180906

MEC323 Lecture 1

office Monday, 3-4pm, EPH338B

Section 9 TA: Maryan Navi

Midterm	35%
Computer Assignment	5%
Assignment	5%
Final	55%

Intro to Static Mechanics

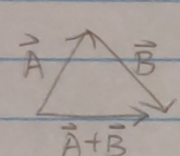
Static: force an object can withstand
Mechanic: material properties needed

Introductory Idea

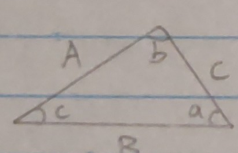
Static implies that there's no $\vec{a}$ (in Newtonian physics)

$\vec{F}=m\vec{a}$ or $m\vec{a}=\Sigma\vec{F}$, when $\Sigma\vec{F}\neq 0 \rightarrow \vec{a}\neq 0$ (dynamics)
$\Sigma\vec{F}=0 \rightarrow \vec{a}=0$ (statics)

Review of Vector & Triangle Rule

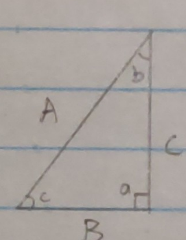


$\vec{A}+\vec{B} = \vec{B}+\vec{A}$ (commutative property)
$(\vec{A}+\vec{B})+\vec{C} = \vec{A}+(\vec{B}+\vec{C})$ (Associative Property)
$\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$ (Vector subtraction)



Sine Law: $\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C}$

Cosine Law: $A=\sqrt{B^2+C^2-2BC\cos a}$
$B=\sqrt{A^2+C^2-2AC\cos b}$
$C=\sqrt{A^2+B^2-2AB\cos c}$



Pythagorean Theorem: $A^2=B^2+C^2$

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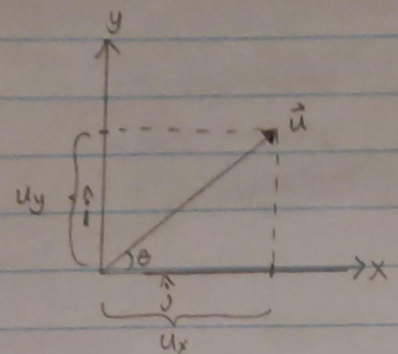
Vectors in Cartesian (2D)



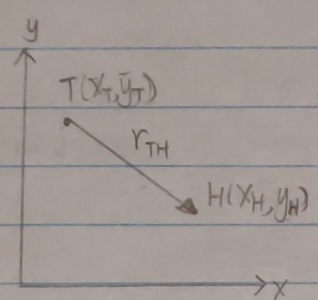
unit vector $\hat{u} = \frac{\vec{u}}{|\vec{u}|}$, dimensionless

$|\vec{u}| = \sqrt{u_x^2 + u_y^2}$

and $\theta = \arctan\left(\frac{u_y}{u_x}\right)$

where $\hat{u} = u_x\hat{i} + u_y\hat{j}$

Position Vectors



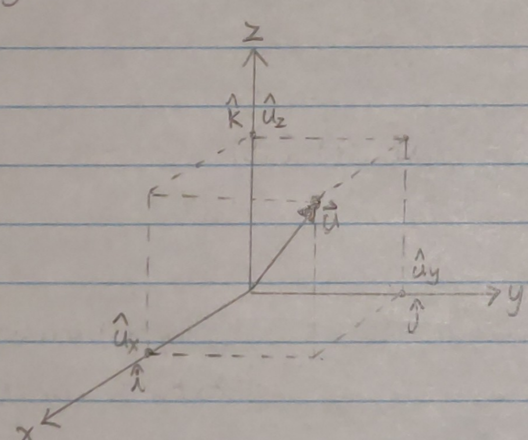
r_{TH} is 'position of H relative to T'.

- How far apart
- direction

$r_{TH} = (X_H - X_T)\hat{i} + (y_H - y_T)\hat{j}$

Vectors in Cartesian (3D)

- x,y,z have to be in RHR orientation.

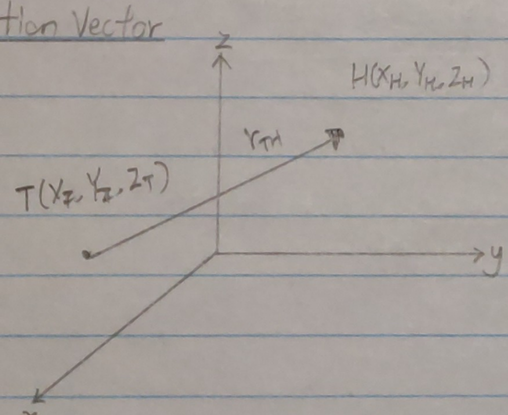


$\vec{u} = \hat{u}_x + \hat{u}_y + \hat{u}_z$

or $\vec{u} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$

& $|u| = \sqrt{u_x^2 + u_y^2 + u_z^2}$

Position Vector

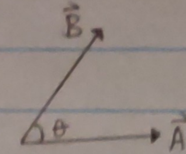


$r_{TH} = (X_H - X_T)\hat{i} + (Y_H - Y_T)\hat{j} + (Z_H - Z_T)\hat{k}$

Vector Dot Product

$$\vec{A}\cdot\vec{B} = \overbrace{|A||B|\cos\theta}^{\text{Scalar}}$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

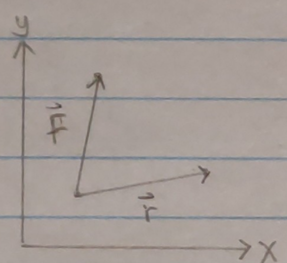


Useful for:

1. determining θ b/w 2 vectors
2. how many components of $\vec{A}$ acts on $\vec{B}$.

⇓

Parallel & Perpendicular Component of a vector in a specific direction:



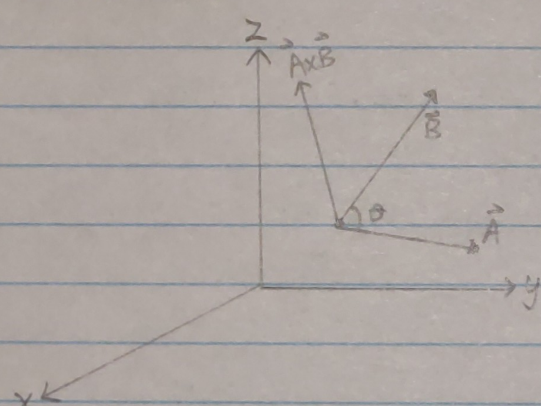
Understand this

$F_{\parallel}$, parallel to $\vec{r}$: $F_{\parallel} = \vec{F}\cdot\vec{r}$

$F_{\perp}$, perpendicular to $\vec{r}$: $F_{\perp} = \sqrt{F^2 - F_{\parallel}^2}$

and $\hat{F} = \hat{F}_{\parallel} + \hat{F}_{\perp}$

Cross Product



$$\vec{A}\times\vec{B} = \overbrace{|\vec{A}||\vec{B}|\sin\theta\,\hat{u}}^{\text{Vector}}$$

$\hat{u}$, unit vector defined by RHR

$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

Useful for:

1. Find direction normal to the plane defined by Vector $\vec{A}$, $\vec{B}$.
2. Area of parrallelgram
3. Find moment or torque.

Anti-Commtative property: $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$, but $\vec{A}\times\vec{B} = (-\vec{B})\times\vec{A}$

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Lecture 2

Today:

4. Equilibrium in 2D and 3D (EMS 3.1, 3.3)
5. Moment of force (EMS 4.1)

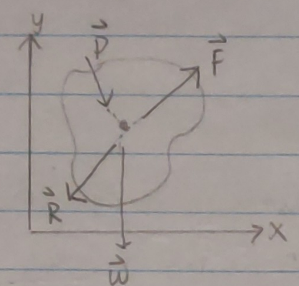
(force apply elsewhere from centroid, creates torque.)

Equilibrium of Particles in 2D and 3D

Equilibrium of particle in 2D:

- A large object may be idealized as a particle when all forces acts on a single point.

e.g.



Here, the lines of action of all forces intersect at a common pt.. This is called a concurrent force system.

- The common Pt. can be anywhere.
  - Gravity however, makes it always in centroid.

- In these system, we can idealize the object as a particle.

Static Equilibrium:

- means $\vec{F}_{net}$ on an object or particle is 0.

This means $\Sigma\vec{F}=0 \Rightarrow \vec{a}=0$

In 2D: $(\Sigma\vec{F}_x)\hat{i} + (\Sigma\vec{F}_y)\hat{j} = \underline{0}$, $\Sigma\vec{F}_x=0$, $\Sigma\vec{F}_y=0$

Free Body Diagram (FBD)

Instructions:

1. Which object/particle to analyze.
2. Isolate object/particle from surrounding.
3. Sketch object/particle separatly
4. Sketch forces on object
   - applied forces
   - forces from surrounding (cable, structure)
   - reaction forces
5. Sketch coordinate system (RHR) and all related dimensions.

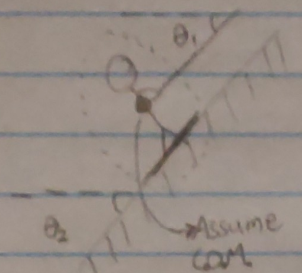
e.g. A skier is being towed up a snow coverd slope at constant velocity.

towed → force on cable

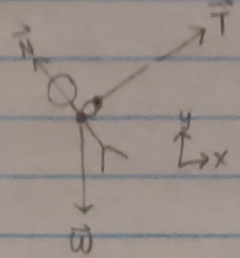
snow coverd slope → no friction on slop

constant velocity → $\vec{a} = 0$

• String only exert tension, no compression.



FBD

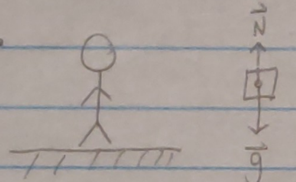


• static equl. woul. emply $\Sigma \vec{F} = \vec{\omega} + \vec{T} + \vec{N} = 0$

Reaction Forces

A reaction force is a force exerted by a support on a body or structure.
Often, reaction force is needed to keep body in static equilibrium.

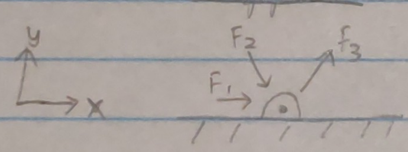
e.g.



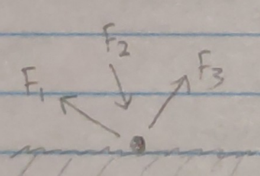
In static equil.
$\Sigma F_y = 0$, $\vec{N} + \vec{g} = 0$

Common Supports and reaction in 2D :

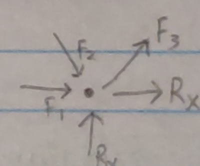
Support | Reaction



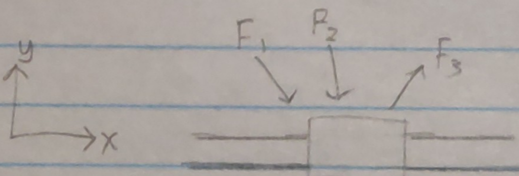
particle pined to surface



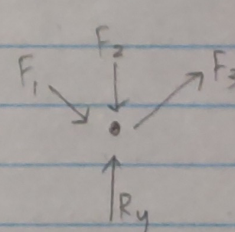
particle on rough surface



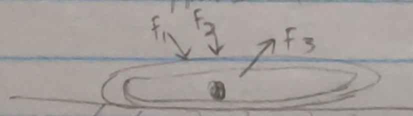
• x and y reaction force



Slider support on a frictionless bar



• reaction in y
• no reaction in x



• freely slide in x but no in y.

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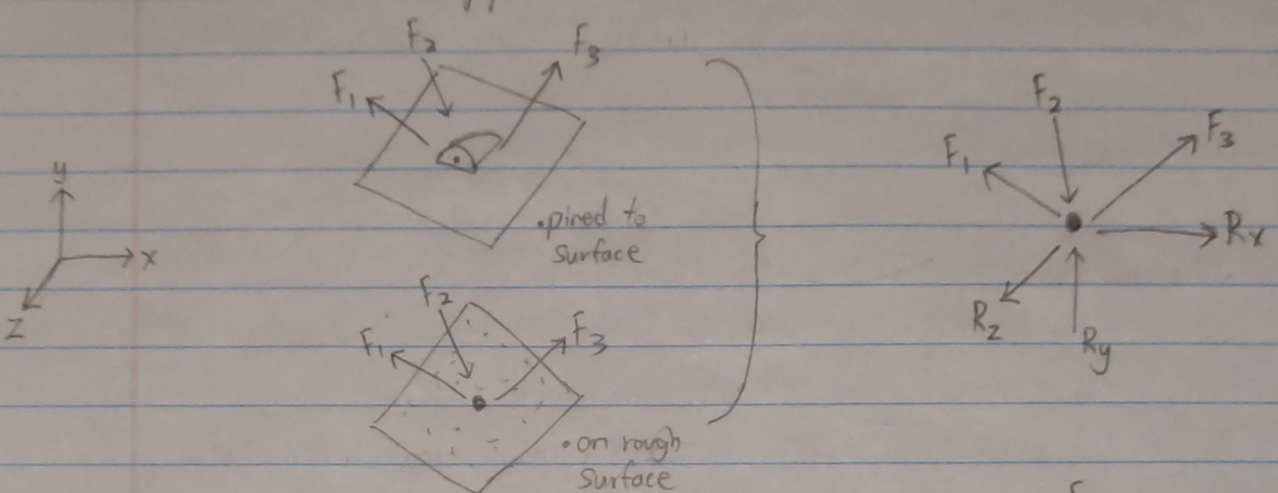
Equilibrium of Particles in 3D

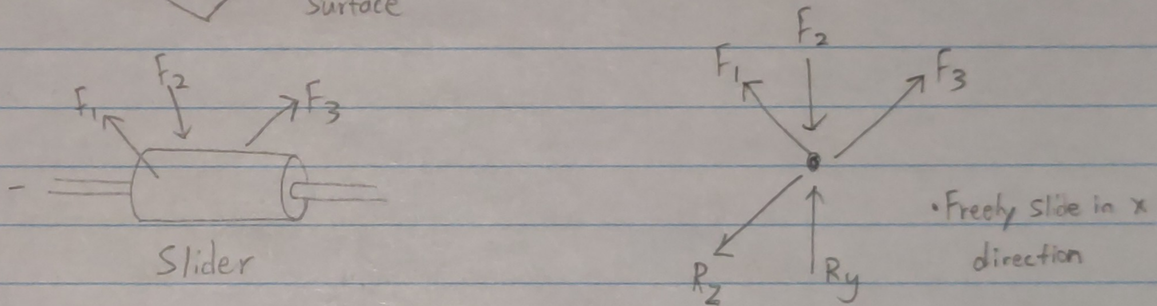
- Similar to eq of particle in 2D.

in 3D, $\Sigma F = 0$ or $(\Sigma F_x)\hat{i} + (\Sigma F_y)\hat{j} + (\Sigma F_z)\hat{k} = 0$, $\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$

Reaction Forces:

Support | Reactions





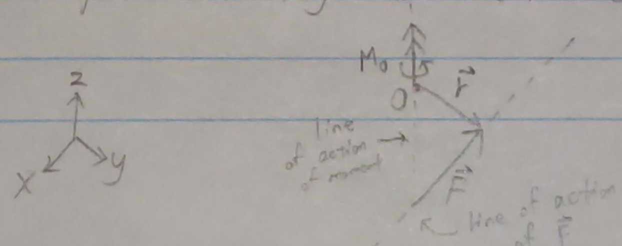
Moment of a force (FMS 4.1)

The moment of a force, or simply moment, is a measure of a force's ability to produce twisting or rotation about a pt. A moment have direction and direction, so it is a vector quantity.

Line of action of a moment

↳ the line of action of a moment is the line about the which the twisting action occurs.

↳ it is parallel to the axis through point O that is perpendicular to the plane containing $\vec{F}$ and the moment arm $\vec{r}$.



$\vec{F}$ applied force

$\vec{r}$ moment arm

$\vec{M}_o$ moment about point O

Vector approch to calculating moment

$\vec{M}_o = \vec{r} \times \vec{F}$ where $\vec{F}$ is the force vector
$\vec{r}$ is the position vector from point O to any pt. alone line of action of $\vec{F}$

Units: SI: N·m
English: ft·lb
in·lb
Dimension: length × force

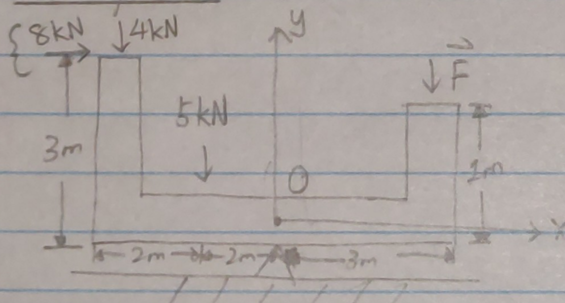
remarks: • order is important $\boxed{\vec{r} \times \vec{F}}$ not $\vec{F} \times \vec{r}$
• direction of moment set by RHR
• $\vec{r}$ is a position vector from O to any point on line of action of $\vec{F}$. (Not the case in scalar)

Scalar Approach to Calculating moment

$M_o = Fd$ here, F is magnitude of force
d is ⊥ distance from O to line of action of F. (moment arm)
M_o has direction dictated by RHR following $\vec{r} \times \vec{F}$

Example 1

Combine 8kN + 4kN into $\vec{F}_1$



Determin F such that the mechanism is $\bar{c}$ static equilibrium.

