

Wave Model: an organized disturbance propagating across space in a medium or field

Phet simulations

Wave: collective motion of particles

### Simple Harmonic Motion

- The simple harmonic oscillator is a mass on a spring undergoing oscillations.

- Hooke's Law  $\vec{F}_s = -k\vec{x}_{\text{stretch}}$ ,  $\vec{x}_{\text{stretch}}$  defined from resting position

$$[k] = \frac{N}{m}$$

- Potential energy of the oscillator  $U_{\text{osc}} = \frac{1}{2}k(\vec{x}_{\text{stretch}})^2$

$$x(t) = A \cos(\omega t + \phi), \quad -A \leq x(t) \leq A$$

$$\begin{array}{cccc} \nearrow & \uparrow & \nearrow \\ \text{Amplitude} & [\omega t] = \text{rad} & \text{Phase} & \omega = 2\pi f \\ w = \sqrt{\frac{k}{m}} & [w] = \text{rad/sec} & \text{constant} & \end{array}$$

$$wT = 2\pi \Rightarrow T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}} \quad f = \frac{1}{T} = \frac{w}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

### Oscillation to uniform circular motion

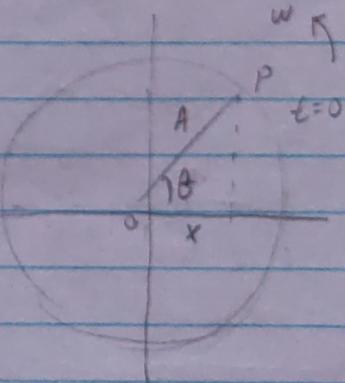
Radius and angular velocity is related to period.

The circle with  $A$  (oscillation amplitude) as the radius is called a reference circle.

Line OP makes an angle  $\theta$  with the  $+x$  at  $t=0$

phase  $\theta = \omega t + \phi$ , position needs direction,

$\phi$  is initial angle phase is easier way to describe status



$V_{\max} = wA$ ,  $x$  component of P is velocity

$$V = -wA \sin(\omega t + \phi)$$

if  $-\omega$ , rotate clockwise, all other instance CCW.

$$x(t) = A \cos \theta = A \cos(\omega t + \phi)$$

$$T = 2\pi \sqrt{\frac{L}{g}}, \quad x = A e^{-bt/2m}$$

$$\omega = \sqrt{\frac{g}{L}}$$

SHO require a force that pulls back

↓

equation in the form  $\vec{F} = -k\vec{x}$  defines a SHM

Angular frequency, period, and frequency only depend on the spring constant and mass.

$$\omega = \frac{2\pi}{T} = 2\pi f = \sqrt{\frac{k}{m}}$$

To pinpoint phase constant require velocity of particle.

$$a(t) = -A\omega^2 \cos(\omega t + \phi) \quad a_{max} = +A\omega^2 = \frac{AK}{m}$$

### Energy of SHO

In SHO mechanical energy is converted from PE to KE and back

$$E_{Total} = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$A^2 = \frac{m}{k}v^2 + x^2 = \frac{v_{max}^2}{\omega^2} + x^2 \quad A^2 = \frac{v_{max}^2}{\omega^2} + x^2$$

### Pendulum Jan 16, 2018

#### Simple Pendulum

consist of particle like mass  $m$ , and string length  $L$ .

$m_{string} < m_{mass}$

driven by gravitational force

Very similar to SHO if  $\theta < 10^\circ$

$$\theta = \theta_{max} \cos(\omega t + \phi), \quad \omega = \sqrt{\frac{g}{L}}, \quad T = 2\pi \sqrt{\frac{L}{g}}$$

$L \uparrow, T \uparrow$

$g \uparrow, T \downarrow$

larger driving force  
period will be smaller.

#### Physical Pendulum

oscillates about a fixed axis, not CM

object can't be approximated as a point mass.

can not be treated as simple pendulum.

$F_g$  provides torque about pivot.

Magnitude of torque is  $mgd \sin\theta$ ,  $d$  is lever arm from pivot to CM.

