



High dimensional time series analysis



3. Time series features

Outline

- 1 Decompositions
- 2 Time series components
- 3 STL decomposition



Feature Extraction And Statistics for Time Series

- works with tidy temporal data provided by the tsibble package.
- produces time series features, decompositions, statistical summaries and visualisations.

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Decompositions

The feasts package supports four common time series decomposition methods:

- Classical decomposition
- STL decomposition
- X11 decomposition
- X-13ARIMA-SEATS decomposition

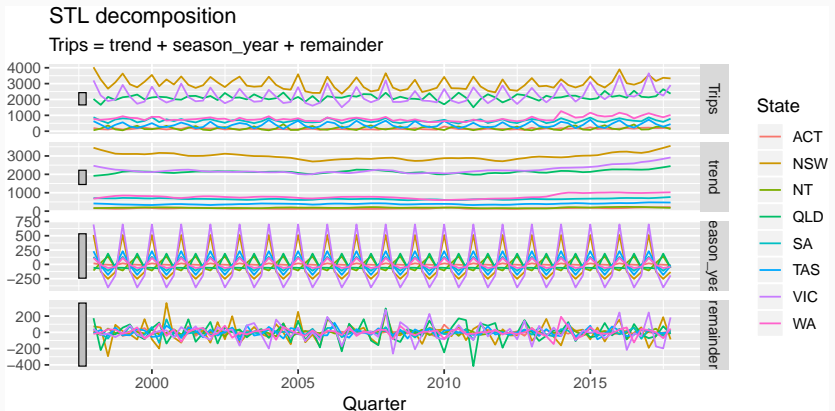
Holidays by state

```
holidays <- tourism %>%  
  filter(Purpose=="Holiday") %>%  
  group_by(State) %>%  
  summarise(Trips = sum(Trips))
```

```
## # A tsibble: 640 x 3 [1Q]  
## # Key:      State [8]  
##   State Quarter Trips  
##   <chr>    <qtr> <dbl>  
## 1 ACT     1998 Q1  196.  
## 2 ACT     1998 Q2  127.  
## 3 ACT     1998 Q3  111.  
## 4 ACT     1998 Q4  170.  
## 5 ACT     1999 Q1  108.  
## 6 ACT     1999 Q2  125.  
## 7 ACT     1999 Q3  178.  
## 8 ACT     1999 Q4  218.  
## 9 ACT     2000 Q1  158.  
## 10 ACT    2000 Q2  155.
```

Decompositions

```
holidays %>% STL(Trips ~ season(window = "periodic")) %>%  
  autoplot()
```



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Time series patterns

Recall

Trend pattern exists when there is a long-term increase or decrease in the data.

Cyclic pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Time series decomposition

$$y_t = f(S_t, T_t, R_t)$$

where y_t = data at period t

T_t = trend-cycle component at period t

S_t = seasonal component at period t

R_t = remainder component at period t

Time series decomposition

$$y_t = f(S_t, T_t, R_t)$$

where y_t = data at period t

T_t = trend-cycle component at period t

S_t = seasonal component at period t

R_t = remainder component at period t

Additive decomposition: $y_t = S_t + T_t + R_t$.

Multiplicative decomposition: $y_t = S_t \times T_t \times R_t$.

Time series decomposition

- Additive model appropriate if magnitude of seasonal fluctuations does not vary with level.
- If seasonal are proportional to level of series, then multiplicative model appropriate.
- Multiplicative decomposition more prevalent with economic series
- Alternative: use a Box-Cox transformation, and then use additive decomposition.
- Logs turn multiplicative relationship into an additive relationship:

$$y_t = S_t \times T_t \times E_t \quad \Rightarrow \quad \log y_t = \log S_t + \log T_t + \log R_t.$$

