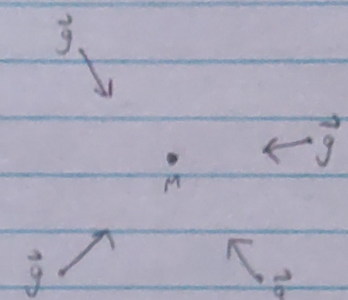
CH13 Newtonian's Gravity

Feb 15, 2018 Lei Yu

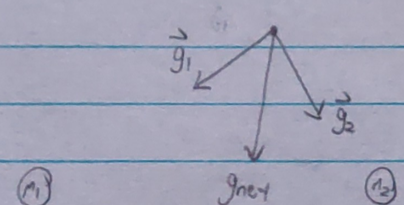


- vector at every point in space
- points toward the COM

- Mass create field
- Mass respond to field

Gravitational Field Strength

strength of gravity caused by mass M → $|\vec{g}| = \frac{GM}{r^2}$, $G = 6.67 \times 10^{-11} \frac{m^3}{kg \cdot sec^2}$



- Gravitational force add up as vectors

$\boxed{\vec{F} = m\vec{g}_{ext}}$

- Orbit equal free fall

net field from sum of fields of point masses

$\vec{g}_1 = -\frac{Gm_1}{r_2}\hat{i}$, $g_{net} = g_1 + g_2 + g_2$

gravitational force $|\vec{F}_G| = \frac{Gm_1m_2}{r^2}$

Newton's law of universal gravitation $|\vec{F}_G| = m_2 g_1 = -m_2 \frac{Gm_1m_2}{r_2}\hat{i} = -\frac{Gm_1m_2}{r_2}\hat{i}$

$|\Sigma F| = m|a|$

$\frac{GM_sM_p}{r^2} = M_p\frac{v^2}{r}$ $\qquad T = \frac{2\pi}{w}$ $\quad w^2 = \frac{GM}{r^3}$

$\frac{GM_s}{r} = v^2$ r, radius of orbit, v; orbital speed; M_s; mass of orbited object

Geostationary Orbit: T = 24 hour

$$\left.\begin{array}{l} \frac{GM_s}{r} = V^2 \\ V = rw \\ V = r\frac{2\pi}{T} \end{array}\right\} \quad \begin{array}{l} \frac{GM_s}{r} = r^2\left(\frac{2\pi}{T}\right)^2 \\ \frac{GM_s}{r} = r^2\left(\frac{2\pi}{T}\right)^2 \\ \frac{GM_s}{r^3} = \left(\frac{2\pi}{T}\right)^2 \end{array}$$

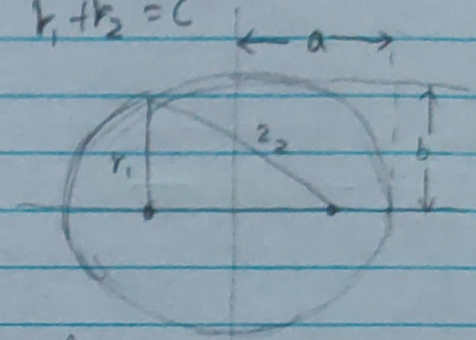
$\therefore T^2 \propto r^3$

Circular Orbit

- velocity is perpendicular to gravitational force

$\frac{GM_s}{r} = v^2$ is satisfied

$r_1 + r_2 = C$



Elliptic Orbit

Kepler's Laws

1. Planet moves in elliptical orbits
2. Planet sweep out equal area in equal times
3. Orbit Period^2 is porportional to semi major axis^3

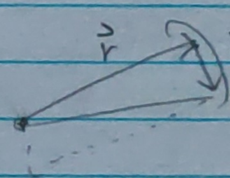
slower  faster

$T^2 \propto a^3$ *not acceleration

Collision $\left\{ \begin{array}{l} \left(\frac{T_A}{T_E}\right)^2 = \left(\frac{a_A}{a_E}\right)^3 \\ \frac{a_A}{a_E} < \frac{1}{2}, \text{no collision} \end{array} \right.$

$$T^2 = \frac{4\pi^2}{G(M_s)} a^3$$

Proof of Kepler's 2nd Law



$\frac{1}{2}$ of a parallelogram

Central force

$|\vec{L}| = |\vec{r} \times \vec{p}| = M_p |\vec{r} \times \vec{v}|$

$$dA = \frac{1}{2} |\vec{r} \times \vec{v}| dt$$

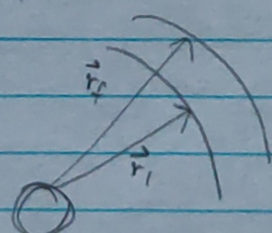
Potential Energy Review

*Conservative force is a state function

$\Delta U = -W_{conservative}$

$W = \vec{F} \cdot \Delta r = |\vec{F}||\vec{r}| \cos\theta$

area under force displacement $W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$



direction is changing momentum not conserved

$\Uparrow$

- energy and angular momentum is conserved

Gravitational PE

$$W_{gravity} = \int_{r_i}^{r_f} F \cdot dr = -\int_{r_i}^{r_f} \frac{Gm_1m_2}{r^2} dr = \frac{Gm_1m_2}{r}\Bigg|_{r_i}^{r_f}$$

