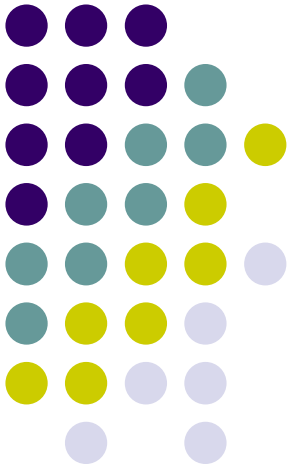
System Programming

04. Integers II (ch2.2-2.3)



2019. Fall

Instructor: Joonho Kwon

jhkwon@pusan.ac.kr

Data Science Lab @ PNU

datalab

data science laboratory

These slides are based on your text book on the slides prepared by Dan Grossman, Ruth Anderson.

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```
Car c = new Car();
c.setMiles(100);
c.setGals(17);
float mpg =
    c.getMPG();
```

Assembly language:

```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

Machine code:

```
01110100000110000
10001101000001000000010
1000100111000010
11000001111110100001111
```

OS:

Computer system:

Integers

- Binary representation of integers
  - Unsigned and signed
  - **Casting in C**
- Consequences of finite width representations
  - Overflow, sign extension
- Shifting and arithmetic operations

Two's Complement Arithmetic

- The same addition procedure works for both unsigned and two's complement integers
  - **Simplifies hardware:** only one algorithm for addition
  - **Algorithm:** simple addition, discard the highest carry bit
    - Called modular addition: result is sum *modulo* 2^w

- **4-bit Examples:**

4	0100	-4	1100	4	0100
+3	+0011	+3	+0011	-3	+1101
=7		=-1		=1	

Why Does Two's Complement Work? (1)

- For all representable positive integers x, we want:

$$\begin{array}{r} \textit{bit representation of } x \\ +\ \textit{bit representation of } -x \\ \hline 0 \end{array}$$ (ignoring the carry-out bit)

  - What are the 8-bit negative encodings for the following?

$$\begin{array}{r} 00000001 \\ +\ ???????? \\ \hline 00000000 \end{array} \qquad \begin{array}{r} 00000010 \\ +\ ???????? \\ \hline 00000000 \end{array} \qquad \begin{array}{r} 11000011 \\ +\ ???????? \\ \hline 00000000 \end{array}$$

Why Does Two's Complement Work? (2)

- For all representable positive integers x, we want:

$$\begin{array}{r} \textit{bit representation of } x \\ +\ \textit{bit representation of } -x \\ \hline 0 \end{array}$$ (ignoring the carry-out bit)

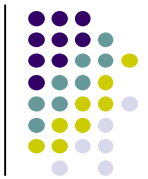
  - What are the 8-bit negative encodings for the following?

```
  00000001        00000010        11000011
+ 11111111      + 11111110      + 00111101
-----------     -----------     -----------
 100000000       100000000       100000000
```

These are the bitwise complement plus 1!

`-x == ~x + 1`

Mapping Signed ↔ Unsigned (1)

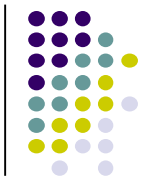


Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

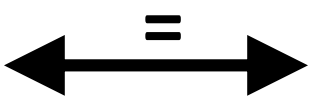
T2U

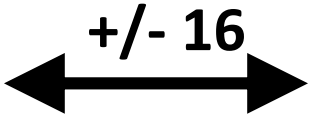
U2T

Mapping Signed ↔ Unsigned (2)



