MONASH University

MONASH BUSINESS SCHOOL

High dimensional time series analysis

2. Time series graphics

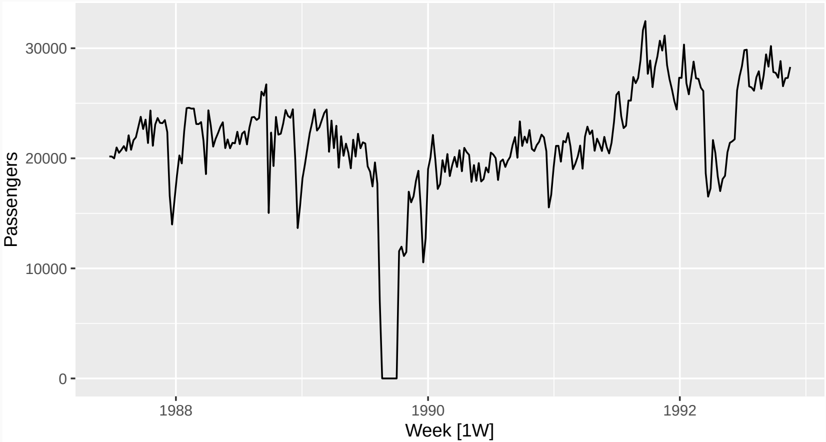
robjhyndman.com/hdtsa

Outline

Outline

Time plots

```
ansett %>%
  filter(Airports=="MEL-SYD", Class=="Economy") %>%
  autoplot(Passengers)
```



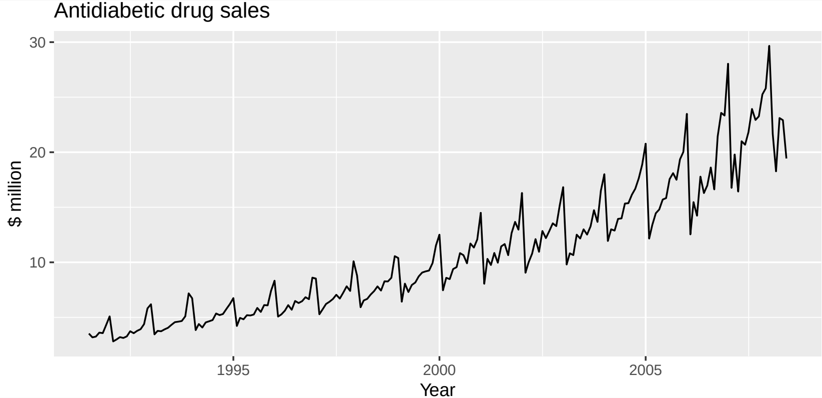
Time plots

```
PBS %>%
  filter(ATC2=="A10") %>%
  select(Month, Concession, Type, Cost) %>%
  summarise(total_cost = sum(Cost)) %>%
  mutate(total_cost = total_cost/1e6) ->
  a10
```

```
## # A tsibble: 204 x 2 [1M]
##        Month total_cost
##        <mth>      <dbl>
##  1 1991 Jul        3.53
##  2 1991 Aug        3.18
##  3 1991 Sep        3.25
##  4 1991 Oct        3.61
##  5 1991 Nov        3.57
```

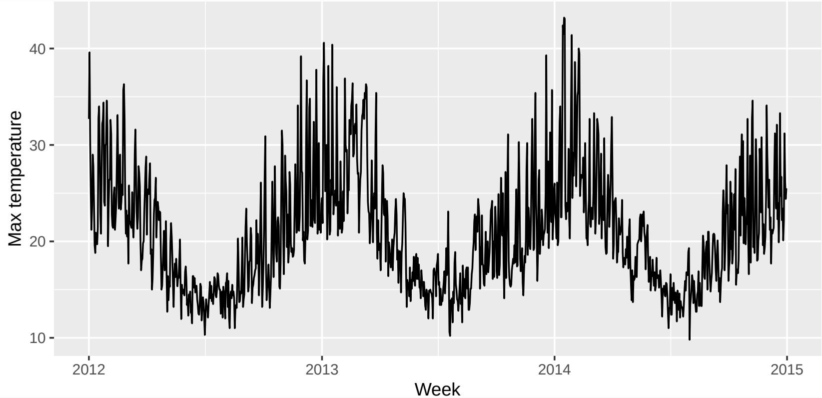
Time plots

```
a10 %>% autoplot(total_cost) +
  ylab("$ million") + xlab("Year") +
  ggtitle("Antidiabetic drug sales")
```



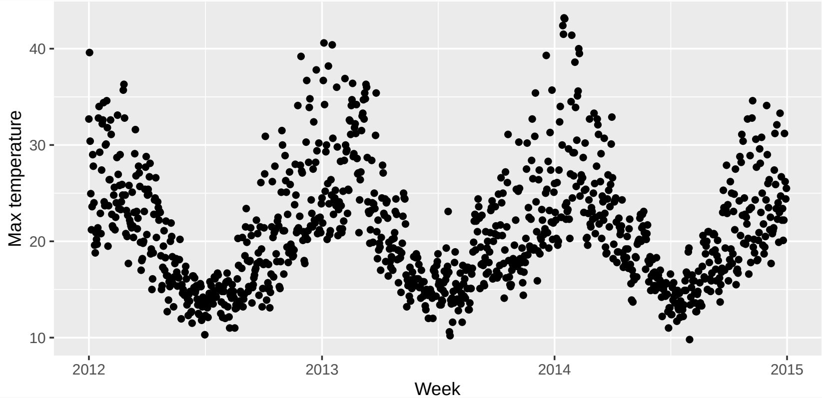
Are time plots best?

```
maxtemp %>%
  autoplot(Temperature) +
  xlab("Week") + ylab("Max temperature")
```