$r\uparrow$, $U_g\uparrow$

$$\Delta U_g = -W_{gravity} = \frac{Gm_1m_2}{r_i} - \frac{Gm_1m_2}{r_f}$$

$U_g = -\frac{Gm_1m_2}{r}$ ← center to center distance

PE of a pair of masses

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Total Energy in Orbit Feb 27, 2018 Lei Yu

$E = K + U$

$E = \frac{1}{2}mv^2 - \frac{GmM}{r}$ $v^2 = \frac{GM}{r}$

$E = \frac{1}{2}m\frac{GM}{r} - \frac{GMm}{r}$

$E_{tot} = -\frac{GMm}{2r}$ (assuming circular orbit)

Escape Speed:

$$\Delta U + \Delta K = 0$$

$$\underbrace{-\frac{GMm}{r_f}}_{U_f} - \Big(\underbrace{-\frac{GMm}{r_i}}_{U_i}\Big) + (0 - \underbrace{\tfrac{1}{2}mv_i^2}_{K_i}) = 0$$

$K_f = 0$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$ initial speed required to escape from a mass M and radius R.

Black hole

- escape speed of black hole is greater than light

Schwarzschild radius of black hole $\rightarrow R_S = \frac{2GM}{c^2}$ (M: mass of black hole; c: speed of light)

CH23
1,2,4

Electric Field Feb 27, 2018

- q can be negative

Property	Gravity	Electricity
Property	M (mass)	q (or Q) electric Charge
Fundamental Constant	G (newton's Constant)	k_e Coulomb's Constant
Field	$\vec{g}$	$\vec{E}$

Two Kinds of Charge

- opposite charge attract
- * Net electric charge cannot be created or destroyed
- Like charge repel
- electric force gets weaker the further away the charges are
- Neutral objects have equal number of positive and negative charges.

Conductor: allow flow of charge freely

Insulator: almost no charge flow

* only e^- move

Insulator

- Can be charged by added / remove from surface

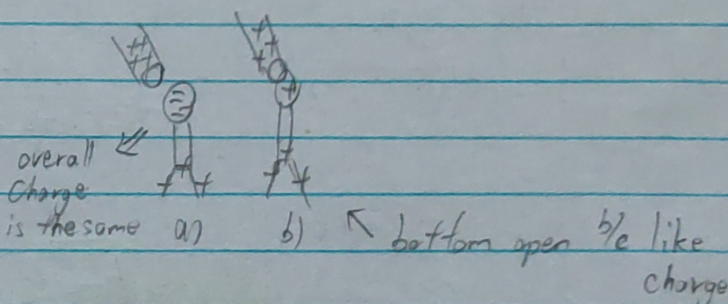
Charging by Induction (no contact)

- happens in metal
- by placing negatively charged object near metal
- metal have to be on the ground?

Electroscope

a) Induction

b) Conduction



Electric Charge

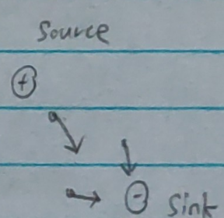
$$[q] = C$$

- nano-coulombs (10^{-9}C) or micro-coulombs (10^{-6}C)

Fundamental unit of charge

- $q_p = +e$

$q_e = -e$ $\quad e = 1.602 \times 10^{-9}$ C



Electric Field

Direction of electric field:

E points away from positive charges, and toward negative charges. $\quad [E] = \frac{N}{C}$ or $\frac{V}{m}$

Force caused by field is $\boxed{\vec{F}_e = q\vec{E}}$ $\quad$ * F=mg?

if $q > 0$, $\vec{F}$ and $\vec{E}$ in same direction

$q < 0$, $\vec{F}$ and $\vec{E}$ in opposite direction

$|\vec{g}| = \frac{GM}{r_2}$ $\quad |\vec{E}| = \frac{k_e Q}{r^2}$, $k_e = 9.0 \times 10^9 \frac{N m^2}{C^2}$