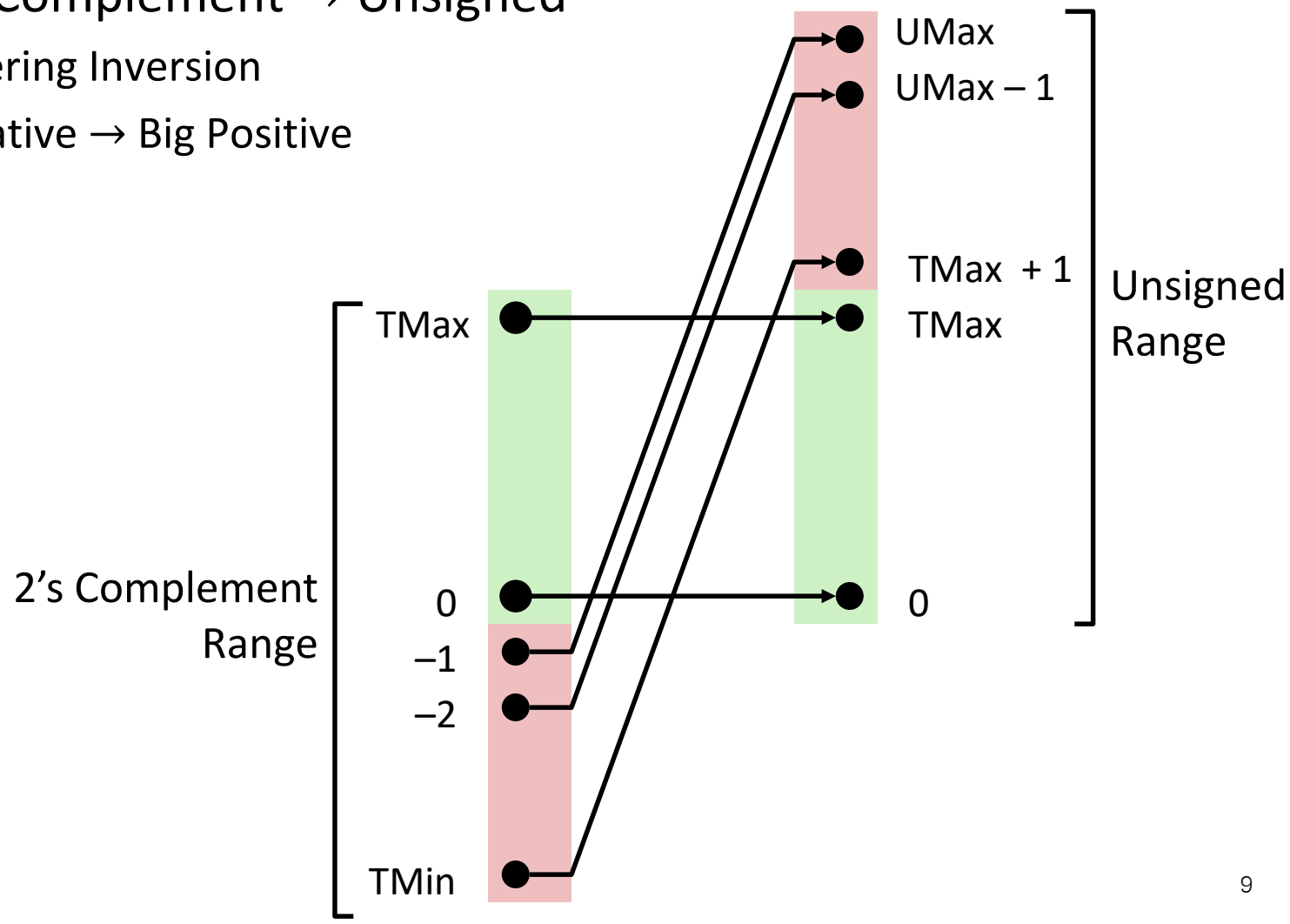
Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15





Signed/Unsigned Conversion Visualized

- Two's Complement → Unsigned
  - Ordering Inversion
  - Negative → Big Positive



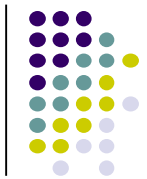
Values To Remember

- Unsigned Values
  - UMin = `0b00…0` = 0
  - UMax = `0b11…1` = $2^w - 1$
- Two's Complement Values
  - TMin = `0b10…0` = -2^{w-1}
  - TMax = `0b01…1` = $2^{w-1} - 1$
  - -1 = `0b11…1`

Example: Values for $w = 64$

	Decimal	Hex
UMax	18,446,744,073,709,551,615	FF FF FF FF FF FF FF FF
TMax	9,223,372,036,854,775,807	7F FF FF FF FF FF FF FF
TMin	-9,223,372,036,854,775,808	80 00 00 00 00 00 00 00
-1	-1	FF FF FF FF FF FF FF FF
0	0	00 00 00 00 00 00 00 00

In C: Signed vs. Unsigned



- Casting
  - Bits are unchanged, just interpreted differently!
    - **int** tx, ty;
    - **unsigned int** ux, uy;
  - *Explicit* casting
    - tx = (**int**) ux;
    - uy = (**unsigned int**) ty;
  - *Implicit* casting can occur during assignments or function calls
    - tx = ux;
    - uy = ty;

Casting Surprises

!!!