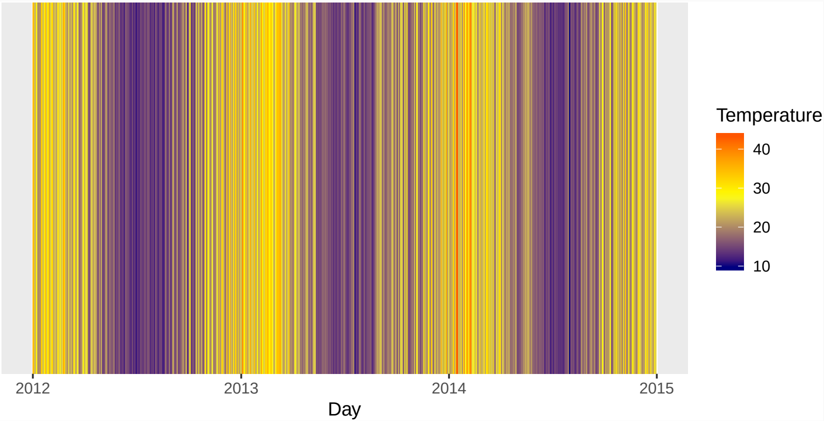


Are time plots best?

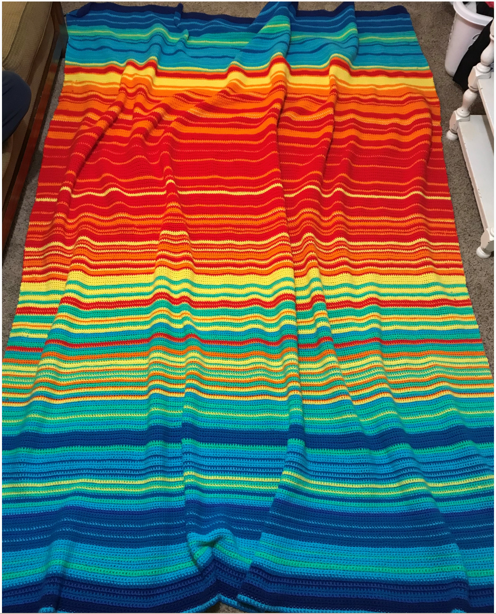
```
maxtemp %>%
  ggplot(aes(x = Day, y = Temperature)) +
  geom_point() +
  xlab("Week") + ylab("Max temperature")
```



Are time plots best?



Are time plots best?



Lab Session 2

- Create time plots of the following time series: Bricks from `aus_production`, Lynx from `pelt`, Google from `gafa_stock`
- Use `help()` to find out about the data in each series.
- For the last plot, modify the axis labels and title.

Outline

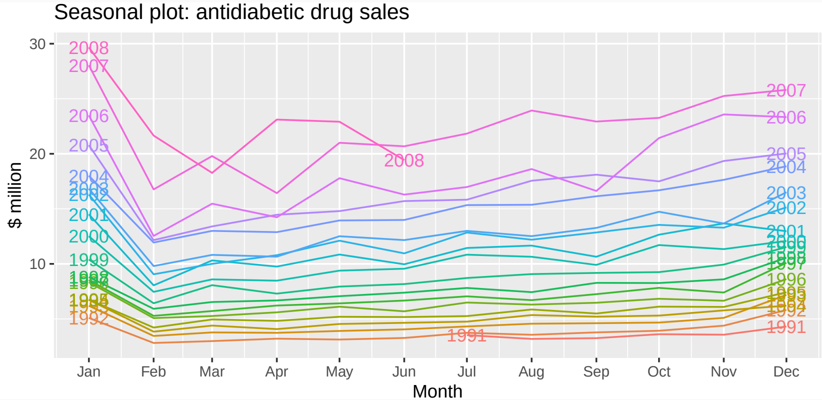
The seasonal period

- Seasonal period = no. observations before seasonal pattern repeats.
- Usually automatically detected using time index.
- Daily & sub-daily time series can have multiple periods.

Data	Minute	Hour	Day	Week	Year
Quarters					4
Months					12
Weeks					52
Days				7	365.25
Hours			24	168	8766
Minutes		60	1440	10080	525960
Seconds	60	3600	86400	604800	31557600

Seasonal plots

```
a10 %>% gg_season(total_cost, labels = "both") +
  ylab("$ million") +
  ggtitle("Seasonal plot: antidiabetic drug sales")
```

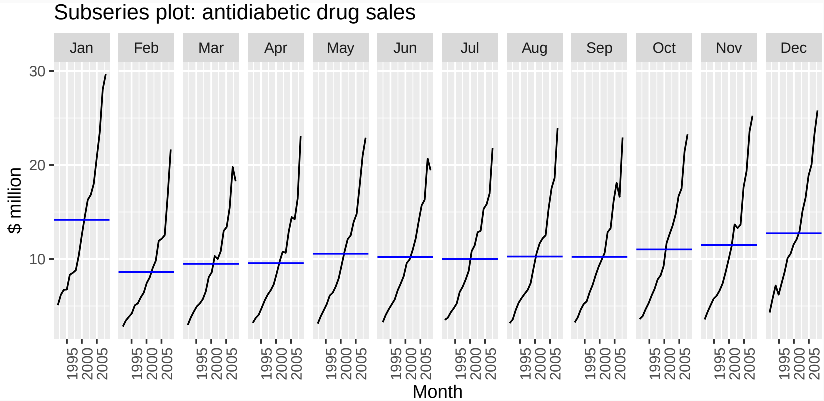


Seasonal plots

- Data plotted against the individual “seasons” in which the data were observed. (In this case a “season” is a month.)
- Something like a time plot except that the data from each season are overlapped.
- Enables the underlying seasonal pattern to be seen more clearly, and also allows any substantial departures from the seasonal pattern to be easily identified.
- In R: `gg_season()`