Superposition principle

$F = Q(\frac{kq_1}{r} - \frac{kq_2}{r}$

- fields are added as vectors $\quad \vec{E}_{net} = E_1 + E_2$

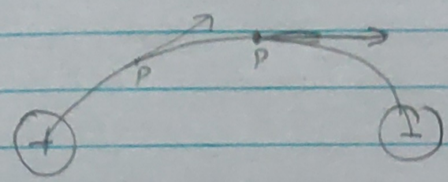
Coulombs Law

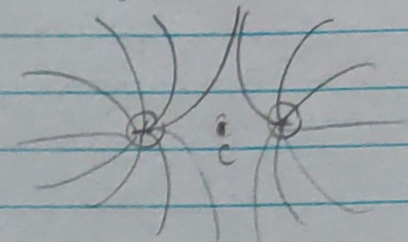
$F = ma$

$|\vec{F}| = \frac{k_e q_1 q_2}{r^2}$, $|\vec{E}| = \frac{k_e q_1}{r^2}$ $\quad F = q|\vec{E}| \Rightarrow |\vec{F}| = \frac{k q_1 q_2}{r^2}$

PCS125 Electric Field Lines Mar 13, 2018 Lei Yu

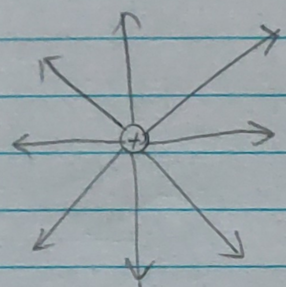
- Electric field at point P is tangent to the field line through that point.





- density of lines represent the strength of the field
- Field lines only end at negative charge or infinity
- Field lines can not cross

- Point C, 0 charge



- further away from charge line density ↓, field strength decreases.
- field line denser closer to charge.

- Field lines should be picture in 3D.

$(8,2,0)\cdot(0,1,0)$
$=2$

Uniformed Electric Field

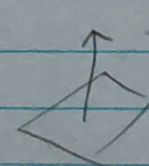
- constant magnitude and field
- called a capacitor

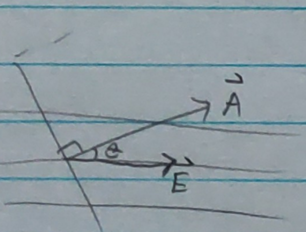
Motion in a Uniform field

- motion in uniformed electric field is similar to kinematics
- as if it's experiencing a constant uniformed acceleration.

Electric Flux

Area Vector: Magnitude: area of surface
Direction: perpendicular to surface





$* \; \Phi_E = E_\perp A = EA_\perp = EA\cos\theta$

$\vec{E}$, direction of field
$\vec{A}$, direction vector of A.

$\theta = 0°$: $\cos(\theta) = 1$, maximum flux
$\theta = 90°$: $\cos(90°) = 0$, no flux

$* \; \Phi_E = \vec{E}\cdot\vec{A}$

Electric Flux through a closed Surface

- a closed surface encloses something, (it has a volume)
- electric flux is positive if it is exiting the surface.
- if entering surface electric flux is negative

Flux is perportion to net charge inside

Electric Flux (General Flux)

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A}$$

Gauss' Law   Mar 15 2018   Lei Yu

- All field lines entering the box end up leaving the box. (Net flux = 0)

$$\oint \vec{E}\cdot d\vec{A} = 4\pi k_e Q_{in} \Rightarrow \oint \vec{E}\cdot d\vec{A} = Q_{in} / \epsilon_0$$

$$\epsilon_0 = 8.85\times 10^{-12} \frac{C}{\quad}$$

Gaussian Surfaces' are fictitious surface we made up to solve a problem

Charge Density

- Charge Q on a line of length L, linear charge density

$$\lambda = Q/L$$

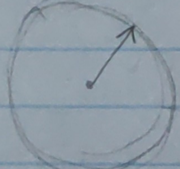
- Surface charge density

$$\sigma = Q/A$$

- Volume charge density

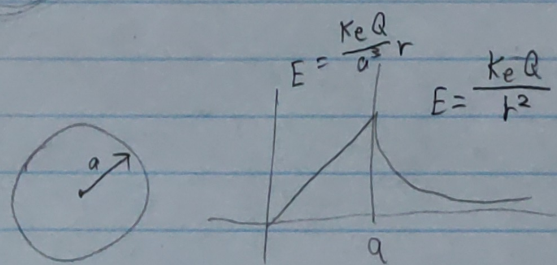
$$\rho = Q/V$$

Gauss' Law with a point charge



$d\phi_1 = |\vec{E}||d\vec{A}|\cos\theta$ add up all the surface

$= |\vec{E}||d\vec{A}|$, $\phi_{net} = |E|(4\pi r^2) = \frac{q}{\epsilon_0} \Rightarrow |\vec{E}| = \frac{q}{4\pi\epsilon_0} = \frac{k_e q}{r^2}$



Capaciter stores electricity

Special Case 2 - Planar Symmetry

- Assume the plate has an area A, and a uniformly distributed charge Q.

