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壳附近原子核 α 衰变预形成因子系统行为研究

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@绵阳 2020.1.13



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outline

- The theoretical models of α decay
- The α preformation probability (P_α)
- The two-potential-approach for P_α
- The cluster-formation model for P_α
- Our works
- Summary



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1. The theoretical models of alpha decay

- α decay has long been perceived as one of the most powerful tools to investigate unstable nuclei, neutron-deficient nuclei, and superheavy nuclei.
- Within Gamow's theory, the α decay process is described as a preformed α particle penetrating the Coulomb barrier.

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma} = \frac{\ln 2}{\lambda} \quad \lambda = P_{\alpha} \nu P$$

P_{α} denotes α preformation factors,

ν is the assault frequency of α particle,

P denote the semiclassical Wentzel-Kramers-Brillouin (WKB) barrierpenetrate probability.



1. The theoretical models of alpha decay

The theoretical models of alpha decay:

I. The cluster model (CM), density-dependent cluster model (DDCM)

任中洲老师课题组:

C. Xu and Z. Ren, Phys. Rev. C 73, 041301 (2006)

C. Xu and Z. Ren, Nucl. Phys. A 760, 303 (2005).....

II. The unified fission model (UFM)

左维老师课题组:

J. Dong, W. Zuo, J. Gu, Y. Wang, and B. Peng, Phys. Rev. C 81, 064309 (2010)

III. Generalized Liquid Drop Model (GLDM)

张鸿飞老师课题组:

H. Zhang, W. Zuo, J. Li, and G. Royer, Phys. Rev. C 74, 017304 (2006)

H. F. Zhang and G. Royer, Phys. Rev. C 77, 054318 (2008)



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1. The theoretical models of alpha decay

IV. The relativistic mean field model (RMF)

龙文辉老师课题组:

W. Long, J. Meng, and S.-G. Zhou, Phys. Rev. C 65, 047306 (2002)

VI. The Coulomb and proximity potential model (CPPM)

张高龙老师课题组:

Y. J. Yao, G. L. Zhang, W. W. Qu and J. Q. Qian, Eur. Phys. J. A 51, 122 (2015)

C. L. Guo, G. L. Zhang and X. Y. Le, Nucl. Phys. A 897, 54 (2013)

VI. Empirical Formulas (EF)

许甫荣老师课题组: C. Qi, F. R. Xu, R. J. Liotta, and R. Wyss, Phys. Rev. Lett. 103, 072501 (09).....

郭建友老师课题组: Z. Y. Wang, Z. M. Niu, Q. Liu and J. Y. Guo, J. Phys. G: Nucl. Part. Phys. 42, 055112 (2015)

.....



2. The α preformation probability (P_α)

Microscopic methods for obtaining P_α

- a) Microscopically, α preformation probabilities can be evaluated through an overlap between the initial state and the α -decaying state. [Phys. Rep. 294 (1998) 265]
- b) The approach of the Tohsaki-Horiuchi-Schuck-Ropke wave function was used to calculate the α preformation probability. [Phys. Rev. C 90 (2014) 034304, Phys. Rev. C 93, (2016) 011306(R)]
- c) The shell model to obtain the α preformation probability.[Phys. Rev. Lett. 69 (1992) 37, Nucl. Phys. A 550(1992)421]
- d) The cluster-formation model to obtain the α preformation probability.[Rom. Rep. Phys. 65 (2013) 1281, J. Phys. G 40 (2013) 065105, Phys. Rev. C 93 (2016) 044326]



2. The α preformation probability (P_α)

Experimental methods for obtaining P_α

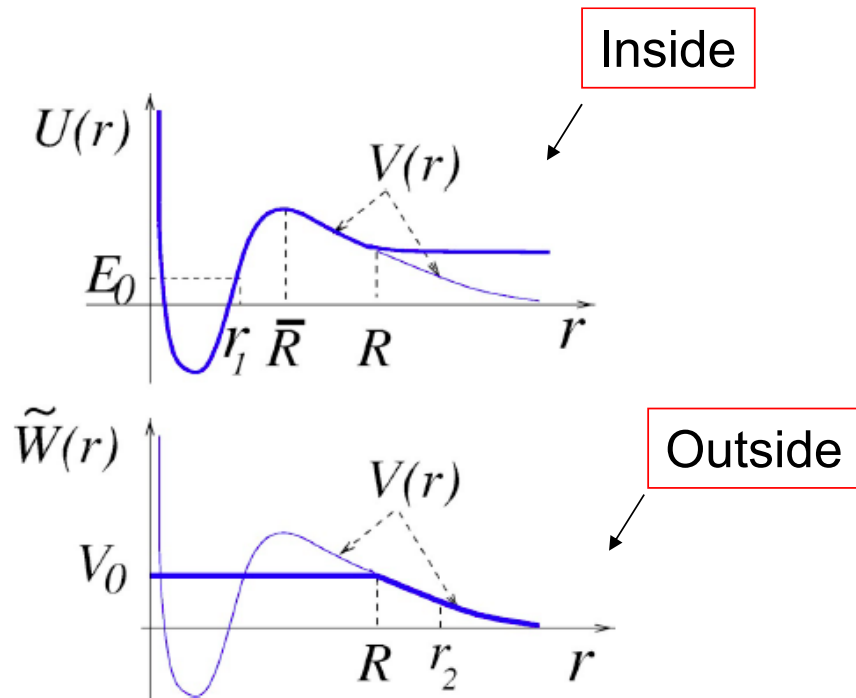
$$P_\alpha = \frac{T_{\text{cal}}}{T_{\text{exp}}}$$

T_{cal} Where is the calculated values without taking P_α



3. The two-potential-approach for P_α

Two-Potential Approach(TPA)



$$U(r) = \begin{cases} V(r) & \text{for } r \leq R, \\ V(R) & \text{for } r > R \end{cases}$$

$$\tilde{W}(r) = \begin{cases} V_0 & \text{for } r \leq R, \\ V(r) & \text{for } r > R \end{cases}$$

Phys. Rev. Lett. 59(1987)262

Phys. Rev. A 69(2004)042705



3. The two-potential-approach for P_α

α -decay half-lives

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma} = \frac{\ln 2}{\lambda}$$

α -decay width

$$\Gamma = \frac{P_\alpha FP}{4\mu}$$

The normalized factor

$$F \int_{r_1}^{r_2} \frac{dr}{2k(r)} = 1$$

The wave number

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2} |Q_\alpha - V(r)|}$$

The penetration probability

$$P = \exp[-2 \int_{r_2}^{r_3} k(r) dr]$$

Classical turning points:

$$V(r_1) = V(r_2) = V(r_3) = Q_\alpha$$



The α -daughter nuclei potential

$$V(r) = V_N(r) + V_C(r) + V_l(r)$$

I. Nuclear potential

$$V_N(r) = -V_0 \frac{1 + \cosh(R/a)}{\cosh(r/a) + \cosh(R/a)}$$

II. Coulomb potential

$$V_C(r) = \begin{cases} \frac{Z_1 Z_2 e^2}{2R} \left[3 - \left(\frac{r}{R} \right)^2 \right] & r \leq R \\ \frac{Z_1 Z_2 e^2}{r} & r > R \end{cases}$$



III. Centrifugal potential

$$V_l(r) = \frac{l(l+1)\hbar^2}{2\mu r^2}$$

The sharp radius

$$R = 1.28 A^{1/3} - 0.76 + 0.8 A^{-1/3}$$



Alpha-cluster preformation factors in alpha decay for even–even heavy nuclei using the cluster-formation model

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J. Phys. G: Nucl. Part. Phys. **40** (2013) 065105

S M Saleh Ahmed *et al*

penetrability was used. Such calculations reflect information about the clustered alpha but not about the mechanism of alpha clustering.

In a previous work [43], we have proposed a new simple model, called the cluster-formation model (CFM), to determine the preformation factor and the clustering amount (CA) of an alpha cluster for ^{212}Po . The determined alpha clustering value was in agreement with that of Varga *et al* (1992) [1, 5], and the preformation factor was in agreement with the value that Ni and Ren [44] extracted by fitting the experimental alpha-decay width to their microscopic calculations for a wide range of nuclei. This agreement motivated us to use the CFM to



4. The cluster-formation model for P_α

- **Ahmed** et al. presented a **new quantum mechanical theory** named cluster-formation model (**CFM**) to calculate the α preformation factors P_α of **even-even nuclei**, which suggests that **the initial state of the parent nucleus should be a linear combination of different possible clusterization states**.
- Very recently, **Ahmed et al.** and **Deng et al.** extended CFM to **odd-A and doubly odd nuclei** through **modifying the formation energy of the interior α cluster** for various types of nuclei (i.e., **even-Z–odd-N, odd-Z–even-N, and doubly odd nuclei**) and **considering the effects of unpaired nucleon**.