## Physical Pendulum, 2 Jan 16, 2018

$$F = ma$$

$$T = I \frac{d\theta}{dt^2}$$

$$-mgdsin\theta = I \frac{d^2\theta}{dt^2}$$

assume  $\theta$  is small

$$\omega = \sqrt{\frac{mgd}{I}}$$

$$T = 2\pi \sqrt{\frac{I}{mgd}}$$

$d$ , distance from pivot to center of mass

## Physical Pendulum, 4 if $I = md^2$

$$I = mL^2, d = L$$

Physical to simple

$$\omega = \sqrt{\frac{mgd}{I}} = \sqrt{\frac{mgL}{mL^2}} = \sqrt{\frac{g}{L}}$$

## Torsional Pendulum

$$T = -k\theta$$

$k$ , torsion constant

## Damped Example

• Retarding Force

$$-R = -b\dot{v}$$

$-b$  is a constant

$-b$  is damping coefficient

$$\sum F_x = -kx - b\dot{x} = m\ddot{x}$$

$$x = A e^{-\frac{bt}{2m}}$$

if  $b$  is small compared to maximum restoring force

$$x = A e^{-(\frac{b}{2m})t} \cos(\omega t + \phi)$$

$A$  is amplitude at  $t=0$

$$e^{-(\frac{b}{2m})t}$$

$A e^{-(\frac{b}{2m})t}$  is amplitude at  $t$ :

• Angular frequency will be  $\omega = \sqrt{\frac{k}{m} - (\frac{b}{2m})^2}$   $\leftarrow b$  have to make sure  $\Gamma$  not negative  
 $\nwarrow$  if negative not an oscillation anymore

$$\omega = \sqrt{\omega_0^2 - (\frac{b}{2m})^2}, \omega_0 \text{ is natural frequency}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

### Types of Damping

if restoring force is such that  $\frac{b}{2m} < \omega_0$ , the system is said to be underdamped.

### Critically damped

- Come to rest fastest, no oscillation

Smaller the mass

higher damping force

Come to rest faster

### Overdamped

- Come to rest slowest, no oscillation.

### Forced Oscillation

external oscillating (driving) force is applied at a frequency close to the natural frequency, can get very large

Driving force  $\rightarrow F = F_0 \cos(\omega t)$  -  $F_0$  is amplitude of driving force  
 $\omega$  is angular frequency of the driving force

$E_{\text{driving}} = E_{\text{lose to internal}}$ , a steady state condition is reached.



$$x = A \cos(\omega t + \phi_x), \quad A = \sqrt{\frac{F_0/m}{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

Resonance occurred when  $\omega_0 = \omega$ .

Wave: Propagated disturbance

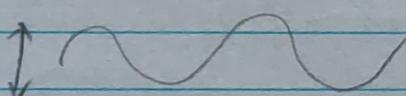
$\omega = \sqrt{\frac{E}{m}}$ , is characteristic of the spring system.

Wave Characteristic:

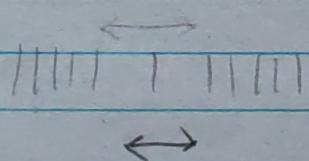
- Speed

Wave:

- A traveling disturbance, carries energy,
- travels through a medium (exception is light),
- light and sound
- Quantum mechanics says that particle also act as waves. (everything is waves)
- disturbance is the thing that moves
- particle says at same position of average.

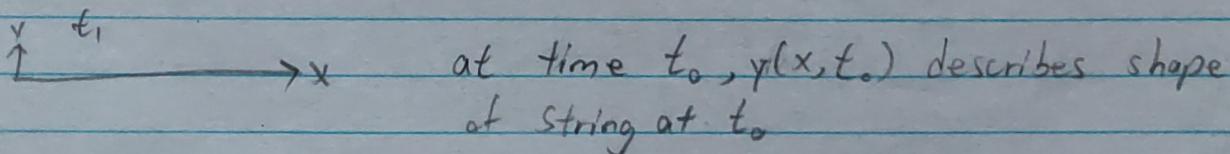


Transverse - the thing that is moving  
(light) moves perpendicular to disturbance

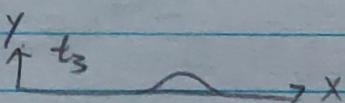


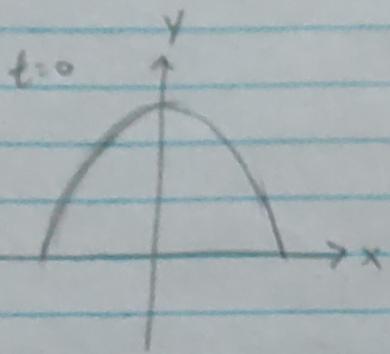
Longitudinal - parallel to disturbance.  
(sound)