Seasonal subseries plots

```
a10 %>%
  gg_subseries(total_cost) + ylab("$ million") +
  ggtitle("Subseries plot: antidiabetic drug sales")
```

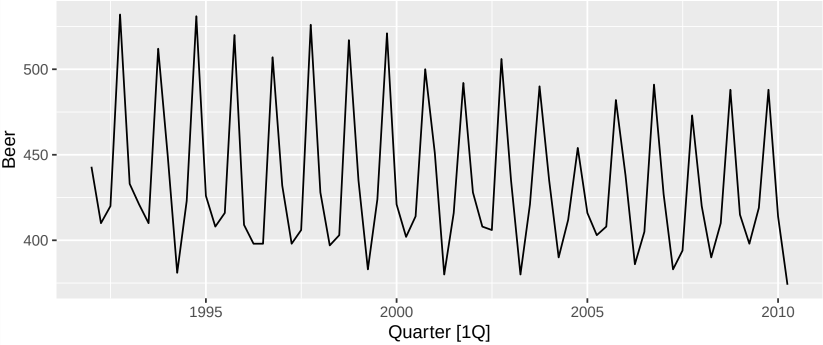


Seasonal subseries plots

- Data for each season collected together in time plot as separate time series.
- Enables the underlying seasonal pattern to be seen clearly, and changes in seasonality over time to be visualized.
- In R: `gg_subseries()`

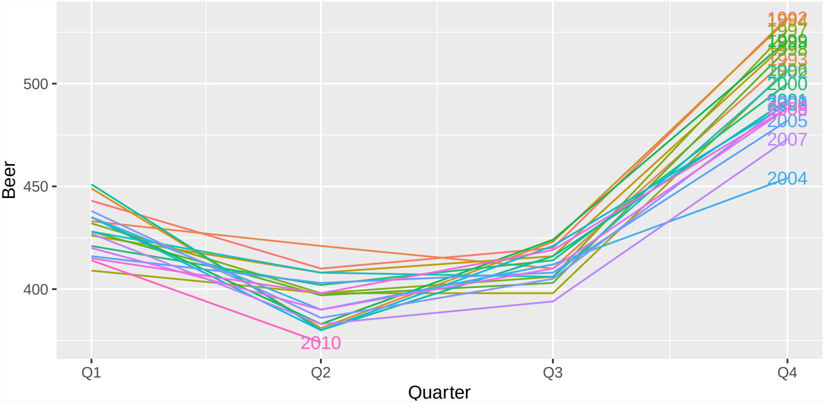
Quarterly Australian Beer Production

```
beer <- aus_production %>%
  select(Quarter, Beer) %>%
  filter(year(Quarter) >= 1992)
beer %>% autoplot(Beer)
```



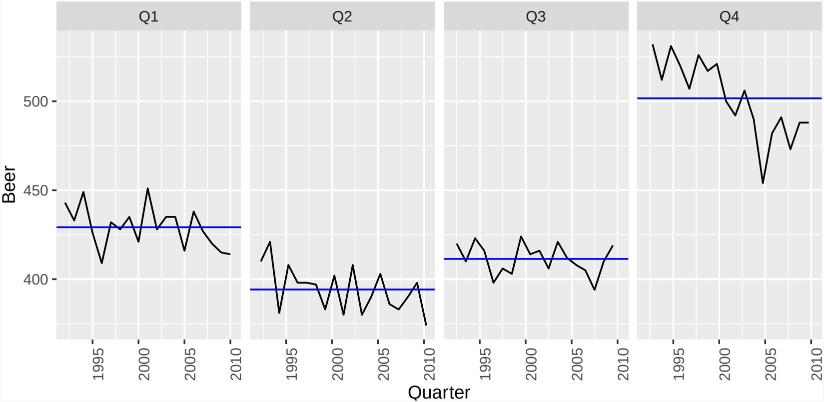
Quarterly Australian Beer Production

```r
beer %>% gg_season(Beer, labels="right")
```



Quarterly Australian Beer Production

```
beer %>% gg_subseries(Beer)
```



Lab Session 3

Look at the quarterly tourism data for the Snowy Mountains

```
snowy <- filter(tourism,
  Region == "Snowy Mountains",
  Purpose == "Holiday")
```

- Use `autoplot()`, `gg_season()` and `gg_subseries()` to explore the data.
- What do you learn?

Outline

Time series patterns

Trend pattern exists when there is a long-term increase or decrease in the data.