Vector Approach: $\vec{M}_o = \vec{r} \times \vec{F}$

$\vec{F}_1 = (8, -4)$kN $\vec{r}_1 = (-4, 3)$m
$\vec{F}_2 = (0, -5)$kN $\vec{r}_2 = (-2, 0)$m
$\vec{F}_3 = (0, -F)$kN $\vec{r}_3 = (3, 1)$m or $(3, 0)$m

$\Sigma M_o = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 + \vec{F}_3 = 0$
$(-8 + 10 - 3F)\hat{k} = 0$
$2 - 3F\hat{k} = 0$
$F\hat{k} = \frac{2}{3}$

Scalar approach: into page ⊖

Static Equi. $\Sigma M_o = 0$

$\Sigma M_o = -8kN(3m) + 5kN(2m) + 4kN(4m) - F(3m)$,
$0 = -24kN{\cdot}m + 10kN{\cdot}m + 16kN{\cdot}m - F(3m)$
$2kN{\cdot}m = F(3m)$
$\frac{2}{3}kN = F$

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Lecture 3

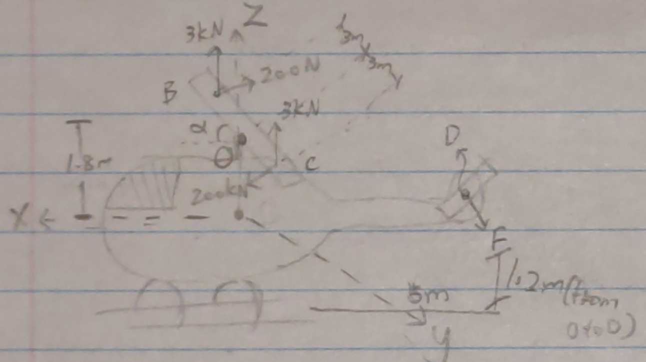
Moment of a Force (cont'd)

Varignon's Theorem (Principle of Moments)

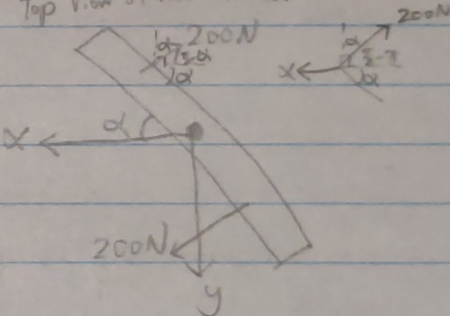
- the moment of a force is equal to the sum of the moment of the vector component of the force.

$$\text{if } \vec{F} = \vec{F}_1 + \vec{F}_2 \quad \text{then } \vec{M}_A = \vec{r} \times \vec{F}$$
$$= \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 + \vec{F}_2$$

Example 2: Forces of 3kN and 200N are exerted at point B and C of the main rotor of a helicoptor, and force F is exerted at point D on the tail rotor. The 3kN forces are parallel to the Z-axis, the 200 N forces are ⊥ to the main rotor and are ∥ to xy plane, F is ∥ to y-axis and $\alpha = 45°$. F = 240 N. Determine resulten moment about O.



Top View of main motor



use (+) vector will take care of signs

$$\Rightarrow \vec{M}_O = (\vec{r}_{OB} \times \vec{F}_B) + (\vec{r}_{OC} \times \vec{F}_C) + (\vec{r}_{OD} + \vec{F}_D)$$

$$\vec{F}_B = -200N \sin 45° \hat{i} - 200N \cos 45° \hat{j} + 3\times10^3 N \hat{k}$$
$$\vec{r}_{OB} = 3m \cos 45° \hat{i} + 3m \sin 45° \hat{j} + 1.8m \hat{k}$$

$$\vec{F}_C = 200N \sin 45° \hat{i} + 200N \cos 45° \hat{j} + 3\times10^3 N \hat{k}$$
$$\vec{r}_{OC} = -3 \cos 45° \hat{i} + 3m \sin 45° \hat{j} + 1.8m \hat{k}$$

$$\vec{F}_D = -240N \hat{j}$$
$$\vec{r}_{OD} = -5m \hat{i} + 1.2m \hat{k}$$

$\therefore M_O = 288 \text{ N·m} \hat{i}$

Moment of a Force about a Line (EMS 4.2)

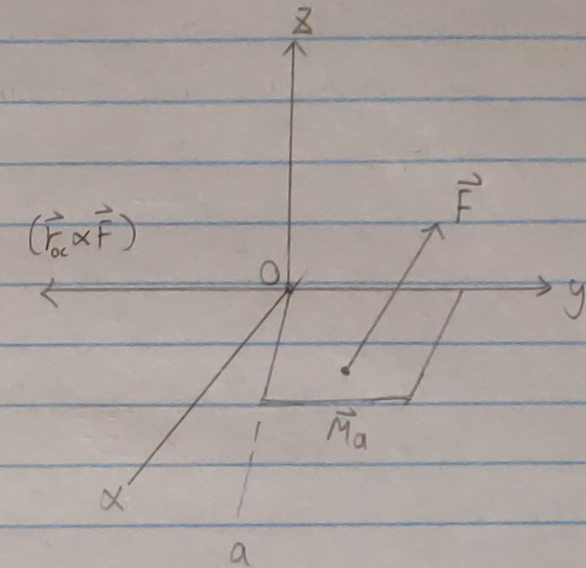
↳ defined as the component of the moment that is in direction of a line.

$\vec{M}_a = M_a \hat{u}$

where $M_a = (\vec{r}_{oc} \times \vec{F}) \cdot \hat{u}$

and $\hat{u}$ is the unit vector at line a.

$\vec{M}_a$ is the u component of the moment, $\vec{r}_{oc} \times \vec{F}$

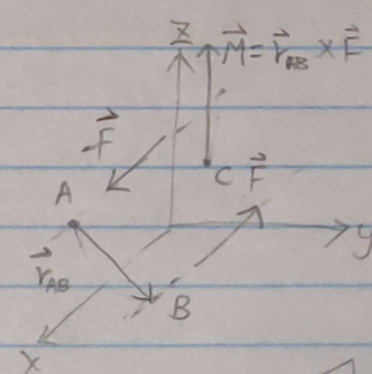


Moment of a Couple (EMS 4.3)

Couple - system of 2 forces of equal magnitude in opposite direction, line of action is ∥ but separated by a certain distance.

$\vec{M} = \vec{r}_{(AB)} \times \vec{F}$ or $\vec{r}_{BA} \times (-\vec{F})$

A, B are any point along the lines of action of $-\vec{F}$ and $\vec{F}$, respectively.



(Scalar approach) $M = Fd$

d is the perpendicular (shortest distance) b/w 2 forces' line of action.

direction of M is dictated by M.

$\vec{M} = \boxed{\vec{r}_{AB}} \times \boxed{\vec{F}}$

a to b ← cross the vector, landed on

Free Vector v.s Fixed Vectors

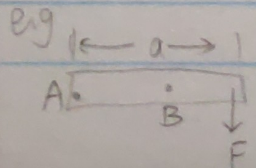
Free Vector: Can be anywhere and does not affect equilibrium. (Can be positioned anywhere)

(e.g couple, applied moment)

Fixed Vector: position affect equilibrium.

(e.g applied force, moment about a point)

e.g



$M_A = -Fa$

$M_B \neq -Fa$

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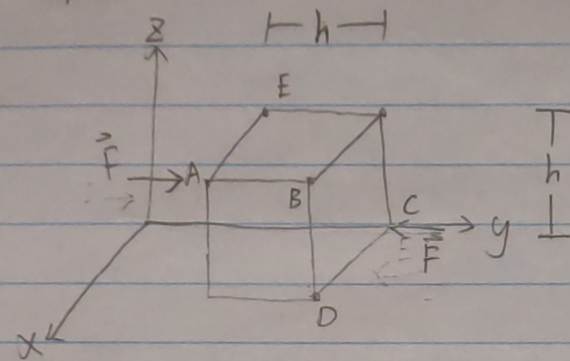
Example 1:

A cube with edge length h is subjected to a force couple whose force have magnitude F. Edges of the cube are parallel to their respective coordinate directions, and the forces are parallel to the y-axis. Determine moment of the couple.



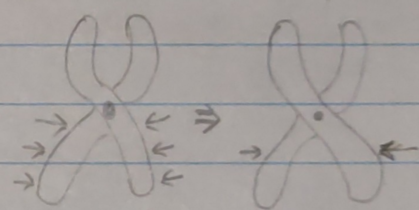
$\vec{M} = (r_{AC} \times \vec{F})$

$= h(-\hat{i} + \hat{j} - \hat{k}) \times (-\vec{F}\hat{j})$

$\therefore \vec{M} = Fh(-\hat{i} + \hat{k})$

Equivalent Force Systems (EMS 44)

we often would like to reduce complicated force systems into simplified equivalant force system, w/ no loss in accuracy.



Principle of Transimissibility of Force

F4.28 - the external effect (translation, rotation) of a force applied to a rigid body, are the same regardless of the pt. of application, alng th ln of actn.

- applied force can be along it's original loa.
- whn mvng awy frm loa. u hv to intrdc a mmnt.

Concurrent force system: whn all frc of n objct intrsct at a commn pt.

Coplanar force system: all frc lie in sm pln, n all mmnt r ⊥ to tht pln.

F4.32 **Parallel force system**: all frc ∥, all mmnt ⊥.

F4.34 **Wrench force system**:

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maryam.navi@ryerson.ca
ma.navi@ryerson.ca

• Solve problems

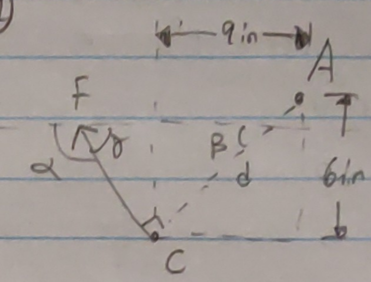
Lab 1

(4.9) Condition & objective

- About pt A: $M_{max} = 10{,}000$ in·lb
- Find max force and α.

* $\vec{F} \perp$ to arm for max frc.

Draw diagram ①



② $d = \sqrt{9^2 + 6^2} = 10.8$ in ← Find d

③ $M_A = Fd$

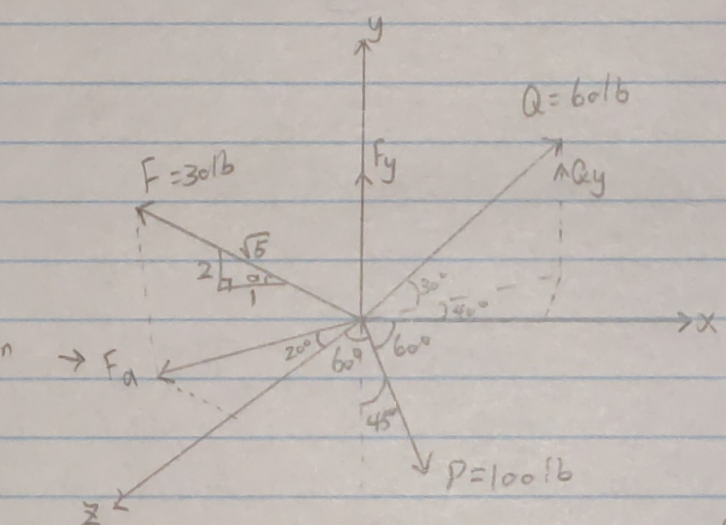
$10{,}000\ \text{lb·in} = F(10.8\ \text{in})$

$\boxed{F = 926\ \text{lb}}$ ← max frc

④ $\cos\beta = \frac{9}{10.8} \rightarrow \beta = 33.6^\circ$ $\gamma = 90 - \beta$

$\alpha = 180 - \gamma = 123.6^\circ$

(2.74)



① Proj of F onto xz pln → F_a

② F into y and a cmpnt

$\sin\alpha$

→ $F_y = F\left(\frac{2}{\sqrt{5}}\right) = 26.83$ lb

$F_a = F\left(\frac{1}{\sqrt{5}}\right) = 13.46$ lb

③ F_a into x, z cmpnt.

③ $F_x = -13.46\ \text{lb} \sin 20^\circ = -4.589$ lb

→ $F_z = F_a \cos 20 = 13.42 \cos 20 = 12.61$ lb

④ Move onto Q

$Q_y = Q \sin 30^\circ = 60 \sin 30^\circ = 30$ lb

$Q_b = Q \cos 30^\circ = 60 \cos 30^\circ = 51.96$ lb

⑤ into x,z

$Q_x = Q_b \cos 40^\circ = 51.96\ \text{lb} \cos 40^\circ = 39.8$ lb

$Q_z = -Q_b \sin 40^\circ = -51.96\ \text{lb} \sin 40^\circ = -33.40$ lb

⑤ Move onto P

$P_x = P \cos 60^\circ = 100 \cos 60^\circ = 50$ lb

$P_y = -P \cos 45^\circ = -100 \cos 45^\circ = -70.7$ lb

$P_z = P \cos 60^\circ = 100 \cos 60^\circ = 50$ lb

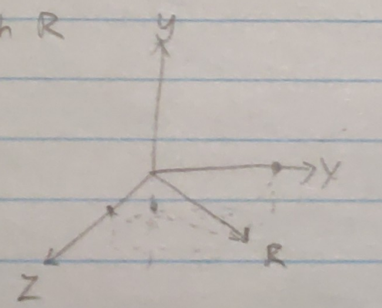
⑥ Sm all cmpnt

$R_x = F_x + Q_x + P_x = -4.589 + 39.8 + 50 = 85.2$ lb

$R_y = F_y + Q_y + P_y = -13.9\hat{j}$

$R_z = F_z + Q_z + R_z = 29.2\hat{k}$

⑦ Sketch R



$R = 85.2\hat{i} - 13.9\hat{j} + 29.2\hat{k}$

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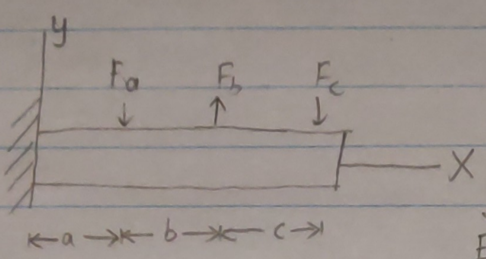
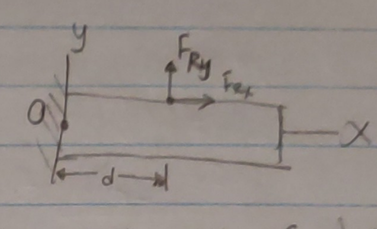
Lec 4

VIII. Equivalent Force System (con't) (EMS 4.4)

Example: (2D) Determine an equivalent force system consisting of a single force only, and specify the x-coordinate of the point where the force intersect the x-axis.



EFS



• Setting arbituary F and d

① Total x force — no x forces $\Rightarrow \therefore F_{Rx} = 0$

② Total y forces — $F_{Ry} = -F_a + F_b - F_c$

③ Where is F_R? $d = ?$

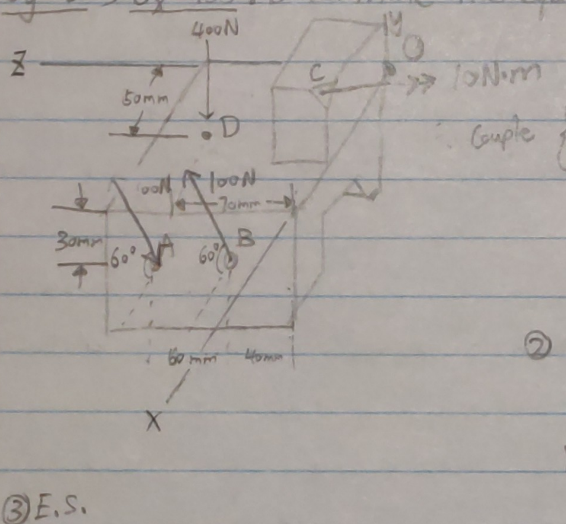
- Reaction force will be the same as original in EFS.

$$(\Sigma M_o)_{original} = (\Sigma M_{Ro})_{equivalent}$$

$$-(F_a \cdot a) + F_b \cdot (a+b) - F_c(a+b+c) = (F_{Ry} \cdot d)$$

$$\frac{1}{-F_a + F_b - F_c}\left[F_b \cdot (a+b) - F_a \cdot a - F_c(a+b+c)\right] = d$$

Example: (3D) A casting supports the forces and moments shown where the 100N forces are all parallel to the x-y plane and the moment at C has direction angle $\theta_x = 60°$, $\theta_y = 60°$, $\theta_z = 135°$. Determine the equivalent force system acting at pt. O.



① Write down vector of all forces

Couple $\begin{cases} \vec{F}_A = 100N(-\cos 60° \hat{i} - \sin 60° \hat{j}) \\ \vec{F}_B = 100N(\cos 60° \hat{i} + \sin 60° \hat{j}) \end{cases}$

$\vec{F}_D = -400N\hat{j}$

$\vec{M}_c = 10N \cdot m(\cos 60° \hat{i} + \cos 60° \hat{j} + \cos 135° \hat{k})$

② Position vector

$r_{OD} = (50\hat{i} + 70\hat{k})mm$

$r_{AB} = (-60\hat{k})mm$

③ E.S.

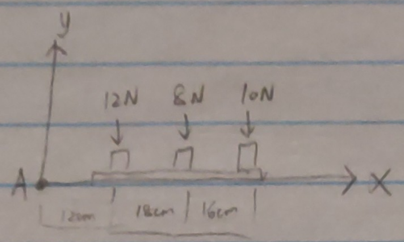
$\vec{F}_{RO} = \vec{F}_A + \vec{F}_B - \vec{F}_D = -400N\hat{j}$

$$\vec{M}_{RO} = \underbrace{(\vec{r}_{AB} \times \vec{F}_B)}_{\text{moment of F. Couple } \vec{F}_A \ \& \ \vec{F}_B} + (\vec{R}_{OD} \times \vec{F}_D) + \underbrace{\vec{M}_c}_{\text{applied moment free vector}} = (38.2\hat{i} + 2\hat{j} - 27.1\hat{k}) N \cdot m$$

Centroid & Composite Bodies

Center of Gravity: defined as the average distribution of weight

e.g. a tray



The resultant force system would be

① $\Sigma(F_y)_{system} = -12N - 8N - 10N$

$\therefore = -30N$

↳ the location of F_R is C.O.G.

EFS of 1 force
⇓
COG

② $\Sigma(M_A)_{original} = \Sigma(M_{R,A})_{equivalent}$

$(-12N)(12cm) + (-8)(30cm) + (-10)(46cm) = (-30N)(\vec{x})$

$28.13cm = \vec{x}$ } the COG

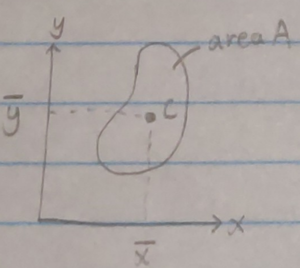
A more general form of COG can be written,

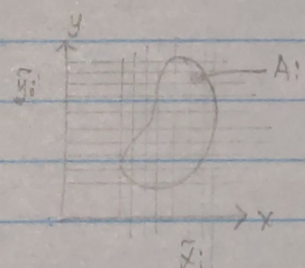
$$\vec{x} = \frac{\sum_{i=1}^{n} \vec{x}_i w_i}{\sum_{i=1}^{n} w_i}$$ where n is # of obj. w/ weight w_i and location $\vec{x}_i$ relative to origin.

$w_i \vec{x}_i$ is also called the **1st moment** of weight w_i.