$y(x, t)$  - displacement ( $y$ ) of the string element at position  $x$  at time  $t$ .



at position  $x_0$ ,  $y(x_0, t)$  describes what happens at  $x_0$  as times goes one



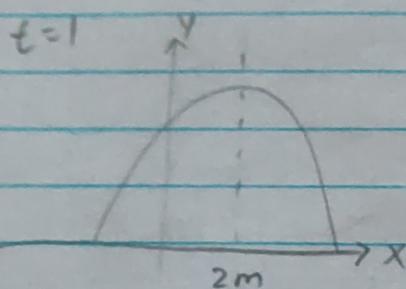


$t=0$

Assume

- shape doesn't change as it propagates
- speed is +2 m/sec
- what's the equation for the wave at  $t=1\text{ sec}$ ?

$$y(x,0) = -ax^2 + b$$



↓ shift right 2m

$$y(x,1) = -a(x-2)^2 + b$$

$$y(x,t) = -a[x - 2\frac{\pi}{2}(t)]^2 + b$$

$f(x)$

- ↓
- $y(x,t) = -a(x-vt)^2 + b$ , wave at different speed  
 $y(x,t) = f(x-vt)$ ,  $f(x)$  shape of wave.

Sinusoidal Wave  $y(x,t) = A \sin [k(x-vt)]$

$\cdot A$ , amplitude

$\frac{2\pi}{\lambda} = k$ , wave number (controls how closely the oscillations are)

Wavelength and Period

• Wavelength ( $\lambda$ ), space between wave repetition

• Period ( $T$ ), time for each iteration.

$$A \sin \theta = A \sin (\theta - 2\pi) \quad \text{and} \quad A \sin kx = A \sin (k(x-\lambda))$$

$$A \sin (kx) = A \sin (k(x-\lambda))$$

$$k\lambda = k_x \quad 2\pi = k\lambda$$

$$A \sin (kx - 2\pi) = A \sin (kx - k\lambda)$$

$$y(x,t) = A \sin \left[ \frac{2\pi}{\lambda} (x - vt) \right] \Rightarrow y(x,t) = A \sin \left[ \frac{2\pi}{\lambda} x - \frac{2\pi}{\lambda} vt \right]$$

$$\bar{w} = \frac{2\pi v}{\lambda}, \quad T = \frac{\lambda}{v}, \quad \text{a wave travels one } \lambda \text{ in one } T$$

$$\lambda = VT, \quad v = \frac{\lambda}{T}, \quad v = \lambda f$$

$$\downarrow \quad \downarrow$$

$$\phi \quad w$$

DCS1>5 CH16 Pg.2 Jan 23, 2018

$$k = \frac{2\pi}{\lambda} \rightarrow \omega = \frac{2\pi}{T}, \frac{\omega}{k} = V$$

Wave snapshot  $(x, t_0)$ :

Graph history  $(x, v, t)$ :

Oscillation speed of the element is

$$v_y = \left. \frac{dy}{dt} \right|_{x=\text{constant}}$$

$$y(x, t) = A \sin [kx - \omega t + \phi]$$

$$\text{or } v_y = -A \omega \cos (kx - \omega t + \phi) \Rightarrow v_{\max} = A\omega$$

Acceleration of element

$$a_y = \left. \frac{dv_y}{dt} \right|_{x=\text{constant}} \quad \text{or } a_y = -\omega^2 A \sin (kx - \omega t + \phi) \quad \text{or } a_{\max} = A\omega^2$$

$$\text{Ex. } y(x, t) = 0.35 \text{ m} \sin \left[ \left( \frac{10\pi \text{ rad}}{\text{sec}} \right) t - \left( \frac{3\pi \text{ rad}}{\text{m}} \right) x + \frac{\pi}{4} \text{ rad} \right]$$

$$\begin{aligned} \sin \theta &= \sin (\pi - \theta) \Rightarrow y(x, t) = 0.35 \sin \left( \pi - (\omega t - kx + \frac{\pi}{4} \text{ rad}) \right) \\ &= 0.35 \sin (kx - \omega t + \frac{3}{4}\pi) \end{aligned}$$

$$\omega = \frac{10\pi \text{ rad}}{\text{sec}} = \frac{2\pi}{T} \quad \Rightarrow T = \frac{1}{5} \text{ second} \quad \Rightarrow f = \frac{1}{T} = 5 \text{ Hz}$$

