System Programming

04. Integers (ch2.2-2.3)

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These slides are based on your text book on the slides prepared by Dan Grossman, Ruth Anderson.

Roadmap

Memory & data
Integers & floats
x86 assembly
Procedures & stacks
Executables
Arrays & structs
Memory & caches
Processes
Virtual memory
Memory allocation
Java vs. C

C:

```c
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```java
Car c = new Car();
c.setMiles(100);
c.setGals(17);
float mpg =
    c.getMPG();
```

Assembly language:

```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

Machine code:

```
0111010000011000
100011010000010000000010
1000100111000010
110000011111101000011111
```

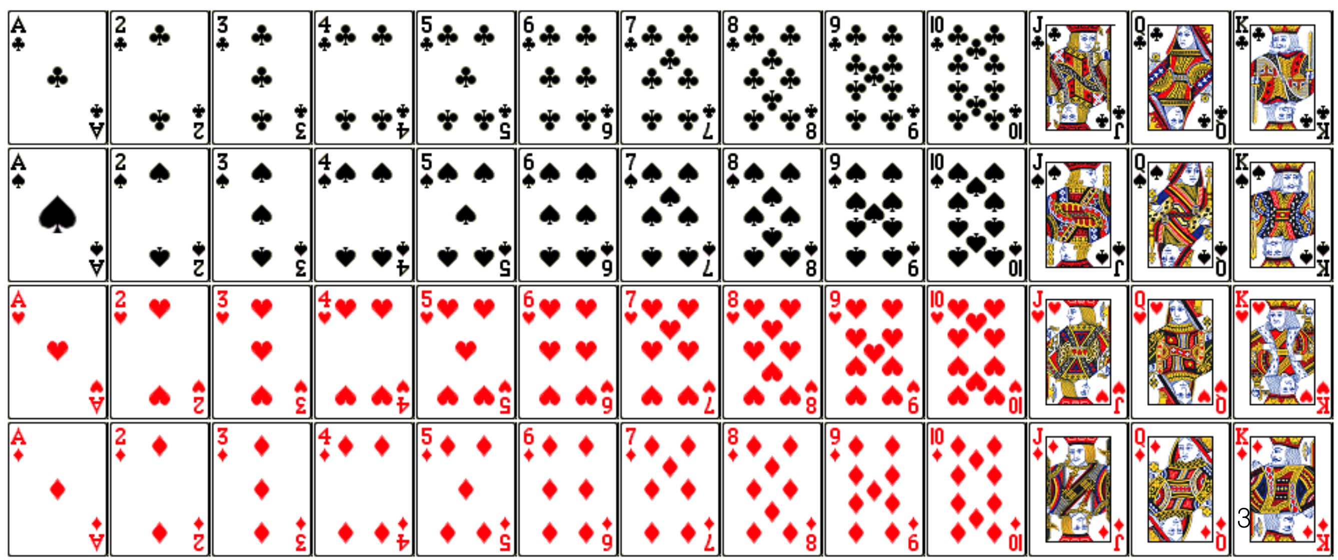
OS:

Windows 8
Mac

Computer system:

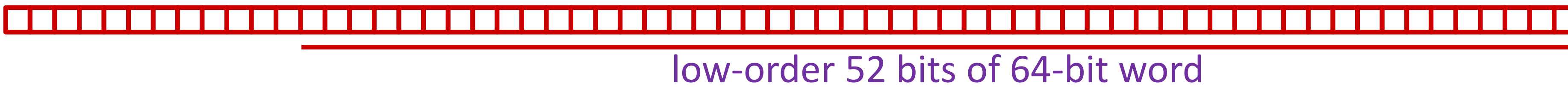
But before we get to integers….

- Encode a standard deck of playing cards
- 52 cards in 4 suits, 13 cards per suit
  - How do we encode suits, face cards?
- What operations do we want to make easy to implement?
  - Which is the higher value card?
  - Are they the same suit?



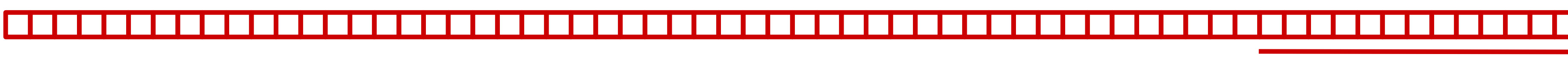
Two possible representations

1) 1 bit per card (52): bit corresponding to card set to 1



- "One-hot" encoding (similar to set notation)
- Drawbacks:
  - Hard to compare values and suits
  - Large number of bits required

2) 1 bit per suit (4), 1 bit per number (13): 2 bits set

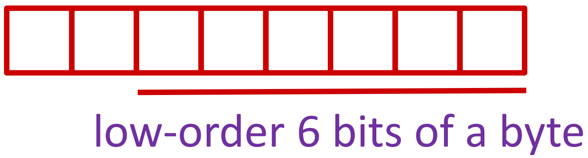


- Pair of one-hot encoded values
- Easier to compare suits and values, but still lots of bits used

- Can we do better?

Two better representations (1)

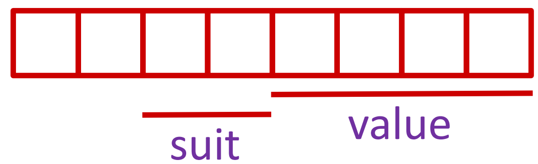
3) Binary encoding of all 52 cards – only 6 bits needed



- $2^6 = 64 \geq 52$
- Fits in one byte (smaller than one-hot encodings)
- How can we make value and suit comparisons easier?

Two better representations (2)

4) Separate binary encodings of suit (2 bits) and value (4 bits)



- Also fits in one byte, and easy to do comparisons

K	Q	J	. . .	3	2	A
1101	1100	1011	. . .	0011	0010	0001

♣	00
♦	01
♥	10
♠	11

Compare Card **Suits**

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turn all *but* the bits of interest in *v* to 0.