$Q = \sigma \cdot A$

E = Constant $\quad |\vec{E}| = \dfrac{Q}{2A\varepsilon_0} = \dfrac{\sigma}{2\varepsilon_0}$

Special Case 3 - Cylindrical Symmetry

- long wire

Conductor

Mar 20, 2018 Lei Yu

- No electric field in metal onece it reache electric equilibrium

- electric field is perpendicular to all conducting surface

Electrostatic Sheilding

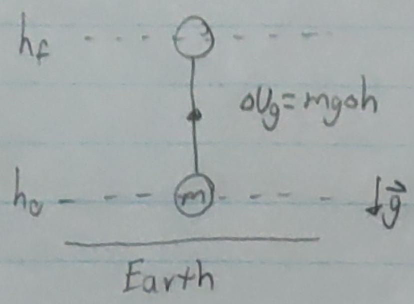
- Hollow space inside a conductor have $\vec{E}_{net} = 0$.
- Electric field inside is zero.

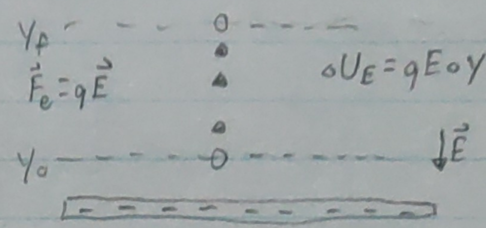
Fields outside a conductor

- When you are very close to a surface of a conducter, it will act like an infinity plate.

$$|\vec{E}| = \frac{\sigma}{\epsilon_0}$$

Potential Energy in an Uniformed Field





$\vec{g} \rightarrow \vec{E}$ $\quad\quad$ $m \rightarrow q$ $\quad\quad$ $G \rightarrow k_e$

Quatity Depending on Source / responding charge

	Force	Energy
Depend both on Source / Responding charge	$\vec{F} = q\vec{E}$	$\Delta U = q\Delta V$
Depend only on Source charge	$E = \frac{\vec{F}}{q}$	$\Delta V = \frac{\Delta U}{q}$

Electric Potential

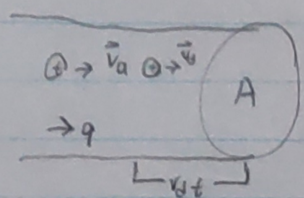
$$[\Delta V] = \frac{[\Delta U]}{[q]} = \frac{J}{C} = V \qquad \Delta V = -\int_1^2 \vec{E} \cdot ds$$

$$1eV = 1e \times 1V = 1.602 \times 10^{-19} J \qquad \Delta V = k_e Q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$V_B = \frac{k_e Q}{r_B} \rightarrow V_{(r)} = \frac{k_e Q}{r}$$

Electric Current and Ohm's Law   Mar 29, 2018 Lei Yu

$I = \frac{\Delta Q}{\Delta t}$ $1A = 1\frac{C}{s}$



$V = Av_d \Delta t$

$N = nAv_d \Delta t$

$\Delta Q = qN$, q is negative (charge of moving particle)

n = # of free electron, moving charges

$\Delta Q = qnAv_d \Delta t$ $\Rightarrow$ $I = \frac{\Delta Q}{\Delta t} = n|q|Av_d$

- electron or proton, all will produce positive current
- ↑ volt ↑ electric field ↑ electric force ↑ drift velocity
- When switch is turned on electric field is created. All electron start to move.

V_d - Drift velocity

- Speed of e^- is fast but randomly (lots of collision)
- electric field adds a small drift.

$V = IR$
$= 4 \times 3$
$= 12V$

4 = 300s

Current Density and Conductivity

$J = \frac{I}{A}$, $[J] = \frac{C}{cm^2 \cdot sec}$

Conductivity $\vec{J} = \sigma \vec{E}$ (current density; conductivity; electric field)

Conductivity is positive, ↑ σ ↑ Conductor.

Resistivity $\rho = \frac{1}{\sigma}$, ↑ ρ ↑ isulator.

- both ρ and σ relate to Temperature and

Ohm's Law

$$\frac{I}{A} = \sigma \cdot \frac{\Delta V}{l} \Rightarrow \Delta V = I \cdot \frac{l}{\sigma A} = I\frac{\rho l}{A} \qquad R = \frac{l}{\sigma A} = \frac{\rho l}{A}$$

$$\Delta V = IR$$

- metal ↑ temperature, ↑ resistivity
- semiconductor ↑ T, ↓ resistivity

$$\rho(T) = \rho(T_0)[1 + \alpha(T - T_0)]$$

Electric Power (27.6)

Power P: the rate at which energy is used in a resistor

$$P = I \Delta V$$
$$\Delta V = IR$$
$$\Rightarrow P = I^2 R = \frac{\Delta V^2}{R}$$

