张量力对原子核双贝塔衰变的研究

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2020绵阳原子核结构理论研讨会

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outline

- Introduction about 0vββ decay
- HFB+QRPA model
- Why tensor force ?
- Present result on 2vββ
- Present result of 0vββ NME
- Summary and perspective

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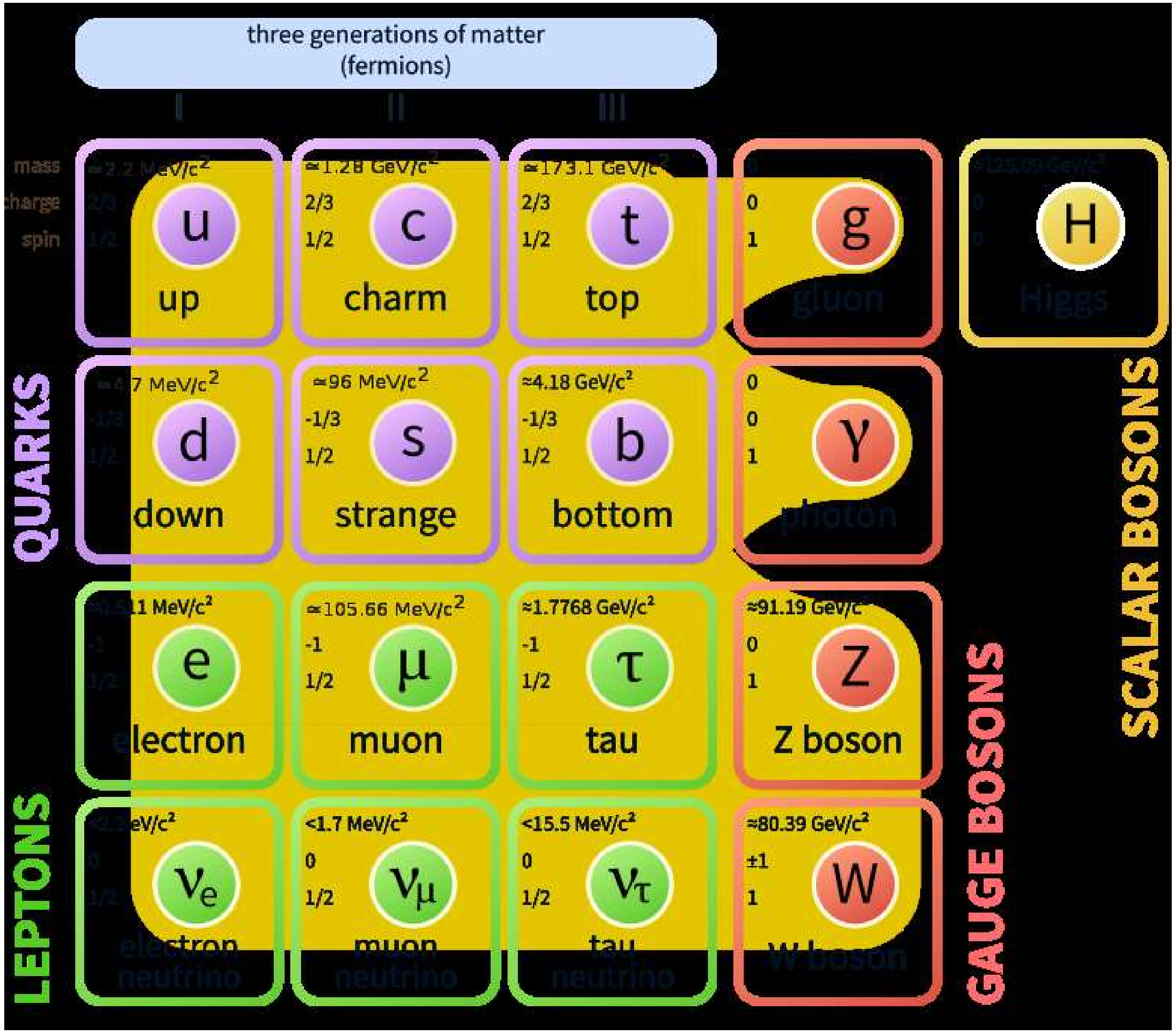
标准模型



是否有超出标准模型的问题呢？

Dirac Fermion and Majorana fermion

物质	反物质
电子	正电子
质子	反质子
中子	反中子
……	……

Dirac Fermion

Ettore Majorana (1906~1938?)

There are several categories of scientists in the world; those of second or third rank do their best but never get very far. Then there is the first rank those who make important discoveries, fundamental to scientific progress. But then there are the geniuses, like Galilei and Newton. Majorana was one of these.

——E. Fermi, 1938, Rome

Dirac Fermion and Majorana fermion

物质	反物质
电子	正电子
质子	反质子
中子	反中子
……	……

Dirac Fermion

Ettore Majorana (1906~1938?)

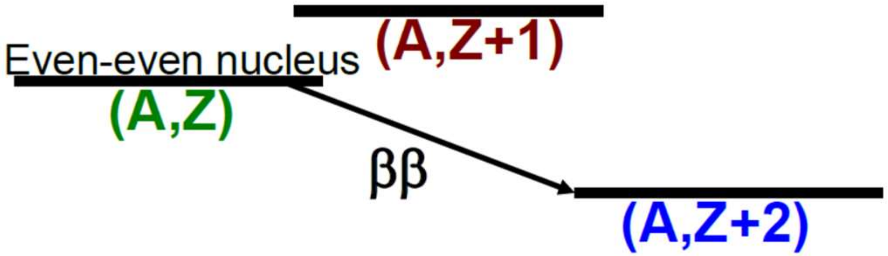
- **Majorana Fermion, antiparticle is itself**
- **2012, Majorana state Confirmed**
- **If exist substantial Majorana Fermion ?**

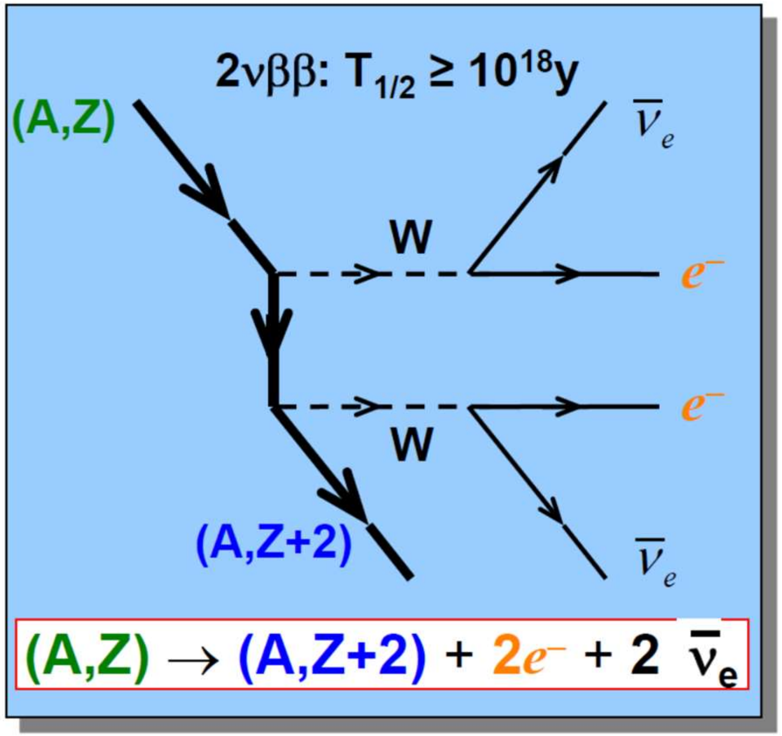
double-beta decay

$2\nu\beta\beta$

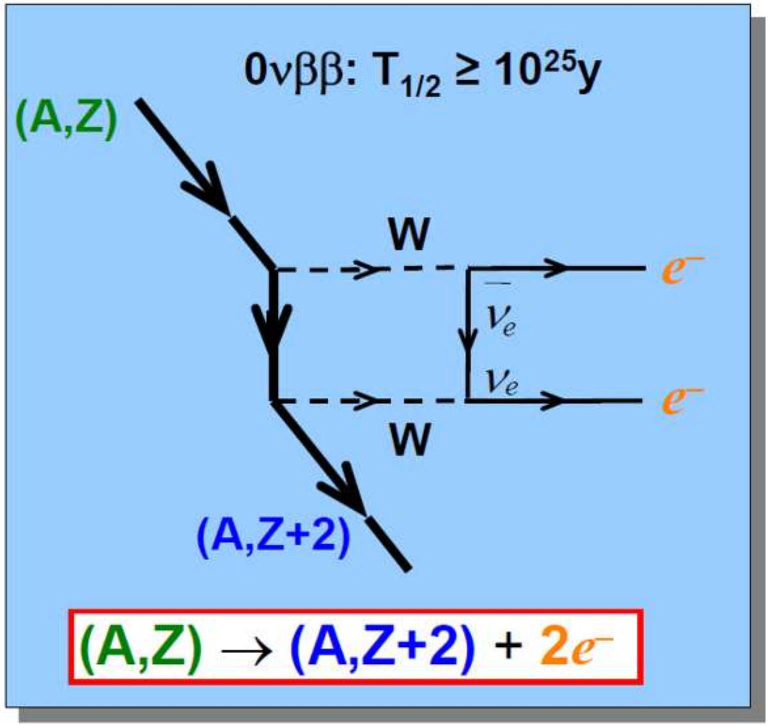
1935

M. Goeppert-Mayer

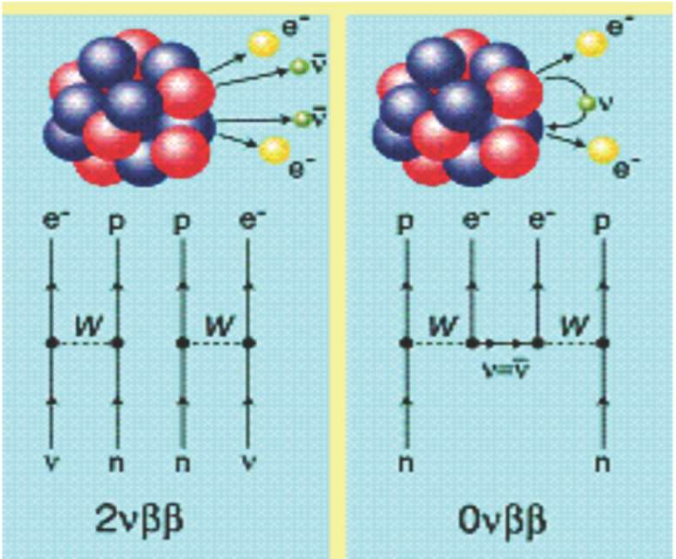




Majorana Neutrino ?



Insight by 0vββ decay



Answer three question simultaneously:

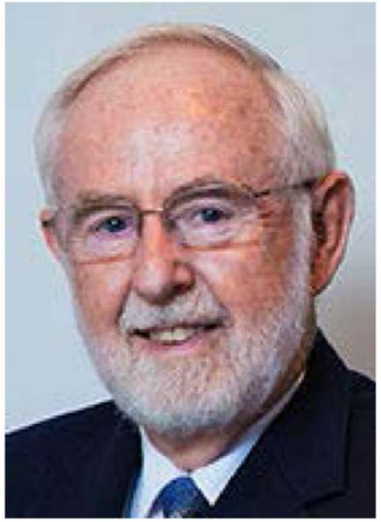
- **once observed, neutrino must be Majorana fermion.**
- **Lepton number do not conserved**

  $n \rightarrow p + \beta^{-} + \bar{\nu}_e, \qquad \nu_e + n \rightarrow p + \beta^{-}.$

- **Neutrino mass can be determined.**

Nobel Prize in Physics 2015

Takaaki Kajita



Arthur B. McDonald

"For the discovery of neutrino oscillations, which shows that neutrinos have mass"

Neutrino Oscillation

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$$

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i} |\nu_\alpha\rangle$$

$|\nu_\alpha\rangle$ 是在某一指定味特征态下的中微子。 ***α***可为电子、**μ**子或**τ**子

$|\nu_i\rangle$ 是在某一指定质量特征态下的中微子。 其中，**i = 1, 2, 3**

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

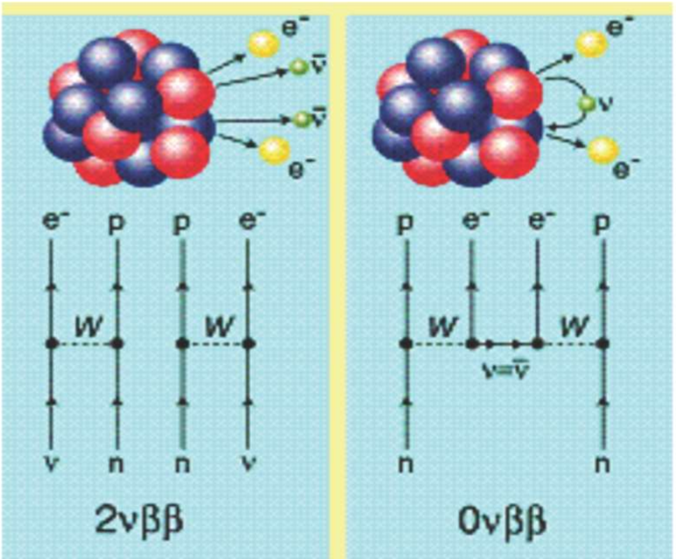
$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

tan2(θ12) = 0.457+0.040/−0.029. ⟹ 太阳中微子问题 ⟹ **Nobel Prize**

θ23= 45±7.1° ⟹ 大气中微子 ⟹ **Nobel Prize**

sin2(2θ13) = 0.092±0.017 ⟹ 大亚湾中微子实验 ⟹ ？ ？ ？

Two modes of double-beta decay



$$\left(T_{1/2}^{2\nu}\right)^{-1} = G_{2\nu}\left(Q_{\beta\beta}, Z\right)\left|M_{2\nu}\right|^2$$

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G_{0\nu}\left(Q_{\beta\beta}, Z\right)\left|M_{0\nu}\right|^2 \frac{\langle m_\nu \rangle^2}{m_e^2}$$

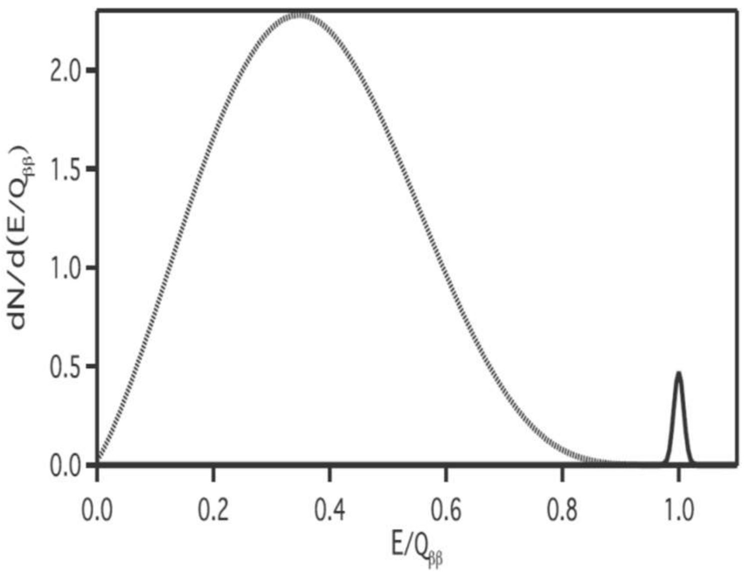


TABLE II. A summary of the $\beta\beta(0\nu)$ proposals and experiments. The Q value is the available energy for the decay as referenced in the text.

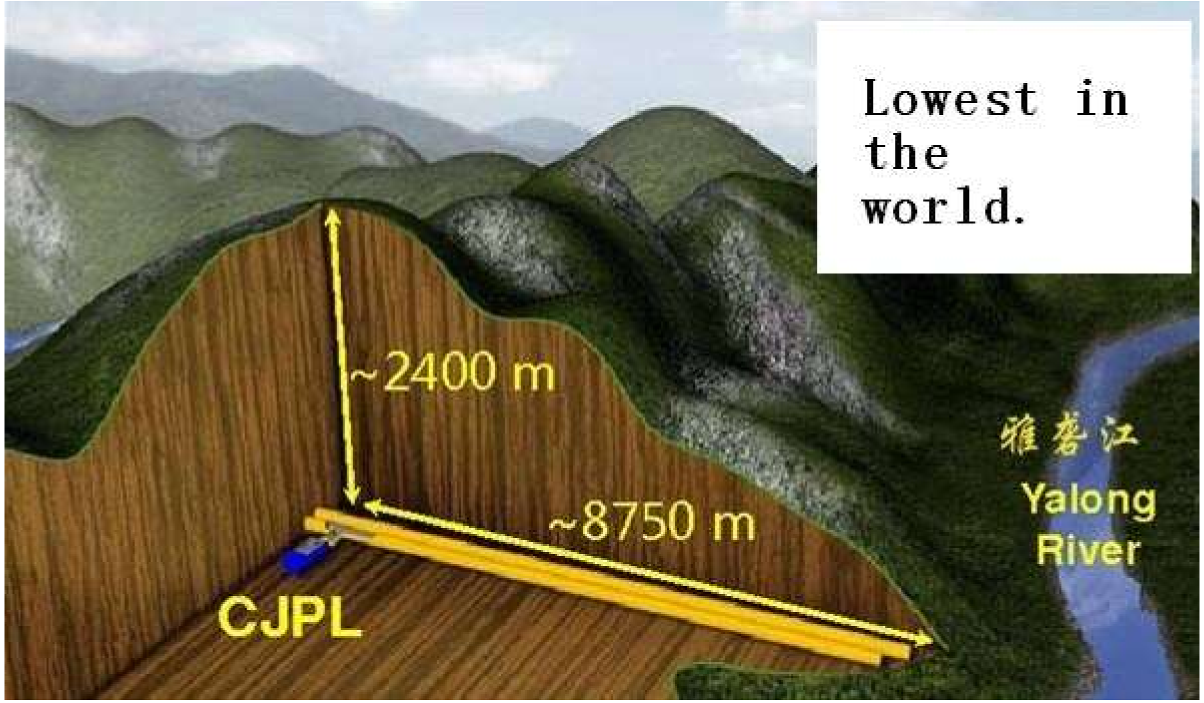
Isotope	Q value (MeV)	Technique	Collaboration
^{48}Ca	4.274	CaF_2 scintillating crystals	CANDLES (Umehara *et al.*, 2008), CARVEL (Zdesenko *et al.*, 2005)
^{82}Se	2.995	ZnSe scintillating bolometers	LUCIFER (Arnaboldi *et al.*, 2011)
		Thin foils and tracking	SuperNEMO (Barabash *et al.*, 2012)
^{76}Ge	2.039	High-purity Ge semiconductor detectors	GERDA (Agostini *et al.*, 2013), MAJORANA (Abgrall *et al.*, 2014)
^{100}Mo	3.034	$CaMoO_4$ bolometers	AMoRE (Lee *et al.*, 2011)
		Thin foils and tracking	MOON (Ejiri *et al.*, 2000)
		$ZnMoO_4$ bolometers	Mo bolometer (Beeman *et al.*, 2012)
^{116}Cd	2.809	CdZnTe semiconductor detectors	COBRA (Dawson *et al.*, 2009)
^{130}Te	2.528	TeO_2 bolometers	CUORE (Alessandria *et al.*, 2011)
		Te dissolved in scintillator	SNO+ (Hartnell, 2012)
^{136}Xe	2.458	Liquid Xe time projection chamber	EXO-200 (Auger *et al.*, 2012), nEXO, LZ (Akerib *et al.*, 2013)
		Gaseous Xe time projection chamber	NEXT (Gómez *et al.*, 2011)
		Xe dissolved in scintillator	KamLAND-Zen (Gando *et al.*, 2013)
		Scintillating liquid Xe within graphene sphere	GraXe (Gómez-Cadenas *et al.*, 2012)
^{150}Nd	3.371	Thin foils and tracking	DCBA (Ishihara *et al.*, 2000)
^{160}Gd	1.730	Cd_2SiO_5:Ce scintillating crystals in liquid scintillator	GSO (Wang, Wong, and Fujiwara, 2002)
Various		Quantum dots in liquid scintillator	Quantum dots (Winslow and Simpson, 2012; Aberle *et al.*, 2013)