```
char hand[5];         // represents a 5-card hand
char card1, card2;    // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( sameSuitP(card1, card2) ) { ... }
```

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

returns int

equivalent

SUIT_MASK = 0x30 =

0	0	1	1	0	0	0	0

suit

value

Compare Card **Suits**

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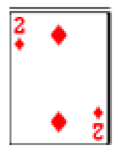
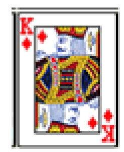




0 0 0 1 0 0 1 0		0 0 0 1 1 1 0 1
&		
0 0 1 1 0 0 0 0	SUIT_MASK	0 0 1 1 0 0 0 0
=		
0 0 0 1 0 0 0 0		0 0 0 1 0 0 0 0

^

0 0 0 0 0 0 0 0

!

0 0 0 0 0 0 0 1

`!(x^y)` equivalent to `x==y`

Compare Card Values

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greaterValue(card1, card2) ) { ... }
```

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

VALUE_MASK = 0x0F =

0	0	0	0	1	1	1	1

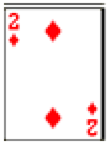
suit value

Compare Card Values

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```



0	0	1	0	0	0	1	0

&

0	0	0	0	1	1	1	1

=

0	0	0	0	0	0	1	0

VALUE_MASK

0	0	1	0	1	1	0	1

&

0	0	0	0	1	1	1	1

=

0	0	0	0	1	1	0	1

$2_{10} > 13_{10}$

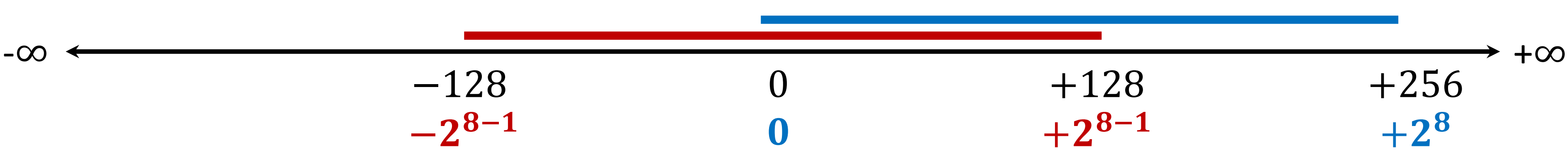
0 (false)

Integers

- **Binary representation of integers**
  - **Unsigned and signed**
  - Casting in C
- Consequences of finite width representation
  - Overflow, sign extension
- Shifting and arithmetic operations

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with w bits
  - Only 2^w distinct bit patterns
  - Unsigned values: $0 \ldots 2^w - 1$
  - Signed values: $-2^{w-1} \ldots 2^{w-1} - 1$
- **Example:** 8-bit integers (*e.g.* `char`)



Unsigned Integers

- Unsigned values follow the standard base 2 system
  - $b_7b_6b_5b_4b_3b_2b_1b_0 = b_7 2^7 + b_6 2^6 + \cdots + b_1 2^1 + b_0 2^0$
- Add and subtract using the normal "carry" and "borrow" rules, just in binary

$$\begin{array}{r} 63 \\ +\underline{\ \ 8} \\ 71 \end{array} \qquad \begin{array}{r} 00111111 \\ +\underline{00001000} \\ 01000111 \end{array}$$

- Useful formula: $2^{N-1} + 2^{N-2} + \ldots + 2 + 1 = 2^N - 1$
  - *i.e.* N ones in a row = $2^N - 1$
- How would you make *signed* integers?