References:

- [1] S. M. S. Ahmed, R. Yahaya, and S. Radiman, Rom. Rep. Phys. 65, 1281 (2013).
- [2] S. M. S. Ahmed, R. Yahaya, S. Radiman, and M. S. Yasir, J. Phys. G 40, 065105 (2013).
- [3] S. M. S. Ahmed, Nucl. Phys. A 962, 103 (2017).
- [4] D. Deng, Z. Ren, D. Ni, and Y. Qian, J. Phys. G 42, 075106 (2015).
- [5] D. Deng and Z. Ren, Phys. Rev. C 93, 044326 (2016).



4. The cluster-formation model for P_α

The theoretical framework of the CFM :

the total clusterization state of parent nuclei: $\Psi = \sum_{i=1}^n a_i \Psi_i$

the superposition coefficient of Ψ_i : $a_i = \int \Psi_i^* \Psi d\tau$

orthogonality condition: $\sum_{i=1}^n |a_i|^2 = 1$

total Hamiltonian: $H = \sum_{i=1}^n H_i$

the total energy: $E = \sum_{i=1}^n |a_i|^2 E_i = \sum_{i=1}^n |a_i|^2 E_{f_i}$

E_{f_i} denotes the formation energy of cluster in the i th clusterization state Ψ_i :

the α preformation factor: $P_\alpha = |a_\alpha|^2 = \frac{E_{f_\alpha}}{E}$

E_{f_α} is the formation energy of the α cluster. E is composed of the E_{f_α} and the interaction energy between α cluster and daughter nuclei.



4. The cluster-formation model for P_α

For the even-even nuclei

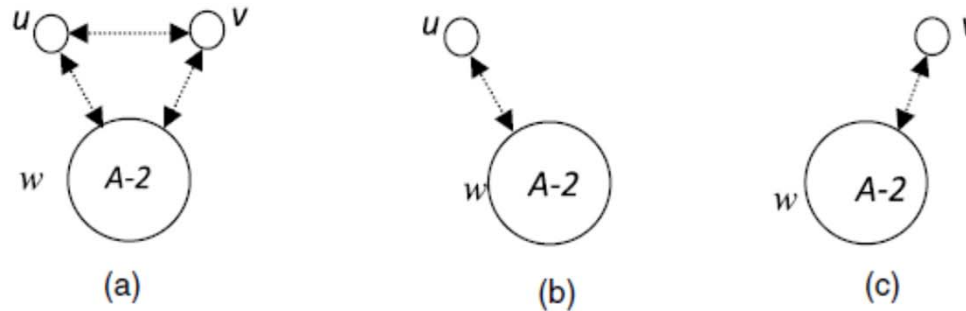


Figure 3. The interactions (a) among the nucleus of $A-2$ and the two nucleons u and v , (b) between the nucleus $A-2$ and the nucleon u , and (c) between the nucleus $A-2$ and the nucleon v .

$$E_{u-v} + E_{u-w} + E_{v-w} = B(A) - B(A-2)$$

For $v=p$, $u=n$

$$E_{p-n} = B(A, Z) - B(A-2, Z-1) - B(A-1, Z)$$

$$E_{p-p} = B(A, Z) + B(A-2, Z-2) - 2B(A-1, Z-1)$$

$$E_{n-n} = B(A, Z) + B(A-2, Z) - 2B(A-1, Z)$$



4. The cluster-formation model for P_α

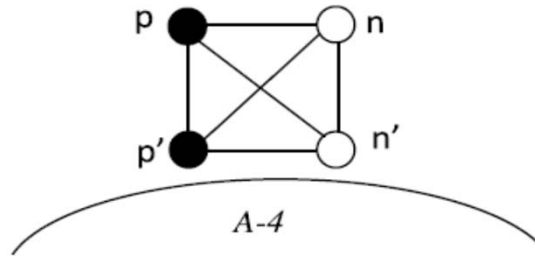


Figure 6. The correlations among the last four nucleons.

$$\begin{aligned}
 B(A, Z) &= B(A - 4, Z - 2) + S_\alpha + E_{p-p} + E_{n-n} + E_{2p-2n} \\
 &= B(A - 4, Z - 2) + E_{p-p} + E_{n-n} + E_{2p-2n} \\
 &\quad + S_{2p}(A - 2, Z) + S_{2n}(A - 2, Z - 2)
 \end{aligned}$$



4. The cluster-formation model for P_{α}

The formation energy $E_{f\alpha}$

$$E_{f\alpha} = E_{2p-2n} + E_{p-p} + E_{n-n} = E_{\alpha d} - S_{\alpha}$$

With $E_{\alpha d}$ and S_{α} being the alpha-decay energy and the separation energy of alpha particle

$$E_{\alpha d} = B(A, Z) - B(A - 4, Z - 2)$$

$$S_{\alpha} = B(A - 4, Z - 2) + B(\alpha) - B(A, Z)$$



4. The cluster-formation model for P_α

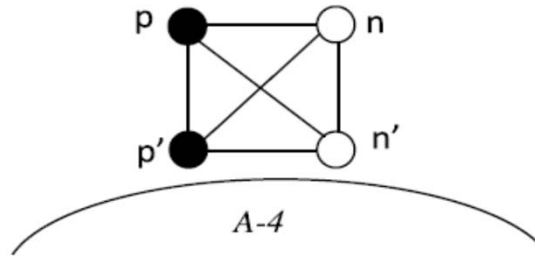


Figure 6. The correlations among the last four nucleons.

$$\begin{aligned} S_{2p}(A-2, Z) &= S_p(A-3, Z-1) + S_{p'}(A-3, Z-1) \\ &= B(A-2, Z) - B(A-4, Z-2) - E_{p-p'} \end{aligned}$$

$$S_{2n}(A-2, Z) = B(A-2, Z) - B(A-4, Z-2) - E_{n-n'}$$

$$S_p(A-3, Z-1) = S_p(A-2, Z) - E_{p-p'}$$

$$S_n(A-3, Z-2) = S_p(A-2, Z-2) - E_{n-n'}$$



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4. The cluster-formation model for P_α

$$E_{2p-2n} = B(A, Z) + B(A-4, Z-2) - B(A-2, Z-2) - B(A-2, Z)$$

$$E_{f\alpha} = 3B(A, Z) + B(A-4, Z-2) - 2B(A-1, Z-1) - 2B(A-1, Z)$$

$$E = B(A, Z) - B(A-4, Z-2)$$



4. The cluster-formation model for P_{α}

- **Case I for even-even nuclei:**

$$E_{f_{\alpha}} = 3B(A, Z) + B(A - 4, Z - 2) - 2B(A - 1, Z - 1) - 2B(A - 1, Z)$$

$$E = B(A, Z) - B(A - 4, Z - 2)$$

- **Case II for even Z -odd N , i.e., even-odd nuclei:**

$$E_{f_{\alpha}} = 3B(A - 1, Z) + B(A - 5, Z - 2) - 2B(A - 2, Z - 1) - 2B(A - 2, Z)$$

$$E = B(A, Z) - B(A - 5, Z - 2)$$

- **Case III for odd Z -even N , i.e., odd-even nuclei:**

$$E_{f_{\alpha}} = 3B(A - 1, Z - 1) + B(A - 5, Z - 3) - 2B(A - 2, Z - 2) - 2B(A - 2, Z - 1)$$

$$E = B(A, Z) - B(A - 5, Z - 3)$$

- **Case IV for doubly odd nuclei:**

$$E_{f_{\alpha}} = 3B(A - 2, Z - 1) + B(A - 6, Z - 3) - 2B(A - 3, Z - 2) - 2B(A - 3, Z - 1)$$

$$E = B(A, Z) - B(A - 6, Z - 3)$$



5. Our works

PHYSICAL REVIEW C **84**, 064608 (2011)

Isospin asymmetry dependence of the α spectroscopic factor for heavy nuclei

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(Received 13 September 2011; revised manuscript received 18 October 2011; published 15 December 2011)

