



北京大学
PEKING UNIVERSITY

2020绵阳原子核结构理论研讨会

不稳定核结构与裂变研究进展

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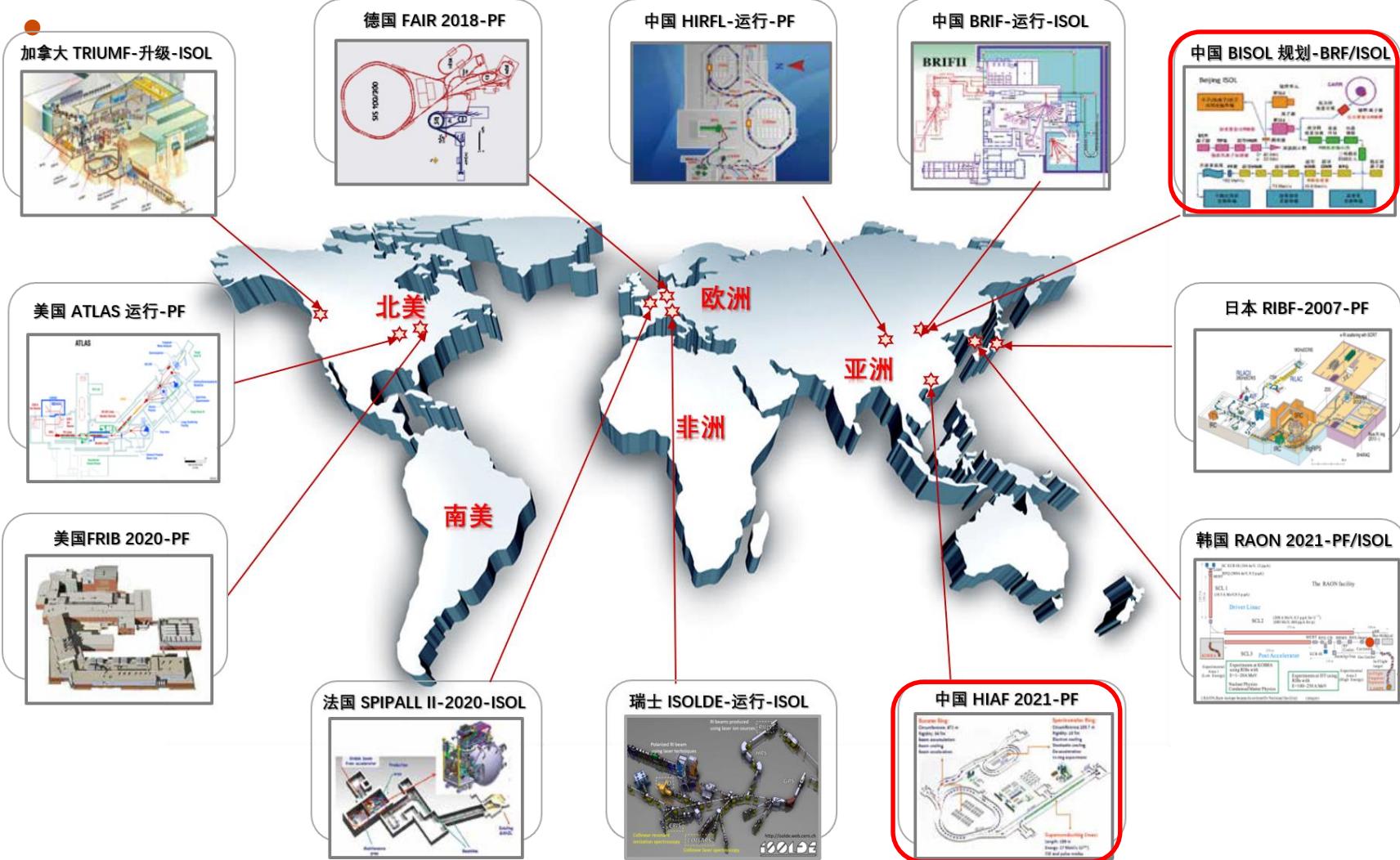


不稳定核结构与裂变研究进展

- 研究背景
- 不稳定核：格点空间HFB，连续谱，弱束缚核，冷原子，有效核力
- 核裂变：复合核裂变，动力学裂变，神经网络



研究背景





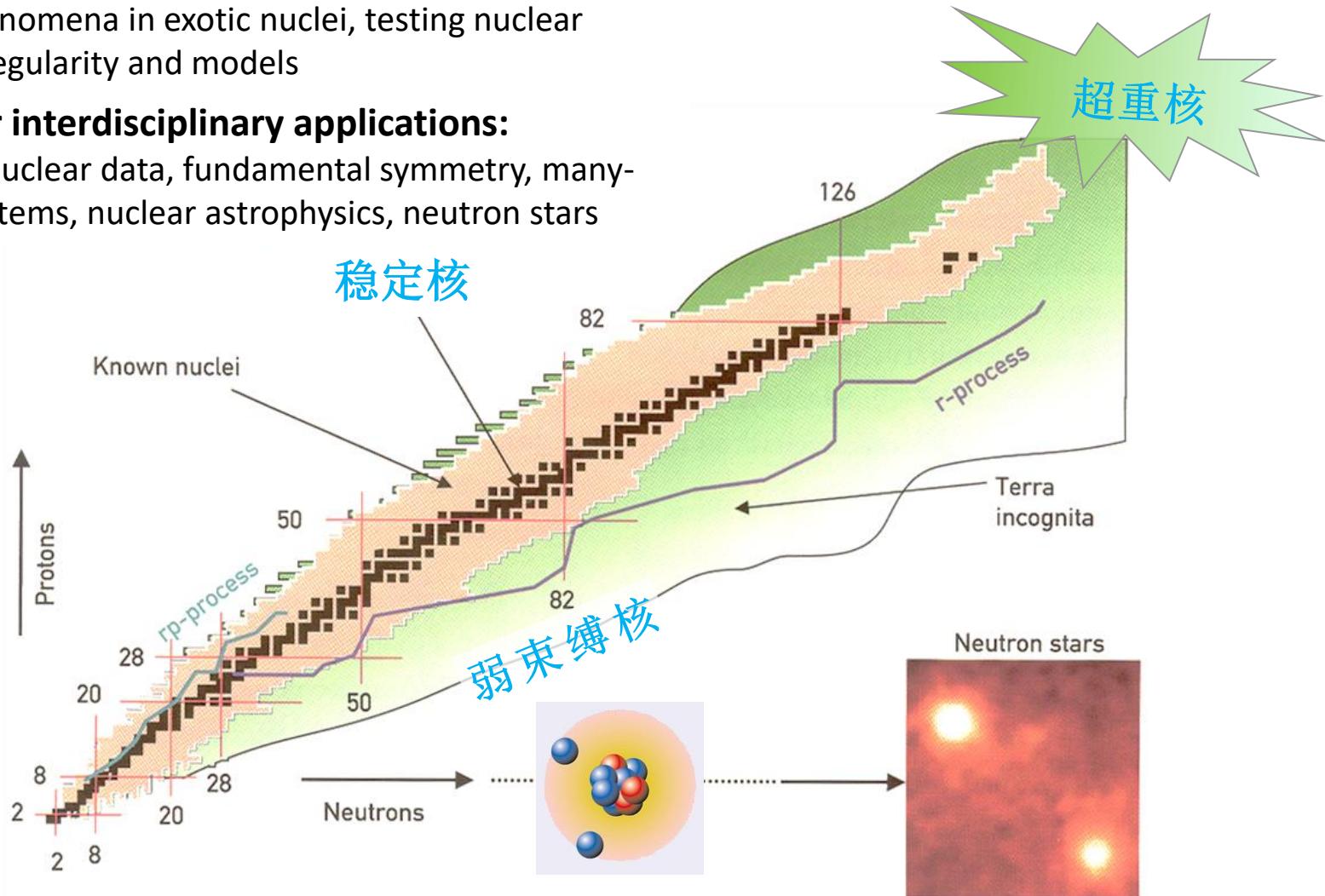
研究背景

Frontiers of nuclear physics in RIB era:

new phenomena in exotic nuclei, testing nuclear forces, regularity and models

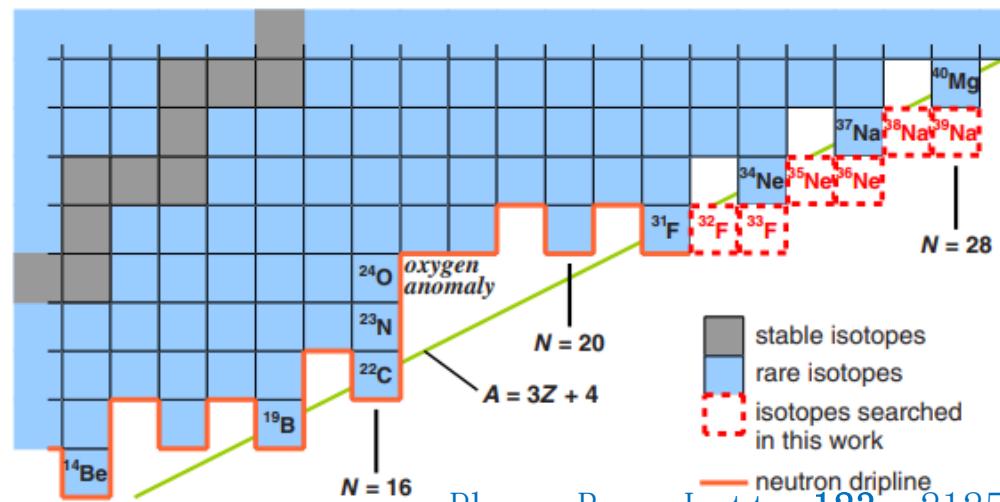
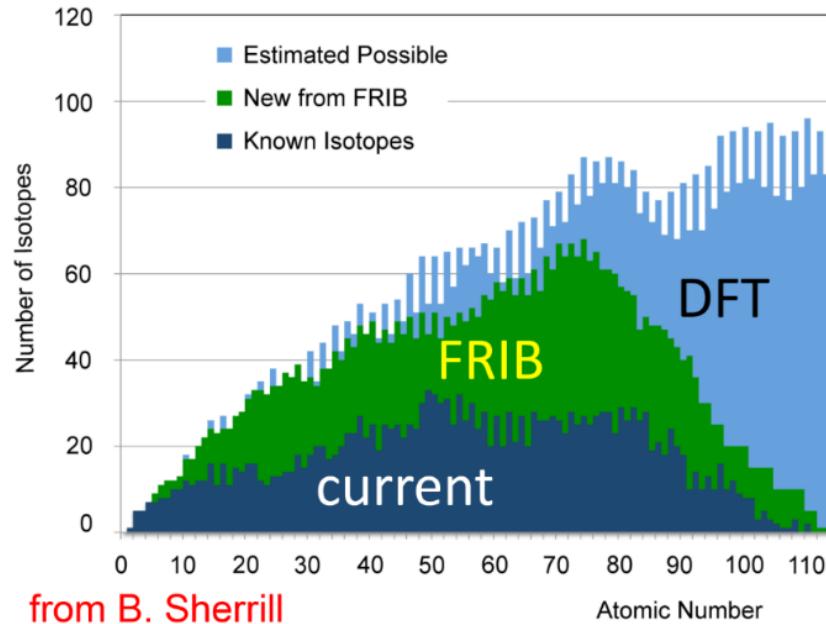
Nuclear interdisciplinary applications:

fission, nuclear data, fundamental symmetry, many-body systems, nuclear astrophysics, neutron stars



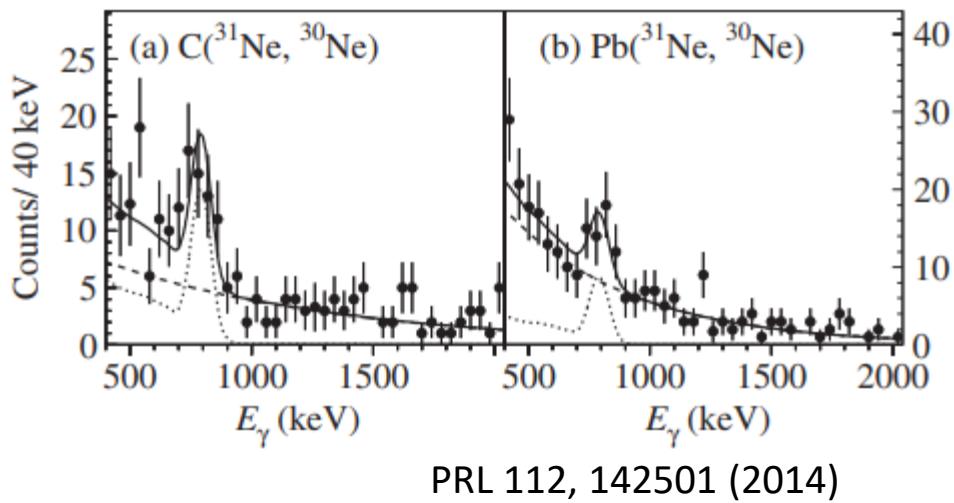
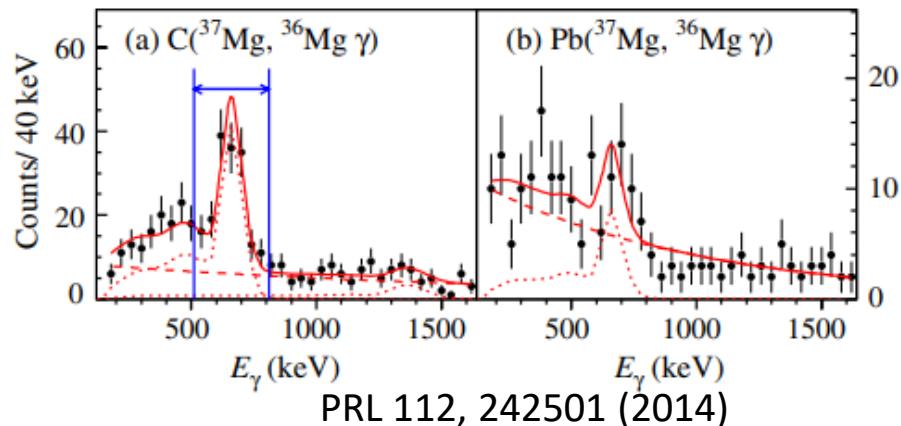
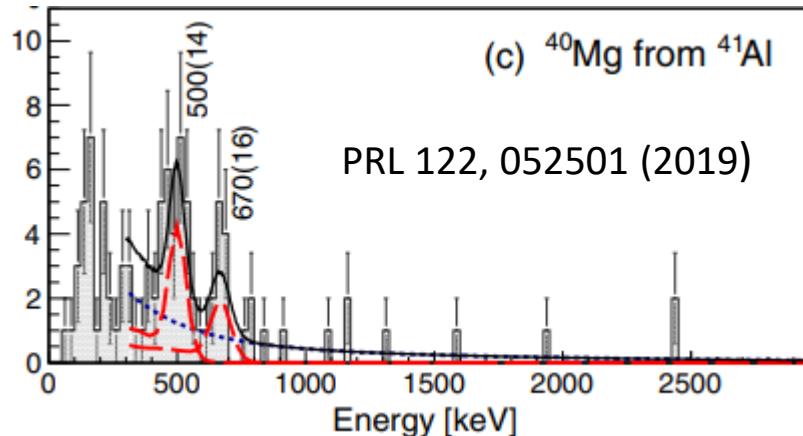


Progress at drip line





Progress at drip line



滴线区的位置确定—benchmark模型
滴线区的奇特结构—晕，efimov
滴线区的激发与decay---晕的激发模式
滴线核反应机制—
(L. Yang, C. J. Lin, et al. PRL 2017)