$$k = \frac{3\pi}{m} = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2}{3} \text{ m}$$

$$V = \frac{\lambda}{T} \quad \text{or} \quad V = \frac{\omega}{k}$$

$$v_{\max} = A\omega \Rightarrow A = ? \text{ m}$$

$$A = 0.35 \text{ m}$$

$$Ex 16.2 \quad A = 15.0 \text{ cm}$$

$$\lambda = 40 \text{ cm}$$

$$f = 5 \text{ Hz}$$



Goal:  $A, k, w, \phi$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{40 \text{ cm}} = \frac{\pi}{20} \text{ cm}^{-1}$$

$$w = 2\pi f = 10\pi$$

$$y(t=0, x=0) = 7.5 \text{ cm}$$

$$v_y(t=0, x=0) < 0$$

$$y(0,0) = 7.5 = A \sin(\omega t + kx + \phi)$$

$$= 15 \sin(-2\pi(5)t + \frac{2\pi}{40}x + \phi)$$

$$\frac{1}{2} = \sin(-10\pi t + \theta)$$

$$10\pi + \frac{\pi}{6} = \theta$$

$$\frac{7}{6}\pi - \frac{\pi}{6} = \frac{\pi}{3} = \phi$$

$$\underbrace{v_y}_{< 0} < 0$$

## PCS125 CH16 Lecture 2 Jan 25, 2018 Lei Y.

Oscillation / particle speed: speed of particle

Wave speed: how fast energy's being transferred away.

### Wave speed

- Wave speed's controlled by the medium
- type of wave does not affect wave speed.

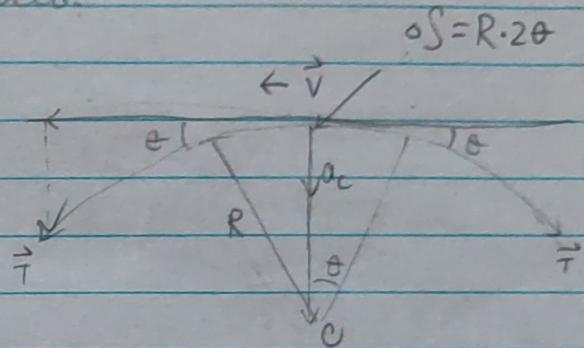
16.6

#### Pulse's POV

- The rope undergoes circular motion

$$\sum F_y = m a_y = \frac{mv^2}{R}$$

$$\begin{aligned} \text{Small angle approx.} \rightarrow 2T \sin \theta &= \frac{mv^2}{R} & m = \mu \circ S = \mu R(2\theta) \\ 2T\theta &\approx mv^2/R & \mu = \text{Mass per length} \\ 2T\theta &\approx \mu R(2\theta)v^2/R \\ T &\approx \mu v^2 \end{aligned}$$



$$v = \sqrt{\frac{T}{\mu}} \quad T, \text{tension of string}$$

### Wave Energy on a String

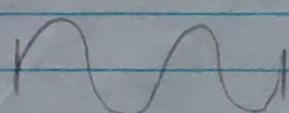
$$dE = \frac{1}{2} (dm) v_{y\max}^2 \quad v_{y\max} = Aw \quad dm = \mu \cdot dx$$

$$dE = \frac{1}{2} (\mu \cdot dx) A^2 w^2 \Rightarrow \frac{dE}{dx} = \frac{1}{2} (\mu A^2 w^2)$$

Power Transmission: the energy going through a cross section per second.

• Wave travels  $v \times (1 \text{ sec})$  per second

• Energy density per unit length:  $\frac{dE}{dx} = \frac{1}{2} (\mu A^2 w^2)$



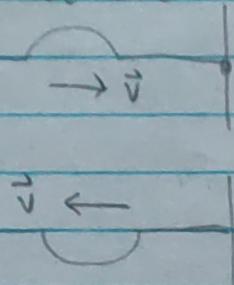
$$P = \frac{1}{2} \mu A^2 w^2 v$$

$$E \propto w^2 A^2$$

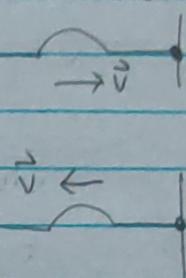
$$P = \frac{dE}{dx} = \frac{dE}{dx} \times v \times (1)$$

