12.5.3 Problems

P12-1. To formulate the performance of a multiple-access network, we need a mathematical model. When the number of stations in a network is very large, the Poisson distribution, $p[x] = (e^{-\lambda} \times \lambda^x)/(x!)$, is used. In this formula, $p[x]$ is the probability of generating x number of frames in a period of time and λ is the average number of generated frames during the same period of time. Using the Poisson distribution:

a. Find the probability that a pure Aloha network generates x number of frames during the vulnerable time. Note that the vulnerable time for this network is two times the frame transmission time (T_{fr}).

b. Find the probability that a slotted Aloha network generates x number of frames during the vulnerable time. Note that the vulnerable time for this network is equal to the frame transmission time (T_{fr}).

P12-2. In the previous problem, we used the Poisson distribution to find the probability of generating x number of frames, in a certain period of time, in a pure or slotted Aloha network as $p[x] = (e^{-\lambda} \times \lambda^x)/(x!)$. In this problem, we want to find the probability that a frame in such a network reaches its destination without colliding with other frames. For this purpose, it is simpler to think that we have G stations, each sending an average of one frame during the frame transmission time (instead of having N frames, each sending an average of G/N frames during the same time). Then, the probability of success for a station is the probability that no other station sends a frame during the vulnerable time.

a. Find the probability that a station in a pure Aloha network can successfully send a frame during a vulnerable time.

b. Find the probability that a station in a slotted Aloha network can successfully send a frame during a vulnerable time.

P12-3. In the previous problem, we found that the probability of a station (in a G-station network) successfully sending a frame in a vulnerable time is $P = e^{-2G}$ for a pure Aloha and $P = e^{-G}$ for a slotted Aloha network. In this problem, we want to find the throughput of these networks, which is the probability that any station (out of G stations) can successfully send a frame during the vulnerable time.

a. Find the throughput of a pure Aloha network.

b. Find the throughput of a slotted Aloha network.

P12-4. In the previous problem, we showed that the throughput is $S = Ge^{-2G}$ for a pure Aloha network and $S = Ge^{-G}$ for a slotted Aloha network. In this problem, we want to find the value of G in each network that makes the throughput maximum and find the value of the maximum throughput. This can be done if we find the derivative of S with respect to G and set the derivative to zero.

a. Find the value of G that makes the throughput maximum, and find the value of the maximum throughput for a pure Aloha network.

b. Find the value of G that makes the throughput maximum, and find the value of the maximum throughput for a slotted Aloha network.

P12-5. A multiple access network with a large number of stations can be analyzed using the Poisson distribution. When there is a limited number of stations in a network, we need to use another approach for this analysis. In a network with N stations, we assume that each station has a frame to send during the frame transmission time (T_{fr}) with probability p. In such a network, a station is successful in sending its frame if the station has a frame to send during the vulnerable time and no other station has a frame to send during this period of time.

a. Find the probability that a station in a pure Aloha network can successfully send a frame during the vulnerable time.

b. Find the probability that a station in a slotted Aloha network can successfully send a frame during the vulnerable time.

P12-6. In the previous problem, we found the probability of success for a station to send a frame successfully during the vulnerable time. The throughput of a network with a limited number of stations is the probability that any station (out of N stations) can send a frame successfully. In other words, the throughput is the sum of N success probabilities.

a. Find the throughput of a pure Aloha network.

b. Find the throughput of a slotted Aloha network.

P12-7. In the previous problem, we found the throughputs of a pure and a slotted Aloha network as $S = Np\ (1-p)^{2(N-1)}$ and $S = Np\ (1-p)^{(N-1)}$ respectively. In this problem we want to find the maximum throughput with respect to p.

a. Find the value of p that maximizes the throughput of a pure Aloha network, and calculate the maximum throughput when N is a very large number.

b. Find the value of p that maximizes the throughput of a slotted Aloha network, and calculate the maximum throughput when N is a very large number.

P12-8. A slotted Aloha network is working with maximum throughput.

a. What is the probability that a slot is empty?

b. How many slots, n, on average, should pass before getting an empty slot?

P12-9. In a bus CSMA/CD network with a data rate of 10 Mbps, a collision occurs 20 μs after the first bit of the frame leaves the sending station. What should the length of the frame be so that the sender can detect the collision?

P12-10. Assume that there are only two stations, A and B, in a bus CSMA/CD network. The distance between the two stations is 2000 m and the propagation speed is 2×10^8 m/s. If station A starts transmitting at time t_1:

a. Does the protocol allow station B to start transmitting at time $t_1 + 8$ μs? If the answer is yes, what will happen?

b. Does the protocol allow station B to start transmitting at time $t_1 + 11$ μs? If the answer is yes, what will happen?

P12-11. Check to see if the following set of chips can belong to an orthogonal system.

$$[+1, +1] \quad \text{and} \quad [+1, -1]$$

P12-12. The random variable R (Figure 12.13) is designed to give stations different delays when a collision has occurred. To alleviate the collision, we expect that different stations generate different values of R. To show the point, find the probability that the value of R is the same for two stations after

a. the first collision. **b.** the second collision.

P12-13. There are only three active stations in a slotted Aloha network: A, B, and C. Each station generates a frame in a time slot with the corresponding probabilities $p_A = 0.2$, $p_B = 0.3$, and $p_C = 0.4$ respectively.

a. What is the throughput of each station?

b. What is the throughput of the network?

P12-14. One of the useful parameters in a LAN is the number of bits that can fit in one meter of the medium ($n_{b/m}$). Find the value of $n_{b/m}$ if the data rate is 100 Mbps and the medium propagation speed is 2×10^8 m/s.