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Cyclic pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

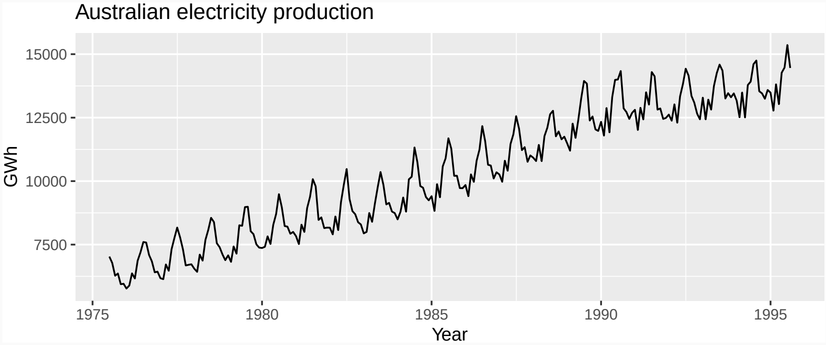
Time series components

Differences between seasonal and cyclic patterns:

- seasonal pattern constant length; cyclic pattern variable length
- average length of cycle longer than length of seasonal pattern
- magnitude of cycle more variable than magnitude of seasonal pattern

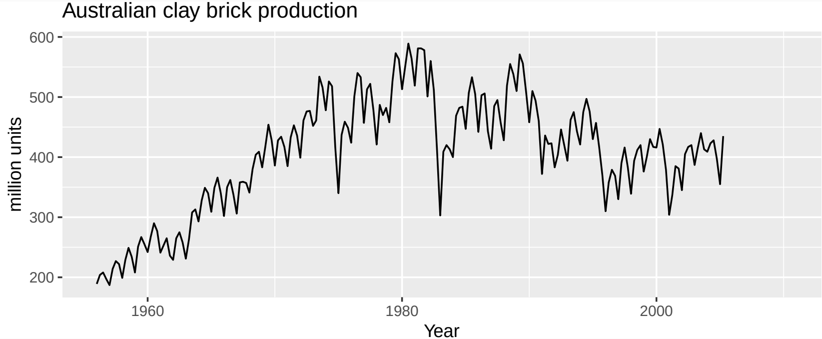
Time series patterns

```
as_tsibble(fma::elec) %>%
  filter(index >= 1980) %>%
  autoplot(value) + xlab("Year") + ylab("GWh") +
  ggtitle("Australian electricity production")
```



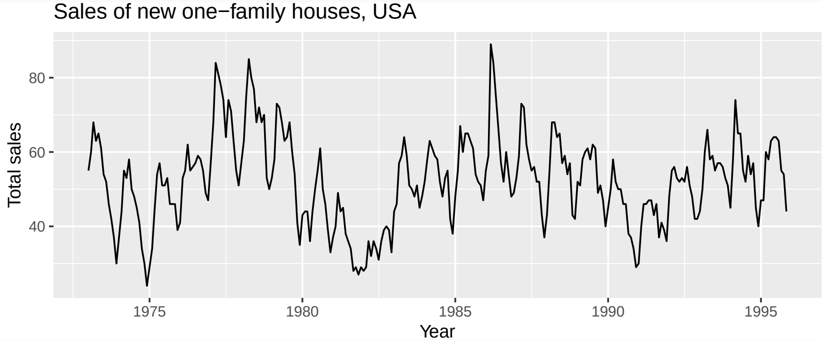
Time series patterns

```r
aus_production %>%
  autoplot(Bricks) +
  ggtitle("Australian clay brick production") +
  xlab("Year") + ylab("million units")
```



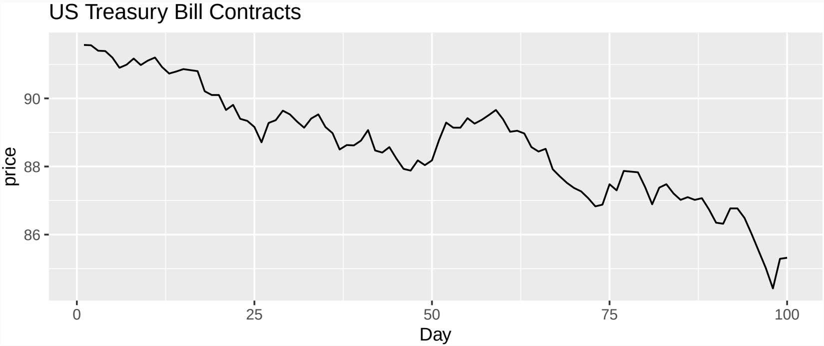
Time series patterns

```
as_tsibble(fma::hsales) %>%
  autoplot(value) +
  ggtitle("Sales of new one-family houses, USA") +
  xlab("Year") + ylab("Total sales")
```



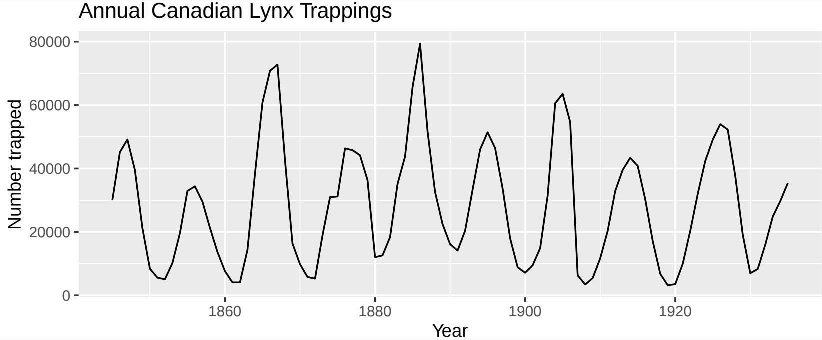
Time series patterns

```
as_tsibble(fma::ustreas) %>%
  autoplot(value) +
  ggtitle("US Treasury Bill Contracts") +
  xlab("Day") + ylab("price")
```



Time series patterns

```
pelt %>%
  autoplot(Lynx) +
  ggtitle("Annual Canadian Lynx Trappings") +
  xlab("Year") + ylab("Number trapped")
```



Seasonal or cyclic?

Differences between seasonal and cyclic patterns:

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Seasonal or cyclic?