- Integer literals (constants)
  - By default, integer constants are considered *signed* integers
    - Hex constants already have an explicit binary representation
  - Use “U” (or “u”) suffix to explicitly force *unsigned*
    - **Examples:** 0U, 4294967259u

- Expression Evaluation
  - When you mixed unsigned and signed in a single expression, then **signed values are implicitly cast to unsigned**
  - Including comparison operators <, >, ==, <=, >=

Casting Surprises

REMINDER: If you mix unsigned and signed in a single expression, then signed values are implicitly cast to unsigned. (The bit pattern does not change, bits are just interpreted differently.).

!!!

- 32-bit examples:
  - TMin = -2,147,483,648, TMax = 2,147,483,647

Constant$_1$	Constant$_2$	Interpretation of bits	Relation
0 0000 0000 0000 0000 0000 0000 0000 0000	**0U** 0000 0000 0000 0000 0000 0000 0000 0000	Unsigned	==
-1 1111 1111 1111 1111 1111 1111 1111 1111	**0** 0000 0000 0000 0000 0000 0000 0000 0000	Signed	<
-1 1111 1111 1111 1111 1111 1111 1111 1111	**0U** 0000 0000 0000 0000 0000 0000 0000 0000	Unsigned	**>**
2147483647 0111 1111 1111 1111 1111 1111 1111 1111	**-2147483648** 1000 0000 0000 0000 0000 0000 0000 0000	Signed	>
2147483647U 0111 1111 1111 1111 1111 1111 1111 1111	**-2147483648** 1000 0000 0000 0000 0000 0000 0000 0000	Unsigned	**<**
-1 1111 1111 1111 1111 1111 1111 1111 1111	**-2** 1111 1111 1111 1111 1111 1111 1111 1110	Signed	>
(unsigned) -1 1111 1111 1111 1111 1111 1111 1111 1111	**-2** 1111 1111 1111 1111 1111 1111 1111 1110	Unsigned	>
2147483647 0111 1111 1111 1111 1111 1111 1111 1111	**2147483648U** 1000 0000 0000 0000 0000 0000 0000 0000	Unsigned	<
2147483647 0111 1111 1111 1111 1111 1111 1111 1111	**(int) 2147483648U** 1000 0000 0000 0000 0000 0000 0000 0000	Signed	>

Integers

- Binary representation of integers
  - Unsigned and signed
  - Casting in C
- **Consequences of finite width representations**
  - **Overflow, sign extension**
- Shifting and arithmetic operations

Arithmetic Overflow

Bits	Unsigned	Signed
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

- **Arithmetic Overflow** - when a calculation produces a result that can't be represented in the current encoding scheme
  - Integer range limited by fixed width
  - Can occur in both the positive and negative directions
- C and Java ignore overflow exceptions
  - You end up with a bad value in your program and no warning/indication… oops!

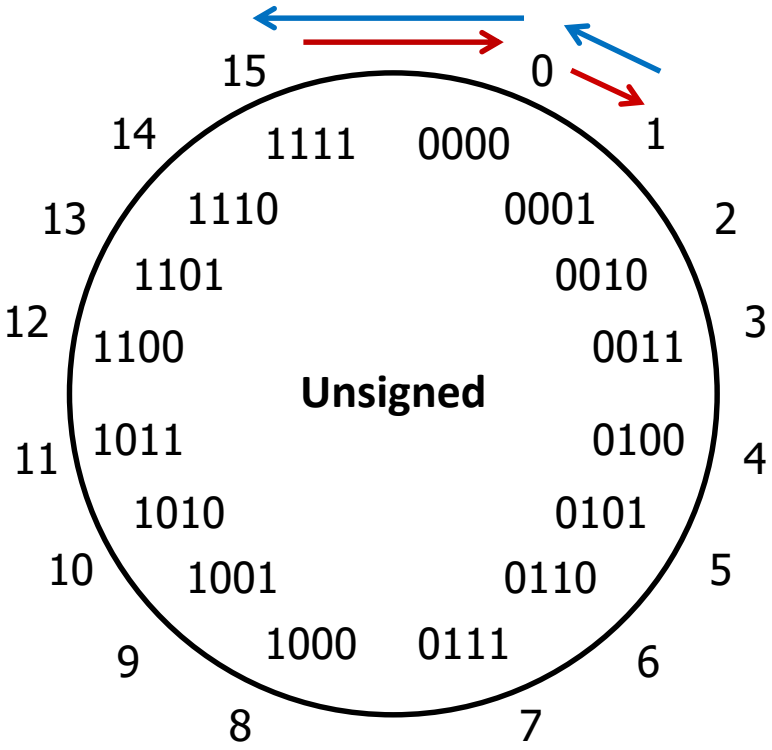
Overflow: Unsigned

- **Addition:** drop carry bit (-2^N)

```
  15        1111
+  2      + 0010
  17       10001
   1
```