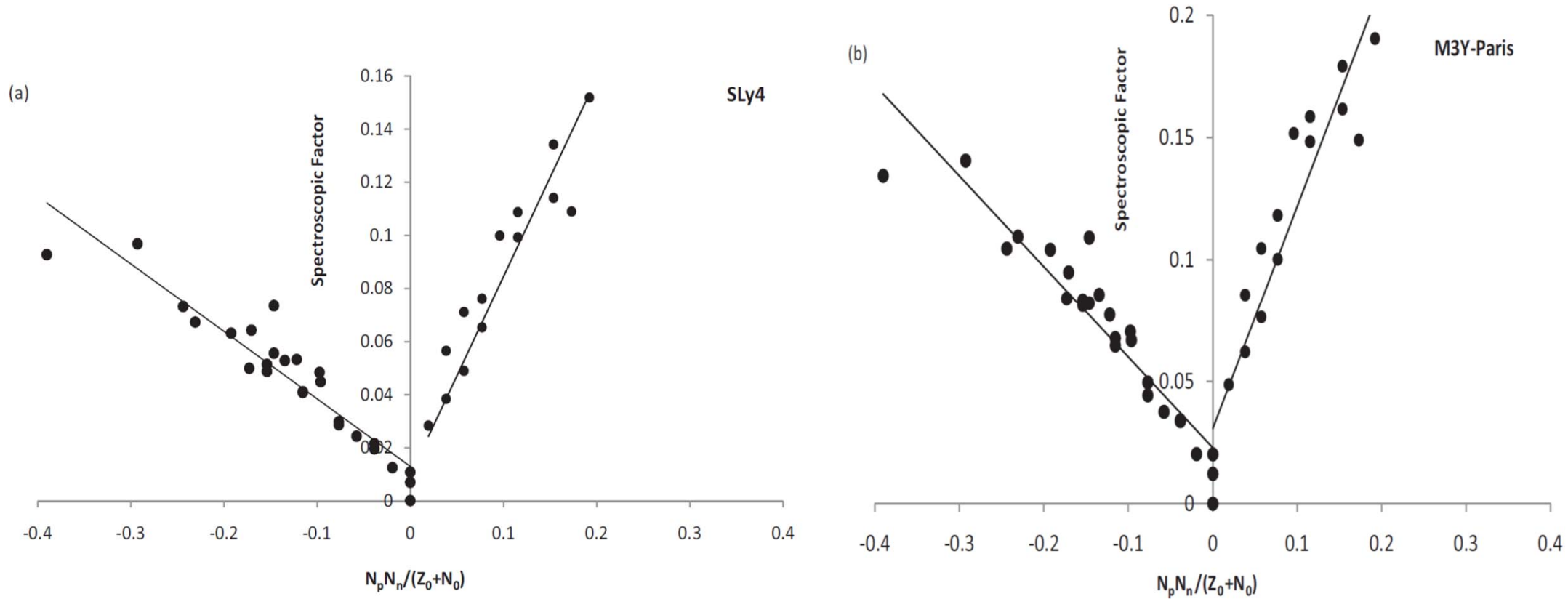
Both the valence nucleons (holes) and the isospin asymmetry dependencies of the preformation probability of an α -cluster inside parents radioactive nuclei are investigated. The calculations are employed in the framework of the density-dependent cluster model of an α -decay process for the even-even spherical parents nuclei with protons number around the closed shell $Z_0 = 82$ and neutrons number around the closed shells $Z_0 = 82$ and $Z_0 = 126$. The microscopic α -daughter nuclear interaction potential is calculated in the framework of the Hamiltonian energy density approach based on the SLy4 Skyrme-like effective interaction. Also, the calculations based on the realistic effective M3Y-Paris nucleon-nucleon force have been used to confirm the results. The calculations then proceed to find the assault frequency and the α penetration probability within the WKB approximation. The half-lives of the different mentioned α decays are then determined and have been used in turn to find the α spectroscopic factor. We found that the spectroscopic factor increases with increasing the isospin asymmetry of the parent nuclei if they have valence protons and neutrons. When the parent nuclei have neutron or proton holes in addition to the valence protons or neutrons, then the spectroscopic factor is found to decrease with increasing isospin asymmetry. The obtained results show also that the deduced spectroscopic factors follow individual linear behaviors as a function of the multiplication of the valence proton (N_p) and neutron (N_n) numbers. These linear dependencies are correlated with the closed shells core (Z_0, N_0). The same individual linear behaviors are obtained as a function of the multiplication of $N_p N_n$ and the isospin asymmetry parameter, $N_p N_n I$. Moreover, the whole deduced spectroscopic factors are found to exhibit a nearly general linear trend with the function $N_p N_n / (Z_0 + N_0)$.



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5. Our works





5. Our works

PHYSICAL REVIEW C **94**, 024338 (2016)

Systematic study of **favoured α -decay** half-lives of closed shell odd- A and doubly-odd nuclei related to ground and isomeric states

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(Received 20 June 2016; published 25 August 2016)

$$P_{\alpha} = P_0 \frac{T_{1/2}^{calc}}{T_{1/2}^{expt}}$$

$$P_{\alpha} = a \frac{N_p N_n}{Z_0 + N_0} + b$$

$$P_{\alpha} = c N_p N_n I + d$$

P_{α} is **model-dependent** and **phenomenological**

$T_{1/2}^{calc}$ is calculated by two-potential approach

P_0 is obtained by **DDCM**, C. Xu and Z. Ren, Nucl. Phys. A **760**, 303 (2005).



5. Our works

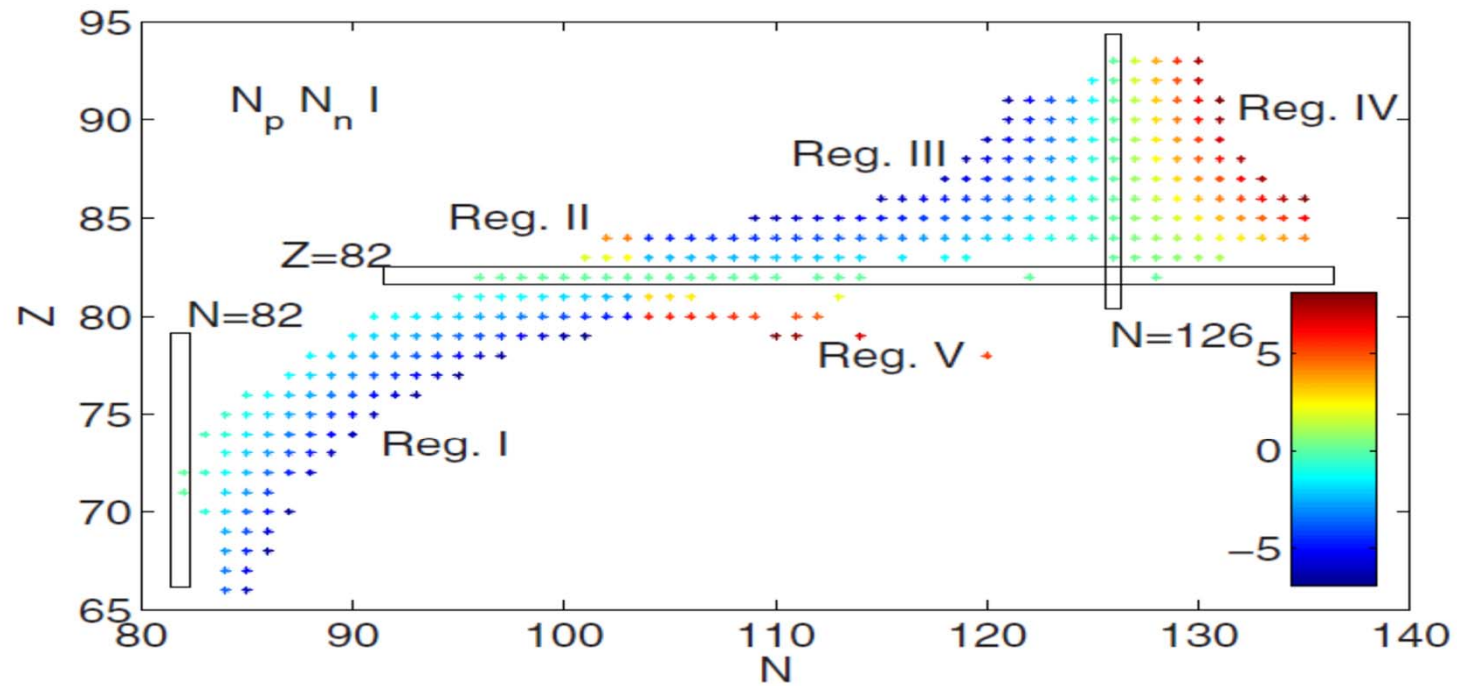


FIG. 2. The colormap of $N_p N_n I$ as a function of neutron numbers N and proton numbers Z of the parent nuclei. The rectangles denote the nuclear shell closures at $Z = 82$ and $N = 82, 126$, and the nuclide chart is divided into five regions.