Half-lives of 0v and 2v double-beta decay

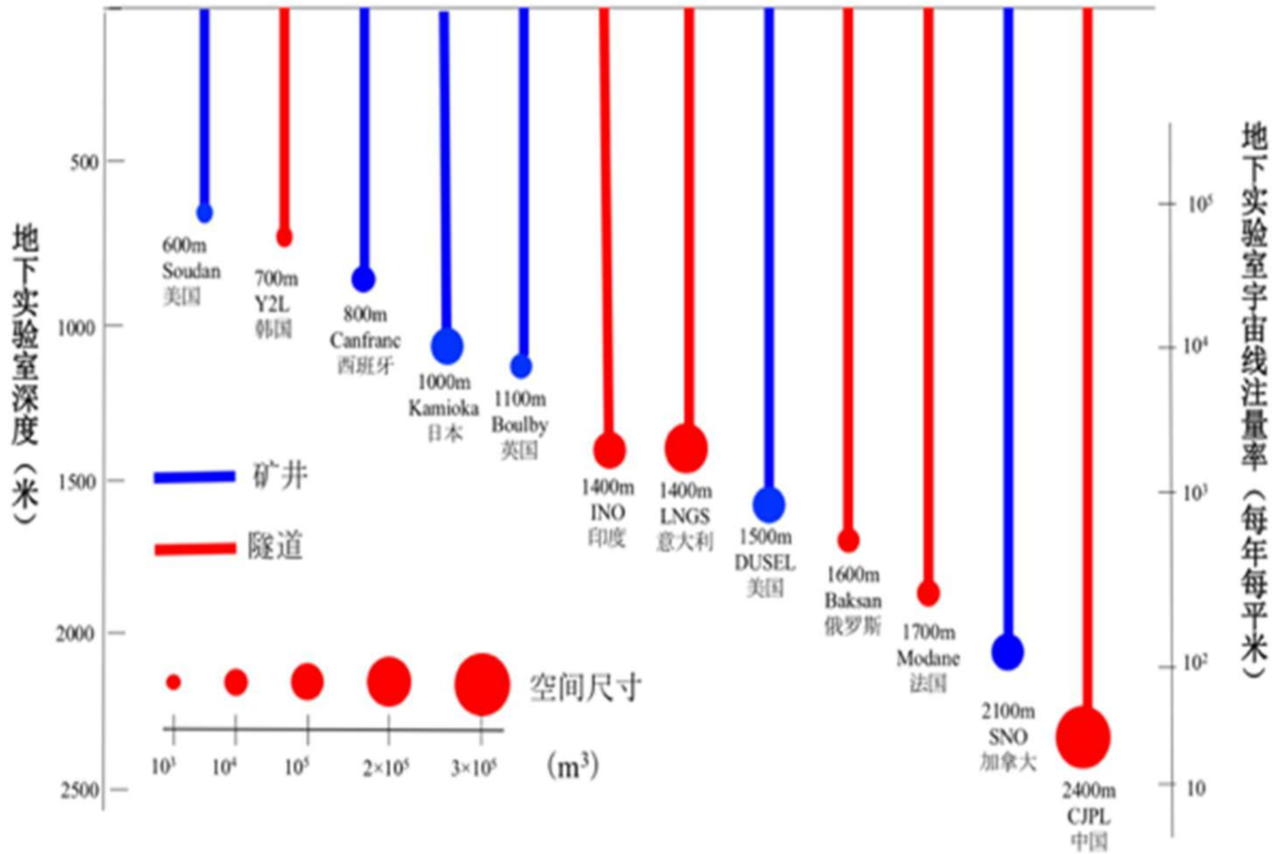
Isotope	$T_{1/2}^{2\nu}$ (yr)	Isotope	$T_{1/2}^{2\nu}$ (yr)
^{48}Ca	$(4.2^{+2.1}_{-1.0}) \times 10^{19}$	^{128}Te	$(2.5 \pm 0.3) \times 10^{24}$
^{76}Ge	$(1.5 \pm 0.1) \times 10^{21}$	^{130}Ba EC-EC(2ν)	$(2.2 \pm 0.5) \times 10^{21}$
^{82}Se	$(0.92 \pm 0.07) \times 10^{20}$	^{130}Te	$(0.9 \pm 0.1) \times 10^{21}$
^{96}Zr	$(2.0 \pm 0.3) \times 10^{19}$	^{150}Nd	$(7.8 \pm 0.7) \times 10^{18}$
^{100}Mo	$(7.1 \pm 0.4) \times 10^{18}$	^{238}U	$(2.0 \pm 0.6) \times 10^{21}$
^{116}Cd	$(3.0 \pm 0.2) \times 10^{19}$		

Isotope	Technique	$T_{1/2}^{0\nu}$ (yr)	$\langle m_{\beta\beta} \rangle$ (eV)	Reference
^{48}Ca	CaF_2 scintillating crystals	$> 5.8 \times 10^{22}$	$< 3.5 - 22$	Umehara *et al.* (2008)
^{76}Ge	^{enr}Ge detector	$> 3.0 \times 10^{25}$	$< (0.2 - 0.4)$	Agostini *et al.* (2013)
^{82}Se	Thin metal foils and tracking	$> 3.2 \times 10^{23}$	$< (0.94 - 1.71)$	Tretyak *et al.* (2011)
^{100}Mo	Thin metal foils and tracking	$> 1.1 \times 10^{24}$	$< (0.3 - 0.9)$	Arnold *et al.* (2013)
^{116}Cd	$^{116}CdWO_4$ scintillating crystals	$> 1.7 \times 10^{23}$	< 1.7	Danevich *et al.* (2003)
^{128}Te	Geochemical	$> 7.7 \times 10^{24}$	$< (1.1 - 1.5)$	Bernatowicz *et al.* (1993)
^{130}Te	TeO_2 bolometers	$> 2.8 \times 10^{24}$	$< (0.3 - 0.7)$	Arnaboldi *et al.* (2008)
^{136}Xe	Liquid Xe scintillator	$> 1.9 \times 10^{25}$	$< (0.16 - 0.33)$	Gando *et al.* (2013)
^{150}Ne	Thin metal foil within time projection chamber	$> 1.8 \times 10^{22}$	N.A.	Barabash *et al.* (2010)

0ν Double-Beta decay in China



双β衰变的实验研究

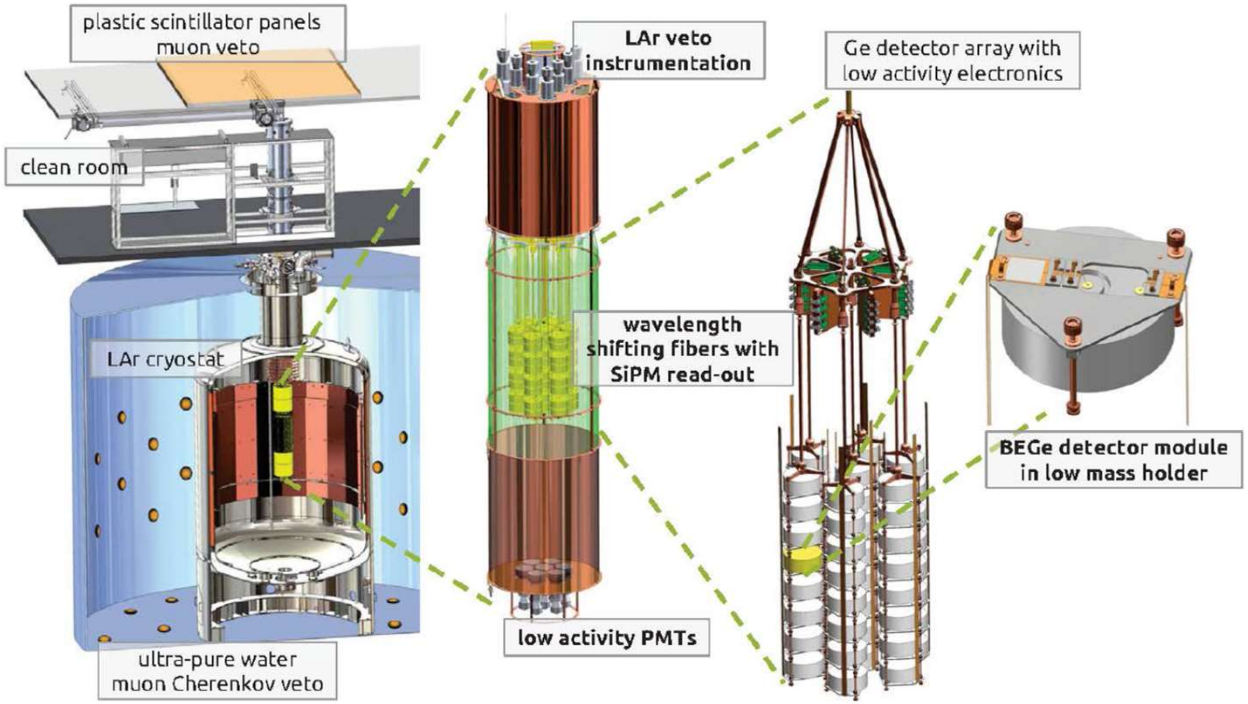


国内计划

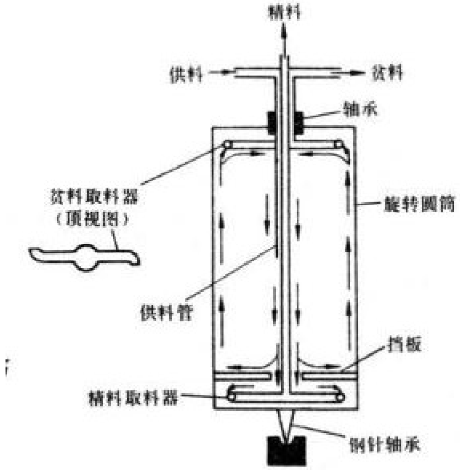
- ➢锦屏山深地实验室在目前国际上综合条件一流
- ➢中科院数理学部战略研究项目0νββ衰变实验

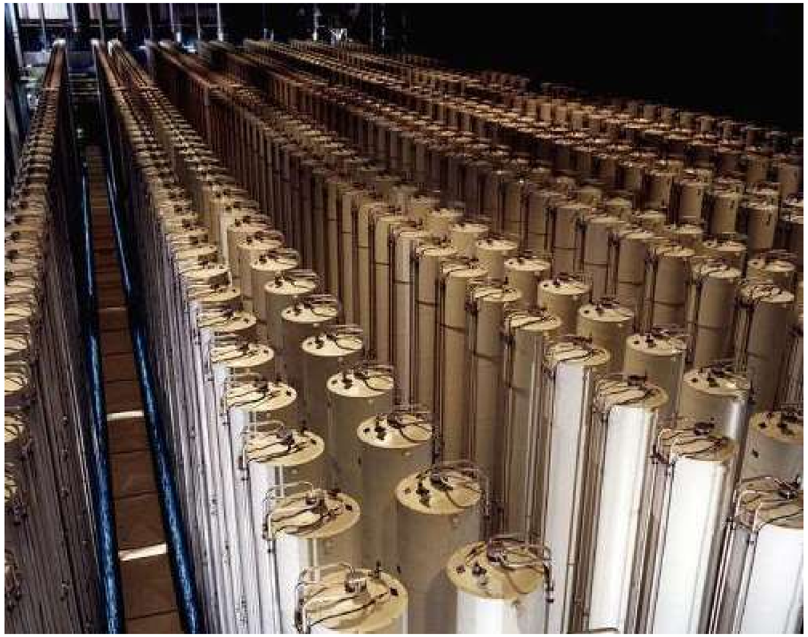
国内研究

主要单位	探测器	核素
复旦大学	晶体量能器	82Se
清华大学	高纯锗探测器	76Ge
上海交通大学	闪烁体探测器	136Xe
中科院近物所	高压TPC	130Te



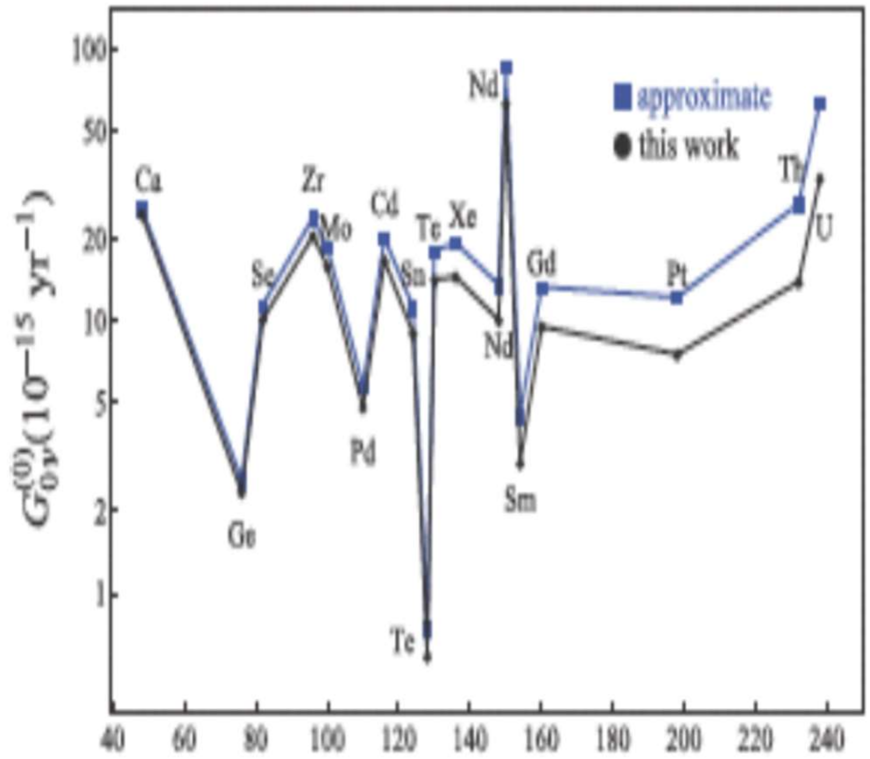
实验最大的难点-核素分离





What is fixed for the 0v double beta decay half-life

Kotila and Iachello, PRC85,034316

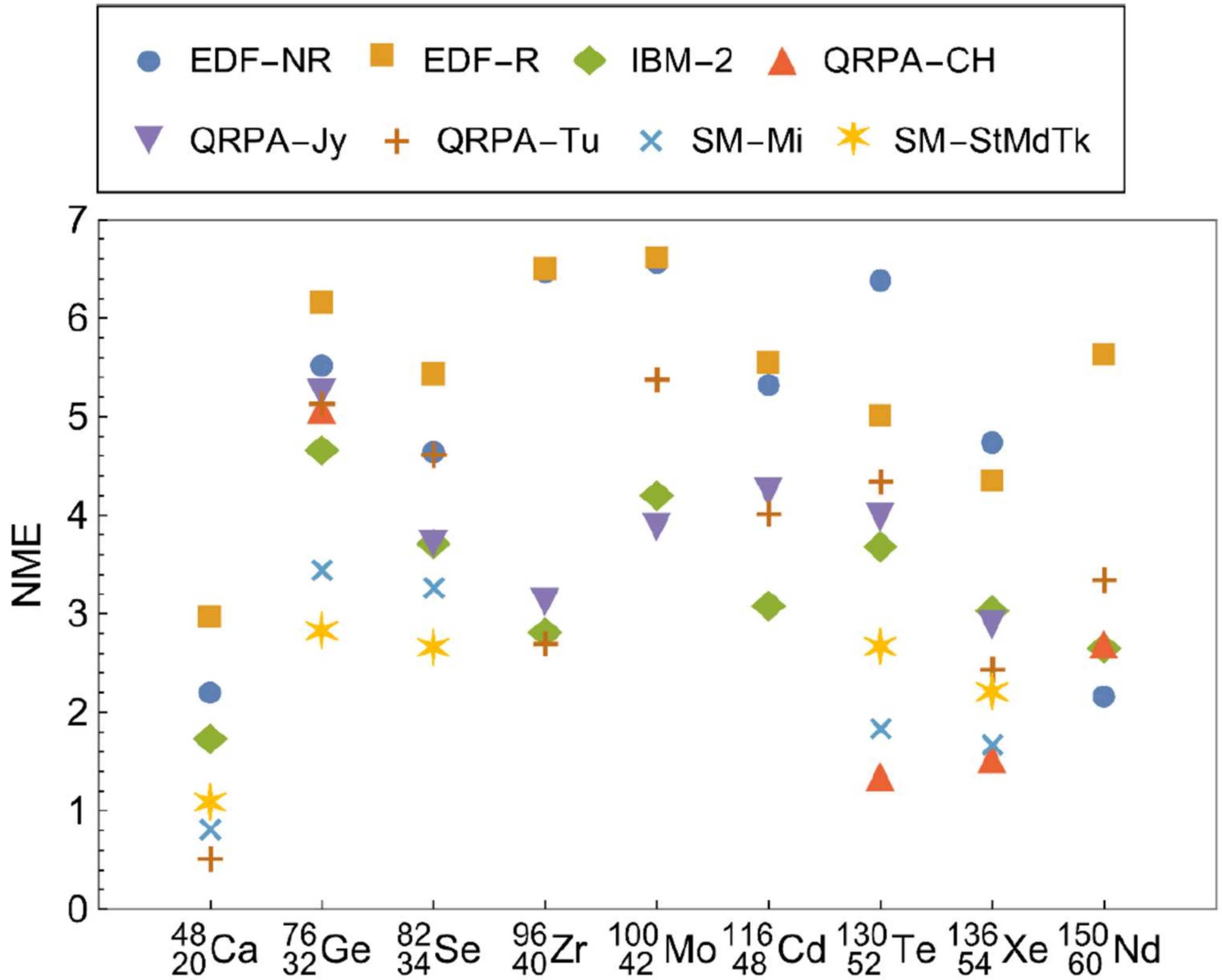


$$\left(T_{1/2}^{0\nu}\right)^{-1} = G_{0\nu}\left(Q_{\beta\beta}, Z\right)\left|M_{0\nu}\right|^{2} \frac{\left\langle m_{\nu}\right\rangle^{2}}{m_{e}^{2}}$$

Various Results

- Categories of methods (whether one calculates the intermediate states or not):
  - non-closure: QRPA, Shell Model(?)
  - closure: various EDF with HFB or GCM, Shell Model
- QRPA can be divided into many groups by the minefield and residual interactions one uses

