

# File Structures

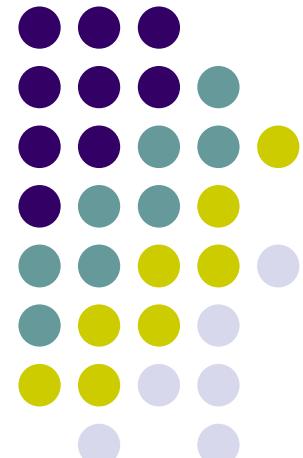
## Ch08. B. Sorting of Large Files

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# Outline



- 8.1 Cosequential operations
- 8.2 Application of the Model to a General Ledger Program
- **8.3 Extension of the Model to Include Multiway Merging**
- 8.4 A Second Look at Sorting in Memory
- 8.5 Merging as a Way of Sorting Large Files on Disk
- Skipped
  - 8.6 Sorting Files on Tape
  - 8.7 Sort-Merge Packages
  - 8.8 Sorting and Cosequential Processing in Unix

# A K-way Merge Algorithm (1/3)



- K-way merge
  - A very general form of cosequential file processing
  - Merge **K sorted input lists** to create a single sorted output list
- Adapt 2-way merge algorithm
  - Instead of List1 and Lists2 keep an array of lists:  
 $\text{List}[1], \text{List}[2], \dots, \text{List}[k]$
  - Instead of item(1) and item(2) keep an array of items:  
 $\text{item}[1], \text{item}[2], \dots, \text{item}[k]$

# A K-way Merge Algorithm (2/3)



- The synchronization step for 2 lists

```
if item(1) < item(2) then ...
if item(1) > item(2) then ...
if item(1) = item(2) then ...
```

- Modify

```
(1) minitem = index of minimum item in item[1],
    item[2], ..., item[k]
(2) output item[minitem] to output list
(3) for i=1 to K do
(4)     if item[i] = item[minitem] then
(5)         get next item from List[i]
```

- If there are no repeated items among different lists, lines (3)-(5) can be simplified to:

get next item from List[minitem]

# A K-way Merge Algorithm (3/3)



- C++/c style

```
// find an index of minimum item
int minItem = MinIndex(Item,k)
// Item(minItem) is the next output
ProcessItem(minItem);
for(i=0; i<k; i++)          // look at each list
    if( Item(minItem) == Item(i)) // advance list i
        MoreItems[i] = NextItemInList(i);
```

- No repeated items

```
// find an index of minimum item
int minI = MinIndex(Item,k)
// Item(minItem) is the next output
ProcessItem(minI);
MoreItems[minI] = NextItemInList(minI);
```

# Review: Ledger code



```
// return current item from this list
int LedgerProcess::Item (int ListNumber)
{
    return AccountNumber[ListNumber];}

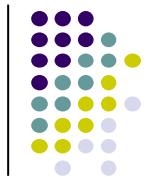
// process the item in this list when it first appears
int LedgerProcess::ProcessItem (int ListNumber)
{
    switch (ListNumber)
    {
        case 1: // process new ledger object
            ledger.PrintHeader(OutputList);
        case 2: // process journal file
            journal.PrintLine(OutputList);
    }
    return TRUE;
}
```

```
//get next item from this list
int LedgerProcess::NextItemInList (int ListNumber)
{
    switch (ListNumber)
    {
        case 1: return NextItemInLedger ();
        case 2: return NextItemInJournal ();
    }
    return FALSE;
}
```

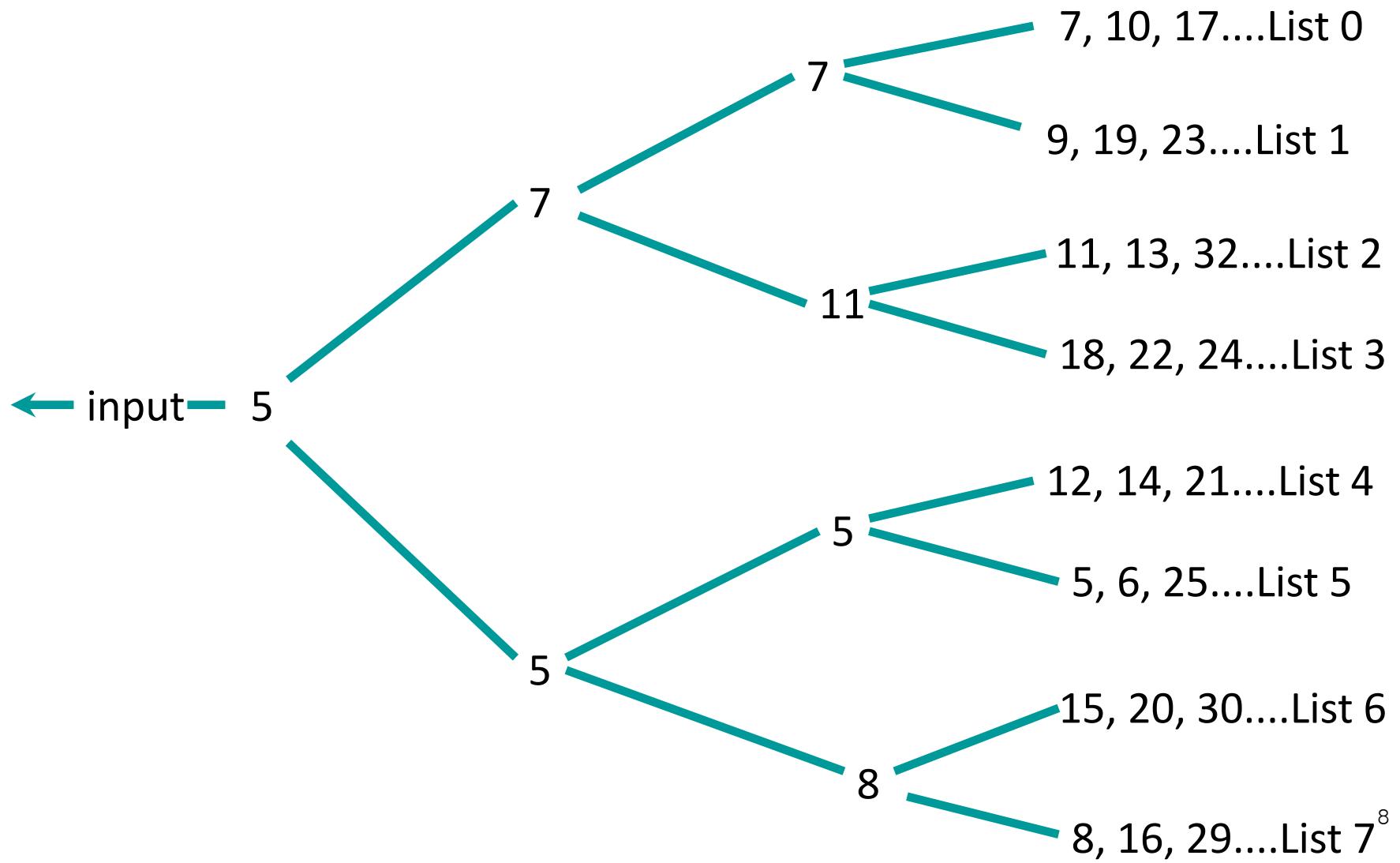
# Selection Tree for Merging Large Number of Lists



- K-way merge
  - nice if K is no larger than 8 or so
  - if  $K > 8$ , the set of comparisons for minimum key is expensive
  - loop of comparison (computing)
- Selection Tree (if  $K > 8$ )
  - time vs. space trade off
  - a kind of “tournament” tree
  - the minimum value is at root node
  - the depth of tree is  $\log_2 K$



# Selection Tree



# CosequentialProcess class



- A single, simple model that can be the basis for the construction of any kind of consequential process
  - supports processing of any type of list
  - Includes operations to match and merge lists
  - Defines the list processing operations required for cosequential processing as virtual methods

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- 8.5 Merging as a Way of Sorting Large Files on Disk
- Skipped
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# A Second Look at Sorting in Memory



- Read the whole file from into memory, perform sorting, write the whole file into disk
- Can we improve on the time that it takes for this RAM sort?
  - perform some of parts in parallel
  - selection sort is good but cannot be used to sort entire file
- Using **Heap** technique!
  - processing and I/O can occur **in parallel**
  - keep all the keys in **heap**
- Heap building while reading a block
- Heap rebuilding while writing a block

# Overlapping processing and I/O



- Heap
  - a kind of binary tree, **complete binary tree**
  - each node has a single key, that key is **less than or equal** to the key at its parent node
  - storage for tree can be allocated sequentially
- so there is **no need for pointers or other dynamic overhead** for maintaining the heap
- Details: Skipped

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# Challenge



- Merging Big Files with Small Memory

How do we *efficiently* merge two sorted files when both are **much larger than our main memory buffer?**

# External Merge Algorithm



- **Input:** 2 sorted lists of length M and N
- **Output:** 1 sorted list of length M + N
- **Required:** At least 3 Buffer Pages
- **IOs:**  $2(M+N)$

# Key (Simple) Idea



- To find an element that is no larger than all elements in two lists, one only needs to compare minimum elements from each list.

