同济大学
TONGJI UNIVERSITY

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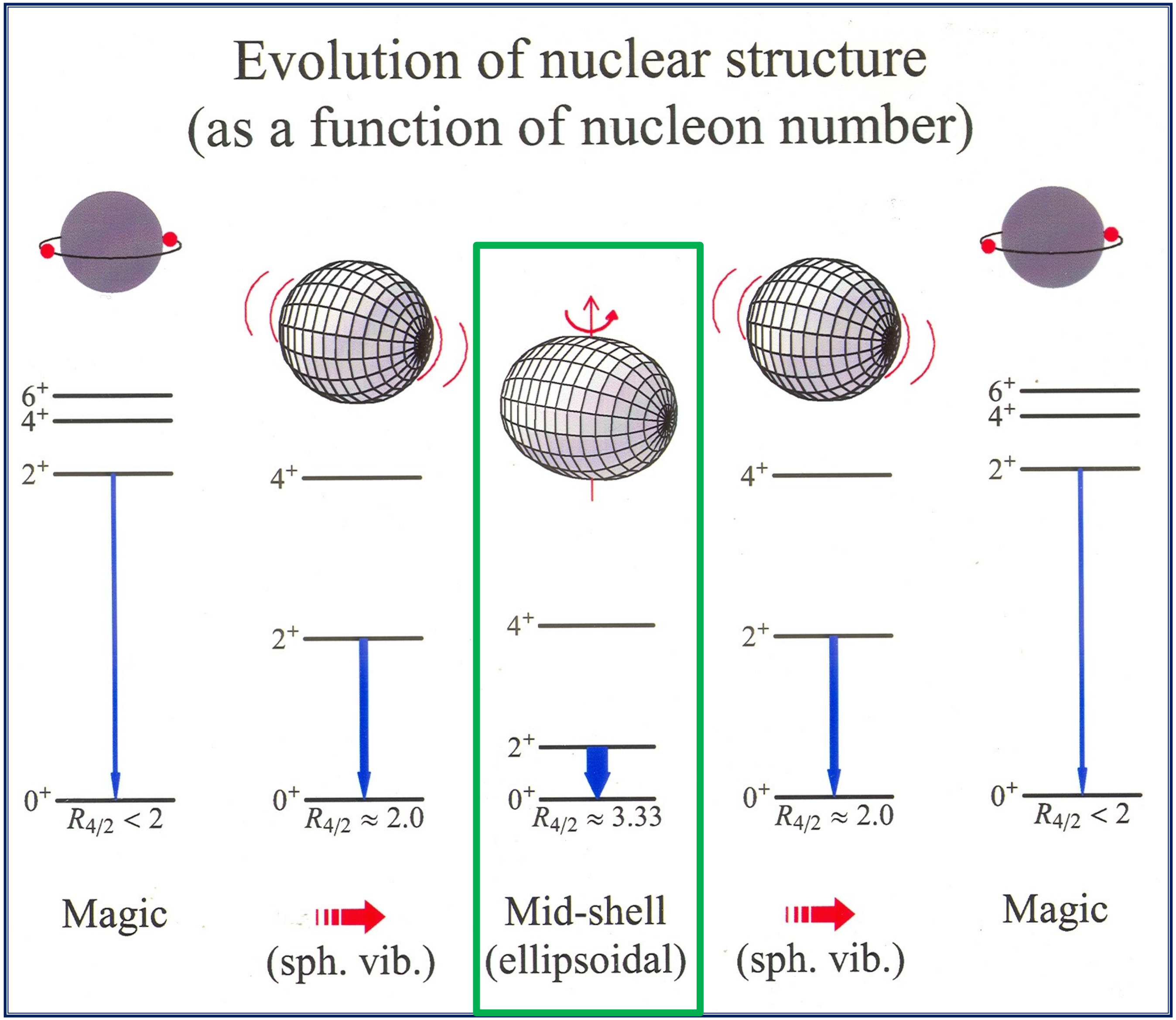
Nucleon-pair coupling in rotational nuclei

GuanJian Fu

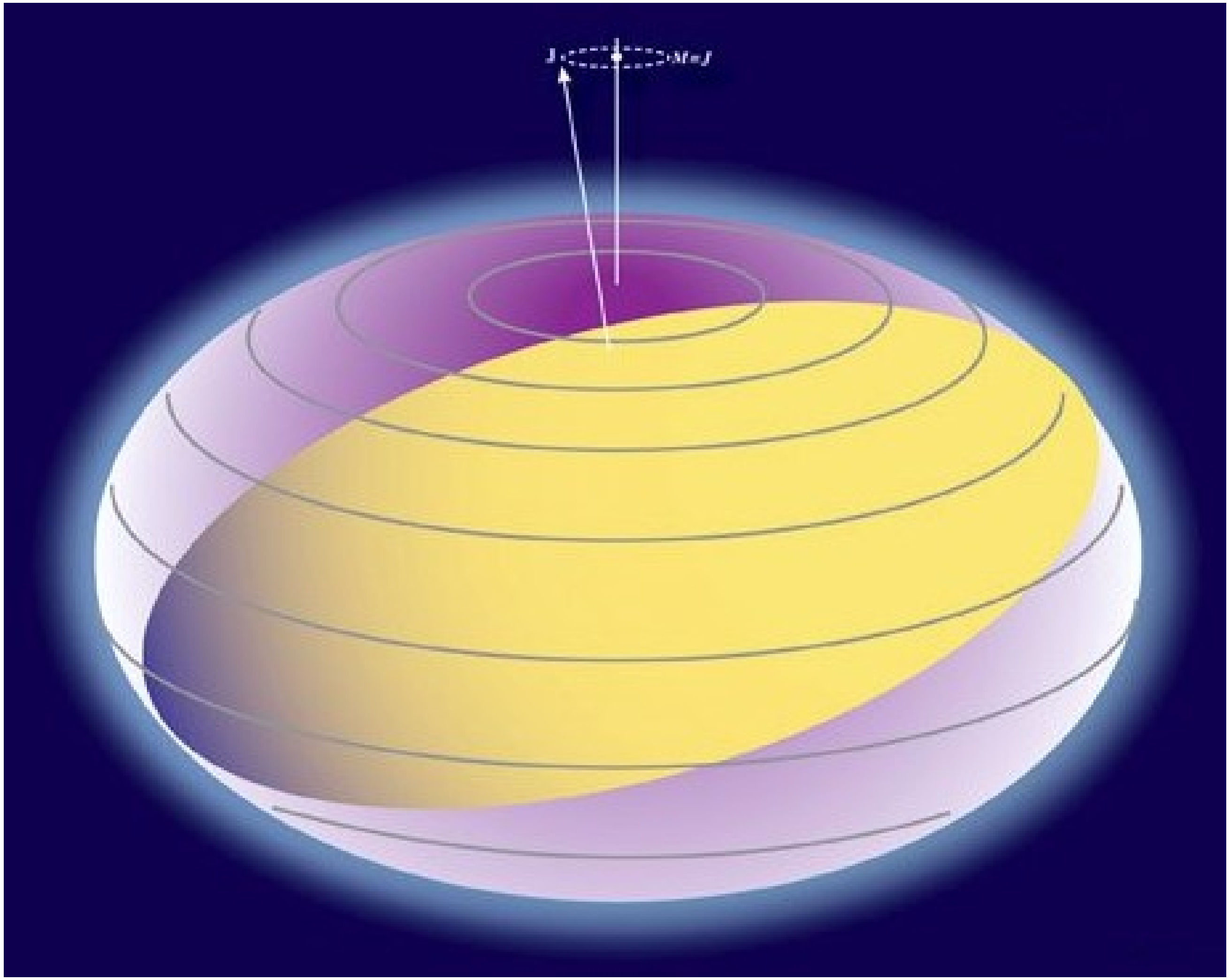
Tongji University

- Introduction: rotational nuclei in the NSM, IBM, and NPA
- The Elliott's SU(3) limit
- For realistic nuclei
- Summary

Nuclear collective behavior

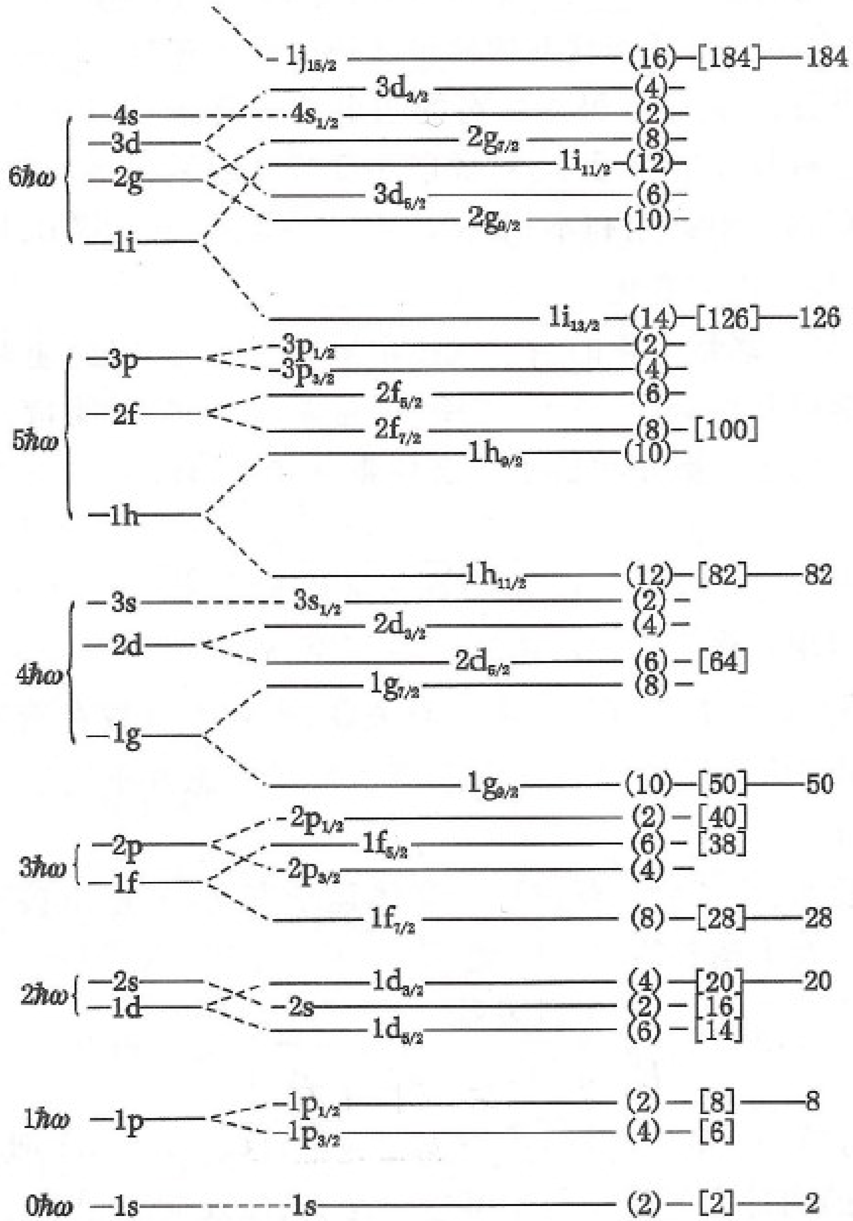


Nuclear collective behavior



Models based on deformation

Rotational motion in the nuclear shell model



Elliott’s SU(3) model:

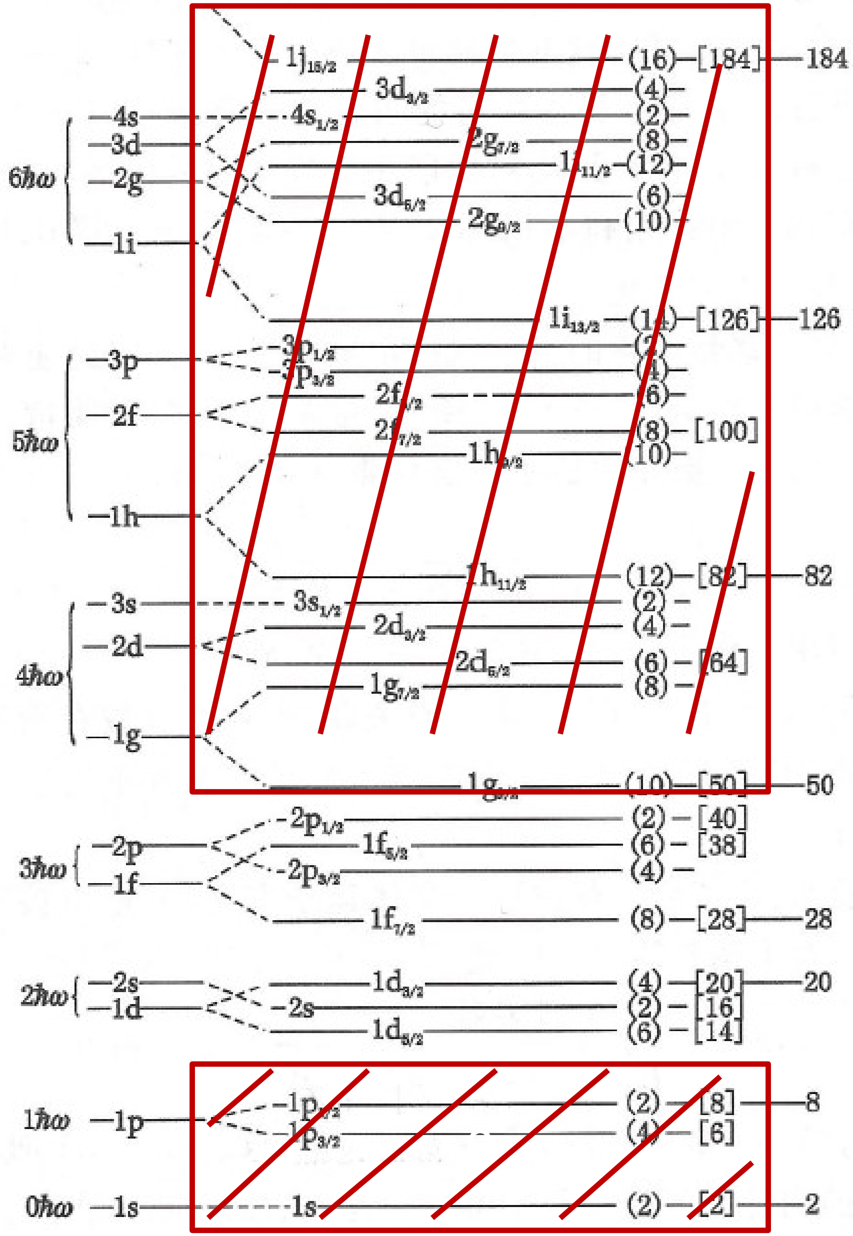
- Quadrupole interaction
- One or many H.O. major shells

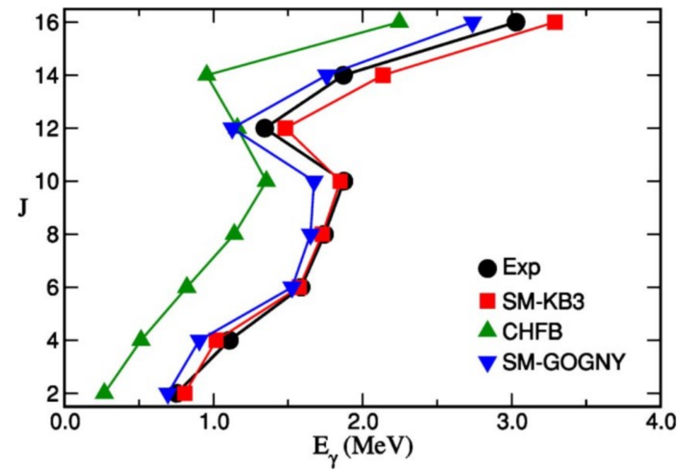
$$-(Q_\pi + Q_\nu)\cdot(Q_\pi + Q_\nu)$$