Centroid of an Area:

average distribution of area





- A centroid is the average position of a distribution of shapes. The distribution can include multi. area, volume.
- Here, A is formed by a collection of areas, A_i, where the centroid of A_i is $\tilde{x}_i, \tilde{y}_i$

$$\bar{x} = \frac{\sum_{i=1}^{n} \tilde{x}_i A_i}{\sum_{i=1}^{n} A_i} \quad \text{and} \quad \bar{y} = \frac{\sum_{i=1}^{n} \tilde{y}_i A_i}{\sum_{i=1}^{n} A_i} \quad \text{and} \quad A_i \to 0 \quad \bar{x} = \frac{\int \tilde{x}\, dA}{\int dA} \quad \bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$\sum_{i=1}^{n} \tilde{x}_i A_i$ and $\int \tilde{x}_i\, dA$ are called 1st moment of the area about the y-axis.

$\sum_{i=1}^{n} \tilde{y}_i A_i$ and $\int \tilde{y}_i\, dA$ " " " " " " " " x-axis.

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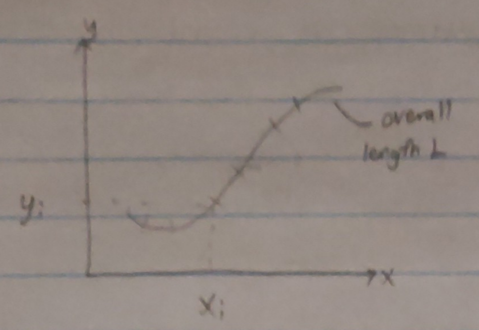
Centroid of a Line:



Then the centroid of the line is

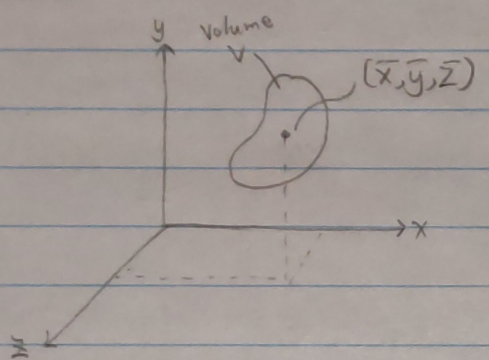
$$\bar{x} = \frac{\sum_{i=1}^{n} \bar{x}_i L_i}{\sum_{i=1}^{n} L_i} \quad \text{and} \quad \bar{y} = \frac{\sum_{i=1}^{n} \bar{y}_i L_i}{\sum_{i=1}^{n} L_i}$$

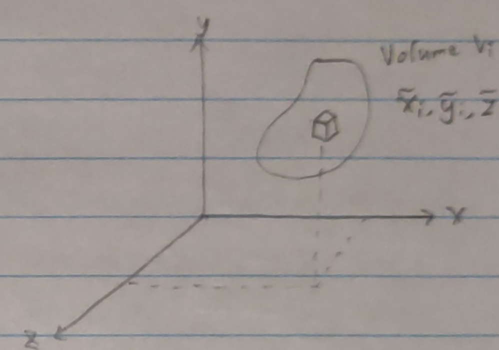
if $L_i \to 0$

$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL} \quad \text{and} \quad \bar{y} = \frac{\int \tilde{y}\, dL}{\int dL}$$

Centroid of a Volume:

· break vol. into small blocks



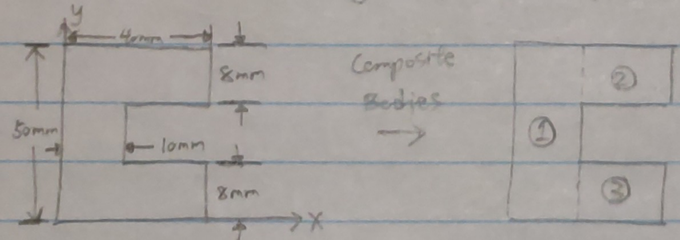


Centroid $C(\bar{x}, \bar{y}, \bar{z})$ of V is located at $\bar{x}, \bar{y}, \bar{z}$. V consist of many V_i, each located at $\tilde{x}_i, \tilde{y}_i, \tilde{z}_i$.

$$\text{So,} \quad \bar{x} = \frac{\sum_{i=1}^{n} \tilde{x}_i V_i}{\sum_{i=1}^{n} V_i}, \quad \bar{y} = \frac{\sum_{i=1}^{n} \tilde{y}_i V_i}{\sum_{i=1}^{n} V_i}, \quad \bar{z} = \frac{\sum_{i=1}^{n} \tilde{z}_i V_i}{\sum_{i=1}^{n} V_i}$$

$$\text{and} \quad \bar{x} = \frac{\int \tilde{x}_i\, dV}{\int dV}, \quad \bar{y} = \frac{\int \tilde{y}_i\, dV}{\int dV}, \quad \bar{z} = \frac{\int \tilde{z}_i\, dV}{\int dV}$$

Example: Centroid of an area using composite shapes



Shape #	Area	$\tilde{x}_i$	$\tilde{y}_i$
1	$500mm^2$	5mm	25mm
2	$240mm^2$	25mm	46mm
3	$240mm^2$	25mm	4mm

$$\bar{x} = \frac{5mm(500mm^2) + 240mm^2(25mm) + 240mm^2(25mm)}{500mm^2 + (240mm^2)2} = 14.796mm$$

$$\bar{y} = \frac{500mm^2(25mm) + 240mm^2(46mm + 4mm)}{500mm^2 + 2(240mm^2)} = 25mm$$

w is in kN/m
F will be in kN

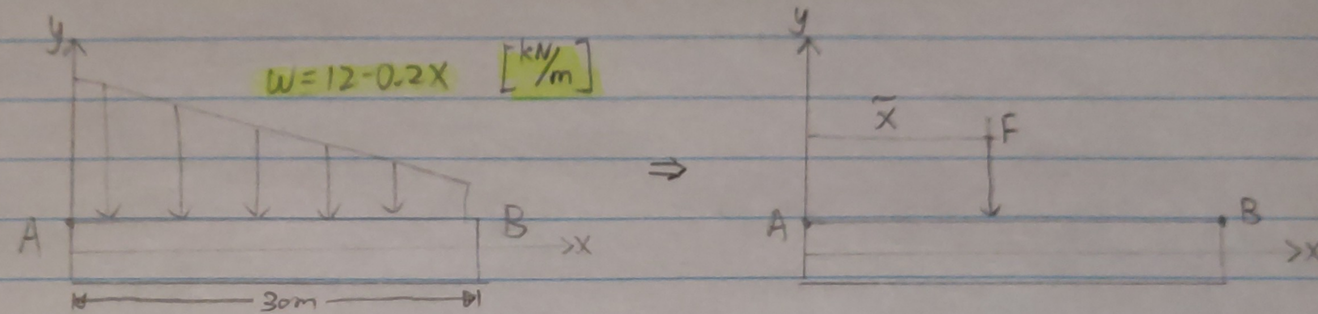
Example 2: Applying concepts from centroids to analyze distributed forces.

If the force on a beam is described by a function, $\tilde{x}$, centroid of small forces

distributed frc → pt frc.

then, $\bar{x} = \dfrac{\int \tilde{x}\, dF}{\int dF} = \dfrac{\int x w\, dx}{\int dx}$, and $F = \int dF = \int w\, dx$

where w is the expression that describes the distribted frces.



in 2D ↗
3D: pressure

$F = \int_0^{30} (12 - 0.2x)\, dx = 270$ kN

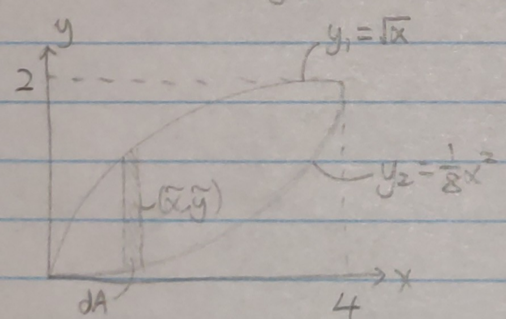
$\bar{x} = \dfrac{\int \tilde{x}\, dx}{\int w\, dx}$

assuming dx very small, $\tilde{x} = x$

$\bar{x} = \dfrac{\int_0^{30} x(12 - 0.2x)\, dx}{\int_0^{30} (12 - 0.2x)\, dx} = \dfrac{3600\ \text{kN}\cdot\text{m}}{270\ \text{kN}} = 13.3\text{m}$

∴ EMS $\bar{x} = 13.3$m
$F = 270$kN

Example 3: Determine the x and y position of the centroid.



$\bar{x} = \dfrac{\int \tilde{x}\, dA}{\int dA}$

$\bar{y} = \dfrac{\int \tilde{y}\, dA}{\int dA}$

$dA = (y_1 - y_2)dx = (\sqrt{x} - \frac{1}{8}x^2)dx$

- assuming slit is very narrow.
$\tilde{x} = x$.

$\bar{x} = \dfrac{\int_0^4 \tilde{x}\, dA}{\int_0^4 dA} = \dfrac{\int_0^4 x(\sqrt{x} - \frac{1}{8}x^2)dx}{\int_0^4 (\sqrt{x} - \frac{1}{8}x^2)dx} = \dfrac{\int_0^4 (x^{3/2} - \frac{1}{8}x^3)dx}{\int_0^4 (x^{1/2} - \frac{1}{8}x^2)dx} = \dfrac{\frac{2}{5}x^{5/2} - \frac{1}{32}x^4 \Big|_0^4}{\frac{2}{3}x^{3/2} - \frac{1}{24}x^3 \Big|_0^4} =$

middle y of slit dA

$\tilde{y} = \frac{1}{2}(y_1 + y_2)$
$= \frac{1}{2}(\sqrt{x} + \frac{1}{8}x^2)$

$\bar{y} = \dfrac{\int \tilde{y}\, dA}{\int dA} = \dfrac{\int_0^4 \frac{1}{2}(\sqrt{x} + \frac{1}{8}x^2)(\sqrt{x} - \frac{1}{8}x^2)dx}{\int_0^4 (\sqrt{x} - \frac{1}{8}x^2)dx} = \frac{9}{10}$

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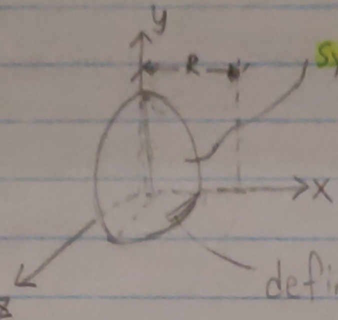
Example 4: Determine the position of the centroid for the solid hemisphere of radius
(30) R shown.



Symmetric about x-axis ($\bar{y} = \bar{z} = 0$)

↳ Centroid is somewhere along x-axis.

defined by $x^2 + y^2 + z^2 = R^2$

$$\bar{x} = \frac{\int \tilde{x}\, dV}{\int dV}$$ we can cut the hemisphere into slices.

- width dx
- volume dV

- assum dx very small

$\tilde{x} = x$

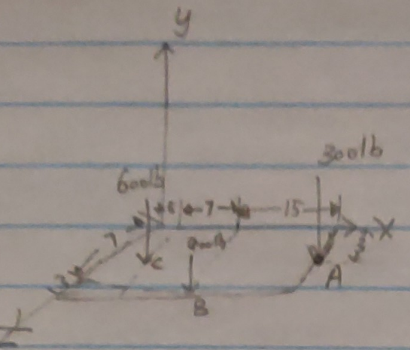
Convert V into smthing smthing dx → by projecting the hemi-sphere onto x-y plane ($z = 0$, $y = \sqrt{R^2 - x^2}$)

$dV = \pi y^2 dx = \pi (R^2 - x^2) dx$, sub back into $\bar{x}$ eqn.

$$\bar{x} = \frac{\int_0^R x \cancel{\pi} (R^2 - x^2) dx}{\int_0^R \cancel{\pi} (R^2 - x^2) dx} = \frac{\int_0^R (R^2 x - x^3) dx}{\int_0^R (R^2 - x^2) dx} = \left. \frac{\frac{1}{2} R^2 x^2 - \frac{x^4}{4}}{R^2 x - \frac{1}{3} x^3} \right|_0^R = \frac{3}{8} R$$

Lab 2

(4.126)



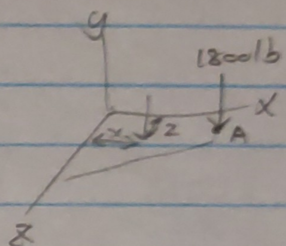
a) EFS at pt A

$F_{Ry} = -1800 lb$
$F_{Rx} = F_{Rz} = 0$ } Just sum up forces

$M_{Ay} = 0$, → force in y ∴ not M in y

Just to line of action { $M_{Ax} = (600lb)(7-3) + 900(7+3-3) = 8700$ ft·lb
$M_{Az} = 600(15+7) + 900(15) = 26,700$ ft.lb

b) Force accounts for moments find in a)



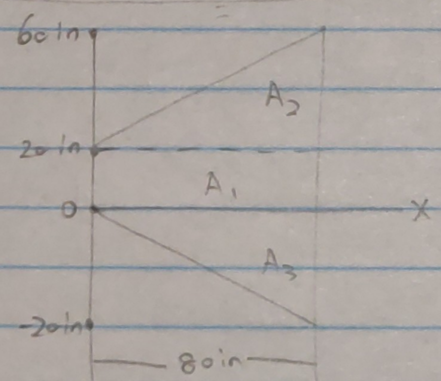
$F_{Ry} = -1800 lb$

$\hat{k}$ $(M_{ROX})_{system 1} = (M_{ROX})_{system 2}$
$1800(3) + 8700 = 1800z$ $z = 7.83$ ft

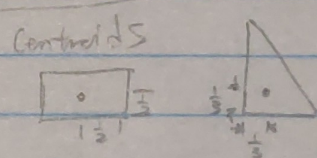
∴ $x = 12.2$ ft
$z = 7.83$ ft

$\hat{i}$ $(M_{ROZ})_{sys 1} = (M_{ROZ})_{sys 2}$
$-(1800)(15+7+5) + 26700 = -1800x$ $x = 12.2$ ft

(7.21) Method of Composite body



Centroids



$\bar{x} = \frac{40(1600) + 53.33(1600) + 53.33(800)}{1600 + 1600 + 800}$
$= 48$ in

$\bar{y} = \frac{10(1600) + 33.33(1600) + 800(-6.667)}{1600 + 1600 + 800}$

$= 16$ in

Region	A_i (in²)	x_i (in)	y_i (in)
1	(80)(20) = 1600	40	10
2	$\frac{1}{2}(80)(40) = 1600$	$\frac{2}{3}(80) = 53.33$	$20 + \frac{1}{3}(40) = 33.33$
3	800	53.33	$\frac{1}{3}(-20) = -6.667$

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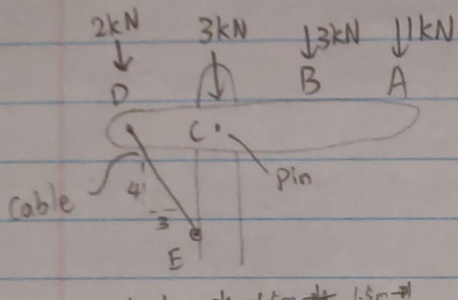
Lec 6

Equilibrium of Rigid Bodies in 2D (EMS 5.4)

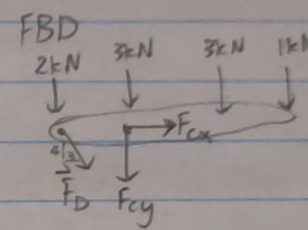
- Sumation of forces and momment = 0

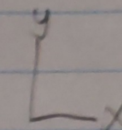
Fig. 5.3. txbk illustrates common 2D reactions

Example 1 : Bar ABCD is supported by a pin at C and a cable from points D to E. Determine the reaction at C and the force supported by the cable.



• F_{cx} and F_{cy} at C





• To satisfy equilibrium $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_c = 0$

$$\Sigma F_x = 0 \Rightarrow \vec{F}_{cx} + \frac{3}{5}F_D = 0$$

$$\Sigma F_y = 0 \Rightarrow -2kN - 3kN - 3kN - 1kN + \vec{F}_{cy} + \frac{4}{5}\vec{F}_D = 0$$

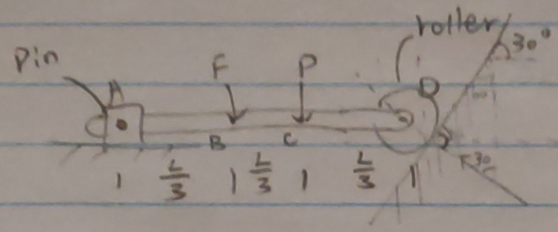
$$\Sigma M_c = 2kN(1m) - 3kN(1.5m) - 1kN(3m) + \frac{4}{5}F_D(1m) = 0$$

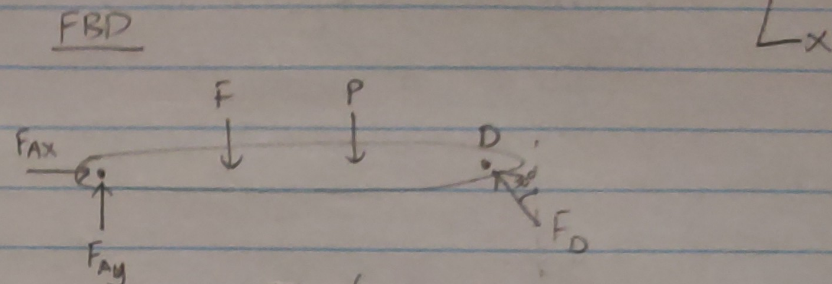
$$F_D = 6.875N$$

$$\vec{F}_{cy} = 9 + \frac{4}{5}(6.875N\cdot m) = 14.5N \qquad \vec{F}_{cx} = -\frac{3}{5}F_D = -\frac{3}{5}(6.875N\cdot m) \doteq -4.13N$$

$$\therefore \vec{F}_{cy} = 14.5N,\ \vec{F}_{cx} \doteq -4.13N,\ \vec{F}_D = 6.875N$$

Example 2: Determine all reactions





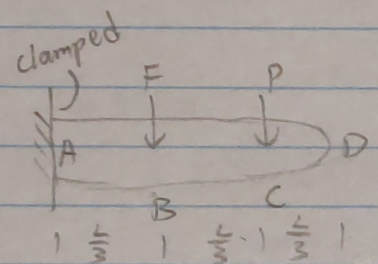
Note: choose pt A so less eqn.

