Attempts in Local Mass Relations

Moqiong Lin

13.01.2020

上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

Outline

Outline

Introduction

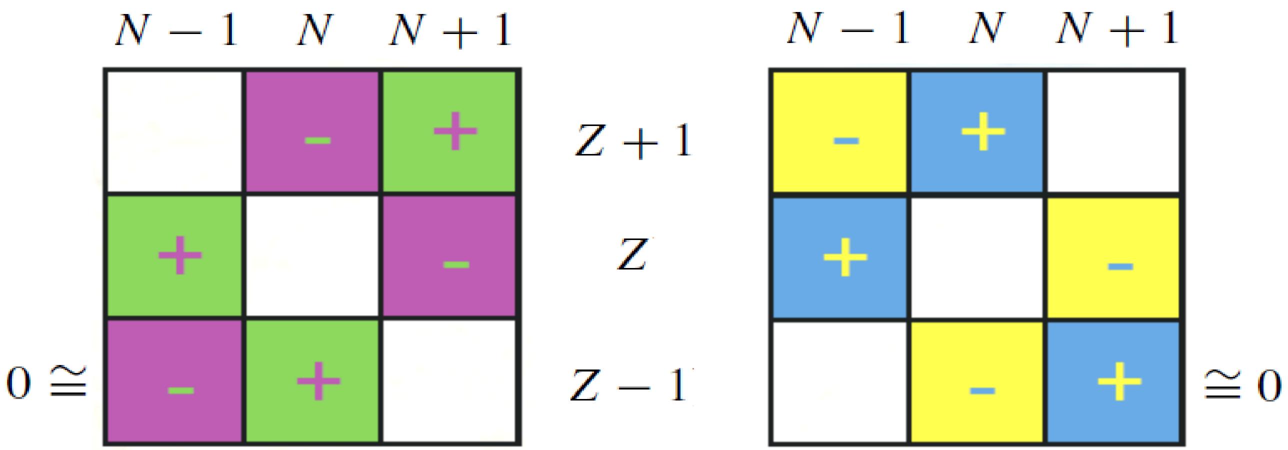
Two popular local relations:

- G-K relations
- δV_{np}

Introduction

Two popular local relations:

- G-K relations
- δV_{np}



$$-M(N-1,Z-1)+M(N,Z-1)+M(N-1,Z) -M(N+1,Z)-M(N,Z+1)+M(N+1,Z+1)\cong 0$$

$$-M(N,Z-1)+M(N+1,Z-1)+M(N-1,Z) -M(N+1,Z)-M(N-1,Z+1)+M(N,Z+1)\cong 0$$

a) Cancel out two-body interactions approximately!

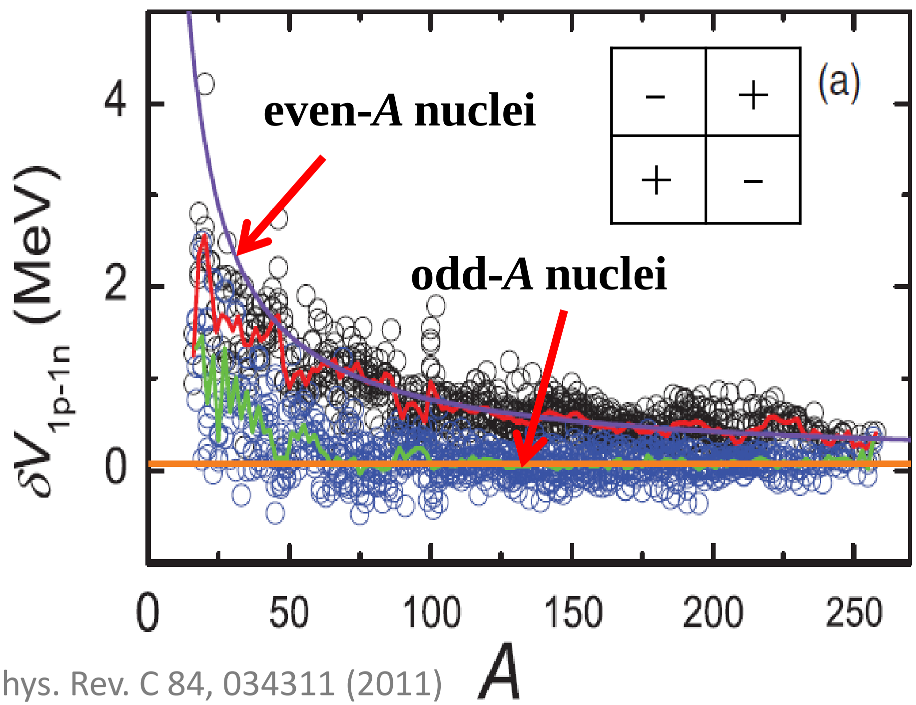
b) RMSD for *A*≥60: ~200 keV

Phys. Rev. C 89, 061304(R) (2014)