- **Subtraction:** borrow ($+2^N$)

```
   1        10001
-  2      - 0010
  -1        1111
  15
```



$\pm 2^N$ because of modular arithmetic

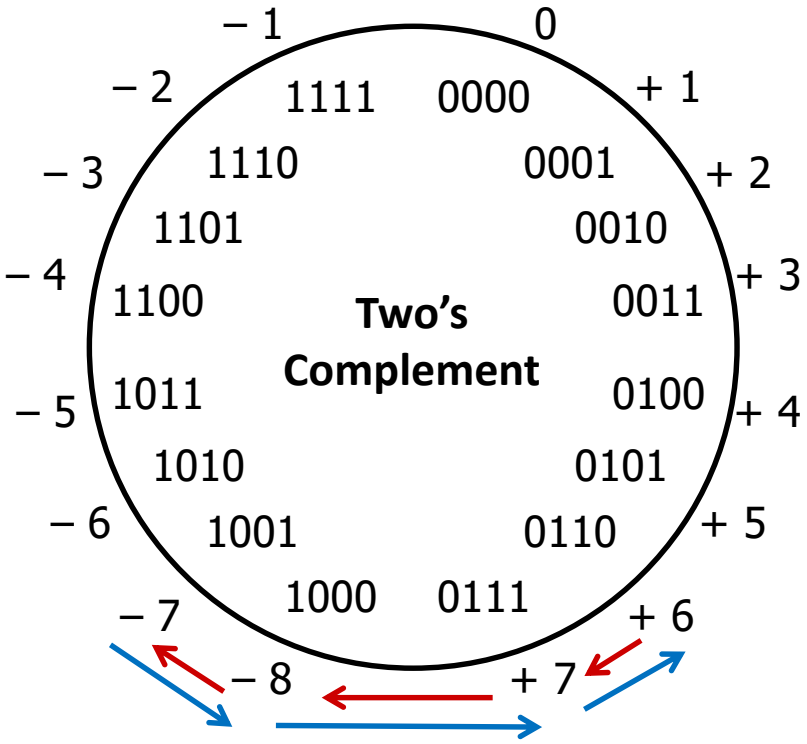
Overflow: Two's Complement

- **Addition:** (+) + (+) = (−) result?

```
   6        0110
 + 3      + 0011
 ---      ------
   9        1001
  -7
```

- **Subtraction:** (−) + (−) = (+)?

```
  -7        1001
 - 3      - 0011
 ---      ------
 -10        0110
   6
```



For signed: overflow if operands have same sign and result's sign is different

Sign Extension (1)

- What happens if you convert a *signed* integral data type to a larger one?
  - *e.g.* `char` → `short` → `int` → `long`

- **4-bit → 8-bit Example:**
  - Positive Case
    - Add 0's?

4-bit: `0010` = +2

8-bit: `00000010` = +2

Question

- Which of the following 8-bit numbers has the same *signed* value as the (signed) 4-bit number **0b1100**?
  - Underlined digit = MSB