Sign and Magnitude (1)

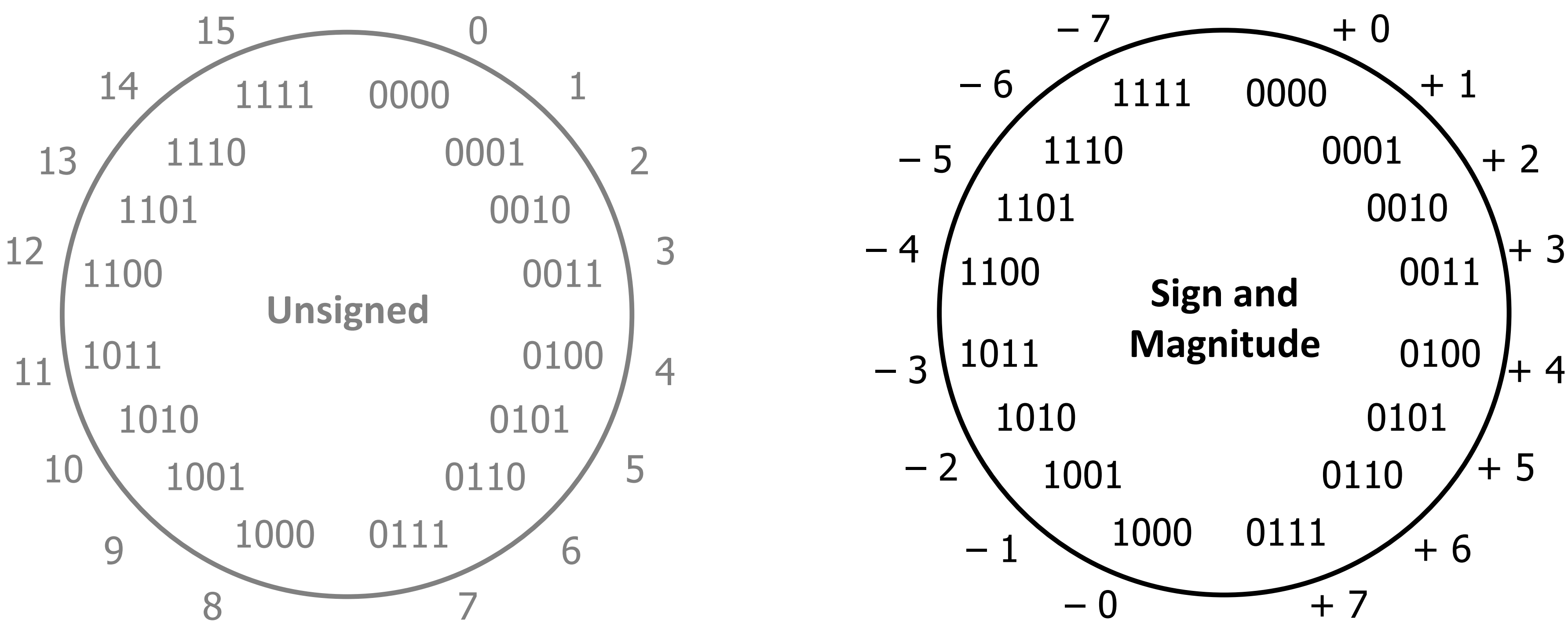
Most Significant Bit

- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive numbers; `sign=1`: negative numbers
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned
  - All zeros encoding is still = 0
- Examples (8 bits):
  - 0x00 = 00000000_2 is non-negative, because the sign bit is 0
  - 0x7F = 01111111_2 is non-negative ($+127_{10}$)
  - 0x85 = 10000101_2 is negative (-5_{10})
  - 0x80 = 10000000_2 is negative... zero???

Sign and Magnitude (2)

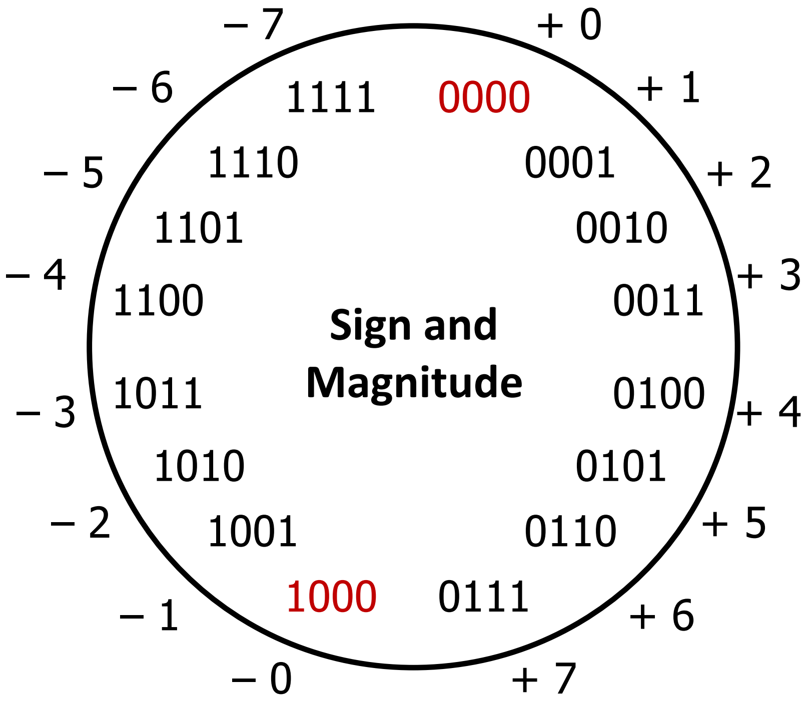
Most Significant Bit

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?