5. Our works

TABLE III. The parameters of Eqs. (9) and (10) that show α preformation probabilities are linearly related to the valence proton-neutron interaction.

Region	Eq. (9)		Eq. (10)	
	a	b	c	d
Odd-A nuclei				
I	-1.162	0.137	-0.055	0.162
III	-1.409	0.068	-0.043	0.069
IV	8.230	0.094	0.212	0.086
Doubly-odd nuclei				
I	-1.831	0.093	-0.011	0.154
III	-3.477	-0.006	-0.128	-0.021
IV	6.676	0.105	0.112	0.137

$$P_{\alpha} = a \frac{N_p N_n}{Z_0 + N_0} + b$$

$$P_{\alpha} = c N_p N_n I + d$$

$$Z_0 = 82, N_0 = 82, 126,$$

$$N_p = Z - Z_0, N_n = N - N_0$$

I is the asymmetry between neutron and proton in parent nuclei



5. Our works

PHYSICAL REVIEW C 96, 024318 (2017)

Systematic study of **unfavored α -decay** half-lives of closed-shell nuclei related to ground and isomeric states

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(Received 5 June 2017; published 22 August 2017)

$$P_{\alpha} = P_0 \frac{T_{1/2}^{calc}}{T_{1/2}^{expt}}$$

$$P_{\alpha} = a \frac{N_p N_n}{Z_0 + N_0} + b$$

$$P_{\alpha} = c N_p N_n I + d$$

P_{α} is **model-dependent** and **phenomenological**

TABLE IV. The parameters of Eqs. (11) and (12) that show α preformation probabilities are linearly related to $N_p N_n$.

Region	a	b	c	d
Odd-A nuclei				
I	−1.65948	−0.11308	−0.06898	0.02948
III	−0.8437	0.05854	−0.03726	0.0402
IV	0.51361	0.00585	0.01281	0.00585
Doubly-odd nuclei				
I	−0.82097	−0.12653	−0.04695	−0.13455
III	−2.72853	−0.02778	−0.09794	−0.04321
IV	0.53443	−0.00317	0.01402	−0.00363



5. Our works

Case of odd-A nuclei α decay

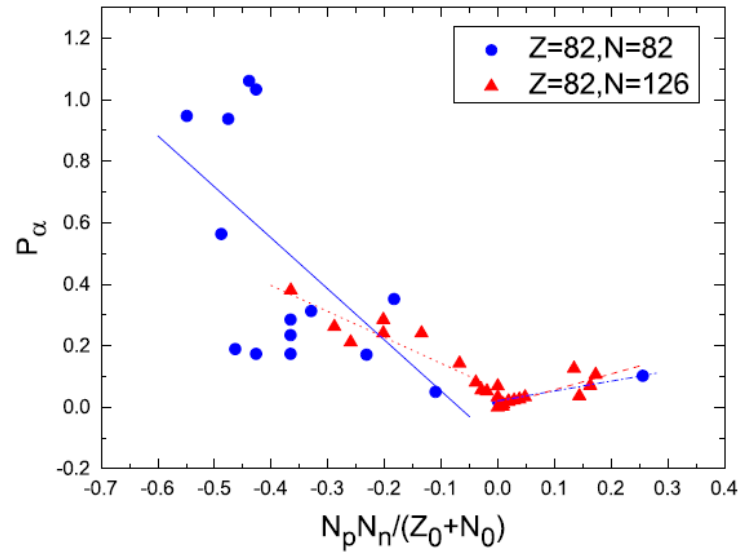


FIG. 2. The linear relationship between α preformation probabilities and $\frac{N_p N_n}{Z_0 + N_0}$. N_p and N_n represent valence protons (holes) and neutrons (holes) of the parent nucleus, respectively. Z_0 and N_0 denote the magic numbers of the proton and neutron, respectively. The blue solid and dash-dotted lines denote the fittings of nuclei in regions I and II, respectively. The red dotted and dashed lines represent the fittings of nuclei in regions III and IV, respectively.

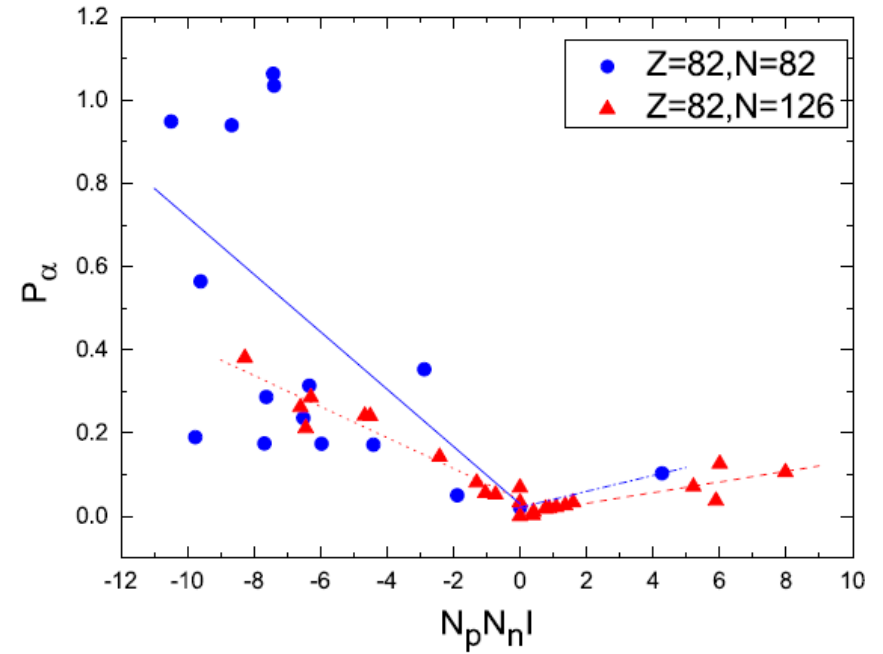


FIG. 3. The same as Fig. 2, but depicting the linear relationship between α preformation probabilities and the product of valence protons (holes), neutrons (holes), and isospin asymmetry as $N_p N_n I$.



5. Our works

Case of doubly odd nuclei α decay

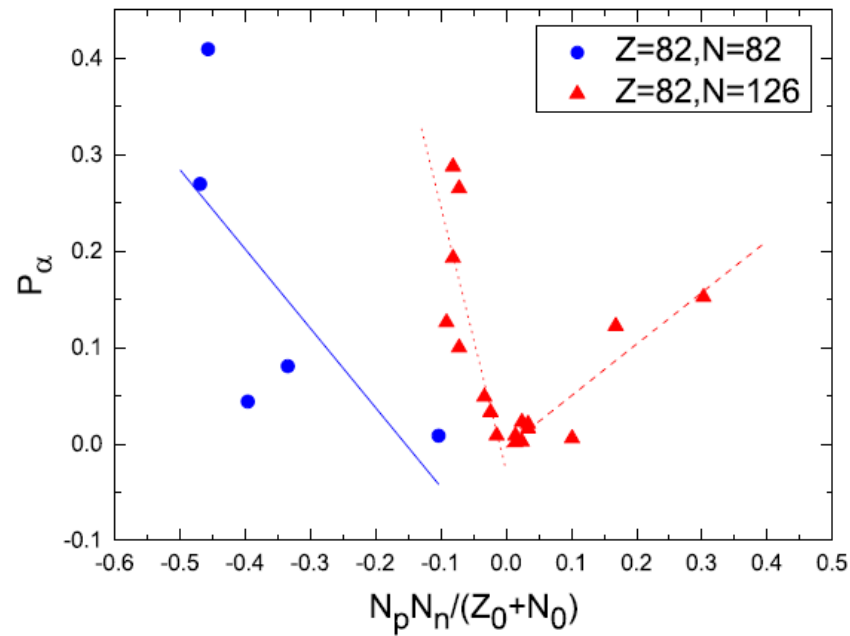


FIG. 4. The same as Fig. 2, but depicting doubly-odd nuclei in accordance with $\frac{N_p N_n}{Z_0 + N_0}$.