☺ a new perspective: the SU(3) symmetry

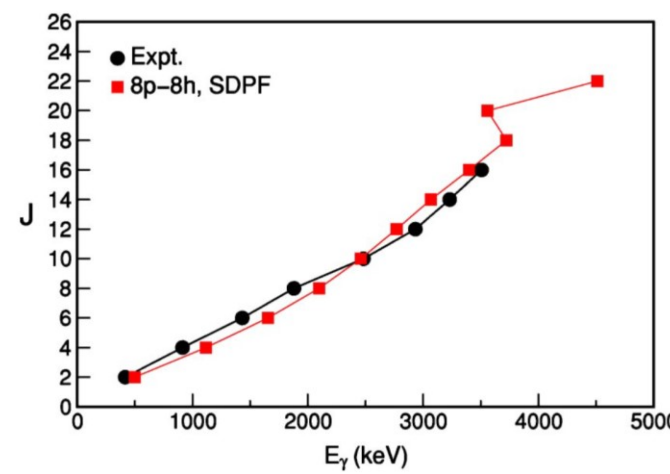
☺ Q operator → quadrupole deformation

Rotational motion in the nuclear shell model





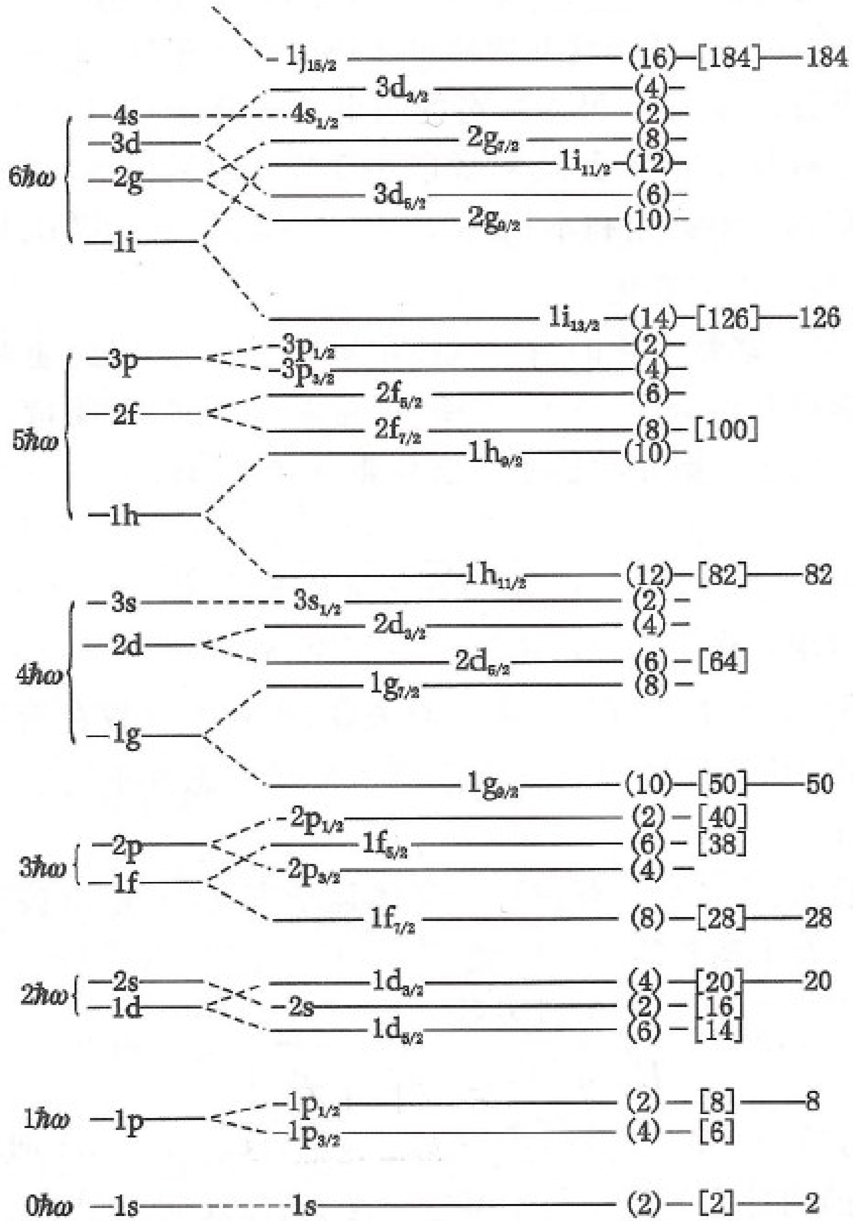
Yrast band in Cr-48

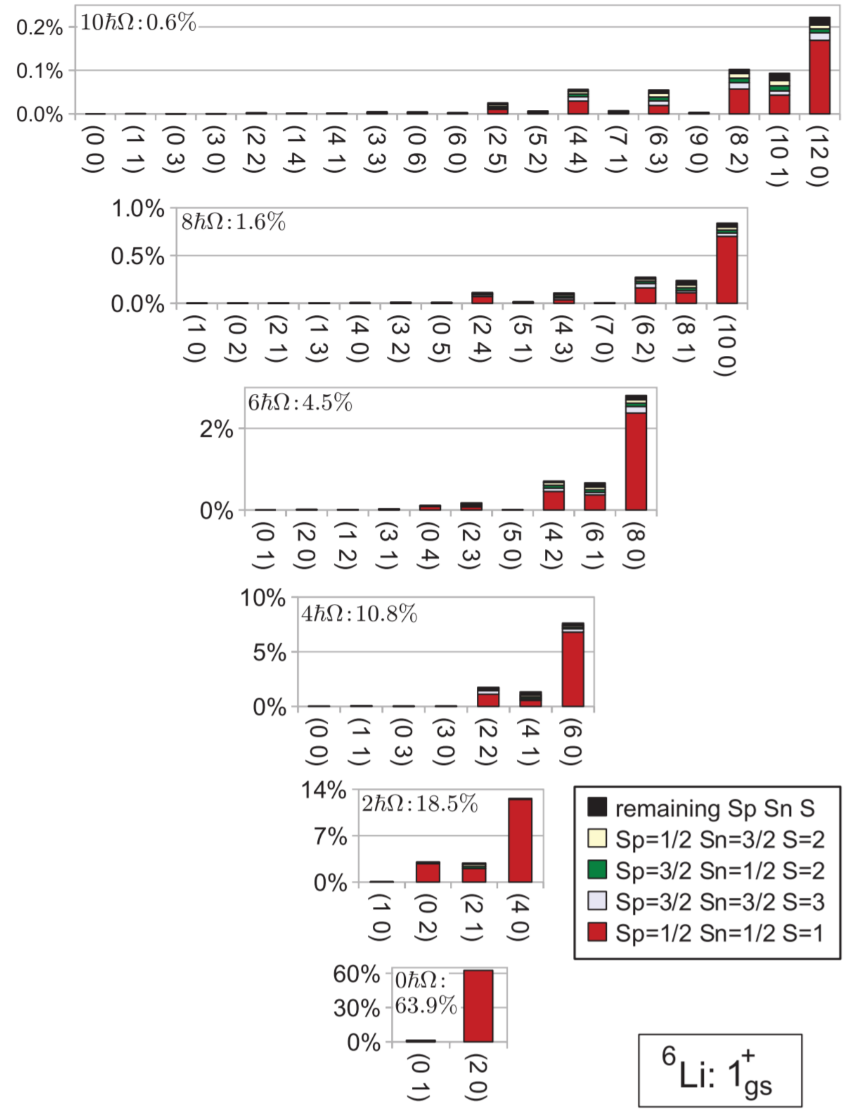


Superdeformed band in Ca-40

E. Caurier *et al.*, Rev. Mod. Phys. 77, 427 (2005).

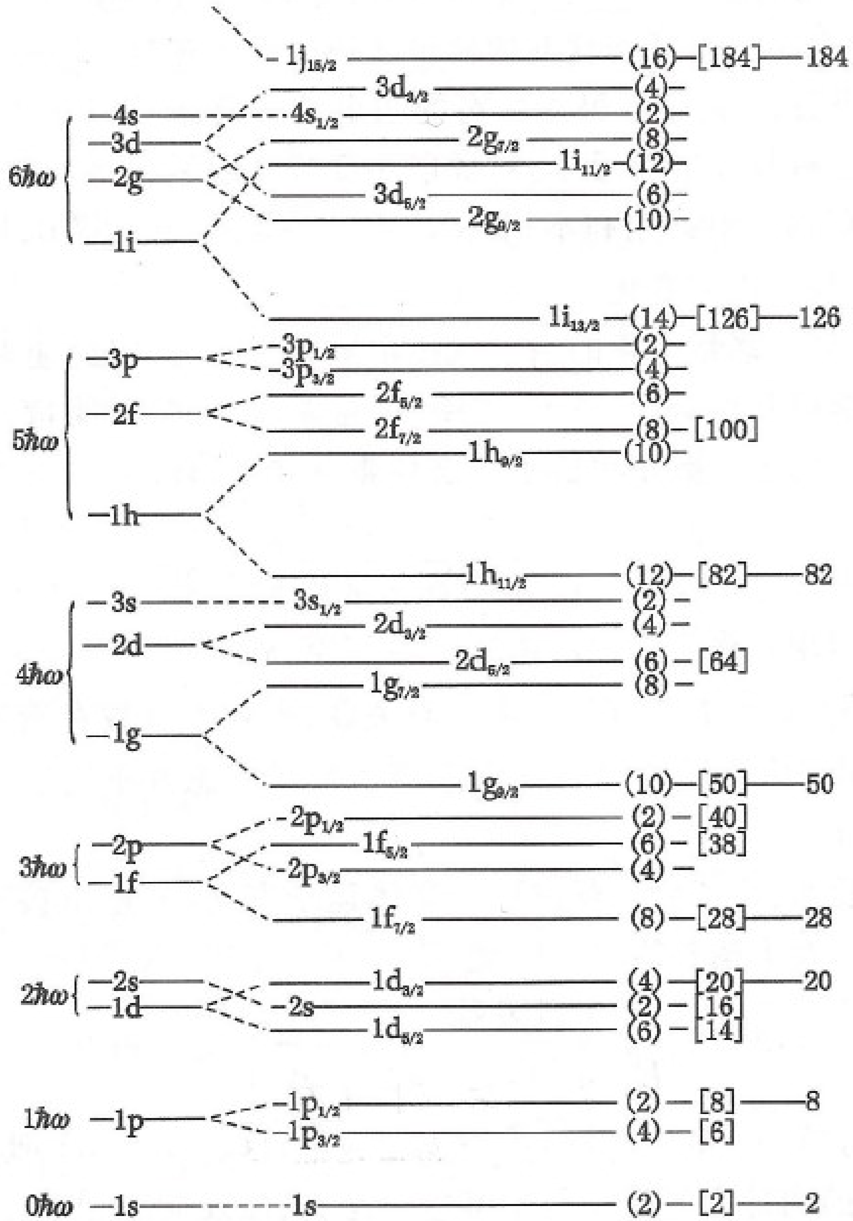
Rotational motion in terms of the shell model





T. Dytrych *et al.*, Phys. Rev. Lett. 111, 252501 (2013).

Rotational motion in the nuclear shell model



Elliott's SU(3) model:

- Quadrupole interaction
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$$-(Q_\pi + Q_\nu) \cdot (Q_\pi + Q_\nu)$$

a new perspective: the SU(3) symmetry

Q operator → quadrupole deformation

For heavy-mass regions:

strong spin-orbit couplings, SU(3) broken

one HO major shell is not enough,

intruder orbits, pseudo- and quasi-SU(3)

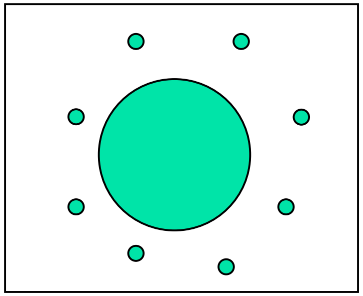
configuration space is too gigantic

Need truncation!

Truncations of the model space

- **Shell model**, full configuration interaction

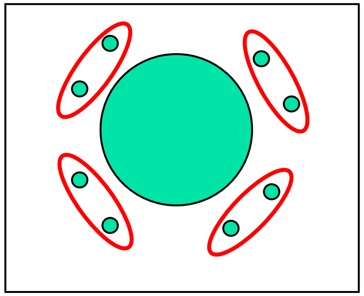
$$|\varphi\rangle = C^+_{j_1 m_1} C^+_{j_2 m_2} \cdots C^+_{j_n m_n} |0\rangle$$



- Cooper pairs with good angular momentum *r*:

$$A^{r\dagger} = \sum_{ab} y(abr) A^{r\dagger}(ab) \ , \quad A^{r\dagger}(ab) = \left(C_a^+ \times C_b^+\right)^r$$

Nucleon-pair approximation (**NPA**):



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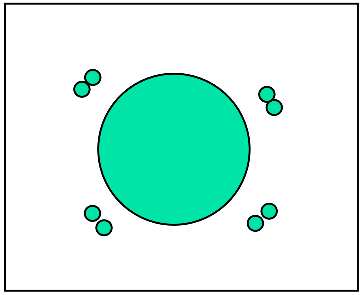
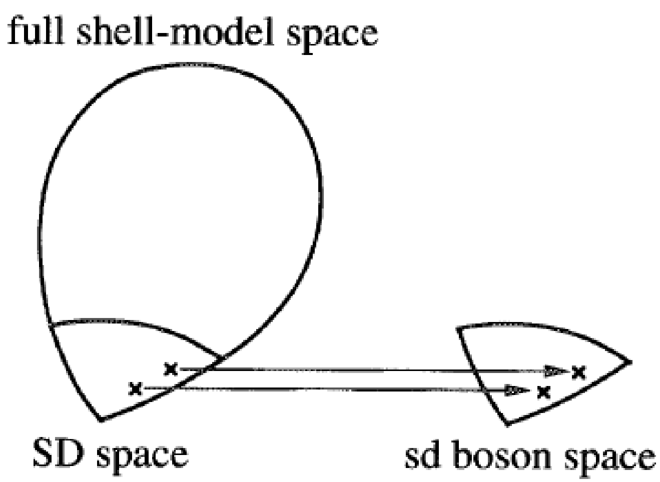
- Cooper pair => boson; Interacting boson model (**IBM**):



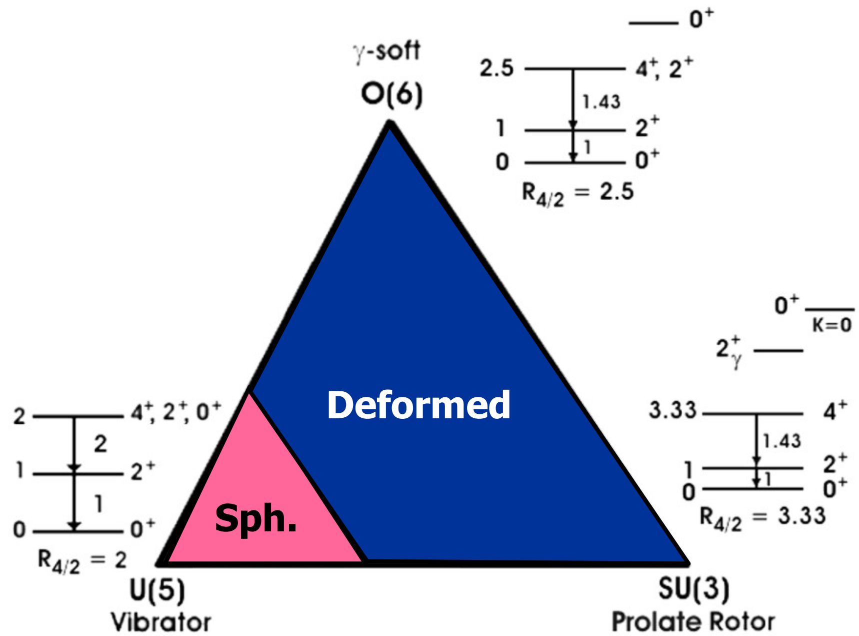
The NPA is a truncation of the shell-model configuration space; further mapping to IBM.