If:

$$\begin{aligned} A_1 &\leq A_2 \leq \dots \leq A_N \\ B_1 &\leq B_2 \leq \dots \leq B_M \end{aligned}$$

Then:

$$\begin{aligned} \min(A_1, B_1) &\leq A_i \\ \min(A_1, B_1) &\leq B_j \end{aligned}$$

for  $i=1\dots N$  and  $j=1\dots M$

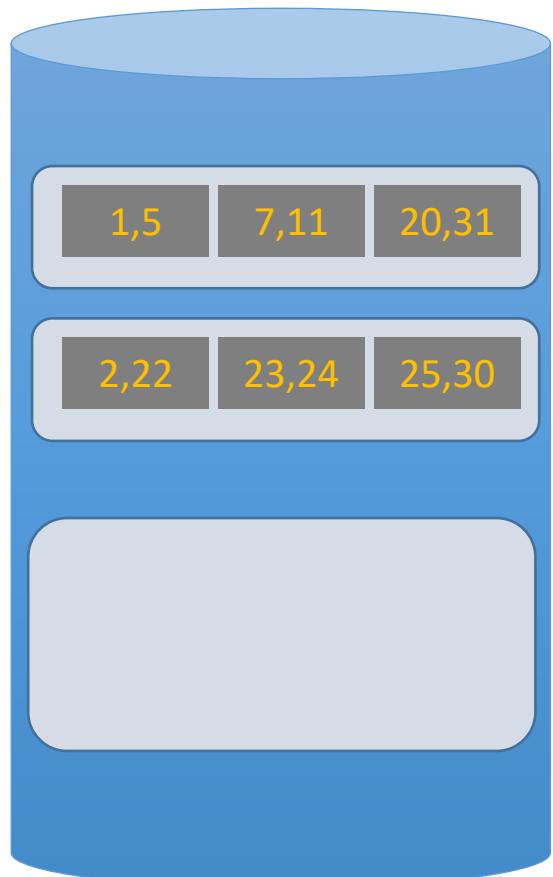
# Review: External Merge Algorithm (1/10)