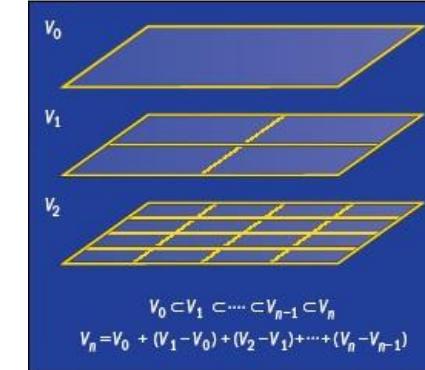
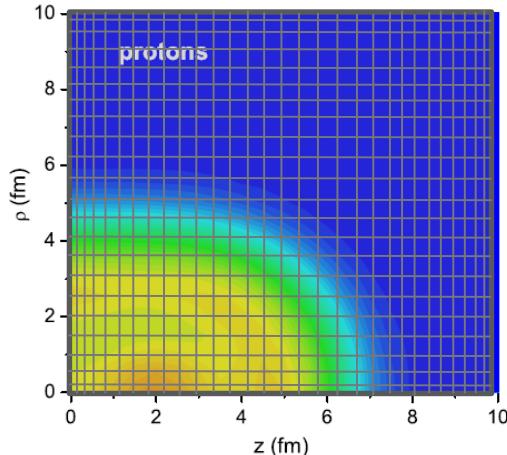
Descriptions of weakly bound nuclei

Problems:

- Self-consistent treatment of continuum, halo, deformation and pairing
- Further descriptions of collective excitations and decays of halo nuclei

Methods:

- Exact outgoing boundary condition for spherical nuclei: Gamow HFB, Green's function method
- Coordinate space HFB for weakly bound deformed nuclei



Axial-symmetric HFB-AX: flexible and efficient
Pei et al. PRC 2008

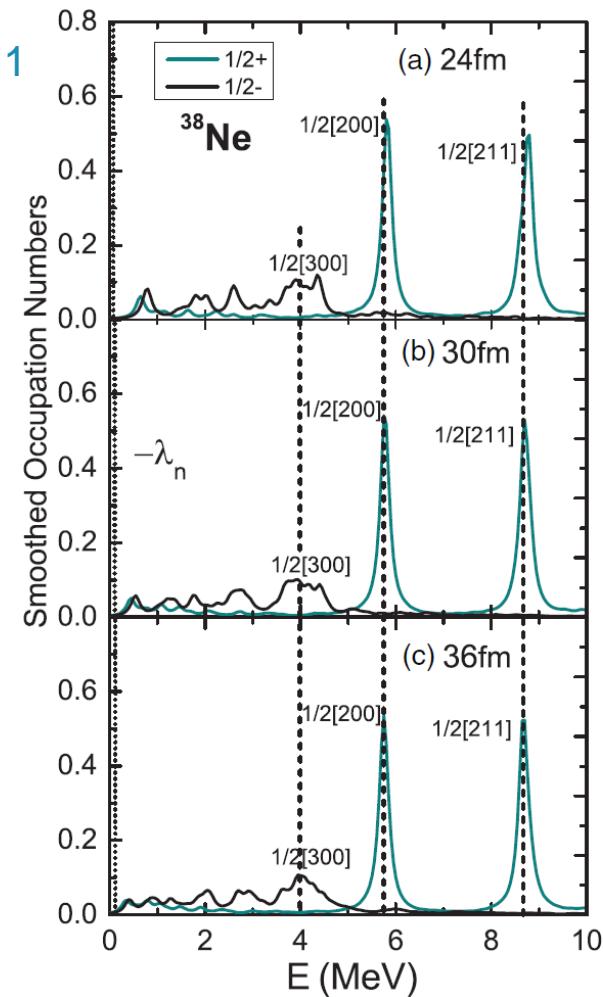
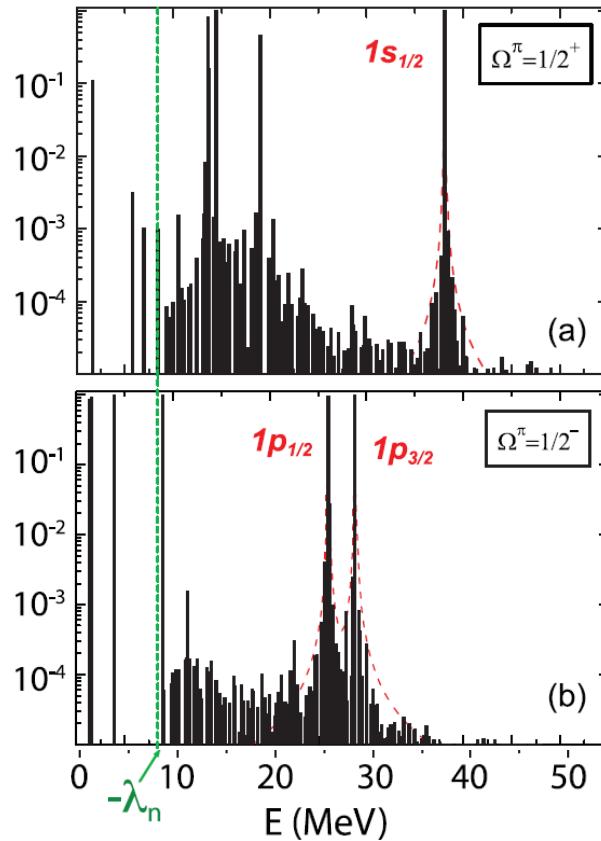
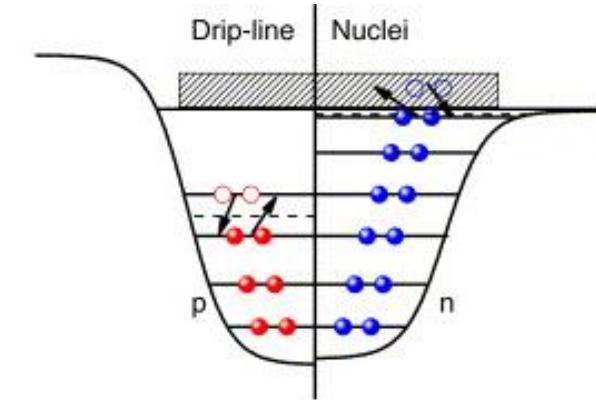
3D-MADNESS HFB: Multi-wavelets
Pei et al. PRC 2014



Coordinate-space HFB for static continuum effects

- Continuum in ground-state of weakly bound nuclei

Pei, A.Kruppa, W.Nazarewicz PRC, 2011



- Continuum discretization in a large box
- Self-consistent to describe deformed halos and continuum
- Non-resonant continuum play an important role

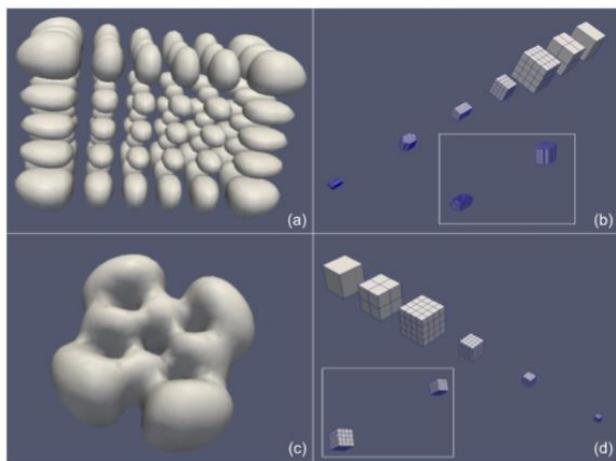
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$$f^n(x) = \sum_{l=0}^{2^n-1} \sum_{i=0}^{k=1} s_{il}^n \phi_{il}^n(x)$$

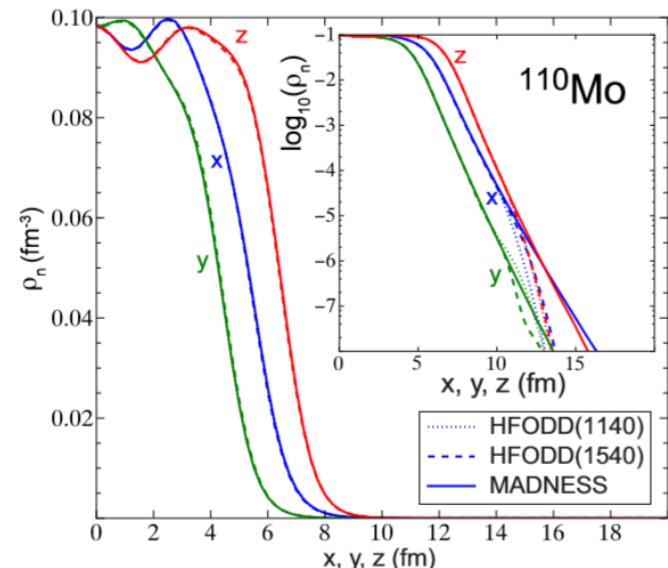
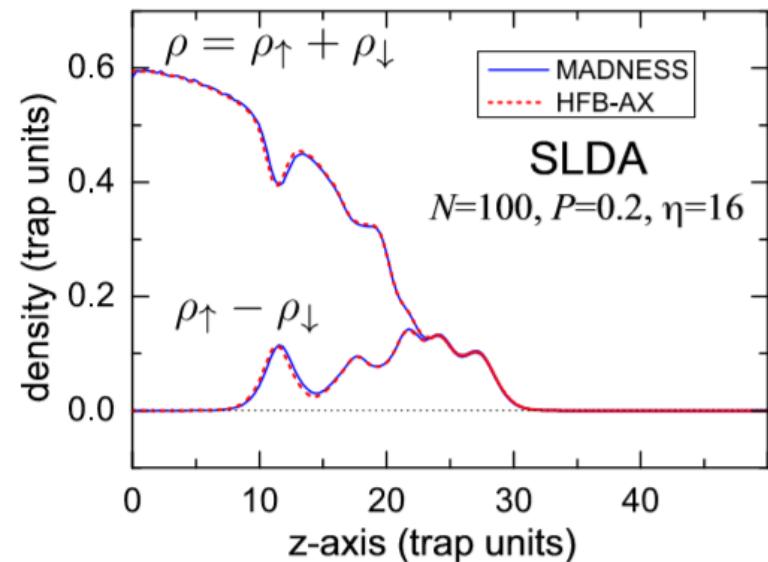
$$f^n(x) = \sum_{i=0}^{k=1} s_{i0}^0 \phi_{i0}^0(x) + \sum_{n=0}^{2^n-1} \sum_{l=0}^{k=1} \sum_{i=0}^{d_{il}^n} \psi_{il}^n(x)$$

d 表示不同level表象之间的差别，
截断小系数 $d_{il} \rightarrow$ 非均匀格点

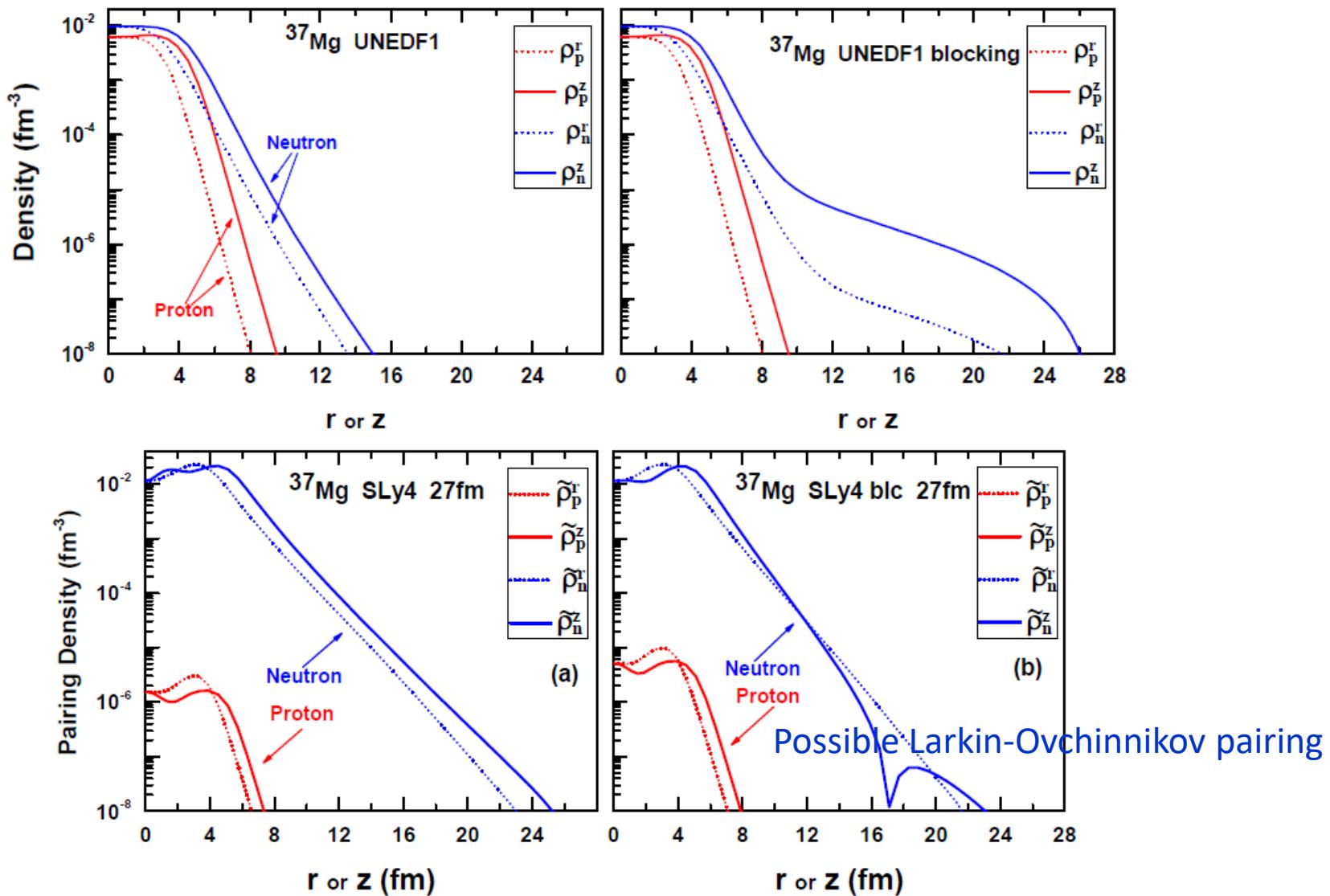
波函数振荡的地方，格点自动加密



J. C. Pei, G. I. Fann, R. J. Harrison, W. Nazarewicz, Yue Shi, and S. Thornton, Phys. Rev. C 90, 024317 (2014)(Highlighted as **Editor's suggestion**)

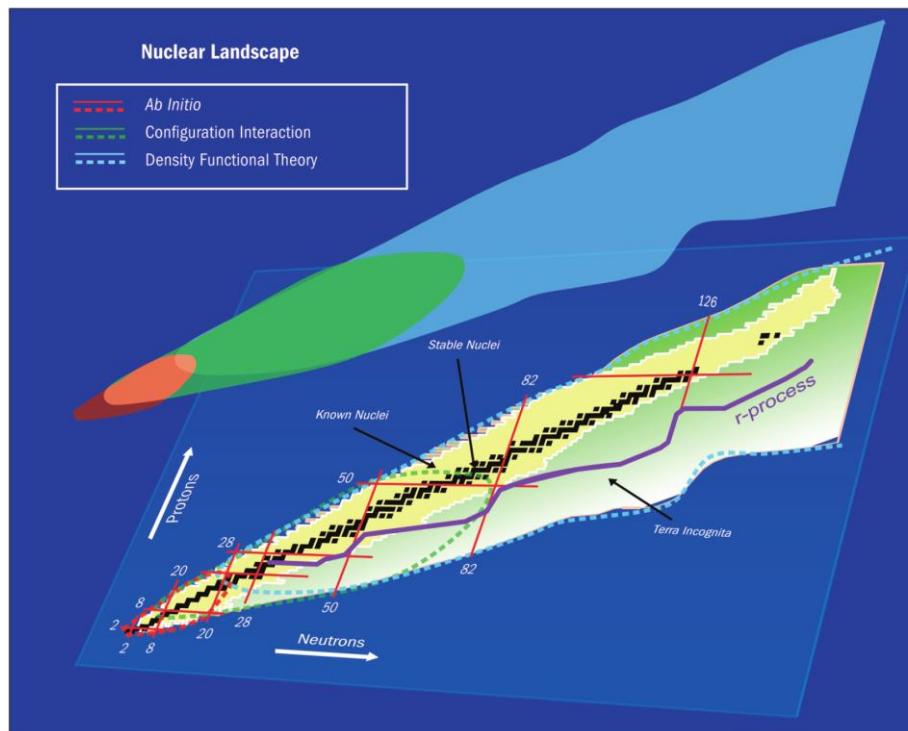


Deformed halo structure in ^{37}Mg



Optimization of Skyrme force

- Nuclear energy density functional is very successful for describing structures and dynamics of the entire nuclear landscape
([M.Bender, et al. RMP 2001](#))
- Our final goal is to develop a high-precision predictive NEDF for various interests, in particular for exotic nuclei, superheavy nuclei, collective motions, fission...