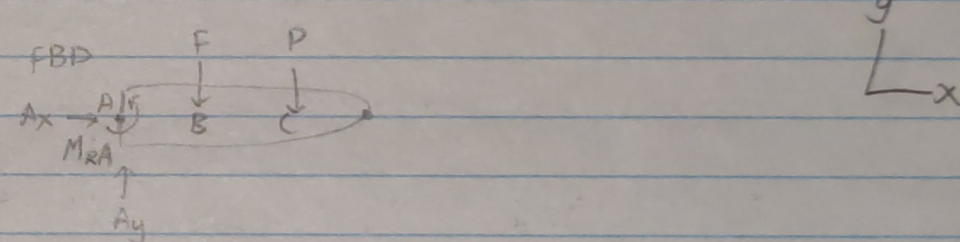
$\Sigma M_A = -F(\frac{L}{3}) - P(\frac{2L}{3}) + F_D \cos 30 (L) = 0 \quad \therefore D = \frac{1}{\cos 30}(\frac{1}{3}F + \frac{2}{3}P)$

$\Sigma F_{Ay} = 0 \Rightarrow \quad A_y - F - P + D\cos 30° = 0 \quad \therefore A_y = F + P + (\frac{1}{3}F + \frac{2}{3}P) = \frac{4}{3}F + \frac{5}{3}P$

$\Sigma F_{Ax} = 0 \Rightarrow \quad A_x - D\sin 30 = 0 \quad \therefore A_x = \tan 30°(\frac{1}{3}F + \frac{2}{3}P)$

Example 3:





$\Sigma M_A = 0 \Rightarrow \quad M_{RA} - F(\frac{L}{3}) - D(\frac{2L}{3}) = 0 \qquad M_{RA} = F\frac{L}{3} - D\frac{2L}{3}$

$\Sigma F_{Ax} = 0 \Rightarrow \quad A_x = 0$

$\Sigma F_y = 0 \Rightarrow \quad A_y - F - P = 0 \rightarrow \qquad A_y = F + P$

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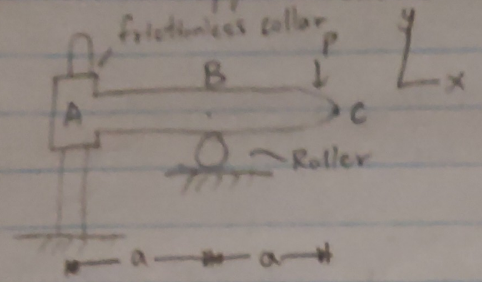
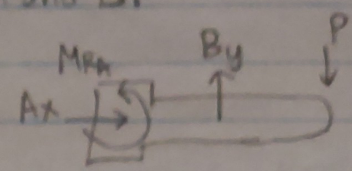
Example 4: Determine support reaction at A and B.



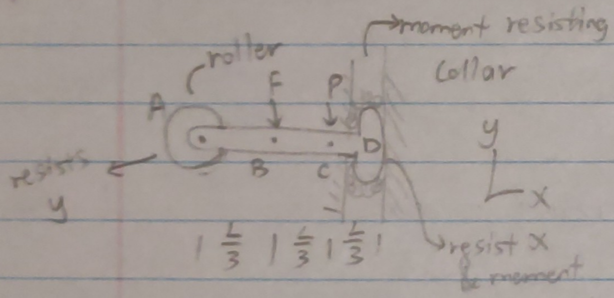


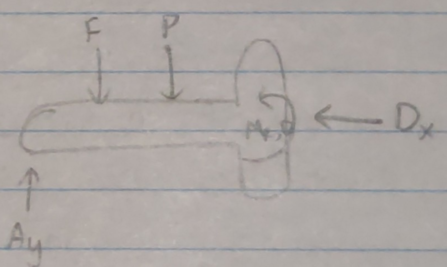
$\Sigma F_x = 0$; $\therefore A_x = 0$

$\Sigma F_y = 0$; $B_y - D = 0 \Rightarrow$ $\therefore B_y = P$

$\overset{\curvearrowleft}{\Sigma} M_A = 0$; $M_{RA} - 2Pa + B_y a = 0$ $\therefore M_{RA} = 2Pa - B_y a = Pa$

Example 5: Determine reaction at A and D.





$\Sigma M_A = 0$: $-F(\frac{L}{3}) - P(\frac{2L}{3}) + M_{R,D} = 0 \Rightarrow M_{R,D} = L(F\frac{1}{3} - P\frac{2}{3})$

$\Sigma F_x = 0$: $D_x = 0$

$\Sigma F_y = 0$: $A_y - F - P = 0$ $\Rightarrow A_y = P + F$

CH

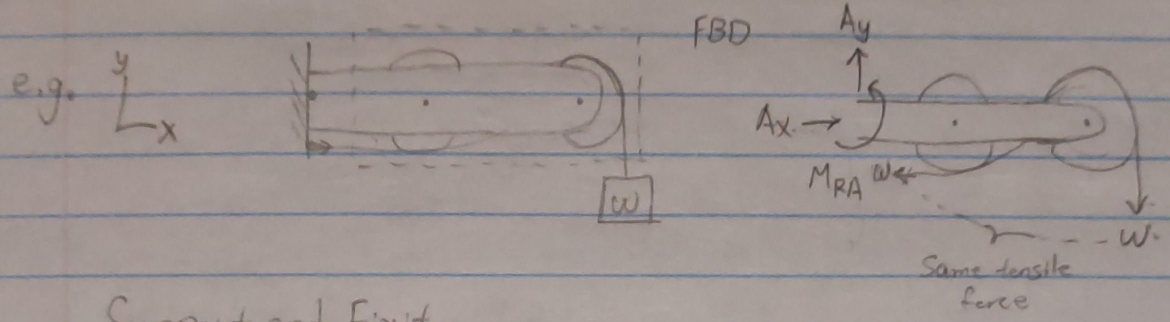
In equilibrium 2D. You'll usually get 3 eqn. x, y, M.

Lec 7: Equilibrium of Bodies in 2D - additional topic

Cables and Pullies:

Since pullies are idealized as frictionless and cables are continuous, perfectly flexible, and weightless, all pullies and cables support the same tesile force.

e.g.



Support and Fixity

(no rigid body motion) Complete fixity - support are sufficient such that body is completly in place.

Partial Fixity - body has ability to move or rotate in at least 1 direction.

(full rigid body motion) No fixity - body has no supports, can move or rotate in any direction.

Static Determinacy & Indeterminacy

Statically determinate body - equilibrium equations are sufficient to determine all unknown force and moments.

" indeterminante " - " " " insufficient " " " " " ".

Equation Counting (rule of thumb):

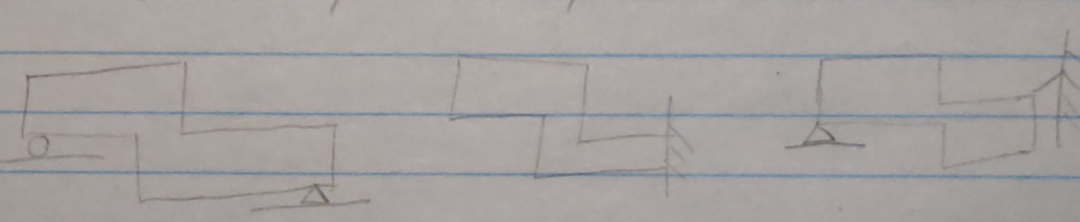
Where n is the number of unknowns, in a 2D system, if n<3

if $n<3$, the body is staticaly determinant

if $n=3$, the body " " " if body has full fixity.

" " " " indeterminant if body is not full fixity.

if $n>0$, the body is statically indeterminants



$n=3$, visual inspection show full fixity, ⇒ statically determinante

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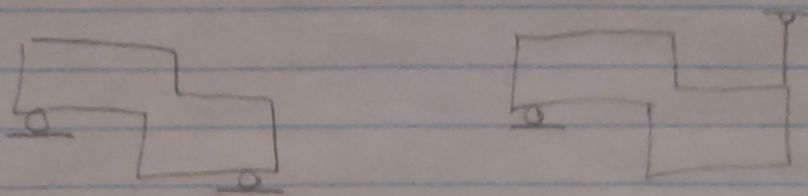
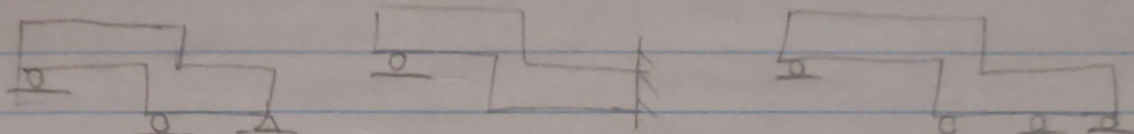
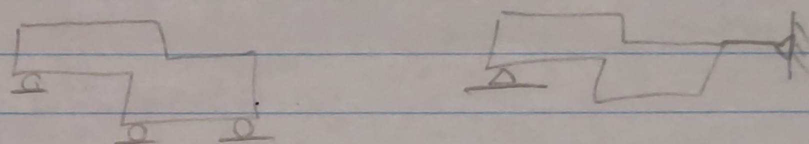


n=2; statically determinante w/ partial fixity



n=4, statically indeterminant



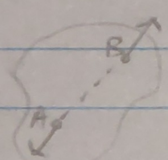
n=3 partial fixity ⇒ statically indeterminant

Two Force and Three Force Members:

only 2 force in system

→ Two Force Members: when in equilibrium, the 2 forces have the same line of action and in opposite direction.

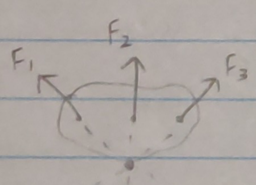
e.g.

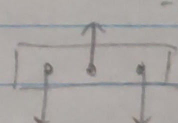


- regardless shap of obj.

Three Force Member: - the line of action of all 3 force must intersect at a common pt.

e.g.

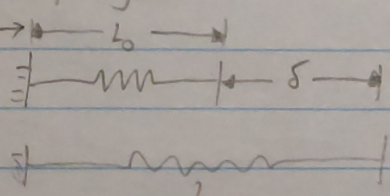




- if paralelle
- pt. intersect at ∞

Springs

original length



final length

Hook's Law

$F_s = k\delta$

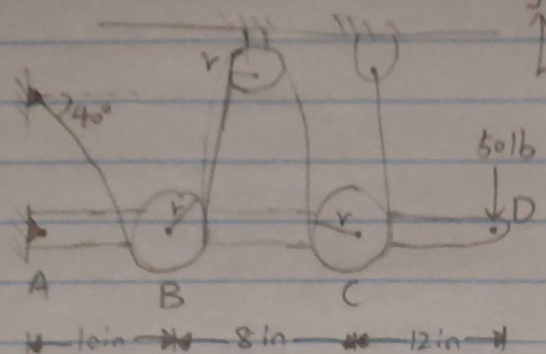
F_s - Spring force

δ - amount of elongation from equilibrium

k - spring constant

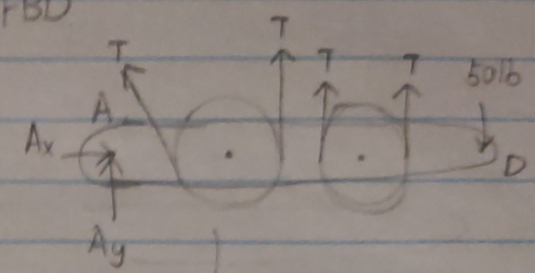
Example 1: Determine reaction at A:

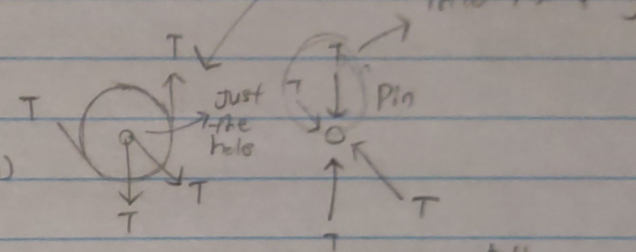
-reaction force on the pin of pullies



$r = 2\,in$

FBD

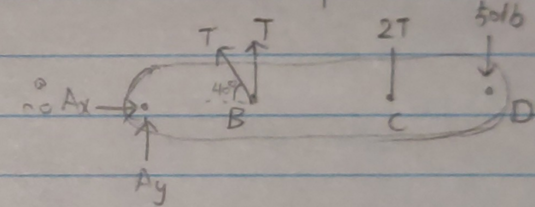




$\Sigma M_A = 0 \Rightarrow$

$0 = T\sin 40°(10in) + T(10in) + 2T(18in) - 50(30in)$

$\boxed{T \doteq 28.6\,lb}$

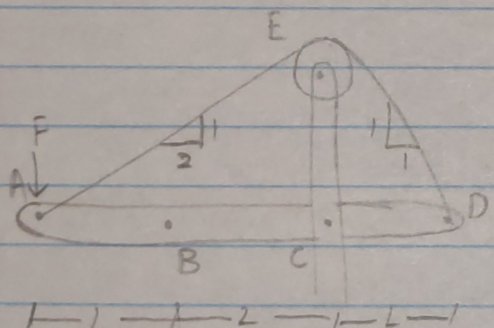


$\Sigma F_x = 0 \Rightarrow$

$0 = A_x - T\cos 40° \Rightarrow \boxed{A_x \doteq 21.9\,lb}$

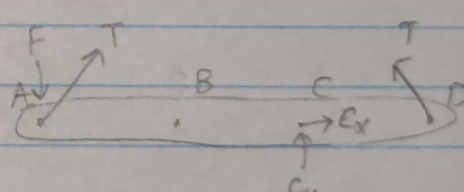
$\Sigma F_y = 0 \Rightarrow A_y + T\sin 40° + 3T - 50lb = 0 \Rightarrow \boxed{A_y \doteq -54.2\,lb}$

Example 2:



a) is ABCD statically determined? ∴ Yes!

FBD



number of unknowns

$n = 3\ (C_x, C_y, T)$

- Cable length can't change
- will not rotate
  - ↳ Fully Fixiated

b) Cable Tension Force as function (F, L), $C_x, C_y = f(F, L)$

$\Sigma M_C = 0;\ F(2L) - T(\frac{1}{\sqrt{5}})(2L) + T(\frac{\sqrt{2}}{2})(L) = 0 \Rightarrow T \doteq 10.7F$

$\Sigma F_x = 0;\ C_x + T(\frac{2}{\sqrt{5}}) - T(\frac{\sqrt{2}}{2}) = 0 \Rightarrow C_x \doteq -2F$

$\Sigma F_y = 0;\ C_y + T(\frac{1}{\sqrt{5}}) + T(\frac{1}{\sqrt{2}}) - F = 0 \Rightarrow C_y \doteq -11.2F$

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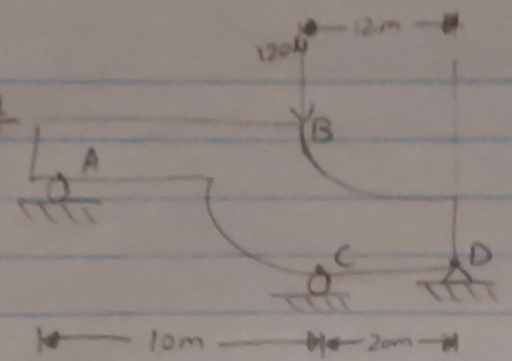
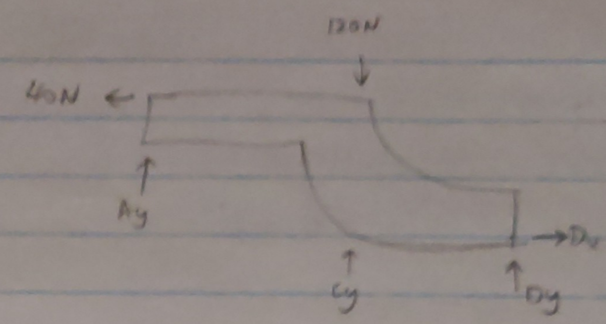
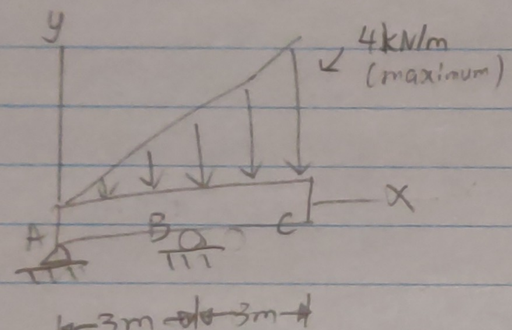
Example 3:



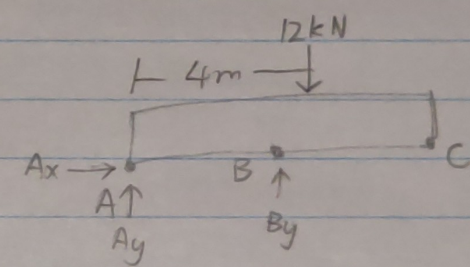


• 4 unknowns, system is statically indeterminante
∴ Cannot be solved.

Example 4:



Determine all support reaction in this system.



n=3

Complete fixity

$F = \frac{1}{2}(4kN/m)(6m)$ Centroid of F $\bar{x} = \frac{2}{3}6m = 4m$

$= 12kN$

$\Sigma F_x = 0; \; A_x = 0$

$\Sigma F_y = 0; \; A_y + B_y - 12kN = 0 \Rightarrow A_y + B_y = 12kN \Rightarrow A_y = -4kN$

$\Sigma M_A = 0; \; B_y(3m) - 12kN(4m) = 0$

$B_y = \frac{48kN \cdot m}{3m}$

$B_y = 16N$

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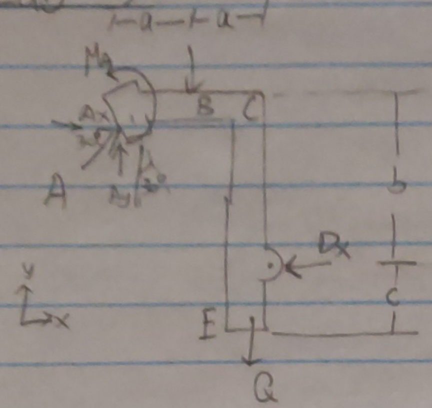
Lab 3

[5-25]

Support & Their reaction



$D_x = ?$

$A = ?$

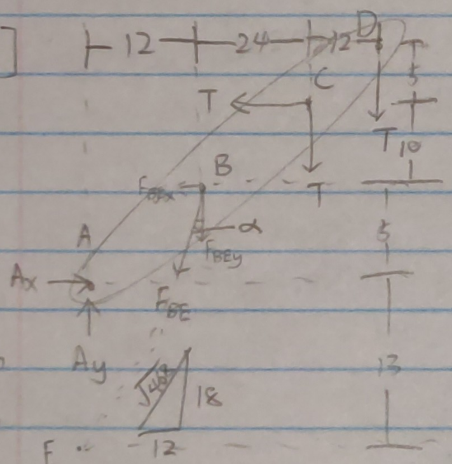
$M_A = ?$

$\Sigma F_x = 0; \quad A\cos 30° - D_x = 0 \Rightarrow D_x = A\cos 30° \Rightarrow \boxed{D_x = (P+Q)\cot 30°}$

$\Sigma F_y = 0; \quad -P - Q + A\sin 30° = 0 \Rightarrow \boxed{A = \frac{P+Q}{\sin 30°}}$

$\Sigma M_A = 0; \quad M_A - (a)P - b\,D_x - (2a)Q = 0 \Rightarrow \boxed{M_A = P(a) + (P+Q)b + (2a)Q}$

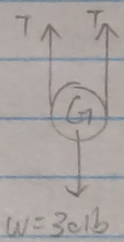
[5-56]



$A_x = ?$

$A_y = ?$

$F_{BE} = ?$



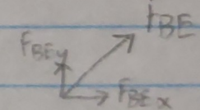
$2T = 30\text{lb}$

$\boxed{T = 15\text{lb}}$

$\sin\alpha = \frac{12}{\sqrt{468}}$

$\cos\alpha = \frac{18}{\sqrt{468}}$

direction of F_{BE}



F_{BE}, direction of reaction force is only dependant of the 2 pins.

