Deformed Random Phase Approximation in the Laboratory Frame

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Outline

- Introduction
- Basic idea of the deformed QRPA in the laboratory frame (dQRPA)
- Theoretical formalism of dQRPA
- Applications of dQRPA
- Numerical comparison between dQRPA and the deformed QRPA in the intrinsic frame (mQRPA) on their E2 behaviors
- Summary

Introduction

- Rotational symmetry restoration in a deformed approach

deformed description

- ✓ Based on the **intrinsic frame**
- ✓ Rotational symmetry **broken**
- ✓ Good spin projection (for axially deformed case)

Symmetry restoration

experimental data

- ✓ Observed in the **laboratory frame**
- ✓ Rotational symmetry **conserved**
- ✓ Good total spin

- Projection (symmetry restoration) after/before variation

Projection **after** variation

Basis in the intrinsic frame

Equation of motion

Wave function

in the intrinsic frame

projection

Wave function

in the laboratory frame

Projection **before** variation

Basis in the intrinsic frame

projection

Basis in the laboratory frame

Equation of motion

Wave function

in the laboratory frame

Intrinsic structure determined

without/with

symmetry consideration

- Deformed QRPA in the intrinsic frame: projection after variation

Deformed quasi-particle states

Basis in the intrinsic frame

$$b_{n\nu}^{+} = \sum_{j} x_{j\nu}^{n} c_{j\nu}^{+} \qquad \beta_{n\nu}^{+} = u_{n\nu} b_{n\nu}^{+} - v_{n\nu} b_{\overline{n\nu}}$$

QRPA phonons in the intrinsic frame

Wave function in the intrinsic frame

$$\Gamma_{K}^{+} = \sum_{\tau} \sum_{n_1\nu_1 < \overline{n_2\nu_2}} \mathcal{X}_{\tau n_1\nu_1,\tau n_2\nu_2} \beta_{\tau n_1\nu_1}^{+} \beta_{\overline{\tau n_2\nu_2}}^{+} - \mathcal{Y}_{\tau n_1\nu_1,\tau n_2\nu_2} \beta_{\overline{\tau n_1\nu_1}} \beta_{\tau n_2\nu_2}$$

Symmetry restoration

Wave function in the laboratory frame

$$|\Psi_{IMK}\rangle = \mathcal{D}_{MK}^{I*}(\omega)\Gamma_{K}^{+}|0\rangle \quad (K=0)$$

$$|\Psi_{IMK}\rangle = \frac{1}{\sqrt{2}}[\mathcal{D}_{MK}^{I*}(\omega)\Gamma_{K}^{+} + (-)^{I-K}\mathcal{D}_{M-K}^{I*}(\omega)\Gamma_{\overline{K}}^{+}]|0\rangle \quad (K>0)$$

Introduction

- A possible version of deformed QRPA as a ‘projection before variation’ apprach

deformed QRPA with symmetry consideration

projection after variation

projection before variation

deformed QRPA in the intrinsic frame (mQRPA)

deformed QRPA in the laboratory frame (dQRPA)

comparison between the two

to be discussed

Basic idea of dQRPA

- Deformed single-particle basis in the laboratory frame

Projection before variation

Basis in the intrinsic frame → Basis in the laboratory frame → Wave function in the laboratory frame

rotational symmetry

character of deformation

spin quantum numbers

Nilsson-like energy levels

Band heads in odd-mass nuclei described by **core+particle** wave functions

↓

'core+particle' wave functions as single-particle basis

- Connections between dQRPA and other QRPA approaches

mQRPA	dQRPA	spherical QRPA
Nilsson orbital	'core+particle' basis	spherical s.p. basis

With the same level scheme & occupation pattern → similar in physics

With the same quantum numbers → similar in formalism

Theoretical formalism of dQRPA

- Definition of the 'core+particle' basis
  - ✓ Wave function of a 'core+particle' system in the adiabatic case (Coriolis force neglected)

Nilsson states

$$\psi^{j}_{nm\nu}(\omega,\vec{r}') = \frac{1}{\sqrt{2}}[\mathcal{D}^{j*}_{m\nu}(\omega)\phi_{n\nu}(\vec{r}')+(-)^{j-\nu}\mathcal{D}^{j*}_{m-\nu}(\omega)\phi_{n\overline{\nu}}(\vec{r}')]$$

  - ✓ The bandhead condition to froze the rotation of the core

$$j = |\nu| \qquad\qquad \psi^{j}_{nm\nu}(\omega,\vec{r}') \rightarrow \psi^{j}_{nm}(\omega,\vec{r}')$$

  - ✓ Treating the 'core+particle' system as a single fermion

$$\psi^{j}_{nm}(\omega,\vec{r}') \rightarrow a^{+}_{njm}|0\rangle \qquad \{a_{n'j'm'}, a^{+}_{njm}\} = \delta_{nn'}\delta_{jj'}\delta_{mm'}$$

Theoretical formalism of dQRPA

- Connection between the ‘core+particle’ basis and the Nilsson orbitals

Nilsson orbitals

$\phi_{n,\nu=j}, \phi_{\overline{n,\nu=j}}$

2-fold degenerate

correspondence

‘core+particle’ basis

$\psi^j_{nm}, \quad m=-j,\cdots,j$

$(2j+1)$ -fold degenerate

common s.p. energy ϵ_{nj}

accomodating 2 particles at most

common Fermi surface

Theoretical formalism of dQRPA

- ‘laboratory form’ of the ‘core+particle’ basis

$$\psi^j_{nm}(\omega,\vec{r}') = \frac{1}{\sqrt{2}}[\mathcal{D}^{j*}_{mj}(\omega)\phi_{nj}(\vec{r}')+(-)^{j-\nu}\mathcal{D}^{j*}_{m-j}(\omega)\phi_{\overline{nj}}(\vec{r}')]$$

property of the Nilsson orbital

$$\phi_{nj}(\vec{r}') = \sum_{j'\geq j} x^n_{j'j}\varphi_{j'j}(\vec{r}')$$

Property of the Euler angles

$$\varphi_{j'j}(\vec{r}') = \sum_{m'} D^{j'}_{m'j}(\omega)\varphi_{j'm'}(\vec{r})$$

$$\psi^j_{nm}(\omega,\vec{r}) = \boxed{\sum_{J=even}}\boxed{\sum_{j'\geq j}} X^{Jj'}_{nj}[\mathcal{D}^{J*}_{.0}(\omega)\varphi_{j'}(\vec{r})]_{jm} \qquad X^{Jj'}_{nj} = C^{j,j',J}_{j,-j,0}x^n_{j'j}$$