Circuit Analysis

In Series

$$R_{eq} = R_1 + R_2 + R_3 \cdots + R_n$$

$$I = I_1 = I_2$$

$$\Delta V = V_1 + V_2 = IR_1 + IR_2 = I(R_1 + R_2) = I R_{eq}$$

In Parrallel

$$\Delta V = \Delta V_1 = \Delta V_2$$

$$I = I_1 + I_2 = \left(\frac{\Delta V_1}{R_1}\right) + \left(\frac{\Delta V_2}{R_2}\right)$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots$$

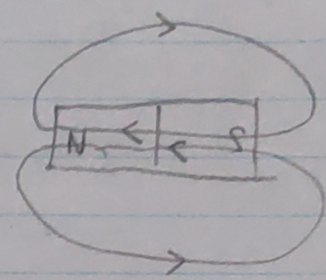
Magnetic Field and Force Apr 3, 2018 Lei Yu

$\odot$ Out of screen

$\otimes$ Into screen

Tesla: $[\vec{B}] = \dfrac{kg}{C \cdot sec} \equiv T$

- Magnetic field lines are alway closed looped



Closed loop

$|\vec{B}|_{Earth} \sim 50\,\mu T$ $|\vec{B}|_{MRI} \sim 1.5T$

Currents produce magnetic fields

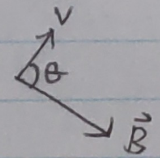
- RH rule to determine direction of current.

Magnetic Force Law

$\vec{F}_B = q(\vec{v} \times B)$

$|\vec{F}| = |q||v||\vec{B}|\sin\theta$

- opposite direction if q is negative

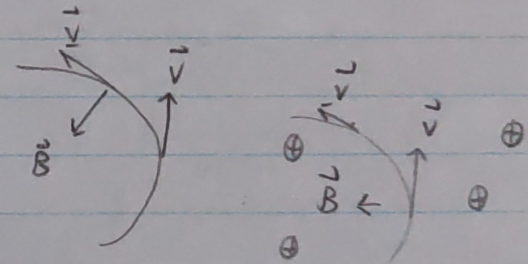


Direction of $\vec{q}v$ is same as I

Motion in a Uniform B-field

Work $= \vec{F}_B \cdot d\vec{S} = 0$

- Circular motion
- Speed is constant



Cyclotron Radius

$|\vec{F}| = m|\vec{a}|$

$|q||\vec{v}||\vec{B}| = m\dfrac{|\vec{v}|^2}{r}$

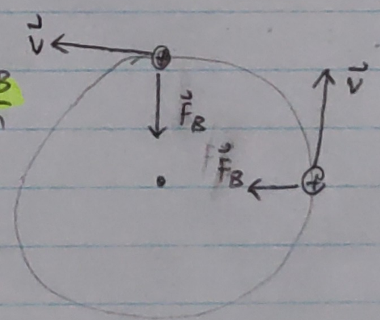
$r = \dfrac{m\vec{v}}{|q||\vec{B}|}$

$T = \dfrac{2\pi r}{V}$

$= 2\pi \dfrac{m}{|q||\vec{B}|}$

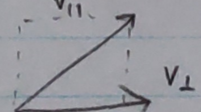
$= \dfrac{2\pi m}{|q||\vec{B}|}$

$w = \dfrac{qB}{m}$



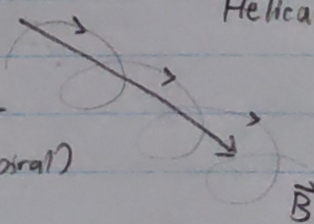
If v and B are not perpendicular

Break $\vec{v}$ into 2 direction



- Combination of straight line and circular motion - helical path (Spiral)

Helical Path

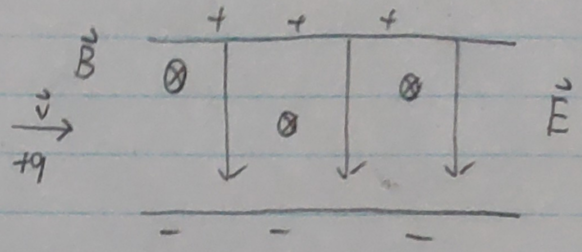


$r = \dfrac{mv_{\perp}}{|q||\vec{B}|}$

- Aurora Borealis
- Charged particle from the sun get traped and funneled to the pole.