Ideas of Effective interactions

- Effective forces

Effective interactions for realistic systems, omitted physics are compensated by parameterizations or renormalization.

e.g. pseudo δ interaction for low-energy s-wave scattering $V(x) = -\alpha \delta(x)$,

$$\psi(x) = \frac{\sqrt{m\alpha}}{\hbar} e^{-m\alpha|x|/\hbar^2} \text{ and } E = -\frac{m\alpha^2}{2\hbar^2}$$

Effective forces:

In-medium effects, soft without hard core, many-body forces, model-space dependent

$$V_{NN}^{\text{NLO}} = C_2 \frac{1}{2} (k^2 + k'^2) + C'_2 \mathbf{k} \cdot \mathbf{k}' + C_2^S \frac{1}{2} (k^2 + k'^2) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + C_2'^S \mathbf{k} \cdot \mathbf{k}' \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \quad \text{Pion-less EFT}$$

$$+ iC_2^{\text{LS}} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{k} \times \mathbf{k}') + C_2^T \boldsymbol{\sigma}_1 \cdot (\mathbf{k}' - \mathbf{k}) \boldsymbol{\sigma}_2 \cdot (\mathbf{k}' - \mathbf{k}) + C_2'^T \boldsymbol{\sigma}_1 \cdot (\mathbf{k}' + \mathbf{k}) \boldsymbol{\sigma}_2 \cdot (\mathbf{k}' + \mathbf{k})$$

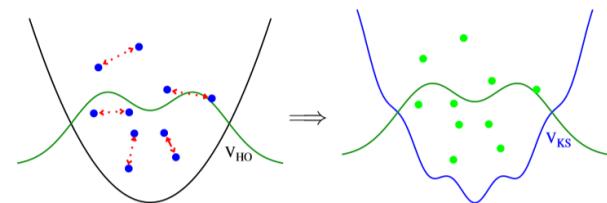
DFT:

unique DFT for ground states, find an exact effective potential include exchange, and correlation energy

$$E[\rho] = T_S[\rho] + E_C[\rho] + E_{XC}[\rho] + E_{Ne}[\rho]$$

Realistic forces:

Start from few nucleon systems, small uncertainties in 2-body and 3-body forces could propagate large errors in *ab initio* calculations of heavy nuclei





NEDF based on Skyrme force

Skyrme interaction is a very low-momentum phenomenological effective interaction.
Similarities with Pion-less EFT.

$$V = \sum_{i < j} v(i, j) + \sum_{i < j < k} v(i, j, k)$$

Tensor term

$$\frac{1}{2} T(\sigma_1 \cdot k \sigma_2 \cdot k - \frac{1}{3} \sigma_1 \cdot \sigma_2 k^2 + \text{conj.})$$

$$\frac{1}{2} U(\sigma_1 \cdot k' \sigma_2 \cdot k - \frac{1}{3} \sigma_1 \cdot \sigma_2 k' \cdot k + \text{conj.})$$

$$V(\vec{r}_1, \vec{r}_2) = t_0(1 + x_0 P_\sigma) \delta(\vec{r})$$

central term

$$+ \frac{1}{2} t_1(1 + x_1 P_\sigma) [\vec{P}^2 \delta(\vec{r}) + \delta(\vec{r}) \vec{P}^2]$$

$$+ t_2(1 + x_2 P_\sigma) \vec{P} \delta(\vec{r}) \vec{P}$$

non-local terms

$$+ iW_0 \vec{\sigma} \bullet [\vec{P} \times \delta(\vec{r}) \vec{P}]$$

spin-orbit term

3-body term in Skyrme force:
Important for saturation properties

$$v_{ijk}^{(3)} = t_3 \delta(\vec{r}_i - \vec{r}_j) \delta(\vec{r}_j - \vec{r}_k)$$

$$V_{ijk}^{(3)} \sim V_{ij}^{(2)}, = \frac{1}{6} t_3 (1 + P_\sigma) \delta(\vec{r}_i - \vec{r}_j) \rho(\frac{\vec{r}_i + \vec{r}_j}{2})$$

Too large incompressibility



$$V_{ijk}^{(3)} \sim V_{ij}^{(2)}, = \frac{1}{6} t_3 (1 + P_\sigma) \delta(\vec{r}_i - \vec{r}_j) \rho(\vec{r})^\gamma$$

Usually a fractional power density dependency is introduced to *simulate 3-body and many body forces*; the power dependency is an open question

γ ranges from 1/6 to 1

$\gamma=1/6$ in SLy4, SkM*, SkP; 0.25 in SkIx

$\gamma=1/3$ in Gogny, Bsk1

UNEDF0=0.32, UNEDF1=0.27



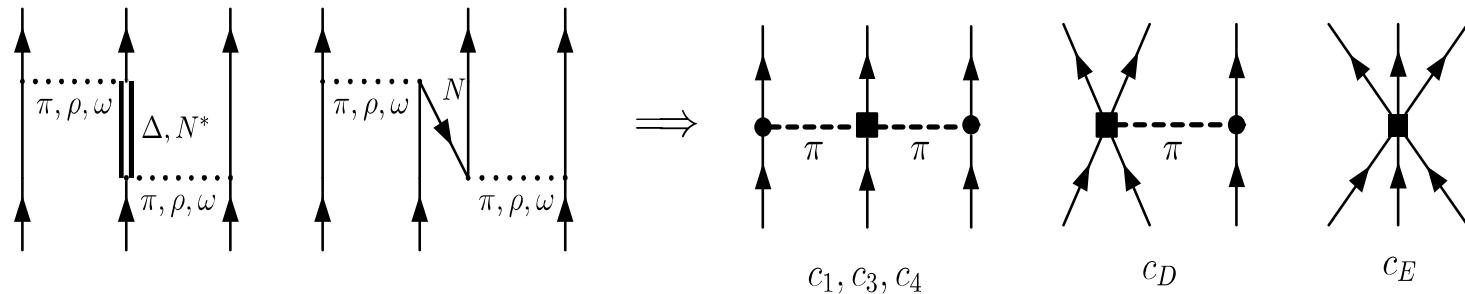
Various developments of Skyrme-like DFT

- UNEDF Skyrme forces have been extremely optimized using POUNDERS
[M. Kortelainen et al., Phys. Rev. C 82, 024313 \(2010\).](#)
- Brussel Skyrme forces with phenomenological corrections obtained high precisions
[S. Goriely, et al., Phys. Rev. C 82, 035804 \(2010\)](#)
- Various extensions of Skyrme forces: additional momentum dependences or density dependencies (advantages are not clear and fine optimizations are needed)
- Very recent: BCPM(Barcelona-Catania-Paris-Madrid), Seattle-DFT, etc:
combined density functionals $\rho^{n/3}$ [A.Bulgac, Phys. Rev. C 97, 044313 \(2018\)](#)
- Other developments: Pionless EFT, density matrix expansion,
Pseudopotential Skyrme forces to 6th order, ab initio EDF
[B. G. Carlsson, et al., PRC 78, 044326 \(2008\)](#)
[M. Stoitsov, et al., PRC 82, 054307 \(2010\)](#)
[M. Grasso, D. Lacroix, and U. van Kolck, Phys. Scr. 91, 063005\(2016\).](#)
[R. J. Furnstahl, Lecture Notes in Physics, Vol.852, 133\(Springer-Verlag, 2012\).](#)
[Zhen Zhang and Lie-Wen Chen, Phys. Rev. C 94, 064326 \(2016\)](#)
[Y. N. Zhang, S. K. Bogner, and R. J. Furnstahl, Phys. Rev. C 98, 064306 \(2018\)](#)

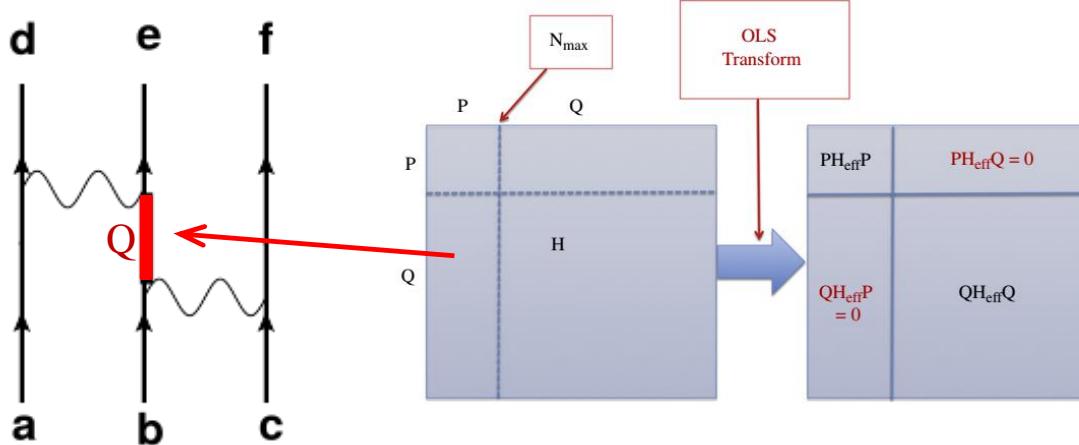
Density dependent forces

- ◆ Skyrme force with a density-dependent term becomes state dependent, in-medium interactions in SCF.
- ◆ 3- and many-body force are generated due to the cutoff of model spaces of 2-body interactions. [P. Grange et al., Phys. Rev. C40, 1040 \(1989\)](#)
- ◆ It is remarkable in softer interactions (e.g., Skyrme, Gogny)

- Real three-body forces from neglected degrees of freedom



- Induced 3- and many-body forces from renormalization



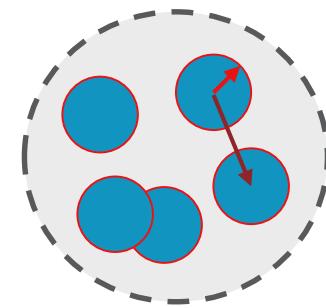
- *Very large 3-body term in soft Skyrme force is induced!*



Refitting procedure

$$\begin{aligned}\varepsilon = & \frac{3\hbar^2}{10m} (3\pi^2)^{2/3} \rho^{5/3} + \frac{\hbar^2 \pi a}{m} \rho^2 + \frac{2\hbar^2 a^2 3^{4/3} \pi^{2/3}}{35m} (11 - 2\ln 2) \rho^{7/3} \\ & + 0.78 \frac{\hbar^2 a^3 3^{5/3} \pi^{10/3}}{2m} \rho^{8/3} \\ \rho = & k_F^3 / 3\pi^2 \quad a = r/d\end{aligned}$$

With an additional higher-order density dependent terms



Huang and C.N. Yang, Phys. Rev. 105, 767(1957).
T. D. Lee and C. N. Yang, Phys. Rev. 105, 1119(1957).
P. Martin and C. De Dominicis, Phys. Rev. 105, 1417(1957).

$$\begin{aligned}v_{ij}^{(2)'} = & \frac{1}{6} t_3 (1 + x_3 P_\sigma) \rho(\mathbf{R})^\gamma \delta(\mathbf{r}_i - \mathbf{r}_j) \\ & + \frac{1}{6} t_{3E} (1 + x_{3E} P_\sigma) \rho(\mathbf{R})^{\gamma + \frac{1}{3}} \delta(\mathbf{r}_i - \mathbf{r}_j).\end{aligned}$$

- Only refit the momentum independent parameters: t_0 , t_3 , t_{3E} , leading regularization terms for saturation properties
- Induced three body and many-body forces are huge in the soft Skyrme force, and a single term may not be sufficient for various systems from dilute halos to high density neutron stars
- Using simulated annealing method, fitting binding energies of 50 nuclei and charge radii of 8 spherical nuclei



X.Y. Xiong, J. C. Pei, W.J. Chen, Phys. Rev. C 93, 024311 (2016)
 Z.W. Zuo, J.C. Pei, X.Y. Xiong, Y. Zhu, Chinese Phys. C 42 064106(2018)

Refitting based on SLy4

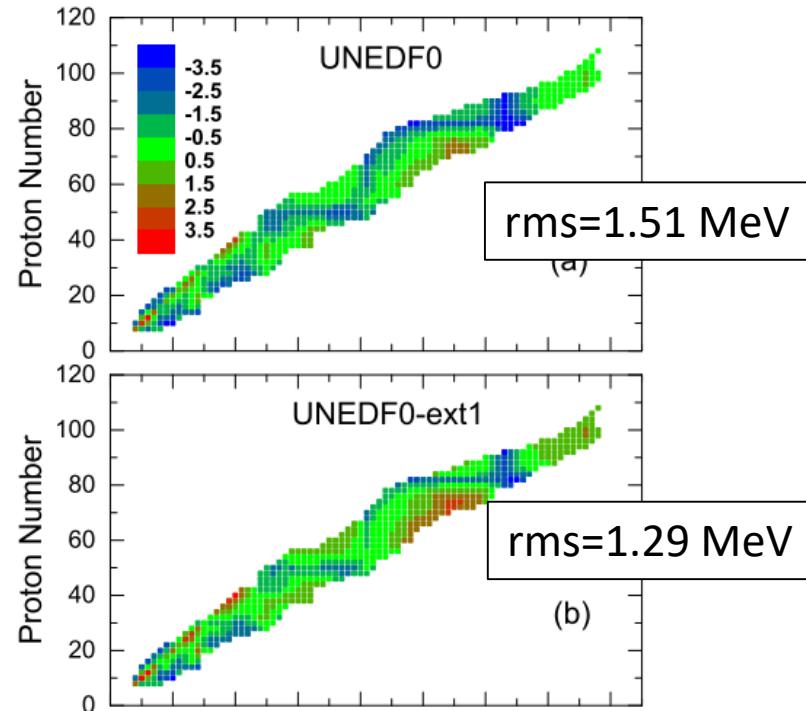
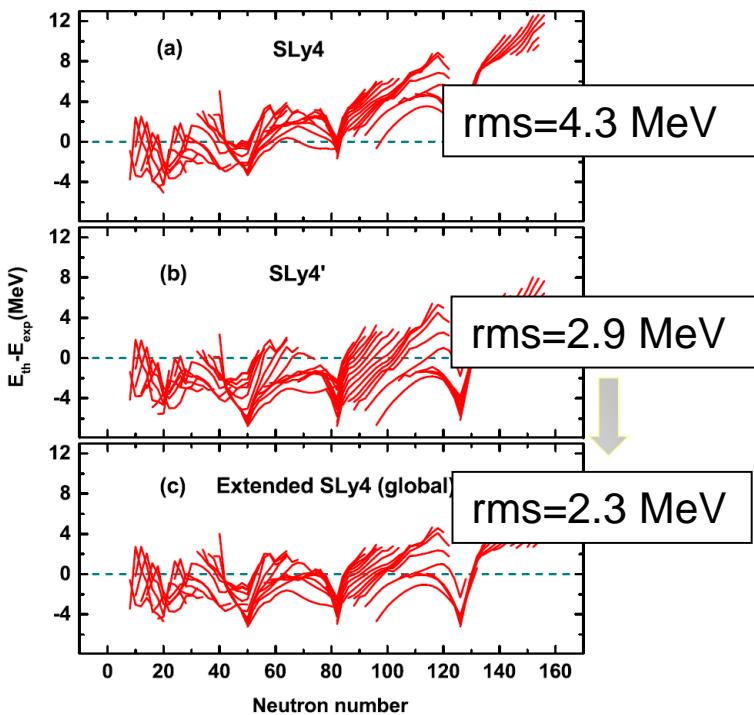
Parameters	SLy4	SLy4' (global)	Group 1 (magic)	Group 2 (light)	Group 3 (heavy)	Group 4 (global)
t_0 (MeV fm ³)	-2488.91	-2493.536	-2132.57	-2106.69	-2310.79	-2319.15
t_3 (MeV fm ^{3(1+1/6)})	13777.0	13809.324	9366.12	9036.30	11549.45	11660.70
t_{3E} (MeV fm ^{3(1+1/2)})	0	0	2756.97	2970.72	1410.64	1334.88
x_0	0.834	0.931	0.968	0.988	0.972	0.863
x_3	1.354	1.496	1.922	1.975	1.715	1.509
x_{3E}	0	0	0.375	0.530	0.324	0.622
e_∞ (MeV)	-15.972	-15.963	-16.063	-16.085	-16.055	-16.039
K_∞ (MeV)	229.9	230.31	247.33	248.84	239.19	238.58
χ^2_R		10.35	2.18(4.04)	5.00(10.04)	0.39(0.62)	7.4(10.66)
χ^2_B		31.67	0.83(2.46)	3.83(6.82)	19.26(78.28)	25.92(84.87)
χ^2_{sum}		42.84	3.67(6.91)	9.79(17.306)	20.52(79.21)	33.82(95.94)