core particle

Rotation ability of the core

deformed behaviour of the basis

$$\boxed{J=0 \longrightarrow j'=j}$$

- Second quantization in the 'core+particle' representation

  ✓ Anti-commutator between creation/annihilation operators

$$\psi^{j}_{nm} \rightarrow a^{+}_{njm}|0\rangle$$

$$\{a_{n'j'm'}, a^{+}_{njm}\} = \delta_{nn'}\delta_{jj'}\delta_{mm'}$$

  ✓ Representation of multipole operators

$$\hat{O}_{\lambda\mu} = \sum_{n_1 j_1 m_1} \sum_{n_2 j_2 m_2} \langle n_1 j_1 m_1|\hat{O}_{\lambda\mu}|n_2 j_2 m_2\rangle a^{+}_{n_1 j_1 m_1} a_{n_2 j_2 m_2}$$

$$= \sum_{n_1 j_1} \sum_{n_2 j_2} \frac{\boxed{\langle n_1 j_1||\hat{O}_{\lambda}||n_2 j_2\rangle}}{\sqrt{2\lambda+1}} [a^{+}_{n_1 j_1} \tilde{a}_{n_2 j_2}]_{\lambda\mu}$$

$$\tilde{a}_{njm} \equiv (-)^{j+m} a_{nj-m}$$

of central importance in the implementation of dQRPA

- Reduced matrix elements in the 'core+particle' basis

$$\langle n_1 j_1 || \hat{O}_\lambda || n_2 j_2 \rangle = \sqrt{2j_1+1} \sum_\mu C^{j_2,\lambda,j_1}_{m_1-\mu,\mu,m_1} \langle n_1 j_1 m_1 | \hat{O}_{\lambda\mu} | n_2 j_2 m_1-\mu \rangle$$

$$\psi^j_{nm}(\omega,\vec{r}) = \sum_{J=even} \sum_{j'\geq j} X^{Jj'}_{nj} [\mathcal{D}^{J*}_{.0}(\omega)\varphi_{j'}(\vec{r})]_{jm}$$

$\hat{O}_{\lambda\mu}$ acts on $\vec{r}$ only:

$$\int d\omega \mathcal{D}^{J_1}_{m_1-\mu_1,0}(\omega)\mathcal{D}^{J_2*}_{m_1-\mu-\mu_2,0}(\omega)$$

$$\langle j'_1 \mu_1 | \hat{O}_{\lambda\mu} | j'_2 \mu_2 \rangle = \frac{C^{j'_2,\lambda,j'_1}_{\mu_2,\mu,\mu_1}}{\sqrt{2j'_1+1}} \langle j'_1 || \hat{O}_\lambda || j'_2 \rangle$$

$$\langle n_1 j_1 || \hat{O}_\lambda || n_2 j_2 \rangle = \sqrt{(2j_1+1)(2j_2+1)} \sum_J \sum_{j'_1 j'_2} X^{Jj'_1}_{n_1 j_1} X^{Jj'_2}_{n_2 j_2} (-)^{j_2+j'_1-J-\lambda} W(j_1 j'_1 j_2 j'_2; J\lambda) \langle j'_1 || \hat{O}_\lambda || j'_2 \rangle$$

Theoretical formalism of dQRPA

➢ Bogoliubov transformation (in parallel with the Nilsson case)

'core+particle' case	Nilsson case
$\alpha^{+}_{njm} = u_{nj} a^{+}_{njm} - v_{nj} a_{\overline{njm}}$	$\beta^{+}_{n\nu} = u_{n\nu} b^{+}_{n\nu} - v_{n\nu} b_{\overline{n\nu}}$
$\alpha^{+}_{\overline{njm}} = u_{nj} a^{+}_{\overline{njm}} + v_{nj} a_{njm}$	$\beta^{+}_{\overline{n\nu}} = u_{n\nu} b^{+}_{\overline{n\nu}} + v_{n\nu} b_{n\nu}$

s.p. energies ϵ_{nj} ⟷ (equal with $j = \nu$) ⟷ s.p. energies $\epsilon_{n\nu}$

$2j+1$ -fold degenerate
Accomodating at most
2 particles ⟷ (equivalent) ⟷ 2-fold degenerate

Particle number A
Pairing strength G ⟷ (equal) ⟷ Particle number A
Pairing strength G

$u_{nj} = u_{n\nu}$
$v_{nj} = v_{n\nu}$

➢ Representation of operators in terms of quasiparticle pairs

$$\hat{O}_{\lambda\mu} = \sum_{n_1 j_1} \sum_{n_2 j_2} \frac{\langle n_1 j_1 || \hat{O}_\lambda || n_2 j_2 \rangle}{\sqrt{2\lambda + 1}} [a^+_{n_1 j_1} \tilde{a}_{n_2 j_2}]_{\lambda\mu}$$

$$\alpha^+_{njm} = u_{nj} a^+_{njm} - v_{nj} a_{\overline{njm}}$$
$$\alpha^+_{\overline{njm}} = u_{nj} a^+_{\overline{njm}} + v_{nj} a_{njm}$$

Formally parallel to the spherical QRPA

$$\tau_{12} \equiv \{\tau n_1 j_1, \tau n_2 j_2\}|_{n_1 j_1 \le n_2 j_2}$$

$$\bar{A}^+_{\lambda\mu}(\tau_{12}) \equiv \frac{[\alpha^+_{\tau n_1 j_1} \alpha^+_{\tau n_2 j_2}]_{\lambda\mu}}{\sqrt{1 + \delta_{n_1 n_2} \delta_{j_1 j_2}}}$$

$$\bar{A}_{\lambda\mu}(\tau_{12}) = (-)^{\lambda+\mu} \frac{[\alpha_{\overline{\tau n_1 j_1}} \alpha_{\overline{\tau n_2 j_2}}]_{\lambda\mu}}{\sqrt{1 + \delta_{n_1 n_2} \delta_{j_1 j_2}}}$$

$$\hat{O}_{\lambda\mu} = \sum_{\tau_{12}} \xi^\lambda_{\tau_{12}} [\bar{A}^+_{\lambda\mu}(\tau_{12}) + (-)^{\lambda-\mu} \bar{A}_{\lambda-\mu}(\tau_{12})]$$

$$\xi^\lambda_{\tau_{12}} = \frac{\langle n_1 j_1 || \hat{O}^\lambda || n_2 j_2 \rangle}{\sqrt{2\lambda + 1}\sqrt{1 + \delta_{n_1 n_2} \delta_{j_1 j_2}}} (u_{\tau n_1 j_1} v_{\tau n_2 j_2} + v_{\tau n_1 j_1} u_{\tau n_2 j_2})$$

