25, 26]
3. To extend the present effective interaction to the application in the heavier N = 83 isotones ......

Spurious center-of-mass motion

$$H^{'} = H_{\mathrm{SM}} + H_{\beta} = H_{\mathrm{SM}} + \beta \left\{ \frac{\left(\sum_{i=1}^{A} \vec{p}_i\right)^2}{2Am} + \frac{1}{2}\frac{m\omega^2}{A}\left(\sum_{i=1}^{A} \vec{r}_i\right)^2 - \frac{3}{2}\hbar\omega \right\}$$

Spurious center-of-mass motion

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Two-body potential[27] $V_\beta = \frac{\beta}{A}\left\{\frac{\vec{p}_1\cdot\vec{p}_2}{m} + \frac{1}{2}m\omega^2\vec{r}_1\cdot\vec{r}_2\right\}$

$$\begin{aligned}
&E_{JT}(j_1j_2;j_3j_4) = \langle n_1 l_1\; j_1, n_2 l_2 j_2 \left| V_\beta \right| n_3 l_3 j_3, n_4 l_4 j_4 \rangle \\
&= \frac{\beta\hbar\omega}{A} \sum_{\lambda SnlNL} \frac{\{1-(-1)^{S+T+l}\}}{\sqrt{(1+\delta_{n_1n_2}\delta_{l_1l_2}\delta_{j_1j_2})(1+\delta_{n_3n_4}\delta_{l_3l_4}\delta_{j_3j_4})}} \\
&\times(2\lambda+1)(2S+1)\sqrt{(2j_1+1)(2j_2+1)(2j_3+1)(2j_4+1)} \\
&\times \left\{\begin{array}{ccc} l_1 & l_2 & \lambda \\ \frac{1}{2} & \frac{1}{2} & S \\ j_1 & j_2 & J \end{array}\right\} \left\{\begin{array}{ccc} l_3 & l_4 & \lambda \\ \frac{1}{2} & \frac{1}{2} & S \\ j_3 & j_4 & J \end{array}\right\} [(2N+L)-(2n+l)] \\
&\times M_\lambda(nlNL : n_1 l_1 n_2 l_2) M_\lambda(nlNL : n_3 l_3 n_4 l_4)
\end{aligned}$$

Spurious center-of-mass motion

$$H^{'} = H_{\text{SM}} + H_{\beta} = H_{\text{SM}} + \beta \left\{ \frac{\left(\sum_{i=1}^{A} \vec{p}_i\right)^2}{2Am} + \frac{1}{2}\frac{m\omega^2}{A}\left(\sum_{i=1}^{A} \vec{r}_i\right)^2 - \frac{3}{2}\hbar\omega \right\}$$

Two-body potential[27] $\quad V_{\beta} = \frac{\beta}{A}\left\{ \frac{\vec{p}_1 \cdot \vec{p}_2}{m} + \frac{1}{2} m\omega^2 \vec{r}_1 \cdot \vec{r}_2 \right\}$

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&E_{JT}(j_1 j_2; j_3 j_4) = \langle n_1 l_1\ j_1, n_2 l_2 j_2 \left| V_{\beta} \right| n_3 l_3 j_3, n_4 l_4 j_4 \rangle \\
&= \frac{\beta\hbar\omega}{A} \sum_{\lambda SnlNL} \frac{\{1-(-1)^{S+T+l}\}}{\sqrt{(1+\delta_{n_1 n_2}\delta_{l_1 l_2}\delta_{j_1 j_2})(1+\delta_{n_3 n_4}\delta_{l_3 l_4}\delta_{j_3 j_4})}} \\
&\times (2\lambda+1)(2S+1)\sqrt{(2j_1+1)(2j_2+1)(2j_3+1)(2j_4+1)} \\
&\times \left\{ \begin{array}{ccc} l_1 & l_2 & \lambda \\ \frac{1}{2} & \frac{1}{2} & S \\ j_1 & j_2 & J \end{array} \right\} \left\{ \begin{array}{ccc} l_3 & l_4 & \lambda \\ \frac{1}{2} & \frac{1}{2} & S \\ j_3 & j_4 & J \end{array} \right\} [(2N+L)-(2n+l)] \\
&\times M_{\lambda}(nlNL : n_1 l_1 n_2 l_2) M_{\lambda}(nlNL : n_3 l_3 n_4 l_4)
\end{aligned}$$