Refitting based on UNEDF0 and SkM*

TABLE I. The refitted parameters of the extended Skyrme forces based on UNEDF0 and SkM* forces. The units for t_0 , t_3 and t_{3E} are MeV · fm³, MeV · fm^{3(1+γ)} and MeV · fm^{3(γ+4/3)}, respectively. Other parameters have not been adjusted.

	UNEDF0	UNEDF0 _{ext1}	UNEDF0 _{ext2}	SkM*	SkM* _{ext1}	SkM* _{ext2}
t_0	-1883.6878	-2007.948	-2140.306	-2645.0	-2035.587	-2325.478
t_3	13901.948	11616.664	13869.309	15595.0	8007.383	11608.668
t_{3E}	0	3216.9303	1402.674	0	4795.359	2534.788
x_0	0.00974	-0.0494	-0.2363	0.09	0.2376	0.2358
x_3	-0.3808	-0.4722	-0.7760	0	-0.07488	0.2720
x_{3E}	0	-0.1540	1.5051	0	0.9955	-0.4692
γ	0.3219	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

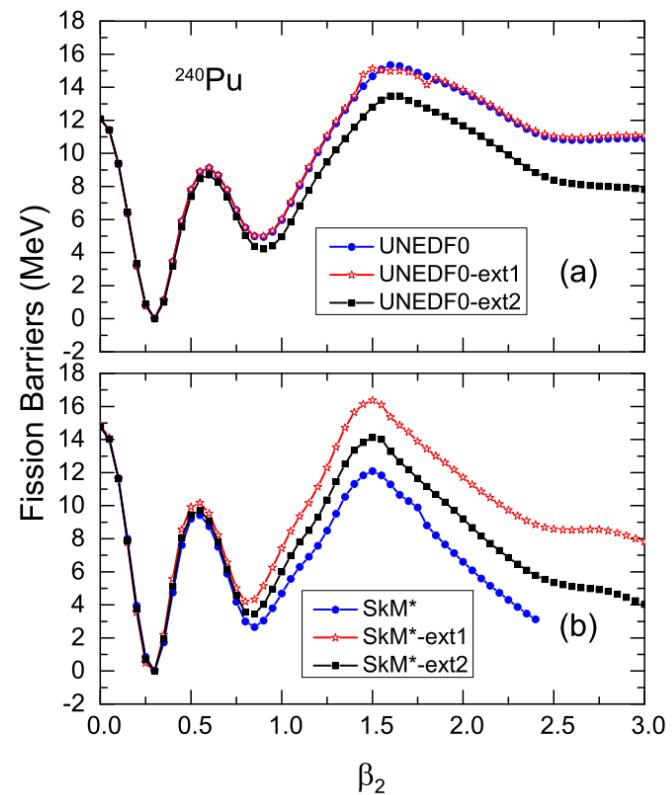
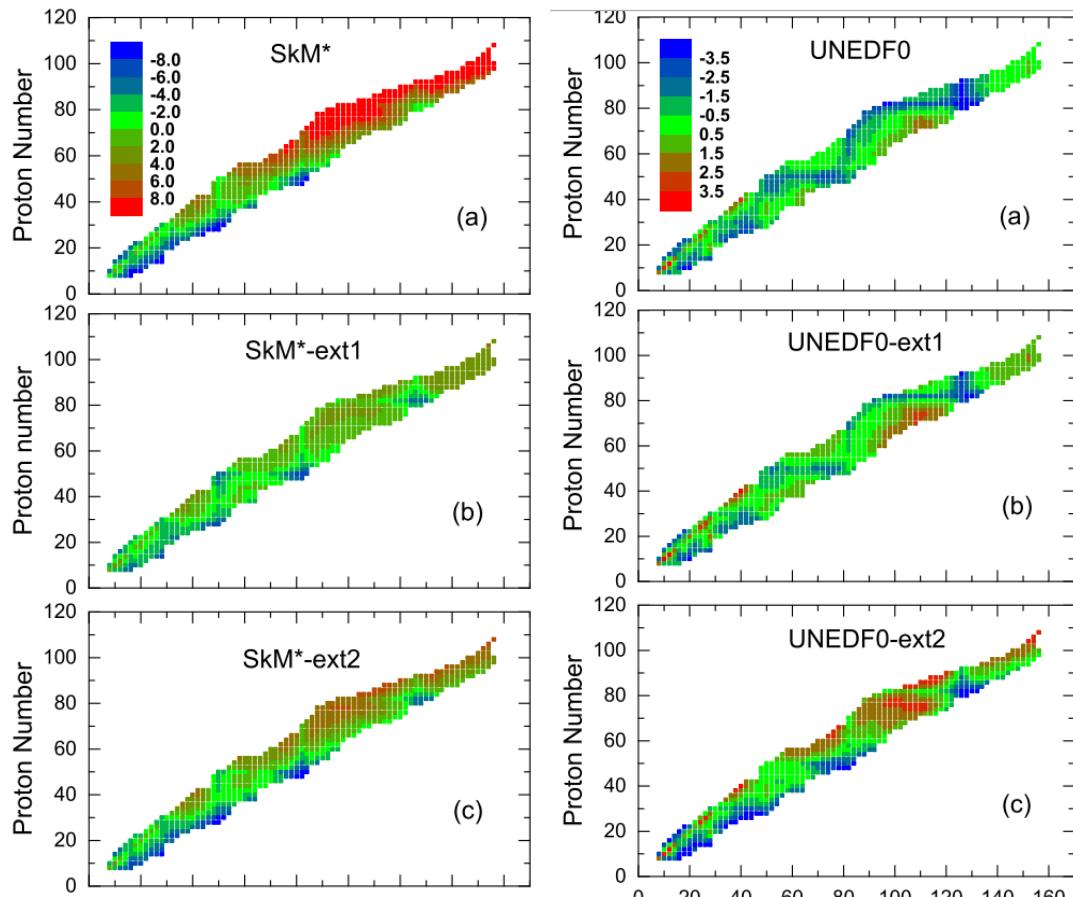
Binding energies



- Calculations of 603 even-even nuclei, reduce the rms by 10~20%
- In light nuclei, binding energies of $N=Z$ nuclei are seriously underestimated.
Missing np correlations ([M. Stoitsov, et al. PRL 98, 132502 \(2007\)](#))
- In heavy nuclei, the shell effects are overestimated

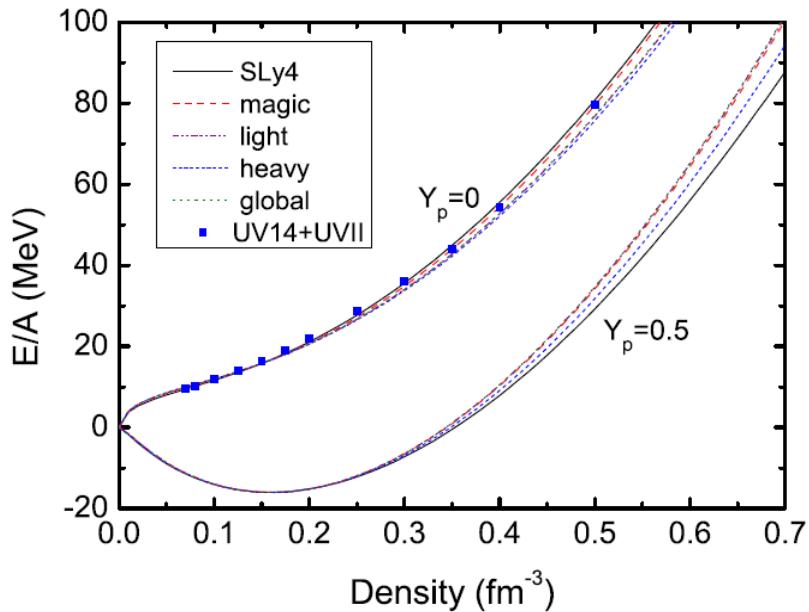


Fission barriers



- Parameter sets which are good at binding energies are not good at fission barriers
- Proton-rich heavy nuclei are less binding, neutron-rich medium nuclei are over binding, indicating conflicting isospin dependences
(surface symmetry energy, [N. Nikola et al, PRC83, 034305 \(2011\)](#))

EOS



Bring more binding energy for high density neutron matter with the high-order term

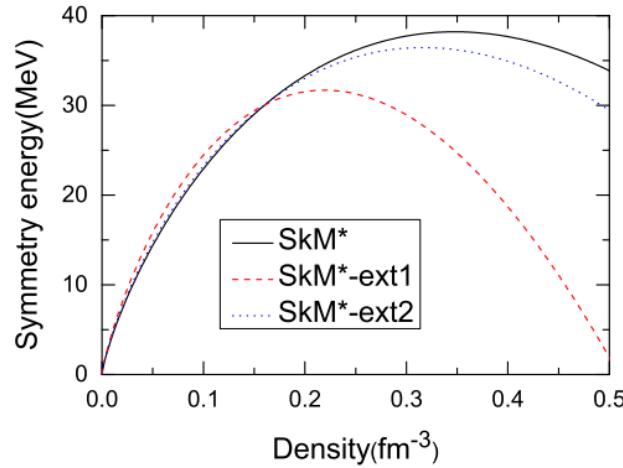
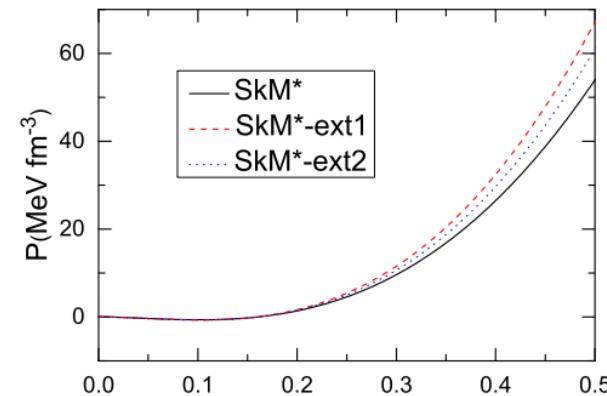
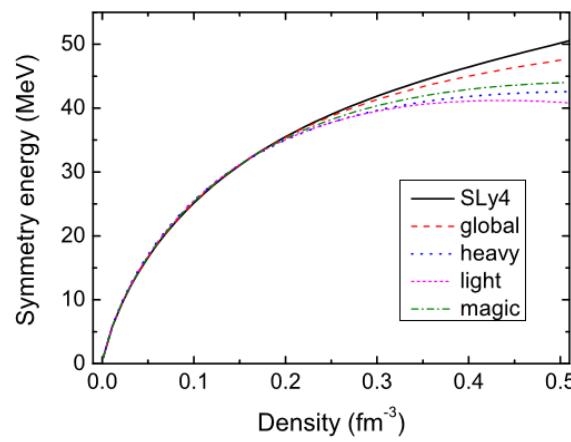
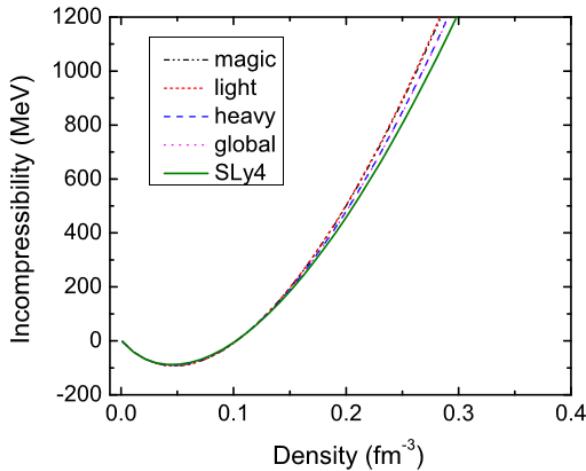
N	SLy4	Extend (light)	<i>ab initio</i>
8	120.81	124.03	135.4(2) ^a
10	169.54	173.47	187.1(6) ^a
12	217.04	221.56	237.1(8) ^a
14	262.69	267.59	286(1) ^a
16	309.28	314.28	334(1) ^a
18	364.83	370.87	386(3) ^a
20	418.56	425.50	432.8(5) ^b
28	656.18	663.84	681(1) ^b
40	1061.23	1070.97	1058(2) ^b
50	1420.19	1428.28	1449(3) ^b

^aNo-core shell-model calculations.

^bCouple-cluster calculations.

- ◆ Benchmarks of neutron droplets in a $\hbar\bar{\hbar}=10$ MeV trap

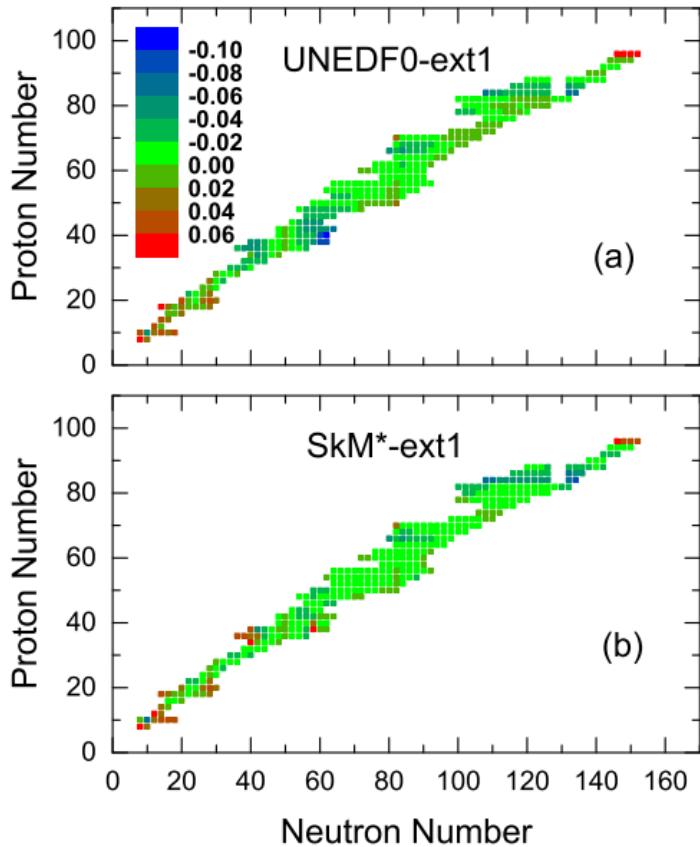
EOS



- High-order density dependent term is needed for high-density EOS, neutron stars
- Increase incompressibility and pressure at high densities
- Reduce symmetry energies at high densities
(soft symmetry energy by π^-/π^+ ratio , Z.G.Xiao et al, PRL 102, 062502 (2009).)