Differences between seasonal and cyclic patterns:

- seasonal pattern constant length; cyclic pattern variable length
- average length of cycle longer than length of seasonal pattern
- magnitude of cycle more variable than magnitude of seasonal pattern

The timing of peaks and troughs is predictable with seasonal data, but unpredictable in the long term with cyclic data.

Outline

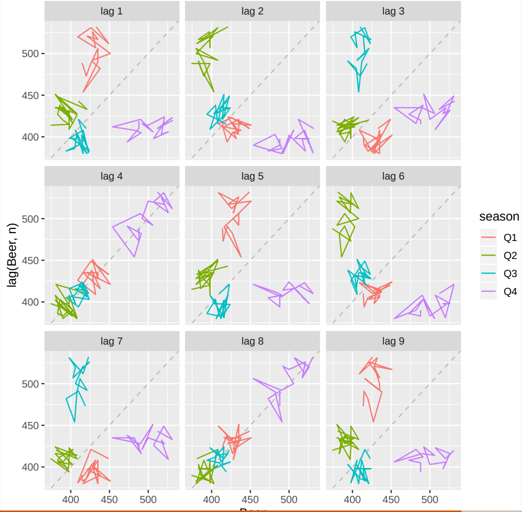
Example: Beer production

```
new_production <- aus_production %>%
  filter(year(Quarter) >= 1992)
new_production
```

```
## # A tsibble: 74 x 7 [1Q]
##    Quarter  Beer Tobacco Bricks Cement Electrici
##      <qtr> <dbl>   <dbl>  <dbl>  <dbl>       <db
##  1 1992 Q1   443    5777    383   1289       383
##  2 1992 Q2   410    5853    404   1501       397
##  3 1992 Q3   420    6416    446   1539       422
##  4 1992 Q4   532    5825    420   1568       384
##  5 1993 Q1   433    5724    394   1450       394
##  6 1993 Q2   421    6036    462   1668       413
```

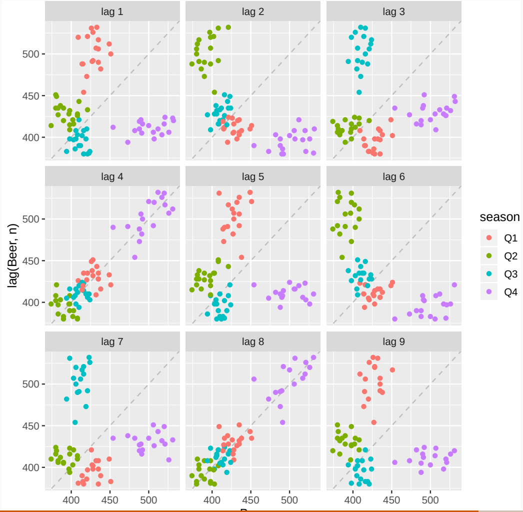
Example: Beer production

```r
new_production %>% gg_lag(Beer)
```



Example: Beer production

```
new_production %>% gg_lag(Beer, geom='point')
```



Lagged scatterplots

- Each graph shows y_t plotted against y_{t-k} for different values of k.
- The autocorrelations are the correlations associated with these scatterplots.

Autocorrelation

Covariance and **correlation**: measure extent of **linear relationship** between two variables (y and X).

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Autocovariance and **autocorrelation**: measure linear relationship between **lagged values** of a time series y.

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Covariance and **correlation**: measure extent of **linear relationship** between two variables (y and X).

Autocovariance and **autocorrelation**: measure linear relationship between **lagged values** of a time series y.

We measure the relationship between:

- y_t and y_{t-1}
- y_t and y_{t-2}
- y_t and y_{t-3}
- etc.

Autocorrelation

We denote the sample autocovariance at lag k by c_k and the sample autocorrelation at lag k by r_k. Then define

$$c_k = \frac{1}{T}\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y})$$

and $$r_k = c_k / c_0$$

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and $$r_k = c_k / c_0$$

- r_1 indicates how successive values of y relate to each other
- r_2 indicates how y values two periods apart relate to each other
- r_k is *almost* the same as the sample correlation between y_t and y_{t-k}.

Autocorrelation

Results for first 9 lags for beer data:

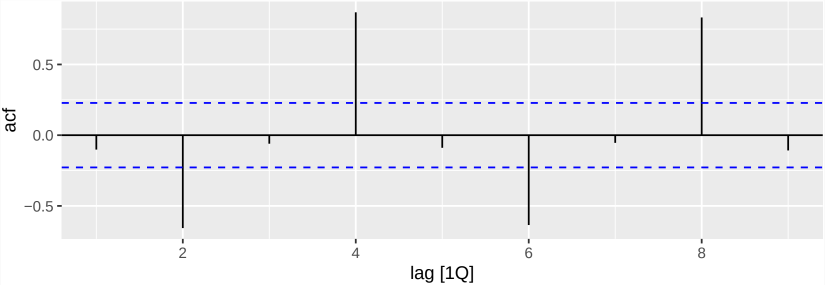
```
new_production %>% ACF(Beer, lag_max = 9)
```

```
## # A tsibble: 9 x 2 [1Q]
##      lag     acf
##    <lag>   <dbl>
## 1     1Q -0.102
## 2     2Q -0.657
## 3     3Q -0.0603
## 4     4Q  0.869
## 5     5Q -0.0892
## 6     6Q -0.635
## 7     7Q -0.0542
## 8     8Q  0.832
```

Autocorrelation

Results for first 9 lags for beer data:

```
new_production %>% ACF(Beer, lag_max = 9) %>% autoplot()
```