Modification of the single-particle enenrgy[27]

$$\tilde{\varepsilon}_j = \varepsilon_j + (2n + l + \tfrac{3}{2})\frac{\beta\hbar\omega}{A} + \frac{1}{2\,(2j+1)} \sum_{JTj_c} (2J+1)(2T+1) E_{JT}(jj_c; jj_c)$$

Spurious center-of-mass motion

$$H^{'} = H_{\rm SM} + H_\beta = H_{\rm SM} + \beta \left\{ \frac{\left(\sum_{i=1}^{A} \vec{p}_i\right)^2}{2Am} + \frac{1}{2}\frac{m\omega^2}{A}\left(\sum_{i=1}^{A} \vec{r}_i\right)^2 - \frac{3}{2}\hbar\omega \right\}$$

Two-body potential[27] $V_\beta = \frac{\beta}{A}\left\{\frac{\vec{p}_1\cdot\vec{p}_2}{m} + \frac{1}{2}m\omega^2\vec{r}_1\cdot\vec{r}_2\right\}$

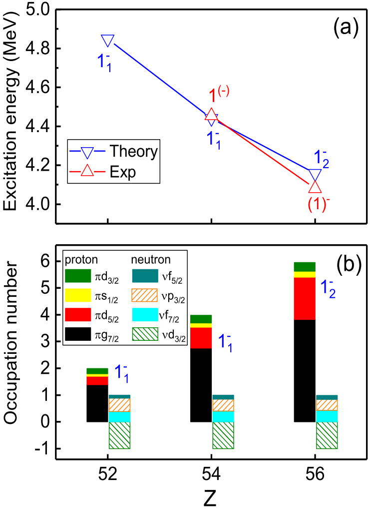
$$\begin{aligned}
&E_{JT}(j_1j_2;j_3j_4) = \langle n_1l_1\ j_1, n_2l_2j_2 \left| V_\beta \right| n_3l_3j_3, n_4l_4j_4\rangle \\
&= \frac{\beta\hbar\omega}{A}\sum_{\lambda SnlNL}\frac{\{1-(-1)^{S+T+l}\}}{\sqrt{(1+\delta_{n_1n_2}\delta_{l_1l_2}\delta_{j_1j_2})(1+\delta_{n_3n_4}\delta_{l_3l_4}\delta_{j_3j_4})}} \\
&\times(2\lambda+1)(2S+1)\sqrt{(2j_1+1)(2j_2+1)(2j_3+1)(2j_4+1)} \\
&\times\left\{\begin{matrix} l_1 & l_2 & \lambda \\ \frac{1}{2} & \frac{1}{2} & S \\ j_1 & j_2 & J \end{matrix}\right\}\left\{\begin{matrix} l_3 & l_4 & \lambda \\ \frac{1}{2} & \frac{1}{2} & S \\ j_3 & j_4 & J \end{matrix}\right\}[(2N+L)-(2n+l)] \\
&\times M_\lambda(nlNL : n_1l_1n_2l_2)M_\lambda(nlNL : n_3l_3n_4l_4)
\end{aligned}$$

Modification of the single-particle enenrgy[27]

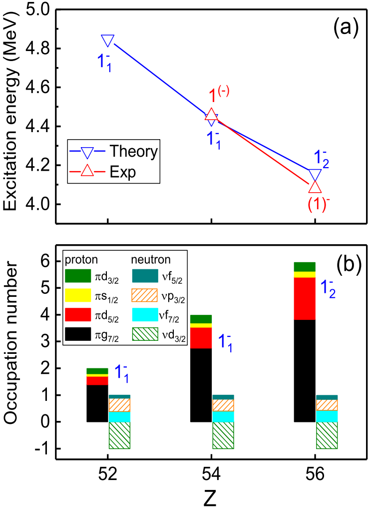
$$\tilde{\varepsilon}_j = \varepsilon_j + (2n+l+\tfrac{3}{2})\frac{\beta\hbar\omega}{A} + \frac{1}{2\,(2j+1)}\sum_{JTj_c}(2J+1)(2T+1)E_{JT}(jj_c;jj_c)$$