Charge radii



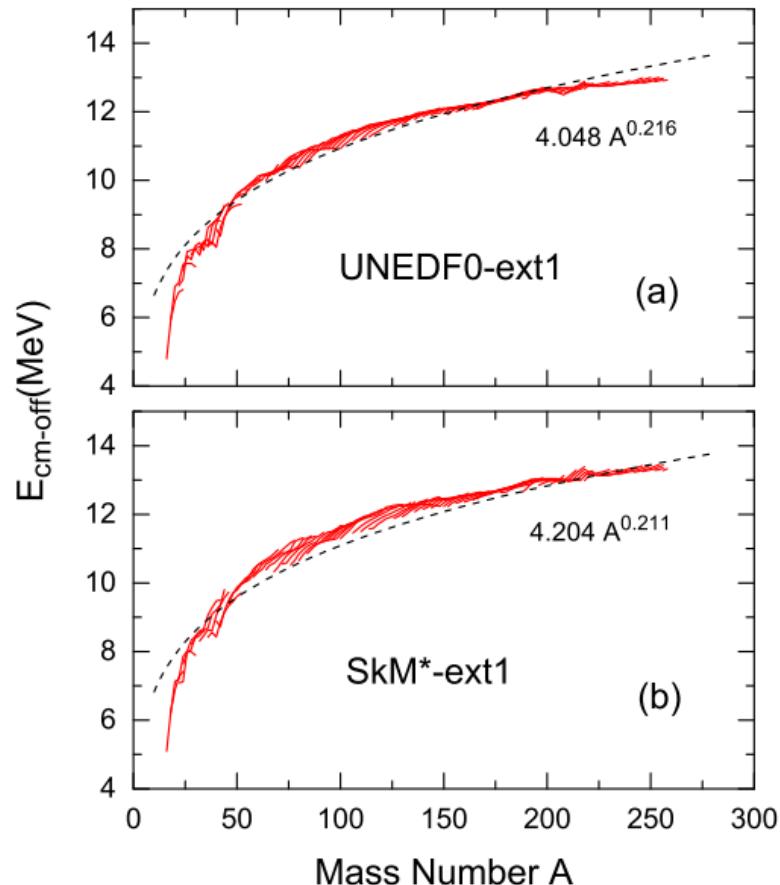
Experiments interests for Ca isotopes
Indicating shell evolutions and deformations, etc.

Charge radii of 309 even nuclei
SkM* (rms=0.023 fm) is slightly better
than UNEDF0 (rms=0.027 fm)

Charge radii in Light nuclei have large
uncertainties

CoM corrections

The two-body center of mass correction



$$E_{c.m.} = \frac{1}{2mA} \sum_{i=1}^A \mathbf{P}_i^2 + \frac{1}{2mA} \sum_{i>j} \mathbf{P}_i \cdot \mathbf{P}_j$$

Important for surface energy and fission barriers,
Bender, et al, Eur. Phys. J. A 7, 467 (2000).

Two-body CoM: $4.05A^{0.21}$

One-body CoM: $-14.58A^{0.047}$

Total CoM: $-18.33A^{-0.208}$

Two-body CoM: $4.20A^{0.21}$

One-body CoM: $-14.92A^{0.046}$

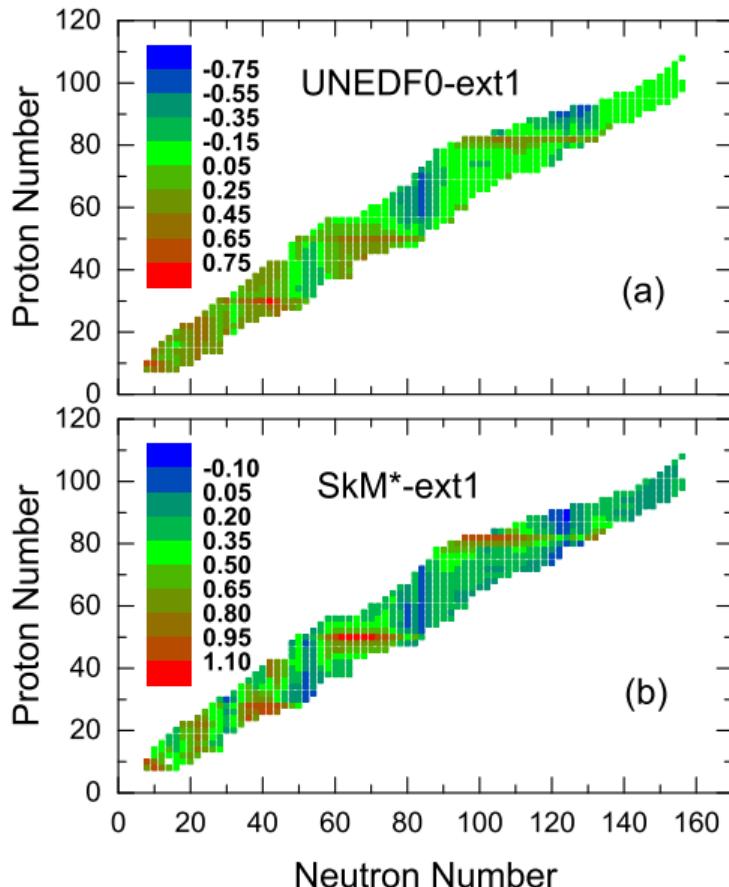
Total CoM: $-18.61A^{-0.213}$

The two-body com corrections is close to the **surface curvature** energy $A^{1/3}$
The usually missing two-body part has different mass dependence, beyond
one-body cm optimizations



Lipkin-Nogami corrections

- Approximate restoration of the particle number conservation



$$\Delta E_{LN} = E_{HF-LN} - E_{HF-BCS}$$

BCS: rms = 1.31

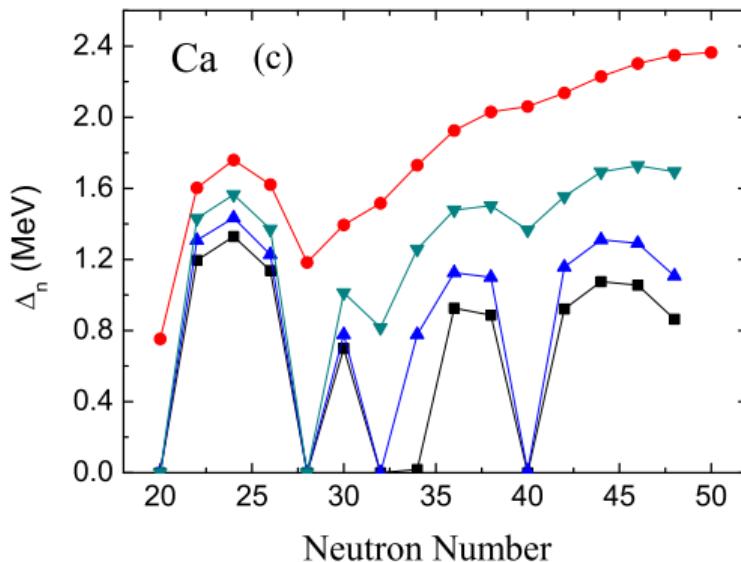
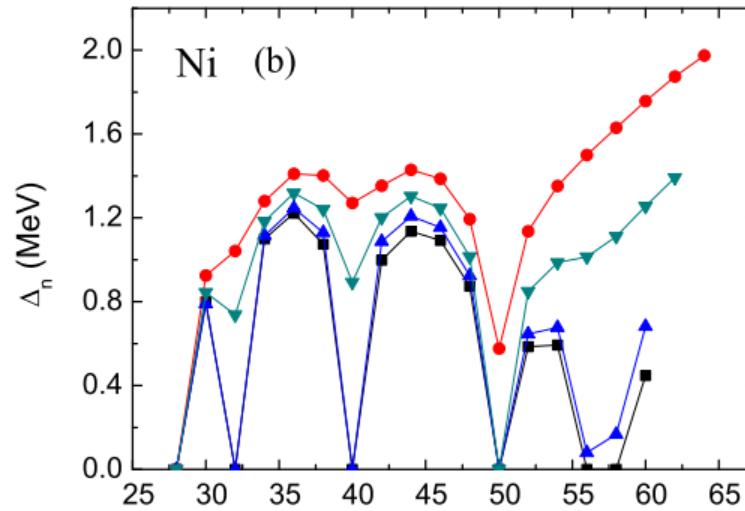
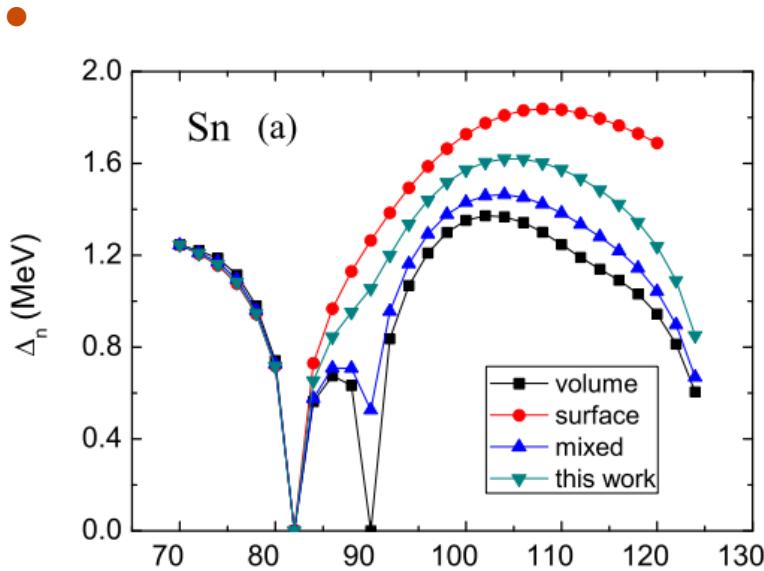
LN: rms=1.29

- LN corrections show shell effects
- Lipkin-Nogami doesn't improve the global binding energies significantly

M. Samyn, et al.,
Phys. Rev. C 70, 044309 (2004).

Angular momentum projection has not been considered presently

Pairing interactions towards dripline



$$V_0[1 - \eta(\rho(\mathbf{r})/\rho_0(\mathbf{r}))^\gamma]$$

$\eta=0.8, \gamma=0.7$

$\eta=0$: volume pairing

$\eta=0.5$: mix pairing

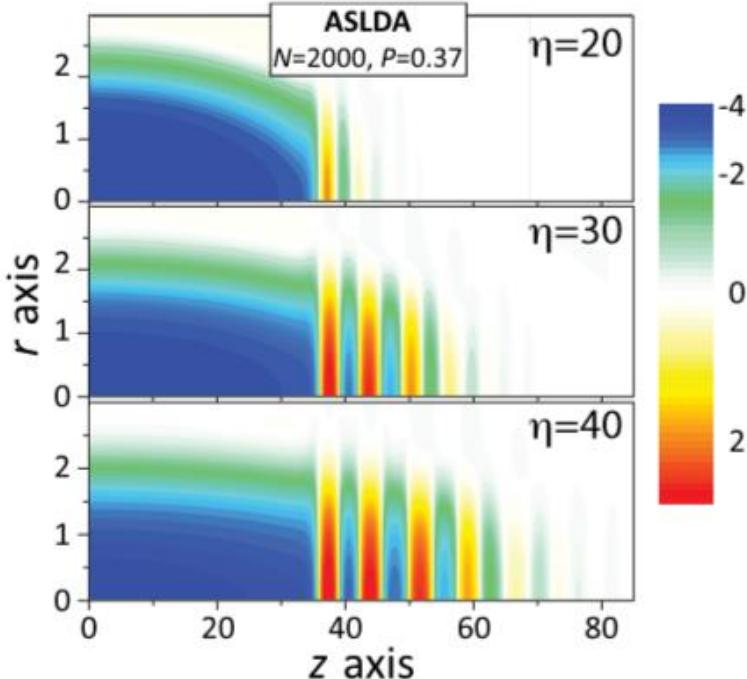
$\eta=1$: surface pairing

HFB for cold Fermi gases

- Spin-polarized systems

$$\begin{bmatrix} h_{\uparrow}(\mathbf{r}) - \lambda_{\uparrow} & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h_{\downarrow}(\mathbf{r}) + \lambda_{\downarrow} \end{bmatrix} \begin{bmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{bmatrix} = E_i \begin{bmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{bmatrix},$$

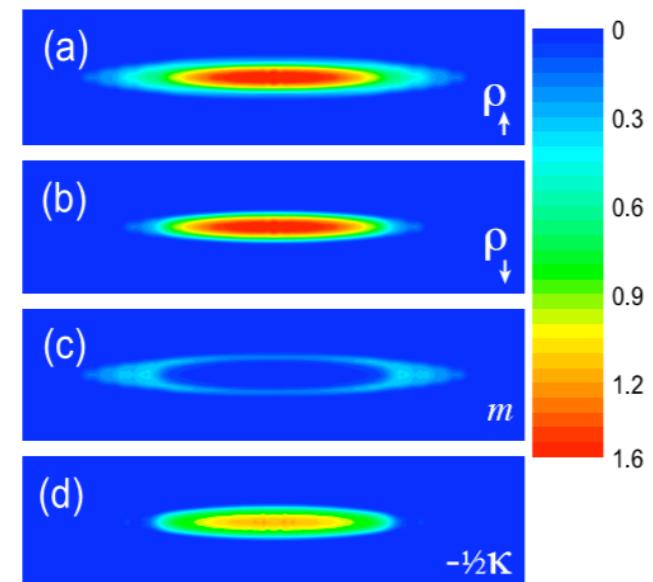
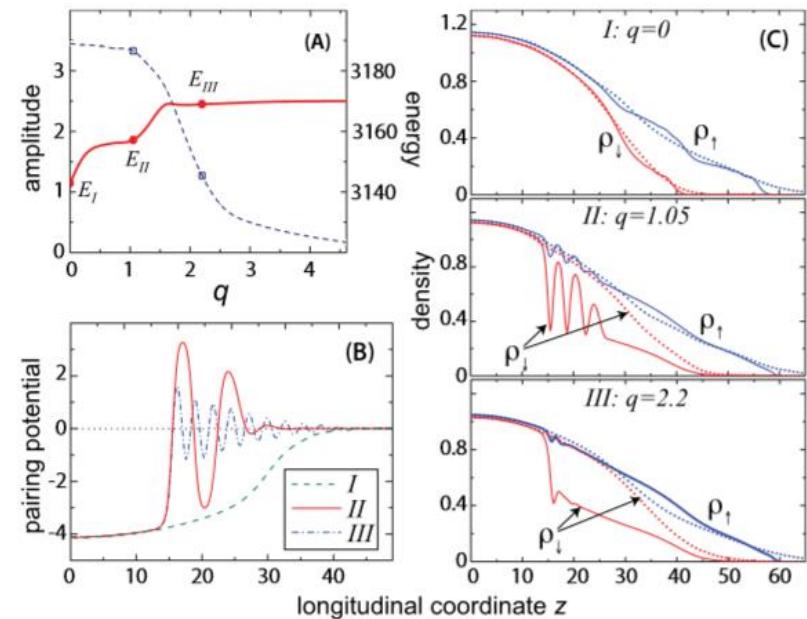
Larkin-Ovchinnikov phases