A. Arima and F. Iachello, Ann. Phys. (NY) 99, 253 (1976); 111, 201 (1978); 123, 468 (1979);
J. Q. Chen, Nucl. Phys. A 626, 686 (1997); Y. M. Zhao and A. Arima, Phys. Rep. 545, 1 (2014).

Microscopic foundation of the IBM



e.g. OAI mapping



space onto the *sd* boson space. The mapping scheme for deriving the IBM Hamiltonian of this type is usually referred to as the Otsuka-Arima-Iachello (OAI) mapping and can be extended as the proton-neutron interacting boson model (IBM-2) as a natural consequence [5,6]. The OAI mapping has been practiced for limited realistic cases of nearly spherical or γ-unstable shapes [11–14] by using zero- and low-seniority states of the shell model [5,6,10] and has been also tested

Microscopic foundation of the IBM

PHYSICAL REVIEW C 81, 044307 (2010)

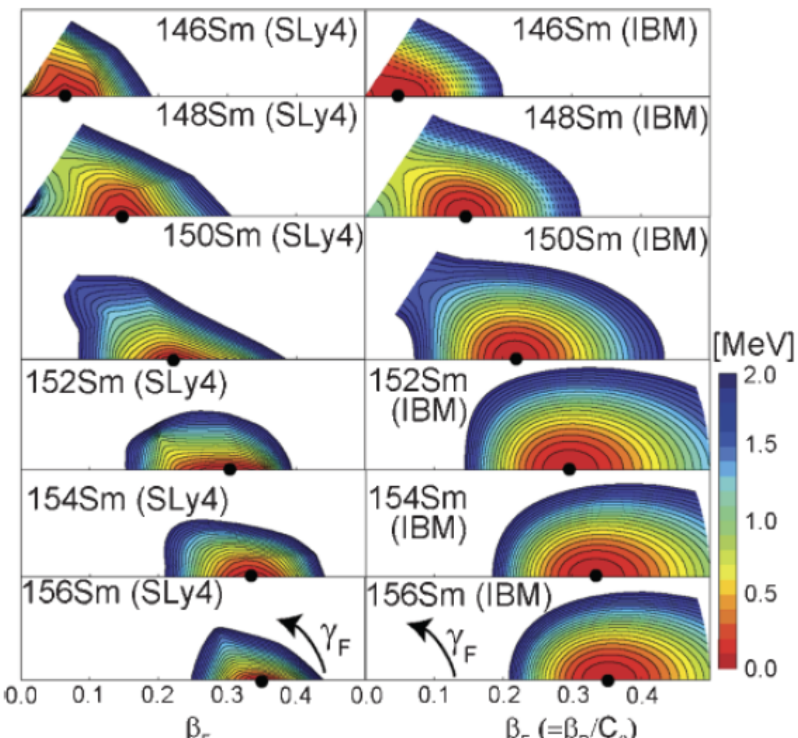
Formulating the interacting boson model by mean-field methods

Kosuke Nomura,[1] Noritaka Shimizu,[1] and Takaharu Otsuka[1,2,3]

PHYSICAL REVIEW C 83, 041302(R) (2011)

Microscopic formulation of the interacting boson model for rotational nuclei

Kosuke Nomura,[1] Takaharu Otsuka,[1,2,3] Noritaka Shimizu,[1] and Lu Guo[4]



The nucleon-boson difference of the rotational response discussed so far suggests that the rotational spectrum of a nucleonic system may not be fully reproduced by the boson system determined by the mapping method of Ref. [8] using the PESs at rest. In fact, it will be shown later that the moment of inertia of a nucleon system differs from the one calculated by the mapped boson Hamiltonian. We then introduce a term into the boson Hamiltonian, so as to keep the PES-based mapping procedure, but incorporate the different rotational responses. This term takes the form of $\hat{L}\cdot\hat{L}$, where

For deformed nuclei, require manually add a term to reproduce moment of inertia…

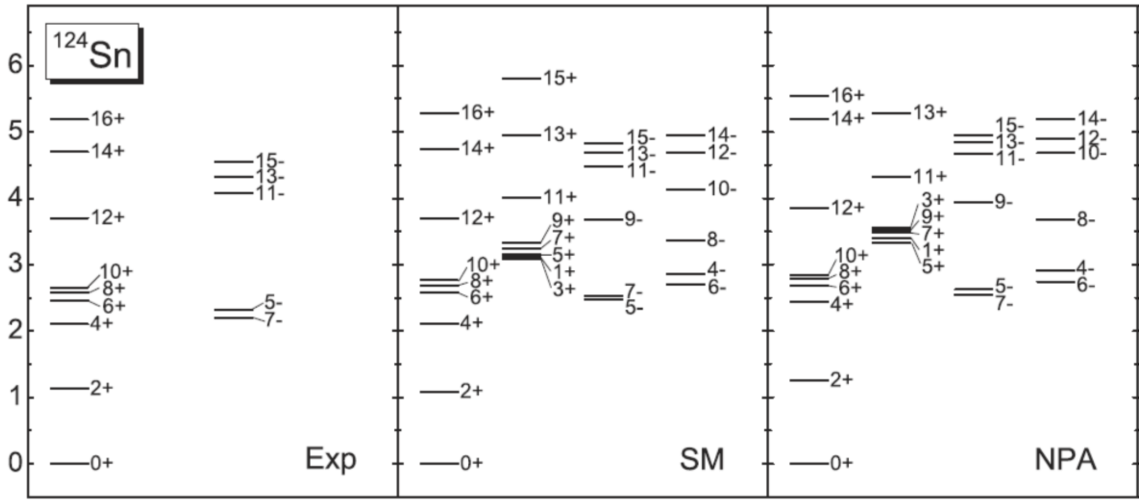
Validity of the NPA

For nearly spherical nuclei:

☺ compare NSM wave functions with NPA wave functions,

overlap > 90% for g.s. with, e.g., the *SD*-pair truncation

	BP	*SD*	*SDG*
	^{46}Ca		
0_1^+	0.990	0.992	0.999
2_1^+	0.960	0.969	0.977
4_1^+	0.968	0.978	0.988
6_1^+	0.970	0.946	0.991
8_1^+			0.946
10_1^+			0.954
0_2^+	0.154	0.963	0.966
2_2^+	0.966	0.843	0.896
3_1^+	0.845	0.082	0.667
4_2^+	0.961	0.139	0.906
5_1^+	0.969		0.334
6_2^+	0.055		0.884
7_1^+			0.930
8_2^+			0.953
9_1^+			0.857



^{130}Te

spin$^{\text{parity}}$	*SD*	favored pairs
0_1^+	0.961	0.980
2_1^+	0.813	0.945
4_1^+	0.533	0.903
6_1^+	0.046	0.956
8_1^+		0.961
10_1^+		0.965

Y. Lei *et al.*, Phys. Rev. C 82, 034303 (2010); 84, 044301 (2011);
Y. Y. Cheng *et al.*, Phys. Rev. C 94, 024321 (2016).

Validity of the NPA

For nearly spherical nuclei:

compare NSM wave functions with NPA wave functions, overlap > 90% for g.s. with, e.g., the *SD*-pair truncation

For rotational nuclei:

can reproduce rotational band (parameters)

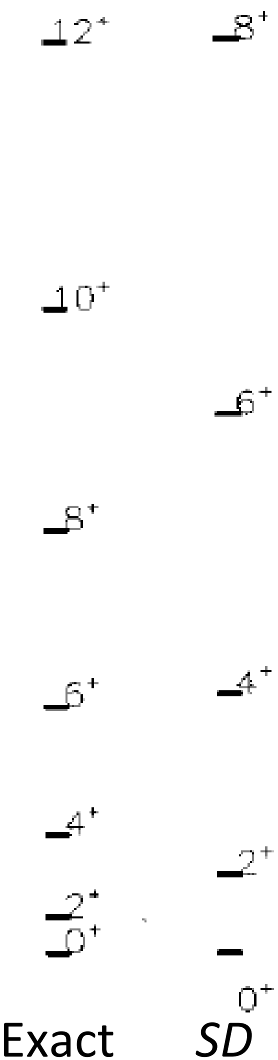
under the same interaction: NPA predicts much smaller moment of inertia and $B(E2)$ than NSM does

Difficulty: Selecting good pairs is a long-standing problem; no one has successfully reproduced rotational bands by the NPA in a large space with effective interactions.

- Structure coefficients of the *SD* pairs?
- Higher-spin pairs?