Introduction

Two popular local relations:

- G-K relations
- δV_{np}: Proton-neutron interaction between the last neutron and the last proton



$$\delta V_{1\mathrm{n}-1\mathrm{p}}(N,Z) = B(N,Z) + B(N-1,Z-1) - B(N-1,Z) - B(N,Z-1)$$

RMSD for $A \geq 60$: ~160 keV without any corrections

Phys. Rev. C 84, 034311 (2011)

Outline

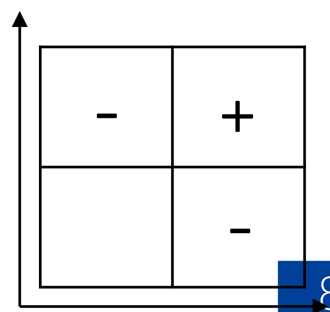
One-nucleon separation energy

$$S_\mathrm{n} = (a_1 + a_2 \cdot A^{1/3}) \cdot (Z/N) + a_3 + \boxed{\delta_\mathrm{pair}} + \boxed{\delta_\mathrm{shell}} + \boxed{\delta_\mathrm{sv} + \delta_\mathrm{ss}}$$

$$S_\mathrm{p} = (a_1 + a_2 \cdot A^{1/3}) \cdot (N/Z) + a_3 + \boxed{\delta_\mathrm{pair}} + \boxed{\delta_\mathrm{shell}} + \boxed{\delta_\mathrm{sv} + \delta_\mathrm{ss}} + \boxed{\delta_\mathrm{Coul}}$$

Pairing term	Shell effect	Symmetry energy terms	Columb effect
$a_\mathrm{pair}/A^{1/2}$ $-a_\mathrm{pair}/A^{1/2}$	$a_\mathrm{shell} \cdot n$	$\delta_\mathrm{sv} \approx 2a_\mathrm{sv}\lvert I\rvert$, $\delta_\mathrm{ss} \approx 2a_\mathrm{ss}A^{-1/3}\lvert I\rvert$	$a_\mathrm{Coul} \cdot Z/A^{1/3}$

N,Z	1-28	29-50	51-82	83-126	127-
n	0	1	2	3	4



Phys. Rev. C **87**, 044313 (2013)

One-nucleon separation energy

$$S_n = (a_1 + a_2 \cdot A^{1/3}) \cdot (Z/N) + a_3 + \boxed{\delta_{pair}} + \boxed{\delta_{shell}} + \boxed{\delta_{sv} + \delta_{ss}}$$

$$S_p = (a_1 + a_2 \cdot A^{1/3}) \cdot (N/Z) + a_3 + \boxed{\delta_{pair}} + \boxed{\delta_{shell}} + \boxed{\delta_{sv} + \delta_{ss}} + \boxed{\delta_{Coul}}$$

Pairing term	Shell effect	Symmetry energy terms	Columb effect
$a_{pair}/A^{1/2}$ $-a_{pair}/A^{1/2}$	$a_{shell} \cdot n$	$\delta_{sv} \approx 2a_{sv}\lvert I\rvert$, $\delta_{ss} \approx 2a_{ss}A^{-1/3}\lvert I\rvert$	$a_{Coul} \cdot Z/A^{1/3}$

	a_1	a_2	a_3	a_{pair}	a_{shell}	$2a_{sv}$	$2a_{ss}$	a_{Coul}	RMSD	keV
S_n	11467	3348	-10523	6556	-1566	13659	-28137	-	325	
S_p	18743	1209	-8582	6178	-1223	-26461	13064	-1182	342	

Phys. Rev. C **87**, 044313 (2013)