J. C. Pei, J. Dukelsky, and W. Nazarewicz, Phys. Rev. A 82, 021603(2010)(Rapid Communication).

J. C. Pei, W. Nazarewicz, M. Stoitsov, EUR. PHYS. J. A 42, 595(2009).

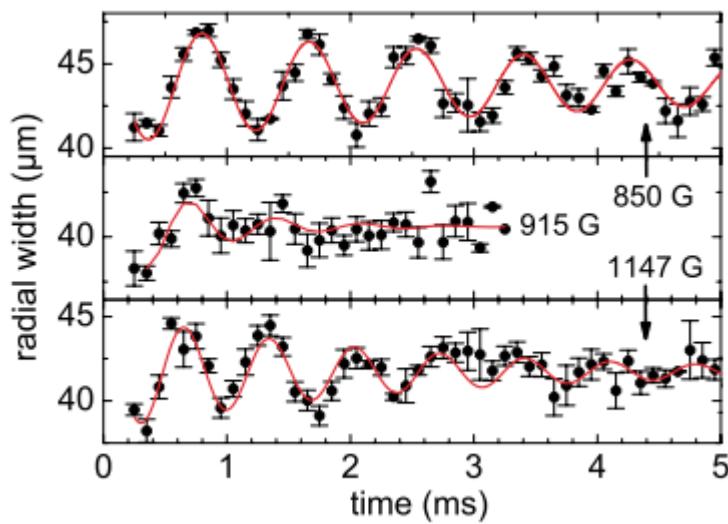
Frustrated structure



Collective modes of cold Fermi gases

- Hydrodynamical results
- Various collective modes

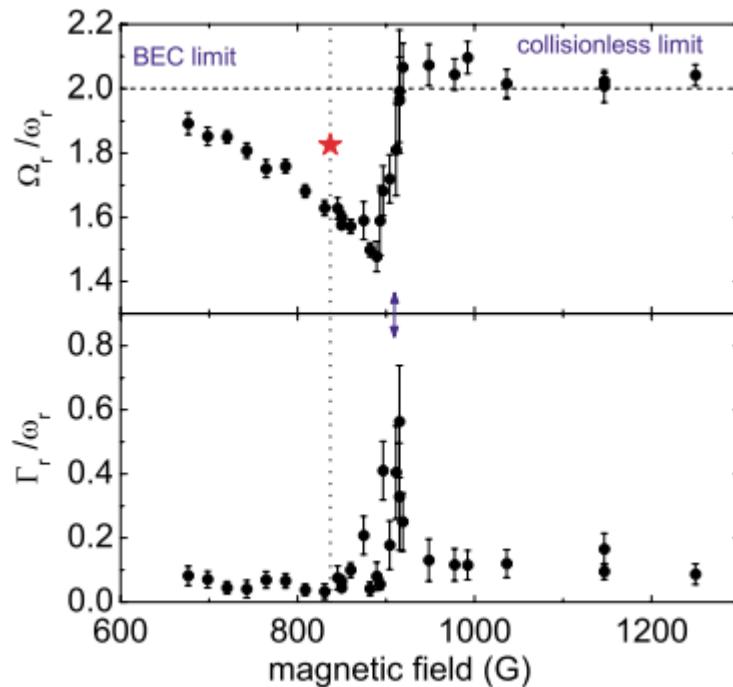
Precise measurement of collective frequencies



M. Bartenstein, PRL 92, 2004

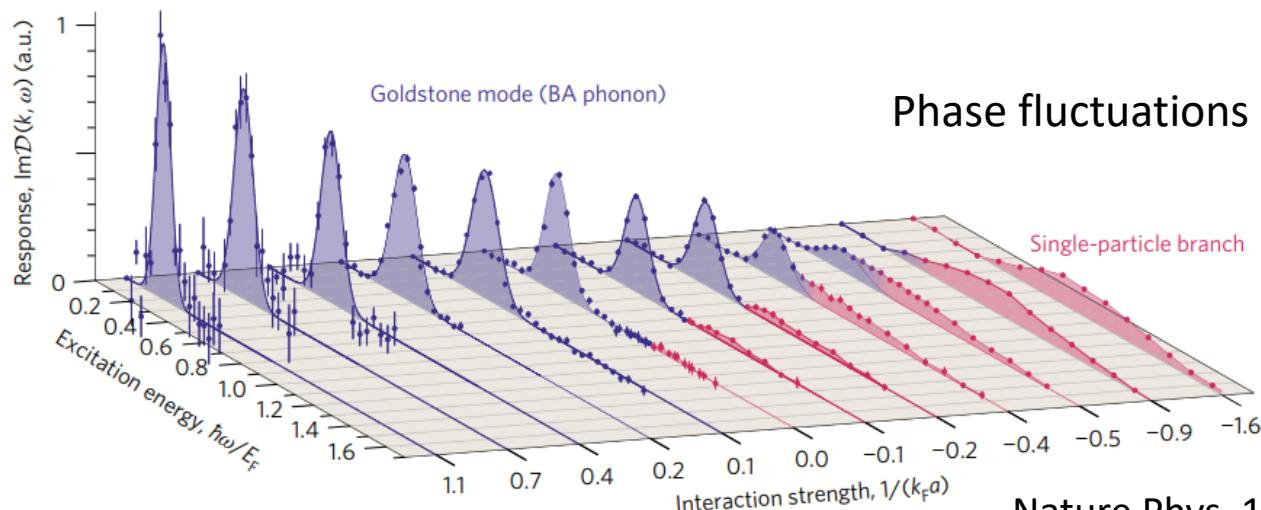
Trap type	Mode	f_1	ω	K
Spherical $\lambda = 1$	Dipole	z	ω_0	0
	Monopole	$R^2 - 2r^2$	$2\omega_0$	$\frac{256}{525\pi}$
	Quadrupole	xy	$\sqrt{2}\omega_0$	0
Axial $\lambda \ll 1$	$M = \pm 2$	$xy, x^2 - y^2$	$\sqrt{2}\omega_0$	0
	$M = \pm 1$	xz, yz	ω_0	0
	Radial	$x^2 + y^2 + \frac{2}{5}\lambda^2 z^2 - \frac{2}{5}R^2$	$\sqrt{10/3}\omega_0$	$\frac{1024}{2625\pi}$
	Axial	$R^2 - 6\lambda^2 z^2$	$\sqrt{12/5}\lambda\omega_0$	$\frac{256}{2625\pi}$

A. Bulgac and G. F. Bertsch, PRL 94, 070401 (2005)

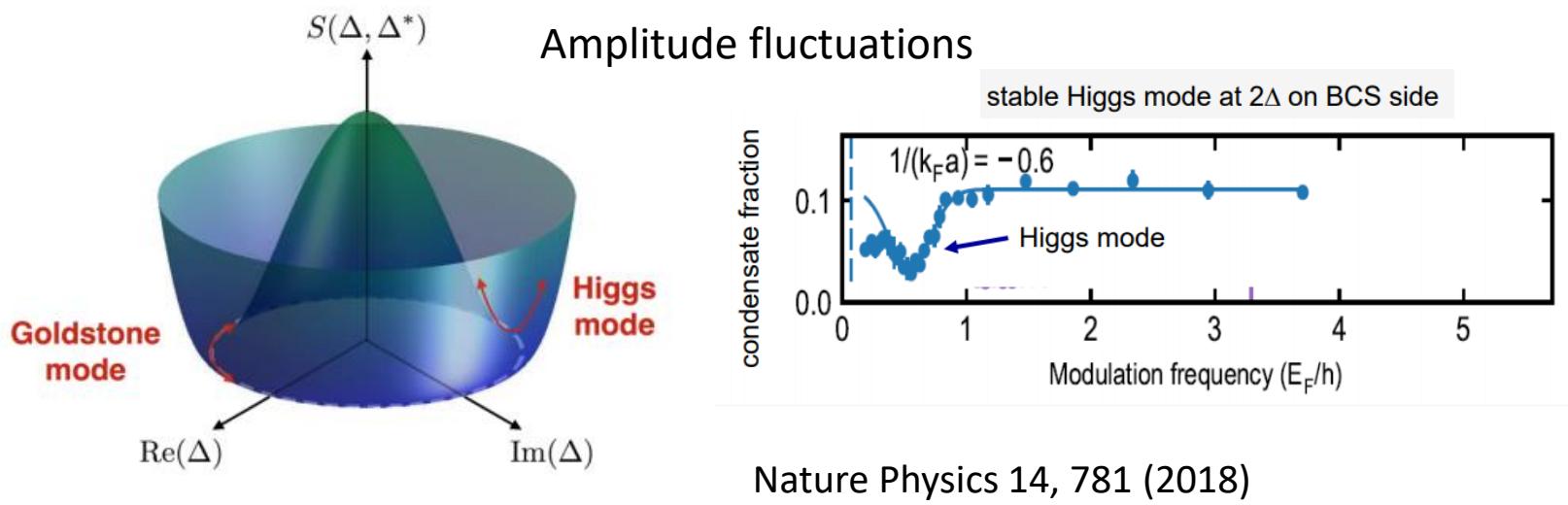




Goldstone and Higgs modes



Nature Phys. 13, 943 (2017)





SLDA-QRPA

- Energy density functional for cold Fermi gases: SLDA with time-odd terms

$$\epsilon(\vec{r}) = \alpha \frac{\tau_c(\vec{r})}{2} + \beta \frac{3(3\pi^2)^{\frac{2}{3}} \rho^{\frac{5}{3}}(\vec{r})}{10} + g_{eff} |\kappa_c(\vec{r})|^2 + V_{ext} \rho(\vec{r}) - (\alpha - 1) \frac{\vec{j}^2(\vec{r})}{2\rho(\vec{r})},$$

$$\frac{1}{g_{eff}(\mathbf{r})} = \frac{\rho^{\frac{1}{3}}(\mathbf{r})}{\gamma} + \Lambda_c(\mathbf{r}),$$

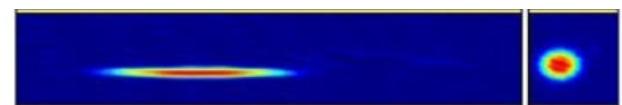
- Various induced terms

$$\begin{aligned} \delta U &= \beta \frac{(3\pi^2)^{\frac{2}{3}}}{3\rho^{\frac{1}{3}}} \delta \rho - \left(\frac{g_{eff}}{3\gamma\rho^{\frac{2}{3}}} \right)^2 (\kappa^* \Delta + \Delta^* \kappa) \delta \rho \\ &\quad + \frac{g_{eff}}{3\gamma\rho^{\frac{2}{3}}} (\Delta \delta \kappa^- + \Delta^* \delta \kappa^+) + \frac{2|\Delta|^2}{9\gamma\rho^{\frac{5}{3}}} \delta \rho, \end{aligned}$$

$$\delta \Delta^\pm = \frac{g_{eff} \Delta}{3\gamma\rho^{\frac{2}{3}}} \delta \rho - g \delta \kappa^\pm,$$

↗ **Dynamical pairing**

$$\delta h_{\text{odd}} = \frac{i}{2\rho} (\alpha - 1) (\nabla \cdot \delta \vec{j} + \delta \vec{j} \cdot \nabla),$$



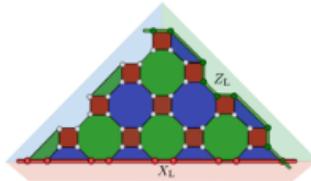
- Solve FAM-QRPA equations

Existing QRPA studies of cold Fermi gases are spherical

$$(E_\mu + E_\nu - \omega) X_{\mu\nu}(\omega) + \delta H_{\mu\nu}^{20}(\omega) = -F_{\mu\nu}^{20}(\omega),$$

$$(E_\mu + E_\nu + \omega) Y_{\mu\nu}(\omega) + \delta H_{\mu\nu}^{02}(\omega) = -F_{\mu\nu}^{02}(\omega),$$

Motivation:
Large deformations and finite-sizes effects
Connect from small to large systems



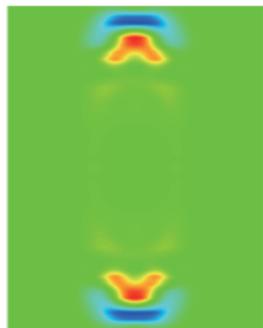
EDITORS' SUGGESTION

Fault-tolerant protection of near-term trapped-ion topological qubits under realistic noise sources

Taking into account an increasingly realistic noise model, a low-resource toolbox of fault-tolerant quantum error detection and correction scheme is provided. By extensive numerical simulations, a thorough microscopic characterization is performed, showing that a flag-qubit-based approach can significantly boost the performance of a trapped-ion processor.

A. Bermudez *et al.*

Phys. Rev. A **100**, 062307 (2019)



KALEIDOSCOPE

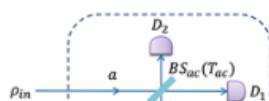
Small-amplitude collective modes of a finite-size unitary Fermi gas in deformed traps

December 3, 2019

Na Fei, J. C. Pei, K. Wang, and M. Kortelainen

Phys. Rev. A **100**, 053613 (2019)

[More Kaleidoscopes](#)



EDITORS' SUGGESTION

Entanglement improvement via a quantum scissor in a realistic environment

Current Issue

Vol. 100, Iss. 6 — December 2019

[View Current Issue](#)

Previous Issues

[Vol. 100, Iss. 5 — November 2019](#)

[Vol. 100, Iss. 4 — October 2019](#)

[Vol. 100, Iss. 3 — September 2019](#)

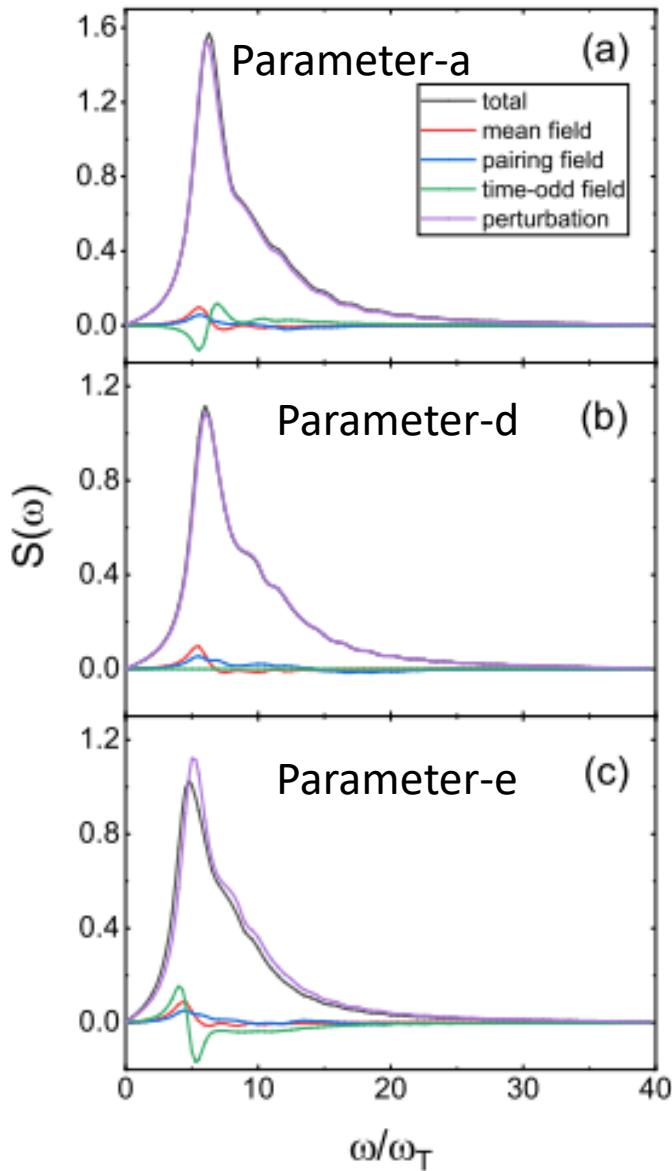
[Vol. 100, Iss. 2 — August 2019](#)

[Browse All Issues »](#)

The APS Editorial Office will be closed December 25 - January 1. During this period we will continue to receive submissions, referee reports, and other correspondence that are sent electronically. Please note that dates of receipt registered by our servers will be recorded. Articles will be published as usual December 26-31. We appreciate your patience and understanding as manuscript and correspondence processing will be somewhat delayed as a result.