Y. M. Zhao *et al.*, Phys. Rev. C 62, 014316 (2000).

Elliott's SU(3)



the *pf* shell

Pair-structure coefficient

- In early applications, one determines pair-structure coefficients using the generalized seniority (**GS**) states of the SM.

  the *S* pair is chosen so that the expectation value of Hamiltonian in the *S*-pair condensate is minimized:

$$\frac{\langle (S_\tau)^N | \hat{H} | (S_\tau)^N \rangle}{\langle (S_\tau)^N | (S_\tau)^N \rangle}, \quad \text{with } \tau = \pi \text{ or } \nu,$$

  D, *G*,… pairs obtained by diagonalizing the Hamiltonian matrix in the space spanned by the generalized-seniority-two (i.e., one-broken-pair) states

  ☺ simple; works well for nearly-spherical nuclei

  ☹ it does not work for deformed nuclei;
  it does not tell us which pairs are important in advance

Pair-structure coefficient

- In early applications, one determines pair-structure coefficients using the generalized seniority (**GS**) states of the SM.

- Conjugate gradient method (**CG**):

  1. treat pair coefficients as free parameters;
  2. minimize g.s. energy by iterative NPA calculations.

☺ numerically optimal solution;
works well for deformed nuclei;

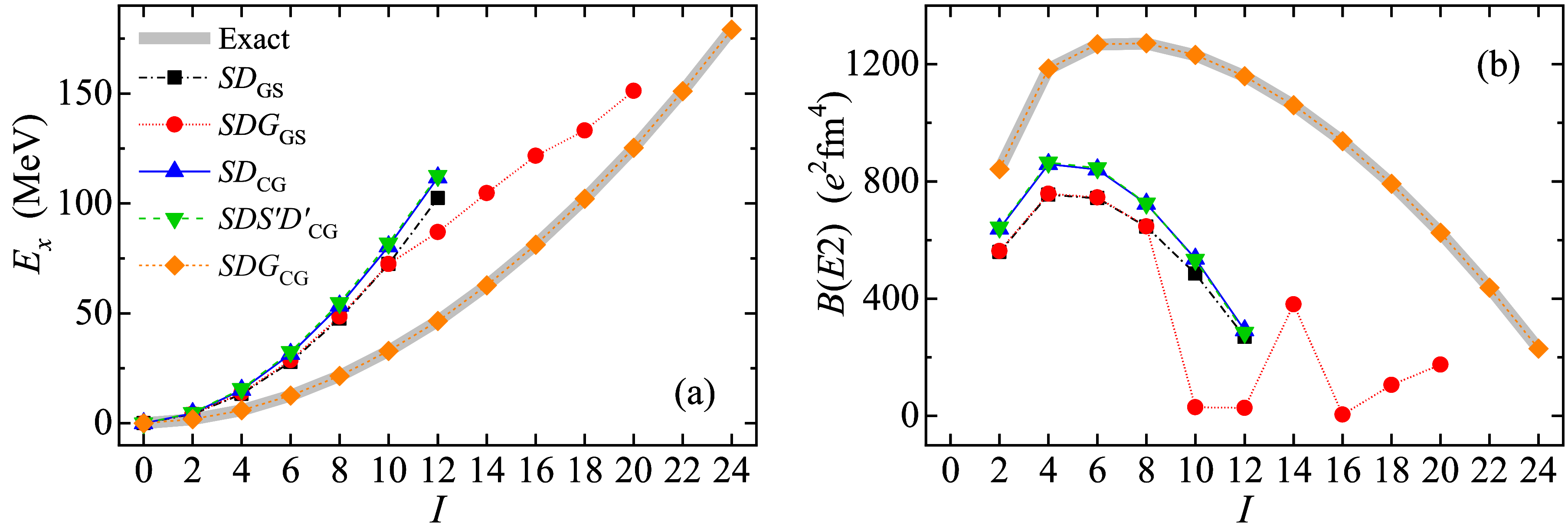
☹ heavy calculation;
it does not tell us which pairs are important in advance

Elliott's SU(3) limit

generalized-seniority based method(**GS**)

conjugate gradient method (**CG**)

Precisely reproduce the *pf*-shell SU(3) in the *SDG*-pair truncation!



6p6n in the *pf* shell under $-(Q_\pi + Q_\nu) \cdot (Q_\pi + Q_\nu)$

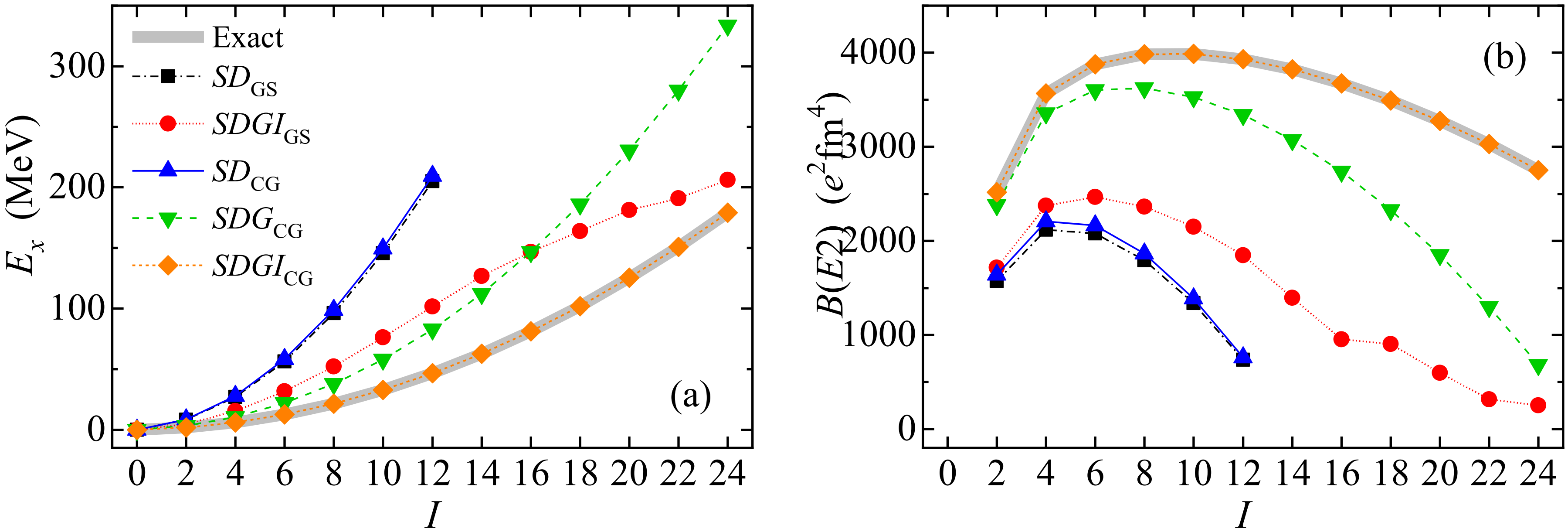
G. J. Fu *et al.*, to be published.

Elliott's SU(3) limit

generalized-seniority based method(**GS**)

conjugate gradient method (**CG**)

Precisely reproduce the *sdg*-shell SU(3) in the *SDGI*-pair truncation!



6p6n in the *sdg* shell under $-(Q_\pi + Q_\nu)\cdot(Q_\pi + Q_\nu)$

G. J. Fu *et al.*, to be published.

Pairs obtained from the HF basis

1. Solve the HF equation in the shell-model space with effective interactions. (SHERPA)

unitary transformation: $\underline{c_a^\dagger} = \sum_\alpha U_{a\alpha} \underline{a_\alpha^\dagger}$.

HF s.p. basis NSM s.p. basis

2. The HF state of 2*N* valence protons is written by a pair-condensate state.

$$\prod_{a=1}^{2N} c_a^\dagger|0\rangle = \left(c_1^\dagger c_2^\dagger + \cdots + c_{2N-1}^\dagger c_{2N}^\dagger\right)^N |0\rangle = \left(\sum_{ab} g_{ab}\, c_a^\dagger c_b^\dagger\right)^N |0\rangle,$$

here $g_{12} = g_{34} = \ldots = g_{(2N-1)(2N)} = 1$, and other $g_{ij} = 0$.

the phase of pair is arbitrary

3. HF pair => pairs with good spin in the NSM basis

$$\sum_{ab} g_{ab}\, c_a^\dagger c_b^\dagger = \sum_{j_\alpha j_\beta JM} y_{JM}(j_\alpha j_\beta)\, A_M^{(J)}(j_\alpha j_\beta)^\dagger,$$

$$y_{JM}(j_\alpha j_\beta) = \sum_{ab} U_{a,\alpha} U_{b,\beta}\, g_{ab} \sum_{m_\alpha m_\beta} C^{JM}_{j_\alpha m_\alpha j_\beta m_\beta}.$$

selecting pairs with large y_{JM}, it tells us what pairs are important in advance!

G. J. Fu and C. W. Johnson, arXiv:1909.08785.

Pairs obtained from the HF basis

e.g., 6p6n in the *pf* shell under $-(Q_\pi + Q_\nu)\cdot(Q_\pi + Q_\nu)$

Solving the HF equation + angular momentum projection = exact solution

HF pair => *SDG* pairs

SDGI pairs for 6p6n, 10p10n(?), 12p12n(?) in the *sdg* shell…

SU(3) boson mapping

$(\text{shell})^n$	(λ,μ)	*sd*-IBM: U(6)	*sdg*-IBM: U(15)	*sdgi*-IBM: U(28)
p^{12}	(0,0)	$[6]+[42]+[2^3]+[1^6]$	$[6]+[42]+[2^3]+[1^6]$	$[6]+[42]+[2^3]+[1^6]$
$(sd)^{12}$	(12,0)	[6]	$[6]+[42]+[2^3]+[1^6]$	$[6]+[42]+[2^3]+[1^6]$
	(0,12)	*	$[6]+[42]+[2^3]+[1^6]$	$[6]+[42]+[2^3]+[1^6]$
$(pf)^{12}$	(24,0)	*	[6]	$[6]+[42]+[2^3]+[1^6]$
$(sdg)^{12}$	(36,0)	*	*	[6]