Theoretical formalism of dQRPA

- The dQRPA equation

  - ✓ Quasi-boson approximation

$$[\bar{A}_{\lambda\mu}(\tau_{12}), \bar{A}^{+}_{\lambda\mu}(\tau'_{12})] = \delta_{\tau_{12},\tau'_{12}}$$

  - ✓ The dQRPA phonon

$$\Gamma^{+}_{\lambda\mu} = \sum_{\tau_{12}} \mathcal{X}_{\lambda}(\tau_{12})\bar{A}^{+}_{\lambda\mu}(\tau_{12}) - \mathcal{Y}_{\lambda}\bar{A}_{\lambda-\mu}(-)^{\lambda-\mu}$$

Formally parallel to the spherical QRPA

  - ✓ Equation of motion

$$\hat{H} = \sum_{\tau}[\sum_{nj}\epsilon_{\tau nj}\hat{N}_{\tau nj} - G_{\tau}\hat{P}^{+}_{\tau}\hat{P}_{\tau}] - \frac{1}{2}\sum_{\tau\tau'}\sum_{\lambda}F^{\lambda}_{\tau\tau'}\sum_{\mu}\hat{Q}_{\tau\lambda\mu}\hat{Q}^{+}_{\tau'\lambda\mu}$$

$$[\hat{H}, \Gamma^{+}_{\lambda\mu}] = \omega_{\lambda}\Gamma^{+}_{\lambda\mu}$$

$$\downarrow$$

$$\langle 0|[\bar{A}_{\lambda\mu}(\tau_{12}), [\hat{H}, \hat{\Gamma}^{+}_{\lambda\mu}]|0\rangle = \omega_{\lambda}\langle 0|[\bar{A}_{\lambda\mu}(\tau_{12}), \hat{\Gamma}^{+}_{\lambda\mu}]|0\rangle$$

$$\langle 0|[\bar{A}^{+}_{\lambda\mu}(\tau_{12}), [\hat{H}, (-)^{\lambda-\mu}\hat{\Gamma}^{+}_{\lambda-\mu}]]|0\rangle = (-)^{\lambda-\mu}\omega_{\lambda}\langle 0|[\bar{A}^{+}_{\lambda\mu}(\tau_{12}), \hat{\Gamma}^{+}_{\lambda-\mu}]|0\rangle$$

✓ The dQRPA equation

$$\langle 0|[\bar{A}_{\lambda\mu}(\tau_{12}), [\hat{H}, \hat{\Gamma}^{+}_{\lambda\mu}]|0\rangle = \omega_\lambda \langle 0|[\bar{A}_{\lambda\mu}(\tau_{12}), \hat{\Gamma}^{+}_{\lambda\mu}]|0\rangle$$

$$\langle 0|[\bar{A}^{+}_{\lambda\mu}(\tau_{12}), [\hat{H}, (-)^{\lambda-\mu}\hat{\Gamma}^{+}_{\lambda-\mu}]]|0\rangle = (-)^{\lambda-\mu}\omega_\lambda \langle 0|[\bar{A}^{+}_{\lambda\mu}(\tau_{12}), \hat{\Gamma}^{+}_{\lambda-\mu}]|0\rangle$$

$$\hat{O}_{\lambda\mu} = \sum_{\tau_{12}} \xi^{\lambda}_{\tau_{12}} [\bar{A}^{+}_{\lambda\mu}(\tau_{12}) + (-)^{\lambda-\mu}\bar{A}_{\lambda-\mu}(\tau_{12})]$$

$$\Gamma^{+}_{\lambda\mu} = \sum_{\tau_{12}} \mathcal{X}_\lambda(\tau_{12})\bar{A}^{+}_{\lambda\mu}(\tau_{12}) - \mathcal{Y}_\lambda \bar{A}_{\lambda-\mu}(-)^{\lambda-\mu}$$

$$[\bar{A}_{\lambda\mu}(\tau_{12}), \bar{A}^{+}_{\lambda\mu}(\tau'_{12})] = \delta_{\tau_{12},\tau'_{12}}$$

Formally parallel to the spherical QRPA

$$\begin{pmatrix} \mathcal{A}_\lambda(\tau_{12},\tau'_{12}) & \mathcal{B}_\lambda(\tau_{12},\tau'_{12}) \\ -\mathcal{B}_\lambda(\tau_{12},\tau'_{12}) & -\mathcal{A}_\lambda(\tau_{12},\tau'_{12}) \end{pmatrix} \begin{pmatrix} \mathcal{X}_\lambda(\tau'_{12}) \\ \mathcal{Y}_\lambda(\tau'_{12}) \end{pmatrix} = \omega_\lambda \begin{pmatrix} \mathcal{X}_\lambda(\tau_{12}) \\ \mathcal{Y}_\lambda(\tau_{12}) \end{pmatrix}$$

$$\mathcal{A}_\lambda(\tau_{12},\tau'_{12}) = \delta_{\tau_{12}\tau'_{12}}(E_{\tau n_1 j_1} + E_{\tau n_2 j_2}) - F^{\lambda}_{\tau\tau'}\xi^{\lambda}_{\tau_{12}}\xi^{\lambda}_{\tau'_{12}}$$

$$\mathcal{B}_\lambda(\tau_{12},\tau'_{12}) = -F^{\lambda}_{\tau\tau'}\xi^{\lambda}_{\tau_{12}}\xi^{\lambda}_{\tau'_{12}}$$

- Reduced transition probabilities to the ground state

$$\langle 0||\hat{T}_\lambda||\lambda\rangle = \sqrt{2\lambda+1}\langle 0|\hat{T}^+_{\lambda\mu}|\lambda\mu\rangle$$

$$\hat{T}_{\lambda\mu} = \sum_\tau e_\tau \hat{Q}_{\lambda\mu}$$

$$= \sqrt{2\lambda+1}\sum_\tau e_\tau \langle 0|[\hat{Q}_{\lambda\mu}, \Gamma^+_{\lambda\mu}]|0\rangle$$

$$= \sqrt{2\lambda+1}\sum_{\tau_{12}} e_\tau \xi^\lambda_{\tau_{12}}[\mathcal{X}_\lambda(\tau_{12}) + \mathcal{Y}_\lambda(\tau_{12})]$$

Formally parallel to the spherical QRPA

$$B(E\lambda, \lambda \to 0) = \frac{1}{2\lambda+1}|\langle 0||\hat{T}_\lambda||\lambda\rangle|^2$$