## Reflection / Transmission

Fixed End



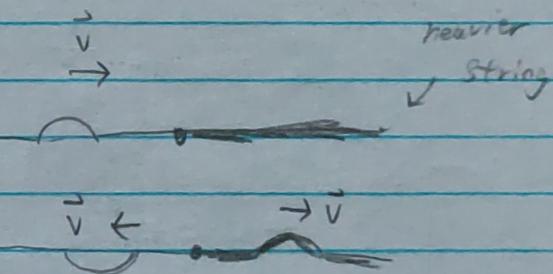
Free End



- When a wave hits a new medium some are reflected some are transmitted.

- If wave travels slower in second medium the reflected pulse will be reflected.

$$A_{\text{inc}} + A_x = A_T$$



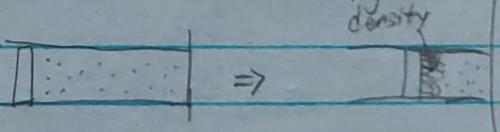
## CH17. Introduction to Sound Waves Jan 30, 2018 Lei Yu

Sound waves are longitudinal wave.

They travel through any material medium

- changes in density and pressure of the air

- lower the frequency, bigger the vibration



Compressed region : compression

low pressure region : rarefaction

## Periodic Sound Wave Displacement

$$S(x, t) = S_{\max} \cos(Kx - \omega t), \quad S \text{ is in the same direction as the wave.}$$

↓ positive x propagation

P<sub>e</sub>

## Periodic Sound Waves: Pressure

Jan 30, 2018 Lei Yu

$$\Delta P = \Delta P_{\max} \sin(kx - wt)$$

$\Delta P$  = maximum pressure change from equilibrium

• Pressure and displacement are out of phase (difference of  $\frac{\pi}{2}$ )

Bulk Modulus to relate Pressure to Displacement

B: resistance to compression

$$\Delta P = -B \frac{\Delta V}{V_i} \Rightarrow \Delta P = -B \frac{\Delta S}{\Delta x}$$

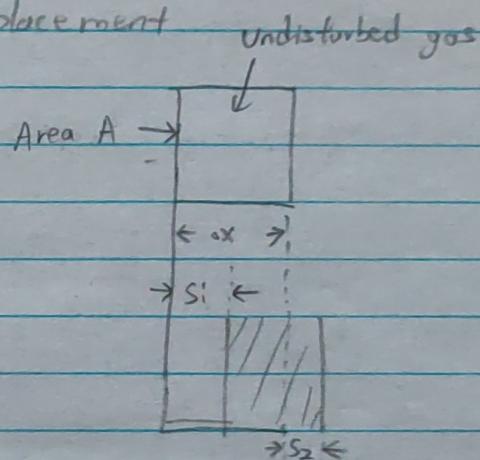
• Pressure  $\uparrow$ , Volume  $\downarrow$

$$V_i = A \cdot \Delta x$$

$$V_f = A \cdot (\Delta x + S_2 - S_1)$$

$$\Delta V = V_f - V_i$$

$$= A(S_2 - S_1)$$



$$\Delta P = -B \frac{[A(S_2 - S_1)]}{\Delta x} = -B \frac{\Delta S}{\Delta x} = -B \frac{ds}{dx} \quad | s \text{ is constant}$$

• When displacement is 0, pressure is max.

Speed of Sound in a gas

[B] = Pascal

$B_{\text{water}} = 10^9 \text{ Pascal}$

$$V = \sqrt{\frac{B}{P}} = \sqrt{\frac{\text{bulk density}}{P}}$$

$$\text{General} \Rightarrow V = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}} \quad \text{string} \Rightarrow V = \sqrt{\frac{T}{\mu}}$$

Amplitude of Pressure and Displacement

$$\Delta P = -B \frac{\Delta S}{\Delta x} \quad \Leftarrow \quad \Delta S = S_{\max} \cos(kx - wt)$$

$$\Delta P = \Delta P_{\max} \sin(kx - wt)$$

$$S = S_{\max} \cos(kx - wt)$$

$$\Delta P_{\max} = B S_{\max} k \quad \Leftarrow \quad w = kv \\ = f v \omega S_{\max}$$

$$v = \sqrt{\frac{B}{\rho}} \Rightarrow B = \rho v^2$$

$Z = fV$ , acoustic impedance

$$V_{\max} = S_{\max} w$$

$$P_{\max} = Z \cdot V_{\max}$$

### 17.3 Power of a Periodic Sound Wave

Power of Wave: energy going through a cross section / area A per unit time.