P12-15. There are only two stations, A and B, in a bus 1-persistence CSMA/CD network with $T_p = 25.6$ μs and $T_{fr} = 51.2$ μs. Station A has a frame to send to station B. The frame is unsuccessful two times and succeeds on the third try. Draw a time line diagram for this problem. Assume that the R is 1 and 2 respectively and ignore the time for sending a jamming signal (see Figure 12.13).

P12-16. There are only three active stations in a slotted Aloha network: A, B, and C. Each station generates a frame in a time slot with the corresponding probabilities $p_A = 0.2$, $p_B = 0.3$, and $p_C = 0.4$ respectively.

a. What is the probability that any station can send a frame in the first slot?

b. What is the probability that station A can successfully send a frame for the first time in the second slot?

c. What is the probability that station C can successfully send a frame for the first time in the third slot?

P12-17. Check to see if the following set of chips can belong to an orthogonal system.

$$[+1, +1, +1, +1], \quad [+1, -1, -1, +1], \quad [-1, +1, +1, -1], \quad [+1, -1, -1, +1]$$

P12-18. We have a pure ALOHA network with a data rate of 10 Mbps. What is the maximum number of 1000-bit frames that can be successfully sent by this network?

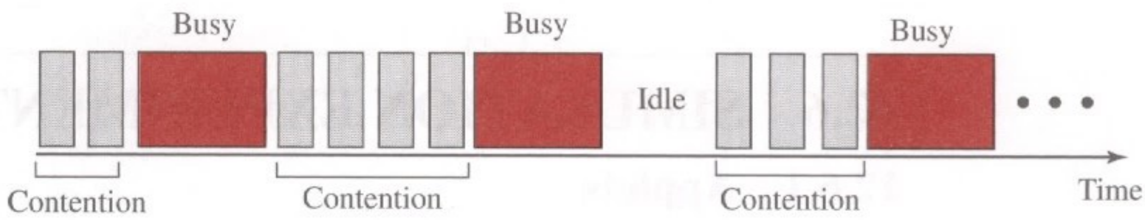
P12-19. Although the throughput calculation of a CSMA/CD is really involved, we can calculate the maximum throughput of a slotted CSMA/CD with the specification we described in the previous problem. We found that the average number of contention slots a station needs to wait is $k = e$ slots. With this assumption, the throughput of a slotted CSMA/CD is

$$S \, \nabla \, (T_{fr})/(\text{time the channel is busy for a frame})$$

The time the channel is busy for a frame is the time to wait for a free slot plus the time to transmit the frame plus the propagation delay to receive the good news about the lack of collision. Assume the duration of a contention slot is $2 \times (T_p)$ and $a = (T_p)/(T_{fr})$. Note that the parameter a is the number of frames that occupy the transmission media. Find the throughput of a slotted CSMA/CD in terms of the parameter a.

P12-20. Assume we have a slotted CSMA/CD network. Each station in this network uses a contention period, in which the station contends for access to the shared channel before being able to send a frame. We assume that the contention period is made of contention slots. At the beginning of each slot, the station senses the channel. If the channel is free, the station sends its frame; if the channel is busy, the station refrains from sending and waits until the beginning of the next slot. In other words, the station waits, on average, for k slots before sending its frame, as shown in Figure 12.30. Note that the channel is either in the contention state, the transmitting state, or the idle state (when no station has a frame to send). However, if N is a very large number, the idle state actually disappears.

Figure 12.30 *Problem P12-20*



a. What is the probability of a free slot (P_{free}) if the number of stations is N and each station has a frame to send with probability p?

b. What is the maximum of this probability when N is a very large number?

c. What is the probability that the jth slot is free?

d. What is the average number of slots, k, that a station should wait before getting a free slot?

e. What is the value of k when N (the number of stations) is very large?

P12-21. We have defined the parameter $\boldsymbol{a}$ as the number of frames that can fit the medium between two stations, or $\boldsymbol{a} = (T_p)/(T_{fr})$. Another way to define this parameter is $\boldsymbol{a} = L_b/F_b$, in which L_b is the bit length of the medium and F_b is the frame length of the medium. Show that the two definitions are equivalent.

P12-22. In a bus 1-persistence CSMA/CD with $T_p = 50$ μs and $T_{fr} = 120$ μs, there are two stations, A and B. Both stations start sending frames to each other at the same time. Since the frames collide, each station tries to retransmit. Station A comes out with $R = 0$ and station B with $R = 1$. Ignore any other delay including the delay for sending jamming signals. Do the frames collide again? Draw a time-line diagram to prove your claim. Does the generation of a random number help avoid collision in this case?

P12-23. Another useful parameter in a LAN is the bit length of the medium (L_b), which defines the number of bits that the medium can hold at any time. Find the bit length of a LAN if the data rate is 100 Mbps and the medium length in meters (L_m) for a communication between two stations is 200 m. Assume the propagation speed in the medium is 2×10^8 m/s.

P12-24. To understand why we need to have a minimum frame size $T_{fr} = 2 \times T_p$ in a CDMA/CD network, assume we have a bus network with only two stations, A and B, in which $T_{fr} = 40$ μs and $T_p = 25$ μs. Station A starts sending a frame at time $t = 0.0$ μs and station B starts sending a frame at $t = 23.0$ μs. Answer the following questions:

a. Do frames collide?

b. If the answer to part *a* is yes, does station A detect collision?

c. If the answer to part *a* is yes, does station B detect collision?

P12-25. Alice and Bob are experimenting with CSMA using a W_2 Walsh table (see Figure 12.29). Alice uses the code [+1, +1] and Bob uses the code [+1, −1]. Assume that they simultaneously send a hexadecimal digit to each other. Alice sends $(6)_{16}$ and Bob sends $(B)_{16}$. Show how they can detect what the other person has sent.