Applications of dQRPA

- E2 transitions

  D. S. Delion, J. Suhonen, Phys. Rev. C 87, 024309 (2013)

- Gamow-Teller strength & $2\nu\beta\beta$ decays
  (with a slightly modified framework called pn-dQRPA)

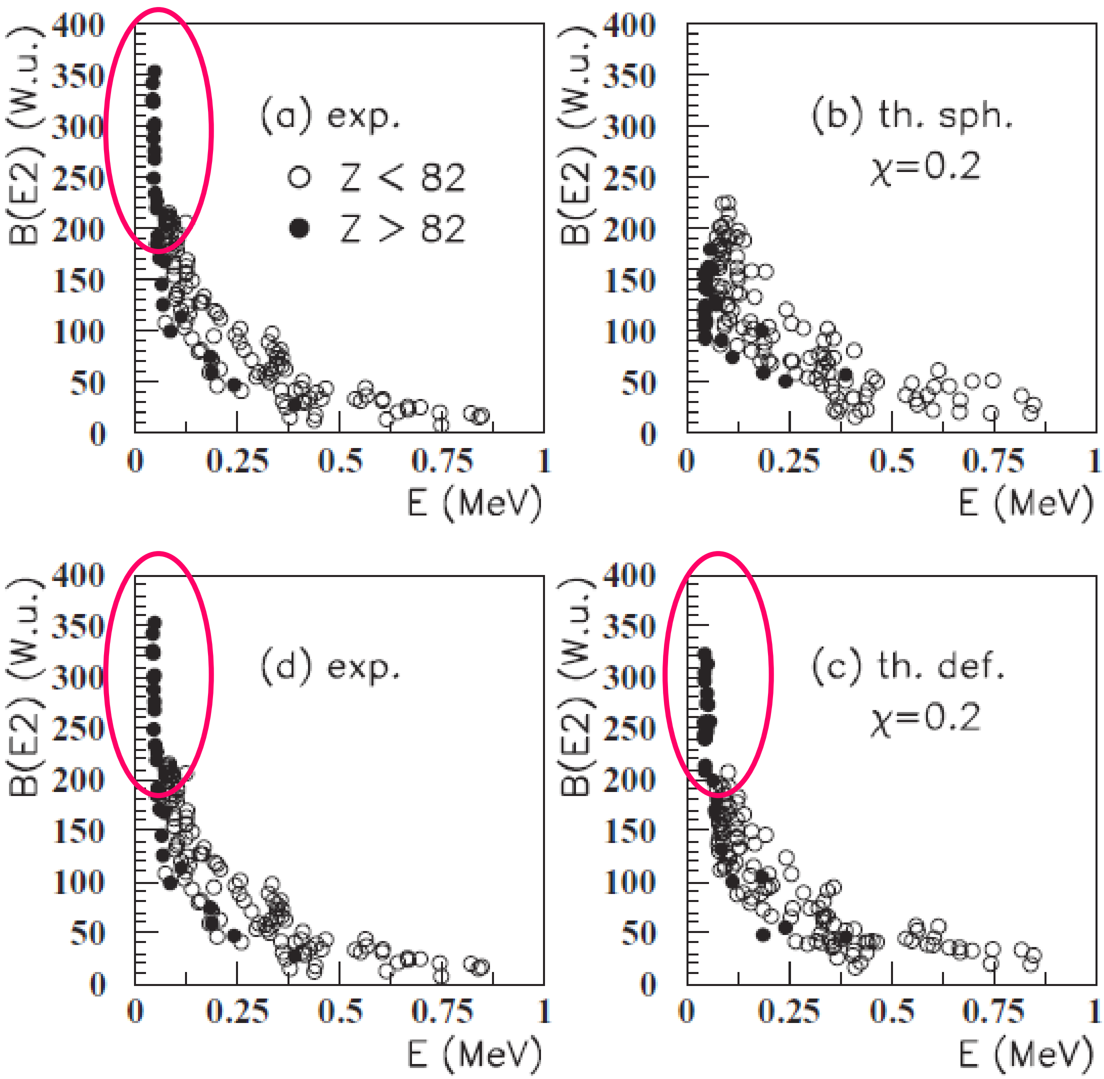
  D. S. Delion, J. Suhonen, Phys. Rev. C 91, 054329 (2015)

  D. S. Delion, J. Suhonen, Phys. Rev. C 95, 034330 (2017)

  D. S. Delion, A. Dumitrescu, J. Suhonen, Phys. Rev. C 100, 024331 (2019)

- E2 transitions

  Systematic reproduction for the $B(E2, 2_1^+ \to 0)$ data in the region $50 \leq Z \leq 100$



- Large E2 strength to the g.s. as an effect of deformation: improvement by the dQRPA *compared to the spherical QRPA*

PRC 87, 024309 (2013)

- Gamow-Teller strength & 2νββ decays

  (with a slightly modified framework called pn-dQRPA)