$$(\text{Energy/Vol})_{\text{Avg}} = \frac{1}{2} \rho V_{\text{max}}^2 = \frac{1}{2} \rho w^2 s_{\text{max}}^2$$

$$(\text{Power})_{\text{Avg}} = (\text{Energy/Vol})_{\text{Avg}} \cdot A \cdot V = \frac{1}{2} \rho A v w^2 s_{\text{max}}^2$$

in  $\Delta t$ , the waves travels  $A v \Delta t$ , thus transmitted energy is

$$P = \frac{\Delta E}{\Delta t} = \frac{(\text{Energy/Vol})_{\text{Avg}} \cdot A v \Delta t}{\Delta t} = \frac{1}{2} \rho A v w^2 s_{\text{max}}^2 \cdot A \text{ (area)}$$

Intensity of a Periodic Sound Wave

Intensity:  $\frac{\text{Power}}{\text{area}}, (I)$

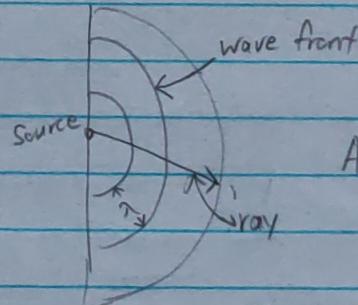
the rate at which energy is being transported by wave

A point source

- spherical wave
- emit sound wave in all directions.

Intensity of Pressure Amplitude

$$I = \frac{(0 P_{\text{max}})^2}{2 \rho V}$$



$$A = 4\pi r^2$$

Intensity decreases as radius increases

Intensity of point source

$$I = \frac{(\text{Power})_{\text{Avg}}}{4\pi r^2}$$

Sound Level

$$\beta = 10 \log\left(\frac{I}{I_0}\right)$$

$I_0$  is reference intensity

$$I_0 = 1.00 \times 10^{-12} \frac{\text{W}}{\text{m}^2}$$

$\beta$  Thresh hold of pain is  $I = 1.0 \text{ W/m}^2$ ;  $\beta = 120 \text{ dB}$

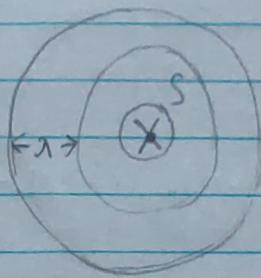
Thresh hold of hearing is  $I_0$ ,  $\beta = 0 \text{ dB}$

## PCS125 Doppler Effect

Feb 1, 2018 Lei Yu

Observer Moving

forward source  
 $\vec{v}_o$  is positive  $\rightarrow \vec{v}_o$



Sphere

$v$  is speed of sound

$$f = \frac{\lambda}{v}, \quad f' = \frac{\lambda}{v+v_o}$$

$$f = \frac{v}{\lambda}, \quad f' = \frac{v+v_o}{v} \cdot f$$

$$f' = \frac{1}{f} = \frac{v+v_o}{\lambda} = \frac{v+v_o}{v} \cdot \frac{v}{\lambda} = \frac{v+v_o}{v} \cdot f$$

Source Moving

Sound barrier: the pressure caused by the compression of wave fronts.

When  $v = v_s$ , sonic boom

$$f' = \left( \frac{v}{v-v_s} \right) f$$

Combining both observer and source

$$f' = \left( \frac{v+v_o}{v-v_s} \right) f$$

$$\frac{v+v_o}{v-v_s}$$

→

$$f' = \left( \frac{v+v_o}{v-v_s} \right) f$$

$$408.0 = \left( \frac{340+v_o}{340-v_s} \right) 400$$

$$1.02 = \frac{340+v_o}{340-v_s}$$

$$346.8 - 1.02v_s = 340 + v_o$$

$$6.8 = 2.02v_s$$

$$3.375 = v_s$$

## CH18 Wave Interference Feb 6, 2018 Lei Yu

Superposition Principle / Interference

displacement and pressure (not amplitude) is the algebraic sum of the individual waves

$$y = A \sin(kx - \omega t)$$

→ ↓

can sum not  $A$

$$y_1 + y_2 = y_f$$

- shape of pulse remain unchanged

$$6 \sin(\pi(2(3) + 3(5)) + 6 \cos[\pi(6 - 13)]) \\ = 6 \sin[\pi(21)] + 6 \cos[\pi(-9)] \\ =$$