Input:  
Two sorted  
files

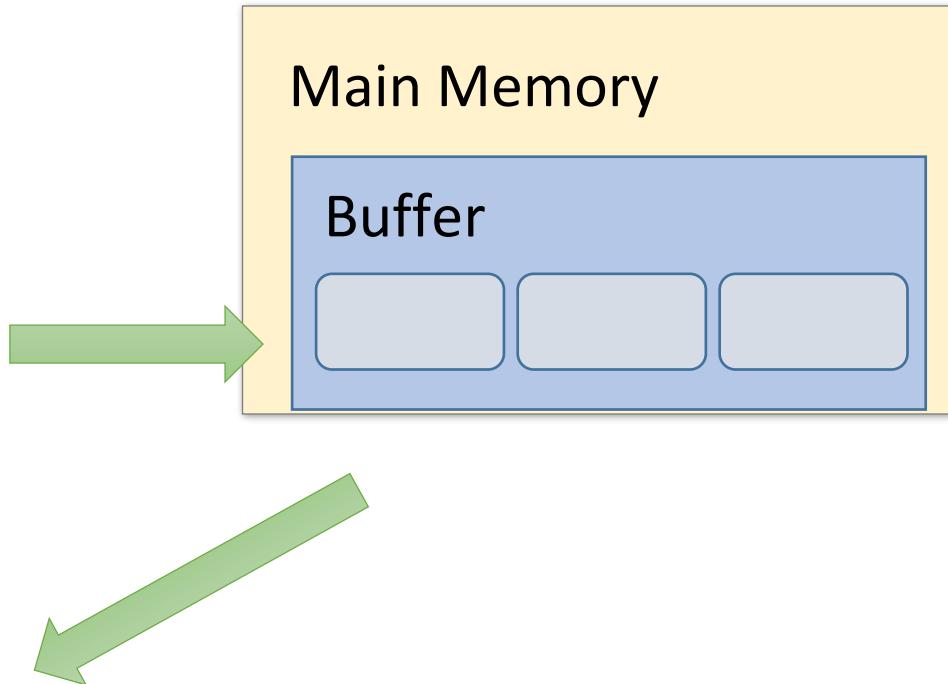
$F_1$

$F_2$



Output:  
One *merged*  
sorted file

Disk



# Review: External Merge Algorithm (2/10)

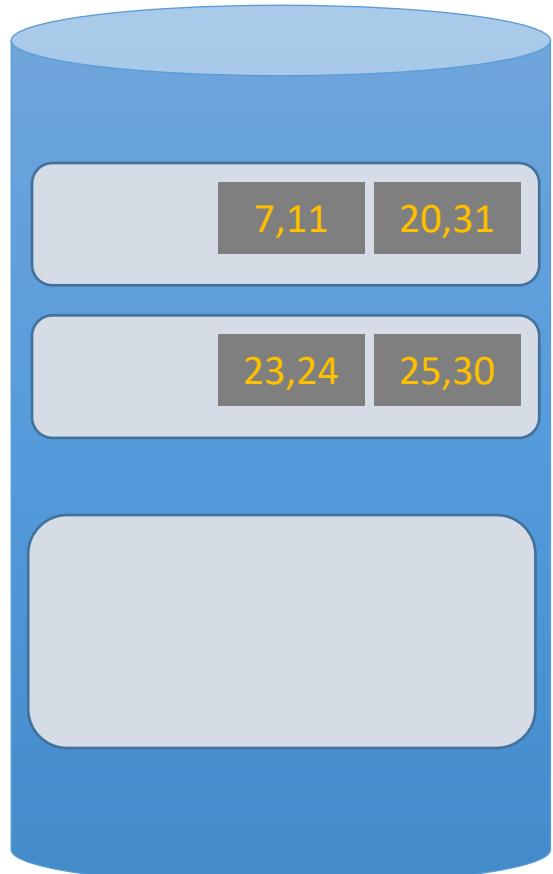


Input:  
Two sorted  
files

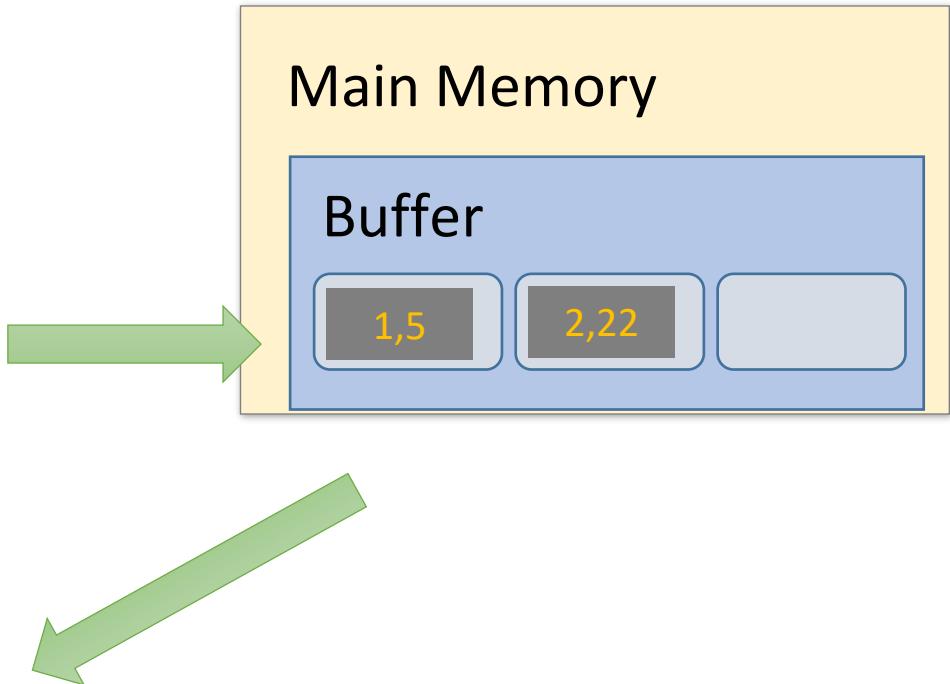
$F_1$

$F_2$

Output:  
One *merged*  
sorted file



Disk

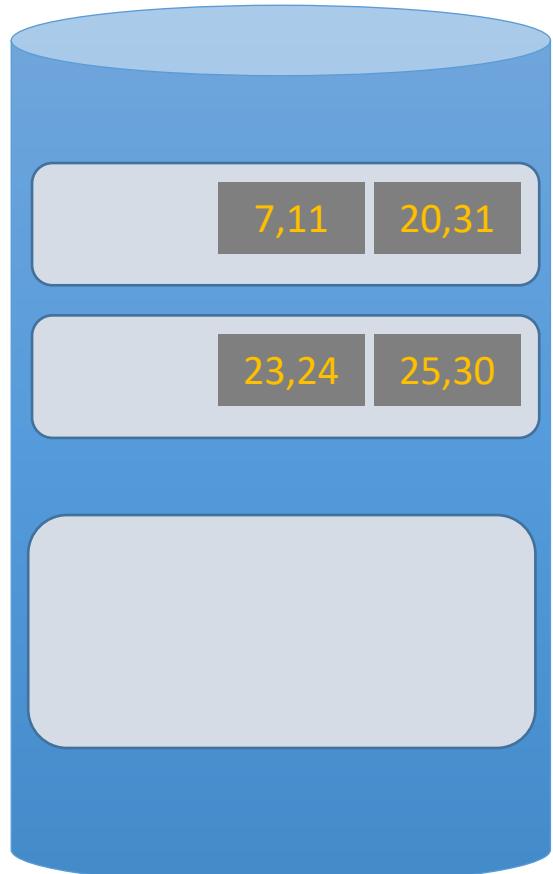


# Review: External Merge Algorithm (3/10)

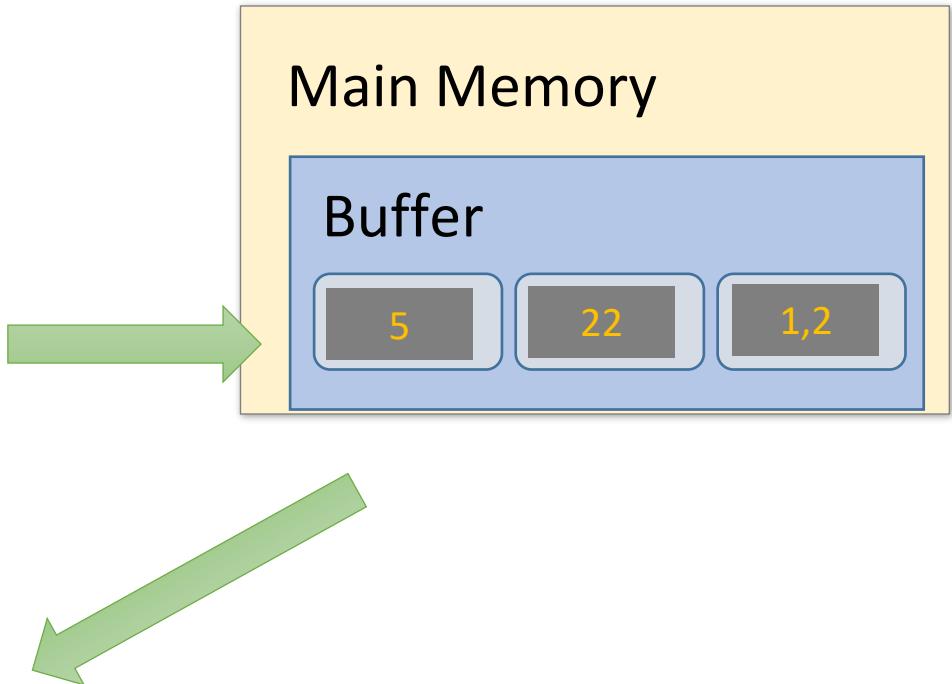


Input:  
Two sorted  
files  
 $F_1$   
 $F_2$

Output:  
One *merged*  
sorted file



Disk



# Review: External Merge Algorithm (4/10)

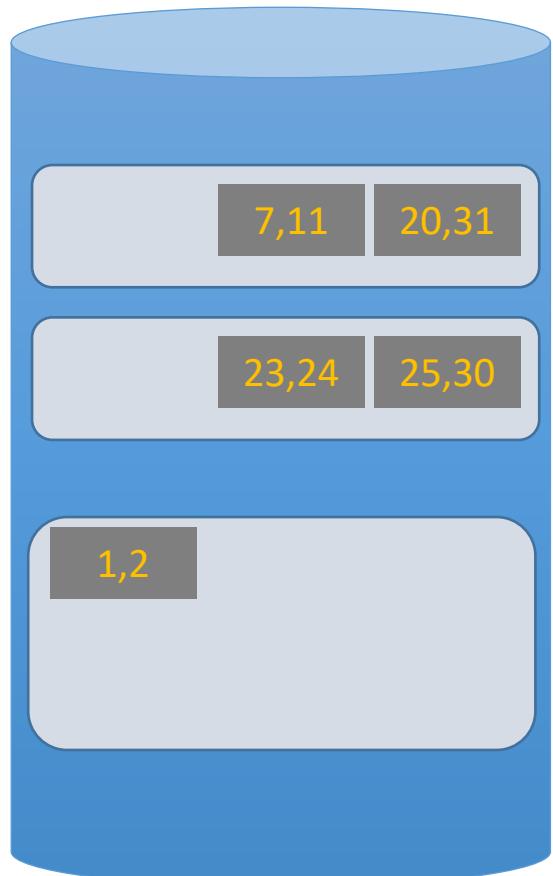


Input:  
Two sorted  
files

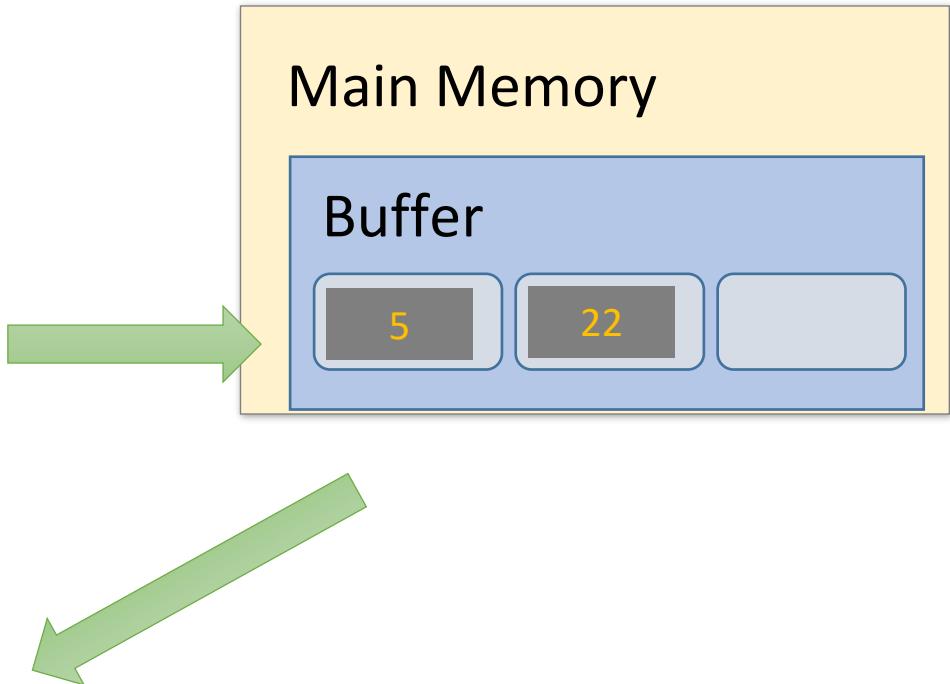
$F_1$

$F_2$

Output:  
One *merged*  