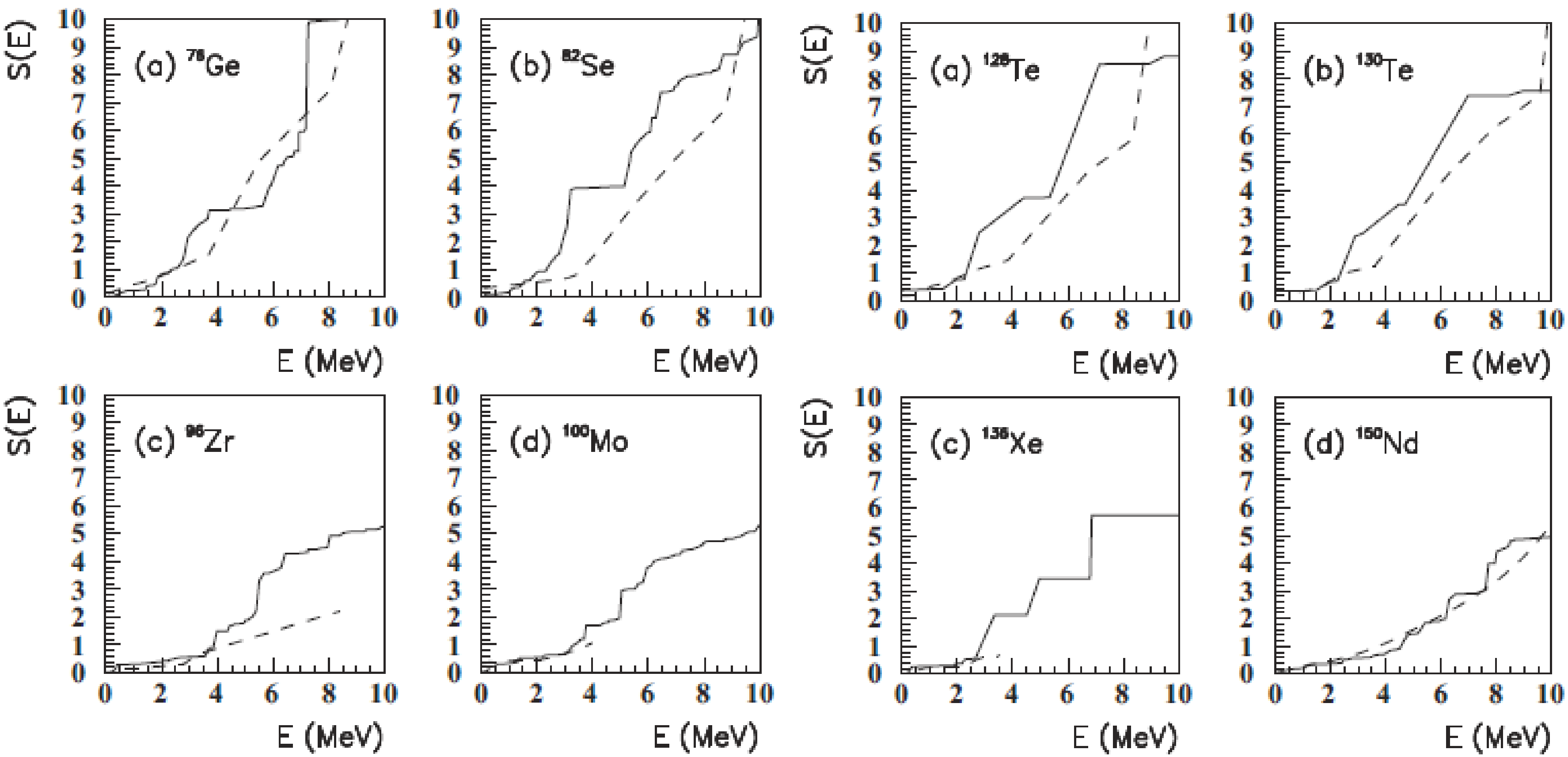
	dQRPA	pn-dQRPA
2qp pairs	2 protons 2 neutrons	1 proton + 1 neutron
residual interactions	pairing + quadrupole	pairing + p-n dipole
transition operator	$\hat{Q}_{2\mu}$	$\hat{\sigma}$

Applications of dQRPA

✓ Gamow-Teller strengths

Cumulative GT strength: theory .vs. experiment

——— Theory - - - - - - Experiment



PRC 95, 034330 (2017)

✓ 2νββ half-lives

PRC 91, 054329 (2015)

TABLE I. $2\nu\beta\beta$ emitters with charge and mass numbers given in the first and second columns. Mother/daughter quadrupole deformation parameter [22] is given in the third/fourth column, theoretical spherical/deformed half-life in fifth/sixth column and experimental value in the last column.

Z	A	β_L	β_R	$\log_{10} T_{\text{th}}^{(\text{sph})}$	$\log_{10} T_{\text{th}}^{(\text{def})}$	$\log_{10} T_{\text{exp}}$
34	82	0.150	0.070	18.83	19.05	19.96
40	96	0.220	0.080	17.71	18.95	19.36
42	100	0.240	0.160	17.70	18.63	18.85
52	128	0.000	0.140	24.99	24.70	24.30
52	130	0.000	−0.110	22.31	21.23	20.84
60	150	0.240	0.210	18.55	18.93	18.91
92	238	0.210	0.210	20.93	21.54	21.30

improvement by the dQRPA *compared to the spherical QRPA*

Numerical comparison between dQRPA and mQRPA on their E2 behaviors

- Motivation
- Correspondence between the subspaces in the two QRPA approaches
- Comparison between the behaviors in subspaces corresponding to each other
- Comparison between the behaviors in full configuration spaces
- Conclusions

Numerical comparison between dQRPA and mQRPA

- Motivation

Conserved rotational symmetry

Effect of deformation

Spherical QRPA .vs. dQRPA .vs. mQRPA

Improvement by dQRPA as the effect of deformation

Any difference as the effect of restored symmetry?

Numerical comparison between dQRPA and mQRPA

- Correspondence between the subspaces in the two QRPA approaches

	mQRPA		dQRPA	
s.p. states	$b^+_{n\nu}$	$\xleftrightarrow{\nu=\pm j}$	a^+_{njm}	same sp energy
2qp pairs	$\beta^+_{n_1\nu_1}\beta^+_{\overline{n_2\nu_2}}$	$\xleftrightarrow[\nu_2=\pm j_2]{\nu_1=\pm j_1}$	$\dfrac{[\alpha^+_{\tau n_1 j_1}\alpha^+_{\tau n_2 j_2}]_{\lambda\mu}}{\sqrt{1+\delta_{n_1 n_2}\delta_{j_1 j_2}}}$	same 2qp energy
subspaces	$K=\nu_1-\nu_2=0$	$\longleftrightarrow$	$\lvert j_1\pm j_2\rvert=0$	same dimensions
	$K=\nu_1-\nu_2=2$	$\longleftrightarrow$	$\lvert j_1\pm j_2\rvert=2$	

Numerical comparison between dQRPA and mQRPA

- Comparison between the behaviors in subspaces corresponding to each other: example of ^{170}Yb

  - ✓ Numerical details
  - ✓ E2 strength functions
  - ✓ 2qp compositions of the first excited states
  - ✓ (reduced) matrix elements of the quadrupole operator

Numerical comparison between dQRPA and mQRPA

✓ Numerical details

- deformed s.p. potential:

$$V = V_0 - \beta_2 m\omega^2 r^2 Y_{20}(\theta, \phi)$$

spherical W. S. potential

axially symmetric deformation

- Pairing strength G_τ :

  determined by pairing gaps extracted from mass difference

- Strength of quadrupole-quadrupole interaction $F^2_{\tau\tau'}$:

  irrespective of τ, τ'