Constructive Interference (building)  $A > A_1, A_2$

Deconstructive Interference (canceling)

particle moving faster → ↓

- Energy does not disappear

### Superposition of Sinusoidal Functions

- Same  $A$  and  $f$
- differ only in phase

$$a = kx - \omega t \quad \text{different phase}$$

$$b = kx - \omega t + \phi \checkmark$$

$$y_1 = A \sin a$$

$$y_2 = A \sin b$$

$$y = y_1 + y_2 = A (\sin a + \sin b) = 2A \cos \frac{a-b}{2} \sin \frac{a+b}{2}$$

will become independent of time

$$\sin a = \sin \left( \frac{a+b}{2} + \frac{a-b}{2} \right) \Rightarrow \sin(x+y) = \sin x \cos y + \sin y \cos x$$

↓

$$\sin \left( \frac{a+b}{2} \right) \cos \left( \frac{a-b}{2} \right) + \sin \left( \frac{a-b}{2} \right) \cos \left( \frac{a+b}{2} \right)$$

$$\sin b = \sin \left( \frac{a+b}{2} - \frac{a-b}{2} \right) \Rightarrow$$

### Review of sinusoidal

$$y = A \sin(kx - \omega t + \phi)$$

$$\phi = 2\pi \left( \frac{r_2 - r_1}{\lambda} \right)$$

$$\lambda = 1.25$$

$$\text{phase, } \phi = kx - \omega t + \phi_0 ,$$

$$\phi = 1.6m(0.5)$$

$$\phi = k(x_2 - x_1) , \text{ phase difference at time } t$$

=

$$\phi = k(r_2 - r_1) , \text{ from point source, } r \text{ is radius from source}$$

## Wave interference Feb 6, 2018 Lei Yu

$$y = y_1 + y_2 = \underbrace{2A \cos\left(\frac{\phi}{2}\right)}_{\text{max amplitude}} \sin(kx - wt + \frac{\phi}{2})$$

### Constructive interference

- When  $\phi = 0$ , or any even multiple of  $\pi$  then  $\cos\left(\frac{\phi}{2}\right) = 1$
- Amplitude of resultant wave is  $2A$ .

### Deconstructive Interference

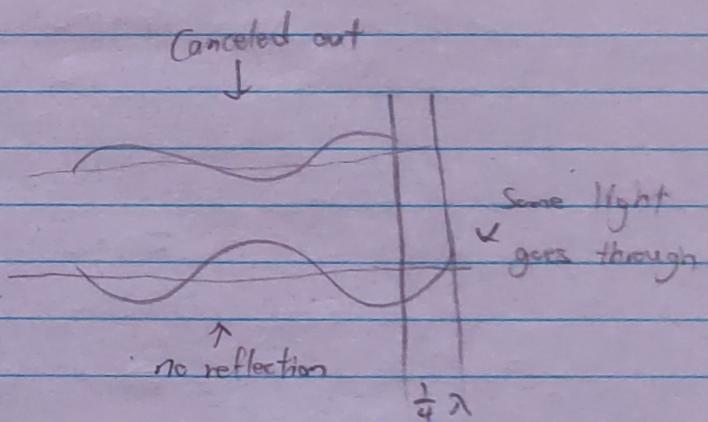
- $\phi = \pi$  or any odd multiple of  $\pi$  then  $\cos\left(\frac{\phi}{2}\right) = 0$ .
- Amplitude of wave is 0

$$y = 2A \cos\left(\frac{\phi}{2}\right) = 0$$

### Anti-reflective Coatings

(coating thickness =  $\frac{1}{4}\lambda$  of light)

- Energy is conserved
- more transmission
- less reflection



### Interference Pattern

$$y_1 = A \sin(kr_1 - wt)$$

$$y_2 = A \sin(kr_2 - wt)$$

$$y_r = y_1 + y_2 = 2A \cos\left(\frac{k(r_2 - r_1)}{2}\right) \sin\left(\frac{k(r_2 + r_1)}{2} - wt\right)$$