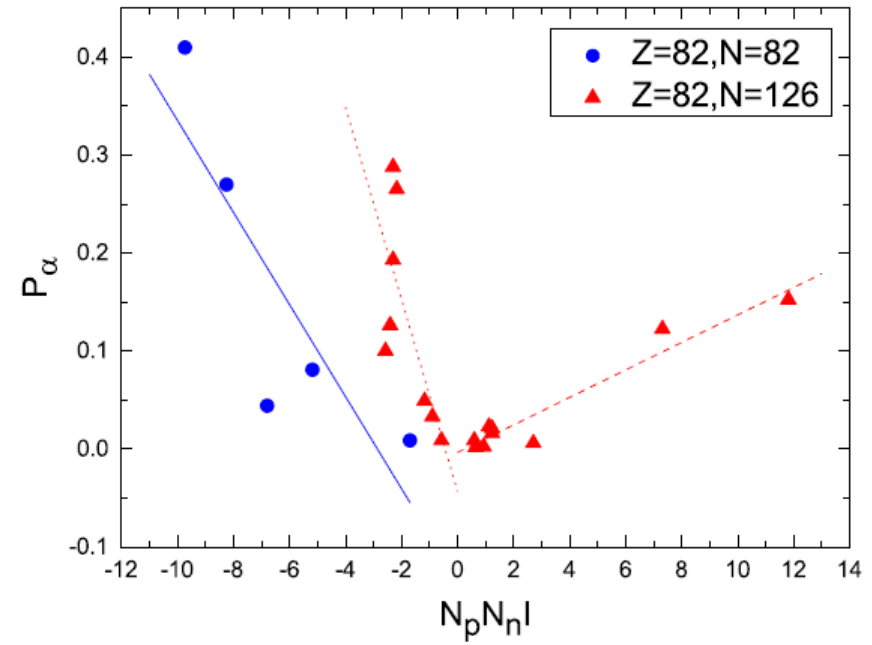


FIG. 5. The same as Fig. 2, but depicting doubly-odd nuclei in accordance with $N_p N_n I$.



5. Our works

- I. Based on the (Cluster-formation model) CFM, we study the α preformation factors of nuclei around $Z = 82, N = 126$ closed shells.
- II. We systematically study the α decay half-lives of nuclei around the $Z = 82, N = 126$ shell closures within the proximity potential 1977 formalism taking $P_\alpha = 1$ and the realistic P_α evaluated by CFM, respectively.

PHYSICAL REVIEW C **97**, 044322 (2018)

Systematic study of α decay of nuclei around the $Z = 82, N = 126$ shell closures within the cluster-formation model and proximity potential 1977 formalism

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(Received 2 February 2018; published 26 April 2018)



5. Our works

- I. Based on the (Cluster-formation model) CFM, we study the α preformation factors of nuclei around $Z = 82$, $N = 126$ closed shells.

Why do we select the CFM to calculate the P_α ?

- ① It is a simple, effective, and microscopic way. Once the binding energies of parent nuclei and neighboring nuclei are known, one can easily evaluate the P_α .
- ② It is interesting to validate whether the realistic P_α within CFM is also linearly dependent on the product of valance protons (holes) and valance neutrons (holes) $N_p N_n$.

X.-D. Sun, P. Guo, and X.-H. Li, Phys. Rev. C **94**, 024338 (2016)

J.-G. Deng, J.-C. Zhao, D. Xiang, and X.-H. Li, Phys. Rev. C **96**, 024318 (2017)



5. Our works

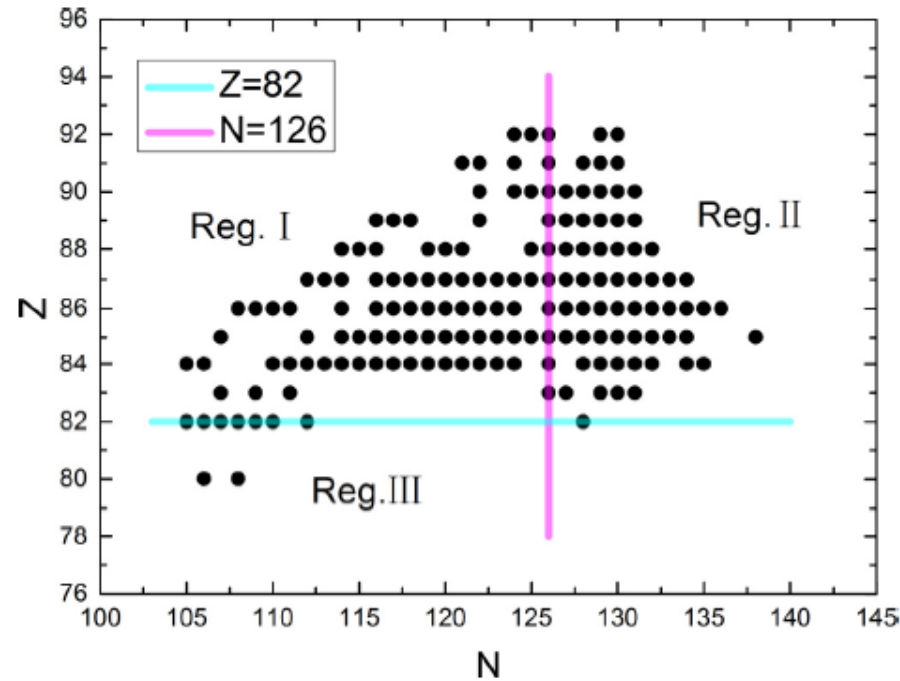


FIG. 1. Nuclide chart is divided into three regions. The cyan and magenta lines denote the $Z = 82$, $N = 126$ nuclear shell closures, respectively.

Case of even-even nuclei α decay

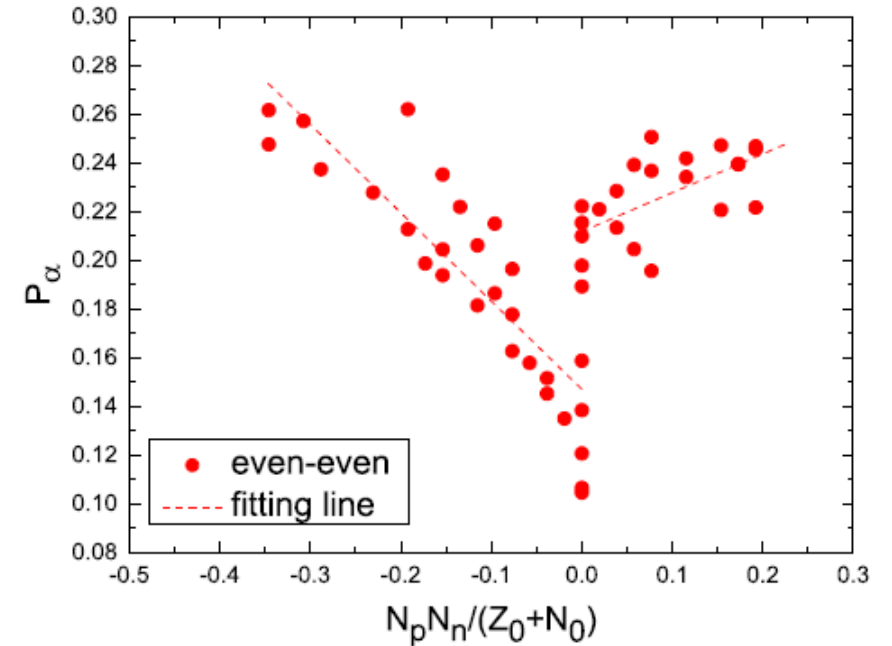


FIG. 2. The linear relationship between α preformation factors and $\frac{N_p N_n}{Z_0 + N_0}$. N_p and N_n represent valence protons (holes) and neutrons (holes) of parent nucleus, respectively. Z_0 and N_0 mean the magic numbers of proton and neutron, respectively. The dash lines represent the fittings of α preformation factors.



