UE Machine Learning
Binary classifier & statistical hypothesis testing
Lab. 3 (Part I)
Quadratic detection of a random signal in white gaussian noise

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Let X be an N-dimensional real random vector such that

$$X = \varepsilon(S+W) + (1-\varepsilon)W$$

where:

- ϵ, S and W are independent
- $S = (S_1, ..., S_N)^T \sim \mathcal{N}(0, \sigma_S^2 \mathrm{I}_N)$ with $\sigma_S \neq 0$, where I_N is the $N \times N$ identity matrix
- $W = (W_1, ..., W_N)^T \sim \mathcal{N}(0, \sigma_W^2 \mathrm{I}_N)$ with $\sigma_W \neq 0$
- $\mathbb{P}[\varepsilon = 1] = \mathbb{P}[\varepsilon = 0] = 1/2$.

Denote by $f_{\chi_N^2}$ the centred χ_N^2 law with N degrees of freedom, whose pdf is:

$$f_{\chi_N^2}(x) = \frac{1}{2^{N/2}\Gamma(N/2)} x^{N/2-1} e^{-x/2} \mathbb{1}_{[0,\infty)}(x)$$

1) Compute the MPE test and its probability of error as a function of :

$$Q_{\chi_N^2}(x) = \int_x^\infty f_{\chi_N^2}(t)\mathrm{d}t$$

2) Check your theoretical result by Monte-Carlo simulations