sorted file



Disk



# Review: External Merge Algorithm (5/10)

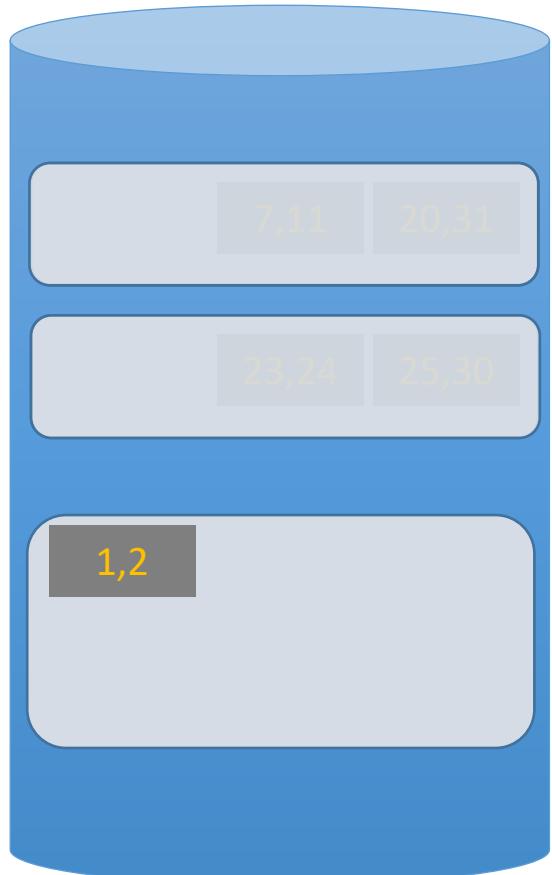


Input:  
Two sorted  
files

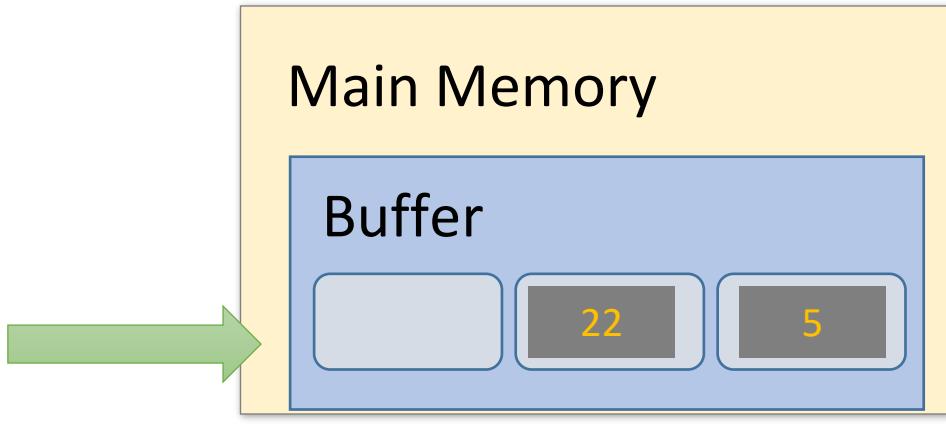
$F_1$

$F_2$

Output:  
One *merged*  
sorted file



Disk

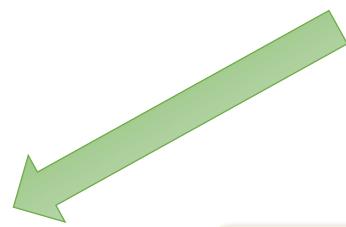


Main Memory

Buffer

22

5



This is all the algorithm  
“sees”... Which file to load a  
page from next?

# Review: External Merge Algorithm (6/10)

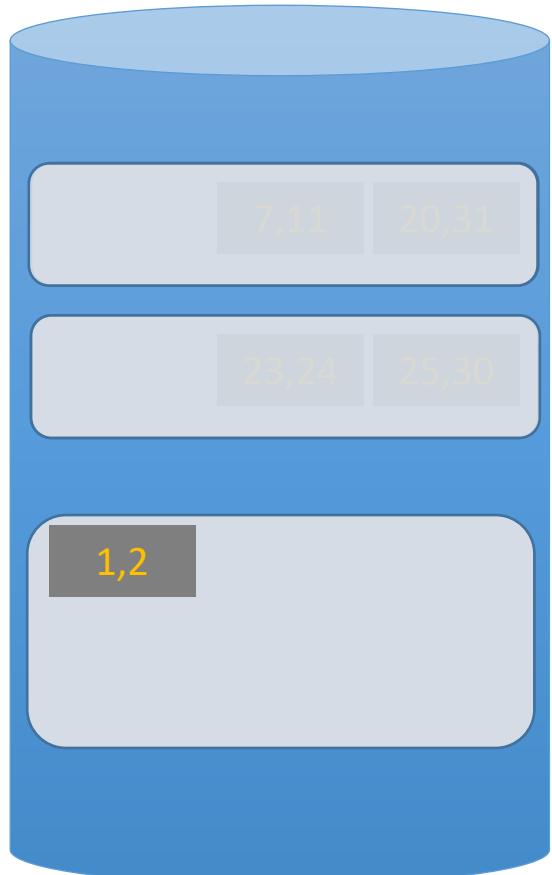


Input:  
Two sorted  
files

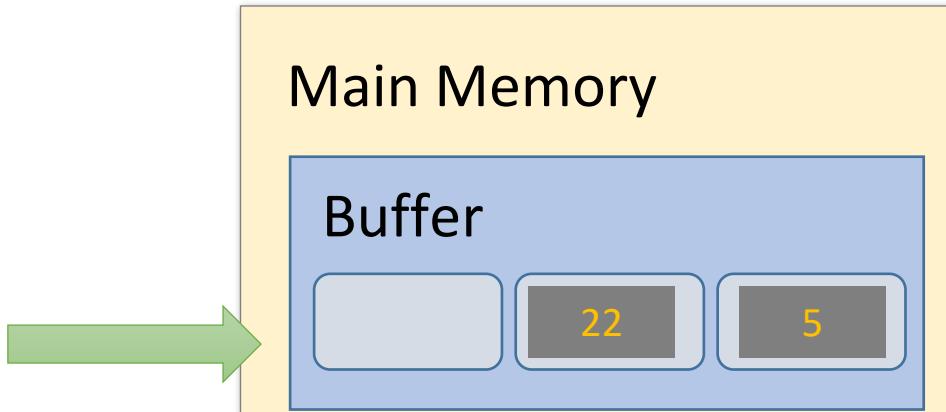
$F_1$

$F_2$

Output:  
One *merged*  
sorted file



Disk

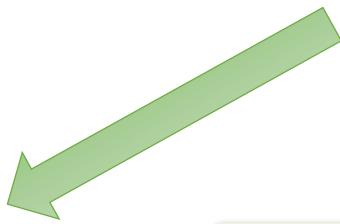


Main Memory

Buffer

22

5



We know that  $F_2$  only contains values  $\geq 22$ ... so we should load from  $F_1$ !