One-nucleon separation energy

- Four parities: ee eo oe oo
- Shell effect
- Symmetry energy
- Pairing term

One-nucleon separation energy

- Four parities: ee eo oe oo
- **Shell effect**
- Symmetry energy
- Pairing term

$$\delta_{shell} = a_{shell} \cdot n$$

$$\delta_{shell}^{(2)} = \pm a_{shell} c(r - \Delta N) Sgn(r - \Delta N)$$

$$Sgn(x) = \begin{cases} 1 & \text{if x>0} \\ 0 & \text{if x<=0} \end{cases}$$

One-nucleon separation energy

- Four parities: ee eo oe oo
- Shell effect
- **Symmetry energy**
- Pairing term

$$\delta_{sv} \approx 2a_{sv}|I|, \quad \delta_{ss} \approx 2a_{ss}A^{-1/3}|I| \Rightarrow \delta_{sv} \approx 2a_{sv}^{(1)}|I| + 4a_{sv}^{(2)}|I|^3, \quad \delta_{ss} \approx 2a_{ss}^{(1)}A^{-1/3}|I| - 4/3a_{ss}^{(2)}A^{-1/3}|I|^2$$

One-nucleon separation energy

- Four parities: ee eo oe oo
- Shell effect
- **Symmetry energy**
- Pairing term

$$\delta_{\mathrm{sv}} \approx 2a_{\mathrm{sv}}|I|,$$
$$\delta_{\mathrm{ss}} \approx 2a_{\mathrm{ss}}A^{-1/3}|I|$$

$$\Rightarrow$$

$$\delta_{\mathrm{sv}} \approx 2a_{\mathrm{sv}}^{(1)}|I| + 4a_{\mathrm{sv}}^{(2)}|I|^3,$$
$$\delta_{\mathrm{ss}} \approx 2a_{\mathrm{ss}}^{(1)}A^{-1/3}|I| - \cancel{4/3a_{\mathrm{ss}}^{(2)}A^{-1/3}|I|^2}$$

One-nucleon separation energy

- Four parities: ee eo oe oo
- Shell effect
- Symmetry energy
- **Pairing term**

One-nucleon separation energy

Parameters and RMSDs in keV

Parameters	a_1				a_2			
S_n	24545	19299	23279	17790	2077	2550	2264	2289
S_p	16625	15828	15671	16539	1867	2203	2289	1951

	a_3				$a_{shell}^{(1)}$	$a_{shell}^{(2)}$
S_n	-17450	-17402	-17476	-17430	-943	-27
S_p	-4834	-8433	-9086	-5623	-449	-47

	$2a_{sv}^{(1)}$	$4a_{sv}^{(2)}$	$2a_{ss}$	a_{Coul}	RMSD
S_n	23613	-105190	-4628	-	**272**
S_p	-42679	-72305	75598	-1589	**301**

One-nucleon separation energy

RMSDs for extrapolations in keV

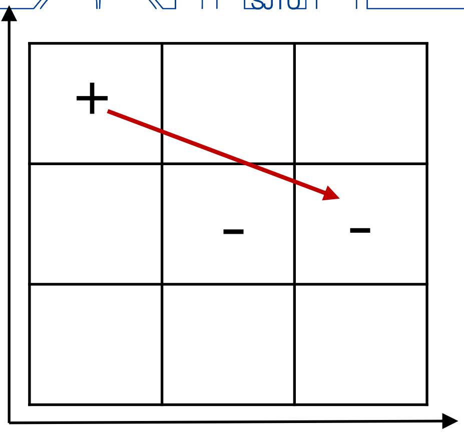
Predictions	DZ28	FRDM	Jiang	Bao	V-03	V-12
RMSD(S_n)	311	341	267	413	**351**	**305**
$N(S_n)$	271	271	112	51	**240**	**51**
RMSD(S_p)	421	439	341	511	**358**	**413**
$N(S_p)$	266	266	115	49	**235**	**49**

One-nucleon separation energy

$$\Delta S_{2\mathrm{n}1\mathrm{p}} = S_{\mathrm{n}}(N,Z) - S_{\mathrm{n}}(N+2,Z-1)$$

$$= a_1 \frac{2Z+N}{N(N+2)} + a_2 (N+Z)^{1/3} \frac{2Z+N}{N(N+2)}$$

$$+ \boxed{\Delta\delta_{\mathrm{pair}}} + \boxed{\Delta\delta_{\mathrm{sv}} + \Delta\delta_{\mathrm{ss}}}$$



Pairing term
$V_{\mathrm{pair}}(N,Z) - V_{\mathrm{pair}}(N-1,Z)$ $- V_{\mathrm{pair}}(N+2,Z-1) + V_{\mathrm{pair}}(N+1,Z-1)$