5. Our works

Case of odd-A nuclei α decay

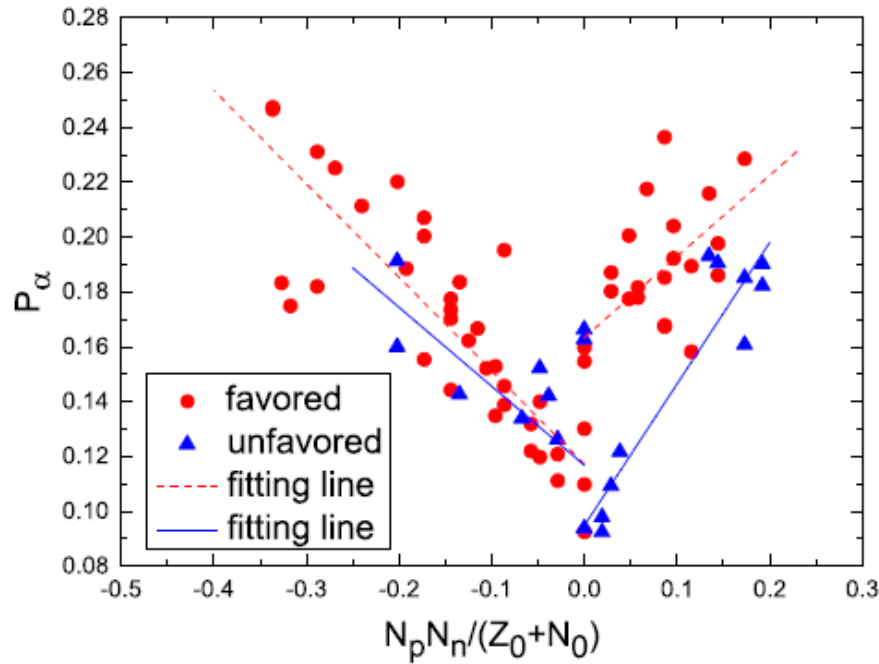


FIG. 3. Same as Fig. 2, but it depicts linear relationships between P_α and $\frac{N_p N_n}{Z_0 + N_0}$ of odd-A nuclei. The red circle and blue triangle represent the cases of favored and unfavored α decay, respectively. The red dash and blue solid lines represent the fittings of α preformation factors for cases of favored and unfavored α decay, respectively.

Case of doubly odd nuclei α decay

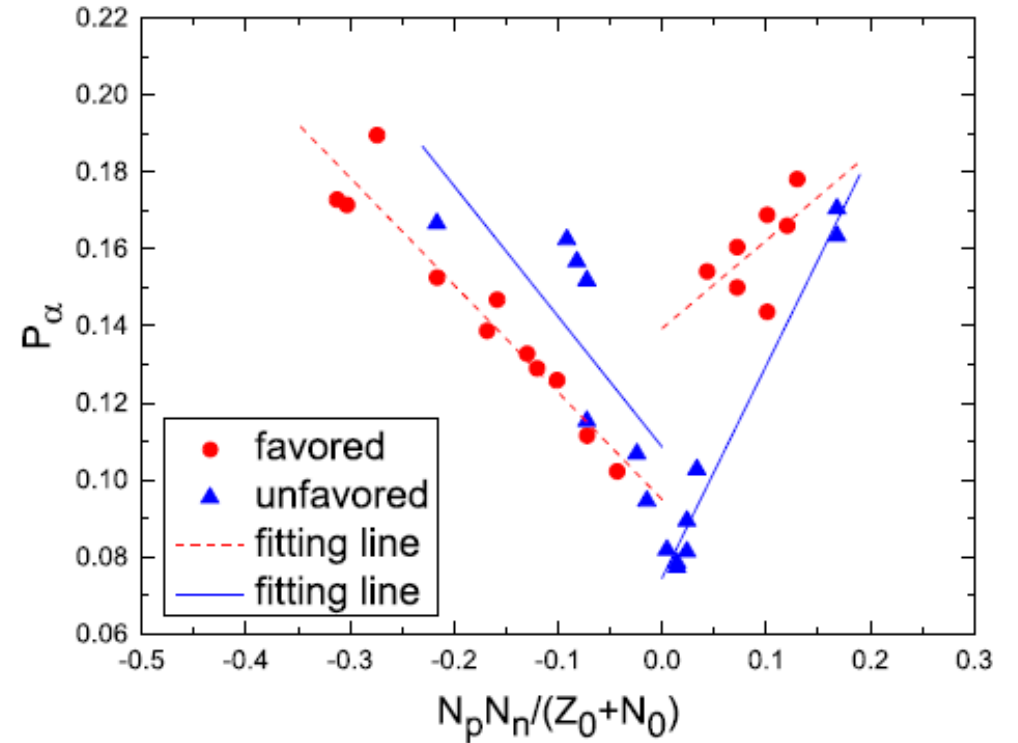


FIG. 4. Same as Figs. 2 and 3, but it depicts linear relationships between P_α and $\frac{N_p N_n}{Z_0 + N_0}$ of doubly odd nuclei.



5. Our works

P_α is obtained by the CFM. We find that the realistic P_α calculated by CFM for nuclei around $Z = 82$, $N = 126$ shell closures are linear with $N_p N_n$.

$$P_\alpha = a \frac{N_p N_n}{Z_0 + N_0} + b$$

$Z_0 = 82$, $N_0 = 126$ represent the magic number of proton and neutron.

$$N_p = Z - Z_0, N_n = N - N_0$$

N_p is valence protons (holes)

N_n is valence neutrons (holes)

TABLE VI. The parameters of Eq. (21) that show α preformation factors are linearly related to $N_p N_n$.

Region	Favored decay		Unfavored decay	
	a	b	a	b
Even-even nuclei				
I	-0.36222	0.14703		
II, III	0.15948	0.21175		
Odd-A nuclei				
I	-0.34101	0.11712	-0.28777	0.11684
II, III	0.29582	0.16333	0.51621	0.09475
Doubly odd nuclei				
I	-0.27858	0.09504	-0.33891	0.10868
II, III	0.22820	0.13944	0.55115	0.07457



5. Our works

Case of odd-A nuclei α decay

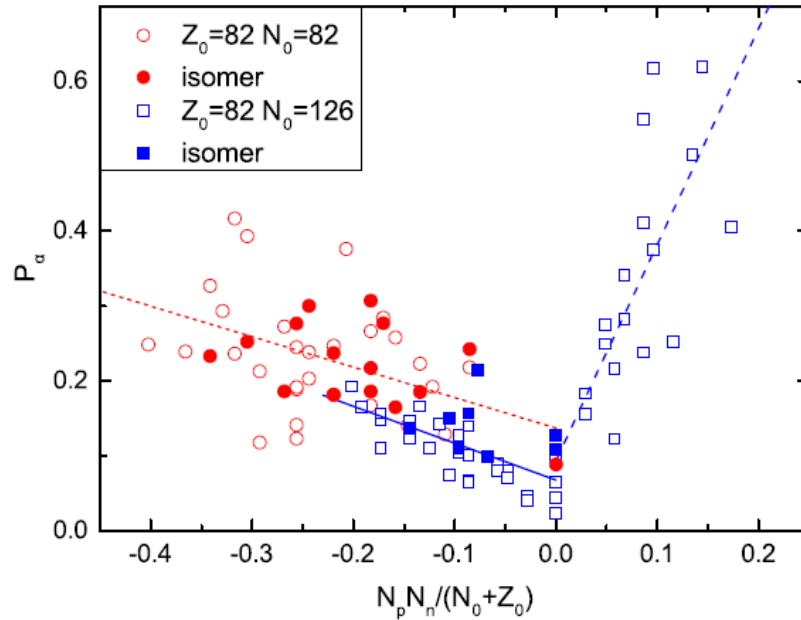


FIG. 3. The α preformation probabilities as a function of $\frac{N_p N_n}{N_0 + Z_0}$, where N_p , N_n represent valence proton numbers and valence neutron numbers of the parent nucleus, respectively, and $N_0(Z_0)$ express neutron (proton) magic numbers. The blue dashed and solid lines denote the fit of nuclei in Regions IV and III, respectively. The red short dashed line denotes the fit of nuclei in Region I.

