Algebra Review

Order of operations

1. Parentheses: $(1 + 1)$
2. Exponents and Square Roots: x^2, $\sqrt[3]{3}$
3. Multiplication and Division: 3×5, $\frac{5}{3}$
4. Addition and Subtraction: $+$, $-$

(e.g.)

$$\frac{3 + 4 + 5}{4} =$$

$$\left(2 + \frac{12}{4}\right)^3 =$$

Summations

$x_1 = 5, x_2 = 4, x_3 = 2, x_4 = 5$

$$\sum x_i =$$

$$\sum x_i{}^2 =$$

$$(\sum x_i)^2 =$$

Factorials

$$n! = n \times (n-1) \times \cdots \times 2 \times 1$$

$$4! =$$

$$\frac{6!}{2!(6-2)!} =$$

Algebra self-assessment

1. $(3 + 2)^3 =$

2. $\dfrac{3+3+2+2}{5} =$

3. $\sqrt[3]{\dfrac{(5-3)^3+(6-2)^2}{3}} =$

Use the following five values for X: $2, 3, 3, 5, 5$

4. $\sum X_i =$

5. $\sum X_i{}^2 =$

6. $\dfrac{5!}{2!(5-2)!} =$

Matrix Algebra Review

Matrix transpose

The transpose of matrix A is denoted as A^T or A'

$$A = \begin{bmatrix} 1 & -3 & 4 \\ 2 & 5 & 2 \end{bmatrix}$$

$$A^T = A' =$$

If a matrix is its own transpose, that matrix is said to be **symmetric**.

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 3 & -6 & 5 \end{bmatrix} = A'$$

Matrix addition

$$A = \begin{bmatrix} 1 & -3 & 4 \\ 2 & 5 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 3 & 2 \end{bmatrix}$$

$$A + B =$$

Matrix multiplication

The number of columns for the first matrix should be the same as the number of rows for the second matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}$$

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -3 & 4 \\ 2 & 5 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 4 \\ -1 & 3 \\ 3 & 2 \end{bmatrix}$$

$$AB =$$

Determinate

The determinate of a square, 2×2 matrix A

$$det(A) = |A| = \begin{vmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{vmatrix} = a_{1,1} * a_{2,2} - a_{1,2} * a_{2,1}$$

For a 3×3 matrix A,

$$det(B) = |B| = \begin{vmatrix} b_{1,1} & b_{1,2} & b_{1,3} \\ b_{2,1} & b_{2,2} & b_{2,3} \\ b_{3,1} & b_{3,2} & b_{3,3} \end{vmatrix} = b_{1,3} \begin{vmatrix} b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2} \end{vmatrix} - b_{2,3} \begin{vmatrix} b_{1,1} & b_{1,2} \\ b_{3,1} & b_{3,2} \end{vmatrix} + b_{3,3} \begin{vmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{vmatrix}$$

$$det(B) = b_{1,3}(b_{2,1}b_{3,2} - b_{2,2}b_{3,1}) - b_{2,3}(b_{1,1}b_{3,2} - b_{1,2}b_{3,1}) + b_{3,3}(b_{1,1}b_{2,2} - b_{1,2}b_{2,1})$$

Determinate

$$|A| = \begin{vmatrix} 2 & 4 \\ -3 & 1 \end{vmatrix} =$$

$$|B| = \begin{vmatrix} 5 & -2 & 3 \\ 4 & 1 & 3 \\ 1 & 6 & 2 \end{vmatrix} =$$

Identity matrix

- A square matrix where every diagonal entry is 1 and all the other entries are 0

- Examples

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse

- B is the inverse of matrix A if $AB = BA = I$.
- B is denoted as A^{-1}.
- When $det(A) \neq 0$, A is invertible.

$$A^{-1} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}^{-1} = \frac{1}{\det(A)} \begin{bmatrix} a_{2,2} & -a_{1,2} \\ -a_{2,1} & a_{1,1} \end{bmatrix}$$