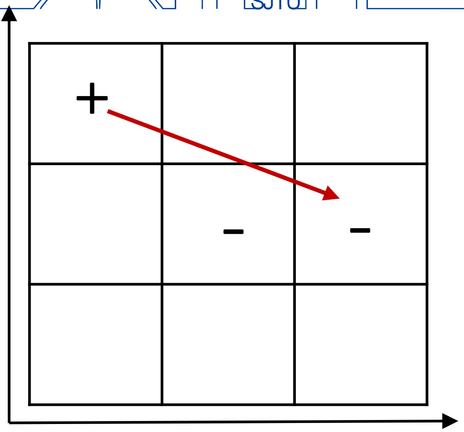
Symmetry energy terms
$\Delta\delta_{\mathrm{sv}} = 2a_{\mathrm{sv}}(N+Z)^{-1}(\lvert N-Z\rvert - \lvert N-Z+3\rvert)$ $\Delta\delta_{\mathrm{ss}} = 2a_{\mathrm{ss}}(N+Z)^{-4/3}(\lvert N-Z\rvert - \lvert N-Z+3\rvert)$

One-nucleon separation energy

$$\Delta S_{2\mathrm{n}1\mathrm{p}} = S_{\mathrm{n}}(N,Z) - S_{\mathrm{n}}(N+2,Z-1)$$

$$= a_1 \frac{2Z+N}{N(N+2)} + a_2 (N+Z)^{1/3} \frac{2Z+N}{N(N+2)}$$

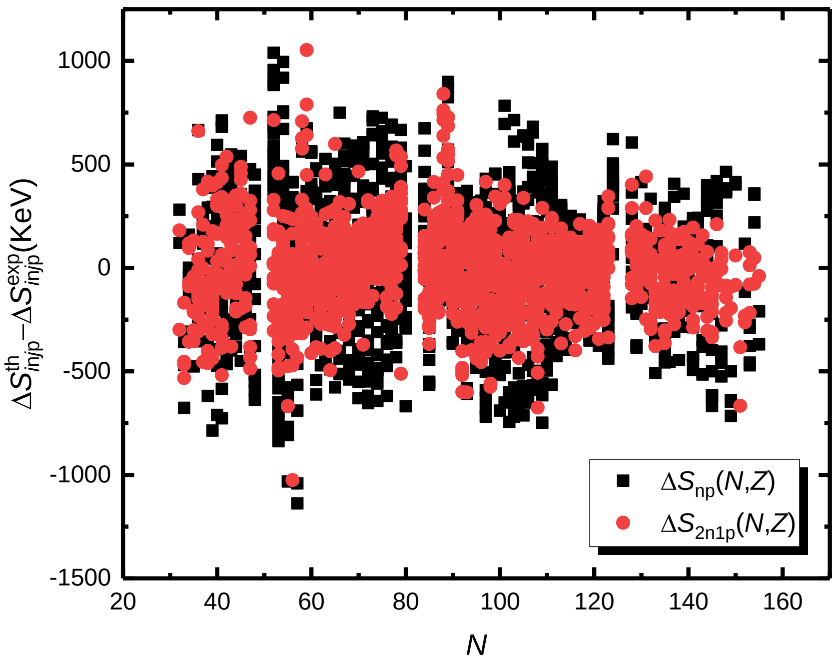
$$+\Delta\delta_{\mathrm{pair}} + \Delta\delta_{\mathrm{sv}} + \Delta\delta_{\mathrm{ss}}$$

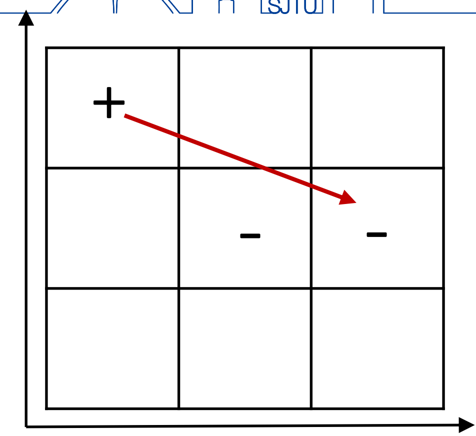


Parameters and RMSDs in keV

Parameters	a_1	a_2	a_{pair}	$2a_{sv}$	$2a_{ss}$	**RMSD**
Even A	38019	-1937	7382	-5831	71042	**210**
Odd A	-27150	1705	2755	-52899	2431	**224**