G. J. Fu *et al.*, to be published.

Shape evolution driven by interactions

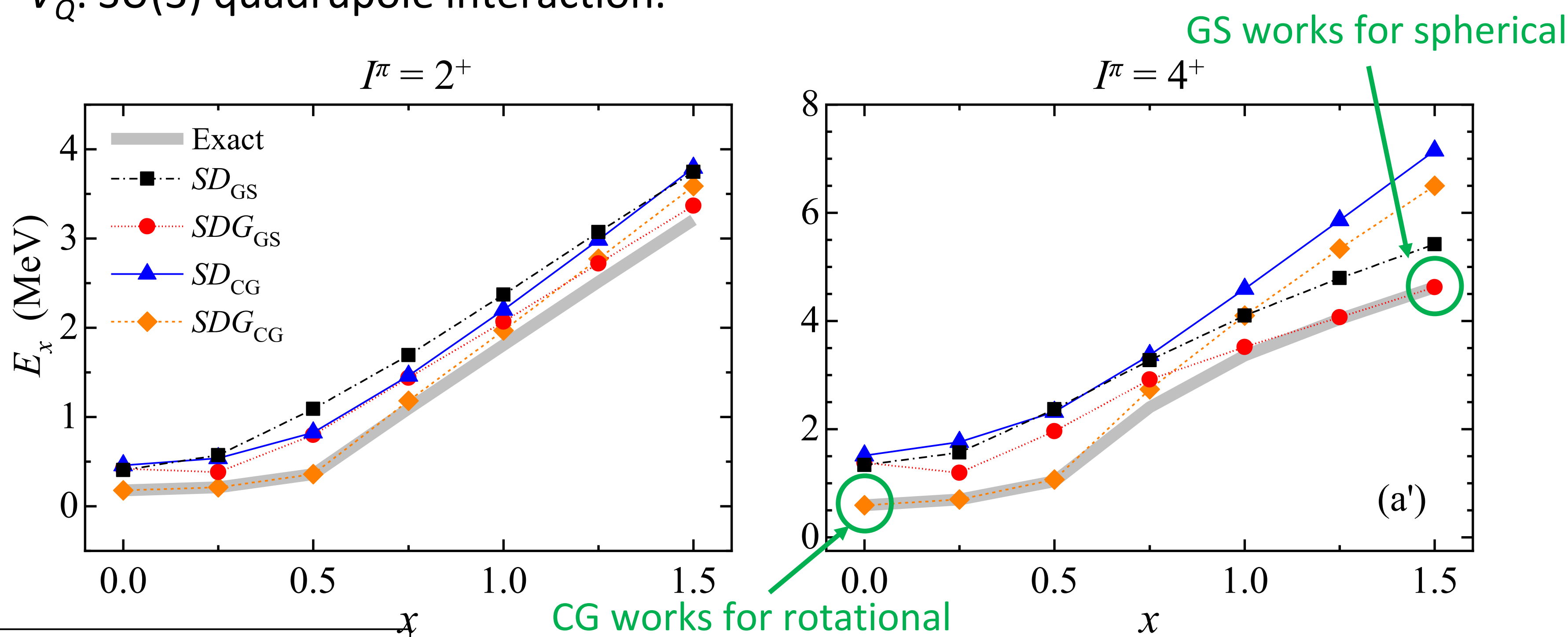
6p6n in the *pf* shell with

$$H(x) = x\left(\sum_{j_\alpha} \varepsilon_{j_\alpha} n_{j_\alpha} + g V_P\right) + \kappa V_Q,$$

$\varepsilon_{j\alpha}$: s.p. energy from kb3g;

V_P: pairing interaction;

V_Q: SU(3) quadrupole interaction.



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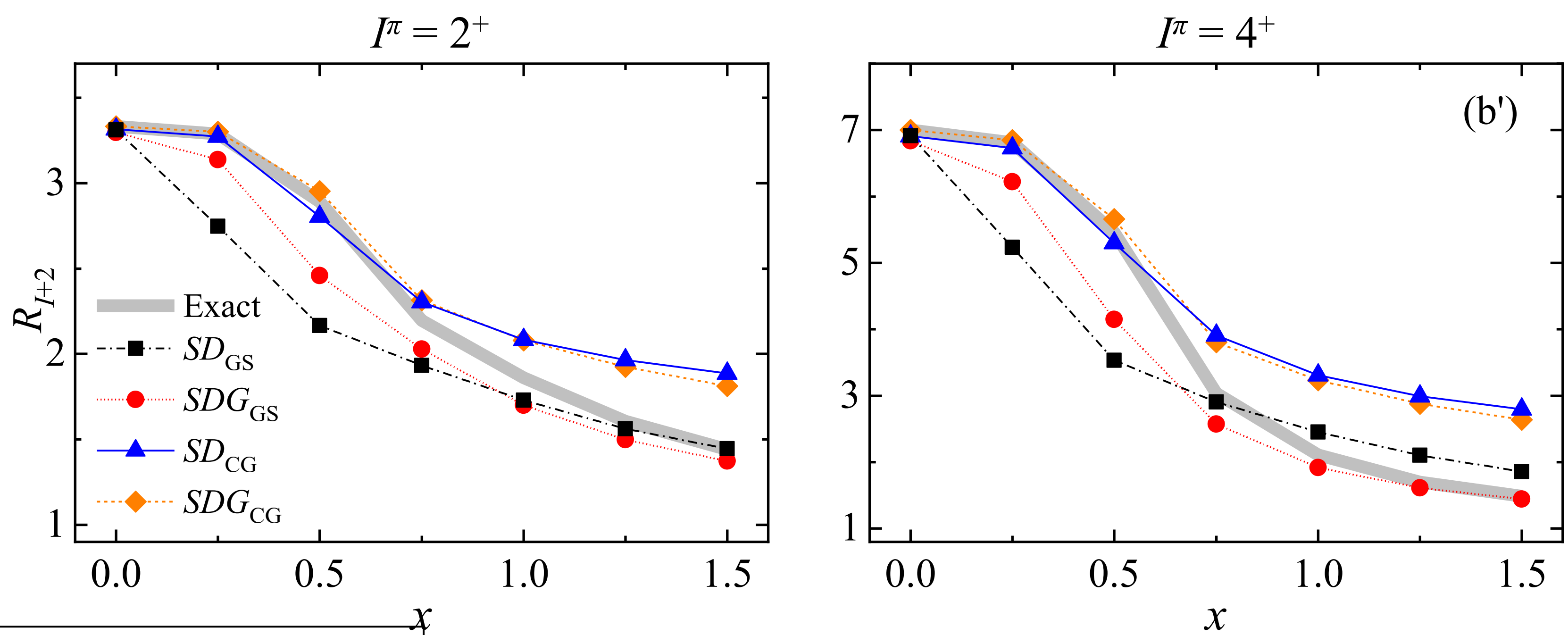

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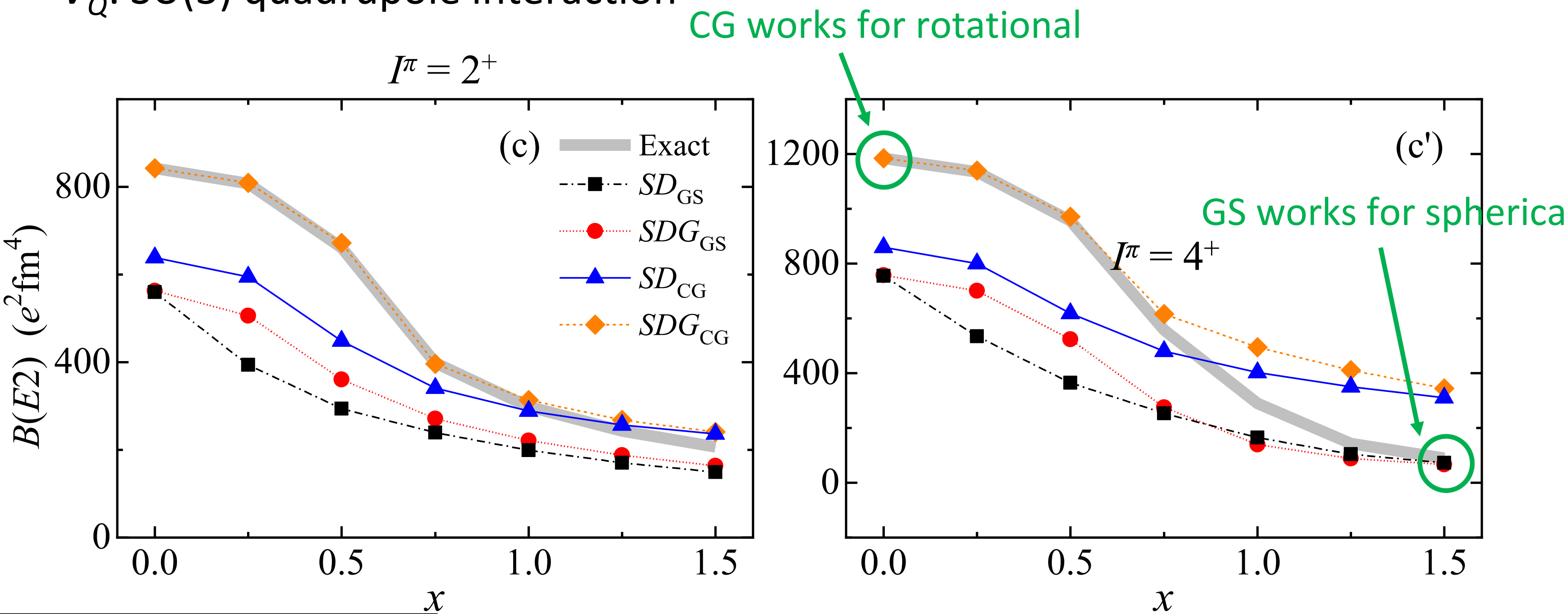
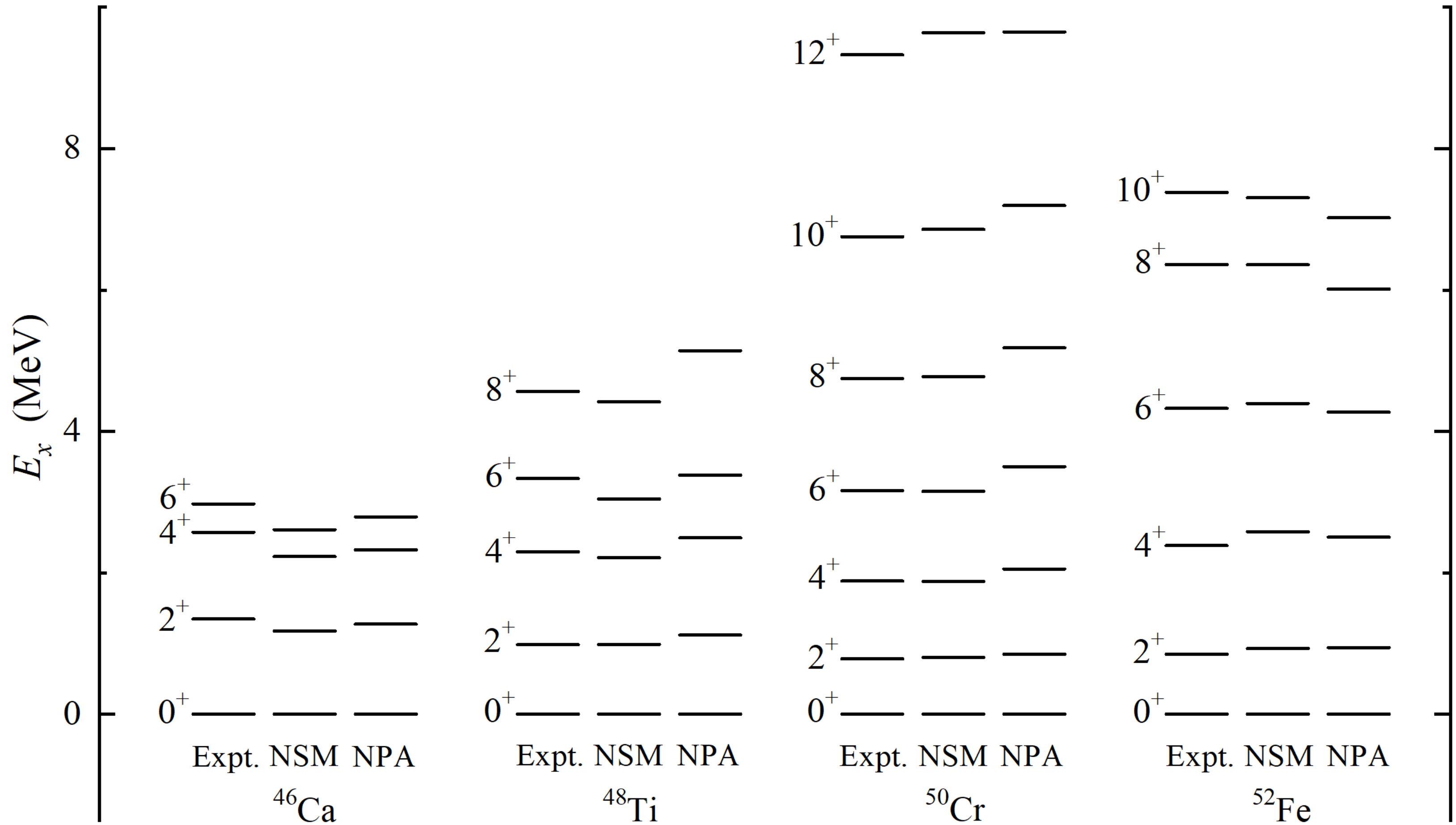