Sign and Magnitude (3)

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)



Sign and Magnitude (4)

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

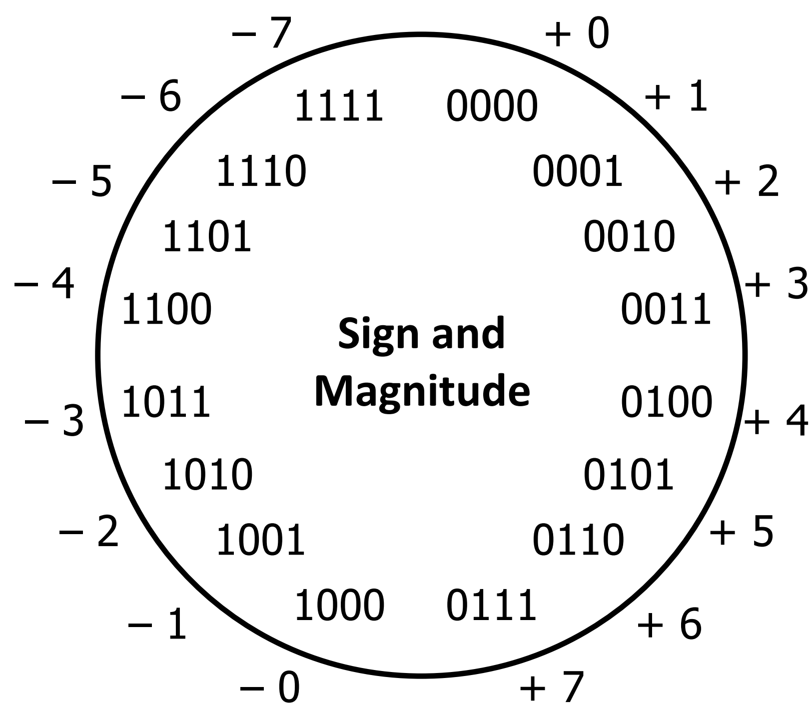
4	0100
− 3	− 0011
1	0001

✓

4	0100
+ −3	+ 1011
−7	1111

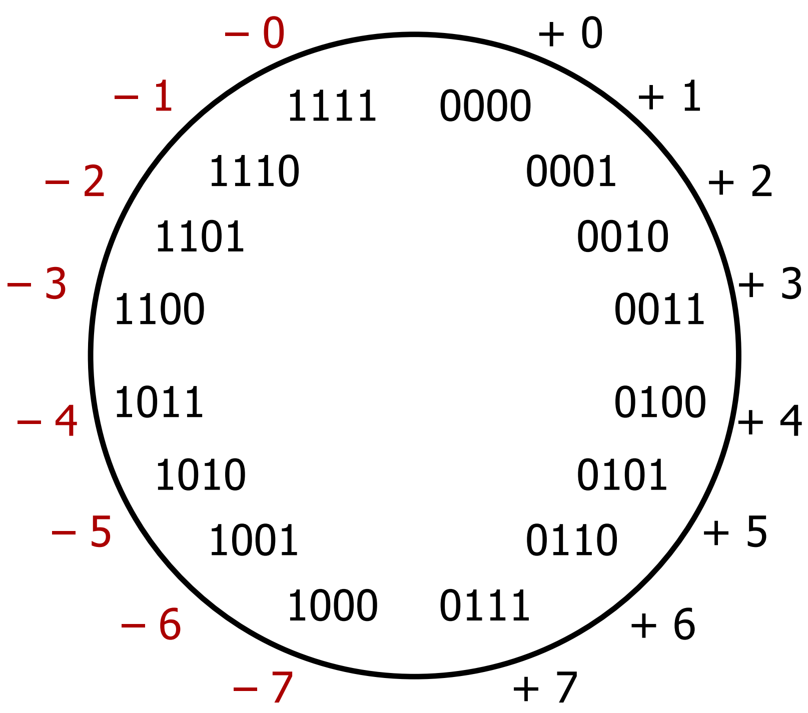
✗

    - Negatives "increment" in wrong direction



Two's Complement (1)

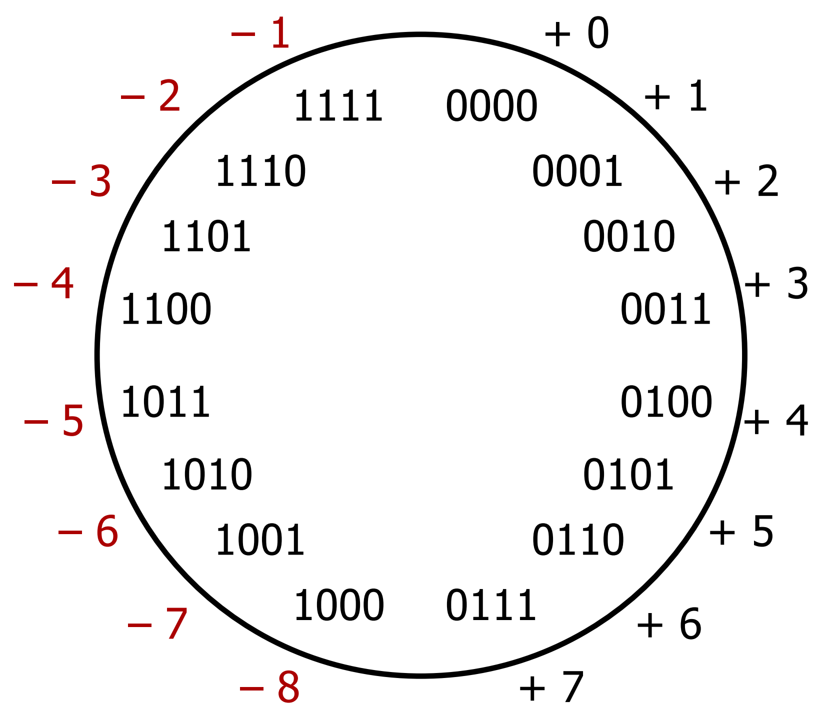
- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works



