Continuous random variables

Continuous Random Variables

[Definition] Random variable X is continuous if there exists a nonnegative function $f(x)$, defined for all real $x \in (-\infty, \infty)$ such that

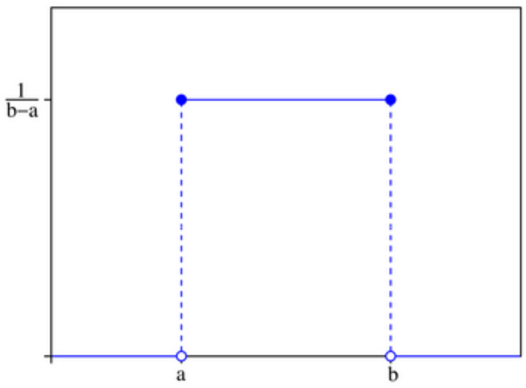
1. $P\{a \leq X \leq b\} = \int_a^b f(x)dx$
2. $P\{X \in (-\infty, \infty)\} = \int_{-\infty}^{\infty} f(x)dx = 1$
3. $P\{X = a\} = \int_a^a f(x)dx = 0$

- $f(x)$ is called the probability density function (PDF)

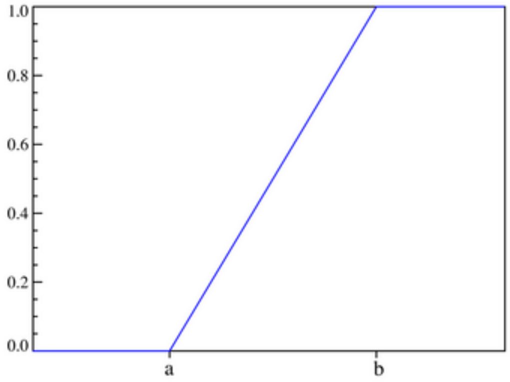
Uniform Random Variables

- $$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & elsewhere \end{cases}$$
- Notation : $X \sim Unif(a, b)$

PDF



CDF



Uniform Random Variables – example

- Example 2.14

$$X \sim Unif(0, 10)$$

(a) $\mathrm{P}\ (X < 3)$

(b) $\mathrm{P}(X > 7)$

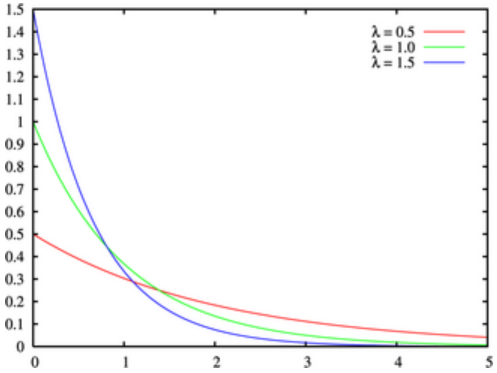
(c) $\mathrm{P}(1 < X < 6)$

- Applications
  - Simulation experiments
  - Random number generations

Exponential Random Variables

- PDF : $f(x) = \lambda \cdot e^{-\lambda x}, \text{if } x \geq 0$
- CDF : $F(x) = \left(1 - e^{-\lambda x}\right)$
- Notation : $X \sim Exp(\lambda)$

PDF



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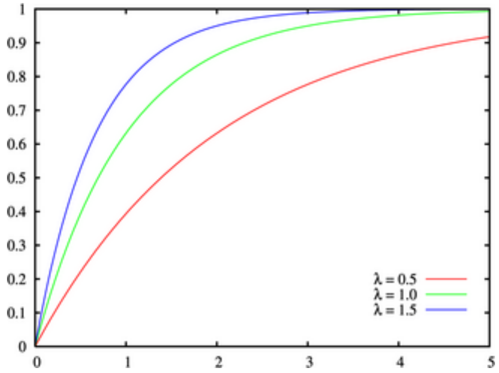


Exponential Random Variables – example

- A certain random variable has a probability density function of the form $f(x) = ce^{-2x}$. Find the following:

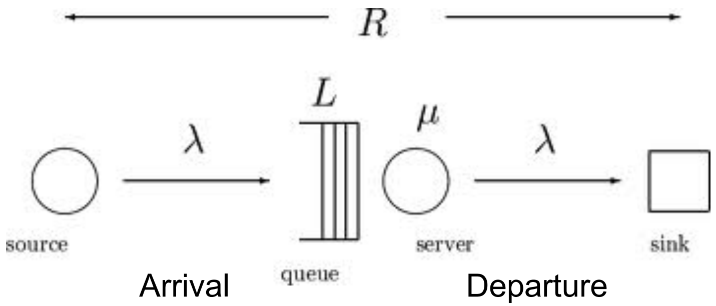
(a) the constant c

(b) $\mathrm{P}(X < 3)$

Exponential Random Variables – applications

- Often used to model the time between events that happen at a constant average rate (λ)
- Commonly encountered in the study of Queuing systems
  - The time between arrivals of customers at a bank,
  - The duration of voice conversation in a telephone network

Queue ?