G. J. Fu *et al.*, to be published.

N = 26 isotones

pf shell + KB3G interaction

NPA: $SDG_{\mathrm{CG}} \bigoplus SDG_{\mathrm{GS}}$

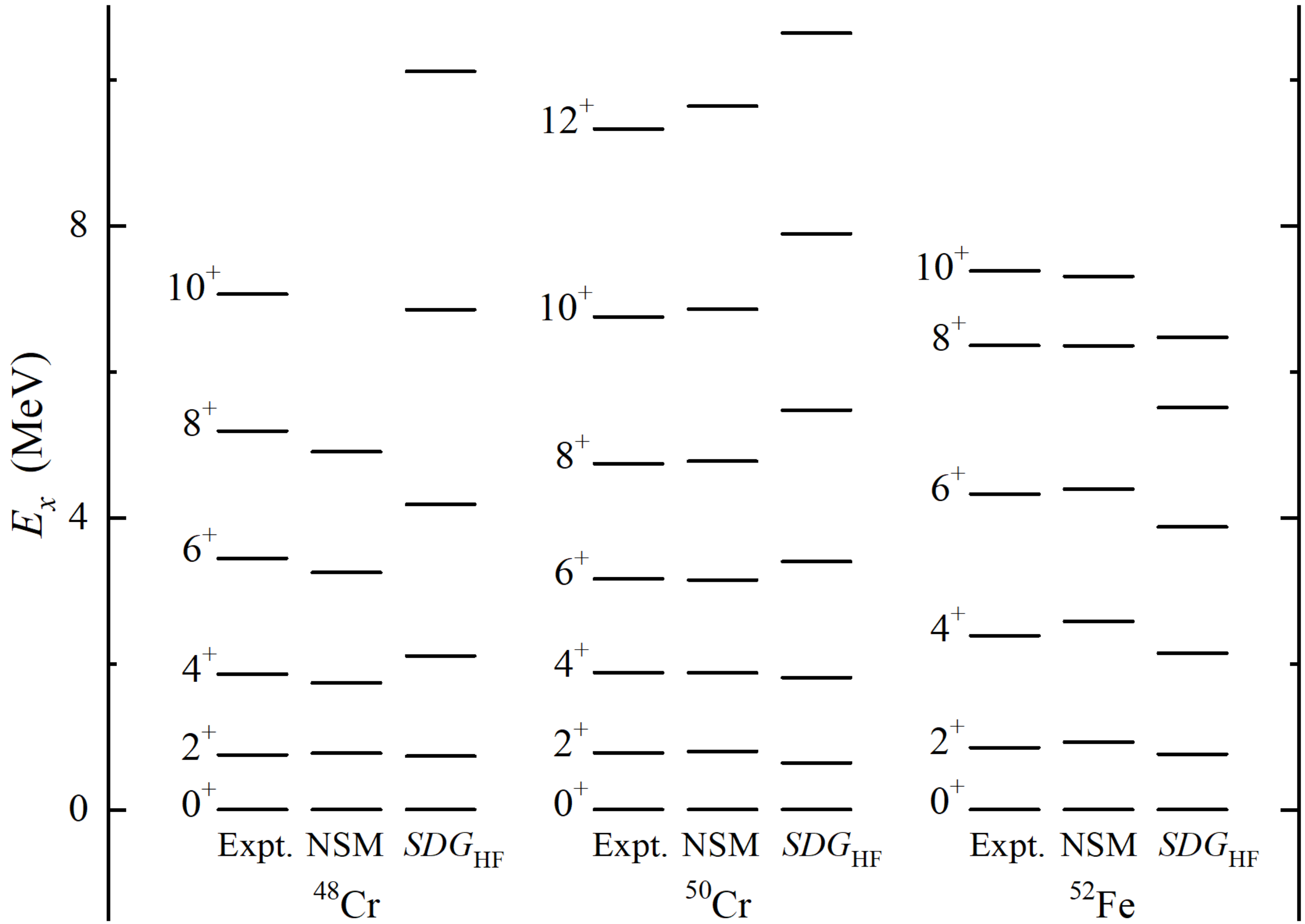


G. J. Fu *et al.*, to be published.

From HF to NPA

HF: selecting pairs from HF states

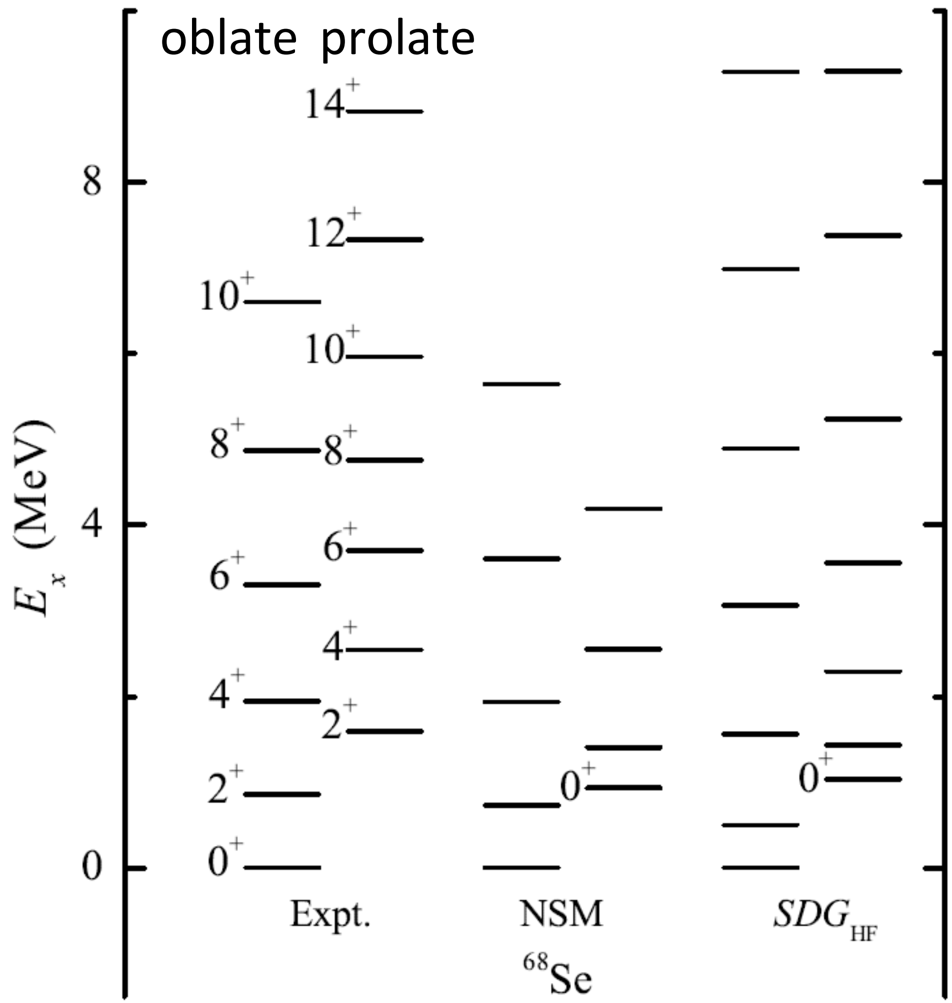
pf shell + KB3G interaction



G. J. Fu *et al.*, to be published.

From HF to NPA

$1p_{1/2}1p_{3/2}0f_{5/2}0g_{9/2}$ shell + JUN45 interaction



G. J. Fu and C. W. Johnson, arXiv:1909.08785.

From HF to NPA

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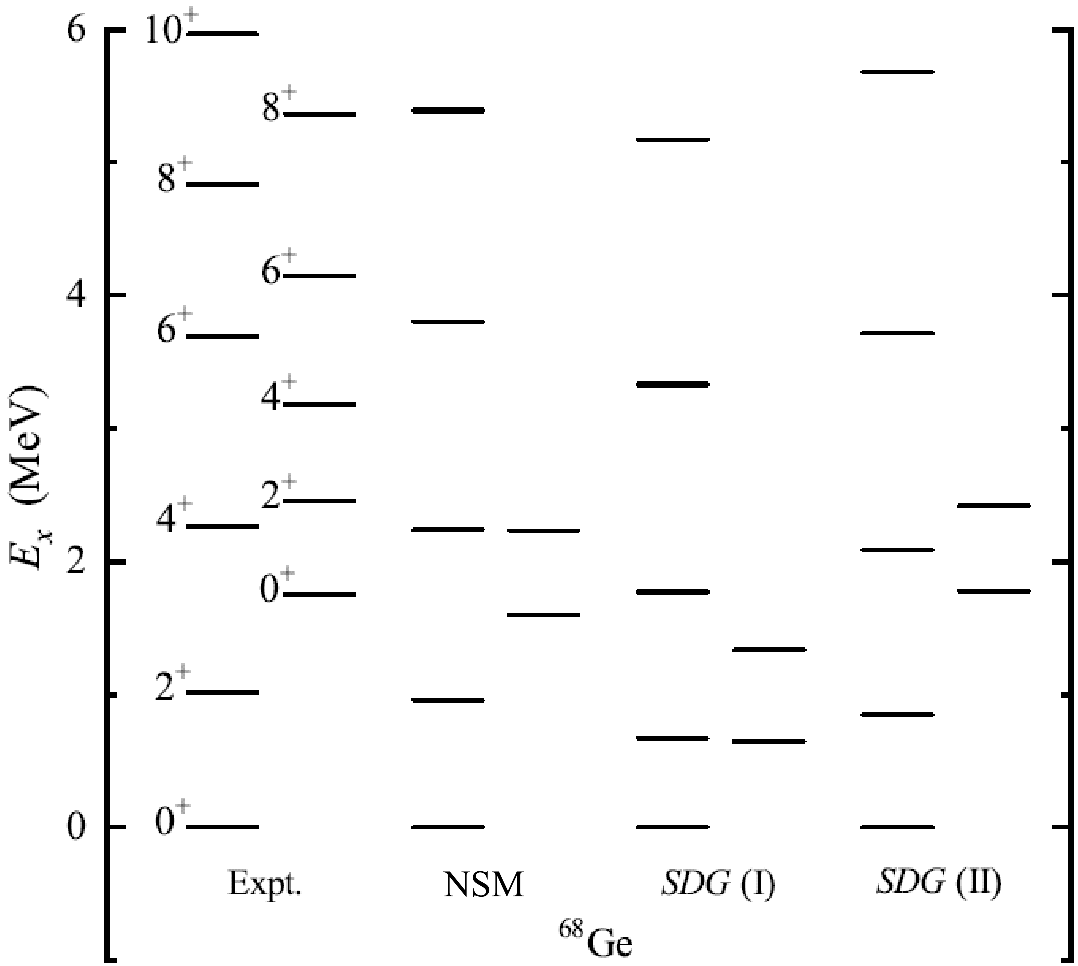


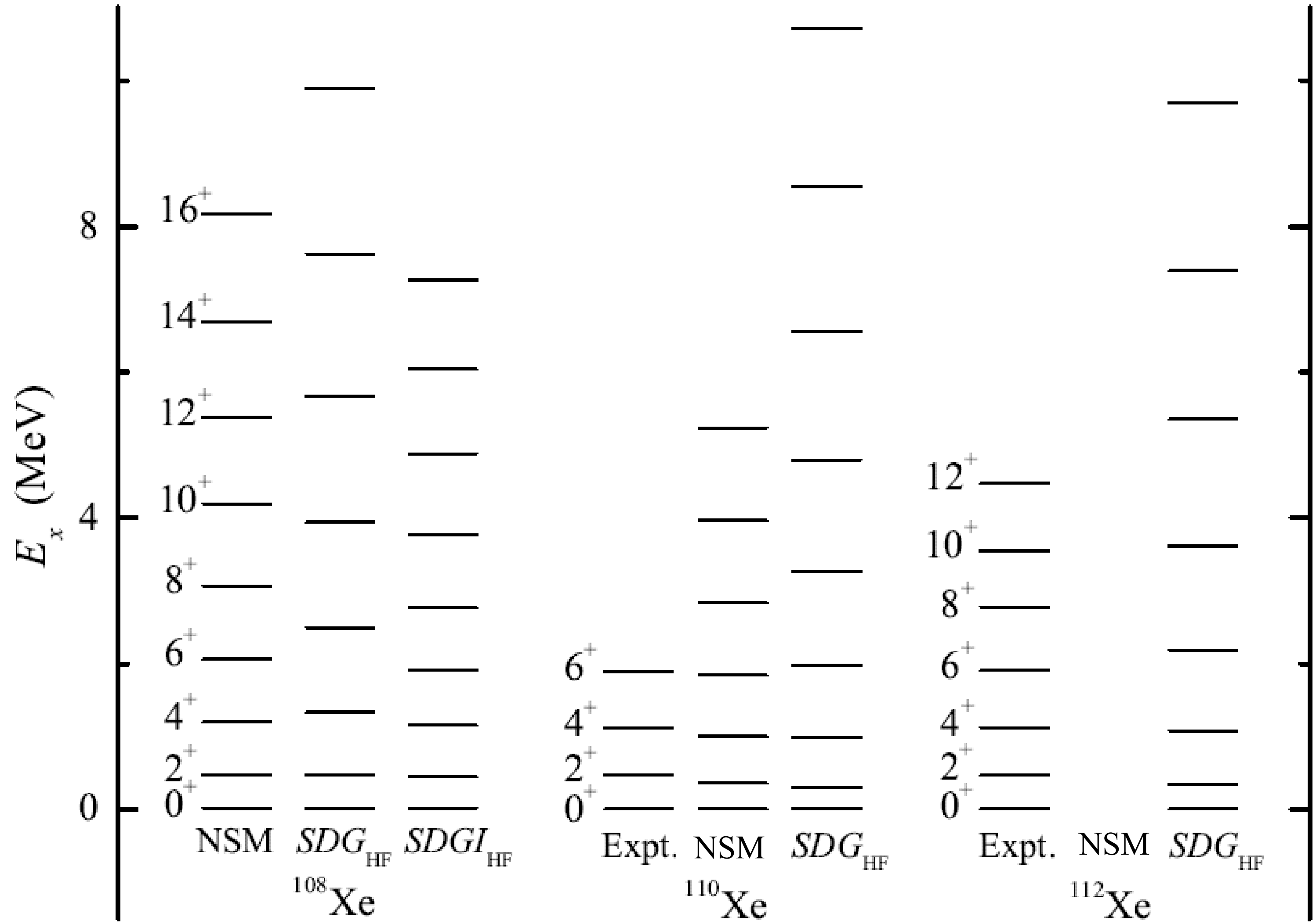
I : two bands are calculated based on two HF solutions, HF1 and HF2, respecitively;

II: we mixed the states obtained from HF1 and HF2.

G. J. Fu and C. W. Johnson, arXiv:1909.08785.

From HF to NPA

$2s_{1/2}1d_{3/2}1d_{5/2}0g_{7/2}0h_{11/2}$ (50-82) shell + Bonn potential



G. J. Fu and C. W. Johnson, arXiv:1909.08785.

Summary

- We study deformed nuclei using the NPA in the framework of the shell model, by selecting good pairs using the **CG** method and from the **HF**.

- We find the ***SDG_{CG}*** **pair truncation precisely reproduces the Elliott's SU(3)** of the 6p6n system in the ***pf*** shell, and the ***$SDGI_{CG}$*** pair truncation reproduces the Elliott's SU(3) in the ***sdg*** shell.

- The CG method works very well for the rotational motion, and the traditional GS method works for nearly spherical nuclei. **CG + GS** reproduces **nuclear shape evolution**.