Velocity Selector

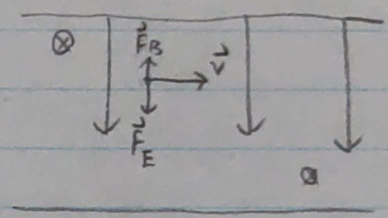
- Shooting a particle through a region with crossed E and B field can allow us to determine its speed



- Works for negative charge too

$$|q||\vec{E}| = |q||\vec{v}||\vec{B}|$$

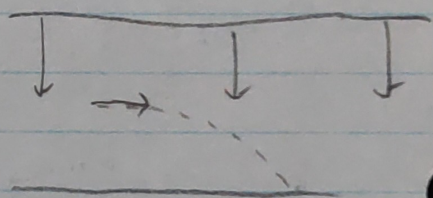
$$|\vec{v}| = \left|\frac{\vec{E}}{\vec{B}}\right|$$



- If particle's speed's just right, the particle will move in a straight line.

$\vec{F}_B = I(\vec{l}\times\vec{B})$, $I|\vec{l}||\vec{B}|\sin\theta$

- If $\vec{v}$ is too small particle will fall



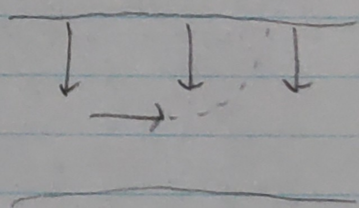
$\Delta V_H = \frac{IB}{nqt'}$

Square

$\vec{B} = \frac{4\mu_0 I}{4\pi a}(\sin\frac{\pi}{4} - \sin(-\frac{\pi}{4}))$

$\sqrt{2}$

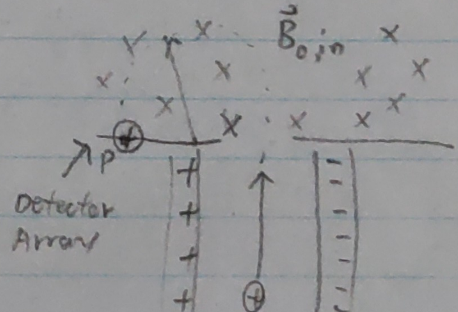
- If $\vec{v}$ is too large particle will rise.



$\vec{B} = \frac{\mu_0 I}{2\pi r}$ ← 2 parallel Line

Mass Spectrometer

- Can be used to separate isotope of an element
- Speed known from velocity selector
- Mass determines the radius of circular path

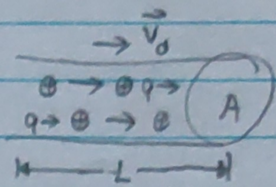


$$r = \frac{mv}{qB},\ v = \frac{E}{B} \Rightarrow m = \frac{rqB^2}{E}$$

Force on a Current Carrying Conductor 29.4 Apr 9, 2018 Lei Yu

Total force on a current carrying wire

Single wire: $\vec{F} = q(\vec{v} \times \vec{B})$ → drift velocity



Wire length L and area A:

$I = \frac{dQ}{dt}$

$$\vec{F}_B = q(\vec{v}_d \times \vec{B})N = q(\vec{v}_d \times \vec{B})nAL$$

N: # of charges in wire

AL: volume of wire

$I = nqAv_d$

$$\vec{F}_B = (qnA\vec{v}_d L) \times \vec{B} = I(\vec{L} \times \vec{B}) \quad \text{or} \quad |\vec{F}_B| = IL|\vec{B}|\sin\theta$$

* RHR: from direction of current to $\vec{B}$. No fliping, only direction of current matter.

current flow

$\rightarrow$ $\rightarrow$ • Attract

$\leftarrow$ $\rightarrow$ • Repel

Multiple Segments

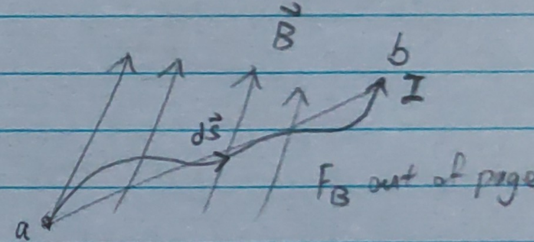
$$\vec{F}_{net} = I\vec{L}_1 \times \vec{B}_1 + I\vec{L}_2 \times \vec{B}_2 + \cdots$$

if $\vec{B}$ is uniform

$$\vec{F} = \int_a^b d\vec{F} = \int_a^b I\,d\vec{s} \times \vec{B},$$

curved wire break segment into infini small.

• will result in a straigh line



• In a closed loop, net force is zero, torque is not zero

Summary

• $\vec{B}$ can also exert force on charged particles.

  • $\vec{F}_B$ depend on velocity ($\vec{F}_e$ doesn't)

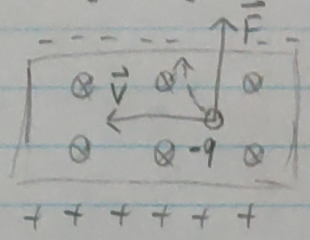
  • $\vec{F}_B$ points $\perp$ to the field ($\vec{F}_e$ ∥ or anti-∥ to the field)

  • $\vec{B}$ do not work on charged particles.