# Review: External Merge Algorithm (7/10)

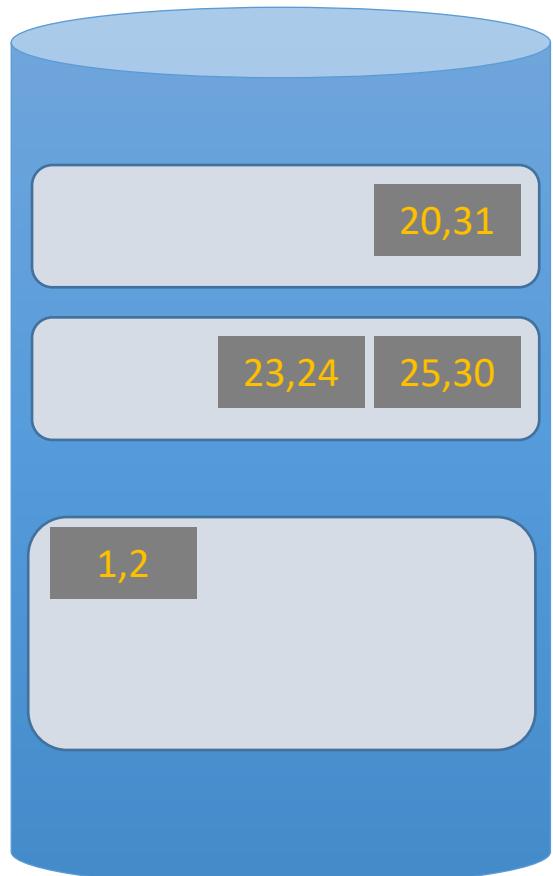


Input:  
Two sorted  
files

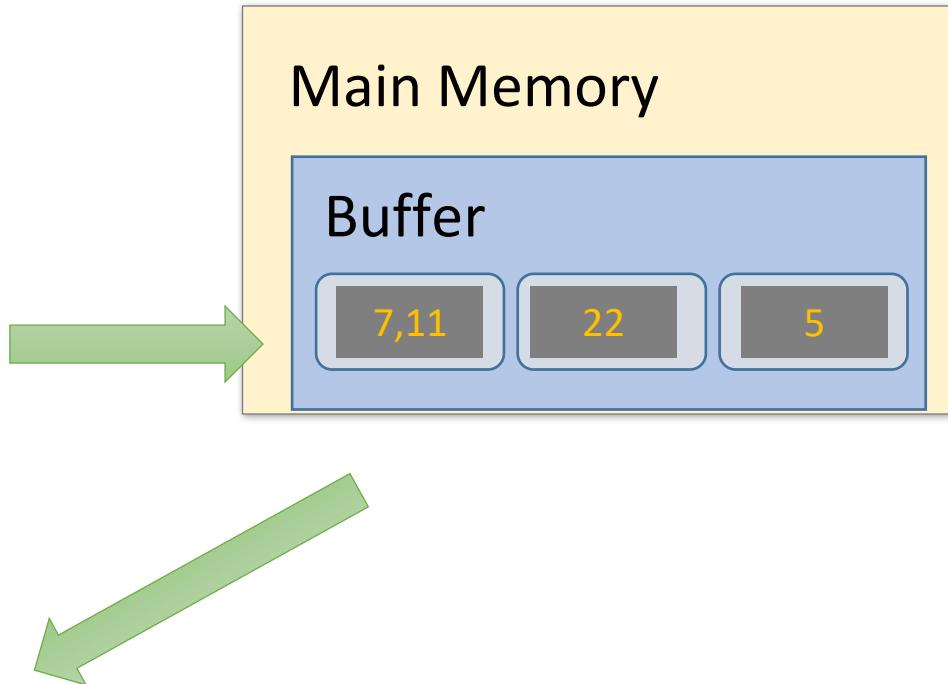
$F_1$

$F_2$

Output:  
One *merged*  
sorted file



Disk



# Review: External Merge Algorithm (8/10)

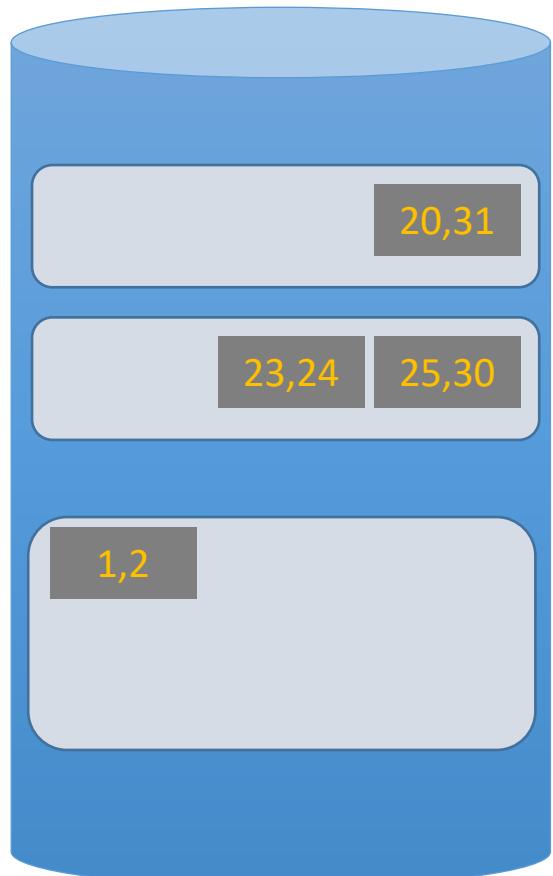


Input:  
Two sorted  
files

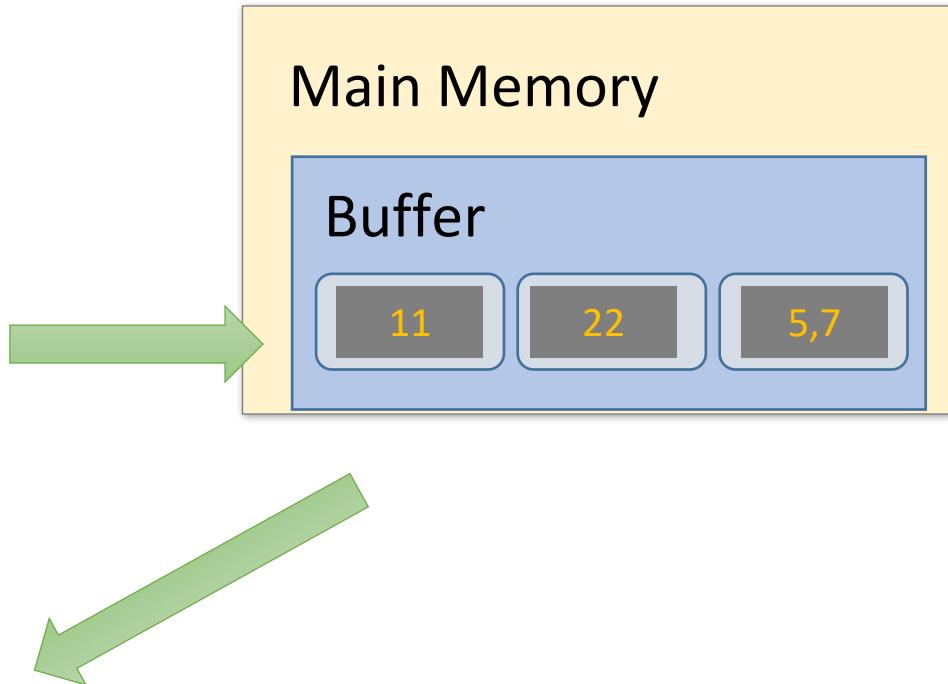
$F_1$

$F_2$

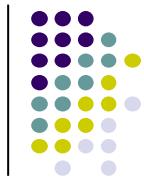
Output:  
One *merged*  
sorted file



Disk



# Review: External Merge Algorithm (9/10)

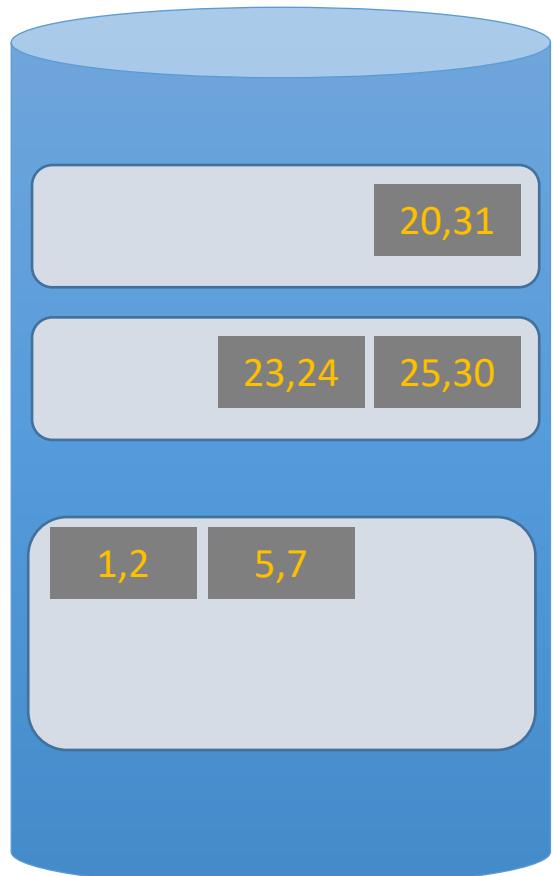


Input:  
Two sorted  
files

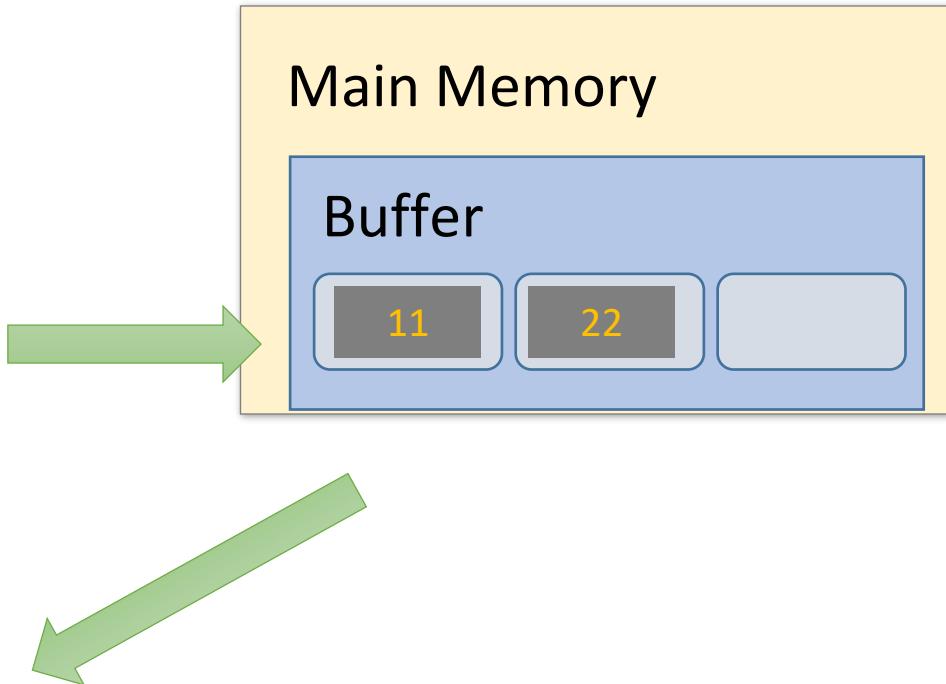
$F_1$

$F_2$

Output:  
One *merged*  
sorted file



Disk



# Review: External Merge Algorithm (10/10)

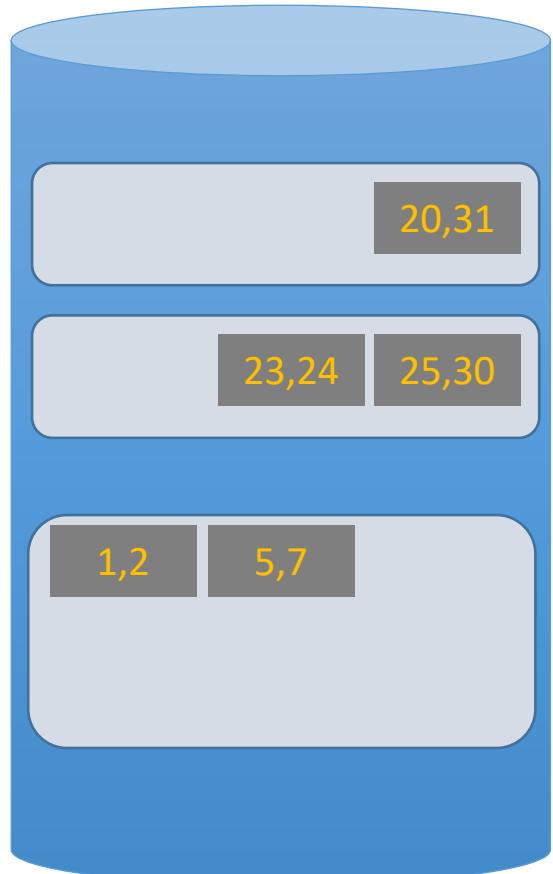


Input:  
Two sorted  
files

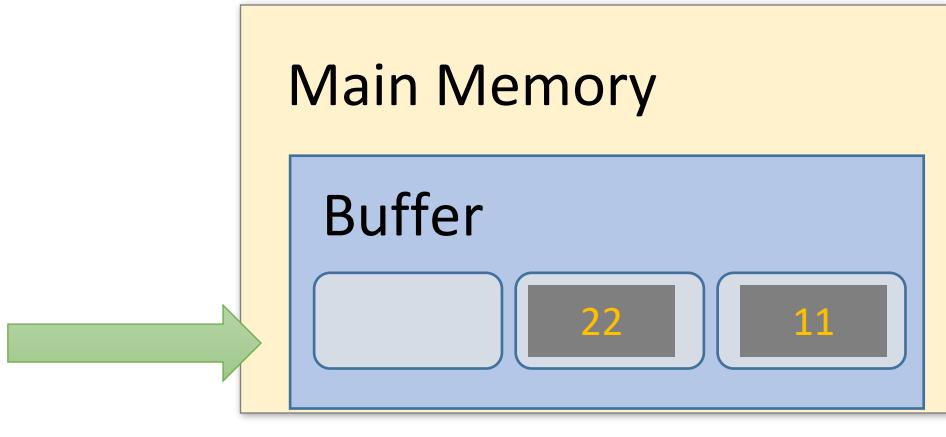
$F_1$

$F_2$

Output:  
One *merged*  
sorted file



Disk



Main Memory

Buffer

22

11



And so on...

# Summary of external merging



- We can merge 2 lists of **arbitrary length** with *only* 3 buffer pages.

If lists of size M and N, then

Cost:  $2(M+N)$  IOs

Each page is read once, written once

With  $B+1$  buffer pages, can merge  $B$  lists. How?

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# External Merge Algorithm



- Suppose we want to merge two **sorted** files both much larger than main memory (i.e. the buffer)
- We can use the **external merge algorithm** to merge files of *arbitrary length* in  $2*(N+M)$  IO operations with only **3 buffer pages!**

Our first example of an “IO aware”  
algorithm / cost model

# Why are Sort Algorithms Important?



- Data requested from DB in sorted order is **extremely common**
  - e.g., find students in increasing GPA order
- **Why not just use quicksort in main memory??**
  - What about if we need to sort 1TB of data with 1GB of RAM...

A classic problem in computer science!

# More reasons to sort...

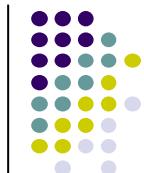


- Sorting useful for eliminating *duplicate copies* in a collection of records (Why?)
- Sorting is first step in *bulk loading* B+ tree index.

*Coming up...*

- *Sort-merge join* algorithm involves sorting

*Coming up...*



# So how do we sort big files?

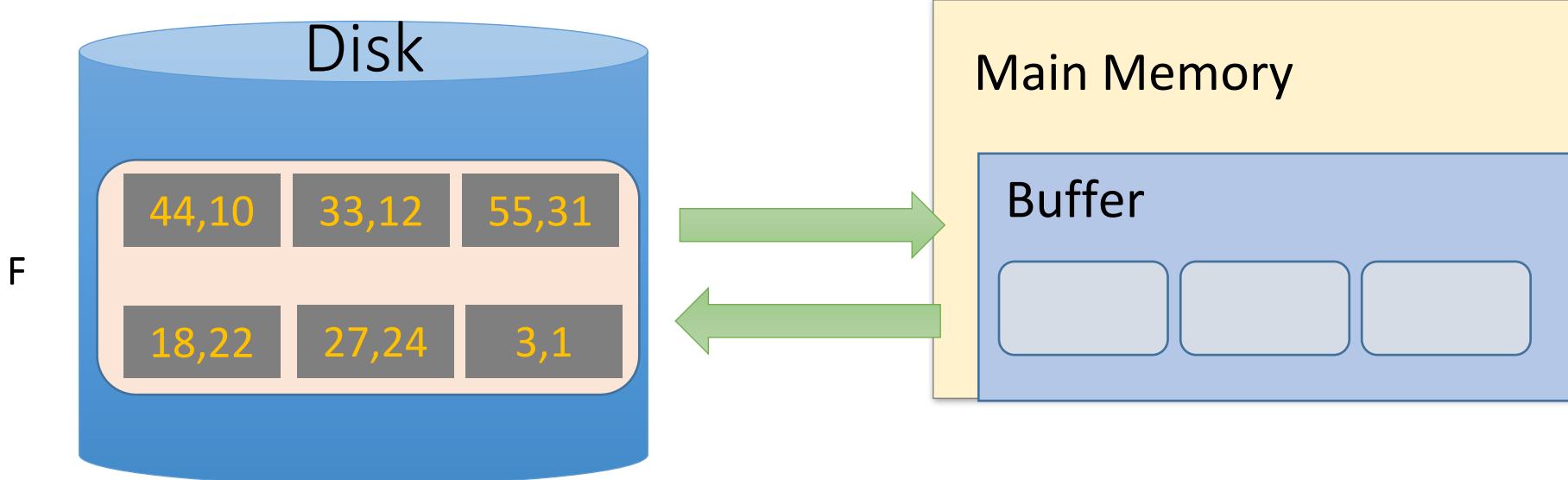
1. Split into chunks small enough to **sort in memory** (“*runs*”)
2. **Merge** pairs (or groups) of runs *using the external merge algorithm*
3. **Keep merging** the resulting runs (*each time = a “pass”*) until left with one sorted file!