One-nucleon separation energy





Parameters	a_1				a_2			
S_n	24545	19299	23279	17790	2077	2550	2264	2289

	a_3			
S_n	-17450	-17402	-17476	-17430

Even *N* Odd *Z*

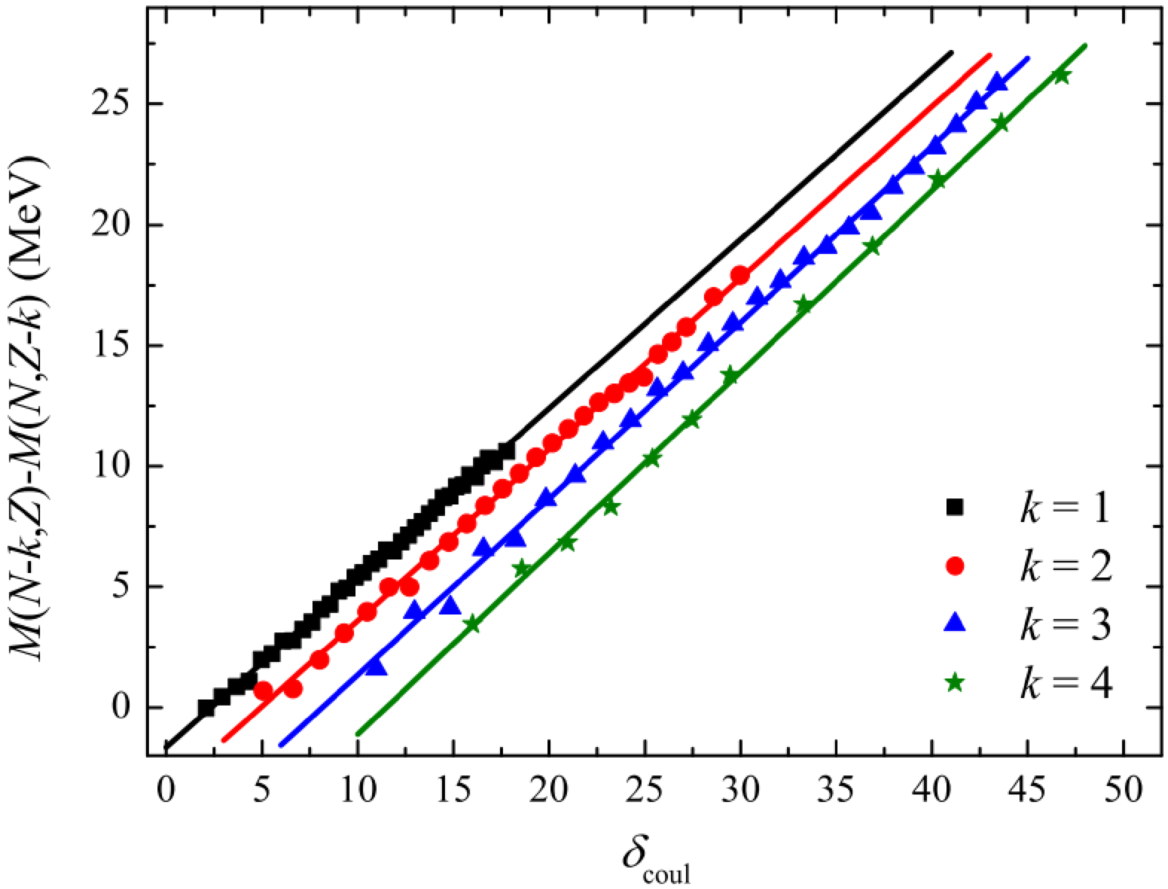
One-nucleon separation energy

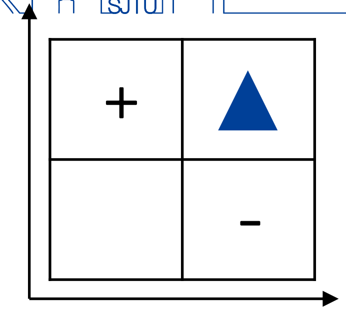
Comparison of the RMSD (in keV) between this work and others.