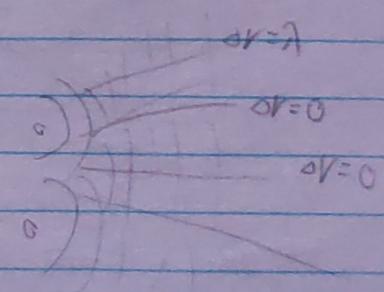
$$= 2A \cos\left(\frac{\phi}{2}\right) \sin\left(\frac{\phi}{2} - wt\right)$$

$$\text{Destructive Interference: } \cos\left(\frac{\phi}{2}\right) = 0, \frac{\phi}{2} = \frac{\pi}{2} + n\pi \Rightarrow \frac{2\pi(r_2 - r_1)}{\lambda} = \frac{\pi(r_2 - r_1)}{\lambda}$$

$$r_2 - r_1 = (2n+1) \frac{\lambda}{2}$$

$$\text{Constructive Interference: } \cos\left(\frac{\phi}{2}\right) = 1 \text{ or } -1$$

$$\text{or } r_2 - r_1 = n\lambda$$

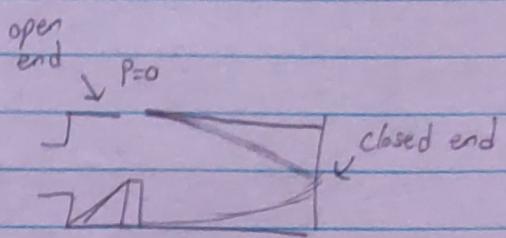


## Standing Wave with Air Column Feb 13, 2018

### Closed End

- the closed end of a pipe is a displacement node.
- pressure antinode

\* Pressure wave is  $90^\circ$  out of phase with displacement wave

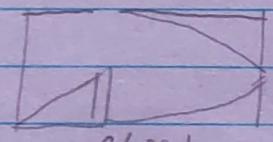


### Open End

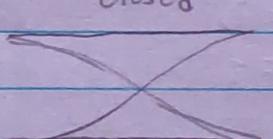
- open end, antinode of displacement wave, node of pressure wave.

one end open

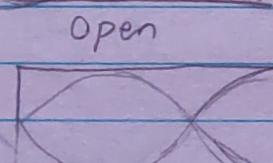
fundamental



$$L = \frac{\lambda}{4} + n * \left(\frac{\lambda}{2}\right), \quad n=0, 1, 2, \dots$$



$$\text{both open, both closed} \quad L = n * \left(\frac{\lambda}{2}\right) \quad n=1, 2 \quad \text{fundamental}$$



$$L = (2n+1) \frac{\lambda}{4}$$

$$f = \frac{nV}{2L}$$

$n=4$ , we have 9th harmonic

$$f = \frac{V}{\lambda} \Rightarrow f = \frac{(2n+1)V}{4L} \quad \text{or} \quad f = \frac{(\text{harmonic } \#)V}{4L}$$

Harmonics:

$$f = \frac{3(340)}{4(0.9)} =$$

$$\text{Harmonic } \# = \frac{f}{f_0}, \quad \frac{\text{frequency}}{\text{fundamental frequency}}$$

Overtones: higher than the fundamental frequency

$$\text{Overtone } \# = \text{Harmonic } \# - 1$$

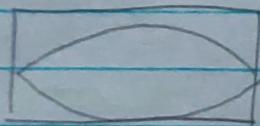
Mass on a spring:  $w = \sqrt{\frac{k}{m}} = 2\pi f$

Simple Pendulum:  $w = \sqrt{\frac{g}{L}}$

Physical Pendulum:  $w = \sqrt{\frac{gmd}{I}}$

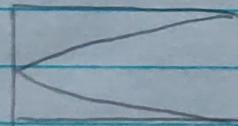
$$k = \frac{2\pi}{T}, w = \frac{2\pi}{T}, V = f\lambda = \frac{w}{k}$$

String:  $V = \sqrt{\frac{T}{\mu}}$  <sup>Tension</sup> Sound  $v = \sqrt{ }$



$$1 \quad \frac{1}{2}\lambda = L \quad \frac{n}{2}\lambda = L$$

$$2 \quad \lambda = L$$



$$1 \quad \frac{1}{4}\lambda = L \quad \frac{2n-1}{2}\lambda = L$$

$$2 \quad \frac{3}{4}\lambda = L$$

$$\frac{5}{4}\lambda = L$$



$$\frac{1}{2}\lambda = L$$

$$\lambda = L$$