理论现状

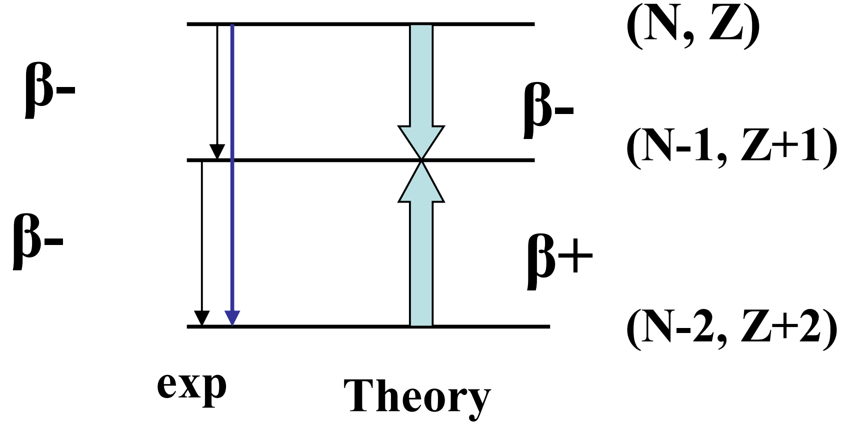


EDF–NR
EDF–R
IBM–2
QRPA–CH
QRPA–Jy
QRPA–Tu
SM–Mi
SM–StMdTk
NME
0
1
2
3
4
5
6
7
$^{48}_{20}$Ca
$^{76}_{32}$Ge
$^{82}_{34}$Se
$^{96}_{40}$Zr
$^{100}_{42}$Mo
$^{116}_{48}$Cd
$^{130}_{52}$Te
$^{136}_{54}$Xe
$^{150}_{60}$Nd

Calculation of NME by QRPA

Theoretical method to calculate the half-life of 2ν decay

$$[T^{2\nu}_{1/2}]^{-1} = G^{2\nu}|M^{2\nu}_{GT}(0^+)|^2$$



$$M^{2\nu}_{GT} = \sum_{mm'} \frac{\langle 0^+_{f,gs}||\boldsymbol{\sigma\tau}_+||1^+_{m'}\rangle\langle 1^+_{m'}|1^+_{m}\rangle\langle 1^+_{m}||\boldsymbol{\sigma\tau}_+||0^+_{i,gs}\rangle}{\frac{1}{2}\left[\Omega^m_{1^+} + \Omega^{m'}_{1^+} + Q_{\beta^-}(A,Z+1) - Q_{\beta^-}(A,Z)\right]},$$

The overlap factor:

$$\langle 1^+_{m_f}|1^+_{m_i}\rangle = \sum_{pn}[X_{pn}(1^+m_i)X_{pn}(1^+m_f) - Y_{pn}(1^+m_i)Y_{pn}(1^+m_f)].$$

Calculation of NME by QRPA

Theoretical method to calculate the half-life of 0v decay

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left|M_{\nu}^{0\nu}\right|^{2}\left(\frac{|\langle m_{\beta\beta}\rangle|}{m_e}\right)^{2}.$$

$$M_{\nu}^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^{2} M_{F}^{0\nu} - M_{T}^{0\nu},$$

$$M_{F}^{(0v)} = \sum_{k}\langle 0_f^+ || \sum_{mn} h_F(r_{mn}, E_k) t_m^- t_n^- || 0_i^+\rangle$$

$$M_{GT}^{(0v)} = \sum_{k}\langle 0_f^+ || \sum_{mn} h_{GT}(r_{mn}, E_k)(\sigma_m \cdot \sigma_n) t_m^- t_n^- || 0_i^+\rangle$$

where

$$h_K(r, E_k) = \frac{2R}{\pi}\int dq \frac{q h_K(q^2)}{q + E_k - (M_i + M_f)/2} j_0(qr),$$

Calculation of NME by QRPA

Theoretical method to calculate the half-life of 0v decay

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left|M_{\nu}^{0\nu}\right|^{2}\left(\frac{|\langle m_{\beta\beta}\rangle|}{m_e}\right)^{2}$$

$$M_{\nu}^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^{2} M_{F}^{0\nu} - M_{T}^{0\nu},$$

$$M_{GT}^{(0v)} = 4\pi \int dq q h_{GT}(q^2) \sum_{k_1 k_2 m n} \sum_{lJ} \frac{1}{(2J+1)} \qquad ($$

$$\times \frac{\langle J_{k_1}||j_l(qr_m)(Y_l\sigma_m)_J t_m^+||0_f^+\rangle\langle J_{k_1}|J_{k_2}\rangle\langle J_{k_2}||j_l(qr_n)(Y_l\sigma_n)_J t_n^-||0_i^+\rangle}{q+(E_{k_1}+E_{k_2})/2-(M_i+M_f)/2}$$

$$j_0(r) = \frac{\sin r}{r} = 1 - \frac{r^2}{3!} + \ldots\ldots$$

$$l = 0 \rightarrow \sigma, r^2\sigma, \ldots.$$

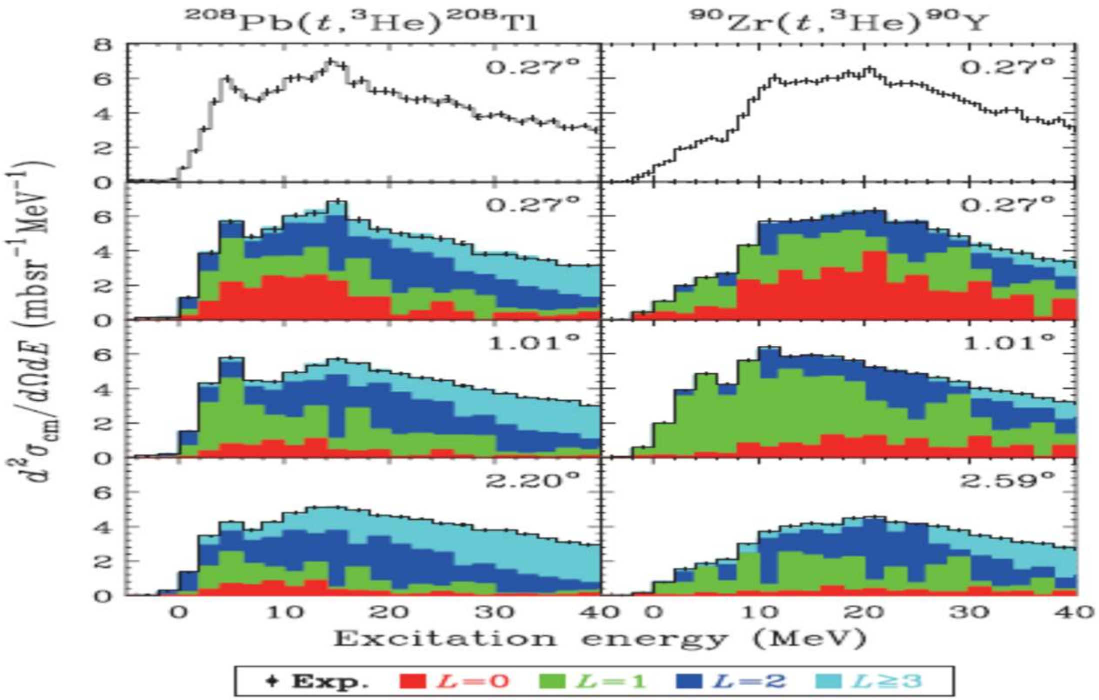
$$l = 1 \rightarrow r(Y_1\sigma)_1, r^3(Y_1\sigma)_1, \ldots.$$

Intermediate states

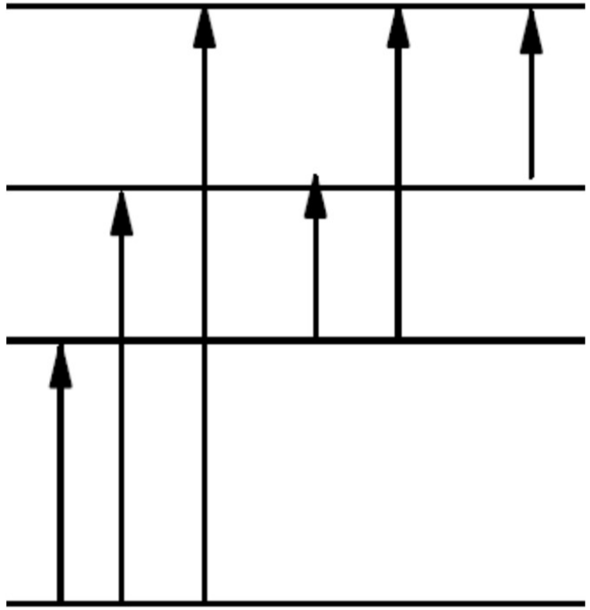
$$\rho(\mathbf{r}) = e f_E(|\mathbf{r} - \mathbf{r}_k|)$$

$$\mathbf{j}(\mathbf{r}) = \frac{e}{2}(\mathbf{v}_k f_E(|\mathbf{r} - \mathbf{r}_k|) + f_E(|\mathbf{r} - \mathbf{r}_k|)\mathbf{v}_k) + \frac{e\hbar}{2M} g_s \nabla \times \mathbf{s}_k f_M(|\mathbf{r} - \mathbf{r}_k|)$$

$$M^{\pm}_{\lambda\mu}(\rho V) = g_{\mathrm{V}} \tau^{\pm} f(r) Y_{\lambda,\mu}\,, \qquad M^{\pm}_{\lambda\mu}(j_{\mathrm{A}}, \kappa) = g_{\mathrm{A}} \tau^{\pm} f(r)(Y_{\kappa}\sigma)_{\lambda\mu}$$

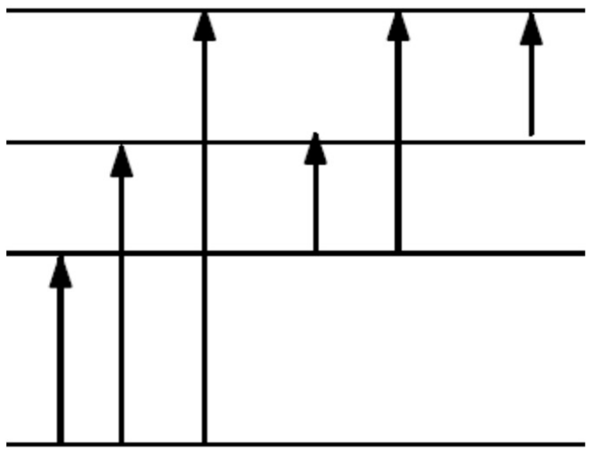


Introduction of collective excitations



Introduction of collective excitations

Realistic excitation is not such single-particle transitions.



$$O^{\nu +} = \sum_{np} X^{\nu}_{np} a^{+}_{p} a_{n} - \sum_{np} Y^{\nu}_{np} a^{+}_{n} a_{p}$$

Closed-shell RPA

$$O^{+}_{\nu} = \sum_{np} X^{\nu}_{np} \alpha^{+}_{p} \alpha^{+}_{n} - \sum_{np} Y^{\nu}_{np} \alpha_{n} \alpha_{p}$$

Open-shell QRPA

$$a^{+}_{k} = u_{k} \alpha^{+}_{k} + v_{k} \alpha_{\bar{k}}$$

Bogolyubov transformation

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- Introduction about 0νββ decay
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- Present result on 2νββ
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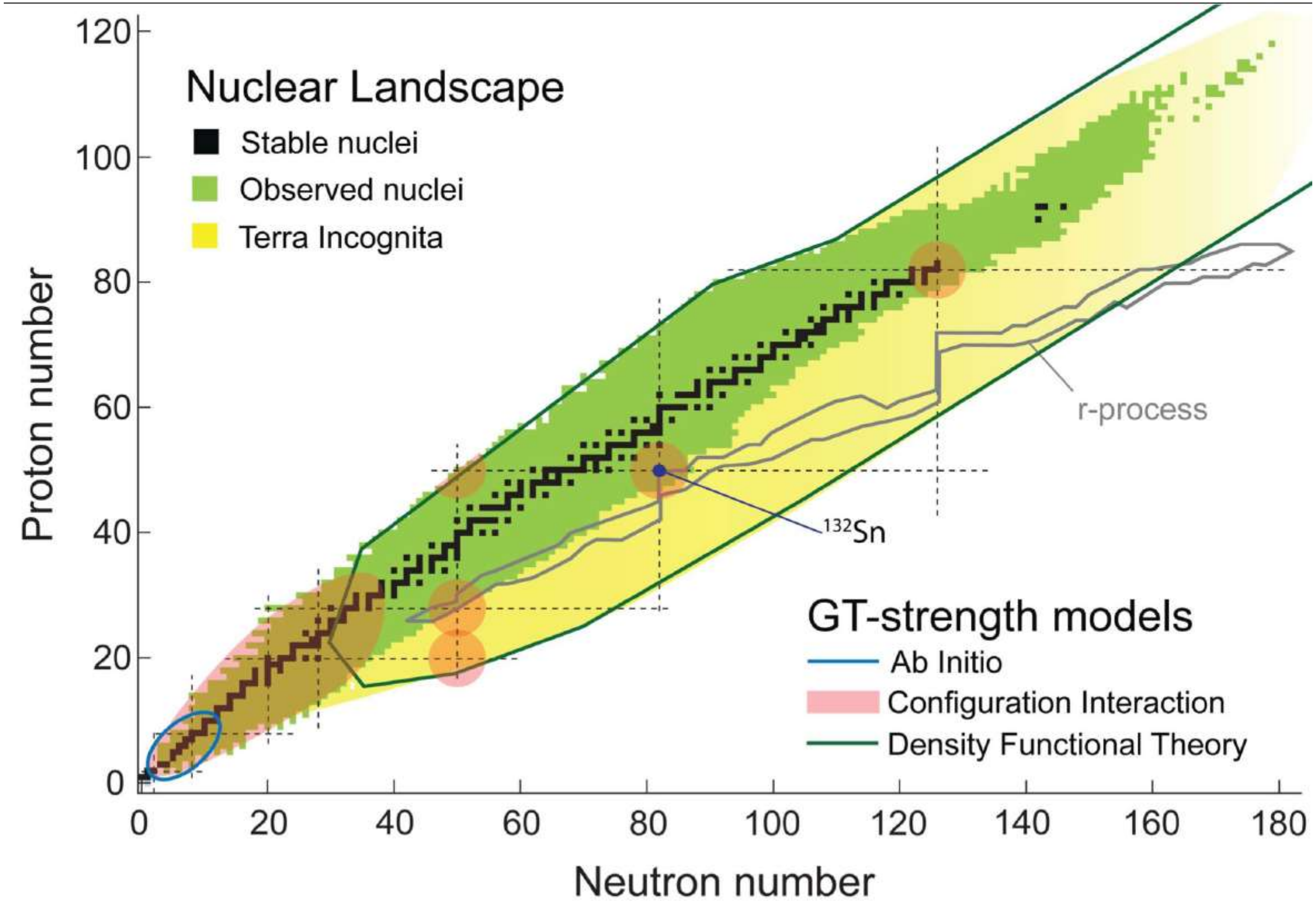
Application of theories and interactions



Brief introduction of nucleon-nucleon force

NN force widely used → Chiral Force → Fit scattering exp.dat describe manybody force

NN force widely used → Realistic Force → Fit very well scattering exp. data

NN force widely used → Effective force → Fit Bulk properties through MF calculation

Effective interactions

- Skyrme interaction

  T.H.R. Skyrme, Phil. Mag. 1, 1043(1956)

- Gogny interaction

  J. Decharge, D. Gogny, PRC 21, 1568(1980)

- Relativistic meson exchange force

  B.D. Serot, J.D. Walecha, Adv. Nucl. Phys. 16, 1(1986)

Skyrme interaction

$$V_{12} = t_0(1 + x_0 P_\sigma)\delta + \frac{1}{2}t_1(1 + x_1 P_\sigma)(\mathbf{k}'^2\delta + \delta\mathbf{k}^2) + t_2(1 + x_2 P_\sigma)\mathbf{k}' \cdot \delta\mathbf{k}$$
$$+ \frac{1}{6}t_3(1 + x_3 P_\sigma)\rho^\gamma\delta + \mathrm{i}W_0(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{k}' \times \delta\mathbf{k}),$$

$$V^T = \frac{T}{2}\left\{\left[(\boldsymbol{\sigma}_1 \cdot \mathbf{k}')(\boldsymbol{\sigma}_2 \cdot \mathbf{k}') - \frac{1}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\mathbf{k}'^2\right]\delta(\mathbf{r}_1 - \mathbf{r}_2)\right.$$
$$\left. + \delta(\mathbf{r}_1 - \mathbf{r}_2)\left[(\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) - \frac{1}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\mathbf{k}^2\right]\right\}$$
$$+ \frac{U}{2}\left\{(\boldsymbol{\sigma}_1 \cdot \mathbf{k}')\delta(\mathbf{r}_1 - \mathbf{r}_2)(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) + (\boldsymbol{\sigma}_2 \cdot \mathbf{k}')\delta(\mathbf{r}_1 - \mathbf{r}_2)(\boldsymbol{\sigma}_1 \cdot \mathbf{k})\right.$$
$$\left. - \frac{2}{3}\left[(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\mathbf{k}' \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2)\mathbf{k}\right]\right\}. \quad (1)$$

密度泛函理论（DFT）

基态的能量密度泛函

$$E = \langle \Psi | H | \Psi \rangle = E(\rho)$$

对密度变分要求能量最低就能得到体系的基态

Hohenberg-Kohn 定理（**applied to nuclear physics**）：

- 基态系统的所有物理性质都由密度唯一决定，
  能量与密度一一对应
- 任意近似密度对应的能量值都大或等于基态对应的
  真正密度所决定的能量值

能量密度泛函的近似求解

对体系密度的定义：

$$\rho(\vec{r}) = \sum_{i=1}^{N} \psi_i^*(\vec{r})\,\psi_i(\vec{r})$$

将能量密度泛函对密度的微分变成对单粒子波函数的变分：

$$\left[-\tfrac{1}{2}\nabla^2 + V_{eff}\left(\vec{r}\right)\right]\psi_i\left(\vec{r}\right) = \varepsilon_i \psi_i\left(\vec{r}\right) \qquad \textbf{Kohn-Sham eq.}$$

平均场近似：

- 体系所有粒子都是独立运动
- 体系所有粒子感受到的相互作用都相同
- 体系的基态是**Slater**行列式

Hartree-Fock近似

Hartree-Fock势：

$$H^{HF}=\sum_{i=1}^{A}h(i)$$

Hartree-Fock基态波函数

$$\left|HF\right\rangle=\prod_{i=1}^{A}a_i^{\dagger}\left|-\right\rangle$$

HF单粒子方程

$$h(i)\varphi_k(i)=\varepsilon_k\varphi_k(i)$$

HF方程求解方法I-基展开

单粒子波函数：

$$\varphi_k = \sum_l D_{lk} \chi_l \to a_k^\dagger = \sum_l D_{lk} c_l^\dagger$$

单粒子密度矩阵

$$\rho_{ll'} = \left\langle HF \left| c_{l'}^\dagger c_l \right| HF \right\rangle = \sum_{i=1}^{A} D_{li} D_{l'i}^*$$