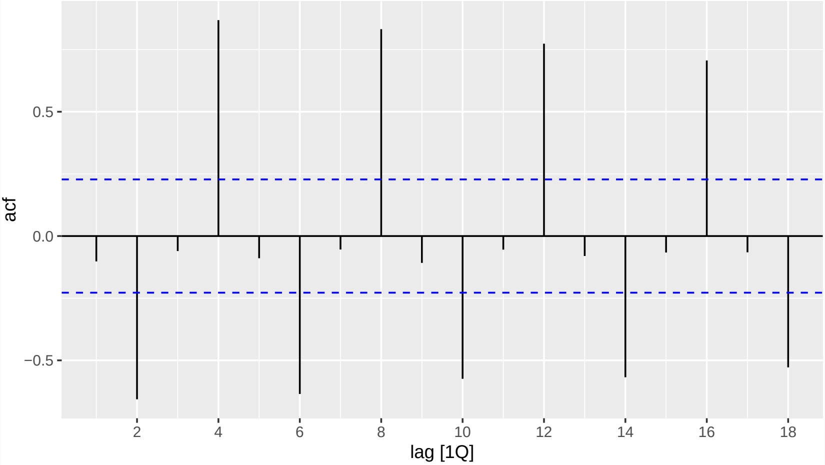


Autocorrelation

- r_4 higher than for the other lags. This is due to **the seasonal pattern in the data**: the peaks tend to be **4 quarters** apart and the troughs tend to be **2 quarters** apart.
- r_2 is more negative than for the other lags because troughs tend to be 2 quarters behind peaks.
- Together, the autocorrelations at lags 1, 2, ..., make up the *autocorrelation* or ACF.
- The plot is known as a **correlogram**

ACF

```
new_production %>% ACF(Beer) %>% autoplot()
```

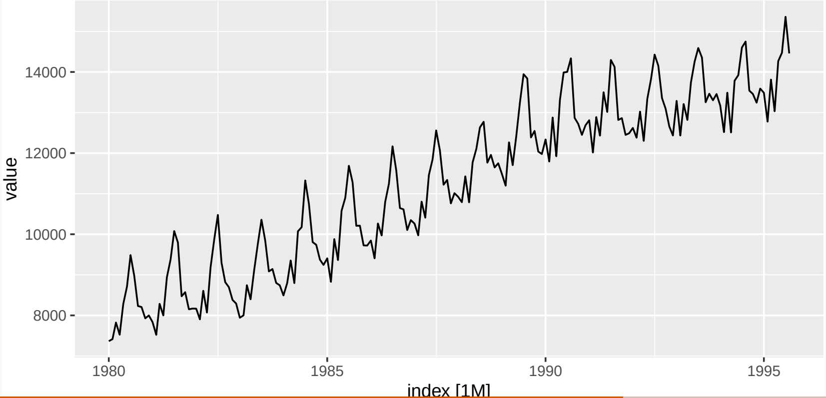


Trend and seasonality in ACF plots

- When data have a trend, the autocorrelations for small lags tend to be large and positive.
- When data are seasonal, the autocorrelations will be larger at the seasonal lags (i.e., at multiples of the seasonal frequency)
- When data are trended and seasonal, you see a combination of these effects.

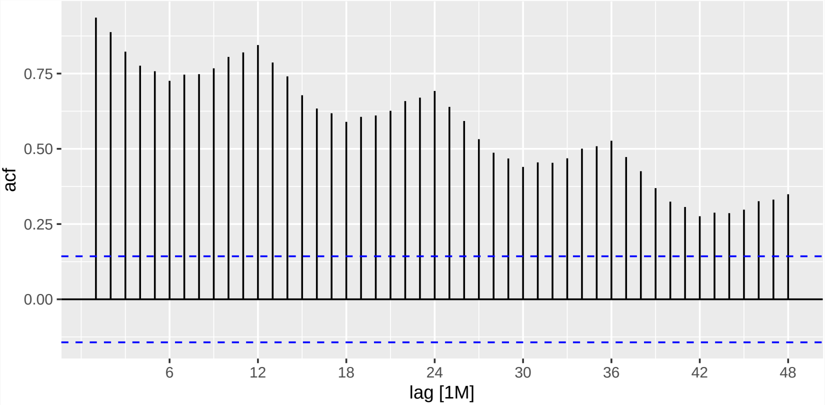
Aus monthly electricity production

```
elec2 <- as_tsibble(fma::elec) %>%
  filter(year(index) >= 1980)
elec2 %>% autoplot(value)
```



Aus monthly electricity production

```
elec2 %>% ACF(value, lag_max=48) %>%
  autoplot()
```



Aus monthly electricity production

Time plot shows clear trend and seasonality.

The same features are reflected in the ACF.

- The slowly decaying ACF indicates trend.
- The ACF peaks at lags 12, 24, 36, . . . , indicate seasonality of length 12.

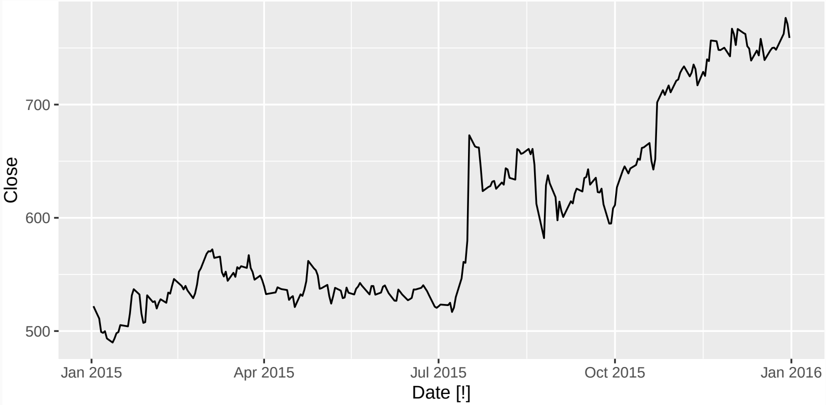
Google stock price

```
google_2015 <- gafa_stock %>%
  filter(Symbol == "GOOG", year(Date) == 2015) %>%
  select(Date, Close)
google_2015
```

```
## # A tsibble: 252 x 2 [!]
##    Date       Close
##    <date>     <dbl>
##  1 2015-01-02  522.
##  2 2015-01-05  511.
##  3 2015-01-06  499.
##  4 2015-01-07  498.
##  5 2015-01-08  500.
##  6 2015-01-09  493.
```