$\Sigma M_A = 0; \quad -F_{BE}\frac{18}{\sqrt{468}}(12\text{in}) + F_{BE}\frac{12}{\sqrt{468}}(5\text{in}) - T(36\text{in}) - T(48\text{in}) + T(15\text{in}) = 0$

$F_{BE} = \boxed{-143.5\text{lb}}$

$\Sigma F_x = 0; \quad A_x - (-143.5)\frac{12}{\sqrt{468}} - 15 = 0 \Rightarrow \boxed{A_x = -64.62\text{lb}}$

$\Sigma F_y = 0; \quad A_y - (-143.5)\frac{18}{\sqrt{468}} - 15 - 15 = 0 \Rightarrow \boxed{A_y = -89.42\text{lb}}$

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Lecture 8: Equilibrium of Bodies in 3D (5.4)

Eqns for Static Equilibrium in 3D

$\Sigma F = 0$ & $\Sigma M_p = 0$ p, denotes a selected pt.

in scalar form...

$$\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma F_z = 0 \quad \& \quad \Sigma M_{px} = 0, \quad \Sigma M_{py} = 0, \quad \Sigma M_{pz} = 0$$

(Fig. 5.23) Common 3D supports in txbk.
(Fig 5.25) for correct FBD

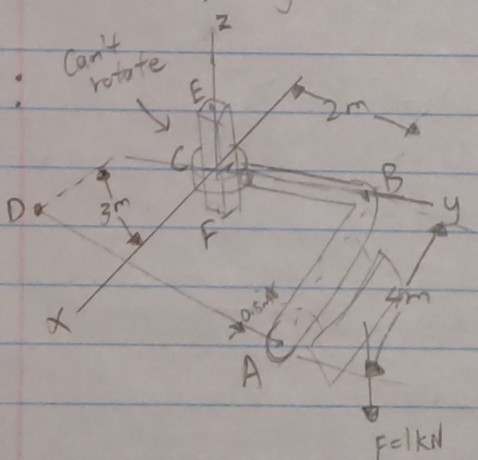
Fixity and Static Determinancy in 3D

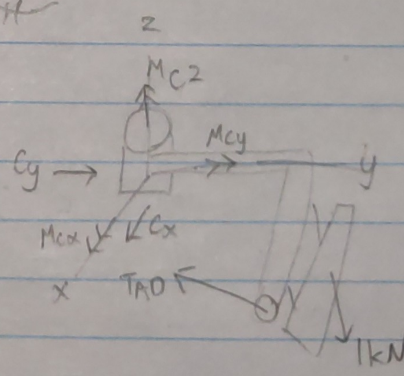
Def. for fixity is same as in 2D. (pg. 21)

For determinancy in 3D (6 eqns, similar to 2D but n at '6')

- if $n < 6$, body is statically determinante, and can have partial fixity ($n = 1,2,3,4,5$) or no fixity ($n = 0$)
- $n = 6$, body is determinant if at full fixity.
- $n > 6$, body is indeterminant.

Example: (P5.102)





$$A \times B = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

$\vec{T}_{AD} = T_{AD}\hat{r} = T_{AD}\left(\frac{\hat{\imath}-2\hat{\jmath}+3\hat{k}}{\sqrt{1^2+2^2+3^2}}\right) = T_{AD}\left(\frac{\hat{\imath}-2\hat{\jmath}+3\hat{k}}{\sqrt{14}}\right)$

$\vec{F} = -1\hat{k}\ kN$

$\Sigma F = 0 \Rightarrow \quad (-1\hat{k})kN + T_{AD}\left(\frac{\hat{\imath}-2\hat{\jmath}+3\hat{k}}{\sqrt{14}}\right) + C_x\hat{\imath} + C_y\hat{\jmath} = 0$

$\Sigma M_C = 0 \Downarrow$

$M_{Cz}\hat{k} + M_{Cy}\hat{\jmath} + M_{Cx}\hat{\imath} + (r_{CA}\times T_{AD}) + (r_{CO}\times F) = 0$

So, $(r_{CA}\times T_{AD}) = \frac{T_{AD}}{\sqrt{14}}(6\hat{\imath} - 12\hat{\jmath} - 10\hat{k})m$

$C_x + \frac{1}{\sqrt{14}}T_{AD} = 0 \Rightarrow C_x = -0.333kN$

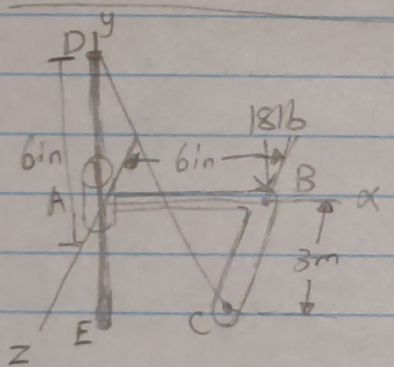
$C_y - \frac{2}{\sqrt{14}}T_{AD} = 0 \Rightarrow C_y = 0.667kN$

$-1kN + \frac{3}{\sqrt{14}}T_{AD} = 0 \overset{③}{\Rightarrow} T_{AD} = 1.25kN$

$(r_{OC}\times F) = (2\hat{\imath}+2\hat{\jmath})\times(-1\hat{k}) = -2\hat{\imath} + 2\hat{\jmath}\ kN\cdot m$

$(-2kN\cdot m + M_{Cx} + \frac{6m}{\sqrt{14}}T_{AD})\hat{\imath} + (+2kN\cdot m + M_{Cy} - \frac{12m}{\sqrt{14}}T_{AD})\hat{\jmath} + (M_{Cx} - \frac{10m}{\sqrt{14}}T_{AD})\hat{k} = 0$

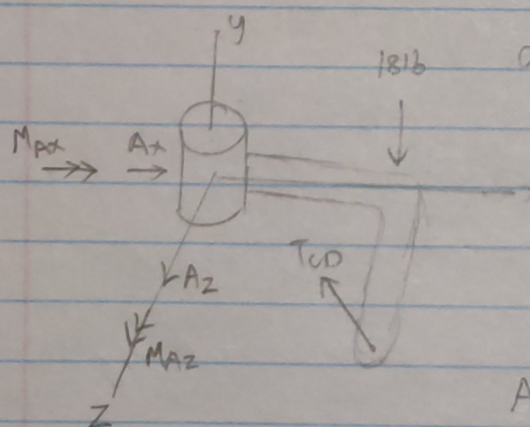
Exmple 2:



$M_{Cz} = 3.33kN\cdot m \Leftarrow \quad M_{Cz} - \frac{10}{\sqrt{14}}T_{AD} = 0$

$M_{Cy} = 2kN\cdot m \Leftarrow \quad 2kN\cdot m + M_{Cy} - \frac{12}{\sqrt{14}}T_{AD} = 0$

$M_{Cx} = 0 \Leftarrow \quad -2kN\cdot m + M_{Cx} + \frac{6}{\sqrt{14}}T_{AD} = 0$



a) n=5, n<6, statically determinant

b) $\vec{T}_{CD} = T_{CD}\left(\frac{-6\hat{\imath}+6\hat{\jmath}-3\hat{k}}{9}\right) =$

$\Sigma F_x = A_x\hat{\imath} + A_z\hat{k} - 18\hat{\jmath} + T_{CD}\left(\frac{-6\hat{\imath}+6\hat{\jmath}-3\hat{k}}{9}\right) = 0$

$(A_x - \frac{2}{3}T_{CD})\hat{\imath} + (A_z - 3T_{CD})\hat{k} + (-18 + \frac{2}{3}T_{CD}) = 0$

$$\left.\begin{array}{l} A_x - \frac{2}{3}T_{CD} = 0 \\ A_z - 3T_{CD} = 0 \\ -18 + \frac{2}{3}T_{CD} = 0 \end{array}\right\} \begin{array}{l} A_x = 18lb \\ A_z = 9lb \\ T_{CD} = 27lb \end{array}$$

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$\Sigma M_p = 0: \; M_{Ax}\hat{\imath} + M_{Az}\hat{k} + (r_{AB} \times (-18\hat{\jmath})) + (r_{AC} \times T_{CD}) = 0$

$\boxed{r_{AB} \times (-18j) = (18\,lb)(6in)\hat{k} =}$

$r_{AC} = 6\hat{\imath} + 3\hat{k}, \quad r_{AB} = 6\hat{\imath}, \quad T_{CD} = \frac{1}{3}T_{CD}(-2\hat{\imath} + 2\hat{\jmath} - \hat{k})$

$\boxed{r_{AC} \times T_{CD} = \frac{1}{3}T_{CD}[-6\hat{\imath} \quad +12\hat{k}] = 2T_{CD}(-\hat{\imath} + \quad +2\hat{k})\,lb}$

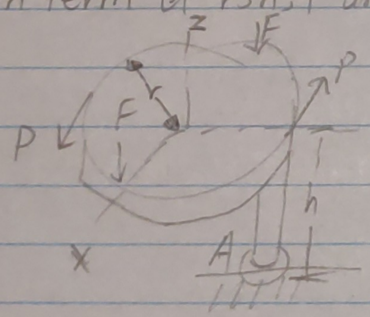
$\Sigma M_A = 0, \quad x \quad (M_{Ax} - 2T_{CD})\hat{\imath} = 0 \qquad \Rightarrow M_{Ax} = 54\,lb\cdot in$

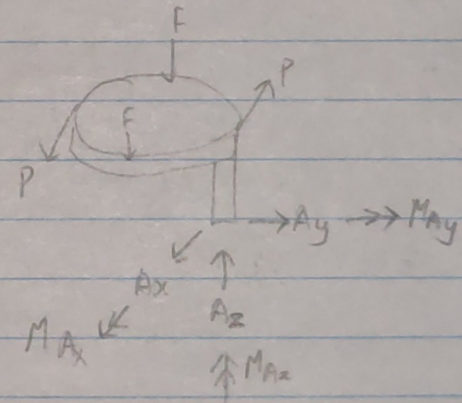
$y \quad \theta$

$z \quad (M_{Az} + 4T_{CD} - (18\,lb)(6\,in))k = 0 \quad \Rightarrow M_{Az} = 0$

Lecture 9: Equilibrium of Bodies in 3D (FMS 5.4) Examples:

Ex. 1: A circular plate of radius r is welded to a post with length h that is built in at point A. Determine the reaction at this point. Express your answers in term of r, h, F and P.





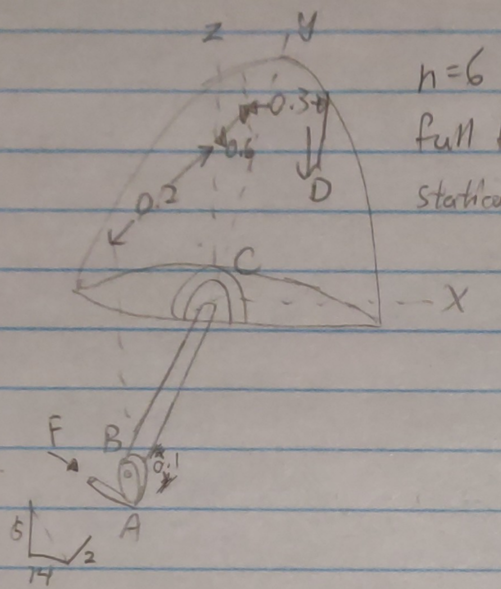
n=6 , fixed
Statically Determinate

$\Sigma \vec{F} = 0 \quad \underbrace{(P + A_x - P)}_{A_x = 0}\hat{\imath} + \underbrace{(A_y)}_{A_y = 0}\hat{\jmath} + \underbrace{(-2F + A_z)}_{A_z = 2F}\hat{k} = 0$

$\Sigma M_A = 0: \quad (M_{Ax} + 2F(r))\hat{\imath} + (M_{Ay} + F(r) - F(r) + P(h) - P(h))\hat{\jmath} + (M_{Az} + P(2r))\hat{k} = \vec{0}$

$M_{Ax} = -2Fr, \; M_{Ay} = 0 \; ; \; M_{Az} = -2Pr$

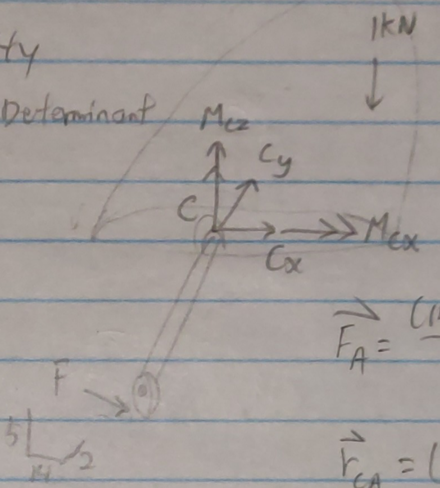
Ex 2: The Central Surface of an aircraft is supported by a thrust bearing a pt. C, and is actuated by a bar connected to point A. The 1kN acts in the negative z direction and the line connecting pt A to B is parallel to the z axis. Determine force F required to maintain Equilibrium.



$n=6$

full fixity

Statically Determinant



$$\vec{F}_A = \frac{(14\hat{i}+2\hat{j}-5\hat{k})F}{\sqrt{225}}$$

$$\vec{r}_{CA} = (-0.2\hat{j}-0.1\hat{k})\,m$$

$$\Sigma\vec{F}=0;\ 0=C_x+F\left(\frac{14}{\sqrt{225}}\right)\hat{i}+\left(C_y+F\frac{2}{\sqrt{225}}\right)\hat{j}+\left(C_z-1kN+F\frac{-5}{\sqrt{225}}\right)\hat{k} \quad \vec{F}_D=-1\hat{k}\ kN$$

4 unkown, 3 eqn.

$$\vec{r}_{CD}=(0.3\hat{i}+0.6\hat{j})\,m$$

$$\Sigma M_C=\vec{0};\quad M_{Cx}\hat{i}+M_{Cz}\hat{k}+(\vec{r}_{CA}\times\vec{F}_A)+(\vec{r}_{CD}\times\vec{F}_D)=0$$

$$\left[M_{Cx}+F\frac{5}{\sqrt{225}}(0.2)-1kN(0.6)+F\frac{2}{\sqrt{225}}(0.1)\right]\hat{i}+\left[-F\left(\frac{14}{\sqrt{225}}\right)0.1+1kN(0.3)\right]\hat{j}$$
$$+\left[M_{Cz}+(F)\frac{14}{\sqrt{225}}0.2\right]\hat{k}=0$$

$C_x=-3kN$

$C_y=-0.429kN$

$C_z=2.07kN$

$M_{Cx}=0.343\ kN\cdot m$

$M_{Cy}=-0.6\ kN\cdot m$

$F=3.21kN$

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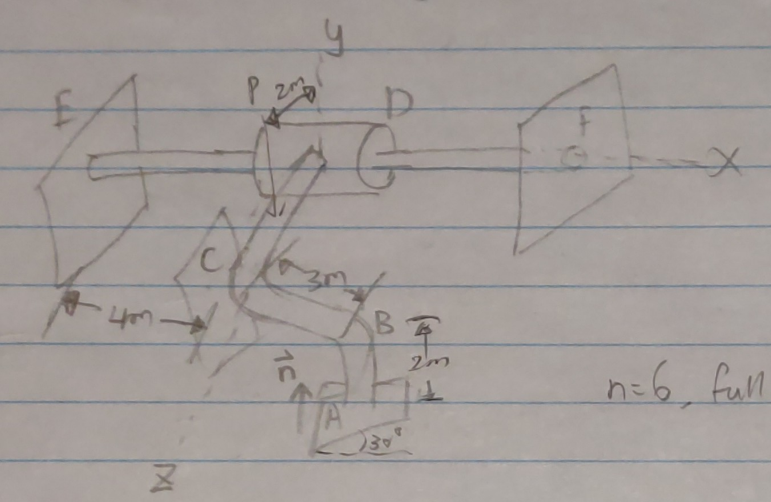
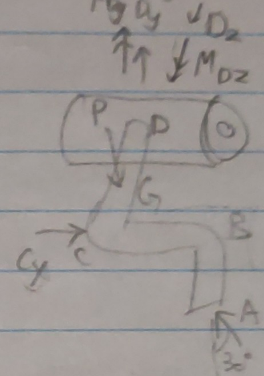
Example 3: Structure A,B,C,D is supported by a collar at D that can rotate and slide along bar EF, which is fixed and frictionless. Structure ABCD makes contact w/ smooth surface at A and C, where the normal vector $\hat{n}$ to the surface at A lies in a plane that is parallel to the xy plane. Force P is parallel to the y axis. If P=10kN. Determine reaction at A, C and D.





n=6, full fixity by inspection

$F_G = -P\hat{j}$ $\quad F_C = C_x\hat{i}$ $\quad F_A = (A)(-\sin 30\hat{i} + \cos 30\hat{j})$

$\vec{r}_{OG} = 2\hat{k}$ $\quad \vec{r}_{OC} = 4\hat{k}$ $\quad \vec{r}_{OA} = (3\hat{i} - 2\hat{j} + 4\hat{k})m$

$\Sigma\vec{F} = 0;\ ((C_x - A\sin 30)\hat{i} - (D_y + A\cos 30 - 10)\hat{j} + (D_z)\hat{k} = \hat{0}$

$\Sigma\vec{M}_O = 0;\ (M_{Dy}\hat{j} + M_{Dz}\hat{k}) + (\vec{r}_{OA} \times \vec{F}_A) + (\vec{r}_{OC} \times \vec{F}_C) + (\vec{r}_{OG} \times F_G) = \hat{0}$