Hamiltonian:

$$H = \sum_{l_1 l_2} t_{l_1 l_2} c_{l_1}^\dagger c_{l_2} + \sum_{l_1 l_2 l_3 l_4} \bar{V}_{l_1 l_2, l_3 l_4} c_{l_1}^\dagger c_{l_2}^\dagger c_{l_4} c_{l_3},$$

HF方程求解方法I-基展开

体系的能量密度函数：

$$E^{HF}(\rho)=\sum_{l_1 l_2} t_{l_1 l_2}\rho_{l_2 l_1}+\frac{1}{2}\sum_{l_1 l_2 l_3 l_4}\rho_{l_3 l_1}\bar{V}_{l_1 l_2,l_3 l_4}\rho_{l_4 l_2}$$

$$\bar{V}_{l_1 l_2,l_3 l_4}=\underbrace{V_{l_1 l_2,l_3 l_4}}_{\text{Hartree项}}-\underbrace{V_{l_1 l_2,l_4 l_3}}_{\text{Fock项}}$$

体系能量对密度变分：

$$h_{kk'}=\frac{\partial E^{HF}(\rho)}{\partial\rho_{k'k}}=\varepsilon_k\delta_{kk'}$$

$$\sum_{l'}h_{ll'}D_{l'k}=\sum_{l'}\left[t_{ll'}+\sum_{i=1}^{A}\sum_{pp'}\bar{V}_{lp',l'p}D_{pi}D^{*}_{p'i}\right]D_{l'k}=\varepsilon_k D_{lk}$$

HF方程

HF方程求解方法II-坐标空间

基态波函数：

$$\phi(x_1, x_2, \ldots, x_A) = \frac{1}{\sqrt{A!}} \det|\phi_i(x_j)|,$$

能量密度泛函的定义：

$$\begin{aligned} E &= \langle \phi, (T+V)\phi \rangle \\ &= \sum_i \left\langle i \left| \frac{p^2}{2m} \right| i \right\rangle + \frac{1}{2} \sum_{ij} \langle ij | \tilde{v}_{12} | ij \rangle \\ &\quad + \frac{1}{6} \sum_{ijk} \langle ijk | \tilde{v}_{123} | ijk \rangle \\ &= \int H(\vec{\mathrm{r}})\, d^3 r, \end{aligned}$$

HF方程求解方法II-坐标空间

相互作用矩阵元的计算：

$$\bar{V}_{ij,ij} = V_{ij,ij} - V_{ij,ji} = \left\langle ij \middle| V\left(1 - P_{12}\right) \middle| ij \right\rangle = \left\langle ij \middle| \bar{V} \middle| ij \right\rangle$$

$$\bar{V} = V\left(1 - P_M P_\sigma P_\tau\right)$$

能量密度泛函：

$$\begin{aligned}
\mathcal{E}_{\text{Skyrme}} = & \frac{1}{2}t_0\left[\left(1+\frac{x_0}{2}\right)\rho^2 - \left(x_0+\frac{1}{2}\right)\sum_q \rho_q^2\right] \\
& + \frac{t_1}{4}\left\{\left(1+\frac{x_1}{2}\right)\left[\rho\tau + \frac{3}{4}(\nabla\rho)^2\right] - \left(x_1+\frac{1}{2}\right)\sum_q\left[\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2\right]\right\} \\
& + \frac{t_2}{4}\left\{\left(1+\frac{x_2}{2}\right)\left[\rho\tau - \frac{1}{4}(\nabla\rho)^2\right] + \left(x_2+\frac{1}{2}\right)\sum_q\left[\rho_q\tau_q - \frac{1}{4}(\nabla\rho_q)^2\right]\right\} \\
& - \frac{1}{16}(t_1x_1 + t_2x_2)J^2 + \frac{1}{16}(t_1 - t_2)\sum_q J_q^2 \\
& + \frac{1}{12}t_3\rho^\gamma\left[\left(1+\frac{x_3}{2}\right)\rho^2 - \left(x_3+\frac{1}{2}\right)\sum_q \rho_q^2\right] \\
& + \frac{1}{2}W_0\left(J\nabla\rho + \sum_q J_q\nabla\rho_q\right).
\end{aligned}$$

HF方程求解方法II-坐标空间

几种密度：

$$\rho_q(\vec{r}) = \sum_{i,\sigma} |\phi_i(\vec{r},\sigma,q)|^2, \qquad \tau_q(\vec{r}) = \sum_{i,\sigma} |\vec{\nabla}\phi_i(\vec{r},\sigma,q)|^2,$$

$$\vec{J}_q(\vec{r}) = (-i) \sum_{i,\sigma,\sigma'} \phi_i^*(\vec{r},\sigma,q)[\vec{\nabla}\phi_i(\vec{r},\sigma',q)\times\langle\sigma|\vec{\sigma}|\sigma'\rangle]$$

能量密度泛函对波函数求微分得：

$$\left[-\vec{\nabla}\cdot\frac{\hbar^2}{2m_q^*(\vec{r})}\vec{\nabla} + U_q(\vec{r}) + \vec{W}_q(\vec{r})\cdot(-i)(\vec{\nabla}\times\vec{\sigma})\right]\phi_i = e_i\phi_i,$$

HF方程

$$\frac{\hbar^2}{2m_q^*(\vec{r})} = \frac{\hbar^2}{2m} + \frac{1}{4}(t_1+t_2)\rho + \frac{1}{8}(t_2-t_1)\rho_q;$$

有效质量

$$U_q(\vec{r}) = t_0[(1+\tfrac{1}{2}x_0)\rho - (x_0+\tfrac{1}{2})\rho_q] + \tfrac{1}{4}t_3(\rho^2-\rho_q{}^2)$$

$$-\tfrac{1}{8}(3t_1-t_2)\nabla^2\rho + \tfrac{1}{16}(3t_1+t_2)\nabla^2\rho_q + \tfrac{1}{4}(t_1+t_2)\tau$$

$$+\tfrac{1}{8}(t_2-t_1)\tau_q - \tfrac{1}{2}W_0(\vec{\nabla}\cdot\vec{J}+\vec{\nabla}\cdot\vec{J}_q) + \delta_{q,+\frac{1}{2}}V_C(\vec{r})$$

$$\vec{W}_q(\vec{r}) = \tfrac{1}{2}W_0(\vec{\nabla}\rho+\vec{\nabla}\rho_q) + \tfrac{1}{8}(t_1-t_2)\vec{J}_q(\vec{r})$$

自旋轨道势

开壳偶偶核的HFB方程

Bogolyubov变换和能量密度泛函：

$$\hat{b}_n^+ = \sum_i (U_{in}\hat{a}_i^+ + V_{in}\hat{a}_i), \qquad E = \langle \Psi | \hat{H} | \Psi \rangle = E[\rho, \kappa, \kappa^*],$$

密度和对密度矩阵：

$$\rho_{ij} = \langle \Phi | \hat{a}_j^+ \hat{a}_i | \Phi \rangle = (V^* V^T)_{ij} = \rho_{ji}^*,$$

$$\kappa_{ij} = \langle \Phi | \hat{a}_j \hat{a}_i | \Phi \rangle = (V^* U^T)_{ij} = -\kappa_{ji}.$$

HFB方程

$$\mathcal{H}\begin{pmatrix} U_n \\ V_n \end{pmatrix} = e_n \begin{pmatrix} U_n \\ V_n \end{pmatrix},$$

$$\mathcal{H} = \begin{pmatrix} h - \lambda & \Delta \\ -\Delta^* & -h^* + \lambda \end{pmatrix}$$

$$\Rightarrow$$

$$-\frac{\hbar^2}{2m}\Delta \phi_n^{(U)}(\mathbf{x}) = (\lambda + e_n)\phi_n^{(U)}(\mathbf{x}),$$

$$-\frac{\hbar^2}{2m}\Delta \phi_n^{(V)}(\mathbf{x}) = (\lambda - e_n)\phi_n^{(V)}(\mathbf{x}).$$

$$h_{ij} = \frac{\delta E}{\delta \rho_{ji}} = h_{ji}^*, \quad \Delta_{ij} = \frac{\delta E}{\delta \kappa_{ij}^*} = -\Delta_{ji},$$

HFB+QRPA theory for open-shell nuclei I

HFB+pnQRPA in canonical basis

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \Omega_{QRPA}\begin{pmatrix} X \\ Y \end{pmatrix}$$

Receive p-h contribution

$$A_{pn,pn} = E_{pp'}\delta_{nn'} + E_{nn'}\delta_{pp'} + V^{ph}_{pn,p'n'}(u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'})$$
$$+ V^{pp}_{pn,p'n'}(u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'})$$

$$B_{pn,p'n'} = V^{ph}_{pn,p'n'}(v_p u_n u_{p'} v_{n'} + u_p v_n v_{p'} u_{n'})$$
$$- V^{pp}_{pn,p'n'}\left(u_p u_n v_{p'} v_{n'} + v_p u_n u_{p'} u_{n'}\right)$$

Reveive p-p contribution

Transition strength:

$$B_v = \left| \sum_{pn} \left(X^v_{pn} u_p v_n + Y^v_{pn} v_p u_n \right) \langle p \| O^{+}_{-} \| n \rangle \right|^2$$

HFB+QRPA theory for open-shell nuclei I

HFB+pnQRPA in canonical basis

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \Omega_{QRPA}\begin{pmatrix} X \\ Y \end{pmatrix}$$

$$A_{pn,pn} = E_{pp'}\delta_{nn'} + E_{nn'}\delta_{pp'} + V^{ph}_{pn,p'n'}(u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}) + V^{pp}_{pn,p'n'}(u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'})$$

$$B_{pn,p'n'} = V^{ph}_{pn,p'n'}(v_p u_n u_{p'} v_{n'} + u_p v_n v_{p'} u_{n'}) - V^{pp}_{pn,p'n'}\left(u_p u_n v_{p'} v_{n'} + v_p u_n u_{p'} u_{n'}\right)$$

The single-particle states are obtained by HFB

J. Dobaczewski, et.al, NPA 422, 103(1984)

For more details of the theory one may refer to:

D.J.Rowe, nuclear collective motion

J. Terasaki. et. al, Phys. Rev. C 71, 034310(2005)

outline

- **Introduction about 0vββ decay**
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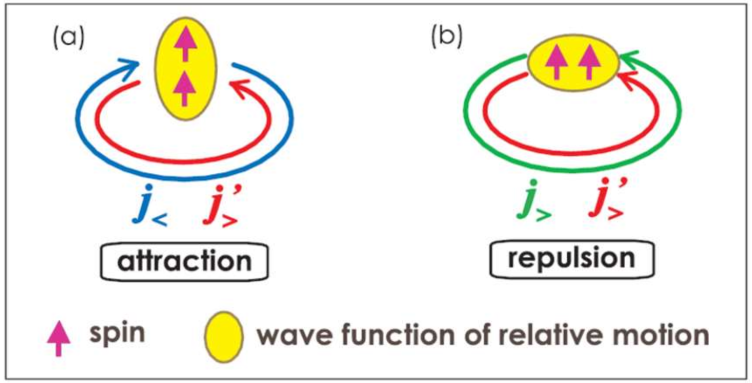
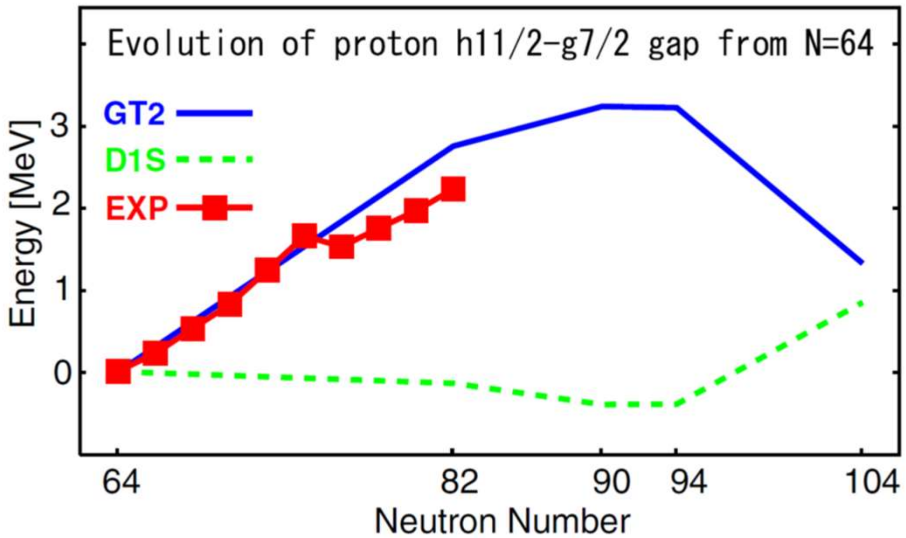
- **Summary and perspective**

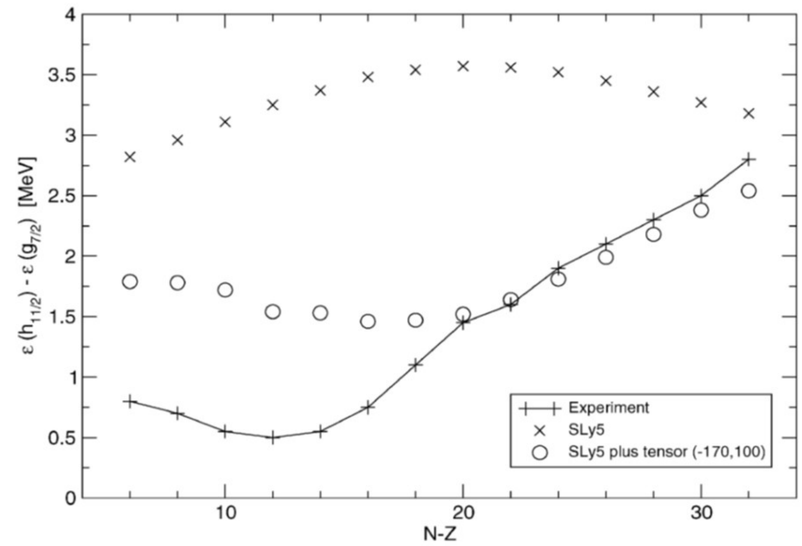
Effect of tensor force on the s-p energies I

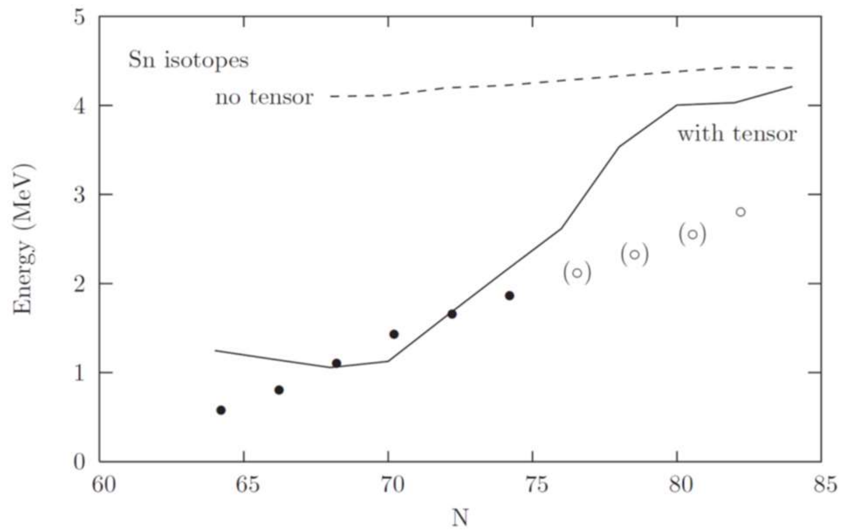




Otsuka, et.al., PRL 97, 162501(2006)

Effect of tensor force on the s-p energies II

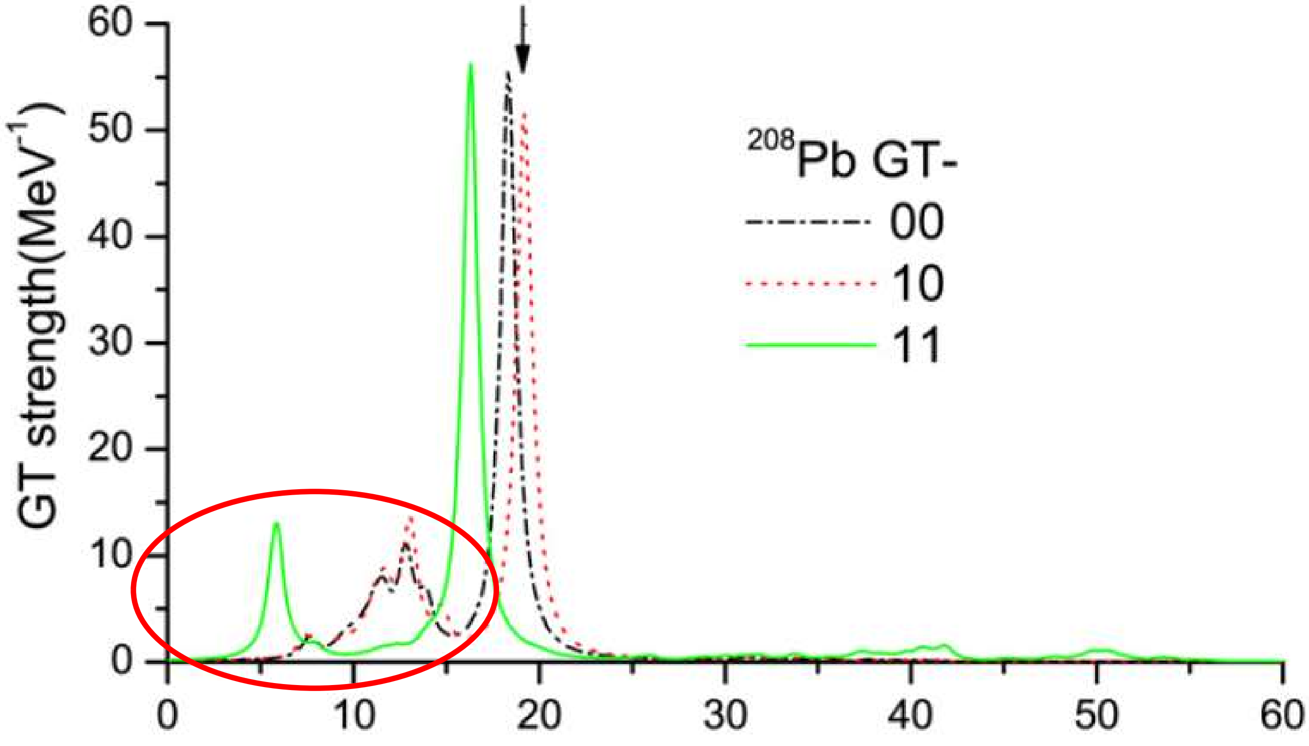




G. Colo, et.al., PLB 646,227(2007)

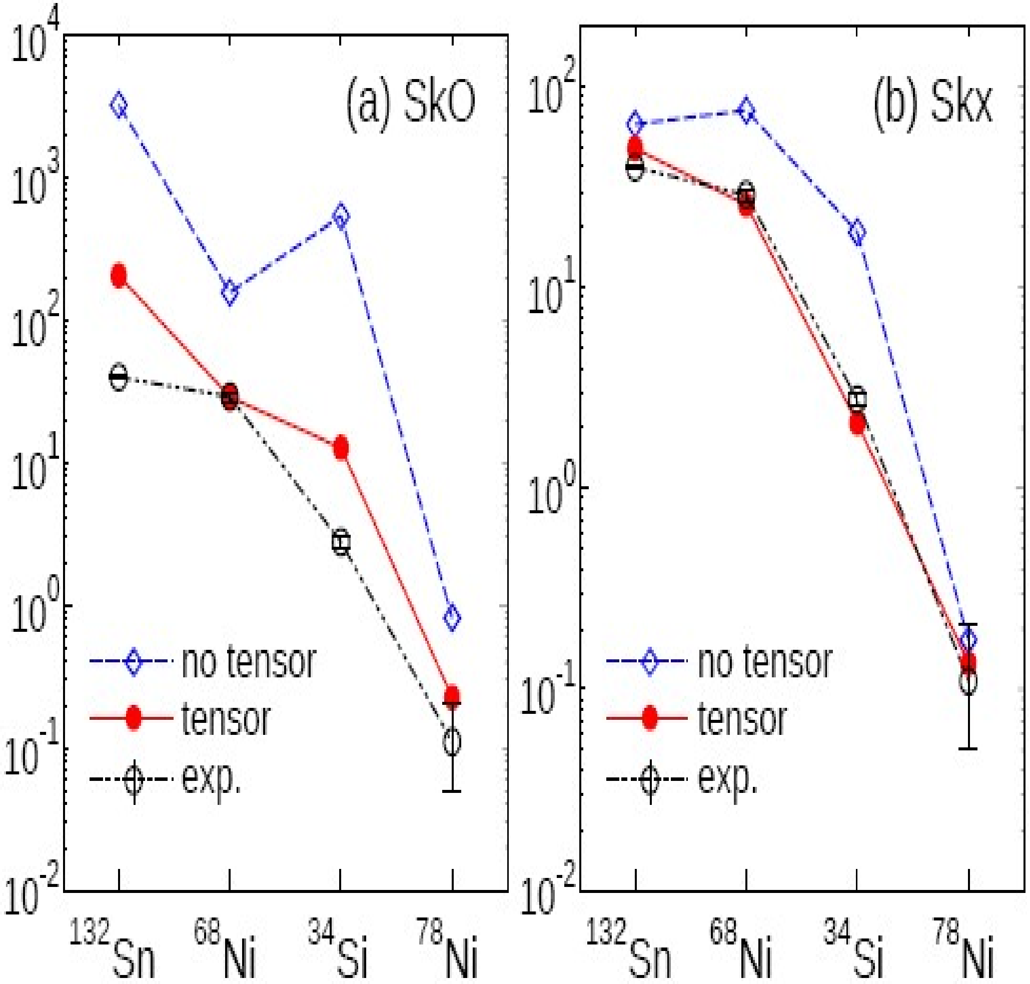
D.M. Brink, et.al., PRC 75, 064311(2007)