Google stock price

```
google_2015 %>% autoplot(Close)
```



Google stock price

```
google_2015 %>%
  ACF(Close, lag_max=100)
# Error: Can't handle tsibble of irregular interval.
```

Google stock price

```
google_2015 %>%
  ACF(Close, lag_max=100)
# Error: Can't handle tsibble of irregular interval.
```

```
google_2015
```

```
## # A tsibble: 252 x 2 [!]
##     Date        Close
##     <date>      <dbl>
##   1 2015-01-02   522.
##   2 2015-01-05   511.
##   3 2015-01-06   499.
```

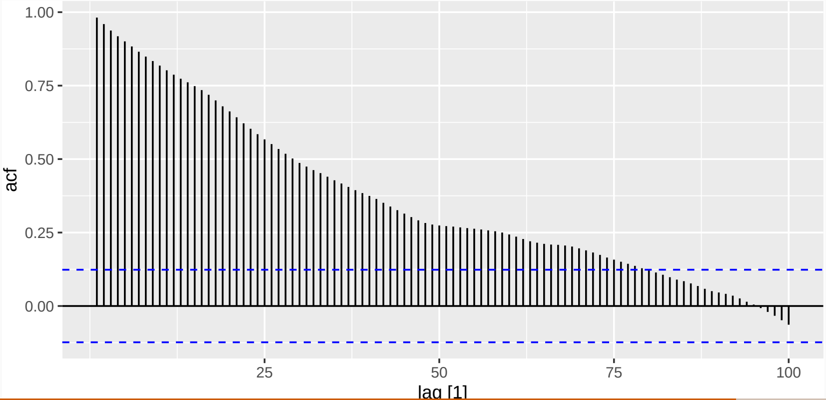
Google stock price

```
google_2015 <- google_2015 %>%
  mutate(trading_day = row_number()) %>%
  update_tsibble(index=trading_day, regular=TRUE)
google_2015
```

```
## # A tsibble: 252 x 3 [1]
##    Date       Close trading_day
##    <date>     <dbl>       <int>
##  1 2015-01-02  522.           1
##  2 2015-01-05  511.           2
##  3 2015-01-06  499.           3
##  4 2015-01-07  498.           4
##  5 2015-01-08  500.           5
##  6 2015-01-09  493.           6
```

Google stock price

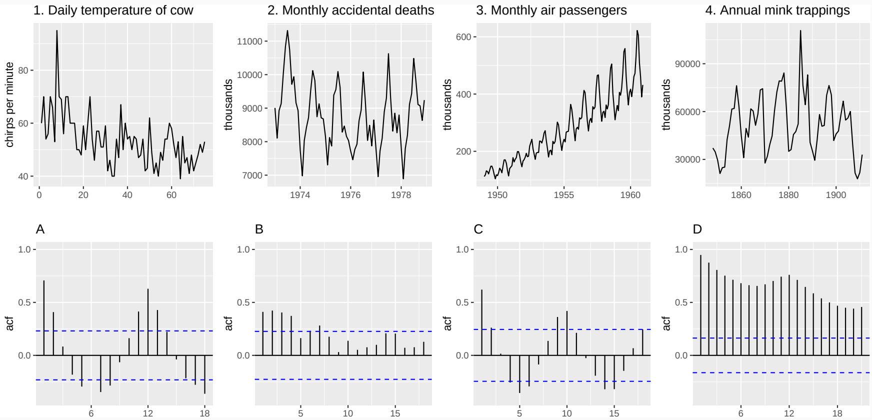
```
google_2015 %>%
  ACF(Close, lag_max=100) %>%
  autoplot()
```



Lab Session 4

Use `gg_lag` and `ACF` on the Snowy Mountains tourism data. What do you learn about the series?

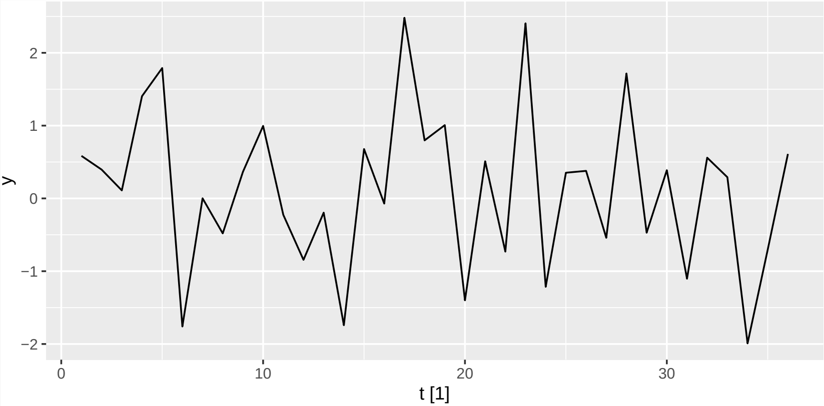
Which is which?