Small systems in spherical trap

- 200 particles



Parameter sets	a [30]	b [10]	c [31]	d [15]	e [32]
α	1.14	1.12	1.104	1.00	0.812
β	-0.553	-0.520	-0.417	-0.430	-0.712
$1/\gamma$	-0.0906	-0.0955	-0.0347	-0.0767	-0.0705
ξ_s	0.422	0.440	0.374	0.376	0.449
η	0.504	0.486	0.651	0.500	0.442

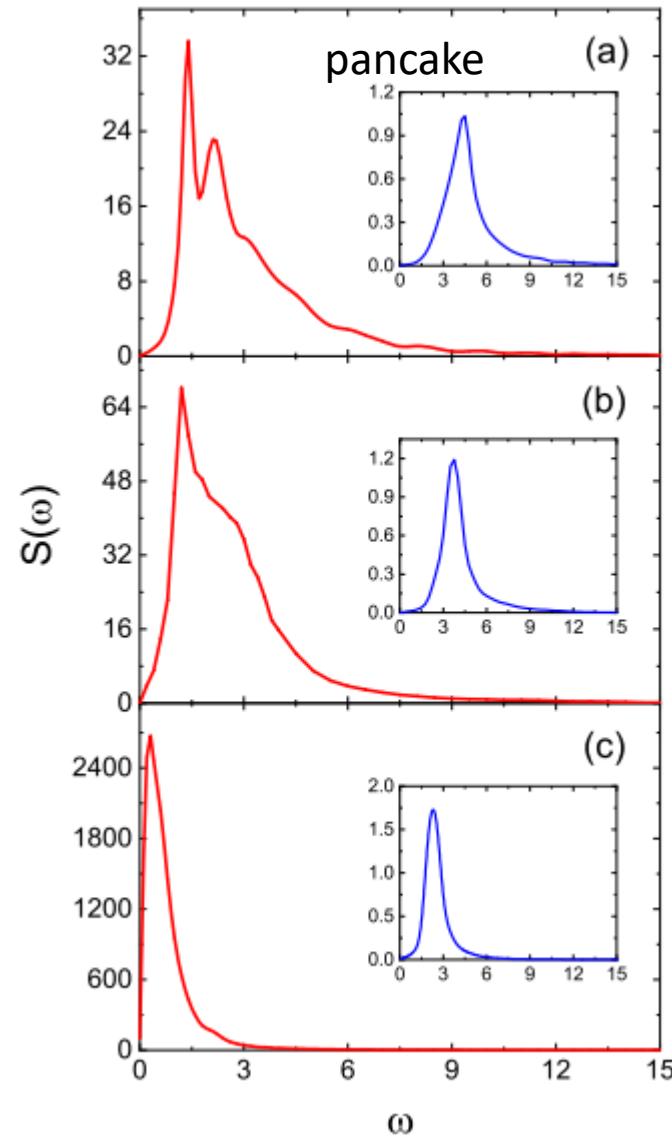
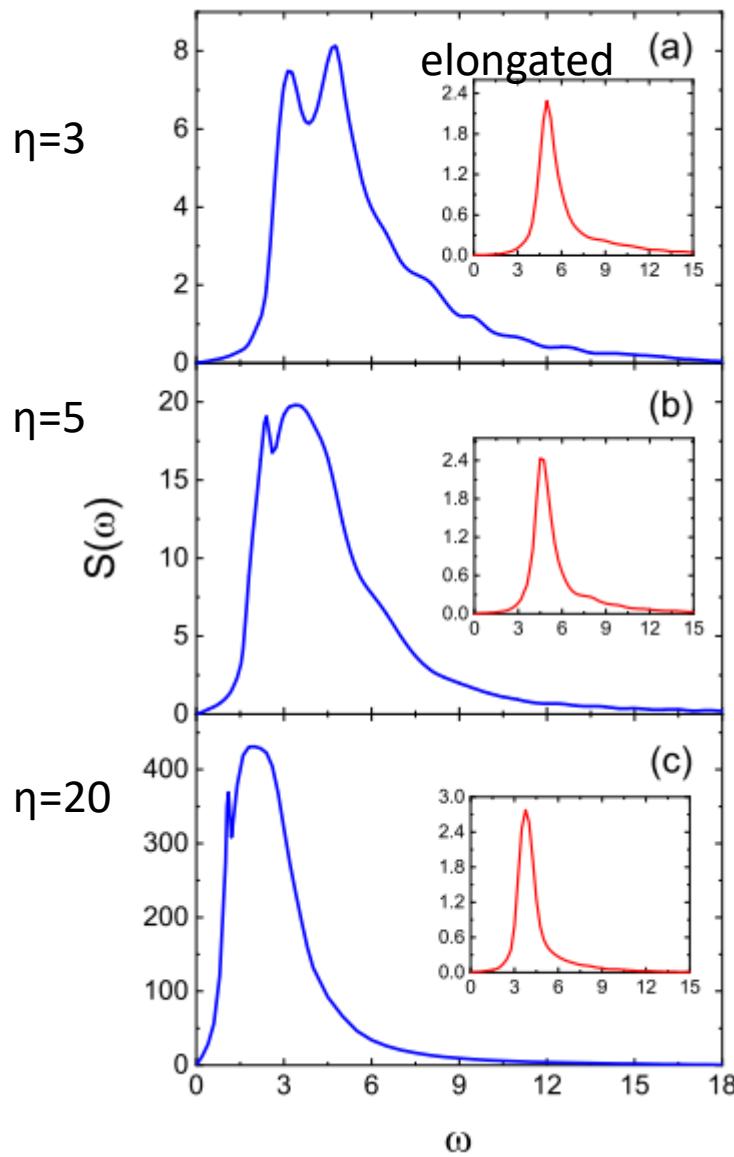
Systems	a	b	c	d	e
ω (FAM-QRPA)	6.261	6.033	6.943	5.951	4.738
m_1/m_0	6.250	6.054	7.021	6.008	4.804
E_{gs}	840.6	859.8	795.4	795.0	857.5
$\sqrt{< r^2 >}$	2.15	2.17	2.17	2.11	2.16

A good testing ground for effective interactions

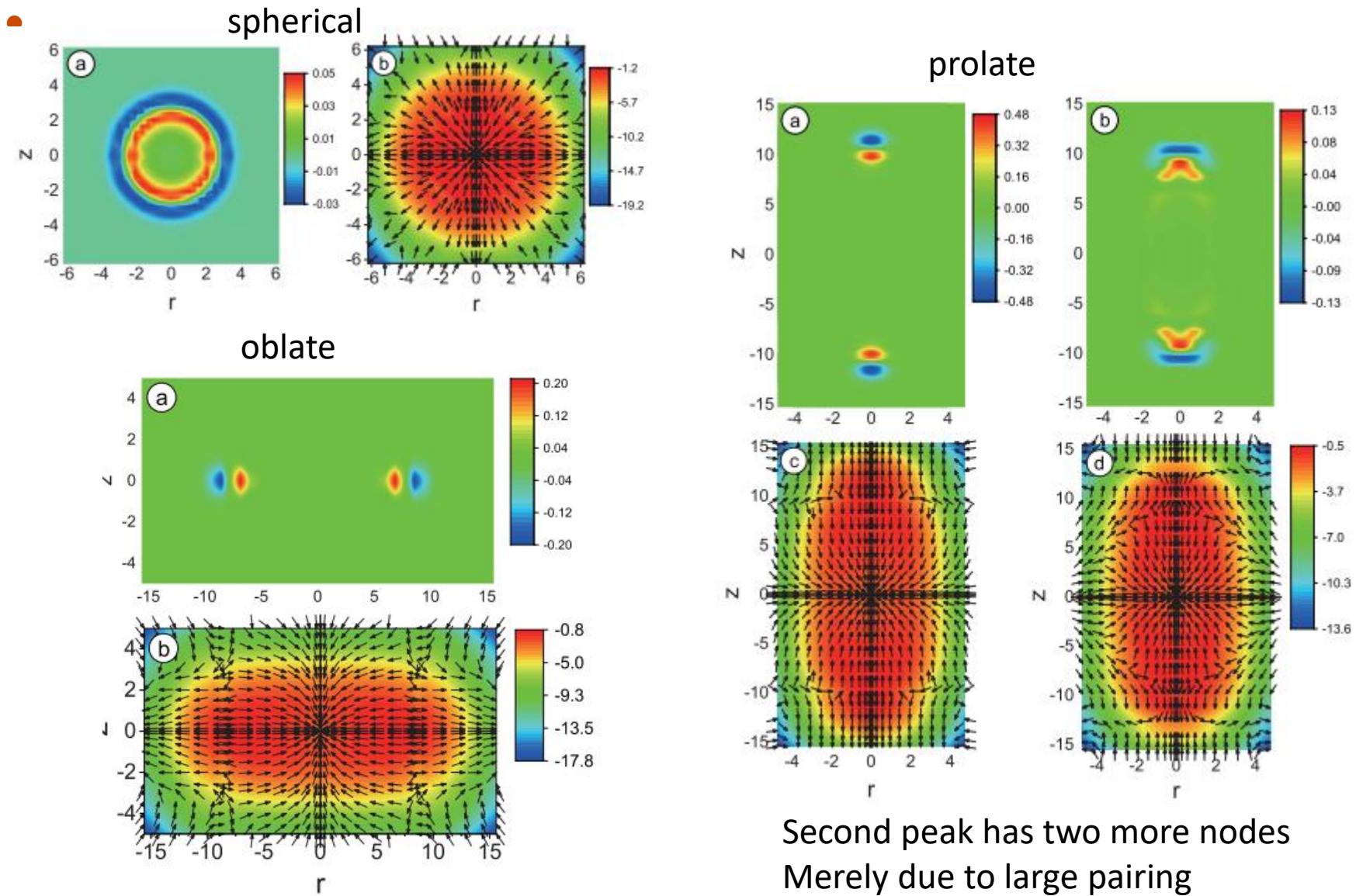
Collective modes of deformed systems

- $V_{\text{ext}}(r) = \frac{1}{2}m\omega_T^2 (r^2 + z^2/\eta^2)$

N.Fei, J.P., K.Wang, M.Kortelainen, to be submitted



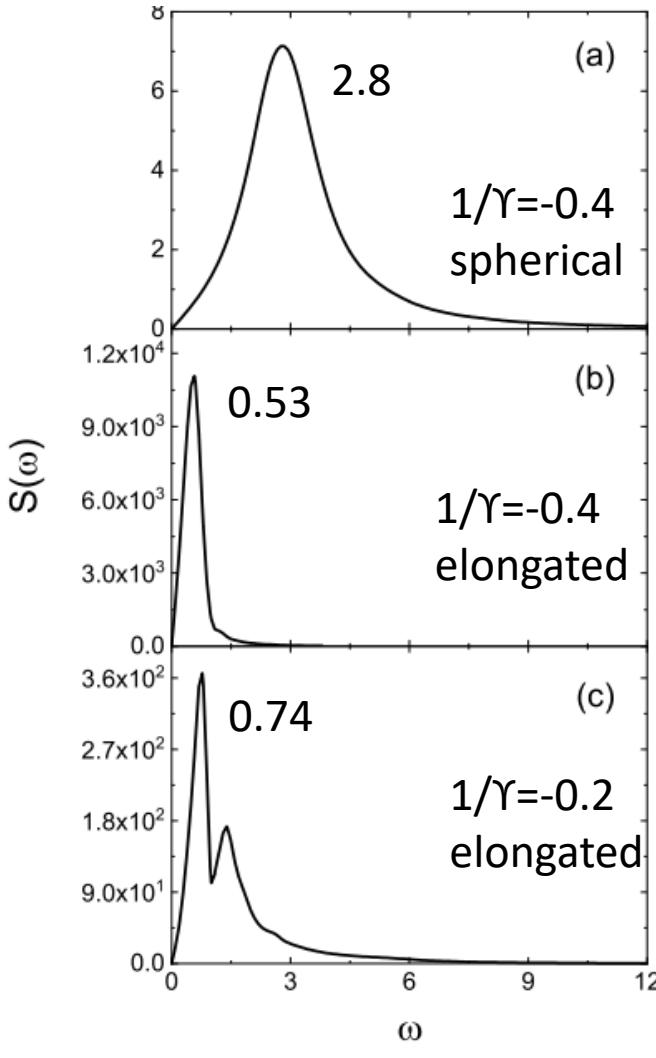
Current properties



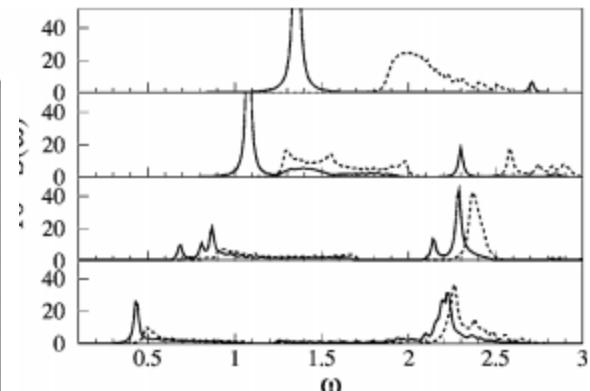
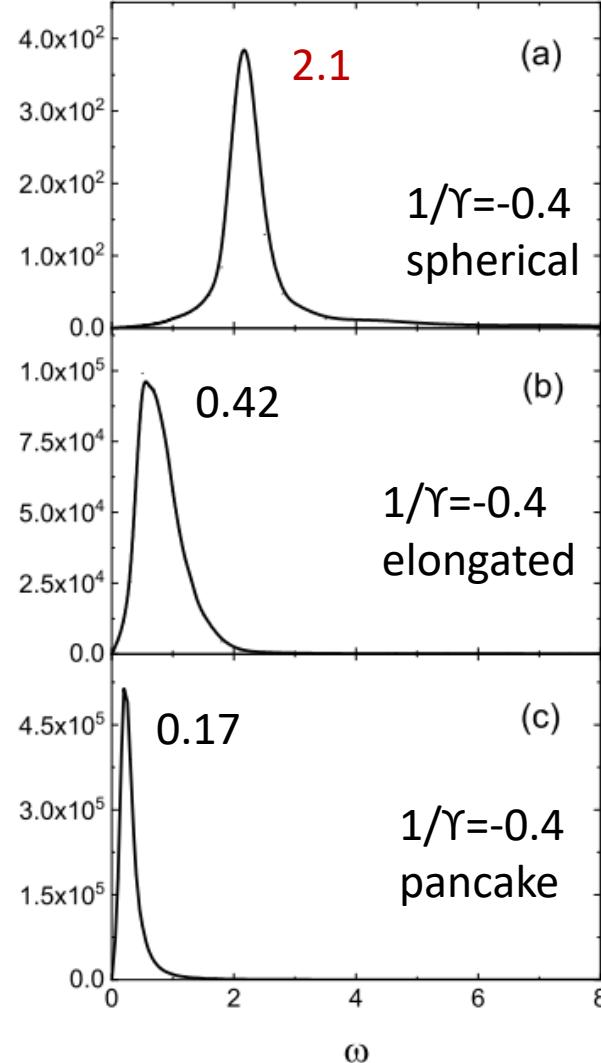
Towards large systems