# External Merge Sort Algorithm (2-way sort)

## (1/6)



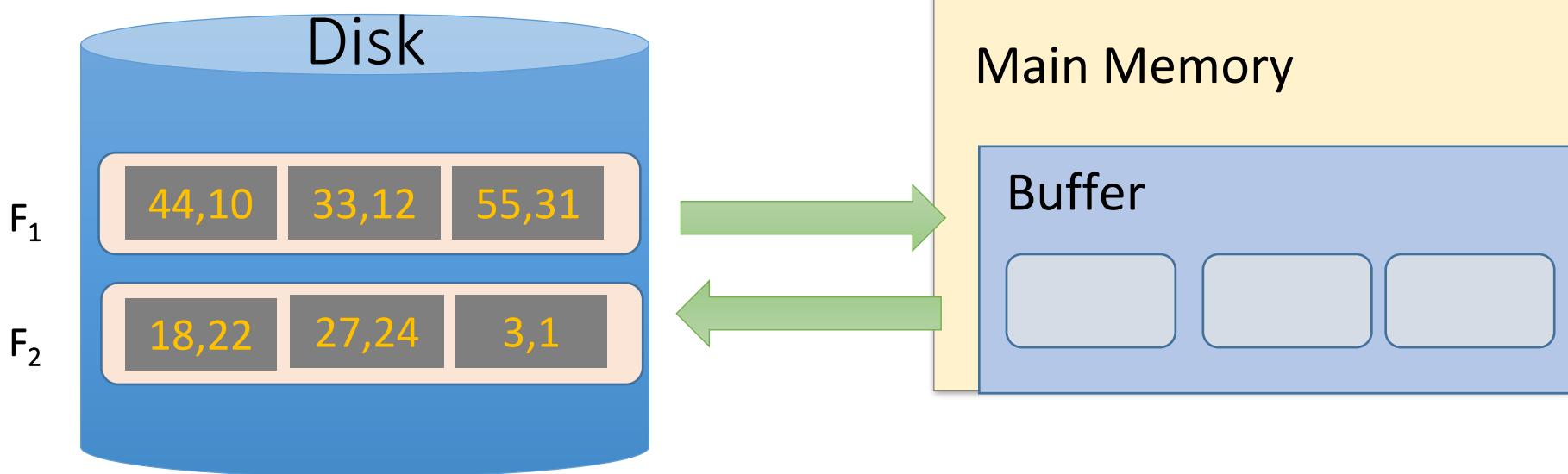
- Example: 3 Buffer pages, 6-page file
  1. Split into chunks small enough to **sort in memory**



# External Merge Sort Algorithm (2-way sort) (2/6)



- Example: 3 Buffer pages, 6-page file
  1. Split into chunks small enough to **sort in memory**



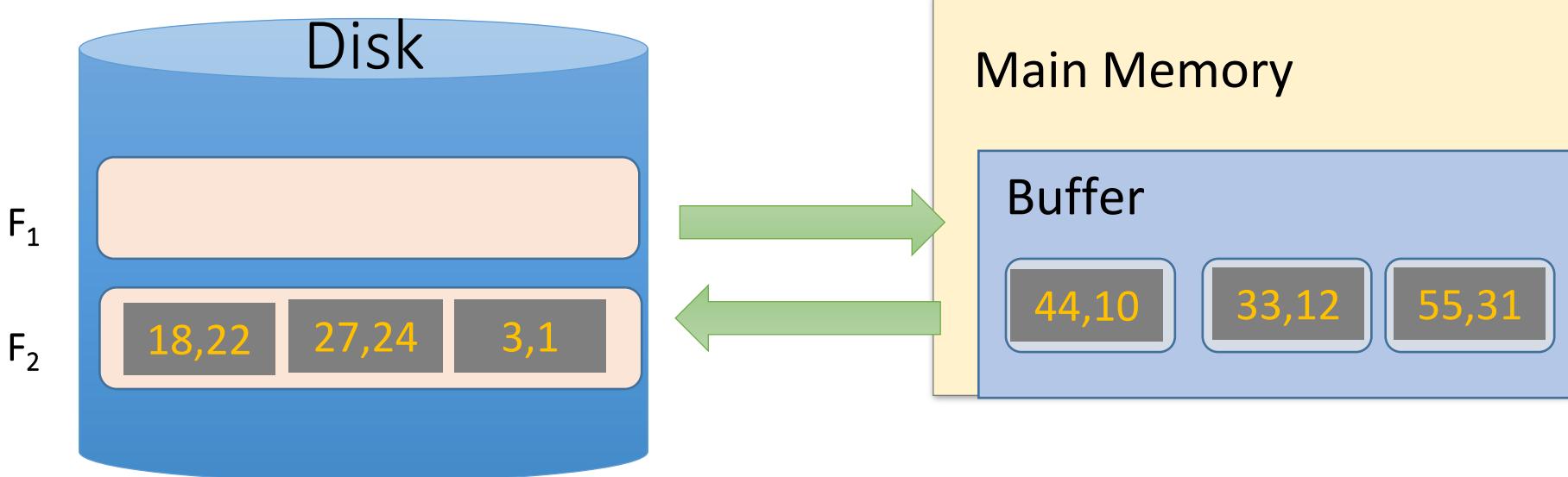
Orange file  
= unsorted

# External Merge Sort Algorithm (2-way sort)

## (3/6)



- Example: 3 Buffer pages, 6-page file
  1. Split into chunks small enough to **sort in memory**



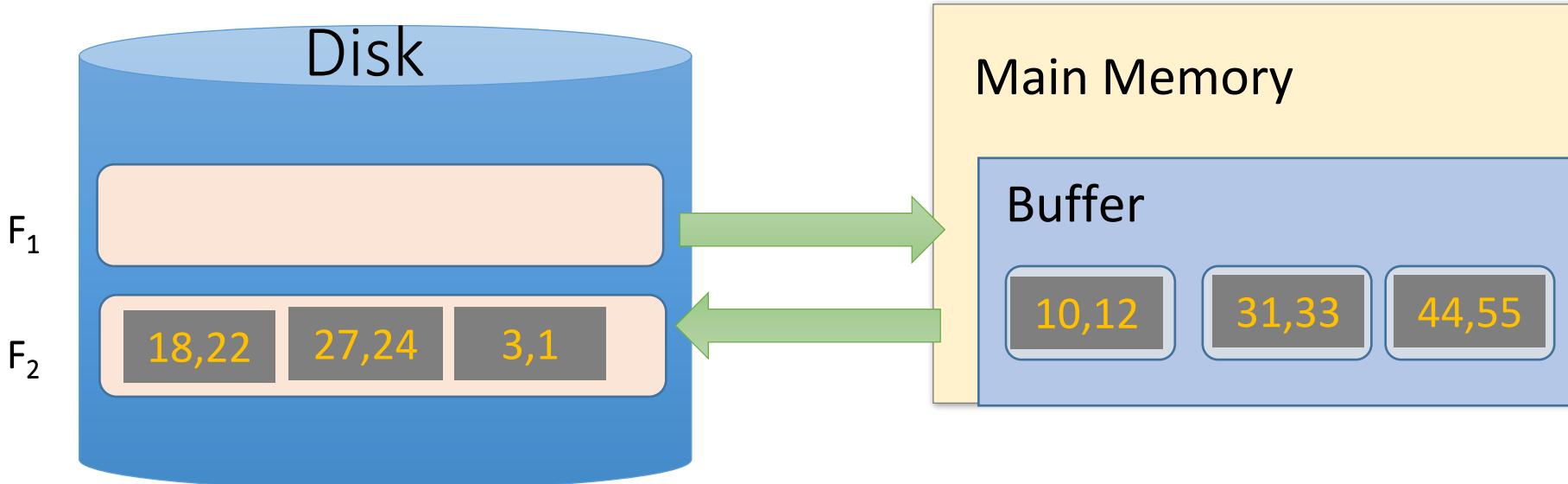
Orange file  
= unsorted

# External Merge Sort Algorithm (2-way sort)

## (4/6)



- Example: 3 Buffer pages, 6-page file
  1. Split into chunks small enough to **sort in memory**

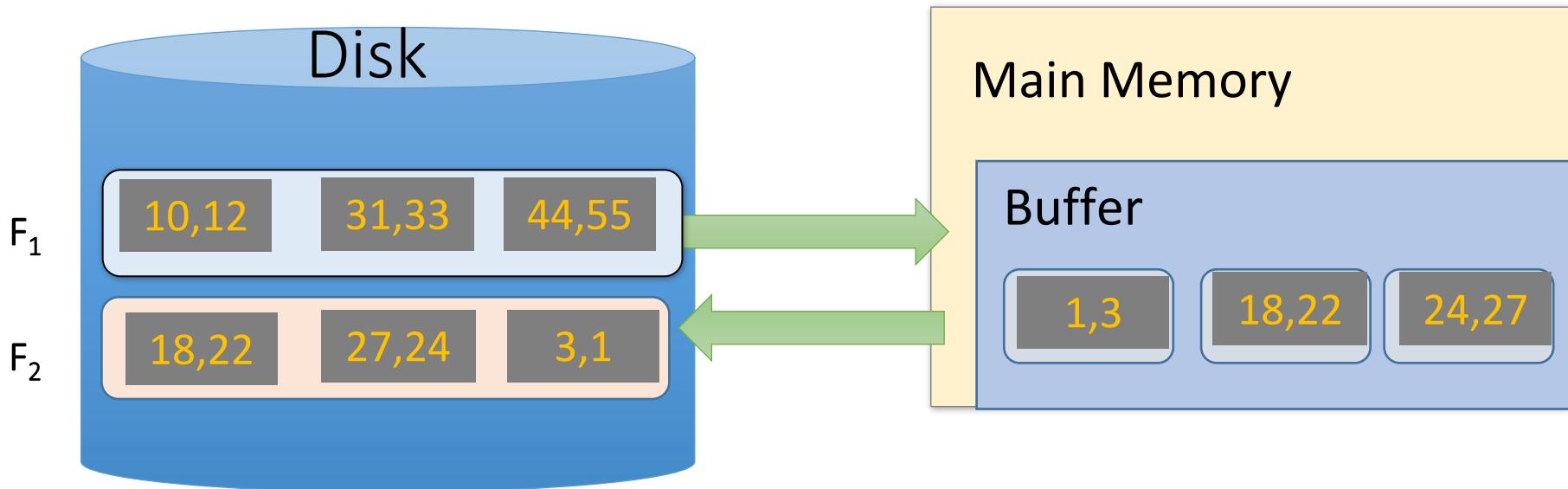


# External Merge Sort Algorithm (2-way sort)

## (5/6)



- Example: 3 Buffer pages, 6-page file
  1. Split into chunks small enough to **sort in memory**