$0 = [(10kN)(2m) - (A\cos 30)(4m)]\hat{i} + [M_{Dy} + C_x 4 - A\sin 30(4)]\hat{j} + [M_{Dz} + A\cos 30(3) - A\sin 30(2)]\hat{k}$

$20kN\cdot m - A\cos 30(4) = 0 \rightarrow A = 5.78kN$

$D_y - A\cos 30 - 10 = 0 \rightarrow D_y = 5.0kN$

$C_x - A\sin 30 = 0 \rightarrow C_x = 2.89kN$

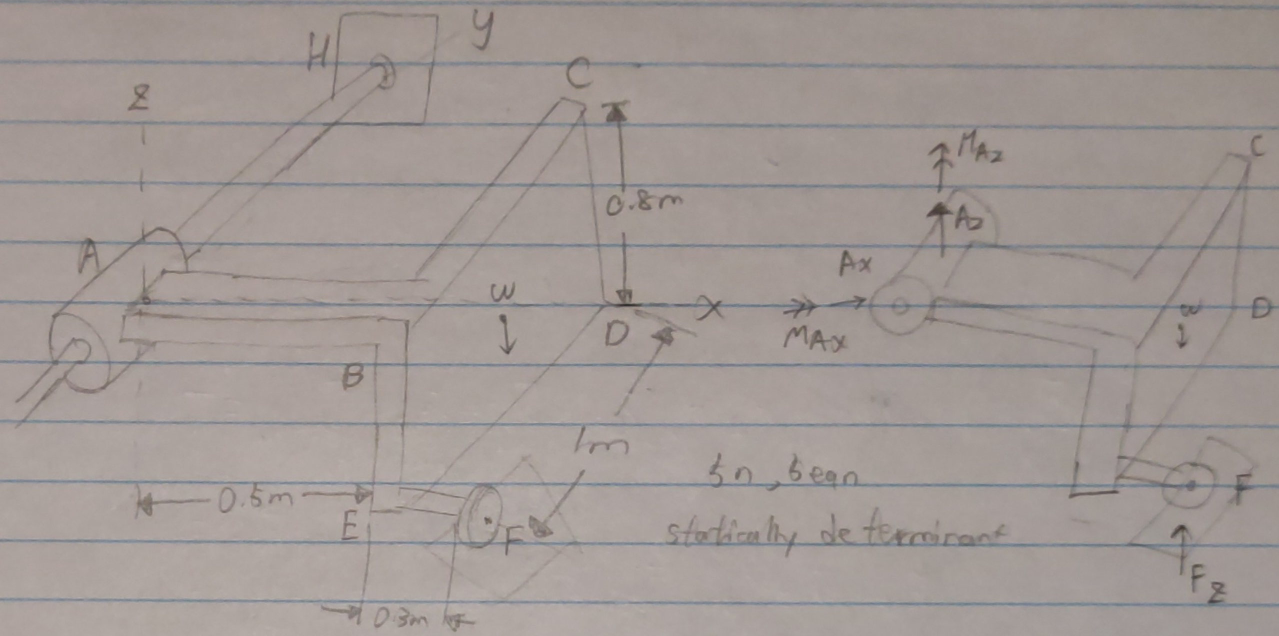
$M_{Dy} + C_x(4) - A\sin 30(4) = 0 \quad M_{Dy} = 0$

$M_{Dz} + A\cos 30(3) - A\sin 30(2) = 0 \rightarrow M_{Dz} = -9.2\,kN\cdot m$

$D_z = 0$

18/004

Example 4: Obj. ABCDEF is a sliding door supported by a frictionless bearing at A and a wheel at F that rest on a frictionless surface. The object has weight $w = 800N$, which acts at the midpoint of the rectangular region BCDE. Determine all support reactions.



5n, 6eqn
statically determinant

$\vec{F}_F = F_z\hat{k}$ $\quad\quad$ $\vec{r}_{AF} = 0.8\hat{\imath} - 0.8\hat{k}$

$\vec{F}_w = -w\hat{k}$ $\quad\quad$ $\vec{r}_{Aw} = 0.5\hat{\imath} + 0.5\hat{\jmath} - 0.4\hat{k}$

$\Sigma\vec{F} = \vec{0}$: $\quad (A_x)\hat{\imath} + (0)\hat{\jmath} + (A_z + F_z - w)\hat{k} = 0$

$A_x = 0$ $\quad\quad$ $A_z + F_z - w = 0$ $\quad\quad$ $A_z = 300$

$\Sigma\vec{M}_A = \vec{0}$: $M_{Ax}\hat{\imath} + M_{Az}\hat{k} + (\vec{r}_A \times \vec{F}_F) + (\vec{r}_{Aw} \times \vec{F}_w) = \vec{0}$

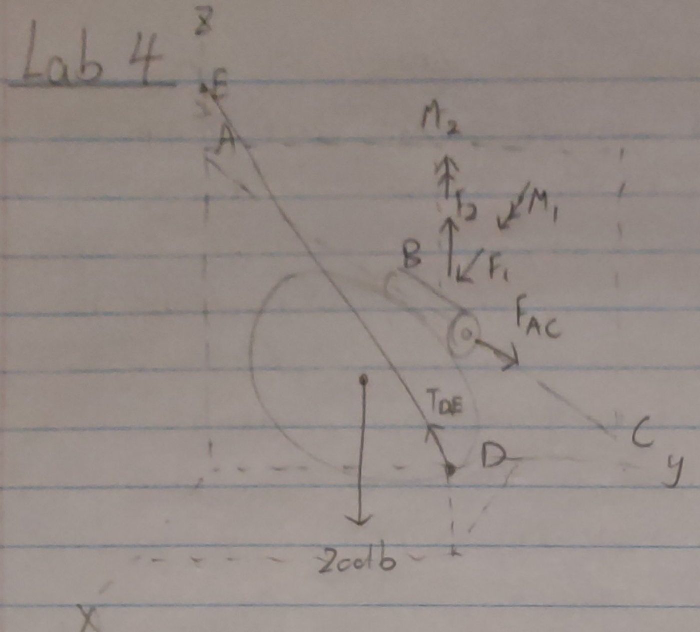
$(M_{Ax} - 400)\hat{\imath} + (400 - 0.8F_z)\hat{\jmath} + (M_{Az})\hat{k} = 0$

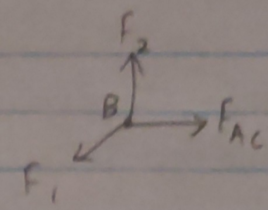
$M_{Ax} = 400$ $\quad\quad$ $F_z = 500$ $\quad\quad$ $M_{Az} = 0$

181004

Lab 4

5.111





D(9, 18, 3) in
F(5, 13, 5) in

- if we dot product by $\hat{r}_{AC}$, M_1 & M_2 = 0. (perpendicular)

* Not traditional way of sol.

$$\sum \vec{M}_B = 0 : \left[\underbrace{M_1}_{\times} + \underbrace{M_2}_{\times} + (\vec{r}_{BF} \times \vec{W}) + (\vec{r}_{BD} \times \underbrace{\vec{T}_{DE}}_{\times})\right] = \vec{0} \quad \cdot \hat{r}_{AC}$$

$\vec{r}_{BF} = 5\hat{i} + \hat{j}$

$\vec{W} = -200\,lb\,\hat{k}$

$\vec{r}_{BD} = 9\hat{i} + 6\hat{j} - 2\hat{k}$

$\hat{r}_{AC} = \dfrac{\vec{r}_{AC}}{|\vec{r}_{AC}|} = \dfrac{24\hat{j} - 10\hat{k}}{26}$

$\vec{T}_{DE} = T_{DE}\left(\dfrac{\vec{r}_{DE}}{|\vec{r}_{DE}|}\right) = \dfrac{-9\hat{i} - 18\hat{j} + 18\hat{k}}{27} T_{DE}$

$$\underbrace{(\vec{r}_{BF} \times \vec{W} + \vec{r}_{BD} \times \vec{T}_{DE})}\cdot \hat{r}_{AC} = 0$$

$$\left[\begin{vmatrix} i & j & k \\ 5 & 1 & 0 \\ 0 & 0 & -200 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 9 & 6 & -2 \\ -9 & -18 & 18 \end{vmatrix} \frac{T_{DE}}{27}\right] = [-200\hat{i} - (-1000)\hat{j}] + [72\hat{i} - 144\hat{j} - 108\hat{k}]\frac{T_{DE}}{27}$$

dot product

$$[(-200)(0) + (1000)(\tfrac{24}{26}) + (0)(\tfrac{-10}{26})] + [72(0) + (-144)(\tfrac{24}{26}) + (-108)(\tfrac{-10}{26})]\frac{T_{DE}}{27} = 0$$

$$\therefore T_{DE} = 27\,\frac{(1000)(\frac{24}{26})}{(-144)\frac{24}{26} + (-108)(\frac{-10}{26})} = 273\,lb$$

Lecture 10: Truss Structures & the Method of Joints

A truss is a structure that consists of two-force members only (2 force along same line of action and no moment). Truss structure in 2D are called plane truss, and have the following characteristics:

- lower weight
- Economically Efficient

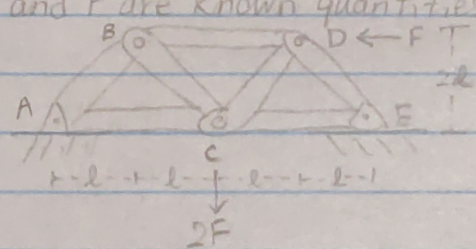
- All members are connected by frictionless pins, location of pins are called joints.
- each member may have no more than 2 joints.
- forces can only be applied to joints
- for this course → weight of each members are negligible

Since all members & pins of a truss structure are in static equilibrium, we can use equil. anologies to study forces in pins and truss.

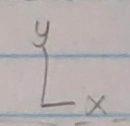
Method of Joints

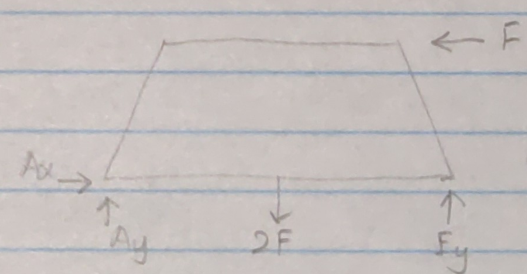
Draw FBDs for all joints & members to obtain a system of eqns.

Ex1. Use MOJ to determine the forces supported by each member of the truss shown. ℓ and F are known quantities.



FBD of entire structure





☆ Start w/ Joint w/ 2 unknowns.

☆ All force on joint start ⊕ away from joint

$\Sigma M_A = 0$: $-2F(2\ell) + F(2\ell) + E_y(4\ell) = 0$

$E_y 4\ell = 4F\ell - 2F\ell$

$E_y = F - \frac{1}{2}F = \frac{1}{2}F$

$\Sigma F_x = 0$; $A_x - F = 0 \Rightarrow A_x = F$

$\Sigma F_y = 0$; $A_y - 2F + E_y = 0 \Rightarrow A_y = \frac{3}{2}F$

Joint A

$\Sigma F_x = 0$

$F + F_{AC} + F_{AB}\cos\alpha = 0$

$\Sigma F_y = 0$

$\frac{3}{2}F + F_{AB}\sin\alpha = 0$

$\therefore F_{AC} \doteq 0.23F$, $F_{AB} \doteq -1.7F$

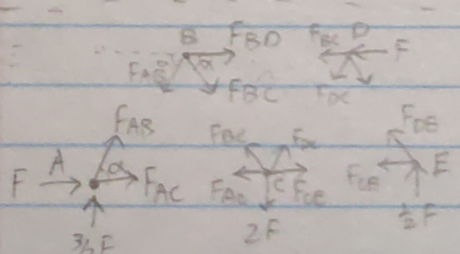
Joint B

$\Sigma F_x = 0$: $F_{BD} + F_{BC}\cos\alpha - F_{AB}\cos\alpha = 0$

$\Sigma F_y = 0$; $-F_{AB}\sin\alpha - F_{BC}\sin\alpha = 0$

$\therefore F_{BC} \doteq 1.7F$, $F_{BD} \doteq -1.5F$

FBD for pins



$\alpha = \tan^{-1}\left(\frac{2\ell}{1}\right) = 63°$

Joint E

$\Sigma F_x = 0$; $-F_{CE} - F_{DE}\cos\alpha = 0$

$\Sigma F_y = 0$; $\frac{1}{2}F + F_{DE}\sin\alpha = 0$

$\Big\}$ $F_{DE} \doteq -0.6F$, $F_{CE} = 0.3F$

Joint D

$\Sigma F_y = 0$, $-F_{CD}\sin\alpha - F_{DE}\sin\alpha = 0$ $\therefore F_{CD} \doteq 0.6F$

8/01

Zero-force Members

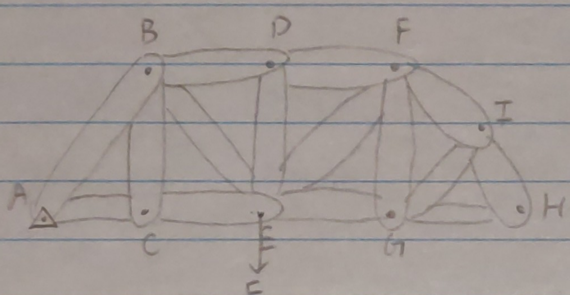
Carries no forces. Creates redundancy in event when other truss fails, or when position shifts. We can quickly identify 0 force member. But not all of them.

if a particular joint:

- has 3 member connected to them
- 2 of them are collinear, and
- the joint has no external force applied to it,

then the non-collinear member is a 0 force member.

Ex2.



0 force mem by inspection.

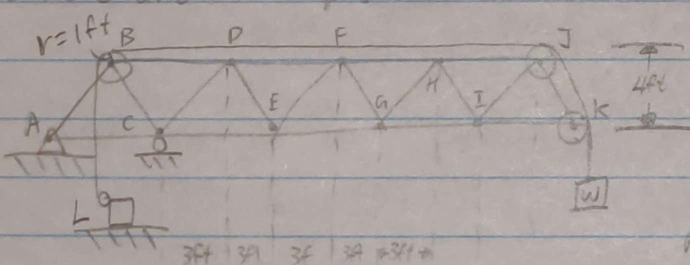
BC, GI

GF, b/c GI is 0 force member

DE, b/c no force on D

Note: these might not be the only ZFM; A full analysis is needed.

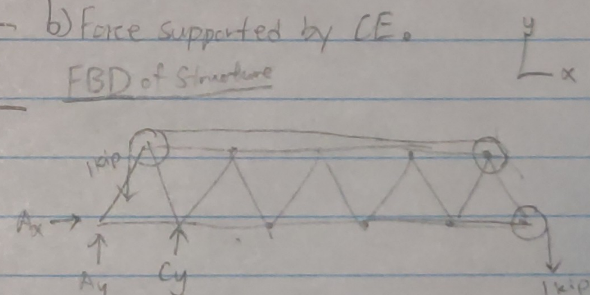
Ex3. The truss has frictionless pulley at pt. B, J, and k. Cable segment BL is verticle and w = 1kip.



a) ZFM by inspection. (No ZFM)

b) Force supported by CE.

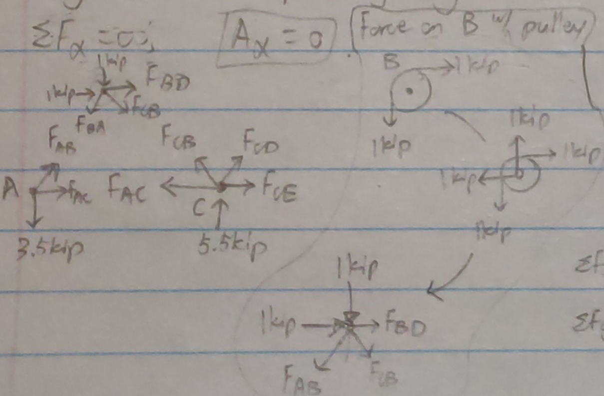
FBD of Structure



$\Sigma M_A = 0$; $C_y(6ft) - 1kip(2ft) - 1kip(3\ ft) = 0$, $C_y \doteq \frac{33}{6} kip \doteq 5.5 kip$

$\Sigma F_y = 0$; $-2kip + A_y + C_y = 0$, $A_y \doteq 2kip - 5.5kip = -3.5 kip$

$\Sigma F_x = 0$; $\boxed{A_x = 0}$



Joint A

$F_{AB} \doteq 4.375 kip$

$F_{AC} = -2.625 kip$

Joint C

$\Sigma F_y = 0$; $F_{CD} \doteq -1.25 kip$

$\Sigma F_x = 0$; $F_{CE} \doteq -5.25 kip$

Joint B

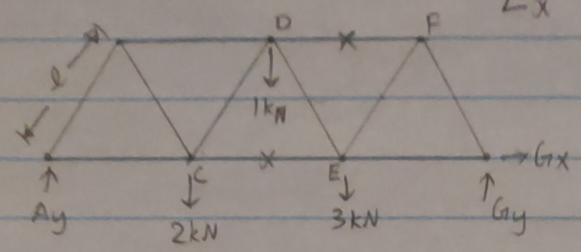
$\Sigma F_x = 0$, $F_{BD} \doteq$ (not useful for us)

$\Sigma F_y = 0$, $F_{BC} \doteq -6.62$

Lab 5

[6.8]

[MoJ]



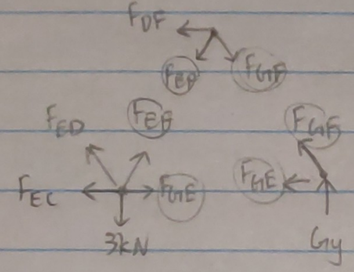
$\Sigma M_G = 0;$

$2kN(2\ell) + 3kN(\ell) - 3A_y\ell + 1kN(1.5\ell) = 0$

$\frac{1}{3\ell}(4kN\cdot\ell + 3kN\cdot\ell + 1.5kN\cdot\ell) = A_y$

$2.833\,kN \doteq A_y$

$\Sigma F_x = 0;\ G_x = 0 \qquad \Sigma F_y = 0;\ A_y + G_y - 2kN - 3kN - 1kN = 0,\ G_y \doteq 3.117kN$



Joint G

$\Sigma F_x = 0;\quad -F_{GE} - F_{GF}\cos 60° = 0 \Rightarrow F_{GE} = 1.8kN$

$\Sigma F_y = 0;\quad G_y + F_{GF}\sin 60° \quad = 0 \Rightarrow F_{GF} = -3.6kN$

~~co CE = -3.53 kN~~

~~DF = -3.6 kN~~

Joint F

$\Sigma F_y = 0;\ -F_{GF}\sin 60° - F_{EF}\sin 60° = 0,\quad F_{EF} = +3.6kN$

$\Sigma F_x = 0;\ -F_{DF} + F_{EF}\cos 60° + F_{GF}\cos 60° = 0,\ \boxed{F_{DF} = -3.6kN}$

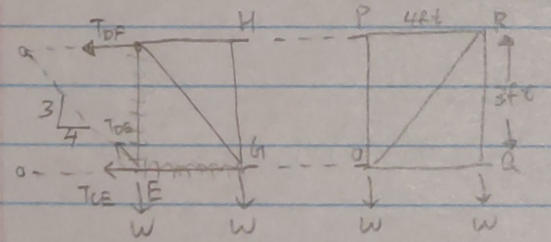
Joint E

$\Sigma F_y = 0;\ F_{ED}\sin 60 + F_{EF}\sin 60 - 3kN = 0,\ F_{ED} = -0.1933kN$

$\Sigma F_x = 0;\ F_{GE} + F_{EF}\cos 60 - F_{ED}\cos 60 - F_{EC} = 0,\ \boxed{F_{EC} = -3.753kN}$

[6.52]

(By Section)



Sections: ① $\Sigma F_y = 0;\ -7(80lb) + T_{DE}(3/5) = 0,\ T_{DE} = 583.3lb$

② $\Sigma M_E = 0;\ T_{DF}(3) - w(4+8+12+16+20+24) = 0$

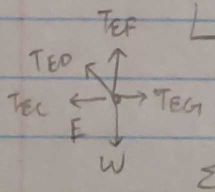
$T_{DF} = \frac{w(4+8+12+16+20+24)}{3}$

$T_{DF} = 14000lb$

③ $\Sigma F_x = 0;\ -T_{DF} - T_{CE} - T_{DE}(4/5) = 0$

$T_{CE} = -18664lb$

start by the part that have least unknown (no support)



still write as negative

$\Sigma F_x = 0;\ -T_{CE} + T_{EG} - T_{DE}(4/5) = 0,\ T_{EG} = -14{,}000lb$

$\Sigma F_y = 0;\ -T_{DE} - T_{CE} - T_{DE}(4/5) = 0$

181015

Truss Structures and the Method of Sections (EMS 6.2)

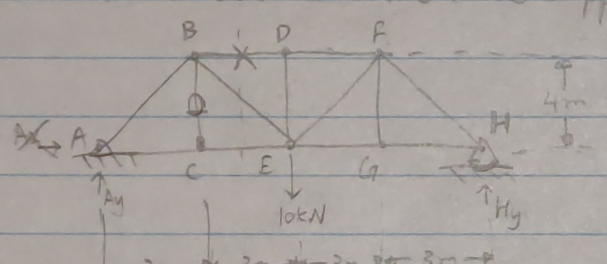
To reduce the workload in solving truss problems, we can use the method of Sections. This method allows one to solve only for the forces in one sections of a truss.