Predictions		AME 2003	AME 2012	H. Jiang	DZ28	FRDM	M. Bao	V-03	V-12
A>60	RMSD	233	284	231	299	301	358	**246**	**252**
	N	217	41	95	251	251	43	**200**	**41**
A>120	RMSD	179	144	203	263	232	336	**213**	**172**
	N	129	24	64	147	147	27	**124**	**26**

Outline

Beta-decay

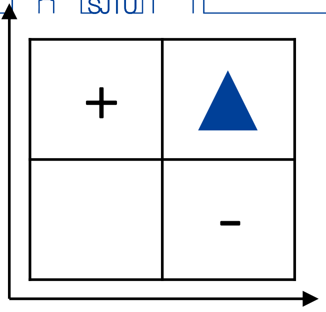




d=1

Phys. Rev. C 94, 044323 (2016)

Beta-decay



d=1

Weisacker mass formula:

$$B(N,Z) = a_{\mathrm{v}}A - a_{\mathrm{s}}A^{\frac{2}{3}} - a_{\mathrm{c}}Z^{2}A^{-\frac{1}{3}}$$

$$-a_{\mathrm{a}}(N-Z)^{2}A^{-1} + V_{\mathrm{p}}(N,Z)$$

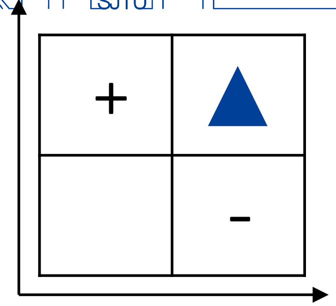
$$M(N-d,Z) - M(N,Z-d)$$

$$= B(N-d,Z) - B(N,Z-d) + dM_{\mathrm{p}} - dM_{\mathrm{n}}$$

$$= -a_{\mathrm{c}}d(A-K-d)(A-d)^{-\frac{1}{3}} - 8a_{\mathrm{a}}Kd$$

$$+V_{\mathrm{p}}(N-d,Z) - V_{\mathrm{p}}(N,Z-d) + d(M_{\mathrm{p}} - M_{\mathrm{n}})$$

Beta-decay



d=1

Weisacker mass formula:

$$B(N,Z) = a_{\mathrm{v}}A - a_{\mathrm{s}}A^{\frac{2}{3}} - a_{\mathrm{c}}Z^{2}A^{-\frac{1}{3}}$$

$$-a_{\mathrm{a}}(N-Z)^{2}A^{-1} + V_{\mathrm{p}}(N,Z)$$

$$M(N-d,Z) - M(N,Z-d)$$

$$= B(N-d,Z) - B(N,Z-d) + dM_{\mathrm{p}} - dM_{\mathrm{n}}$$

$$= -a_{\mathrm{c}}d(A - \boxed{K} - d)(A-d)^{-\frac{1}{3}} - 8a_{\mathrm{a}}Kd$$

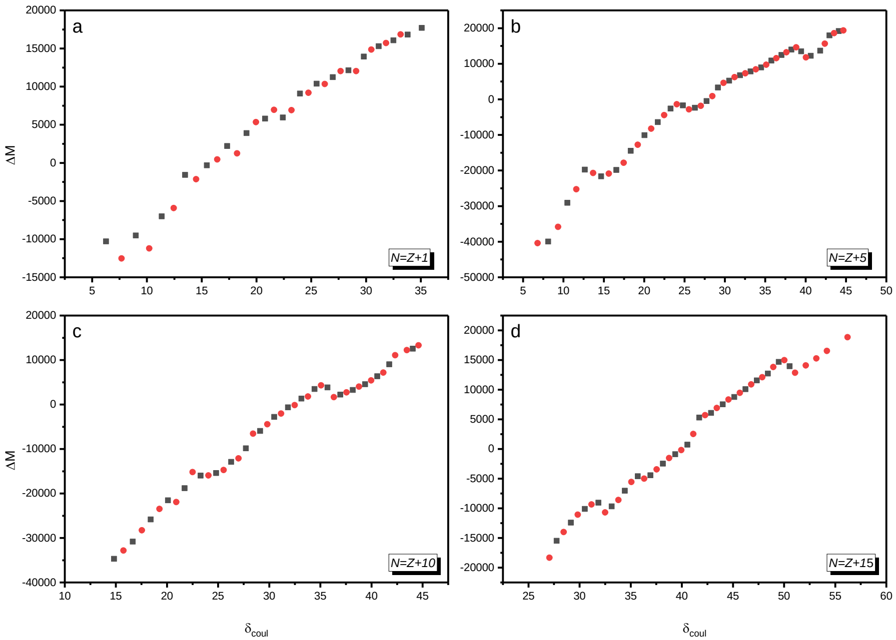
$$+V_{\mathrm{p}}(N-d,Z) - V_{\mathrm{p}}(N,Z-d) + d(M_{\mathrm{p}} - M_{\mathrm{n}})$$