And similarly for  $F_2$

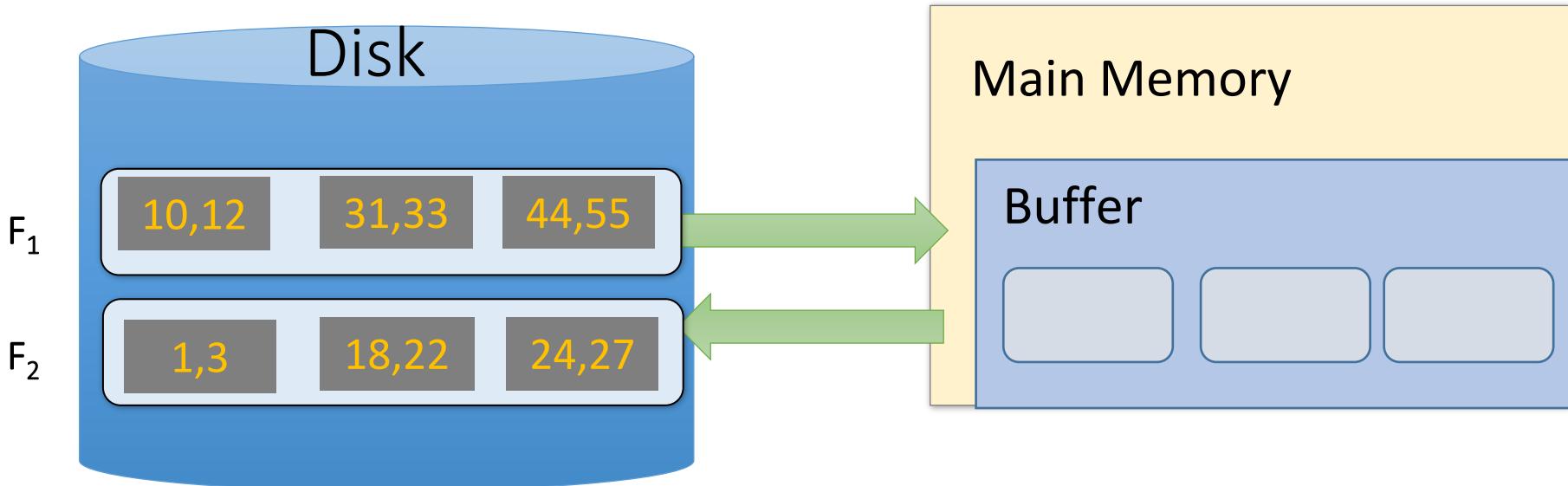
Each sorted file is  
a called a *run*

# External Merge Sort Algorithm (2-way sort)

## (6/6)



- Example: 3 Buffer pages, 6-page file



2. Now just run the **external merge** algorithm & we're done!

# Calculating IO Cost

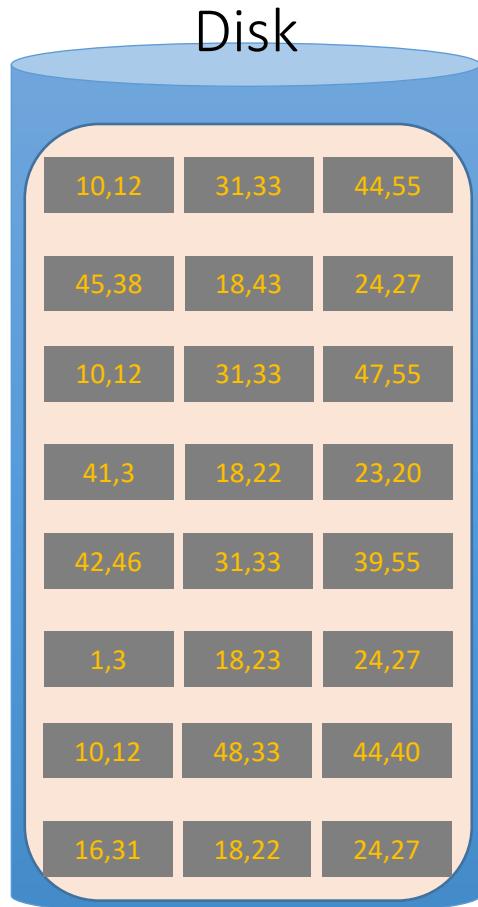


- For 3 buffer pages, 6 page file:
  - Split into **two 3-page** files and **sort in memory**
    - = **1 R + 1 W** for each file =  $2 * (3 + 3) = 12$  IO operations
  - Merge each pair of sorted chunks using the external merge algorithm
    - =  $2 * (3 + 3) = 12$  IO operations
  - **Total cost = 24 IO**

# Running External Merge Sort on Larger Files (1/6)



- Assume we still only have 3 buffer pages (*Buffer not pictured*)

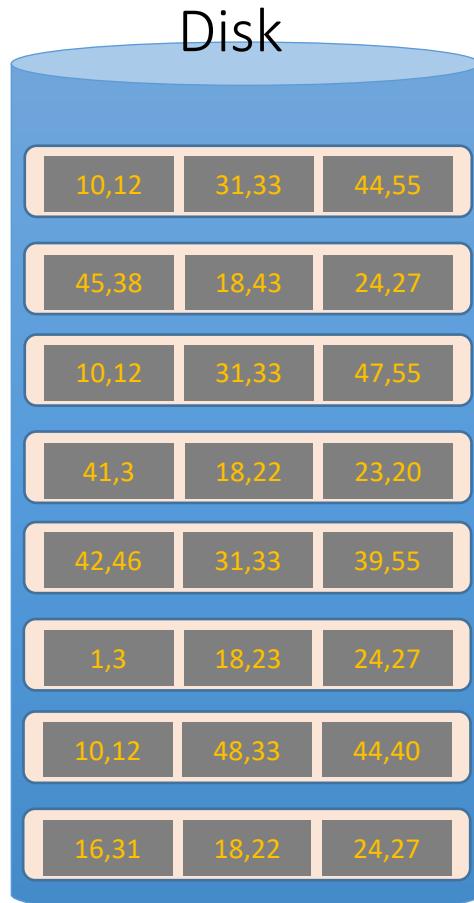


# Running External Merge Sort on Larger Files

## (2/6)



- Assume we still only have 3 buffer pages (*Buffer not pictured*)

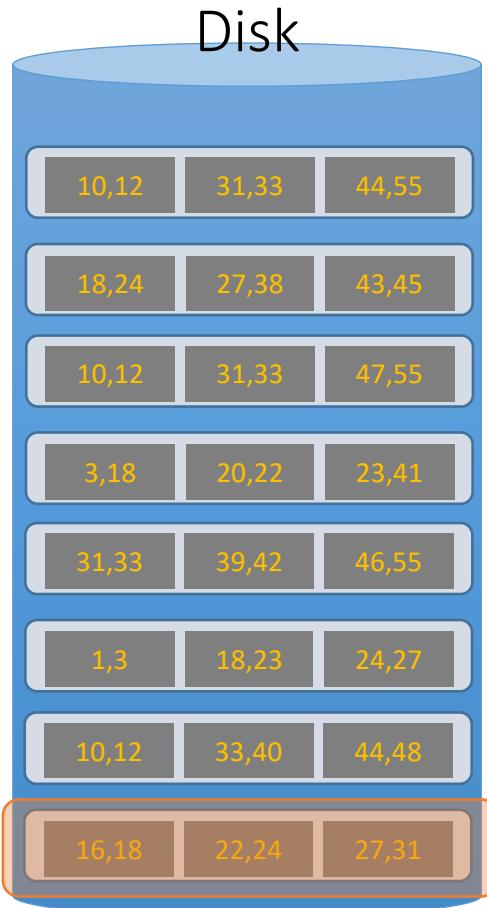


1. Split into files small enough to sort in buffer...

# Running External Merge Sort on Larger Files (3/6)



- Assume we still only have 3 buffer pages (*Buffer not pictured*)



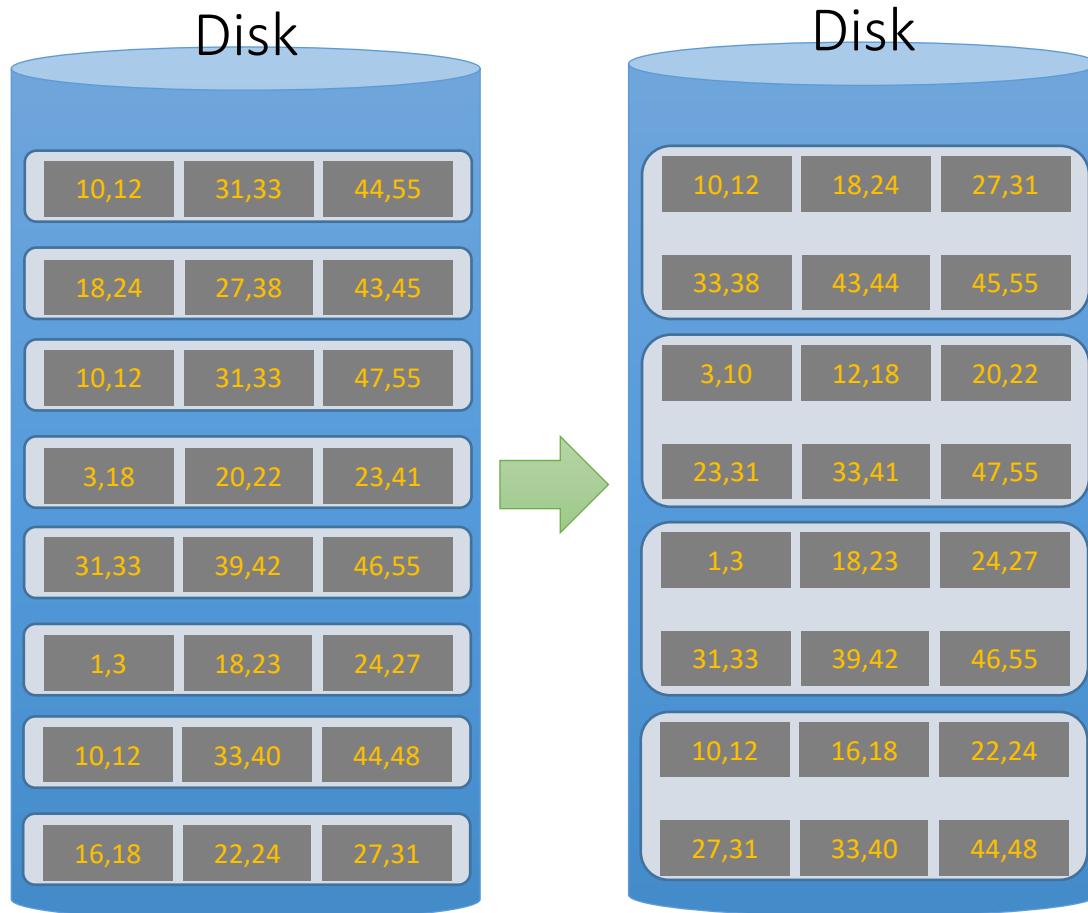
1. Split into files small enough to sort in buffer... and sort

Call each of these sorted files a *run*

# Running External Merge Sort on Larger Files (4/6)



- Assume we still only have 3 buffer pages (*Buffer not pictured*)