A. 0b 0000 1100

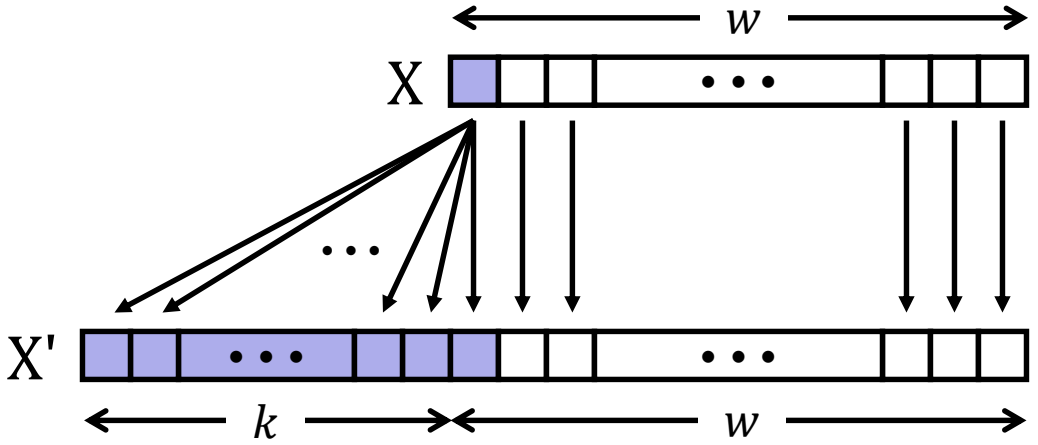
B. 0b 1000 1100

C. 0b 1111 1100

D. 0b 1100 1100

Sign Extension (2)

- **Task:** Given a w-bit signed integer X, convert it to w+k-bit signed integer X′ *with the same value*
- **Rule:** Add k copies of sign bit
  - Let x_i be the i-th digit of X in binary
  - $\mathrm{X}' = \underbrace{x_{w-1}, \ldots, x_{w-1}}_{k \text{ copies of MSB}}, \underbrace{x_{w-1}, x_{w-2}, \ldots, x_1, x_0}_{\text{original X}}$



Sign Extension (3)

0 0 1 0 4-bit 2

0 0 0 0 0 0 1 0 8-bit 2

1 1 0 0 4-bit -4

? ? ? ? 1 1 0 0 8-bit -4

Sign Extension (4)

0 0 1 0 4-bit 2

0 0 0 0 0 0 1 0 8-bit 2

1 1 0 0 4-bit -4

0 0 0 0 1 1 0 0 8-bit 12

Just adding zeroes to the front does not work

Sign Extension (5)

0 0 1 0 4-bit 2

0 0 0 0 0 0 1 0 8-bit 2

1 1 0 0 4-bit -4

1 0 0 0 1 1 0 0 8-bit -116

Just making the first bit=1 also does not work

Sign Extension (6)

0 0 1 0 4-bit 2

0 0 0 0 0 0 1 0 8-bit 2

1 1 0 0 4-bit -4

1 1 1 1 1 1 0 0 8-bit -4

Need to extend the sign bit to all “new” locations

Sign Extension Example

- Convert from smaller to larger integral data types
- C automatically performs sign extension
  - Java too

```
short int x =   12345;
int       ix = (int) x;
short int y =  -12345;
int       iy = (int) y;
```

Var	Decimal	Hex	Binary
x	12345	30 39	00110000 00111001
ix	12345	00 00 30 39	00000000 00000000 00110000 00111001
y	-12345	CF C7	11001111 11000111
iy	-12345	FF FF CF C7	11111111 11111111 11001111 11000111

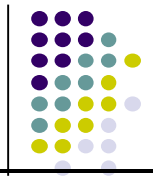
Integers

- Binary representation of integers
  - Unsigned and signed
  - Casting in C
- Consequences of finite width representations
  - Overflow, sign extension
- **Shifting and arithmetic operations**

Shift Operations

- Left shift (`x<<n`) bit vector `x` by `n` positions
  - Throw away (drop) extra bits on left
  - Fill with `0`s on right
- Right shift (`x>>n`) bit-vector `x` by `n` positions
  - Throw away (drop) extra bits on right
  - Logical shift (for unsigned values)
    - Fill with `0`s on left
  - Arithmetic shift (for signed values)
    - Replicate most significant bit on left
    - Maintains sign of `x`

Shift Operations



- Left shift (`x<<n`)
  - Fill with `0`s on right
- Right shift (`x>>n`)
  - Logical shift (for unsigned values)
    - Fill with 0s on left
  - Arithmetic shift (for signed values)
    - Replicate most significant bit on left

	x	0010 0010
	x<<3	0001 0000
logical:	x>>2	0000 1000
arithmetic:	x>>2	0000 1000

	x	1010 0010
	x<<3	0001 0000
logical:	x>>2	0010 1000
arithmetic:	x>>2	1110 1000

- Notes:
  - Shifts by `n<0` or `n≥w` (bit width of x) are *undefined*
  - **In C:** behavior of `>>` is determined by compiler
    - In gcc / C lang, depends on data type of `x` (signed/unsigned)
  - **In Java:** logical shift is `>>>` and arithmetic shift is `>>`

Shifting Arithmetic?