D. H. Gloeckner and R. D. Lawson, Phys. Lett. 53B, 313 (1974)

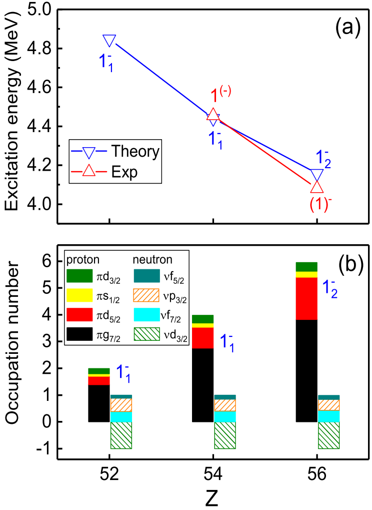
1^- spuriosities



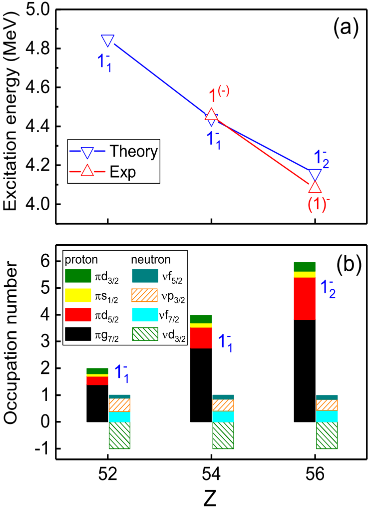
1^- spuriosities



1^- spuriosities



1^- spuriosities



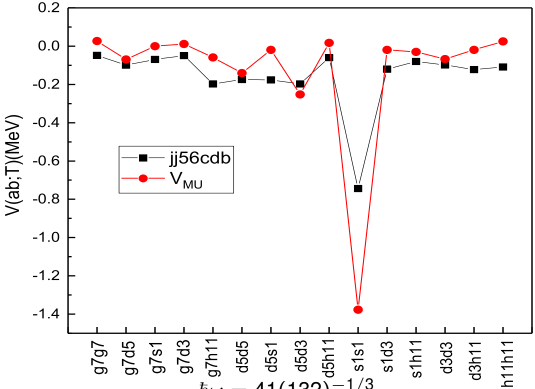
- Spurious center-of-mass motion has an effect of the order $1/A$ [28]
- Such spuriosities could be neglected for heavy nuclei

Monopole interaction

Monopole-based universal interaction V_{MU} Phys. Rev. Lett. 104, 012501 (2010)

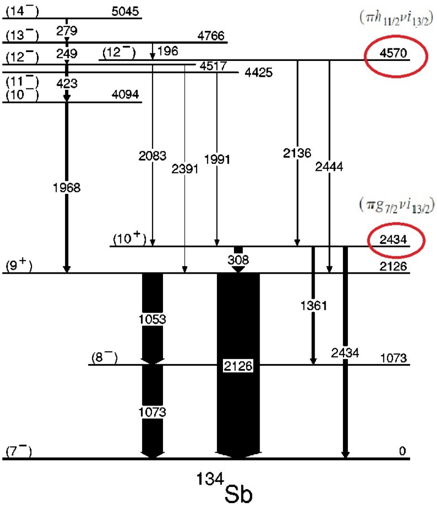
$$V_{MU} \rightarrow V_C\,(\text{Gaussian}) + V_T\,(\pi + \rho)$$

- Central Gaussian force $V_C = \sum_{S,T} f_{S,T} P_{S,T} \exp\left[-(r/\mu)^2\right]$
- Tensor force $V_T = (\vec{\tau}_1 \cdot \vec{\tau}_2)\left(\left[\vec{s}_1 \vec{s}_2\right]^{(2)} \cdot Y^{(2)}\right) f(r)$



$\hbar\omega = 41(132)^{-1/3}$

Neutron $\nu i13/2$ excitation in $N = 83$ isotones



Phys. Rev. C 63, 024322 (2001)

The End

Thank you for your attention!!

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