Self-assessment

Let $A = \begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

1. $A - B =$

2. $det(A) =$

3. $det(B) =$

4. $AB =$

5. $A^{-1}B^{-1} =$

Calculus Review

Summations

❑ $\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$

❑ $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

❑ The Binomial Theorem:

$(x + y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-1} y^i$ where $\binom{n}{i} = \frac{n!}{(n-i)!i!}$

$\sum_{i=1}^{4} i^2 =$

$(x + 1)^4 =$

Geometric series

❑ $S = \sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + \cdots$ where $a \neq 0$.

S converges to $\frac{a}{1-r}$ if $|r| < 1$, but diverges otherwise.

$\sum_{k=1}^{\infty} (\frac{1}{2})^k =$

❑ The Taylor series of e^x:

$\sum_{i=0}^{\infty} \frac{x^i}{i!} = 1 + x + \frac{x^2}{2!} + \cdots$ converges to e^x for $-1 \leq x \leq 1$.

Common derivatives

- ❏ $\frac{d}{dx} x^n = nx^{n-1}$

- ❏ $\frac{d}{dx} \ln x = \frac{1}{x}$

- ❏ $\frac{d}{dx} a^x = a^x \ln a$

- ❏ $\frac{d}{dx} e^x = e^x$

Rules

❑ Product rule
$$[f(x)g(x)]' = f'g + fg'$$

❑ Quotient rule
$$[\frac{f(x)}{g(x)}]' = \frac{gf' - fg'}{(g(x))^2}$$

❑ Chain rule
$$[f(g(x))]' = f'^{(g(x))}g'(x)$$

Examples

Find the derivatives of $f(x)$ for the following:

1. $f(x) = 8x^7 - 5x^5 + 2x^2 - 10$

2. $f(x) = \frac{1}{2x^3}$

3. $f(x) = xe^{-x}$

4. $f(x) = -\ln(2x)^3$

Integrals

$$\int f(x)dx = F(x) + C, \text{where } F'(x) = f(x)$$

❏ Common integrals

1. $\int x^n dx = \frac{1}{n+1}x^{n+1} + C, n \neq 1$

2. $\int \frac{1}{x}dx = \ln x + C$

3. $\int e^x dx = e^x + C$

$$\int (3x^5 - \frac{2}{x} + e^{-x})dx =$$

Rules

❑ The fundamental theorem of calculus:

$\int_a^b f(x)dx = F(b) - F(a)$, where F is an antiderivative of f

❑ Integration using substitution

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

❑ Integration by parts

$$\int_a^b udv = [uv]_a^b - \int_a^b vdu$$

Examples

1. $\int_0^t \frac{1}{2} x e^{-x^2} \, dx =$

2. $\int_0^2 x^2 e^{-x} \, dx =$

3. $\int_{-\infty}^{\infty} e^{-|x| + \frac{x}{2}} \, dx =$

Partial derivatives

❑ $\dfrac{\partial f}{\partial x}(a, b) = \lim\limits_{h \to 0} \dfrac{f(a+h,b)-f(a,b)}{h}$

❑ $\dfrac{\partial f}{\partial y}(a, b) = \lim\limits_{h \to 0} \dfrac{f(a,b+h)-f(a,b)}{h}$

❑ Given the function $f(x, y) = \dfrac{1}{x^2} + 6\sqrt{y} - xy$, find both partial derivatives.

Integration over regions

Consider the rectangular region defined by $a \leq x \leq b$ and $c \leq y \leq d$. Then the iterated integral would be:

❑ $\int_a^b [\int_c^d f(x,y)dx]dy = \int_c^d [\int_a^b f(x,y)dy]dx$

❑ Integrate $f(x,y) = 24xy$, for $0 \leq x \leq 1, 0 \leq y \leq 1$, and $0 < x + y < 1$