Two's Complement (2)

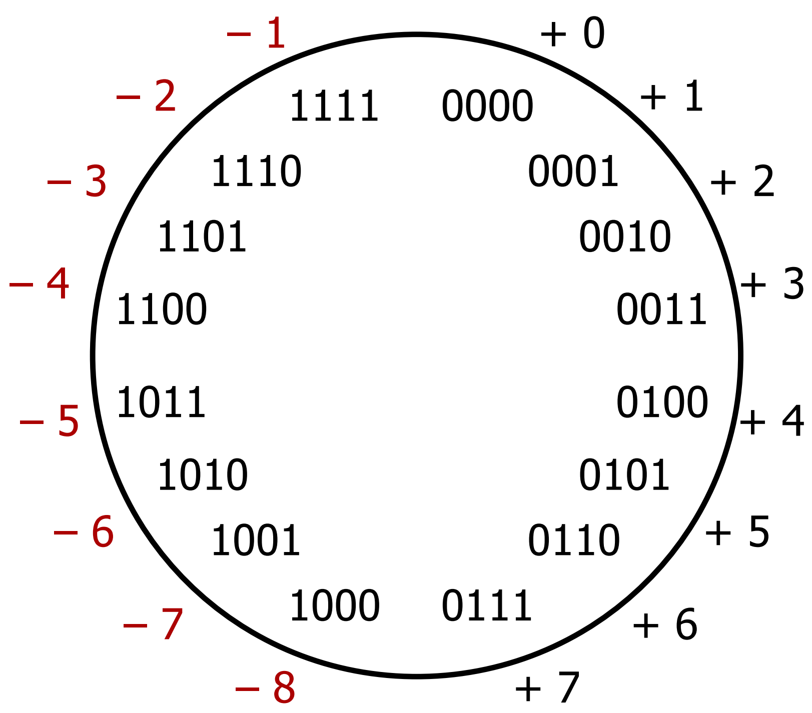
- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works
  2) "Shift" negative numbers to eliminate –0

- MSB *still* indicates sign!
  - This is why we represent one more negative than positive number (-2^{N-1} to $2^{N-1}-1$)



Two's Complement Negatives (1)

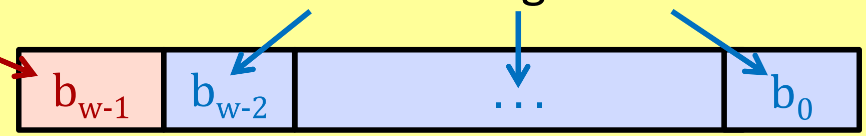
- How should we represent -1 in binary?



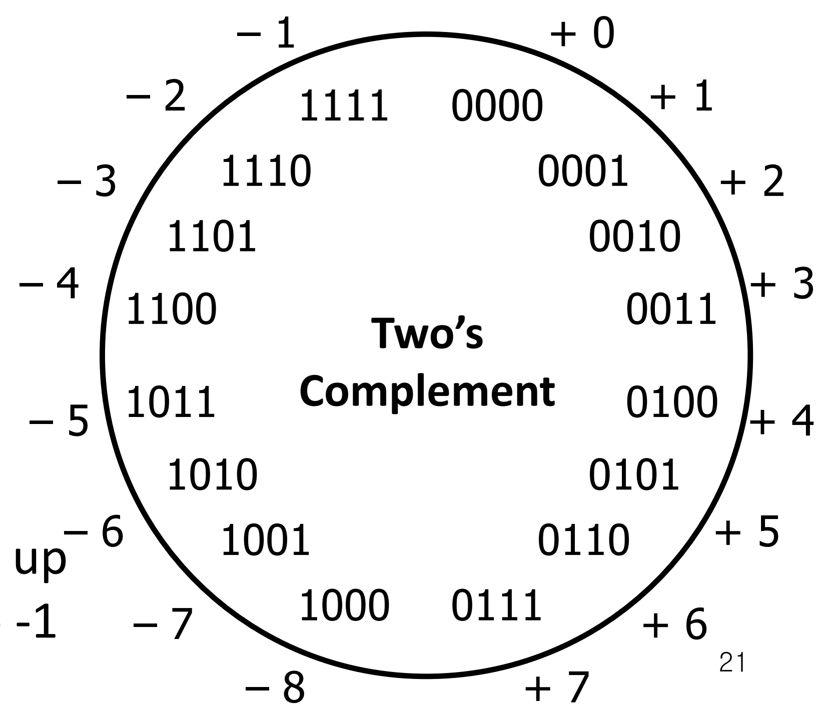
Two's Complement Negatives (2)