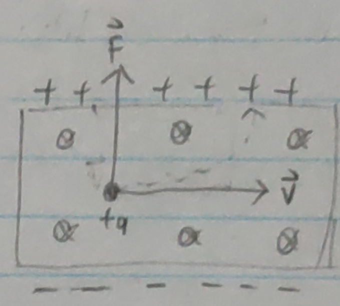
$$\vec{F}_B = I\vec{L} \times \vec{B}$$

The Hall Effect 29.6 Apr 9, 2018 Lei Yu

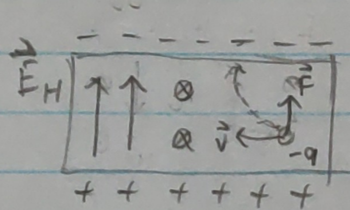
CHarge negative



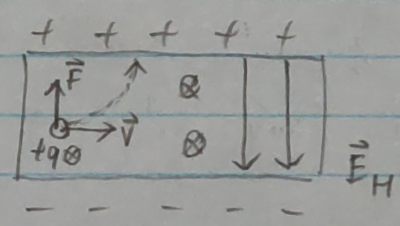
Charge Positive



- Due to deflected charges, a build up of charges on top / bottom
- The charge build up creates the Hall electric Field

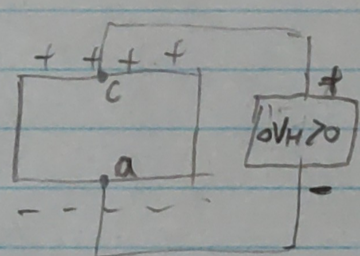


Charge negative



Charge positive

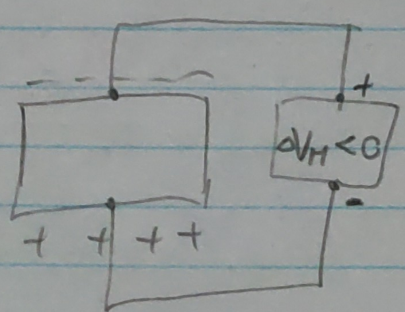
Hall Voltage



$V_c - V_a > 0$

$V_c > V_a$

Positive Charge Carrier



$V_c - V_a < 0$

$V_c < V_a$

Negative Charge Carrier

When does build up stop?

- stops when edge become sufficiently charged. $F_E = F_B$

$$q|\vec{E}_H| = q|\vec{v}||\vec{B}| \Rightarrow |\vec{E}_H| = |\vec{v}_d||\vec{B}|$$

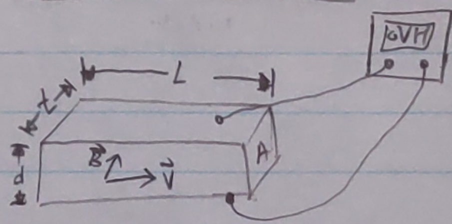
Measuring Drift Velocity

Hall voltage

drift velocity

$$|\Delta V_H| = |\vec{E}_H| d \Rightarrow |\Delta V_H| = |\vec{v}_d||\vec{B}| d$$

Conductor Dimension



L = length along current flow

t = "thickness" along $\vec{B}$

d = direction perpendicular to L and t

(Hall E field direction)

sub A = td

Measuring Concentration of Carriers

$$|\Delta V_H| = |\vec{v}_d||\vec{B}| d$$
$$I = nA|q||\vec{v}_d|$$

$$\Rightarrow |\Delta V_H| = \frac{I|\vec{B}|d}{nAq} = \frac{I|\vec{B}|d}{ntd|q|} = \boxed{\frac{I|\vec{B}|}{nt|q|}}$$

Summary

- in current carrying conductor, magnetic field pointing perpendicular deflects charge
- build up of charge creates Hall electric field and associated ΔV_H.

$$|\vec{v}_d| = \frac{|\vec{E}_H|}{|\vec{B}|}$$

$$|\Delta V_H| = \frac{I|\vec{B}|}{nt|q|}$$

$$|\Delta V_H| = |\vec{v}_d||\vec{B}| d$$

- Hall Votage used to determine:
  - Unknown B fields
  - Density of charge carriers
  - Drift velocity of charge carriers

The Biot-Savart Law 30.1, 30.2

Apr 10, 2018 Lei Yu

"permeability of free space"

$\mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$

current by the wire ↓ length of line ↓ wire to point ↓ (unit vector)

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{|\vec{r}|^2}$$

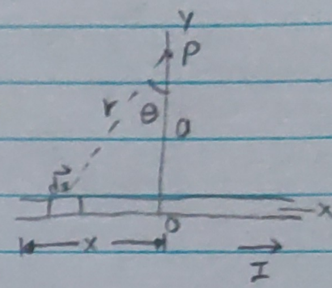
distance from wire → point

• The 'Law' is alway true.