This is the method:

1. Select the member whose force you want to find.
2. Draw FBD of the entire truss and find reaction force.
3. Take cut through the members in question to divide the truss into 2 sections.
4. Draw FBD of 1 of the section
5. Write equil. eqn., and solve.

Example 1: Use MoS to determine the force supplied by member BD.

Method of Sections



$\Sigma M_A = 0;\ -10kN(6m) + H_y(12m) = 0$

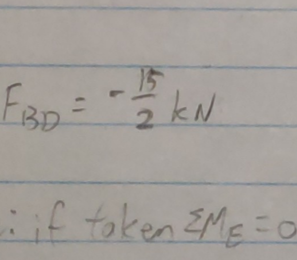
$H_y = 5kN$

$\Sigma F_x = 0;\ A_x = 0$

$\Sigma F_y = 0;\ A_y - 10kN + H_y = 0$

$A_y = 5kN$

if cut go through 3 members it can be solved



$\Sigma F_y = 0;\ A_y - F_{BE}\left(\frac{4}{5}\right) = 0$

$F_{BE} = 5kN\left(\frac{5}{4}\right)$

$\Sigma M_B = 0;\ -A_y(3m) + F_{CE}(4m) = 0$

$F_{CE} = \frac{3}{4}(5)kN$

-each cut produce 1 unknown

-3 equilibrium eqn.

$\Sigma F_x = 0;\ F_{BD} + F_{CE} + F_{BE}\left(\frac{3}{5}\right) = 0$

$F_{BD} = -F_{CE} - \frac{3}{5}(F_{BE})$

$= \frac{-3}{4}(5kN) - \frac{3}{5}\left(\frac{5}{4}5kN\right)$

$= 5\left(\frac{-3}{4} - \frac{3}{4}\right)$

$F_{BD} = -\frac{15}{2}kN$

$\therefore F_{BD} = -\frac{15}{2}kN$

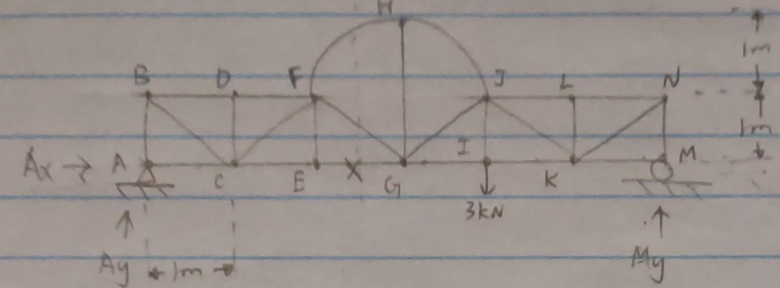
Note: if taken $\Sigma M_E = 0$. Solve in 1 step.

$\Sigma M_E = 0;\ -A_y(6m) - F_{BD}(4m) = 0$

$F_{BD} = \frac{-3}{2}(5kN)$

$F_{BD} = \frac{-15}{2}kN$

Example 2: Determine the force supported by EG.



$\Sigma F_x = 0;\ A_x = 0$

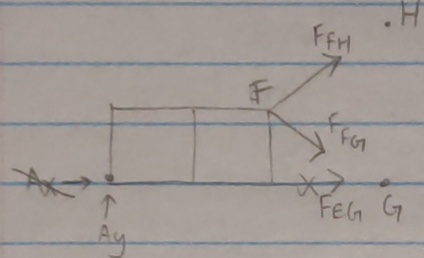
$\Sigma M_A = 0;\ -3kN(4m) + M_y(6m) = 0$

$M_y = \frac{12}{6} kN$

$M_y = 2kN$

$\Sigma F_y = 0;\ A_y - 3kN + M_y = 0$

$A_y = 1kN$

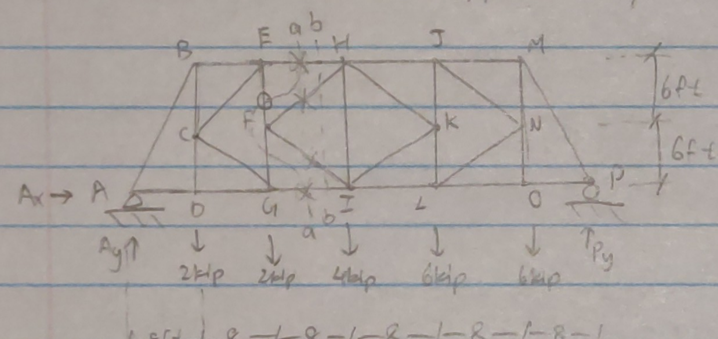


$\Sigma M_F = 0;\ -A_y(2m) + F_{EG}(1m) = 0$

$F_{EG} = 2kN$

$\therefore F_{EG} = 2kN$

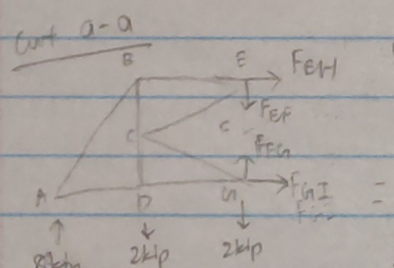
Example 3: Determine the force supported by member EH, FH, FI and GI.



$\Sigma F_x = 0,\ A_x = 0$

$\Sigma M_p = 0;\ -A_y(48) + 2(40) + 2(32) + 4(24) + 6(16) + 6(8) = 0$

$8kip = A_y$

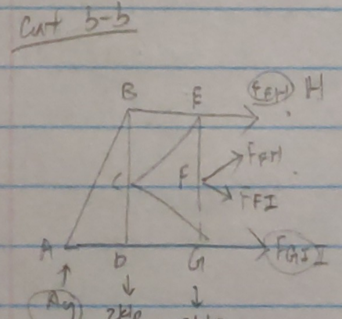


$\Sigma M_G = 0;\ 2kip(8) - 8(16) - F_{EH}(12) = 0$

$F_{EH} = \frac{16 - 8(16)}{12}$

$\doteq -9.3kip$

$\Sigma F_x = 0;\ F_{EH} + F_{GI} = 0 \Rightarrow F_{GI} \doteq 9.3kip$



$\Sigma M_H = 0;\ 2(8) + 2(16) - 8(24) + 9.3kip(12) + 6(F_F)\frac{4}{5} + 8(F_{FI})\frac{3}{5} = 0$

$F_I(\frac{24}{5} + \frac{24}{5}) = 32.4$

$F_I = 3.375kip$

$\Sigma F_x = 0;\ F_{EH} + F_{GI} + F_{FH}(\frac{4}{5}) + F_{FI}(\frac{4}{5}) = 0$

$F_{FI} = [-F_{EH} - F_{GI} - F_{FH}(\frac{4}{5})]\frac{5}{4}$

$= -3.3kip$

$\boxed{F_{FH} = -3.3 \quad F_{FI} = -3.3}$

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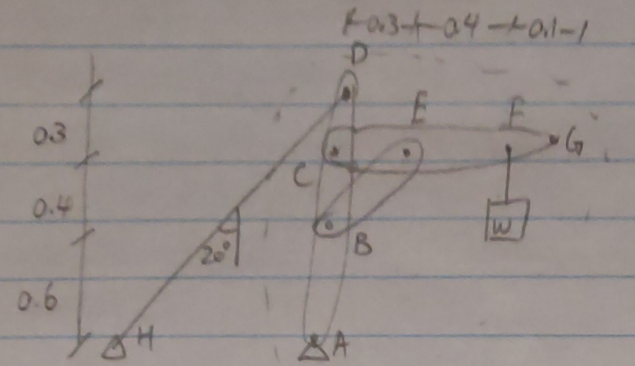
Frames and Machine (EMS)

Example 1



A fram used for supporting a weight $W = 800N$ is shown. Determine force supported by member ABCD, CEFG, and BE. and Cable DH.

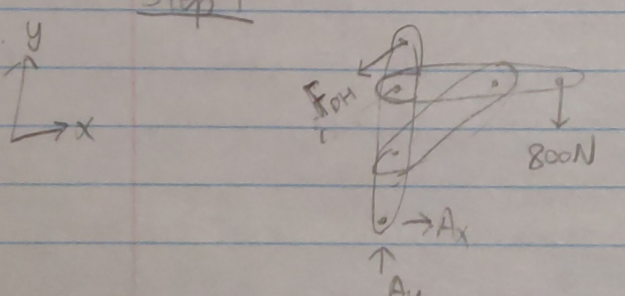
Frame - a stationary structure that is composed of members.

Machine - a structure of multiple members, member can move relative to each other. (we assume static equilibrium works)

Solution Procedure

1. Where possible, draw FBD of entire structure to obtain reaction force and moments. (not always possible)
2. Disassemble frame or machine into individual members, and draw a FBD for each member, indicate all forces and moments.
3. Apply static equil. eqn to each member's FBD and solve for unknown.

Step 1



$\Sigma M_A = 0;\ F_{DH}(\sin 20)(1.3m) - 800N(0.7N) = 0$

$F_{DH} = 1259N$

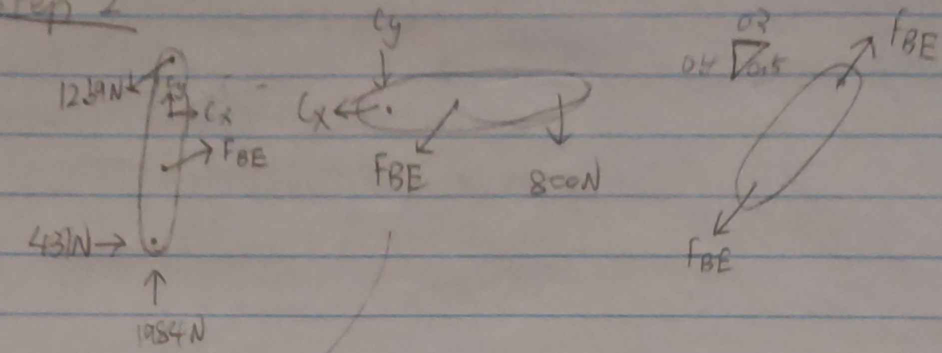
$\Sigma F_x = 0;\ A_x - F_{DH}(\sin 20°) = 0$

$A_x = 431N$

$\Sigma F_y = 0;\ A_y - 800N - F_{DH}(\cos 20) = 0$

$A_y = 1984N$

Step 2



$\Sigma M_C = 0, \quad -F_{BE}\left(\frac{0.4}{0.5}\right)(0.3m) - 800N(0.7m) = 0$

$F_{BE} \doteq -2333N$

$\Sigma F_x = 0 \quad -C_x - F_{BE}\left(\frac{0.3}{0.5}\right) = 0 \qquad C_x = 1400N$

$\Sigma F_y = 0 \quad -C_y - F_{BE}\left(\frac{0.4}{0.5}\right) - 800N = 0 \qquad C_y \doteq 1067N$

Friction

Coulomb's Law of Friction

$|F| \leq \mu_s N$, before sliding (<) and impending movement (=).

$|F| = \mu_k N$, after sliding begin

where, N is normal force b/w 2 surfaces, defined ⊕ in compression, hence $N > 0$ always.

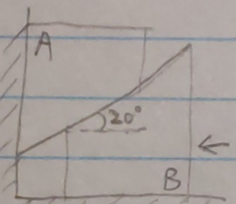
F is friction force, w/ directions alway opposing relative motion b/w 2 surface.

μ_s is static coef. f frict.

μ_k is kinetic coef. f frictn.

Example 1:

Treating Boxes as particles

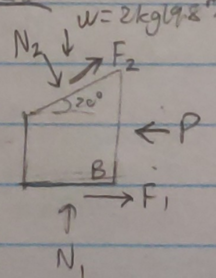


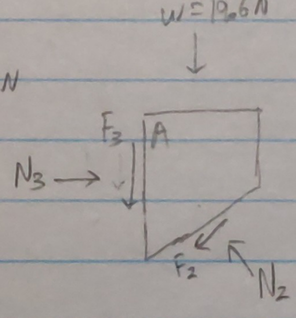
Block A and B each has mass 2kg. $\mu_s = 0.4$.

Find P that cause impending motion of block B to the left.

FBD

$w = 2kg(9.8\,m/s^2) = 19.6N$





Block A

$\Sigma F_y = 0; \; N_2\cos 20 - F_2 \sin 20° - F_3 - 19.6 = 0$ ①

$\Sigma F_x = 0; \; N_3 - N_2(\sin 20) - F_2\cos 20 = 0$ ②

Block B

$\Sigma F_y = 0; \; N_1 - 19.6 - N_2\cos 20 + F_2 \sin 20 = 0$ ③

$\Sigma F_x = 0; \; -P + F_1 + N_2(\sin 20) + F_2\cos 20 = 0$ ④

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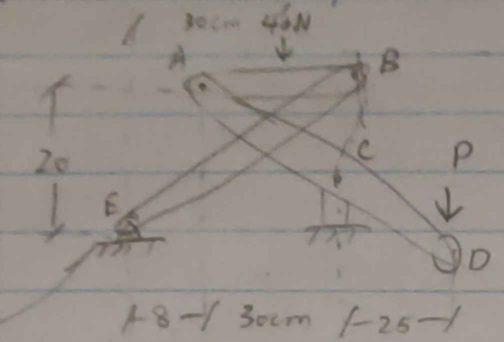
Example for machine (Pg.38)

Example 2: Determine force P to maintain equilibrium



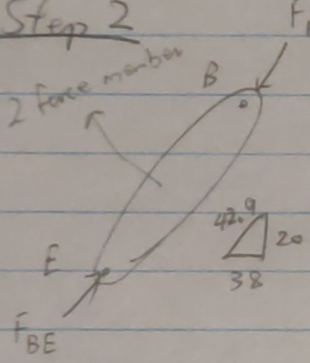
2 force member
1 reaction at E

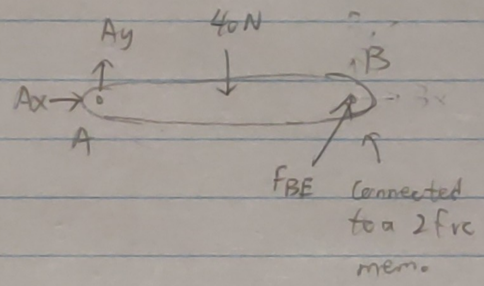
Step 1

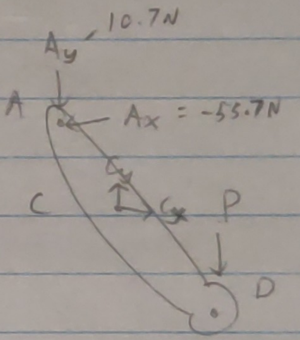
not useful b/c their will be 4 unkrows,
F_{BE}, C_y, C_x, P

Step 2

-change direction for members







AB

$\Sigma M_A = 0;\ F_{BE}\left(\frac{20}{42.9}\right)(30cm) - 40N(22cm) = 0 \qquad F_{BE} \doteq 64N$

$\Sigma F_x = 0;\ A_x + F_{BE}\left(\frac{38}{42.9}\right) = 0,\ A_x \doteq -55.7N$

$\Sigma F_y = 0;\ A_y - 40N + F_{BE}\left(\frac{20}{42.9}\right) = 0,\ A_y \doteq 10.7N$

ABC

$\Sigma M_C = 0;\ A_x(20cm) + A_y(30cm) - P(25cm) = 0,\ \boxed{P \doteq 32N}$

For impending motion $|F| = \mu_s N$

$$\left.\begin{aligned} F_1 &= \mu N_1 = 0.4N_1 \\ F_2 &= \mu N_2 = 0.4N_2 \\ F_3 &= \mu N_3 = 0.4N_3 \end{aligned}\right\} \text{Actually 4 unknowns } (N_1, N_2, N_3, P)$$