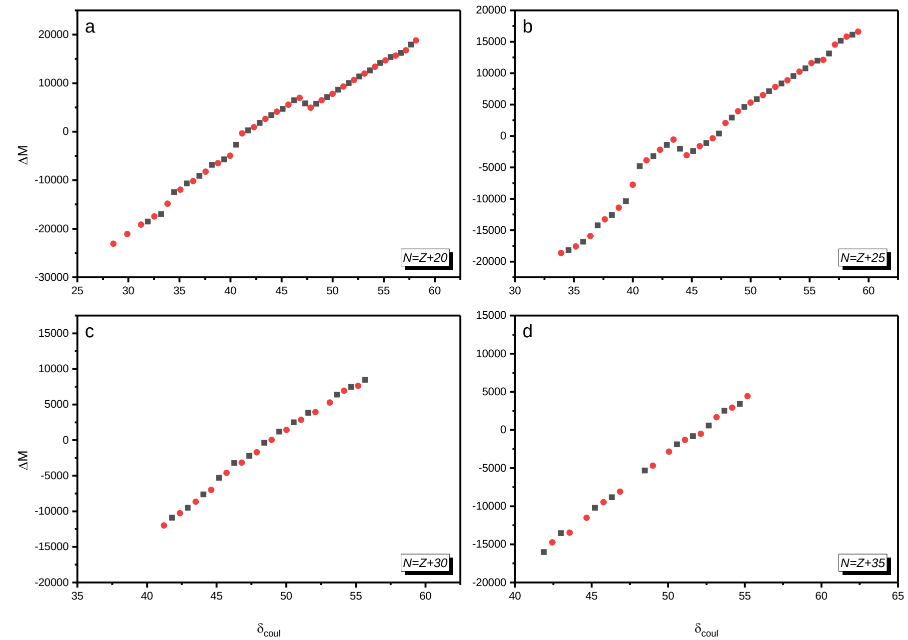
$K=N-Z=0$ gives out Mirror Nuclei.

Phys. Rev. C 94, 044323 (2016)

Beta-decay

d=2





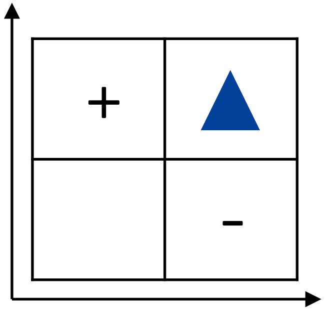
d=1 is similar but with odd-even staggerings.

Beta-decay

$$\Delta M(N,Z) = a_c \delta_{\mathrm{Coul}} + a_{\mathrm{a}} K d + \delta V_{\mathrm{p}}$$

$$+ a_{\mathrm{sh}}^{(e)} (n_{\mathrm{sh}} - p_{\mathrm{sh}}) + a_{\mathrm{sh}}^{(o)} (n_{\mathrm{sh}} - p_{\mathrm{sh}})$$