- Accomplished with one neat mathematical trick!

b_{w-1} has weight -2^{w-1}, other bits have usual weights $+2^i$



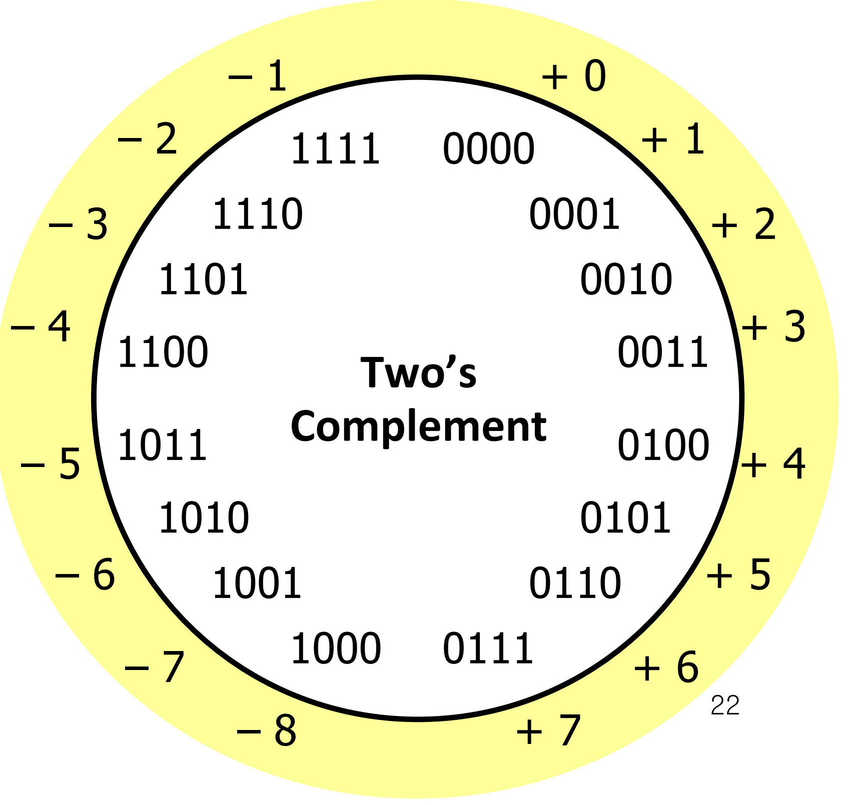
- 4-bit Examples:
  - 1010_2 unsigned:
    $1*2^3+0*2^2+1*2^1+0*2^0$ = **10**
  - 1010_2 two's complement:
    $-1*2^3+0*2^2+1*2^1+0*2^0$ = **–6**
- -1 represented as:
  $1111_2 = -2^3+(2^3 – 1)$
  - MSB makes it super negative, add up all the other bits to get back up to -1



Why Two's Complement is So Great

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero
- All zeros encoding = 0

- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!
    `( ~x + 1 == -x )`



Question

- Take the 4-bit number encoding `x = 0b1011`
- Which of the following numbers is NOT a valid interpretation of `x` using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement

A. -4

B. -5

C. 11

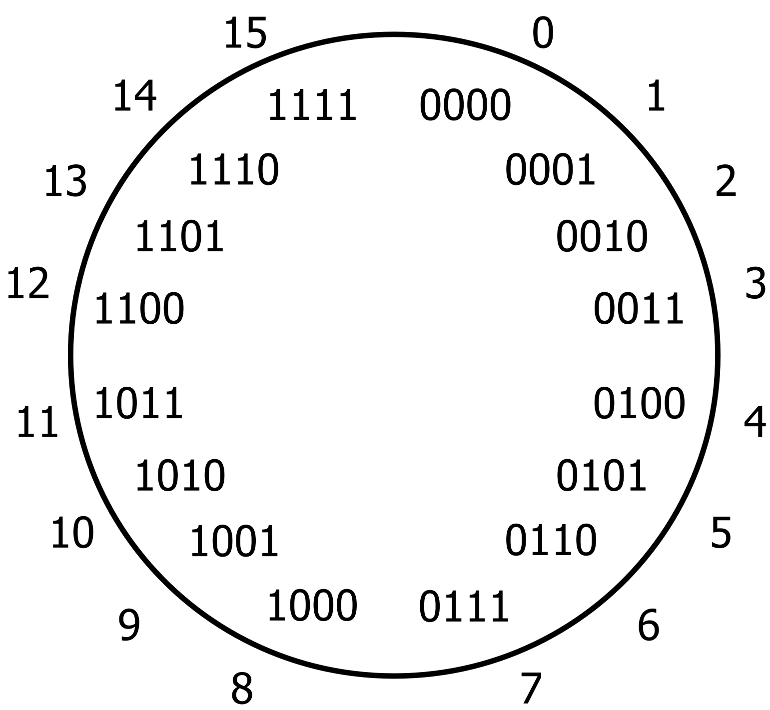
D. -3

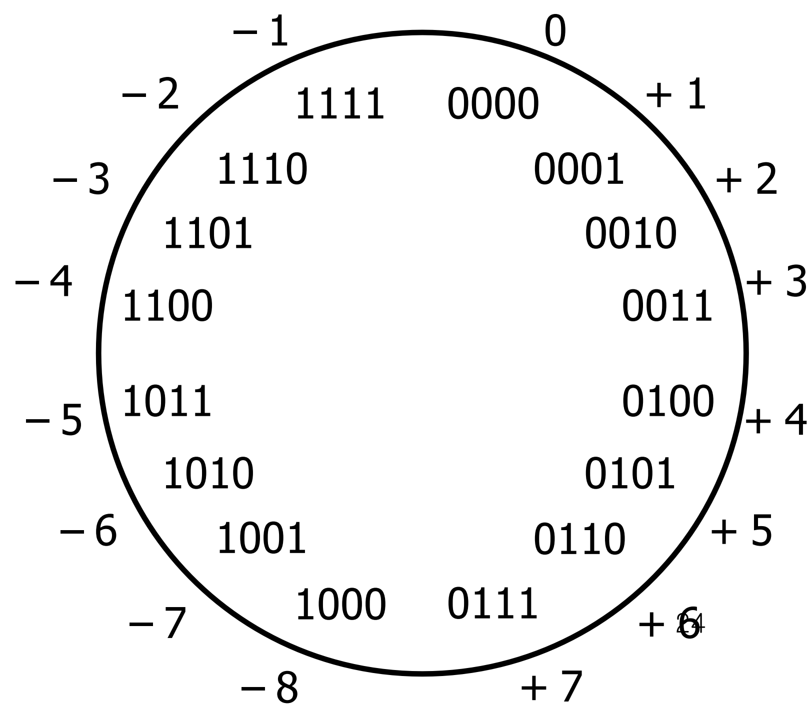
4-bit Unsigned vs. Two's Complement (1)

1 0 1 1

$2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1$

$-2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1$





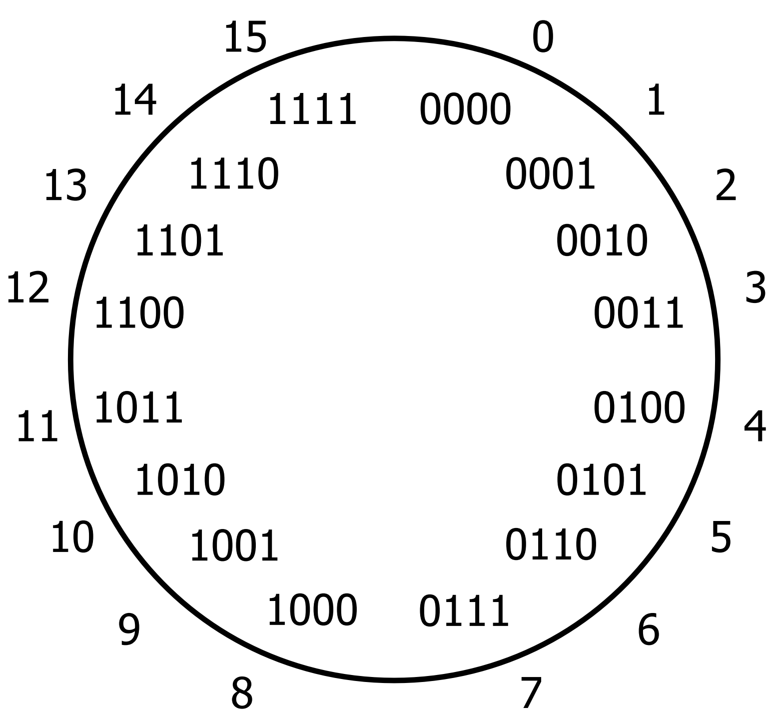
4-bit Unsigned vs. Two's Complement (2)