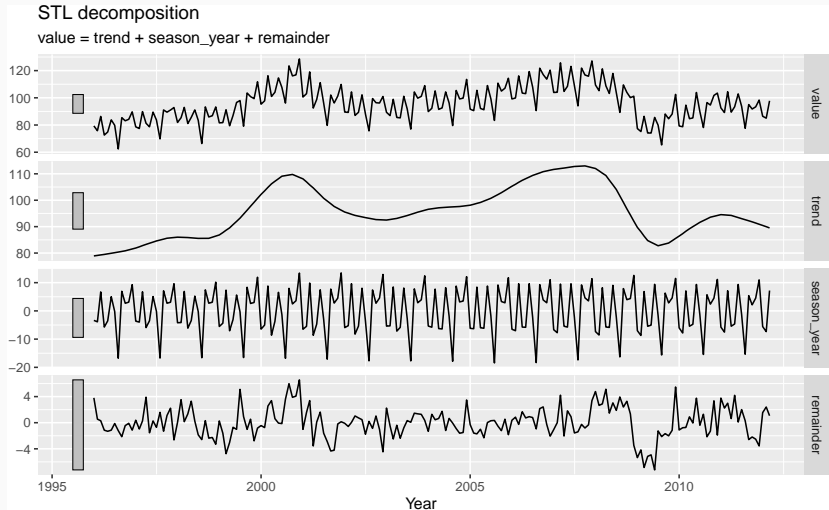
Decomposition dable

```
elecequip <- as_tsibble(fpp2::elecequip)
dcmp <- elecequip %>% STL(value ~ season(window = 7))
dcmp
```

```
## # A dable:                195 x 6 [1M]
## # STL Decomposition: value = trend + season_year +
## #   remainder
##       index value trend season_year remainder season_adjust
##       <mtx> <dbl> <dbl>         <dbl>         <dbl>         <dbl>
## 1 1996 Jan   79.4  78.9      -3.37          3.81          82.7
## 2 1996 Feb   75.8  79.1      -3.87          0.547         79.7
## 3 1996 Mar   86.3  79.3       6.73          0.301         79.6
## 4 1996 Apr   72.6  79.5      -5.74         -1.15          78.3
## 5 1996 May   74.9  79.7      -3.53         -1.31          78.4
## 6 1996 Jun   83.8  79.9       5.03         -1.14          78.8
## 7 1996 Jul   79.8  80.1      -0.222        -0.119         80.0
## 8 1996 Aug   62.4  80.4     -16.8         -1.21          79.2 12
## 9 1996 Sep   85.4  80.6       6.04         -2.15          78.5
```

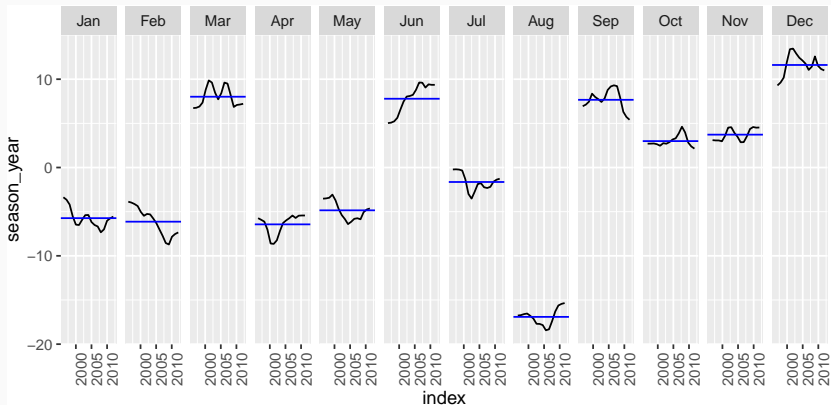
Euro electrical equipment

```
autoplot(dcmp) + xlab("Year")
```



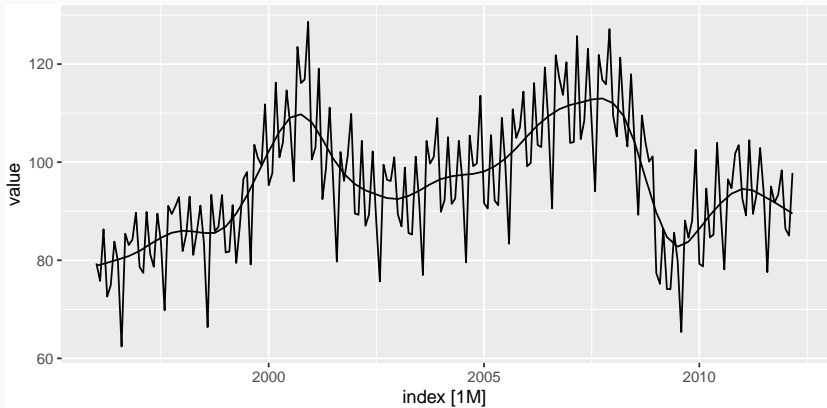
Euro electrical equipment

```
dcmp %>% gg_subseries(season_year)
```



Euro electrical equipment

```
autoplot(elecequip, series="Data") +  
  autolayer(dcmp, trend, series="Trend-cycle")
```



Your turn

Repeat the decomposition using

```
elecequip %>%  
  STL(value ~ season(window=7) + trend(window=11)) %>%  
  autoplot()
```

What happens as you change `season(window = ???)` and `trend(window = ???)`?