- What are the following computing?
  - `x>>n`
    - `0b 0100 >> 1 = 0b 0010`
    - `0b 0100 >> 2 = 0b 0001`
    - Divide by 2^n
  - `x<<n`
    - `0b 0001 << 1 = 0b 0010`
    - `0b 0001 << 2 = 0b 0100`
    - Multiply by 2^n
- Shifting is faster than general multiply and divide operations

Left Shifting Arithmetic 8-bit Example

- No difference in left shift operation for unsigned and signed numbers (just manipulates bits)
  - Difference comes during interpretation: $x*2^n$?

		Signed	Unsigned
`x = 25;`	00011001 =	25	25
`L1=x<<2;`	00011001**00** =	100	100
`L2=x<<3;`	000110010**00** =	-56	200
`L3=x<<4;`	0001100101**000** =	-112	144

signed overflow

unsigned overflow

Right Shifting 8-bit Examples

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Logical Shift: $x/2^n$?

```
xu = 240u;          11110000          = 240

R1u=xu>>3;          000 11110 000     =  30

R2u=xu>>5;          00000 111 10000   =   7
```

rounding (down)

Right Shifting 8-bit Examples

- **Reminder:** C operator >> does *logical* shift on unsigned values and *arithmetic* shift on signed values
  - Arithmetic Shift: $x/2^n$?

```
xs = -16;          11110000          = -16

R1s=xu>>3;         111111110000      =  -2

R2s=xu>>5;         11111111110000    =  -1
```

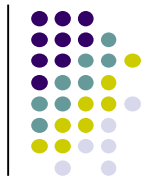
rounding (down)

Question

For the following expressions, find a value of **`signed char`** `x`, if there exists one, that makes the expression `TRUE`. Compare with your neighbor(s)!

- Assume we are using 8-bit arithmetic:
  - `x == (unsigned char) x`
  - `x >= 128U`
  - `x != (x>>2)<<2`
  - `x == -x`
    - Hint: there are two solutions
  - `(x < 128U) && (x > 0x3F)`

Question: A



- Assume we are using 8-bit arithmetic:
  - `x == (unsigned char) x` — works for all x (Both sides treated as unsigned)
  - `x >= 128U` (unsigned; 0b1000 0000) — any x < 0 (need 1ST bit set to one)
  - `x != (x>>2)<<2` — will fill lowest two bits with zero — any x where lowest two bits are not 0b00
  - `x == -x` — "flip bits and add one" — ① x = 0b0.....0 = 0 ② x = 0b10....0 = -128
    - Hint: there are two solutions
  - `(x < 128U) && (x > 0x3F)` (unsigned) — any x where upper two bits are exactly 0b01

x cannot have a one in first bit in order to be < 128U (0b1000 0000)

x needs a one in the second bit position in order to be > 0x3F (0b0011 1111)

Using Shifts and Masks (1)

- Extract the 2nd most significant *byte* of an `int`:
  - First shift, then mask: `(x>>16) & 0xFF`

x	00000001	00000010	00000011	00000100
x>>16	00000000	00000000	00000001	00000010
0xFF	00000000	00000000	00000000	11111111
(x>>16) & 0xFF	00000000	00000000	00000000	00000010

  - Or first mask, then shift: `(x & 0xFF0000)>>16`

x	00000001	00000010	00000011	00000100
0xFF0000	00000000	11111111	00000000	00000000
x & 0xFF0000	00000000	00000010	00000000	00000000
(x&0xFF0000)>>16	00000000	00000000	00000000	00000010

Using Shifts and Masks (2)

- Extract the *sign bit* of a signed `int`:
  - First shift, then mask: `(x>>31) & 0x1`
    - Assuming arithmetic shift here, but this works in either case
    - Need mask to clear `1`s possibly shifted in

x	00000001 00000010 00000011 00000100
x>>31	00000000 00000000 00000000 00000000
0x1	00000000 00000000 00000000 00000001
(x>>31) & 0x1	00000000 00000000 00000000 00000000

x	10000001 00000010 00000011 00000100
x>>31	11111111 11111111 11111111 11111111
0x1	00000000 00000000 00000000 00000001
(x>>31) & 0x1	00000000 00000000 00000000 00000001