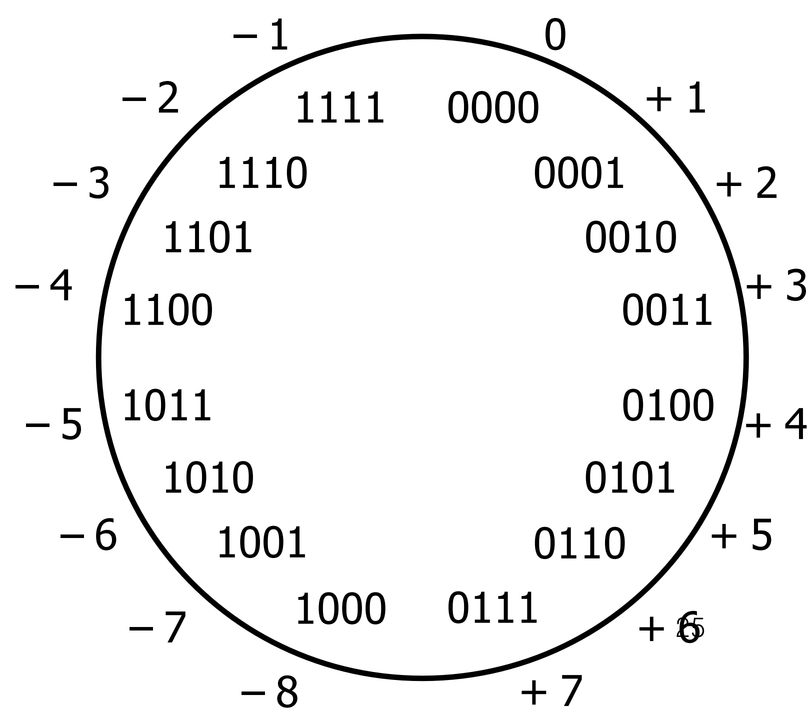
1 0 1 1

$2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1$

$-2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1$

11 ← **(math) difference = 16 = 2^4** → -5





4-bit Unsigned vs. Two's Complement (3)

1 0 1 1

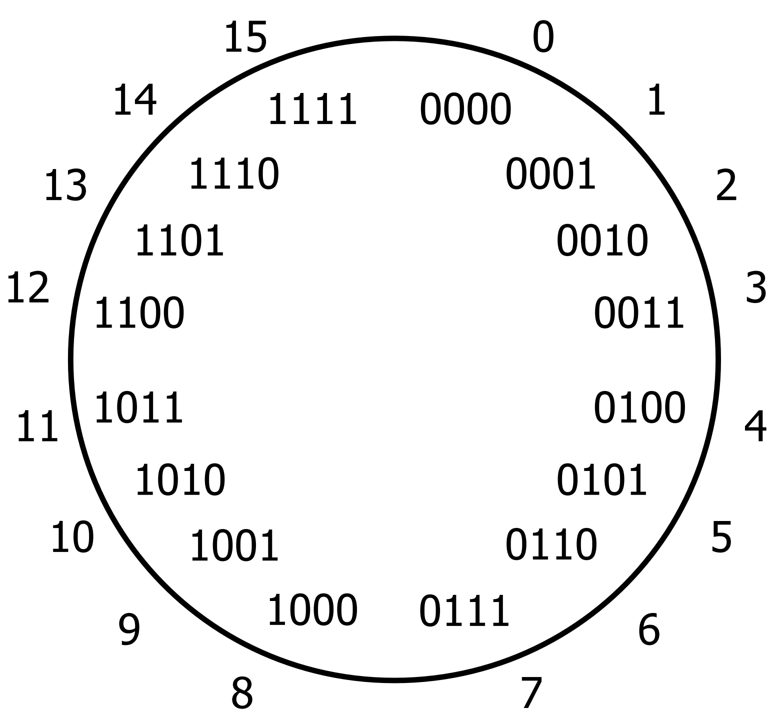
$2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1$

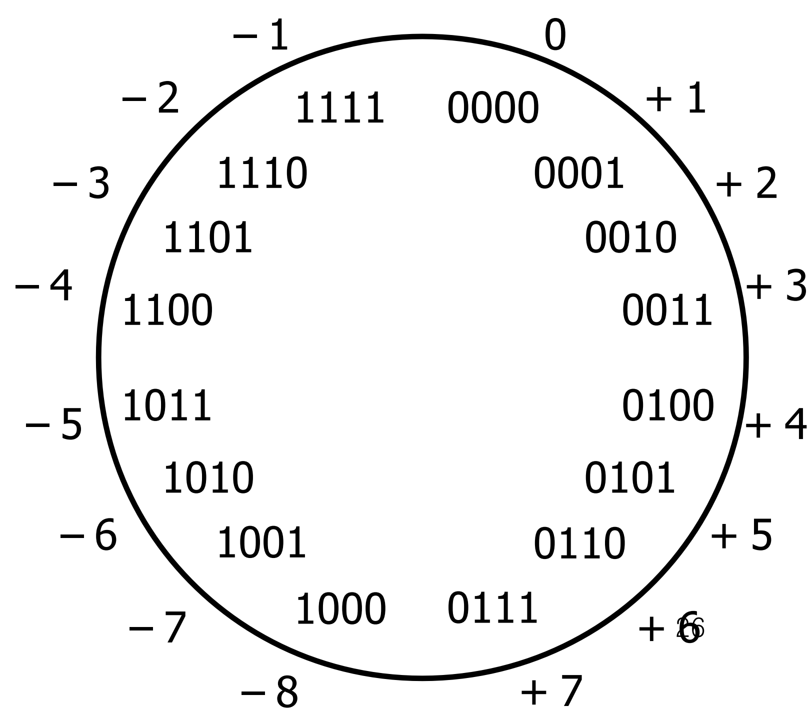
11

$-2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1$

-5

(math) difference = 16 = 2^4





Summary

- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations

- Integers represented using unsigned and two's complement representations
  - Limited by fixed bit width
  - We'll examine arithmetic operations next lecture

Q&A