GT states and Tensor force



Bai , Sagawa, Zhang, Zhang, Colò, Xu, Phys. Lett. B675, 28(2009)

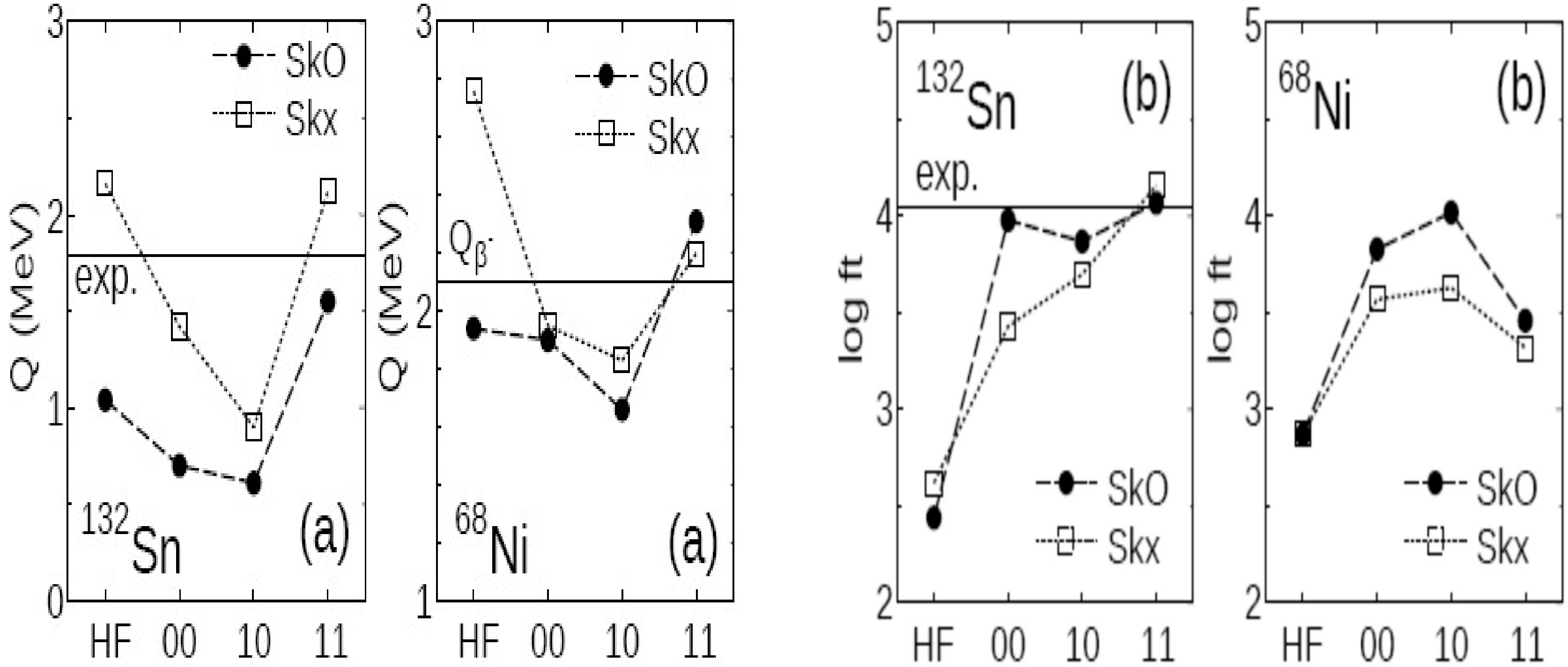
β-decay half-life in closed-shell nuclei



Tensor RPA correlations improve the β-decay half-life calculations in closed-shell nuclei.

F. Minato and C. L. Bai, Phys. Rev. Lett 110, 122501(2013)

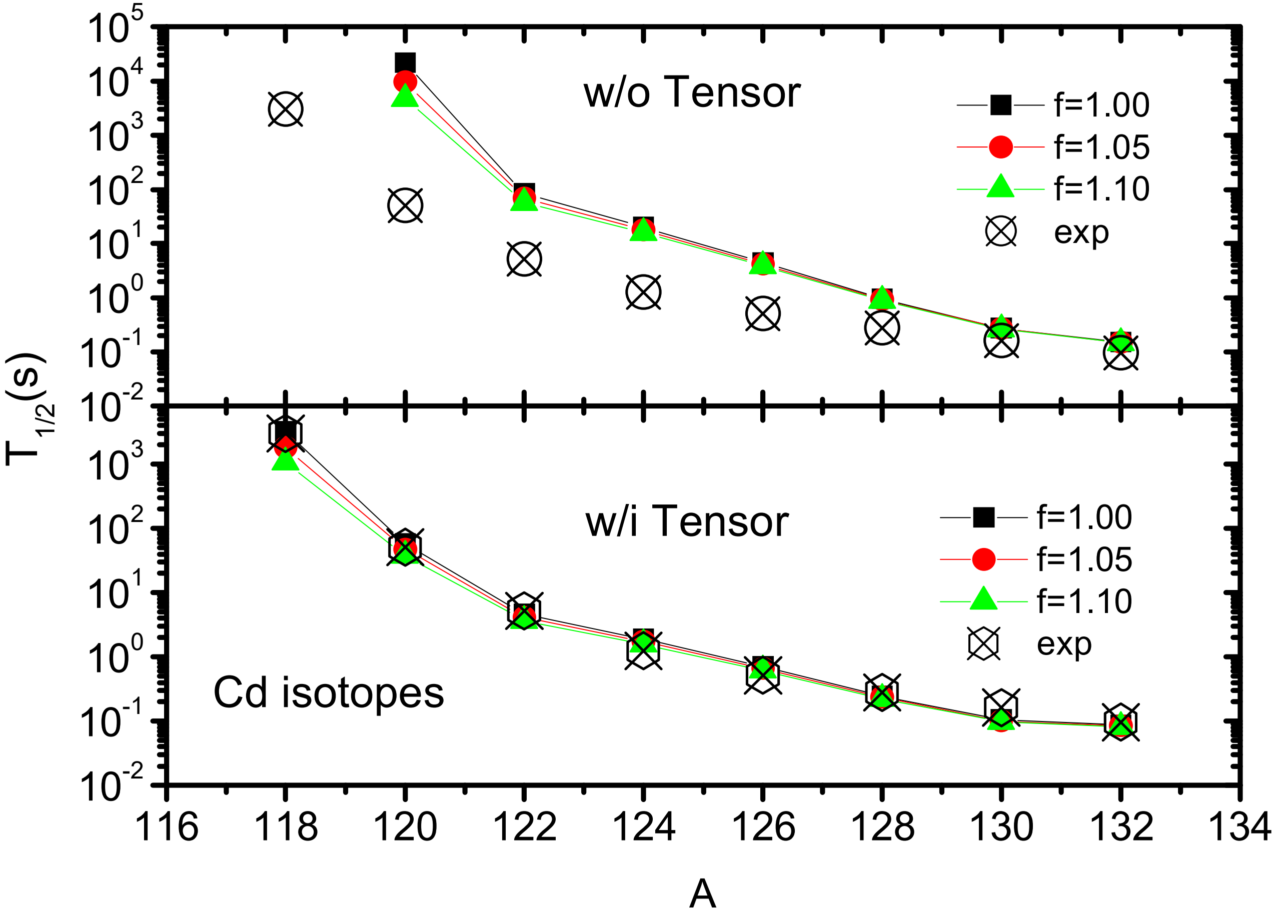
β-decay half-life in closed-shell nuclei



The inclusion of tensor force in self-consistent RPA calculation improve the Result of Q and logft values.

F. Minato and C. L. Bai, Phys. Rev. Lett 110, 122501(2013)

β-decay results for open shell



C.L. Bai, D.L. Fang, H.Q. Zhang, X.Z. Zhang, and F.R. Xu, in preparing.

logft and Q value

	SKO		SKO+T		Exp.[42]	
	Q_i	$\log ft$	Q_i	$\log ft$	Q_i	$\log ft$
^{120}Cd	1.211	4.17	2.171	4.03	2.281	4.10
^{122}Cd	1.922	4.21	2.973	4.05	3.431	3.95
^{124}Cd	2.579	4.25	3.761	4.06		
^{126}Cd	3.191	4.29	4.538	4.08		
^{128}Cd	3.769	4.33	5.312	4.09	6.241	4.17
^{130}Cd	4.328	4.39	1.637	3.88	6.741	4.10
^{132}Cd	2.118	4.20	2.064	3.84		

C.L. Bai, D.L. Fang, H.Q. Zhang, X.Z. Zhang, and F.R. Xu, in preparing.

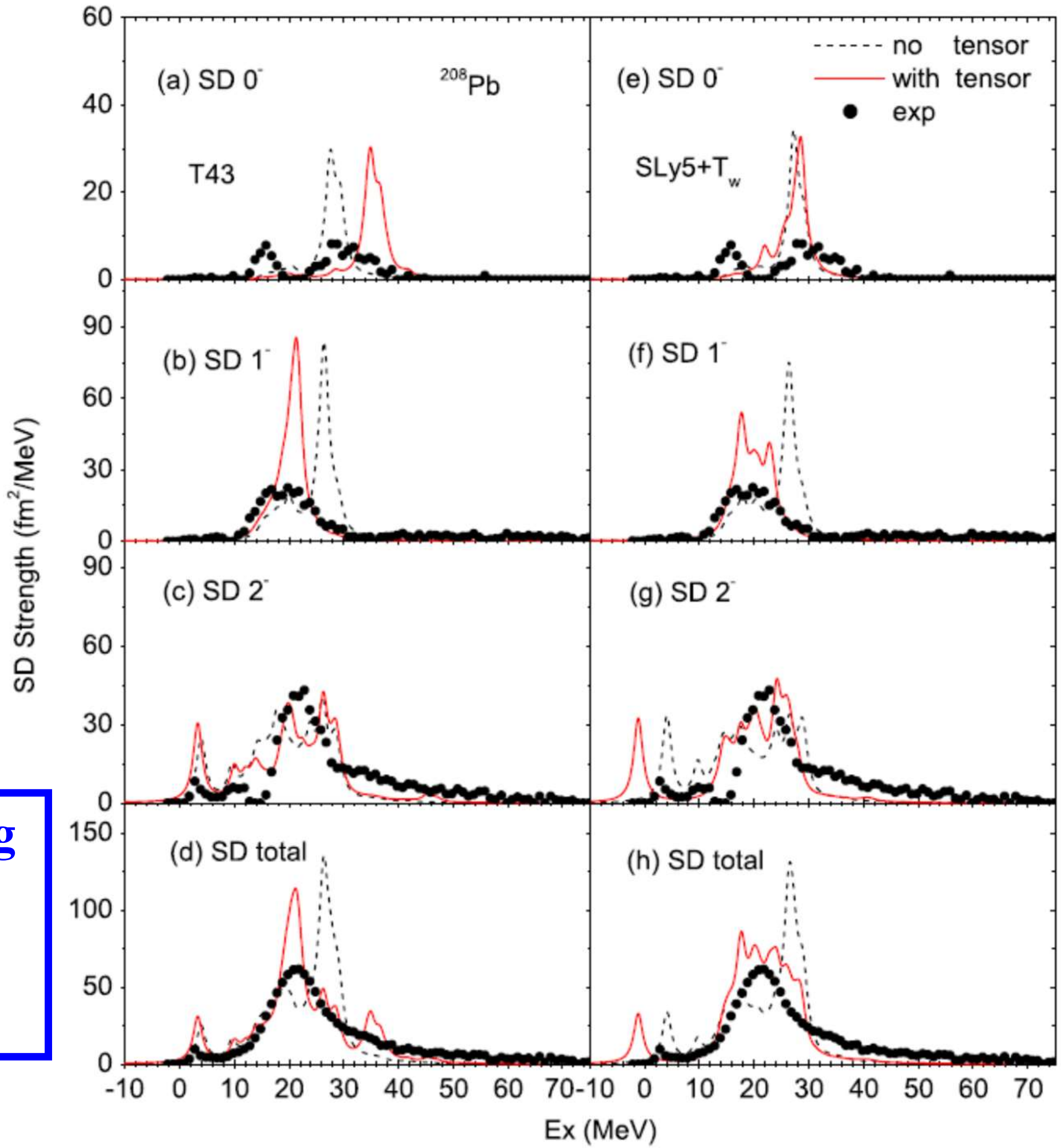
Tensor effects on Charge-exchange SD

Bai, Zhang, Sagawa, Zhang, Colò, Xu, PRL 105,072501(2010)

Exp. From
T. Wakasa et. al.,
PRC 84, 014614(2011)

In T43, T >0, U<0
In SLy5+Tw, T>0, U>0

Tensor force may have strong Effect on the first forbidden Transition, especially from SD 2- state.