- We can select good pairs from the HF. Our preliminary results show that the SDG_{HF} pair truncation can well reproduce low-lying **rotational bands**.

Thanks for your attentions !

Triaxially and octupole deformed nuclei

HF pair => pairs with good spin in the NSM basis

$$\sum_{ab} g_{ab}\, c_a^\dagger c_b^\dagger = \sum_{j_\alpha j_\beta JM} y_{JM}(j_\alpha j_\beta)\, A_M^{(J)}(j_\alpha j_\beta)^\dagger,$$

$$y_{JM}(j_\alpha j_\beta) = \sum_{ab} U_{a,\alpha} U_{b,\beta}\, g_{ab} \sum_{m_\alpha m_\beta} C^{JM}_{j_\alpha m_\alpha j_\beta m_\beta}.$$

- Generally, for given *J* one obtains 2*J*+1 different pairs for different *M.*
- A HF state can have arbitrary orientation with the same physical meaning, but y_{JM} will be changed.

✓ The truth is the 2*J*+1 pairs are not linearly independent.
orthogonalization: diagonalize the norm matrix of pair

Numerically, axially deformed: 1 pair
triaxially deformed: 2 pairs
octupole deformed: parity-nonconserving pairs

✓ y_{JM} are rotation invariant

Improved pairing in the HF basis

The HF state of $2N$ valence protons is written by a pair-condensate state.

$$\prod_{a=1}^{2N} c_a^\dagger |0\rangle = \left(c_1^\dagger c_2^\dagger + \cdots + c_{2N-1}^\dagger c_{2N}^\dagger\right)^N |0\rangle = \left(\sum_{ab} g_{ab}\, c_a^\dagger c_b^\dagger\right)^N |0\rangle,$$

$$\text{here } g_{12} = g_{34} = \ldots = g_{(2N-1)(2N)} = 1, \text{ and other } g_{ij} = 0.$$

the phase of pair is arbitrary

Consider pairing in the HF basis:

$$|\varphi\rangle = \left(\sum_a g_a c_a^\dagger c_{\bar{a}}^\dagger\right)^N |0\rangle$$

where g_a is obtained by minimizing the energy