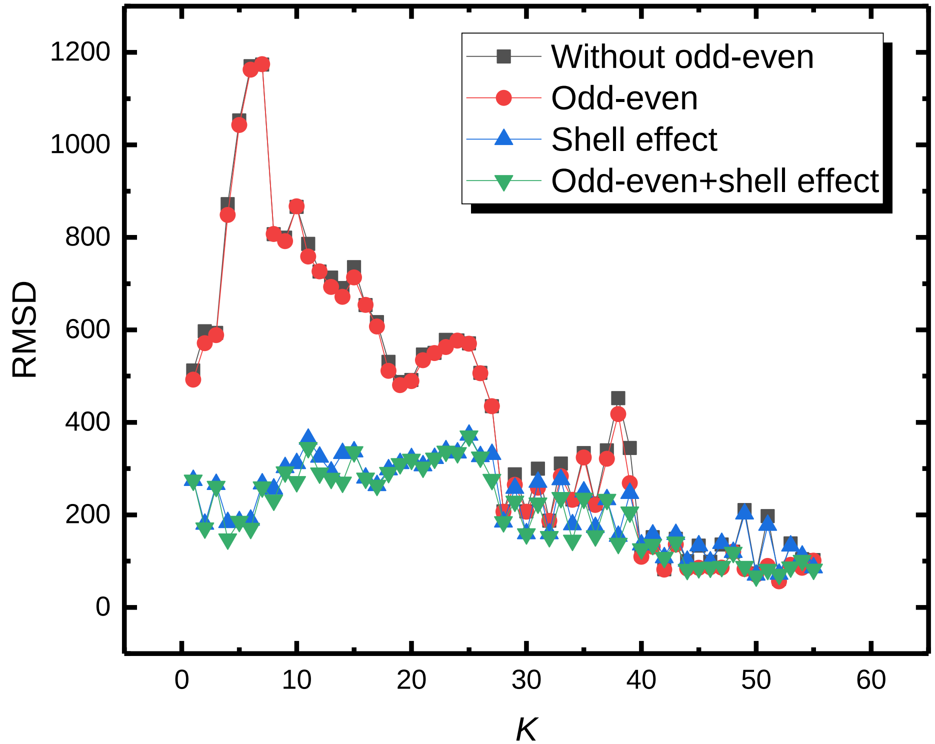
N	9-20	21-28	29-50	51-82	83-126	127-
n_{sh}	1	2	3	4	5	6
Z	9-20	21-28	29-50	51-82	83-126	
p_{sh}	0	1	2	3	4	



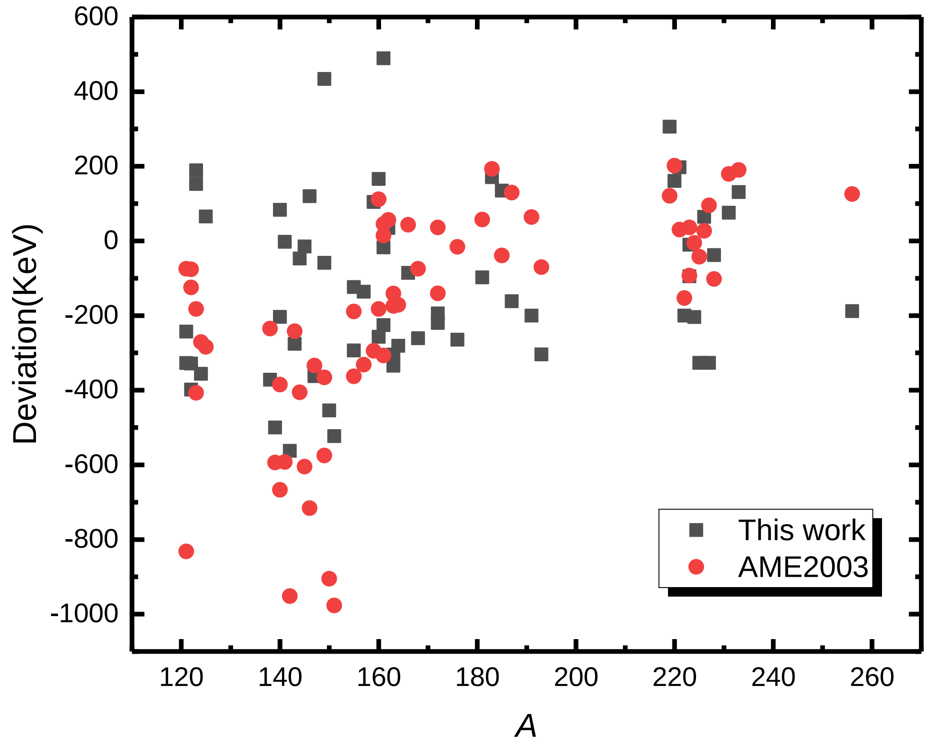
d=1

Beta-decay

$$\Delta M(N,Z) = a_c \delta_{\text{Coul}} + a_{\text{a}} K d + \delta V_{\text{p}}$$
$$+ a_{\text{sh}}^{(e)}(n_{\text{sh}} - p_{\text{sh}}) + a_{\text{sh}}^{(o)}(n_{\text{sh}} - p_{\text{sh}})$$



Beta-decay



$A \geq 60$	AME2003	This work	$A \geq 120$	AME2003	This work
RMSD	443	406	RMSD	354	281
N	95	95	N	61	61

Outline

Thanks for Your Attention!