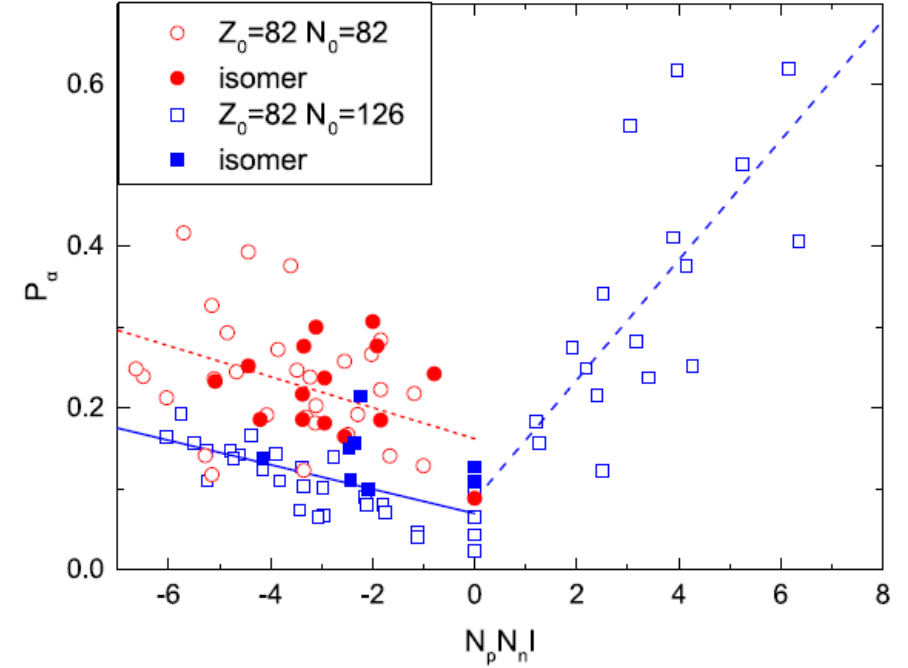


FIG. 4. Same as Fig. 3, but as a function of the product of valence proton numbers and valence neutron numbers and isospin asymmetry $N_p N_n I$.



5. Our works

Case of doubly odd nuclei α decay

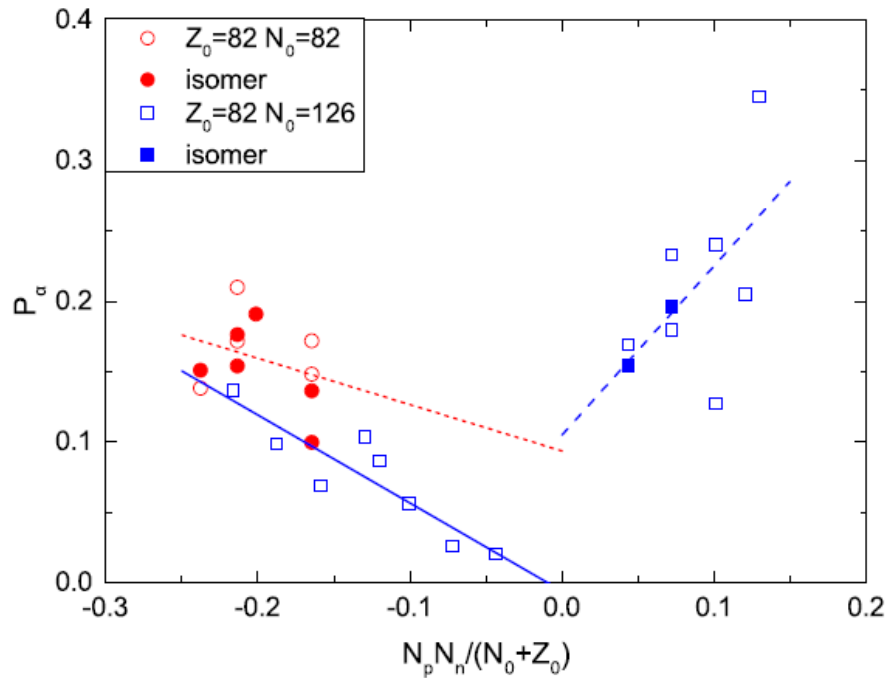


FIG. 5. Same as Fig. 3, but for doubly-odd nuclei as a function of $\frac{N_p N_n}{N_0 + Z_0}$.

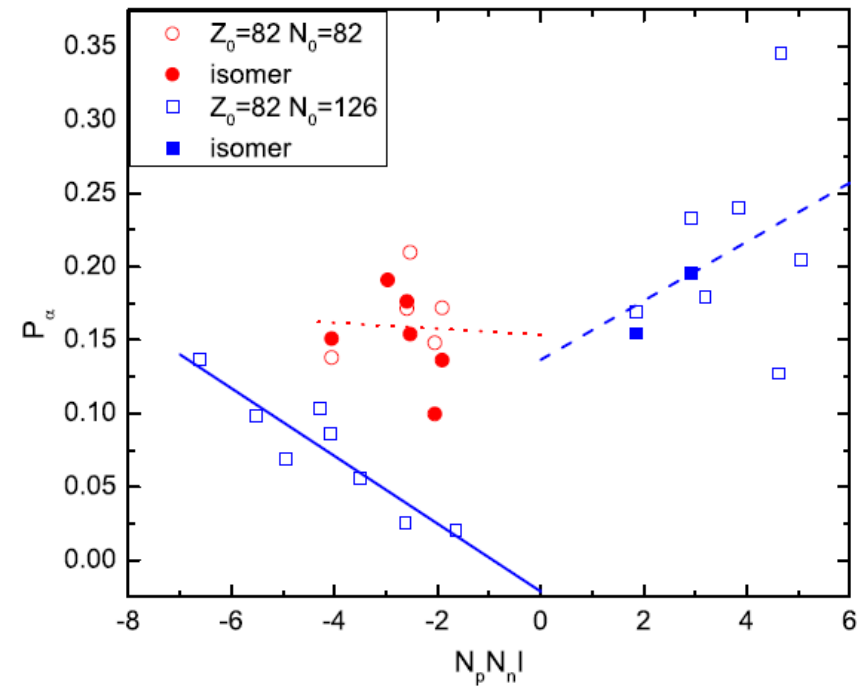


FIG. 6. Same as Fig. 3, but for doubly-odd nuclei as a function of $N_p N_n I$.



5. Our works

- II. We systematically study the α decay half-lives of nuclei around the $Z = 82$, $N = 126$ shell closures within the proximity potential 1977 formalism taking $P_\alpha = 1$ and the realistic P_α evaluated by CFM, respectively.

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma} = \frac{\ln 2}{\lambda}$$

$$\lambda = P_\alpha \nu P$$

In CPPM, the P_α is left out of consideration or assumed as $P_\alpha = 1$.

$$\lambda = \nu P$$



5. Our works

$T_{1/2}^{calc1}$, $T_{1/2}^{calc2}$ and $T_{1/2}^{calc3}$ denote calculated α decay half-life by proximity potential Prox.1977 formalism without considering P_α , with taking P_α by CFM, and with fitting P_α .

σ_1 , σ_2 , and σ_3 denote standard deviations between $T_{1/2}^{calc1}$, $T_{1/2}^{calc2}$, $T_{1/2}^{calc3}$ and $T_{1/2}^{expt}$, respectively.

$$\sigma = \sqrt{\frac{\sum \left(\log_{10} T_{1/2}^{calc} - \log_{10} T_{1/2}^{expt} \right)^2}{n}}$$

TABLE VII. The standard deviations between α decay half-lives of calculations and experimental data.

Nuclei	Favored decay			Unfavored decay		
	σ_1	σ_2	σ_3	σ_1	σ_2	σ_3
Even-even nuclei	0.583	0.380	0.383			
Odd-A nuclei	0.659	0.370	0.366	0.897	0.542	0.536
Doubly odd nuclei	0.813	0.215	0.213	1.631	0.940	0.926



- **Conclusions:**

- I. The P_α of nuclei around shell closures is linear with $N_n N_p$ in all cases.
- II. Our works show that the valence proton-neutron interaction plays a key role in the α preformation for nuclei around $Z = 82, N = 126$ shell closures whether the P_α is model dependent or microcosmic.



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5. Our works

Systematic study of α decay of nuclei around $Z = 82$, $N = 126$ shell closure within a generalized liquid drop model

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In this work, we systematically study the α decay preformation factors P_α and α decay half-lives of 152 nuclei around $Z = 82$, $N = 126$ closed shells based on a generalized liquid drop model while P_α is extracted from the ratio of the calculated α decay half-life to the experimental one. The results show that there is an obvious linear relationship between P_α and the product of valance protons (holes) and valance neutrons (holes) $N_p N_n$. Combining with our previous works [Sun *et al.*, *Phys. Rev. C* **94**, 024338 (2016); Deng *et al.*, *ibid.* **96**, 024318 (2017); Deng *et al.*, *ibid.* **97**, 044322 (2018)] and the work of Seif *et al* [Seif *et al.*, *Phys. Rev. C* **84**, 064608 (2011)], we suspect that this phenomenon of linear relationship for the nuclei around those closed shells is model independent. It confirms the importance of valance protons (holes) and valance neutrons (holes) in the formation of α clusters. Meanwhile, we use the fitted P_α obtained by fitting the calculated α decay half-lives to the experimental ones to calculate the α decay half-lives of these nuclei. The calculated results are agree with the experimental data well.