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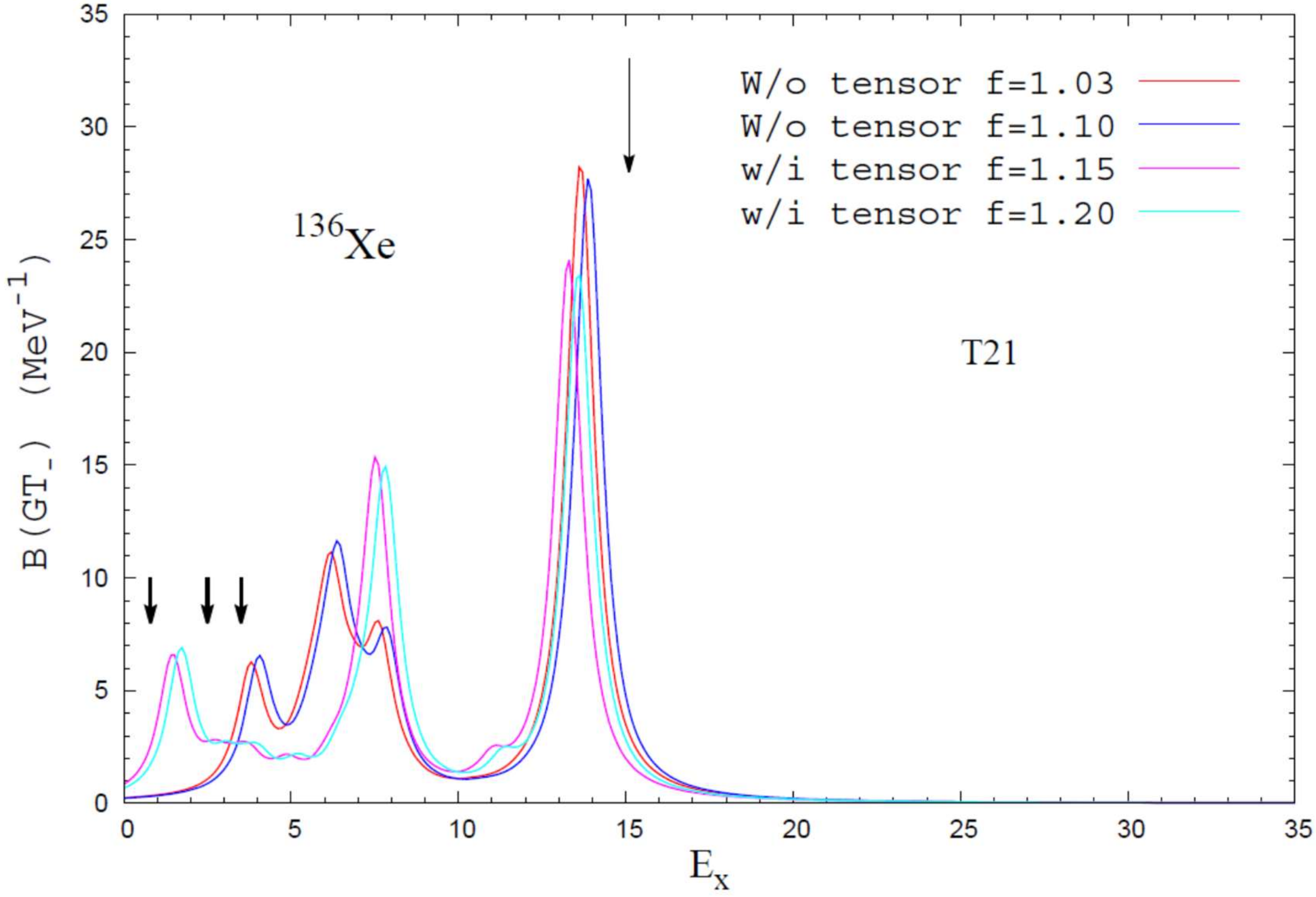
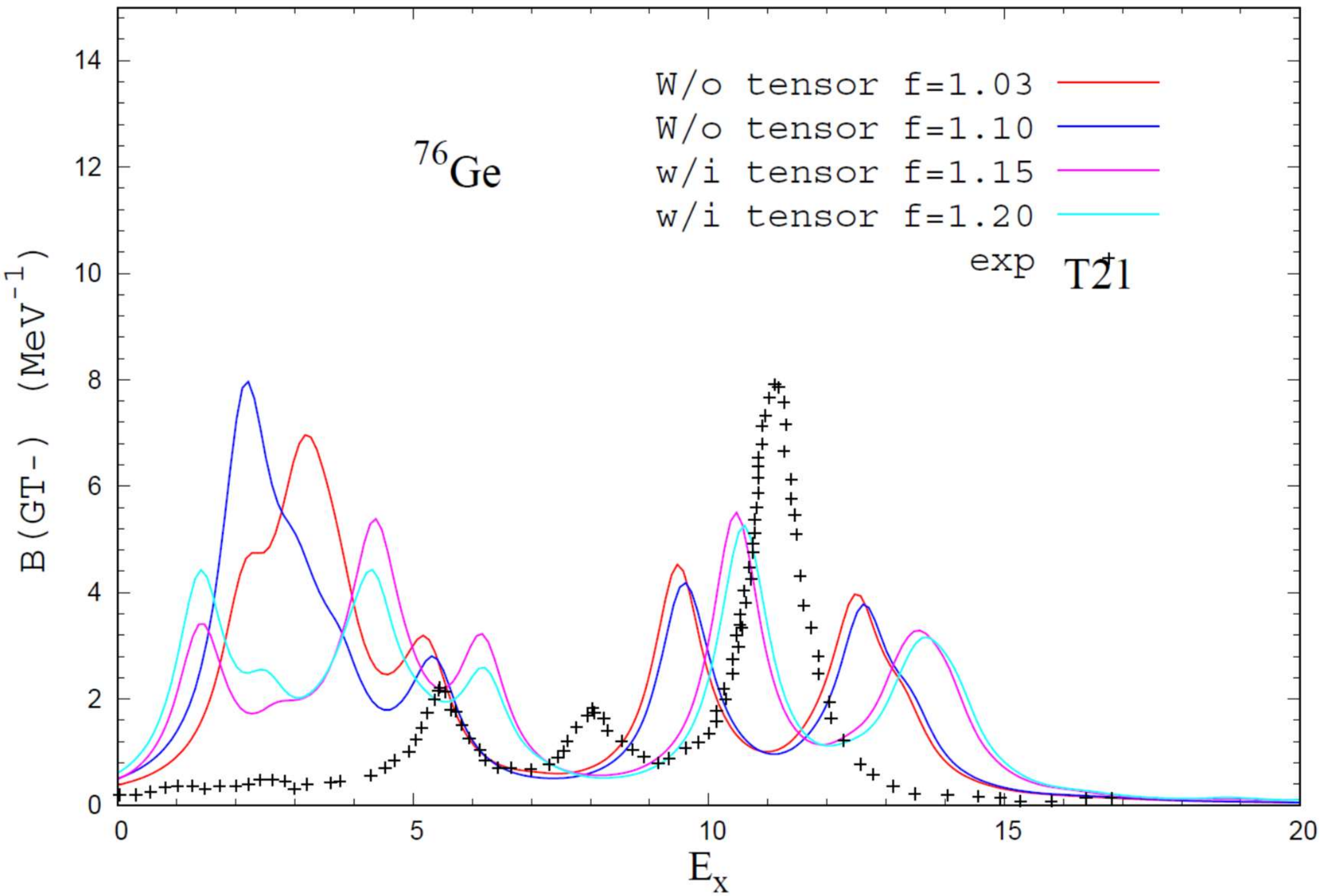
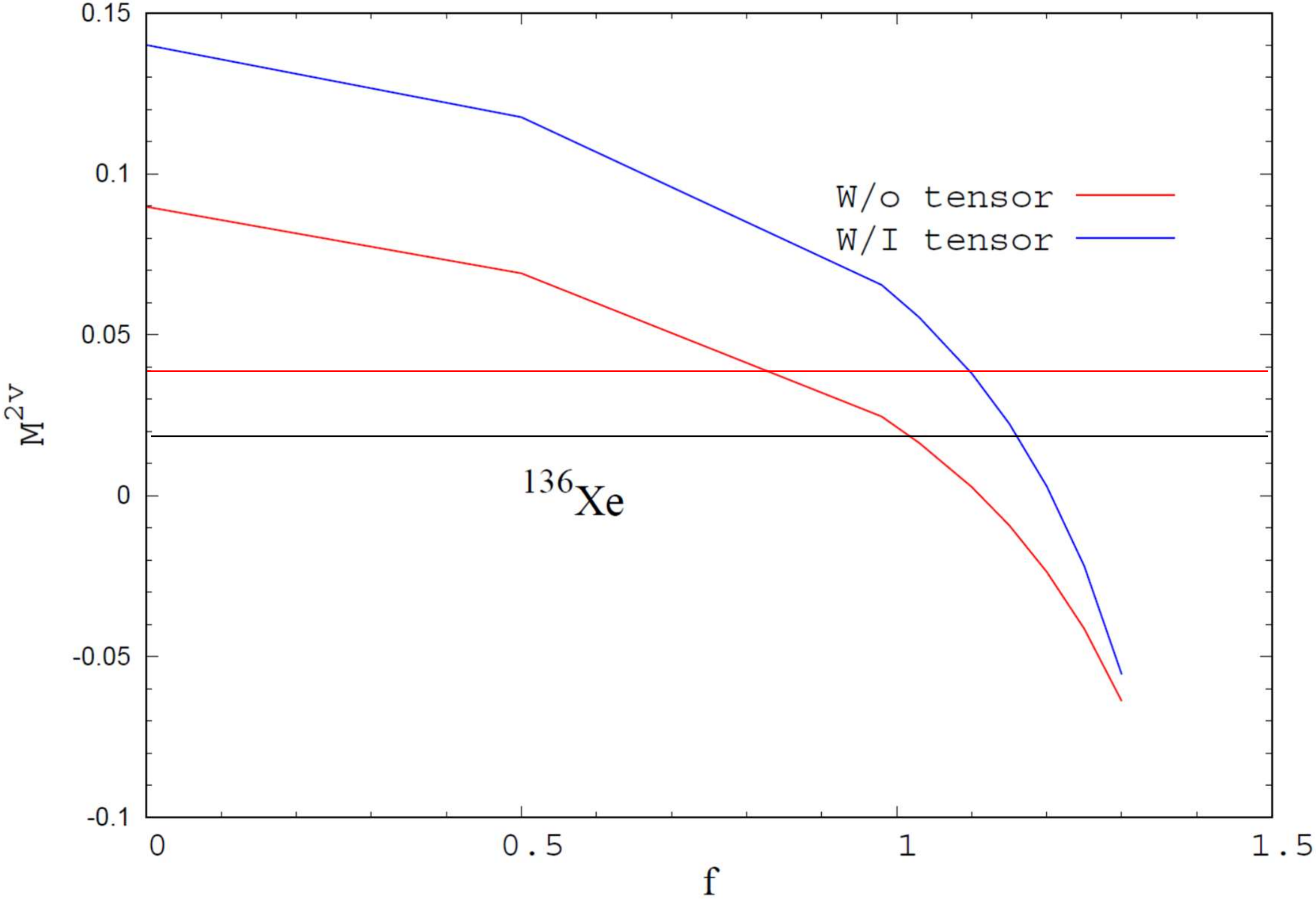
GT strength distribution



GT strength distribution

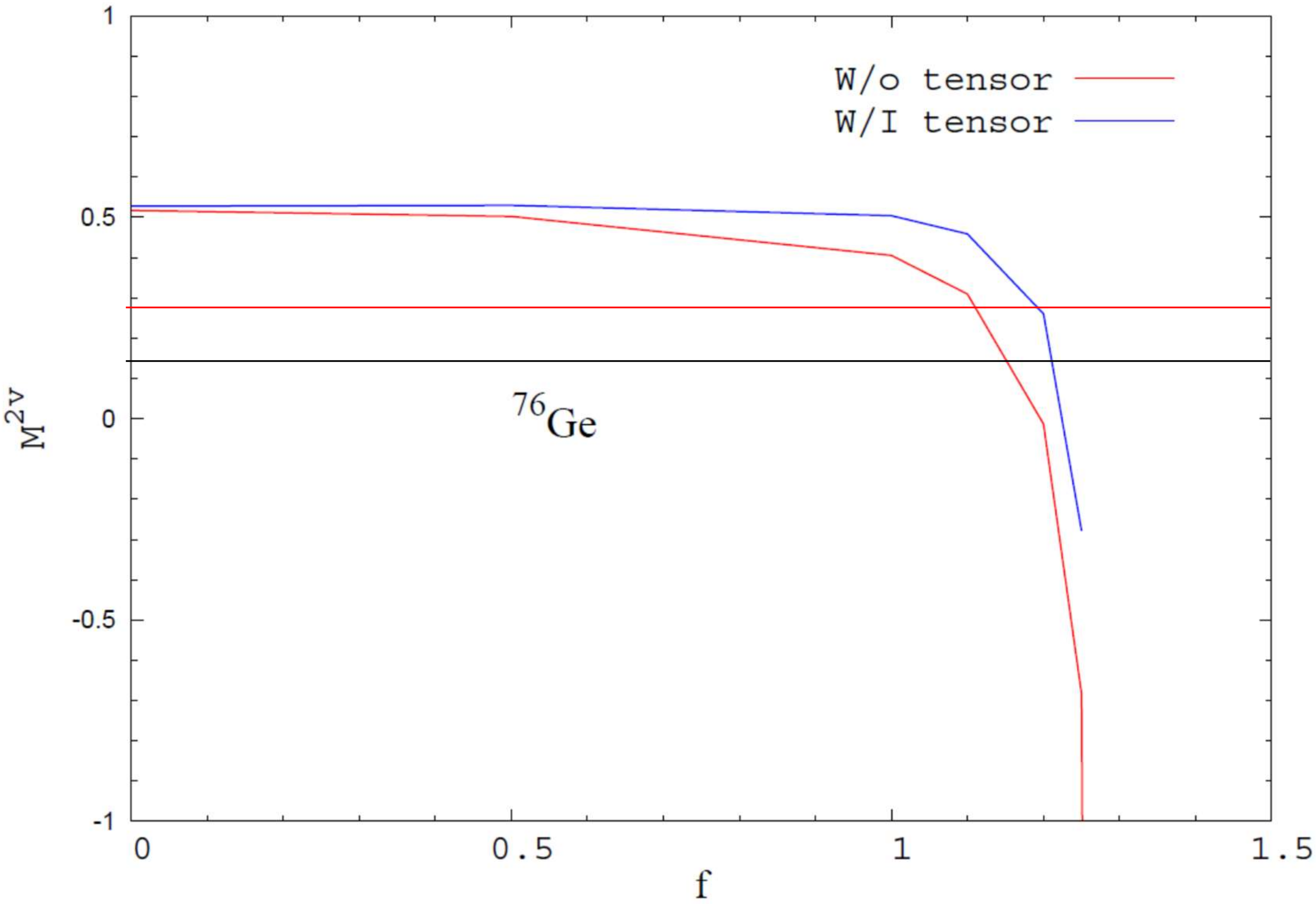


Present results on the 2νββ matrix elements



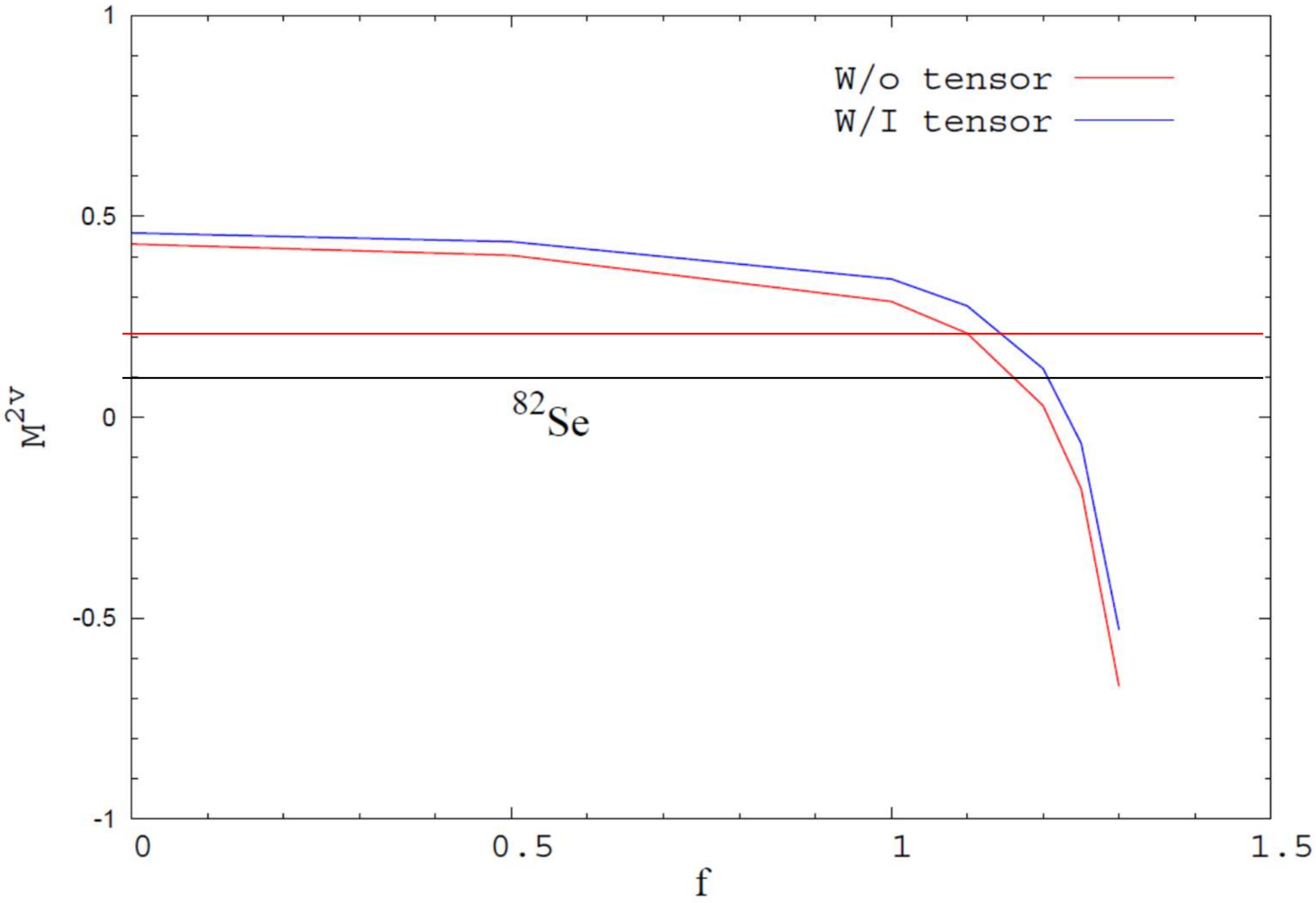
Increased Dramatically

Present results on the 2νββ matrix elements



f is set at 1.15 and 1.21 with g_A=1.26 for 0νββ

Present results on the 2νββ matrix elements



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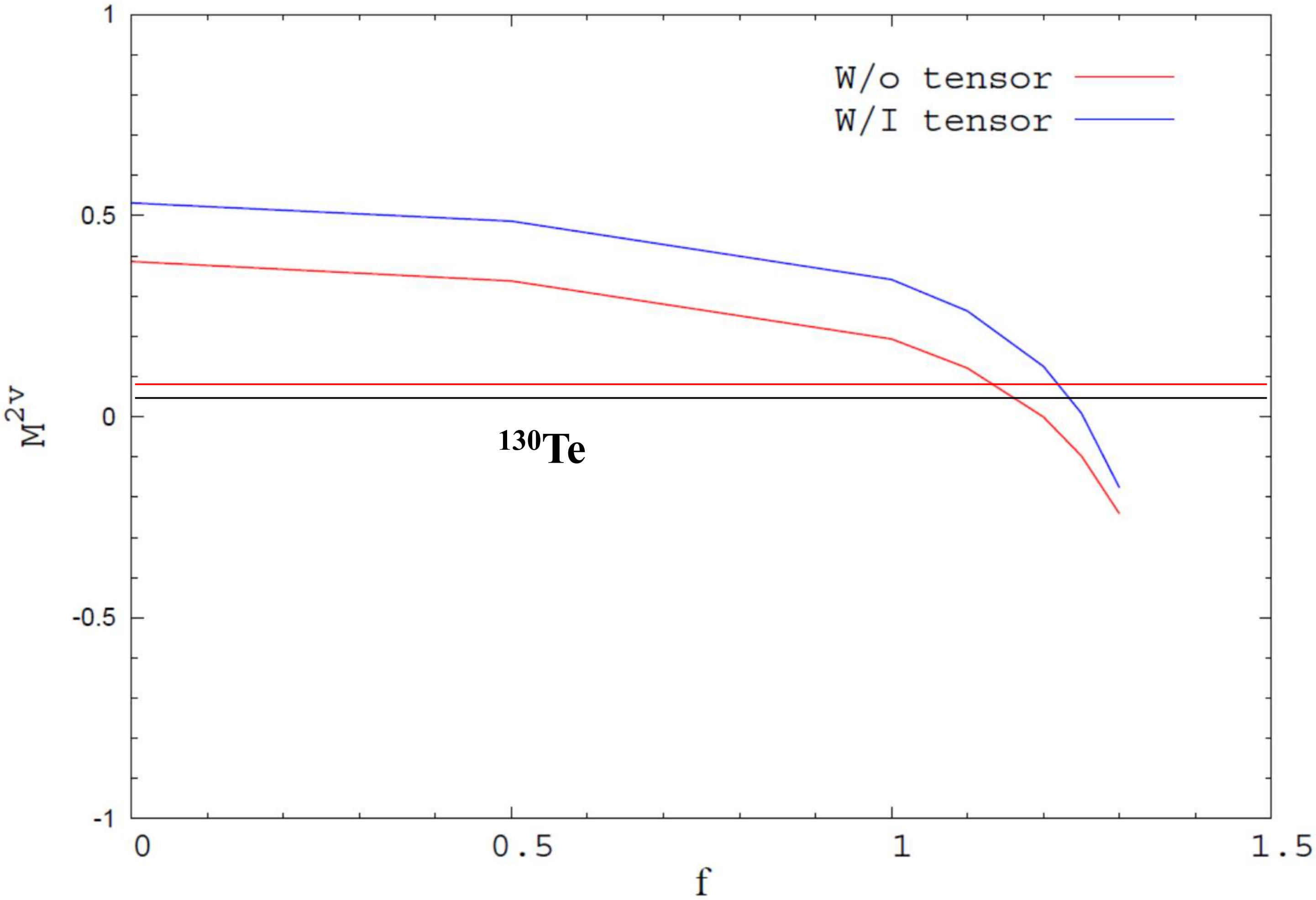