Circular Loop ⇒ $|\vec{B}_{net}| = \frac{\mu_0 I}{2r}$

$$\boxed{\vec{B} = \frac{\mu_0 I L}{2\pi a \sqrt{L^2 + 4a^2}} \hat{k}}$$

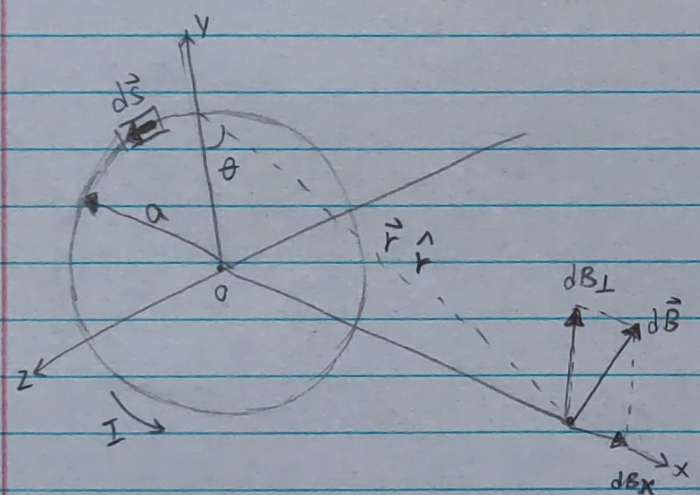
Field of a wire of length L a distance from the wire. (along axis which bisect it)



Infinite Wire

$$|\vec{B}| = \frac{\mu_0 I}{2\pi a}$$

1) At a perpendicular wire
a) From a very long wire
$a << L$



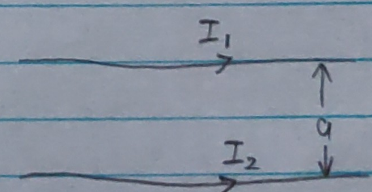
$$|\vec{B}| = \frac{\mu_0 I a^2}{2[a^2 + x^2]^{3/2}} \hat{i}$$

$$|\vec{B}| = \frac{\mu_0 I}{2a} \hat{i} \quad \text{(at center of circle, } x=0\text{)}$$

$$|\vec{B}| = \frac{\mu_0 I a^2}{2x^3} \hat{i} \quad \text{(very far from loop) } x >> a$$

Force between ~~current carrying~~ wire

Using $\vec{B}_2 = \frac{\mu_0 I_2}{2\pi a}$, $\vec{B}$ from wire two at wire 1 position.



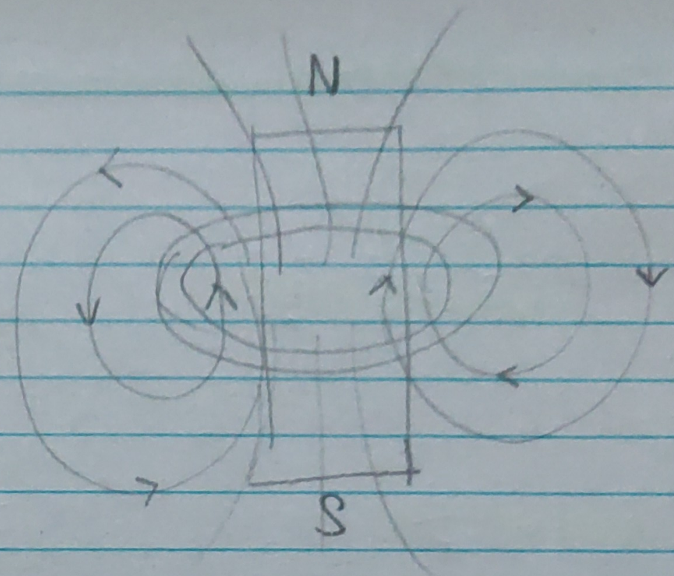
Force on wire 1 due to B_2

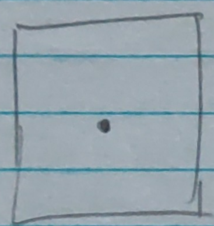
$$|\vec{F}_1| = I_1 l |\vec{B}_2| \sin\theta$$

$$|\vec{F}_1| = \frac{\mu_0 I_1 I_2 l}{2\pi a}$$

* Loop will have higher $\vec{B}$ than infinit wire

$\vec{B}$ of a current loop is similar to a bar magnet.





each side

$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$ $\theta_1 = 45°$, $\theta_2 = -45°$

4 side

$B = \frac{4\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$

$(T^2 \propto R^3)$