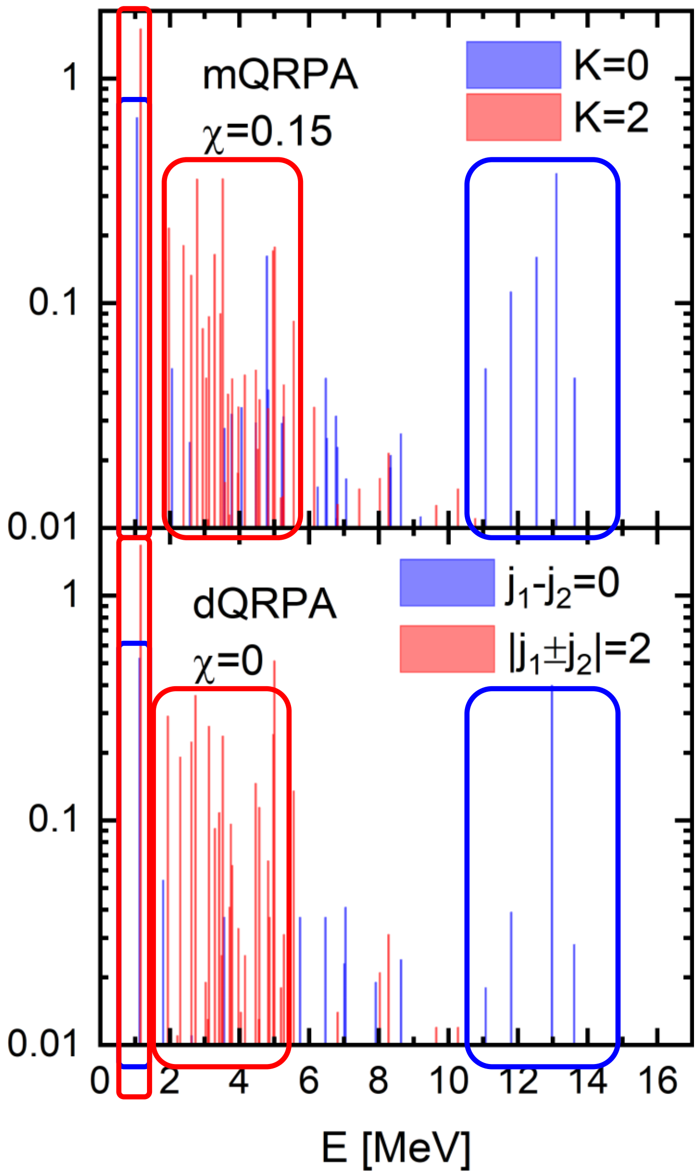
  determined by fitting the first excitation energy to data

- Model space:

  15 s.p. levels below/above Fermi surface

Numerical comparison between dQRPA and mQRPA

✓ E2 Strength functions: example of ^{170}Yb



- Similar overall behavior in corresponding subspaces

$$K = 0 \leftrightarrow j_1 - j_2 = 0$$
$$K = 2 \leftrightarrow |j_1 \pm j_2| = 2$$

- Collective first excited states: β/γ-vibrational character

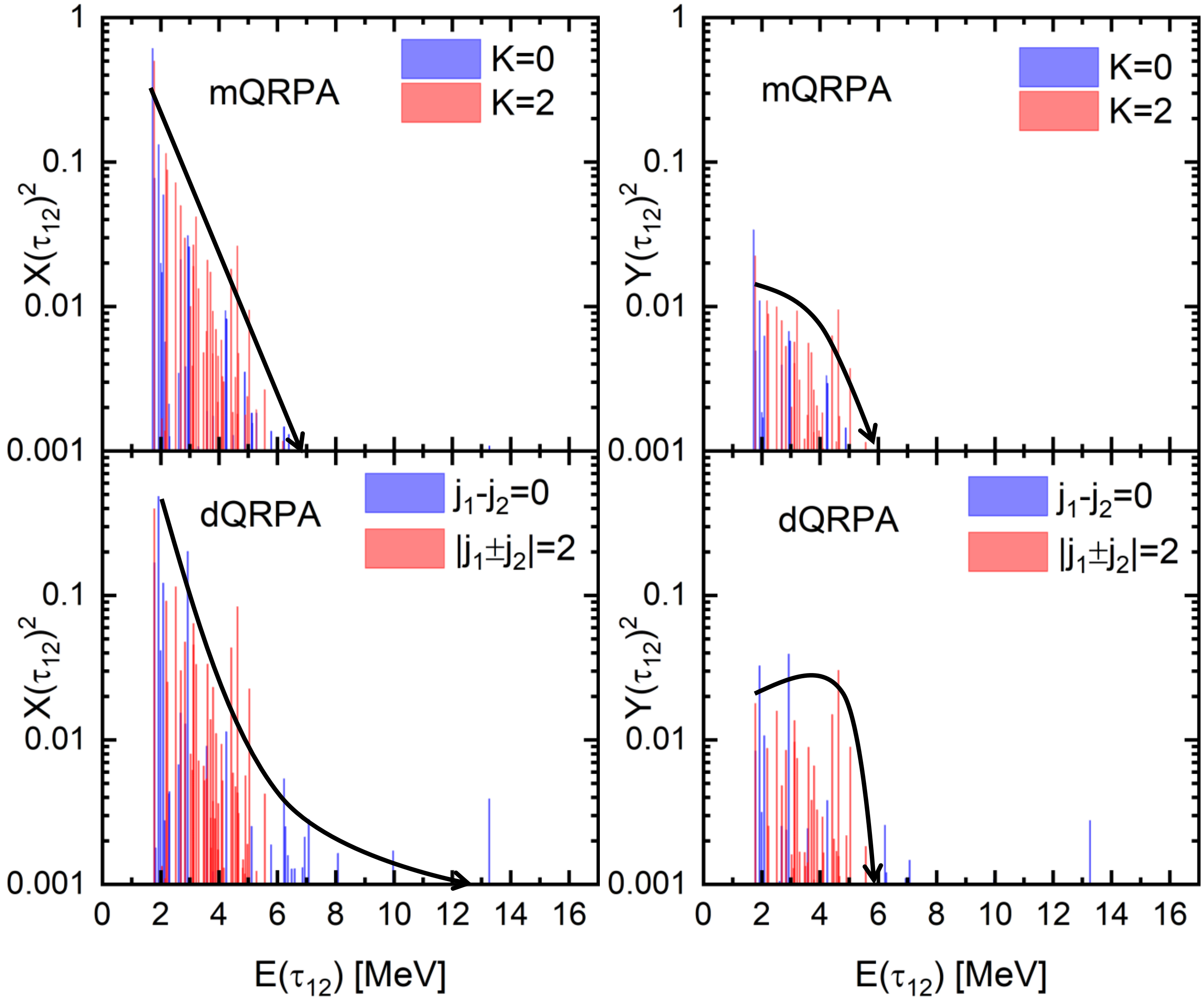
- **Feature of dQRPA:**
  w/o effective charge