① $N_2 \cos 20 - 0.4N_2 \sin 20 - 0.4N_3 - 8.6N = 0$

② $-N_2 \sin 20 - 0.4N_2 \cos 20 + N_3 = 0$

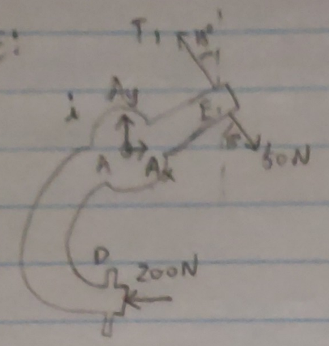
↳ 2 unknown 2 eqn, $N_2 \doteq 38N$, $N_3 \doteq 27N$

Sub N_2 into ③ → $N_1 \doteq 30N$ → sub N_1 into ④ → $\boxed{P = 47N}$

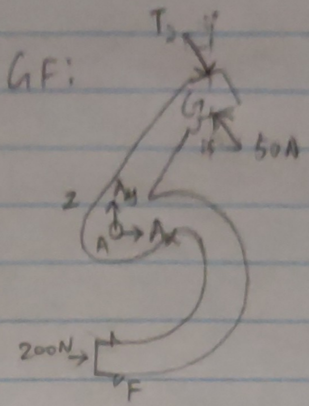
18\(\smash{\cdot}\)018

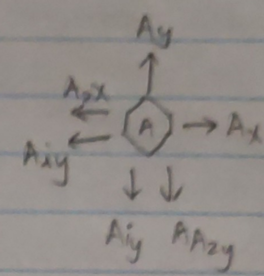
Lab 6

[6.96] a) DE:



GF:





b) DE: $\Sigma M_A = 0;\ -200(70) - 50(60) + T_1(60) = 0 \Rightarrow T_1 = 283N$

FG: $\Sigma M_A = 0;\ 200(70) + 50(60) - T_2(60) = 0 \Rightarrow T_2 = 283N$

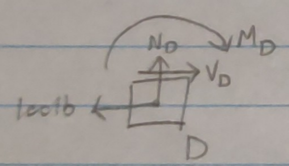
c) DE: $\Sigma F_x = 0;\ A_{1x} - 200 - T_1 \sin 15 + 50 \sin 15 = 0 \Rightarrow A_{1x} = 260N$

$\Sigma F_x = 0;\ A_{1y} + T_1 \cos 15 - 50 \cos 15 = 0 \qquad A_{1y} = -225N$

GF: $\Sigma F_x = 0;\ A_{2x} + 200 + T_2 \sin 15 - 50 \sin 15 = 0 \qquad A_{2x} = -260N$

$\Sigma F_y = 0;\ A_{2y} - T_2 \cos 15 + 50 \sin 15 = 0 \qquad A_{2y} = 225N$

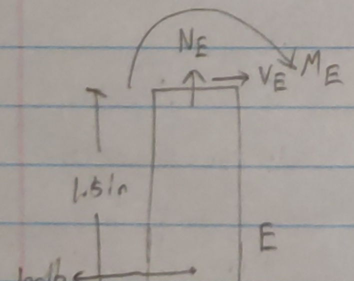
[8.10]



D: $\Sigma F_x = 0;\ V_D - 100lb = 0,\ V_D = 100lb$

$\Sigma F_y = 0;\ N_D = 0$

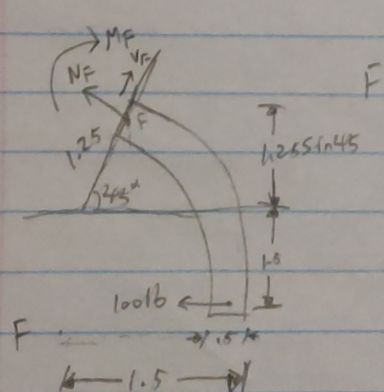
$\Sigma M_D = 0;\ M_D = 0$



E: $\Sigma F_x = 0;\ V_E - 100 = 0,\ V_E = 100lb$

$\Sigma F_x = 0;\ N_E = 0$

$\Sigma M_E = 0;\ -M_E - 100(1.5) = 0,\ M_E = -150lb$



F: $\Sigma F_x = 0;\ -N_F \sin 45 + V_F \cos 45 - 100 = 0$

$\Sigma F_y = 0;\ +N_F \cos 45 + V_F \sin 45 = 0$

$\left.\right\} N_F = -70.71lb,\ V_F = 70.71lb$

$\Sigma M_F = 0;\ -100lb(1.5 + 1.25 \sin 45) + M_F = 0$

$M_F = -238.4lb$

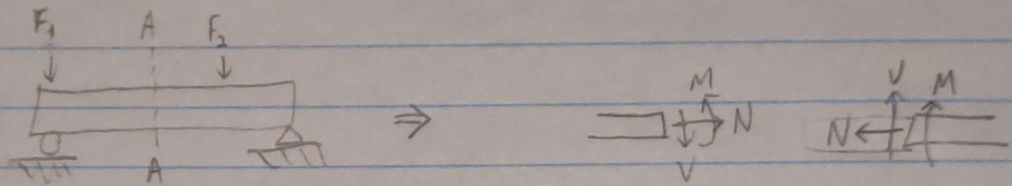
Internal Forces In Structural Members (EMS 8.1)

Internal Forces: are forces & moments that develops within members and materials due to the external forces applied. As the internal forces that a member must support become larger, the member must be larger and/or be constructed of stronger materials so that it has sufficient strength.

Int. frcs. for slender members in 2d.

member is slender if cross section is small compared to length.

e.g.



V, shear force
M, bending moment
N, normal/axial

Forces that develop in cross section are called internal forces.

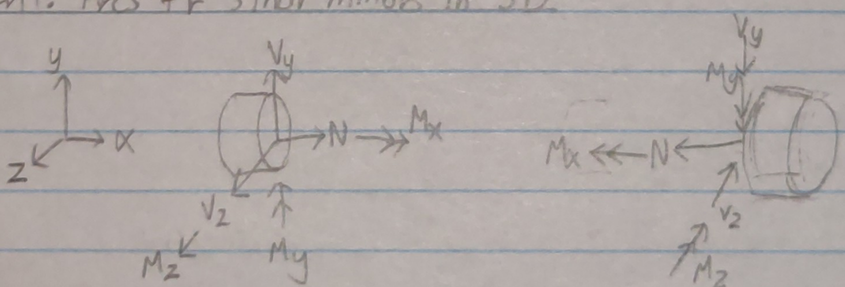
↳ they ensure material on the left do not move relative to right.

N, normal, axial (positive as shown)   Remark: in 2D, x direction is axial direction

V, shear force (⊕ as shown)

M, bending moment (⊕ as shown)

Int. frcs fr Slndr mmbrs in 3D.



N, normal or axial force
V, shear force
M_y M_z, bending mmnt
M_x, torque
(⊕ as shown)

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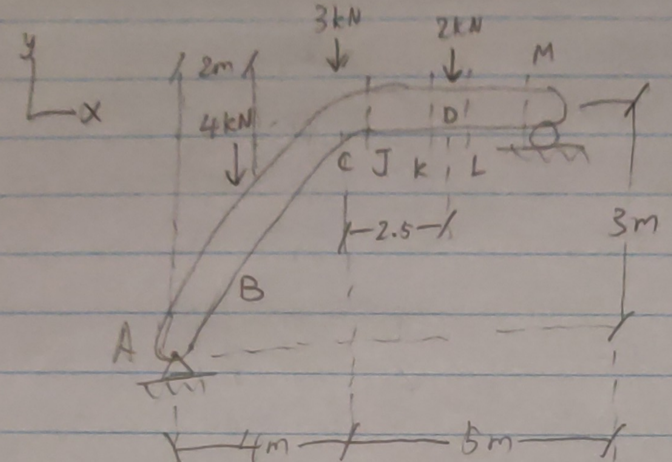
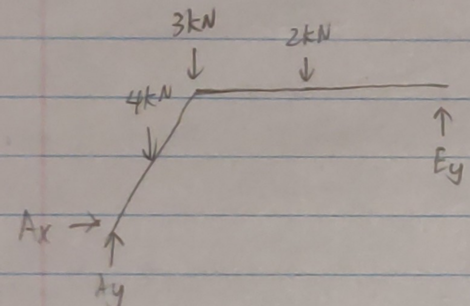
Example. Determine internal forces acting on cross section J, K, L, and M (2D)



External forces act on B, C, D
Cut are taken immediately to the left or right of C, D, and E



in summation eq. write forces & moment wrt x, y axis

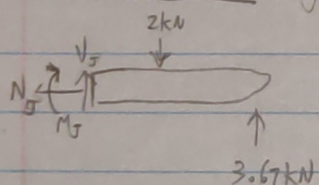
$\Sigma M_A = 0;\ E_y(9m) - 4kN(2) - 3(4) - 2(6.5) = 0$

$E_y \doteq 3.67kN$

$\Sigma F_y = 0;\ A_y + E_y - 4 - 3 - 2 = 0,\ A_y \doteq 5.33kN$

$\Sigma F_x = 0;\ A_x = 0$

Cross Section J (right side)

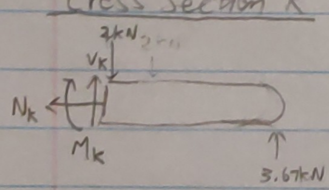


$\Sigma F_x = 0;\ N_D = 0$

$\Sigma F_y = 0;\ V_J + 3.67 - 2kN = 0,\ V_J = -1.67kN$

$\Sigma M_J = 0;\ -2(2.5) + 3.67(5) - M_J = 0,\ M_J \doteq 13.33 kN\cdot m$

Cross Section K

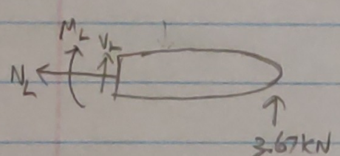


$\Sigma M_D = 0;\ 3.67(2.5) - M_K = 0,\ M_K = 9.17 kN\cdot m$

$\Sigma F_x = 0;\ N_K = 0$

$\Sigma F_y = 0;\ V_K - 2 + 3.67 = 0,\ V_K = -1.67kN$

Cross Section L

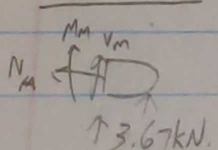


$\Sigma M_L = 0;\ 3.67(2.5) - M_K = 0,\ M_K = 9.17 kN\cdot m$

$\Sigma F_x = 0;\ N_L = 0$

$\Sigma F_y = 0;\ V_L = -3.67kN$

Section M

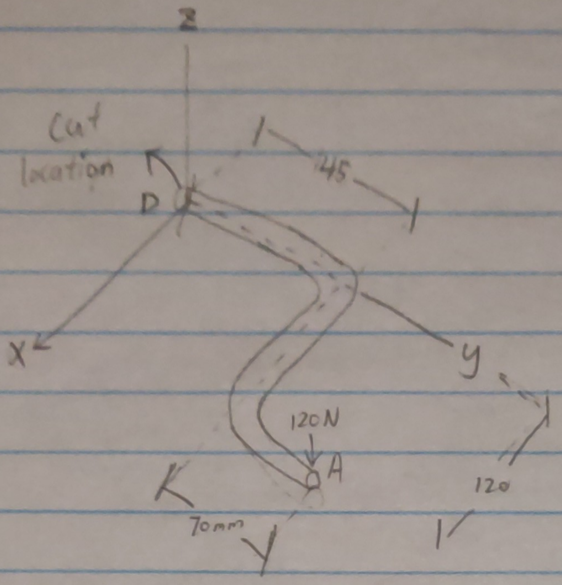


$\Sigma F_x = 0;\ N_M = 0$

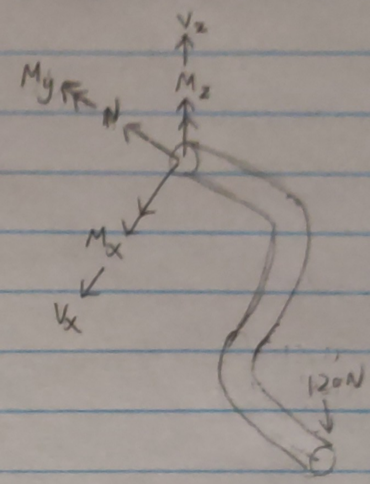
$\Sigma M_M = 0;\ M_m = 0$

$\Sigma F_y = 0;\ V_m = -3.67kN$

Exmple: Determine internal forces acting on section D.
(3D)



FBD (right section)



$\vec{F} = -120\hat{k}$ N $\quad \vec{r}_{DA} = (120\hat{i} + 115\hat{j})$ mm

$$= \begin{vmatrix} 120 & 115 & 0 \\ 0 & 0 & -120 \end{vmatrix}$$

$$= (-13800, -(-14400), 0)$$

$$= -13800\hat{i} + 14400\hat{j}$$

$\Sigma F_y = 0$; $\boxed{V_x = 0}$

$\Sigma F_x = 0$; $\boxed{N = 0}$

$\Sigma F_z = 0$; $V_z - 120\text{N}$, $\boxed{V_z = 120\text{N}}$

$\Sigma M_D = 0$; $-M_y\hat{j} + M_x\hat{i} + M_z\hat{k} + \vec{r}_{AB} \times \vec{F} = 0$

$-M_y\hat{j} + M_x\hat{i} + M_z\hat{k} + 14400\hat{j} - 13800\hat{i} = 0$

$\Sigma M_{Dx} = 0$; $M_x - 13800 = 0$, $\boxed{M_x = 13800 \text{ N}\cdot\text{mm}}$

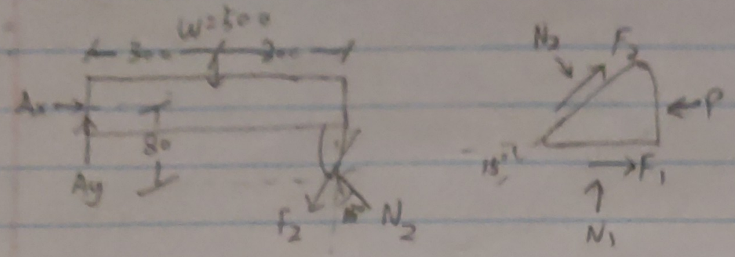
$\Sigma M_{Dy} = 0$; $-M_y + 14400 = 0$, $\boxed{M_y = +14400 \text{ N}\cdot\text{mm}}$

$\Sigma M_{Dz} = 0$; $\boxed{M_z = 0}$

18625

Lab 7

9.17



a) AB $\Sigma M_A=0$ ① $-500(300)+N_2\cos15(600)-N_2(\sin15)(80)-F_2\cos15(80)-F_2\sin15(600)=0$

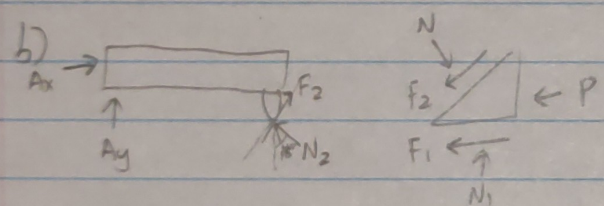
Impending Motion: ② $F_1=\mu_s N_1=0.3N_1$, $F_2=\mu_s N_2=0.3N_2$

① & ② $N_2=306.7N$ $F_2=92.01N$

Wedge C $\Sigma F_y=0$; $N_1-N_2\cos15+F_2\sin15=0$, $N_1=272.4N$, $F_1=81.73N$

$\Sigma F_x=0$; $F_1+N_2\sin15+F_2\cos15-P=0$, $P=250N$

b)



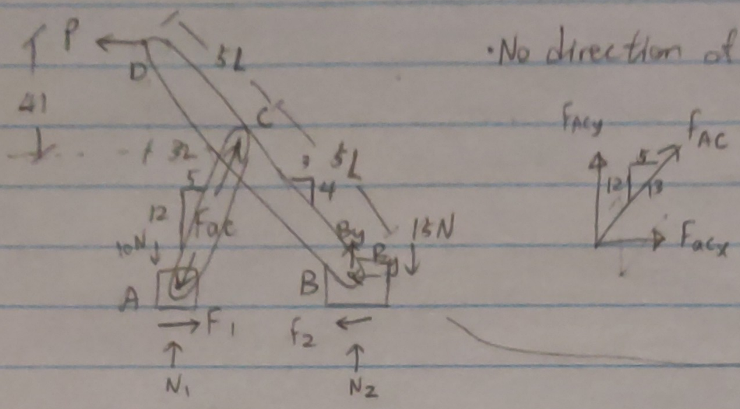
AB $\Sigma M_A=0$ $-500(300)+N_2\cos15(600)-N_2(\sin15)(80)+F_2\cos15(80)+F_2\sin15(600)=0$

$N_2=238.6N$ $F_2=71.59N$

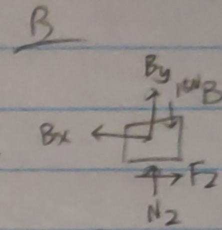
Wedge $\Sigma F_y=0$; $N_1-N_2\cos15-F_2\sin15=0$ $N_1=249.0N$ $P=-82.1N$

$\Sigma F_x=0$; $-F_1+N_2\sin15-F_2\cos15-P=0$ $F_1=74.70N$

9.45



• No direction of motion, assume direction for F_1, F_2



Bar BCD

$$\Sigma M_B = 0; \quad P(8L) - \tfrac{5}{13}(F_{AC})(4L) - \tfrac{12}{13}F_{AC}(3L) = 0$$
$$\Sigma F_x = 0; \quad \tfrac{5}{13}F_{AC} - B_x - P = 0$$
$$\Sigma F_y = 0; \quad B_y + \tfrac{12}{13}F_{AC} = 0$$

$F_{AC} = \frac{13}{7}P$ → revise FBD

$\boxed{B_x = \frac{-2}{7}P}$

$B_y = \frac{12}{7}P$

Block A

10N F_{ac} F_1 N_1

$$\Sigma F_x = 0; \quad F_1 - \tfrac{5}{13}\underset{\left(\frac{13}{7}\right)P}{F_{ac}} = 0$$
$$\Sigma F_y = 0; \quad N_1 - 10N - \tfrac{12}{13}F_{ac} = 0$$

$$\tfrac{5}{7}P = \mu_s\left(10 + \tfrac{12}{7}P\right) \Rightarrow P = 15N$$

when P = 15N block A starts moving

Block B

$$\Sigma F_x = 0; \quad \tfrac{-2}{7}P + F_2 = 0$$
$$\Sigma F_y = 0; \quad N_2 + \tfrac{12}{7}P - 15 = 0$$

$N_2 = 15 - \frac{12}{7}P$

$F_2 = \frac{2}{7}P$

Impending Motion

$F_2 = \mu_s N_2$

$$\tfrac{2}{7}P = (0.3)\left(15 - \tfrac{12}{7}P\right) \rightarrow \boxed{P = 5.625\,N}$$

∴ P = 5.625N causes motion

Block B moves left.