Outline

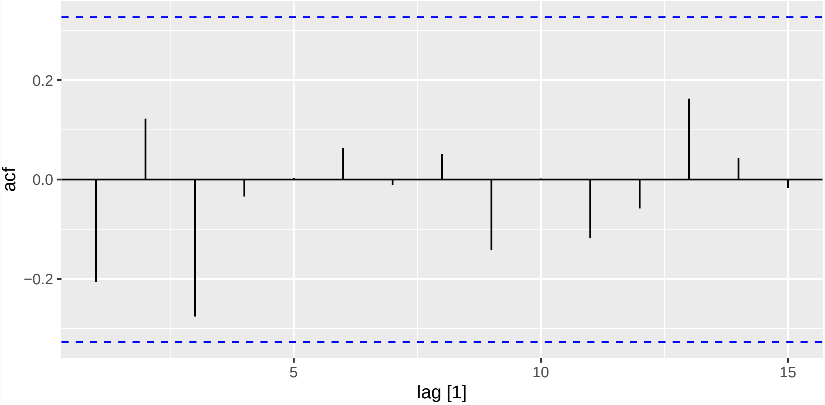
Example: White noise

```
wn <- tsibble(t = seq_len(36), y = rnorm(36),
              index = t)
wn %>% autoplot(y)
```



Example: White noise

r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}
-0.206	0.123	-0.276	-0.034	0.003	0.063	-0.011	0.051	-0.142	0.002



Sampling distribution of autocorrelations

Sampling distribution of r_k for white noise data is asymptotically $\mathrm{N}(0,1/T)$.

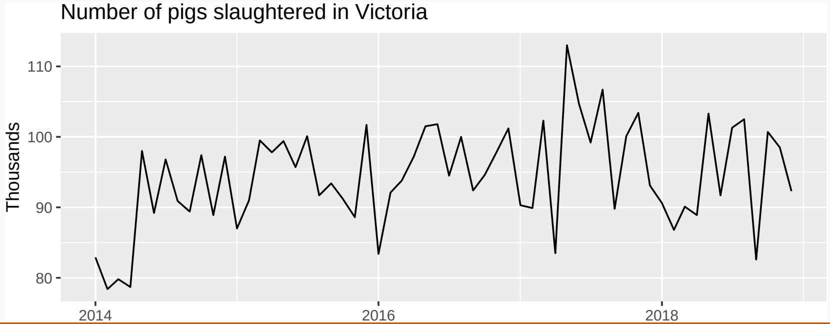
Sampling distribution of autocorrelations

Sampling distribution of r_k for white noise data is asymptotically N($0,1/T$).

- 95% of all r_k for white noise must lie within $\pm 1.96/\sqrt{T}$.
- If this is not the case, the series is probably not WN.
- Common to plot lines at $\pm 1.96/\sqrt{T}$ when plotting ACF. These are the **critical values**.

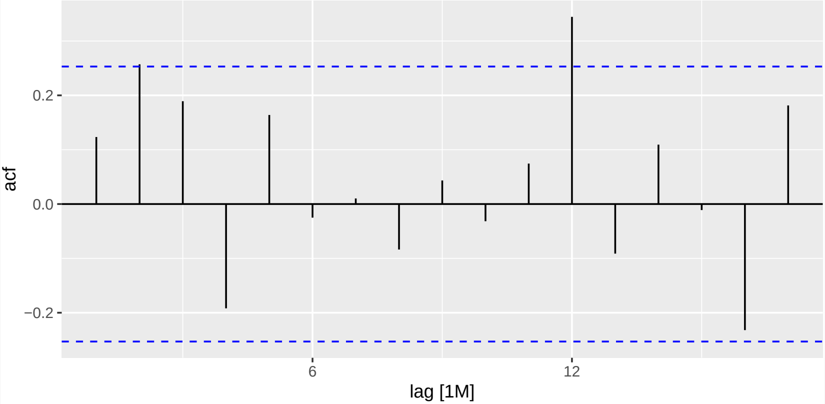
Example: Pigs slaughtered

```
pigs <- aus_livestock %>%
  filter(State == "Victoria", Animal == "Pigs",
         year(Month) >= 2014)
pigs %>% autoplot(Count/1e3) +
  xlab("Year") + ylab("Thousands") +
  ggtitle("Number of pigs slaughtered in Victoria")
```



Example: Pigs slaughtered

```
pigs %>% ACF(Count) %>% autoplot()
```



Example: Pigs slaughtered

Monthly total number of pigs slaughtered in the state of Victoria, Australia, from January 2014 through December 2018 (Source: Australian Bureau of Statistics.)

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Monthly total number of pigs slaughtered in the state of Victoria, Australia, from January 2014 through December 2018 (Source: Australian Bureau of Statistics.)

- Difficult to detect pattern in time plot.
- ACF shows significant autocorrelation for lag 2 and 12.
- Indicate some slight seasonality.

Example: Pigs slaughtered

Monthly total number of pigs slaughtered in the state of Victoria, Australia, from January 2014 through December 2018 (Source: Australian Bureau of Statistics.)

- Difficult to detect pattern in time plot.
- ACF shows significant autocorrelation for lag 2 and 12.
- Indicate some slight seasonality.

These show the series is **not a white noise series**.

Google stock prices

You can compute the daily changes in the Google stock price in 2018 using

```
dgoog <- gafa_stock %>%
  filter(Symbol == "GOOG", year(Date) >= 2018) %>%
  mutate(trading_day = row_number()) %>%
  update_tsibble(index=trading_day, regular=TRUE) %>%
  mutate(diff = difference(Close))
```

Does `diff` look like white noise?