⇩

Enhanced collectivity

Numerical comparison between dQRPA and mQRPA

✓ 2qp compositions of the first excited states: example of ^{170}Yb

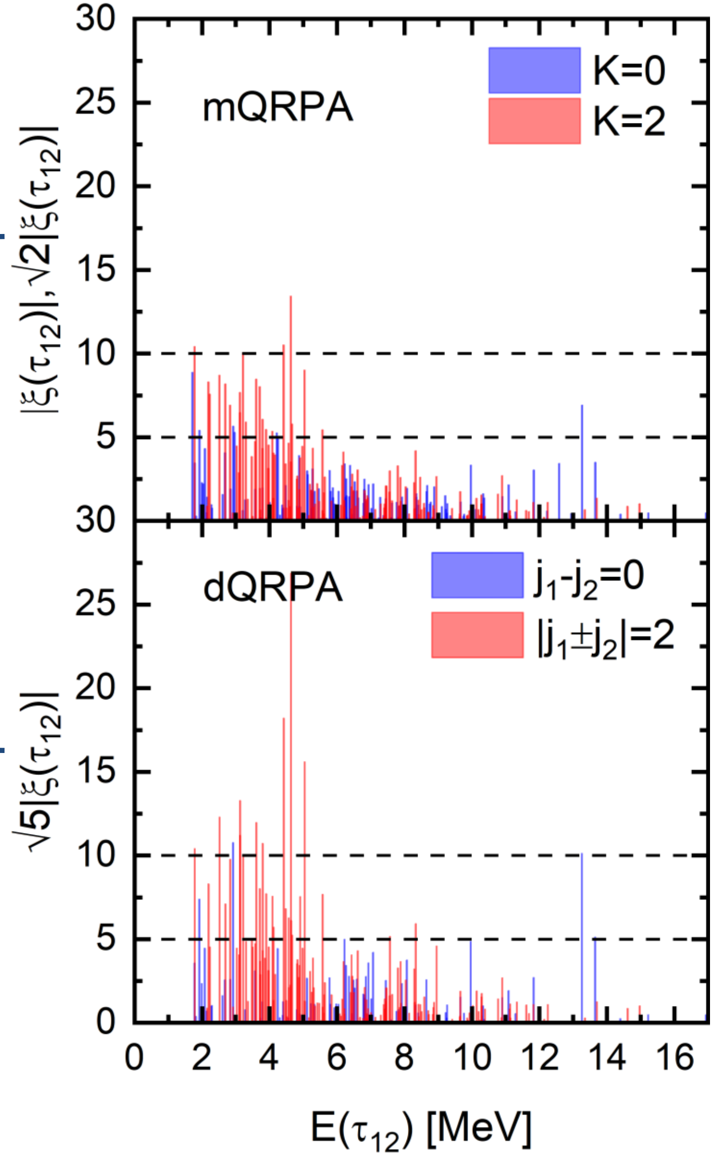


- Similar overall behaviour
- **Feature of dQRPA:** Diffused 2qp compositions ⇩ **Enhanced collectivity**

Numerical comparison between dQRPA and mQRPA

✓ (reduced) matrix elements of the quadrupole operator: example of 170Yb

Playing the same role in the B(E2) calculation



- Similar overall behavior
- **Feature of dQRPA:** Stronger quadrupole correlation

Enhanced collectivity

Numerical comparison between dQRPA and mQRPA

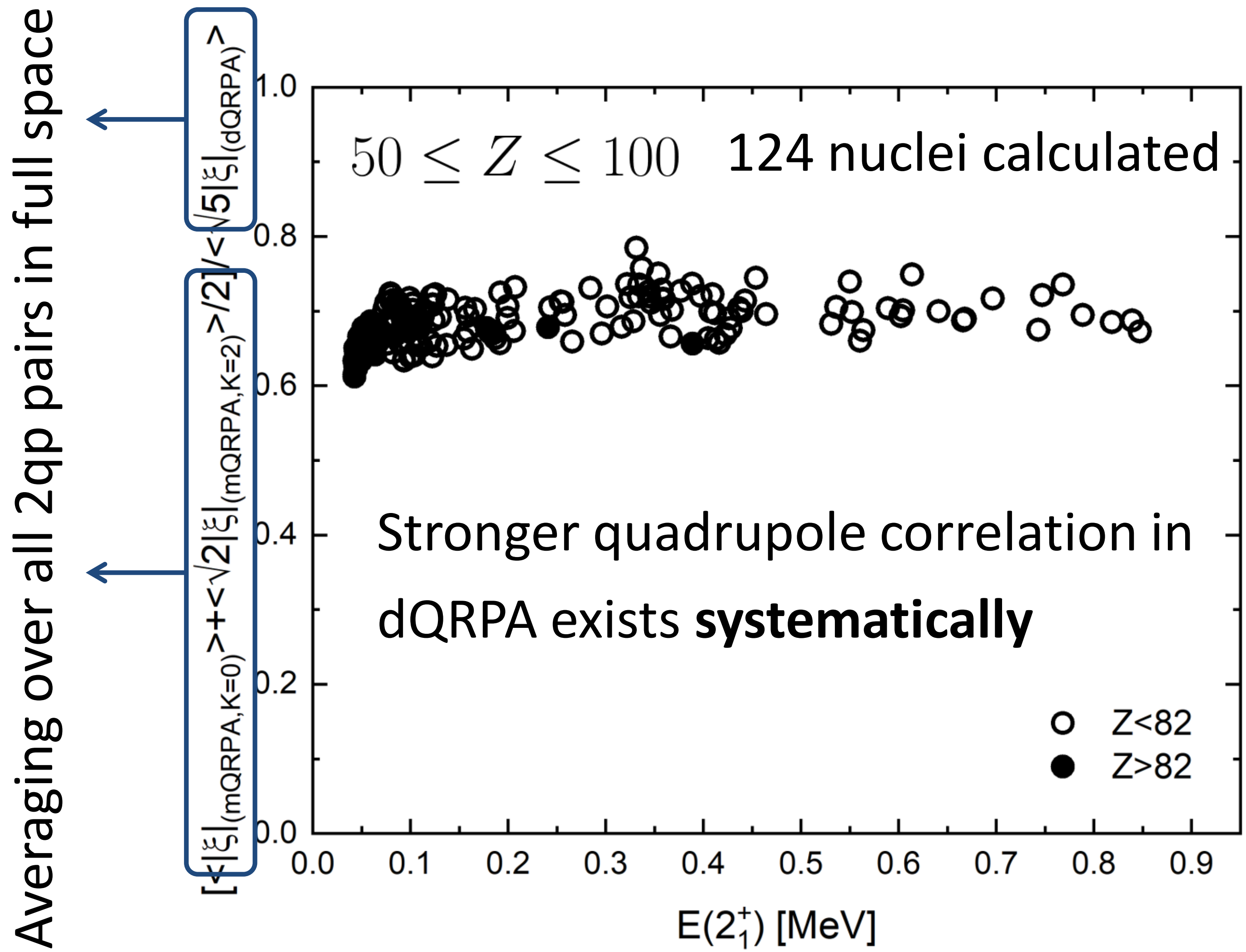
- Comparison between behaviors in full configuration spaces: systematic calculations for a range of nuclei