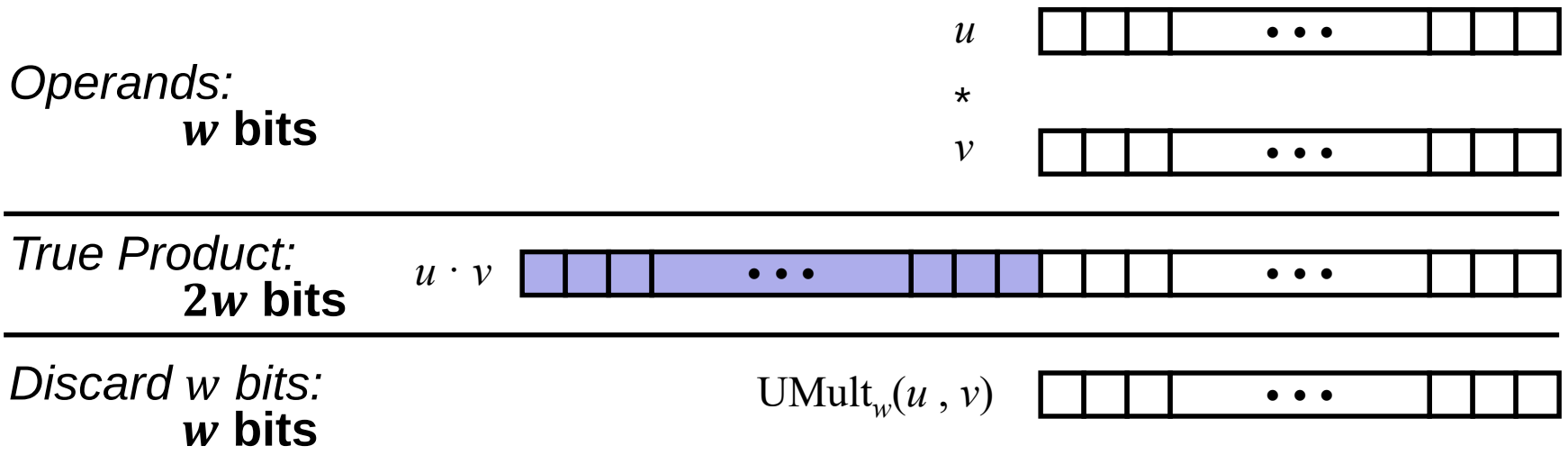
Using Shifts and Masks (3)

- **Conditionals as Boolean expressions (assuming x is 0 or 1)**
  - For **`int`** `x`, what does `(x<<31)>>31` do?
    - !123 = 0x 00, !!123 = 0x01

x=!!123	00000000 00000000 00000000 00000001
x<<31	10000000 00000000 00000000 00000000
(x<<31)>>31	11111111 11111111 11111111 11111111
!x	00000000 00000000 00000000 00000000
!x<<31	00000000 00000000 00000000 00000000
(!x<<31)>>31	00000000 00000000 00000000 00000000

  - Can use in place of conditional:
    - In C: `if(x) {a=y;} else {a=z;}` equivalent to `a=x?y:z;`
    - `a=(((x<<31)>>31)&y) | (((!x<<31)>>31)&z);`