More details ➔ Chapter 8

The Definition of *e*

- The mathematical constant *e* is the base of the natural logarithm function. Its value to the 29th decimal digit is:
  - *e* = 2.71828 18284 59045 23536 02874 7135...

- Alongside the number π and the imaginary unit *i*, *e* is one of the most important numbers in mathematics. It has a number of equivalent definitions; some of them are 1), 2) below.

1) $$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

2) $$e = \sum_{n=0}^{\infty}\frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \cdots$$

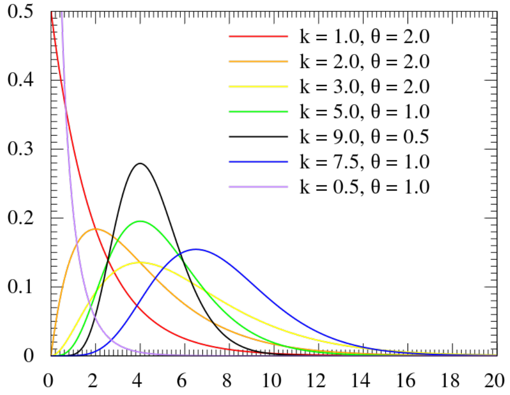
Gamma Random Variables

- PDF : $f(x) = \frac{\lambda \cdot e^{-\lambda x}(\lambda x)^{\alpha-1}}{\Gamma(\alpha)}, \text{if } x \geq 0$

$$\Gamma(\alpha) = \int_0^{\infty} e^{-x} x^{\alpha-1} dx$$

- Notation: $X \sim Gamma(\alpha, \lambda)$

PDF



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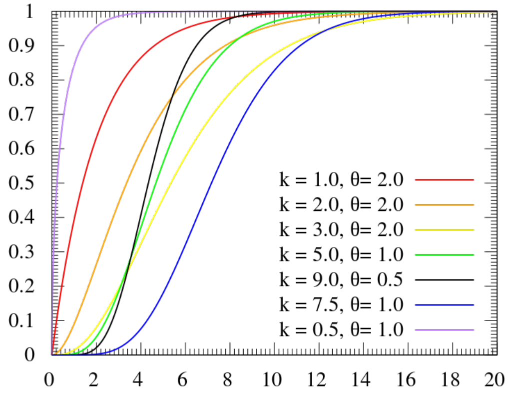


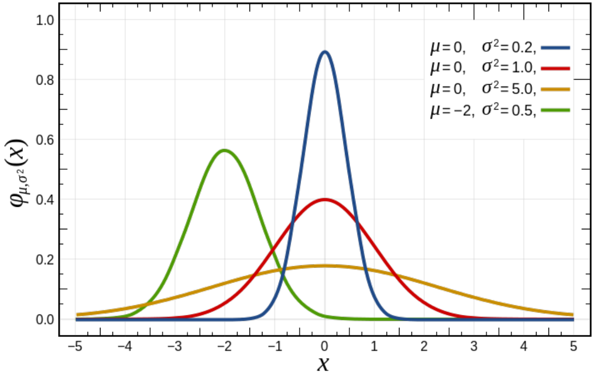
Gamma Random Variables – applications

- Model the size of insurance claims and rainfalls accumulated in a reservoir.
- Model the multi-path fading of signal power in wireless communication.
- Describe the distribution of inter-spike intervals in neuroscience.
- The copy number of a constitutively expressed protein in bacterial gene expression.
- Peak calling step in ChIP-chip and ChIP-seq data analysis in genomics.
- Widely used as a conjugate prior in Bayesian statistics.

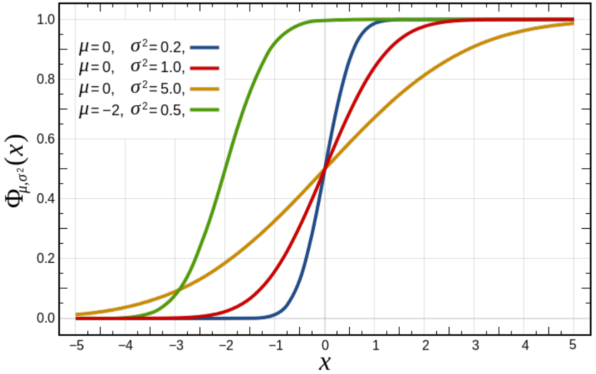
Normal Random Variables

- PDF : $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, -\infty < x < \infty$
- Notation: $X \sim N(\mu, \sigma)$

PDF



CDF



Normal Random Variables – applications

- Measures of size of living tissue (length, height, skin area, weight in biology.
- Certain physiological measurements, such as blood pressure of adult humans in medicine.
- Exchange rates, price indices, and stock market indices in finance.
- Measurement errors in physical experiments
- In standardized testing
- Many scores including percentile ranks, normal curve equivalents, and T-scores.