Increased Dramatically

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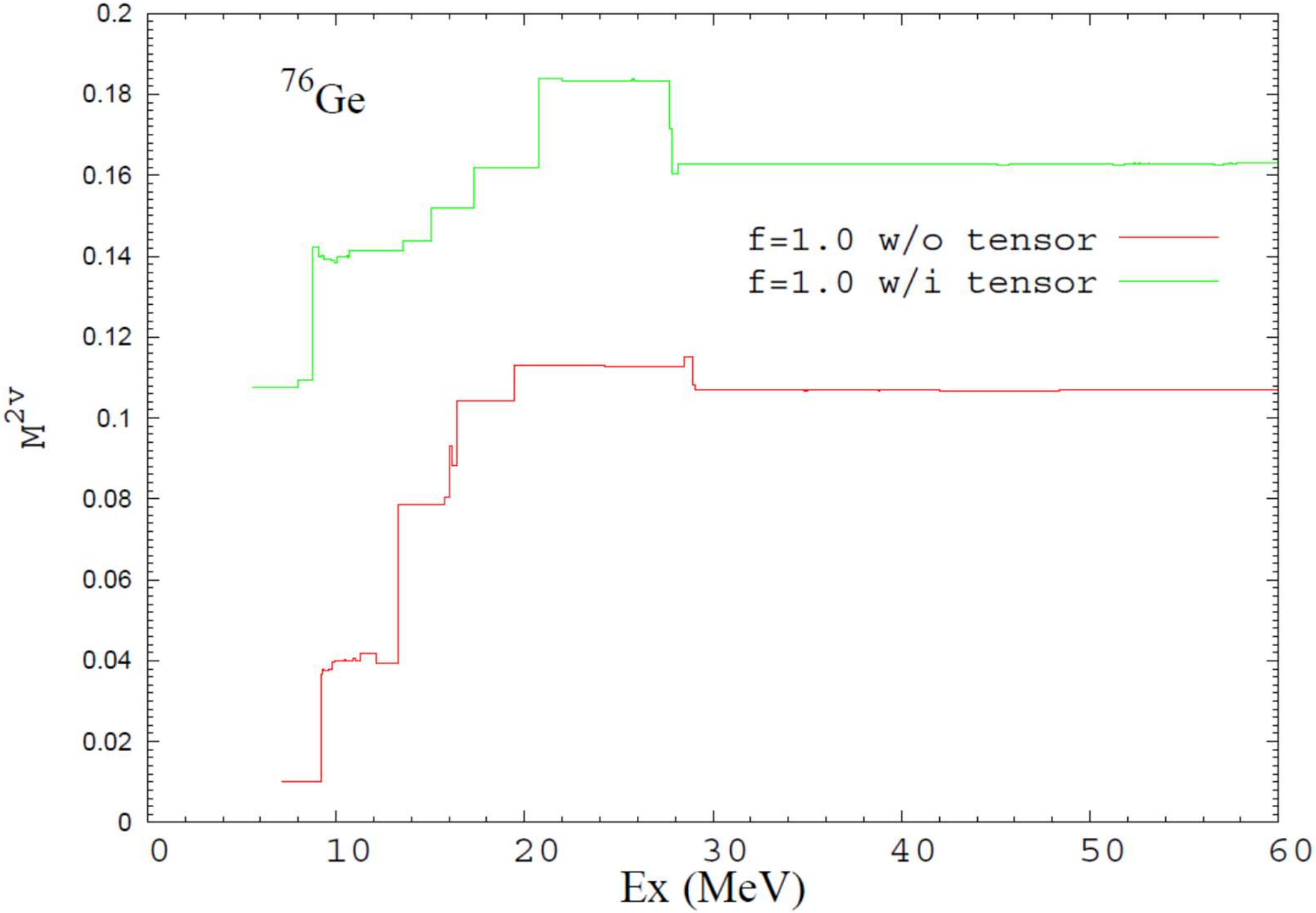

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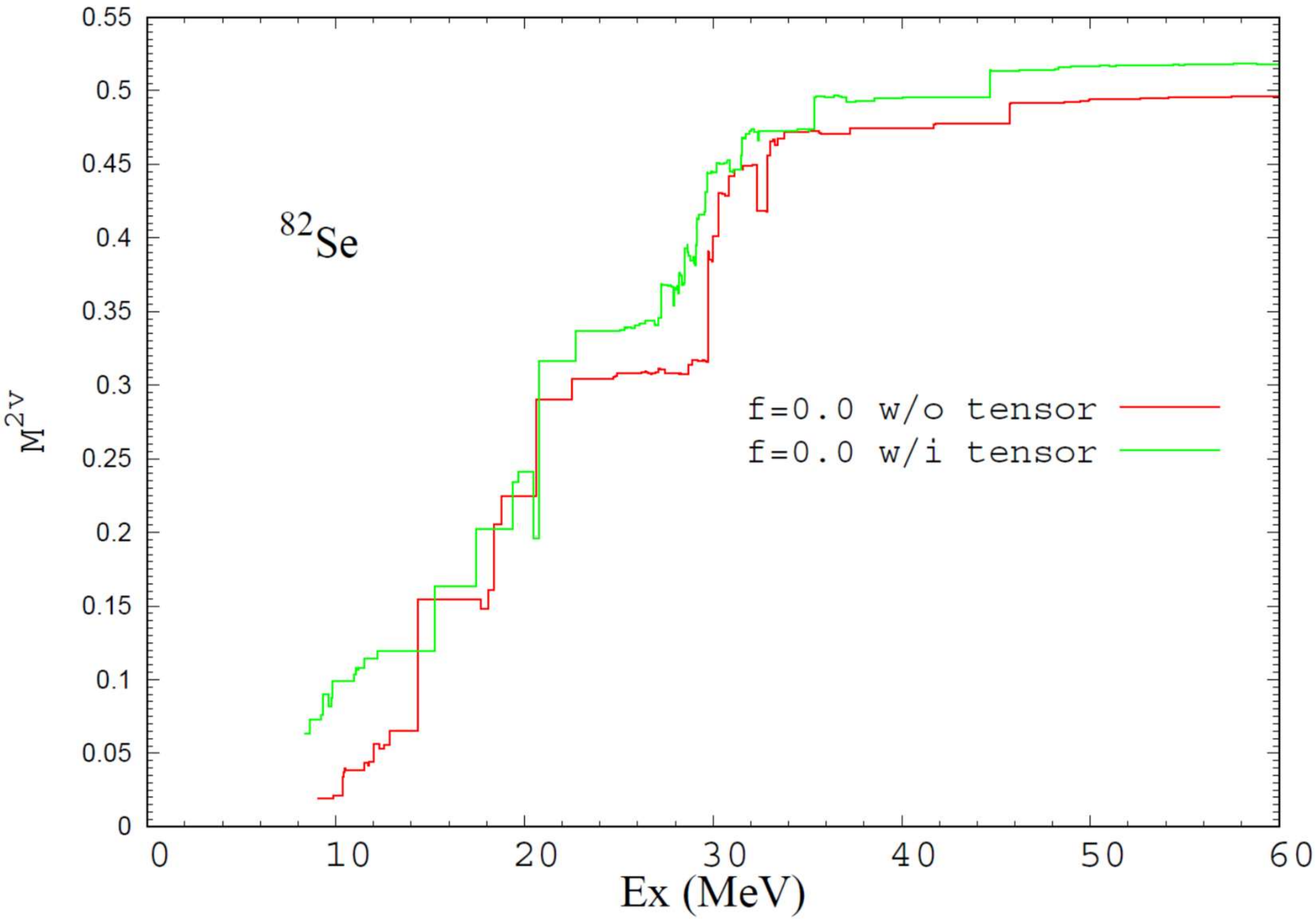

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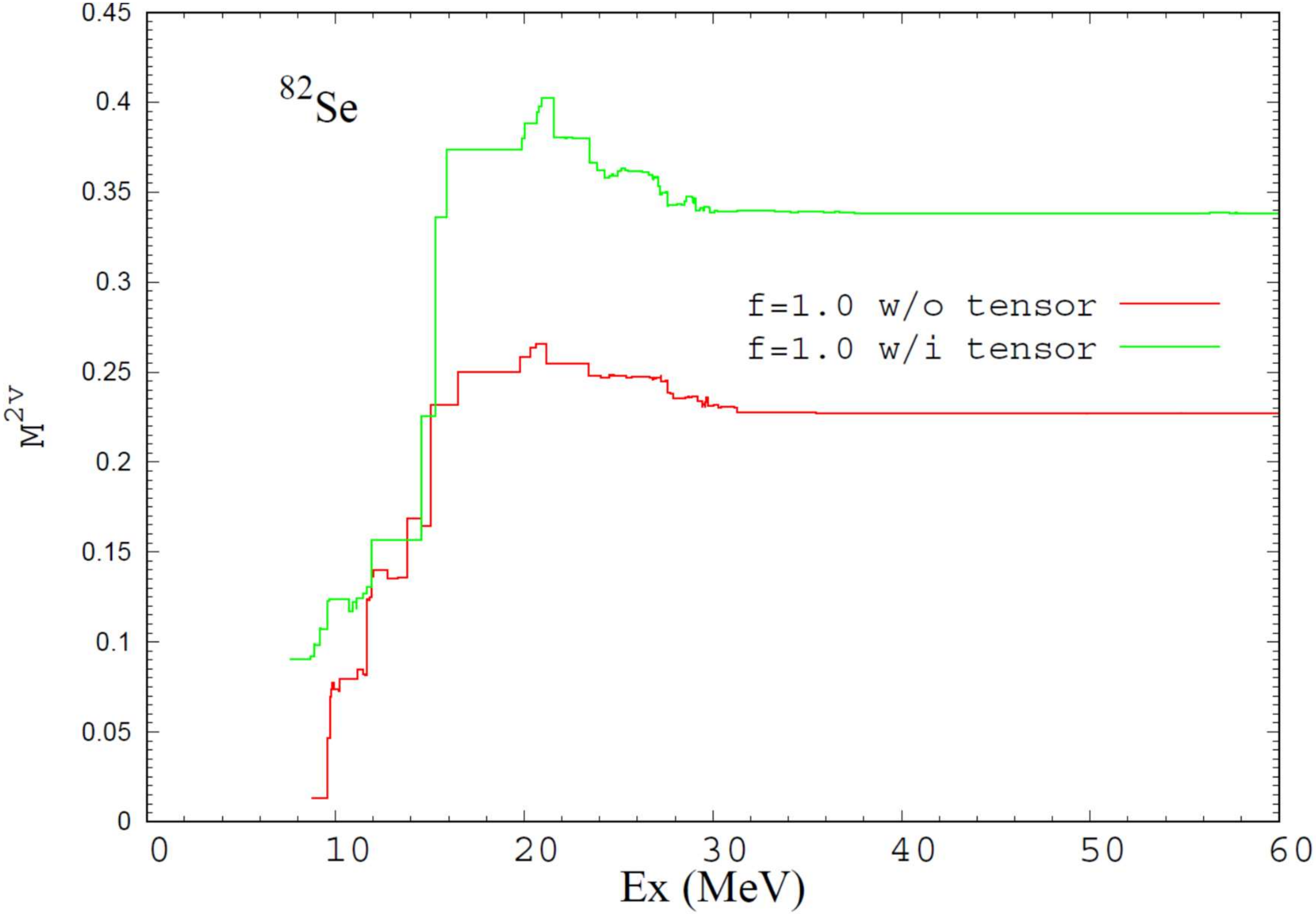
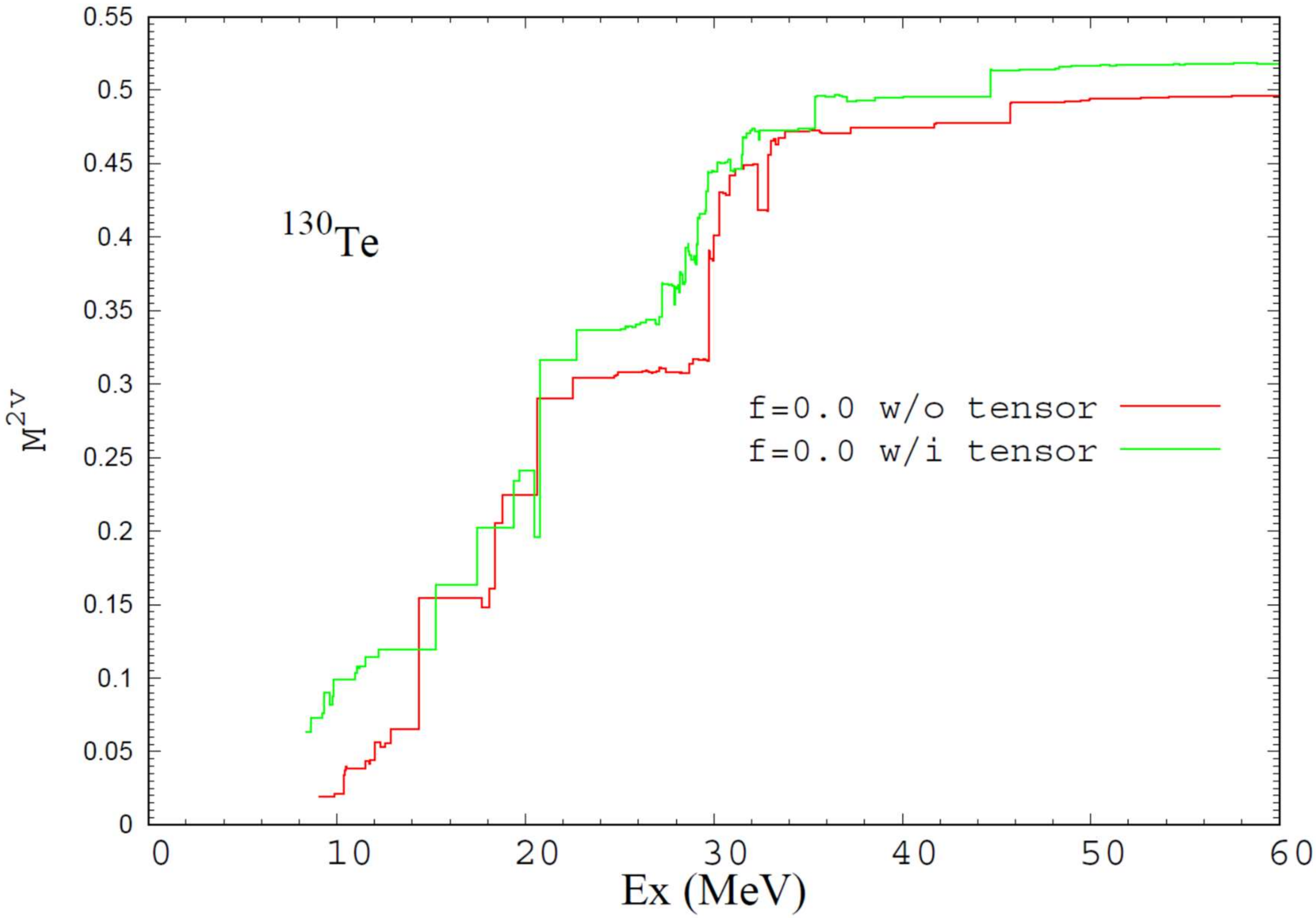