  - ✓ Ratio between the average magnitudes of (reduced) quadrupole matrix elements

  - ✓ Description to the γ-vibrational states

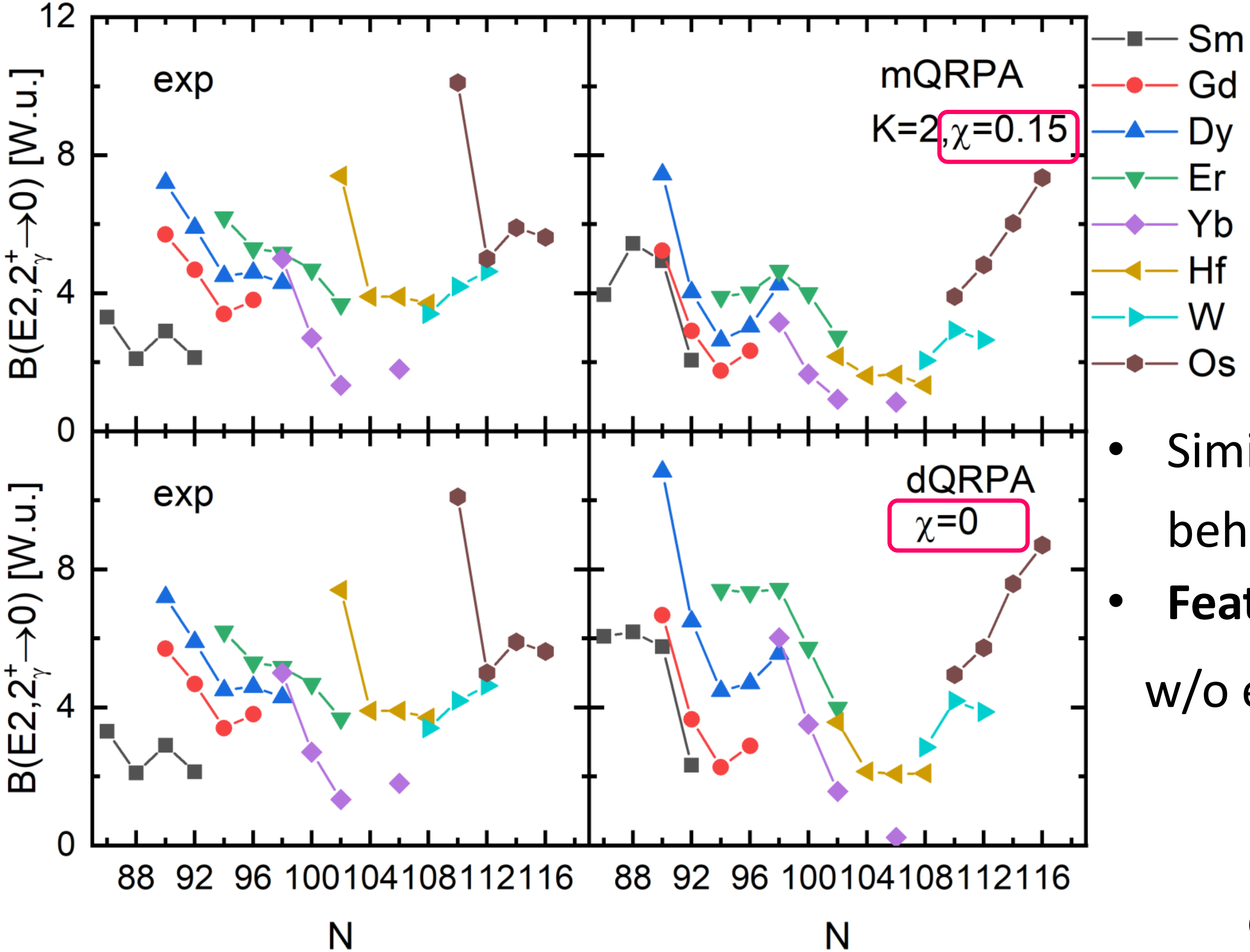
Numerical comparison between dQRPA and mQRPA

✓ Ratio between the average magnitude of (reduced) quadrupole matrix elements



Numerical comparison between dQRPA and mQRPA

✓ Description to the γ-vibrational states for well deformed rare-earth nuclei with $B(E2, 2_\gamma^+ \to 0)$ data



- Similar overall behavior
- **Feature of dQRPA:** w/o effective charge

⇩

Enhanced collectivity

Numerical comparison between dQRPA and mQRPA

✓ Conclusions

- Compared with mQRPA, dQRPA provides

Stronger quadrupole correlations

⇩

Enhanced collectivity

⇩

Overall reproduction to $B(E2, 2^+_\gamma \to 0)$ data

w/o effective charge

- With an effective charge adopted in mQRPA, the two approaches give **similar overall behavior** when describing E2 transitions.

Summary

- Characters of dQRPA framework:
  - ✓ Using the 'core+particle' wave function with good angular momentum as the single-particle basis
  - ✓ Gives QRPA vacuum & phonons with good rotational symmetry
  - ✓ Belongs to the 'projection before variation' category, with phonon structures determined under symmetry consideration
  - ✓ With the formalism similar to the spherical QRPA, and the physics similar to the intrinsic deformed QRPA (mQRPA)

- Implementation of dQRPA
  - ✓ Nilsson+BCS calculation identical to mQRPA
  - ✓ Calculation of the reduced matrix elements of multipole operators
  - ✓ Bosons, phonons & QRPA equations formally identical to the spherical QRPA
- Comparison between the behaviors of dQRPA and mQRPA:
  - ✓ dQRPA provides stronger quadrupole correlation and enhanced E2 collectivity than mQRPA
  - ✓ With an effective charge adopted in mQRPA, the two approaches show similar overall behaviors on the descriptions of E2 transitions

Thanks for your attention and advices!