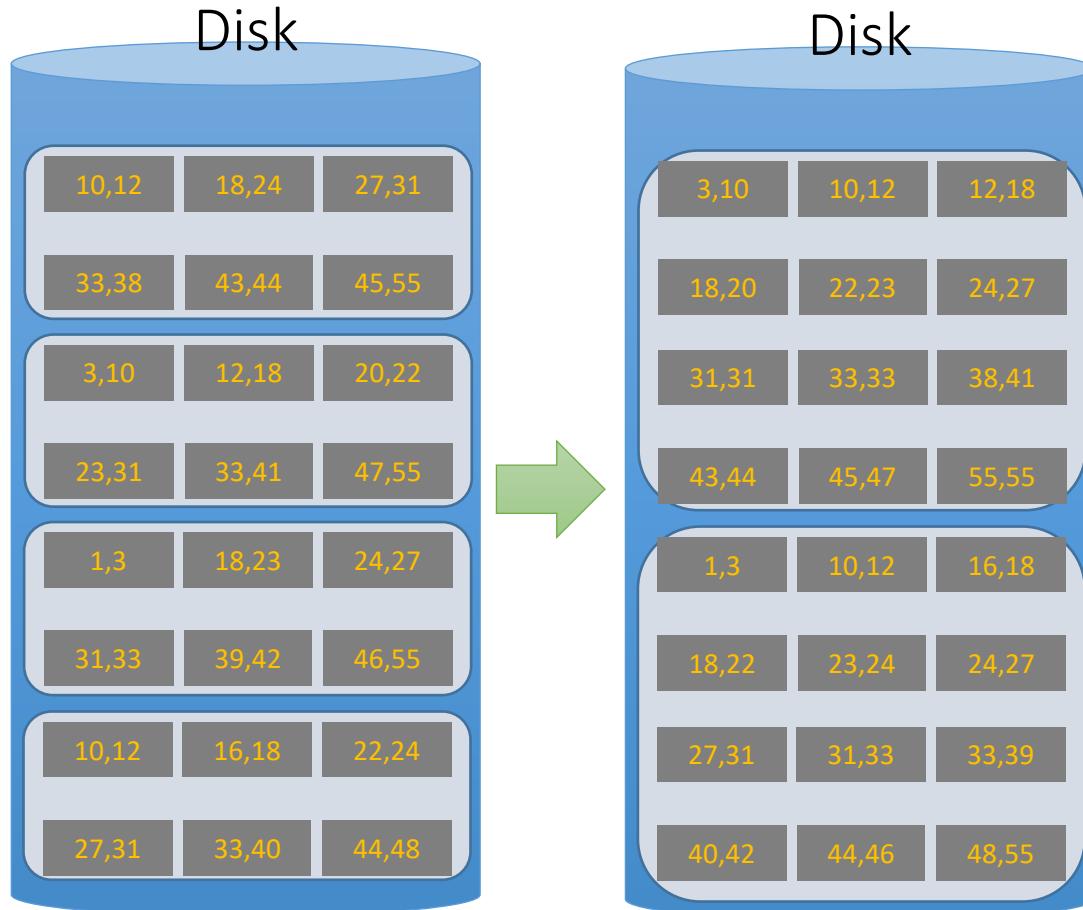


2. Now merge pairs of (sorted) files... **the resulting files will be sorted!**

# Running External Merge Sort on Larger Files (5/6)



- Assume we still only have 3 buffer pages (*Buffer not pictured*)



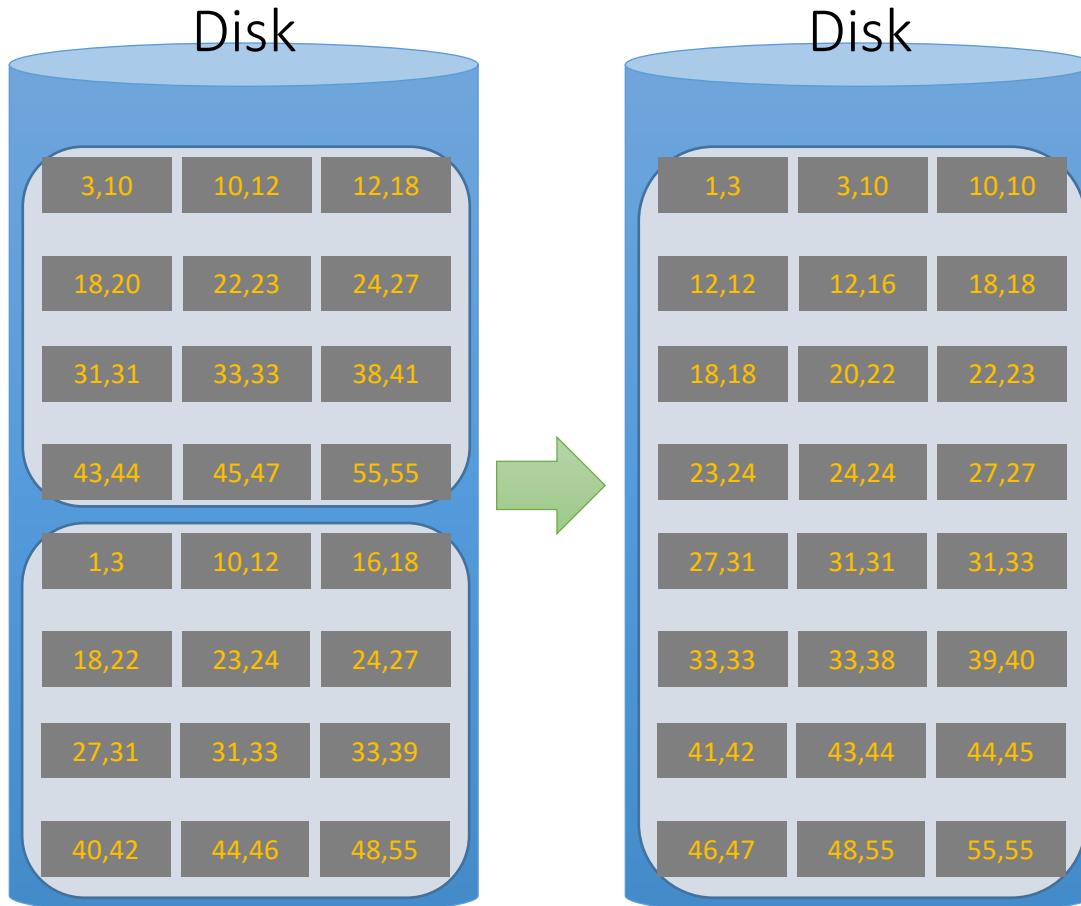
3. And repeat...

Call each of these steps a *pass*

# Running External Merge Sort on Larger Files (6/6)



- Assume we still only have 3 buffer pages (*Buffer not pictured*)

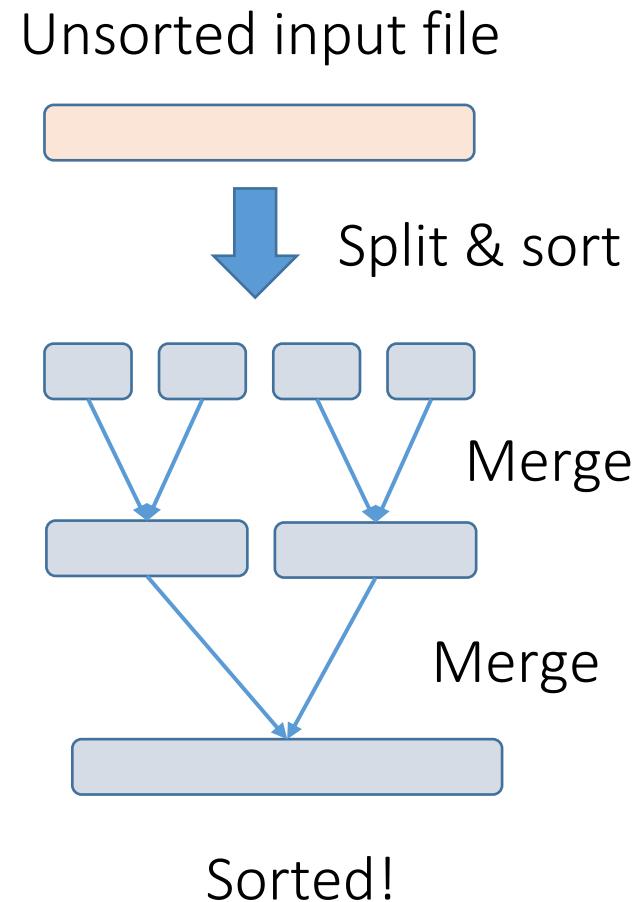


4. And repeat!

# Simplified 3-page Buffer Version



- Assume for simplicity that we split an  $N$ -page file into  $N$  single-page runs and sort these; then:
  - First pass: Merge  $N/2$  pairs of runs each of length 1 page
  - Second pass: Merge  $N/4$  pairs of runs each of length 2 pages
  - In general, for  $N$  pages, we do  $\lceil \log_2 N \rceil$  passes
    - +1 for the initial split & sort
  - Each pass involves reading in & writing out all the pages =  $2N$  IO



→  $2N * (\lceil \log_2 N \rceil + 1)$  total IO cost!

# Using B+1 buffer pages to reduce # of passes (1/2)



- Suppose we have  $B+1$  buffer pages now; we can:
  - 1. Increase length of initial runs.** Sort  $B+1$  at a time!
    - At the beginning, we can split the  $N$  pages into runs of length  $B+1$  and sort these in memory

IO Cost:

$$2N(\lceil \log_2 N \rceil + 1)$$

$$2N\left(\left\lceil \log_2 \frac{N}{B+1} \right\rceil + 1\right)$$

Starting with runs  
of length 1

Starting with runs of  
length  $B+1$

# Using $B+1$ buffer pages to reduce # of passes (2/2)



- Suppose we have  $B+1$  buffer pages now; we can:
  - 2. Perform a  $B$ -way merge.**
    - On each pass, we can merge groups of  $B$  runs at a time (vs. merging pairs of runs)!

$$2N(\lceil \log_2 N \rceil + 1)$$



$$2N\left(\left\lceil \log_2 \frac{N}{B+1} \right\rceil + 1\right)$$

Starting with runs  
of length 1

Starting with runs of  
length  $B+1$



$$2N\left(\left\lceil \log_B \frac{N}{B+1} \right\rceil + 1\right)$$

Performing  $B$ -way  
merges

# Q&A