200 particles



2000 particles



M. Grasso, et al.

PRA 72, 043617 (2005)

- Strong pairing shift up resonance energy
- Resonances becomes narrow in large systems
- Deformation scaling kept in oblate cases
- Reasonable finite-size in spherical systems
- Elongated traps have too high energies



Towards large deformations

- Or towards low-dimensional systems: quasi-1D and quasi-2D systems

Deformation scaling is not kept in elongated traps, pairing too large?

$$g_{1D} = \frac{2\hbar^2 a_{3D}}{ma_\rho^2} \frac{1}{(1 - Aa_{3D}/a_\rho)}.$$

$$a_{1D} = -\frac{a_\rho^2}{a_{3D}} \left(1 - A \frac{a_{3D}}{a_\rho} \right) > 0.$$

X.J. Liu, H. Hu, and P.D. Drummond,
Phys. Rev. A 76, 043605(2007)

$$a_{2D} = a_z \left(\frac{2\sqrt{\pi/b}}{e^\gamma} \right) \exp \left[-\sqrt{\frac{\pi}{2}} \frac{a_z}{a_{3D}} \right], \quad \text{A.A. Orel, Paul. Dyke, M. Delehaye, C.J. Vale and H. Hu, New J. Phys. 13, 113032(2011)}$$

$$g = 4\pi\hbar^2 a_s/m. \quad n = \frac{k_F^3}{3\pi^2} = \int \frac{d^3k}{(2\pi)^3} \left(1 - \frac{\varepsilon_k}{E_k} \right), \quad \rightarrow \text{SLDA parameters } \alpha, \beta, \gamma$$

A.Bulgac PRA 2007

Coupling strength need to be adjusted in large deformations (not a perfect DFT)



微观裂变动力学模型

- 原则上裂变过程是非常复杂的，大幅度，非绝热，非平衡，量子多体动力学过程。

- 绝热裂变理论

TDGCM基于集体坐标的位能曲面动力学演化，波函数的自由度为少数集体形变坐标，演化过程中不考虑单粒子激发, no-damping。能较好的描述裂变产物分布。

[D. Regnier, N. Dubray, N. Schunck, and M. Verriere, PRC 93, 054611\(2016\)](#)

不足：不能描述裂变碎片的激发，静态位能面不能很好的描述激发核裂变。

类似的有朗之万方程动力学over-damped演化模型，基于位能曲面的演化和蒙特卡洛行走，但考虑了耗散，能量相关的能级密度，碎片的中子发射。

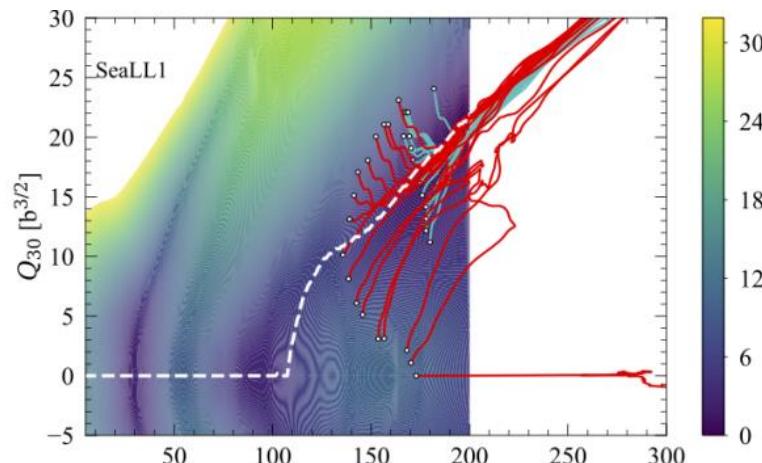
[J. Randrup and P. Möller, Phys. Rev. C 88, 064606 \(2013\)](#)

- 非绝热裂变理论

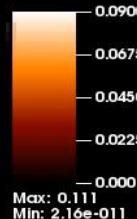
基于微观的时间相关的Hartree-Fock理论，是一个自洽的理论框架，原则上可以考虑各种自由度的时间演化。可以描述碎片的激发和裂变机制。

不足：不能给出合理的碎片分布，集体自由度的波动不足，结果是*deterministic*

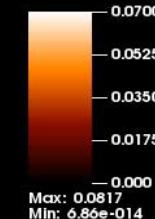
[A.Bulgac, et al. arXiv:1806.00694](#)



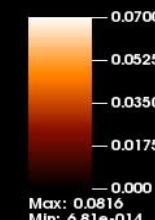
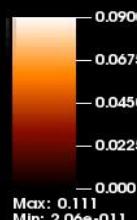
● **Fission of ^{240}Pu at excitation energy $E_x = 8.05; 7.91; 8.08 \text{ MeV}$**

Neutron density (fm^{-3})

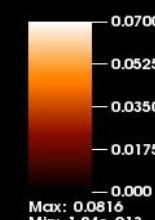
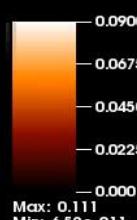
25% volume pairing, 75% surface pairing

Proton density (fm^{-3})

50% volume pairing, 50% surface pairing



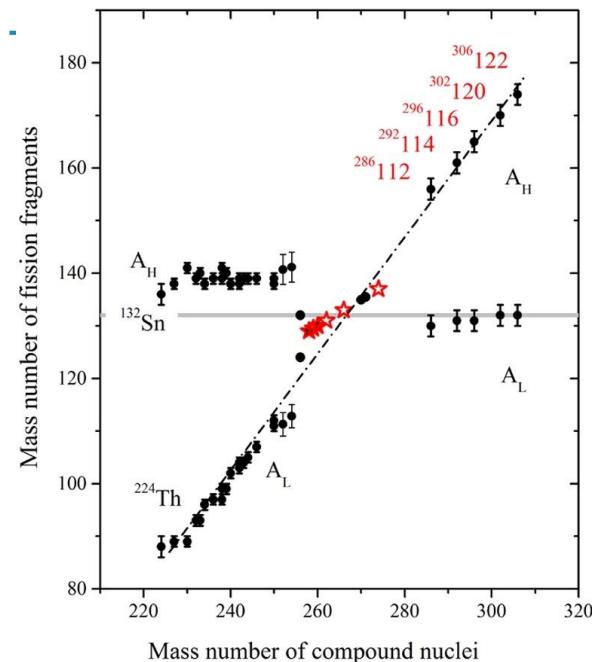
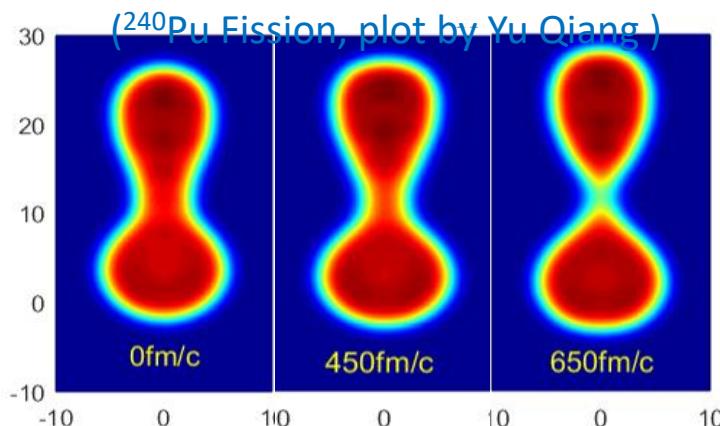
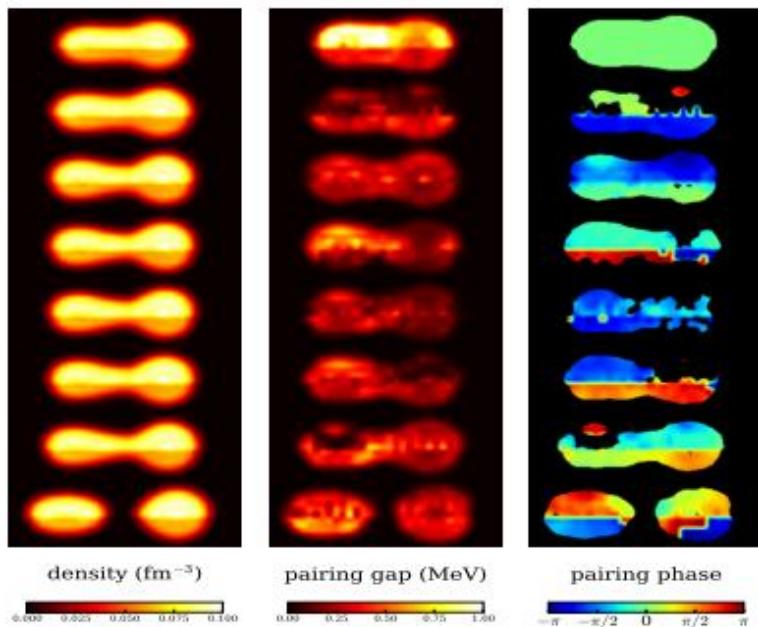
100% volume pairing



Time= 0.000000 fm/c

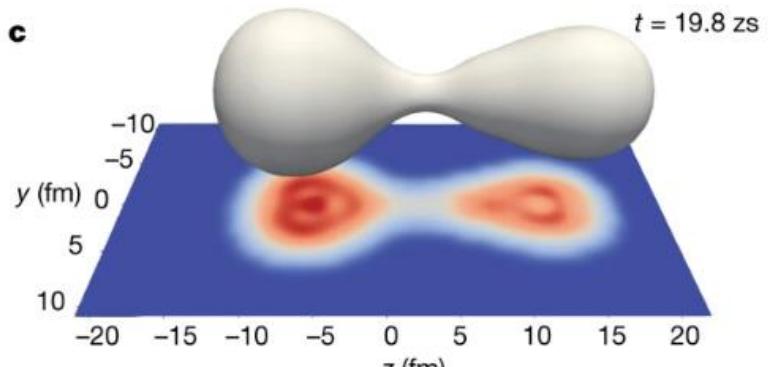
● 动力学对效应

^{240}Pu fission with the normal pairing gap



M.G.Itkis, et al. Nucl. Phys. A 944, 204(2015)

pear-shaped fragments are favorable
for mass-asymmetric fission

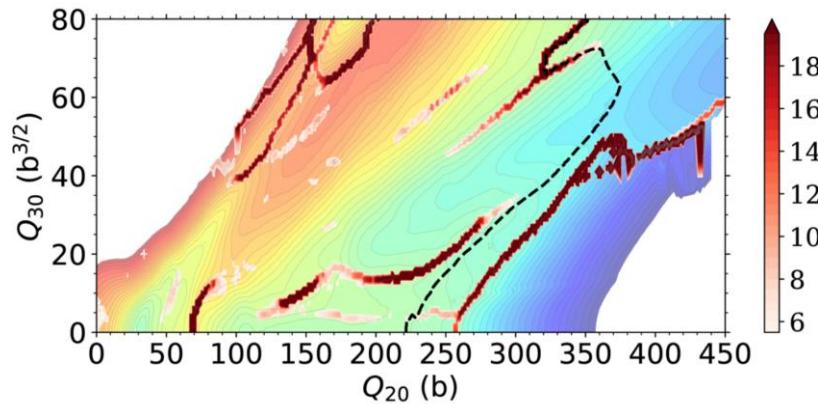


G. Scamps, C. Simenel, Nature 564, 382(2018)



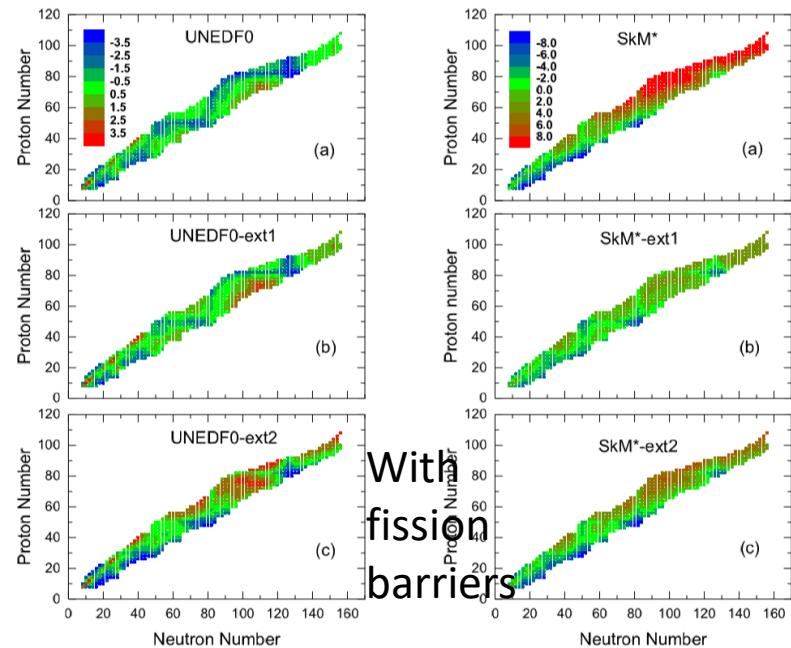
微观动力学模型的挑战

- TDGCM如何加入非绝热效应：不同的组态混合，加入QRPA计算
大形变位能面不连续
更合理的质量参数
裂变位垒用形状参数约束



裂变位能面与核结构描述可能不自洽：
改进优化有效核力

Z.W.Zuo, J.C. Pei, X.Y.Xiong, Y.Zhu,
Chin.Phys.C Vol 42, 064106(2008)



- 改进可能：密度的演化而不是态的演化，
加入非厄密演化，与TDHF结合



其他问题

- 能级密度的参数不确定性，需要微观计算
- 微观计算朗之万方程的输运系数
- 是否可能采用有限温度裂变位垒，巨正则系综。更合理的是微正则系综（能级密度方法）在低能区有区别
- 裂变后产物的角动量分布，能量分布
- 有dissipation效应时的隧穿过程
- 横向自由的对裂变的影响
- TDHF加入两体碰撞项
- 核数据神经网络进一步提供高精度核数据

总之，目前微观模型对裂变描述还有很大误差。

只能发展有效的裂变模型，不同模型有不同自由度和不同使用范围。



Summary

- Development of 2D and 3D Coordinate space HFB for exotic nuclei: deformed halo and continuum properties, odd-A halo, FAM-QRPA for halo excitations. The unique and most advanced HFB approaches
 - Development of high precision Skyrme forces with higher order terms for exotic nuclei
 - Development of microscopic fission theory with Finite-temperature HFB for survival probability of superheavy nuclei; microscopic study fission dynamics with TDHF theory
 - Development of coordinate space HFB for cold atoms, ground states, spin- polarized systems and collective states with our unique HFB solver
 - Development of Bayesian Neural Network evaluation for incomplete fission yields , to provide application-level nuclear data
-
- Outlook:
 1. Collaborate with experiments on drip line nuclei: decays and more precise observables
 2. Development of novel DFT beyond Skyrme from ab initio
 3. Looking for more progress in microscopic fission theory



- Collaborations:

F.R. Xu, W. Nazarewicz, M. Kortelainen,
Y.Zhu, X.Y.Xiong, Z.W.Zuo, C.Q.He, Y.Qiang, N.Fei, Z.Wang, K. Wang.....
And many others for discussions

Thank you for your attention !