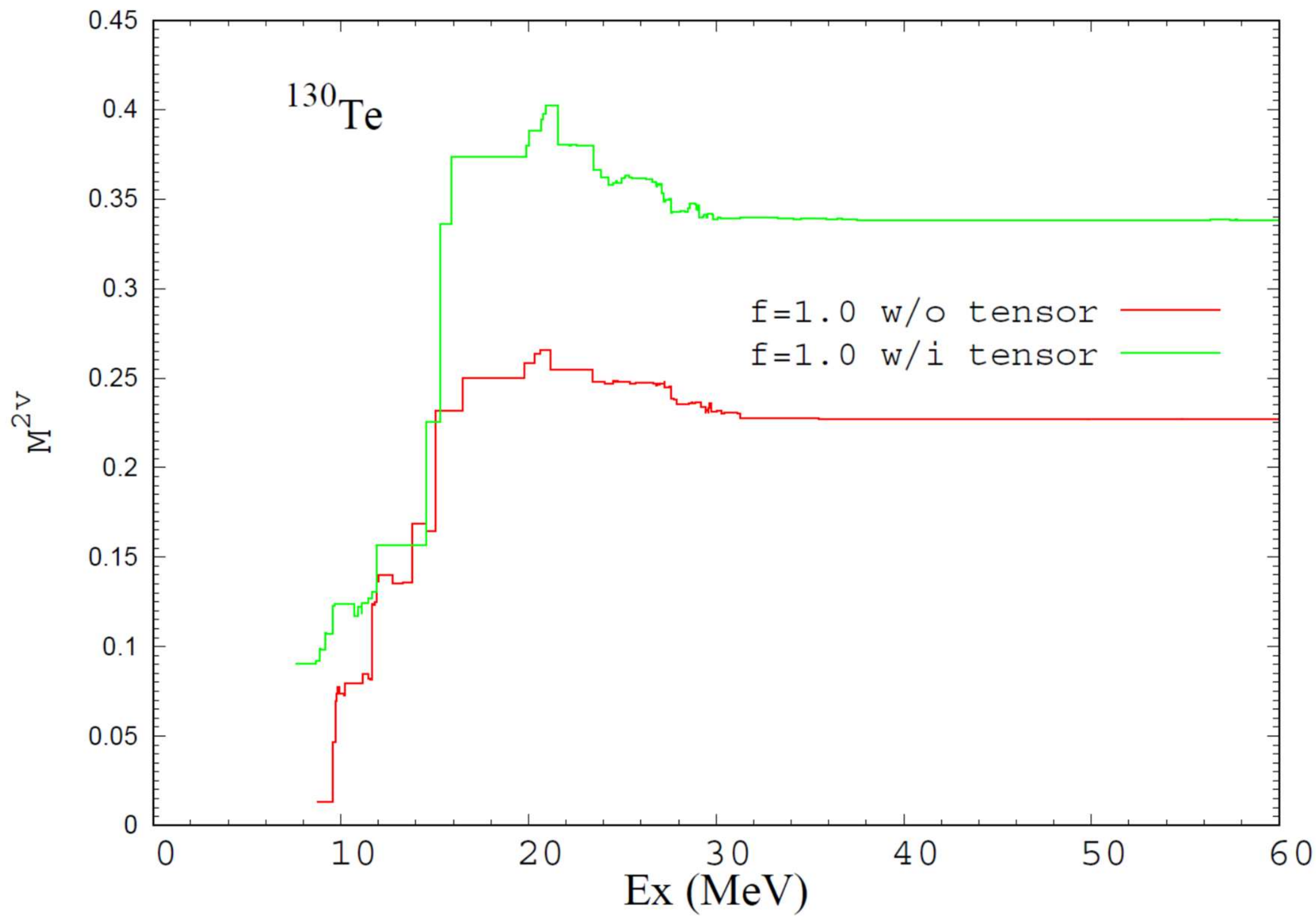
Increased Dramatically

Present results on the 2νββ matrix elements



Increased Dramatically

Present results on the 2νββ matrix elements



Increased Dramatically

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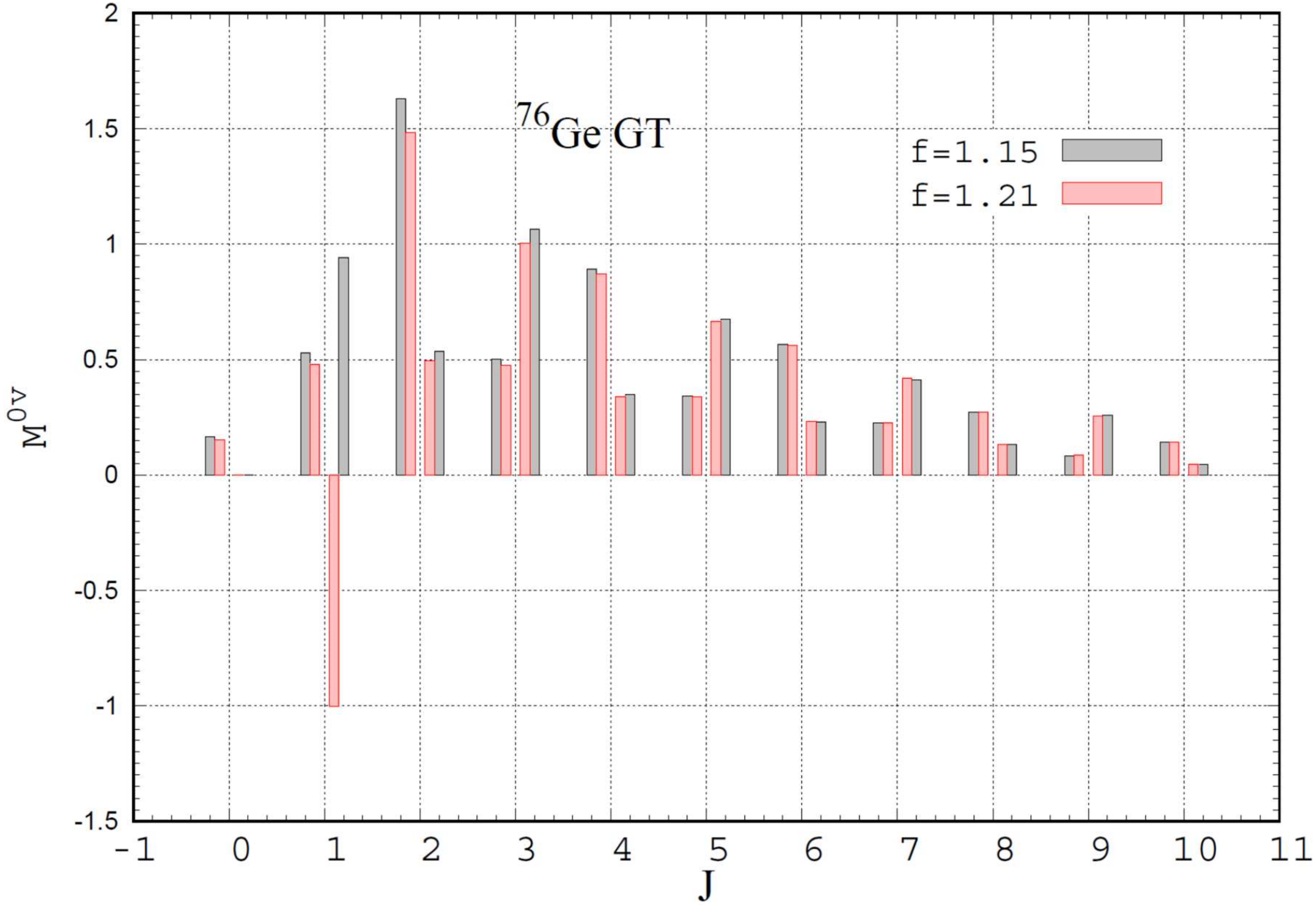
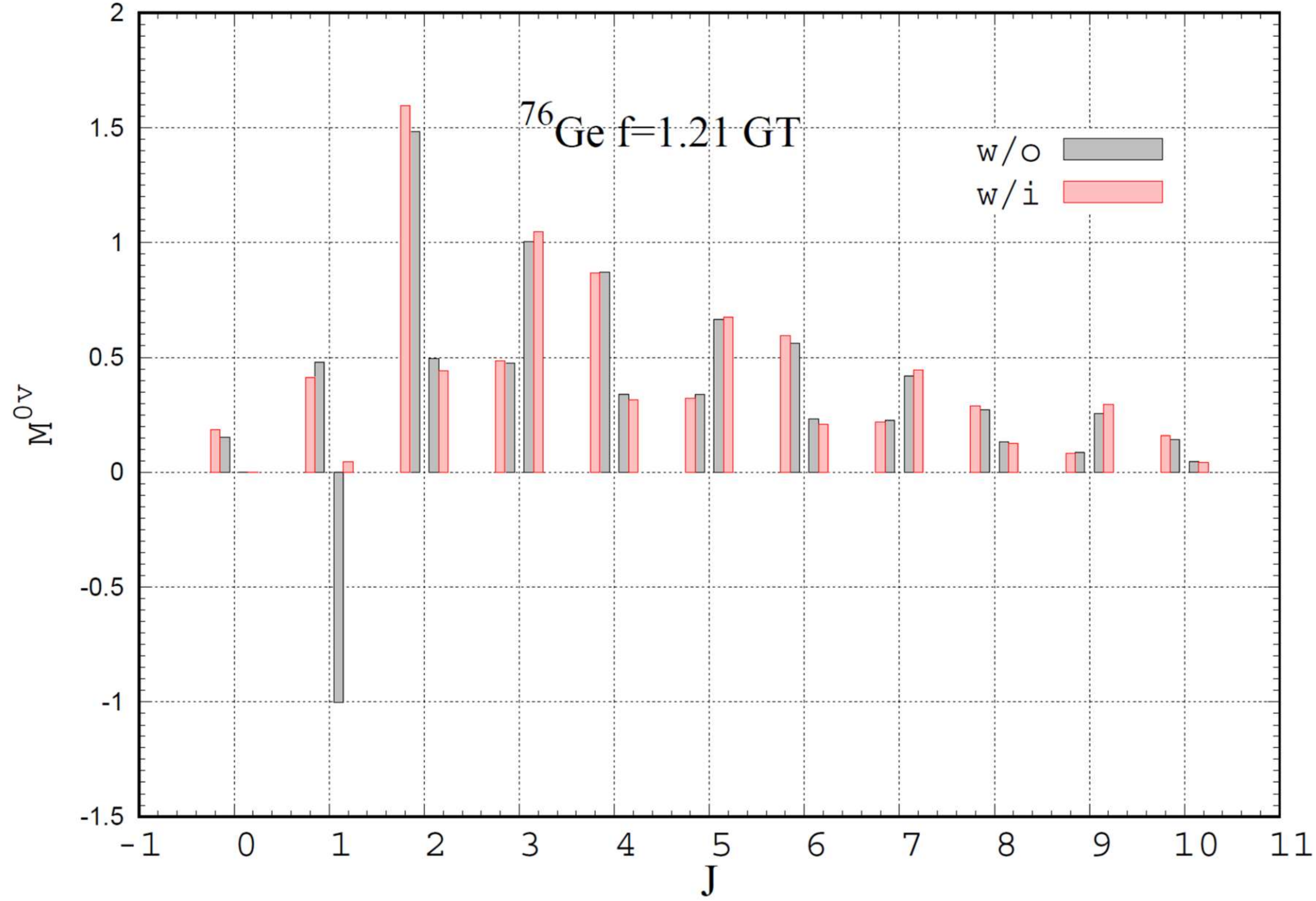
0νββ NME for ^{76}Ge—Effect of IS pairing

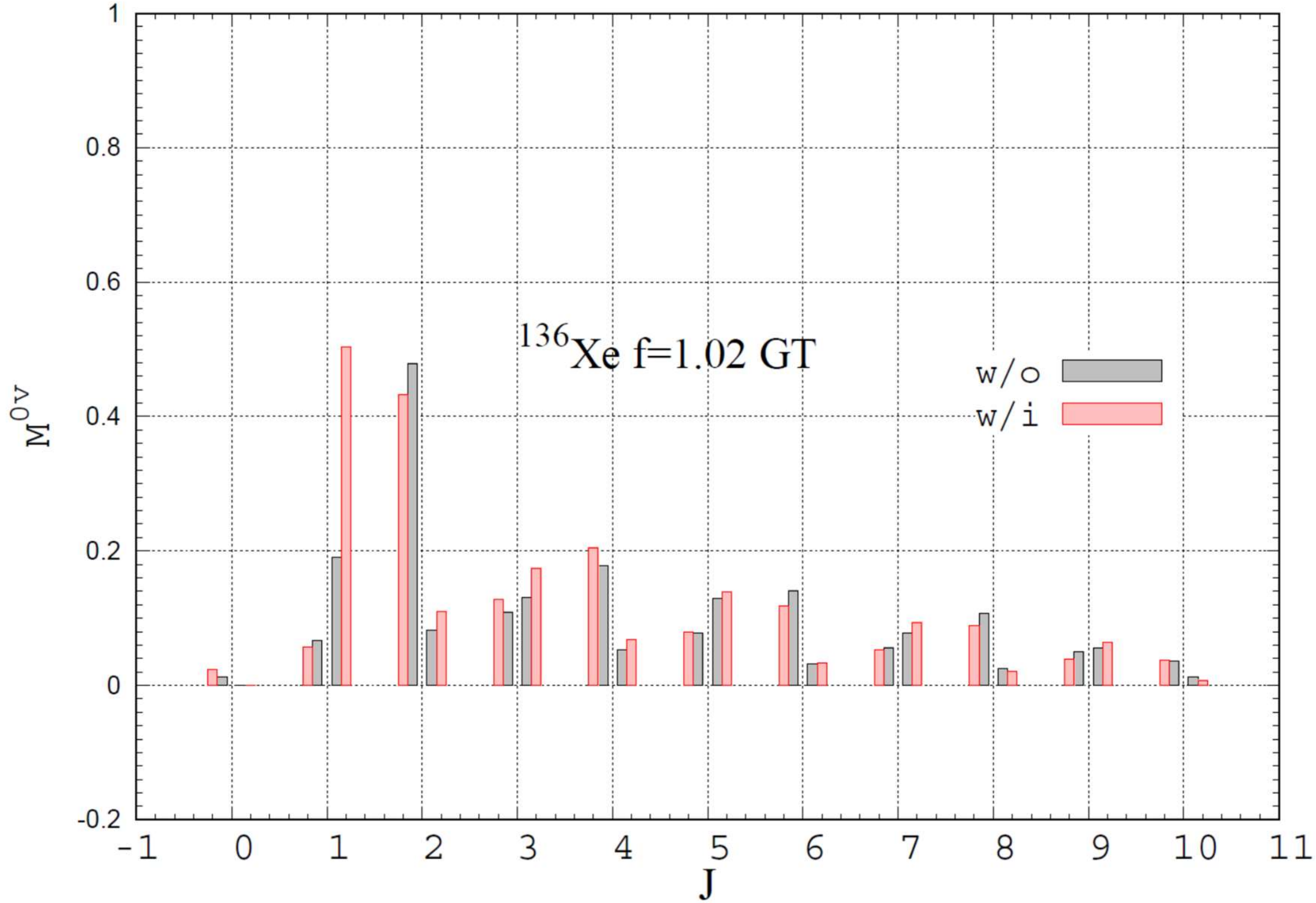


0vββ NME for ^{76}Ge—Effect Tensor force

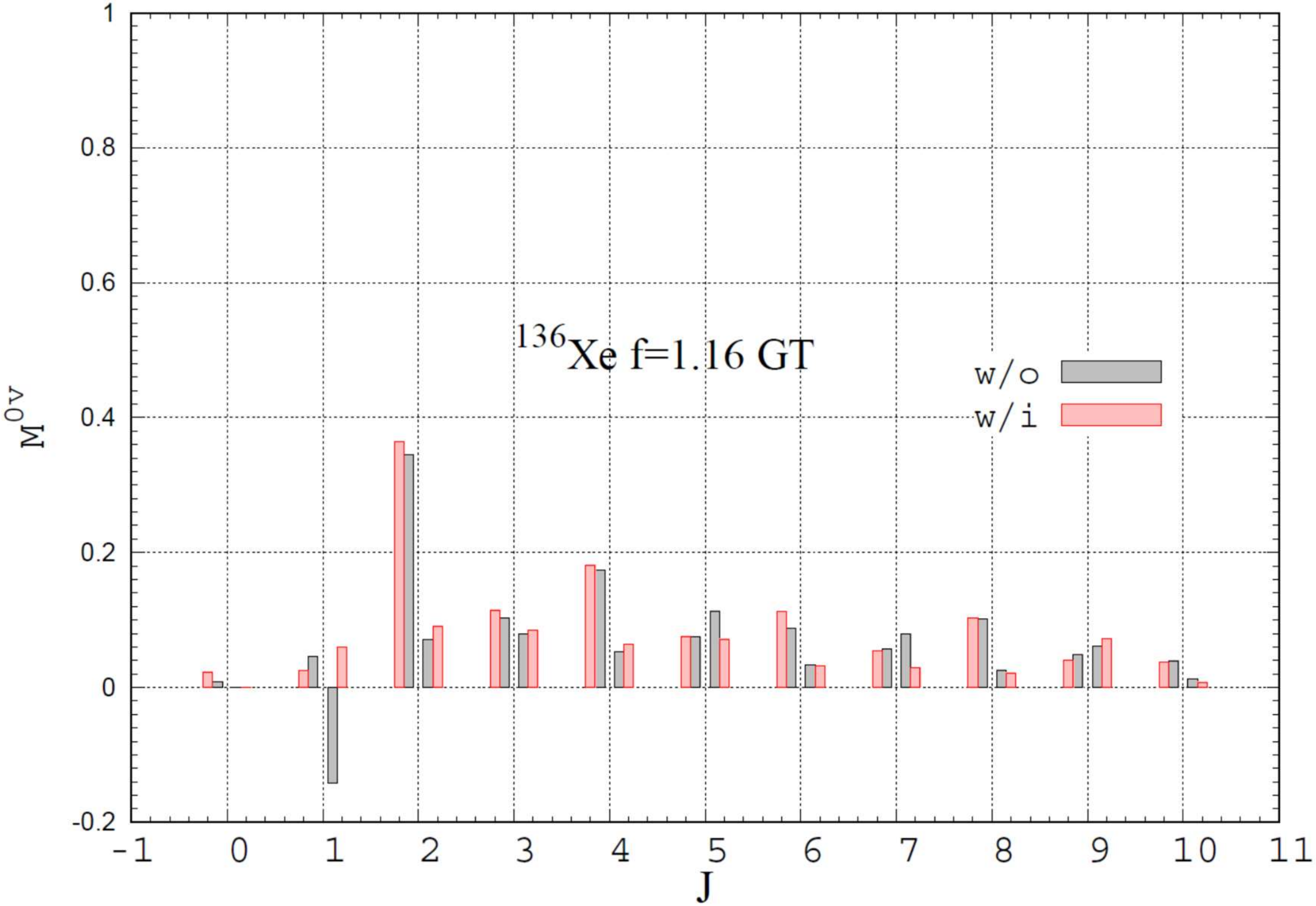


Effect of the tensor force concentrated on the 1^+ state.

0vββ NME for ^{136}Xe—Effect Tensor force



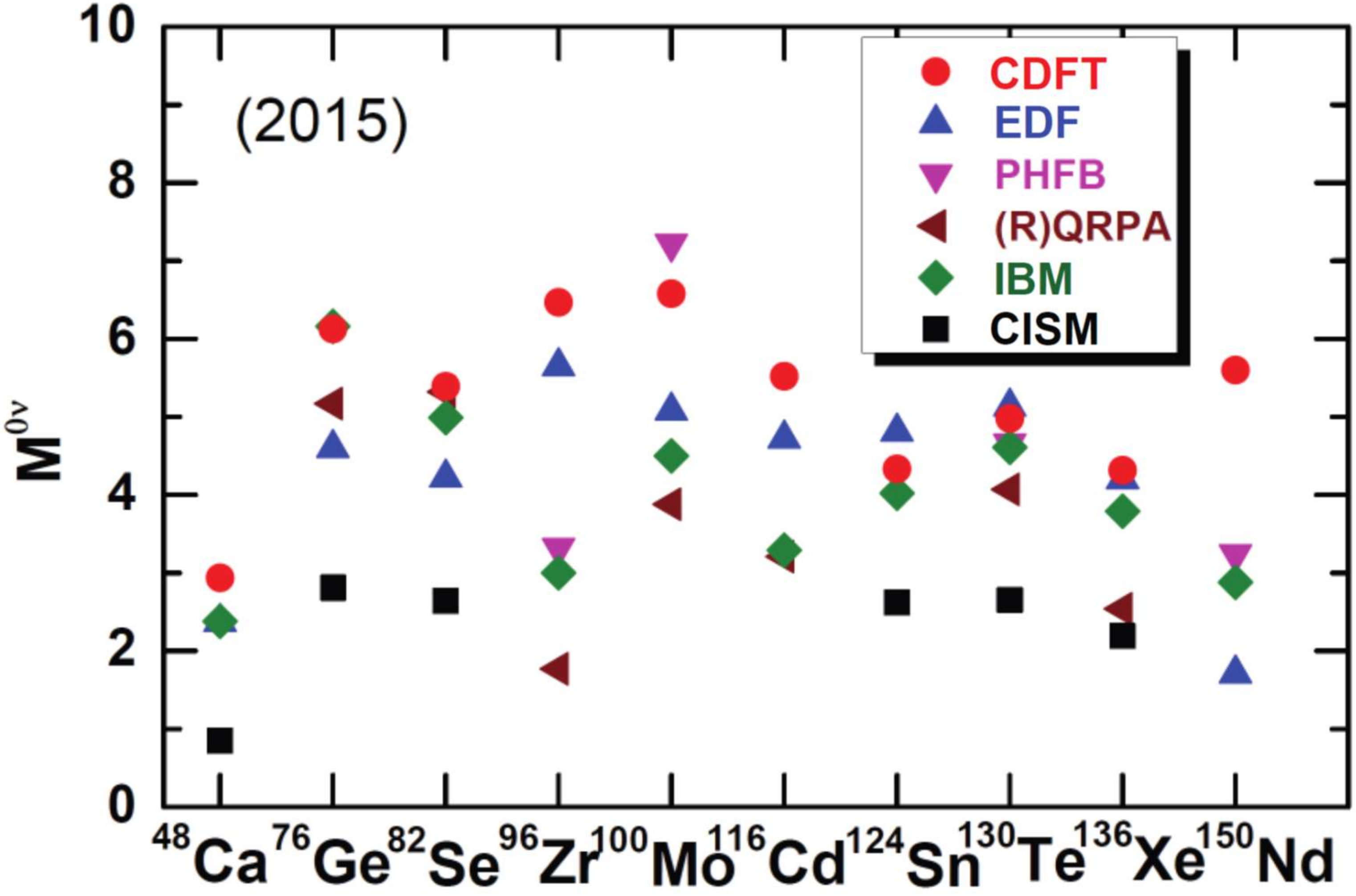
0vββ NME for ^{136}Xe—Effect Tensor force



0νββ NME

nuclei	f	w/o tensor			w/i tensor		
		GT	FM	total	GT	FM	total
^{76}Ge	1.15	6.24	1.23	7.01	6.76	1.21	7.52
	1.21	4.79	1.24	5.57	5.54	1.21	6.30
^{136}Xe	1.02	2.10	0.25	2.25	2.47	0.26	2.64
	1.16	1.47	0.24	1.62	1.66	0.27	1.83

Results of different models



Song et al., Phys. Rev. C 95, 024305(2017)

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Summary and perspective

- Tensor force affect increases the 2v NME for the nuclei studied
- With the same strength of IS pairing, tensor interaction increase 0v NME by about 10%
- The deformation should be included
- NME of heavy neutrino will be calculated

Collaborators

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Dongliang Fang	IMPC, CAS

Many Thanks for your attention