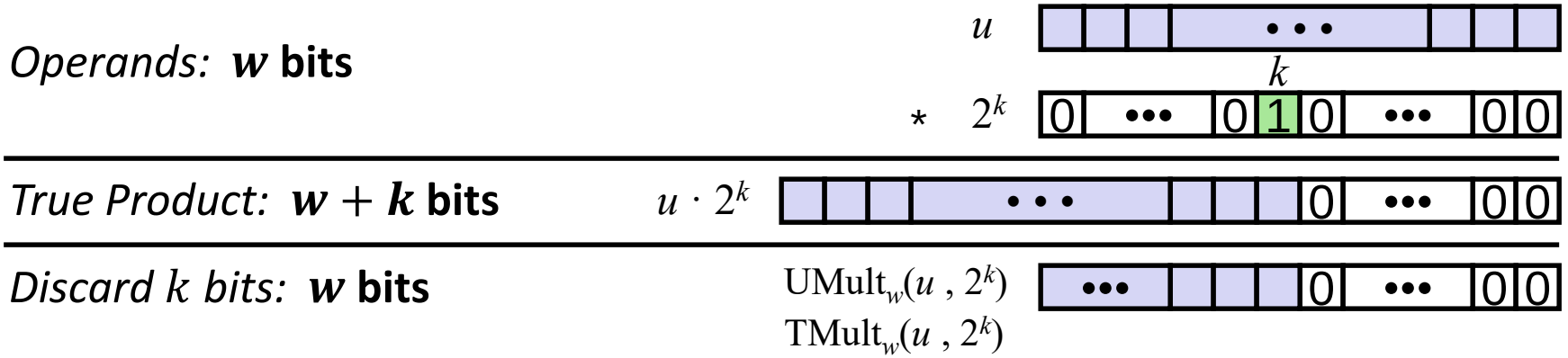
Unsigned Multiplication in C



- Standard Multiplication Function
  - Ignores high order w bits
- Implements Modular Arithmetic
  - $\text{UMult}_w(u , v) = u \cdot v \mod 2^w$

Multiplication with shift and add

- Operation `u<<k` gives `u*`$\mathbf{2^k}$
  - Both signed and unsigned



- Examples:
  - `u<<3 == u * 8`
  - `u<<5 - u<<3 == u * 24`
  - Most machines shift and add faster than multiply
    - ***Compiler generates this code automatically***

Why Should I Use Unsigned?

- *Don't* use without understanding implications
  - Easy to make mistakes

```
unsigned i;
for (i = cnt-2; i >= 0; i--)
  a[i] += a[i+1];
```

  - Can be very subtle

```
#define DELTA sizeof(int)
int i;
for (i = CNT; i-DELTA >= 0; i-= DELTA)
  . . .
```

Counting Down with Unsigned

- Proper way to use unsigned as loop index

```
unsigned i;
for (i = cnt-2; i < cnt; i--)
  a[i] += a[i+1];
```

- See Robert Seacord, *Secure Coding in C and C++*
  - C Standard guarantees that unsigned addition will behave like modular arithmetic
    - 0 – 1 → *UMax*
- Even better

```
size_t i;
for (i = cnt-2; i < cnt; i--)
  a[i] += a[i+1];
```

  - Data type **size_t** defined as unsigned value with length = word size
  - Code will work even if **cnt** = *UMax*
  - What if **cnt** is signed and < 0?

Why Should I Use Unsigned? (cont.)

- *Do* Use When Performing Modular Arithmetic
  - Multiprecision arithmetic
- *Do* Use When Using Bits to Represent Sets
  - Logical right shift, no sign extension

Code Security Example

```
/* Kernel memory region holding user-accessible data */
#define KSIZE 1024
char kbuf[KSIZE];

/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel(void *user_dest, int maxlen) {
    /* Byte count len is minimum of buffer size and maxlen */
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
}
```

- Similar to code found in FreeBSD's implementation of getpeername
- There are legions of smart people trying to find vulnerabilities in programs

Typical Usage

```
/* Kernel memory region holding user-accessible data */
#define KSIZE 1024
char kbuf[KSIZE];

/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel(void *user_dest, int maxlen) {
    /* Byte count len is minimum of buffer size and maxlen */
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
}
```

```
#define MSIZE 528

void getstuff() {
    char mybuf[MSIZE];
    copy_from_kernel(mybuf, MSIZE);
    printf("%s\n", mybuf);
}
```

Malicious Usage

```
/* Declaration of library function memcpy */
void *memcpy(void *dest, void *src, size_t n);
```

```
/* Kernel memory region holding user-accessible data */
#define KSIZE 1024
char kbuf[KSIZE];

/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel(void *user_dest, int maxlen) {
    /* Byte count len is minimum of buffer size and maxlen */
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
}
```

```
#define MSIZE 528

void getstuff() {
    char mybuf[MSIZE];
    copy_from_kernel(mybuf, -MSIZE);
    . . .
}
```

Summary

- Sign and unsigned variables in C
  - Bit pattern remains the same, just *interpreted* differently
  - Strange things can happen with our arithmetic when we convert/cast between sign and unsigned numbers
    - Type of variables affects behavior of operators (shifting, comparison)
- We can only represent so many numbers in *w* bits
  - When we exceed the limits, *arithmetic overflow* occurs
  - *Sign extension* tries to preserve value when expanding
- Shifting is a useful bitwise operator
  - Right shifting can be arithmetic (sign) or logical (0)
  - Can be used in multiplication with constant or bit masking

Q&A