5. Our works

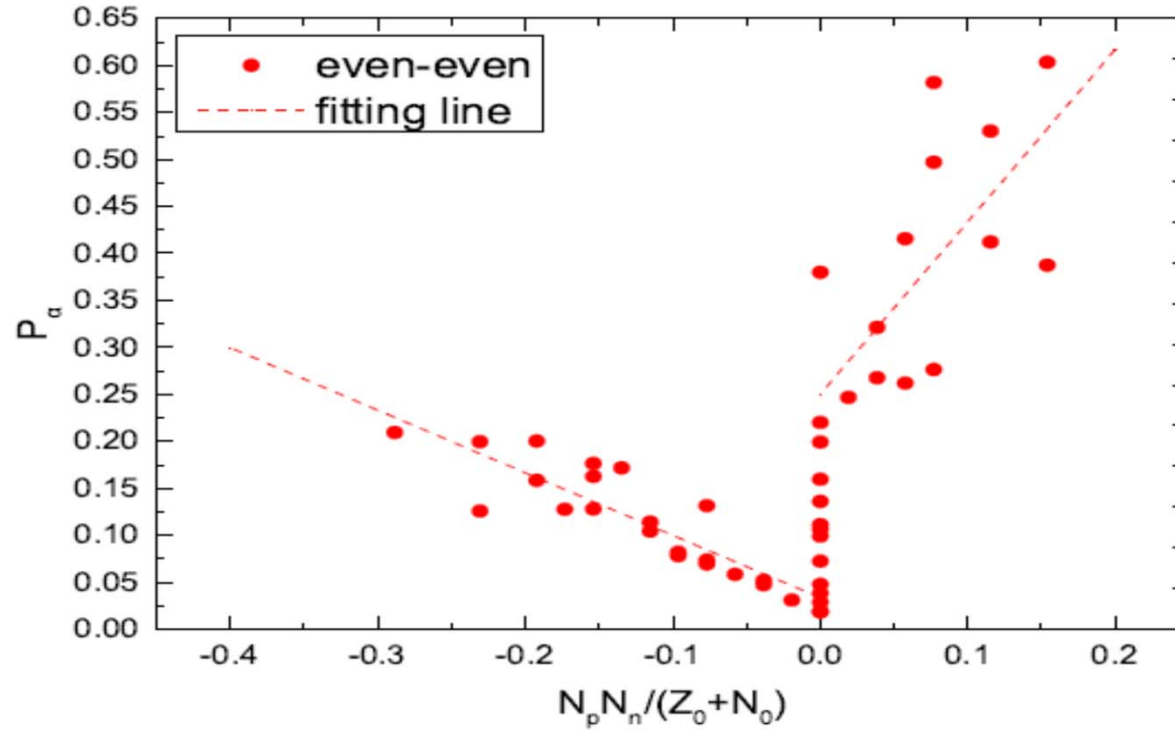


FIG. 1. (color online) The α preformation factors of even-even nuclei around $Z_0 = 82$ and $N_0 = 126$ shell closures as a function of $\frac{N_p N_n}{N_0 + Z_0}$, where N_p and N_n denote valence protons (holes) and neutrons (holes) of parent nucleus, respectively. The dash lines are the fittings of α preformation factors.



5. Our works

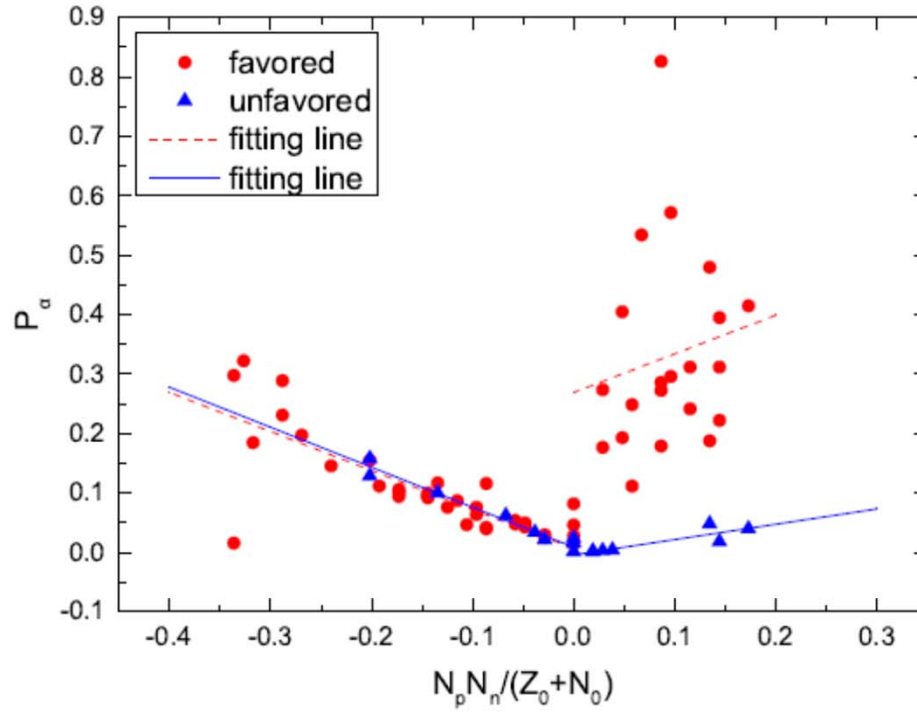


FIG. 2. (color online) Same as Fig. 1, but it represents the α preformation factors as a function of $\frac{N_p N_n}{N_0 + Z_0}$ of odd- A nuclei.

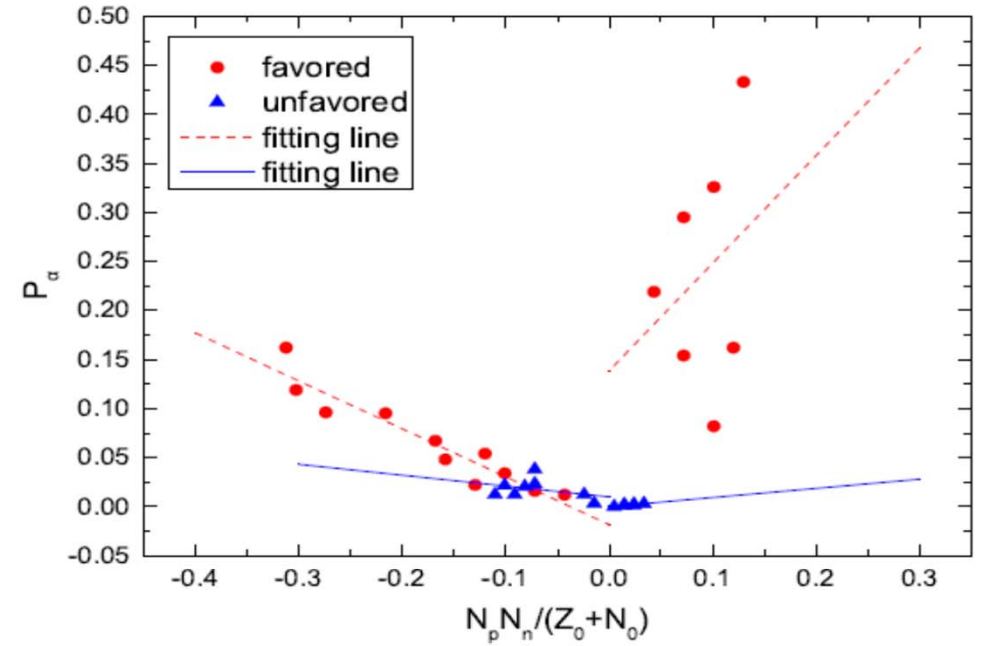


FIG. 3. (color online) Same as Fig. 1, but it represents the α preformation factors as a function of $\frac{N_p N_n}{N_0 + Z_0}$ of doubly-odd nuclei.