Outline

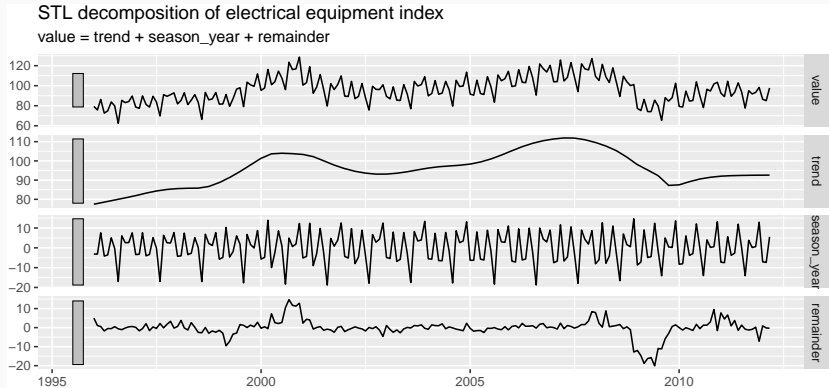
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STL decomposition

- STL: “Seasonal and Trend decomposition using Loess”
- Very versatile and robust.
- Unlike X-12-ARIMA, STL will handle any type of seasonality.
- Seasonal component allowed to change over time, and rate of change controlled by user.
- Smoothness of trend-cycle also controlled by user.
- Robust to outliers
- Not trading day or calendar adjustments.
- Only additive.
- Take logs to get multiplicative decomposition.
- Use Box-Cox transformations to get other decompositions.

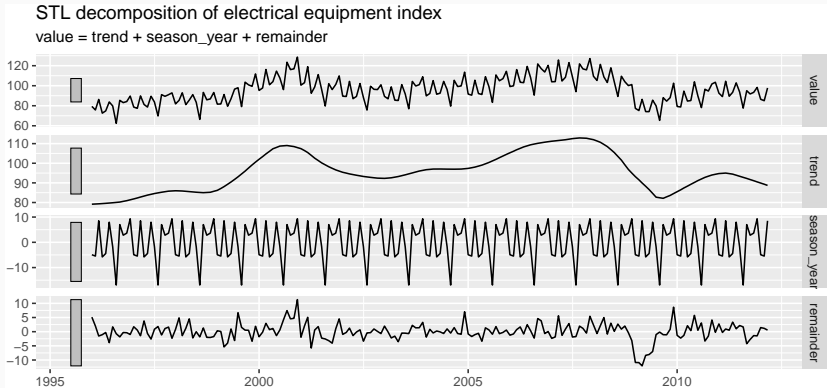
STL decomposition

```
dcmp <- elecequip %>%  
  STL(value ~ season(window = 5), robust = TRUE)  
autoplot(dcmp) +  
  ggtitle("STL decomposition of electrical equipment index")
```



STL decomposition

```
fit <- elecequip %>%  
  STL(value ~ season(window="periodic"), robust=TRUE)  
autoplot(fit) +  
  ggtitle("STL decomposition of electrical equipment index")
```



STL decomposition

```
elecequip %>%
```

```
  STL(value ~ season(window = 5))
```

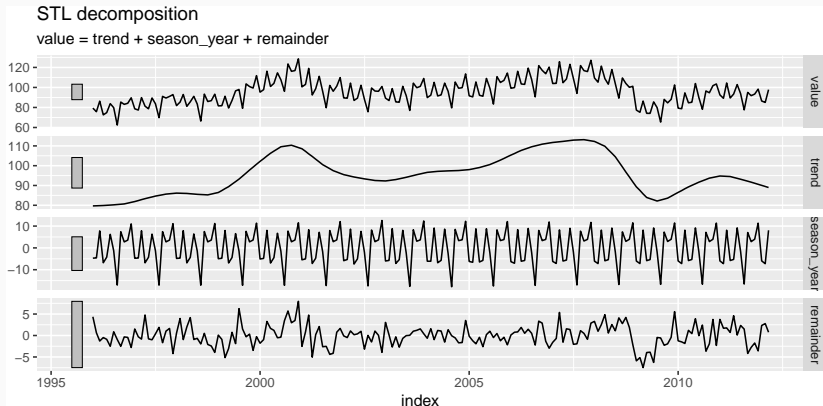
```
elecequip %>%
```

```
  STL(value ~ trend(window=15) + season(window="periodic"),  
        robust = TRUE)
```

- `trend(window = ?)` controls wiggleness of trend component.
- `season(window = ?)` controls variation on seasonal component.

STL decomposition

```
elecequip %>% STL(value) %>% autoplot()
```



- `STL()` chooses `season(window=13)` by default
- Can include transformations. # Features

Feature extraction and statistics

```
tourism %>% features(Trips, feat_stl)
```

```
## # A tibble: 304 x 10
##   Region State Purpose trend_strength seasonal_streng~
##   <chr>   <chr> <chr>          <dbl>          <dbl>
## 1 Adela~ Sout~ Busine~          0.451          0.380
## 2 Adela~ Sout~ Holiday          0.541          0.601
## 3 Adela~ Sout~ Other           0.743          0.189
## 4 Adela~ Sout~ Visiti~          0.433          0.446
## 5 Adela~ Sout~ Busine~          0.453          0.140
## 6 Adela~ Sout~ Holiday          0.512          0.244
## 7 Adela~ Sout~ Other           0.584          0.374
## 8 Adela~ Sout~ Visiti~          0.481          0.228
## 9 Alice~ Nort~ Busine~          0.526          0.224
## 10 Alice~ Nort~ Holiday          0.377          0.827
## # ... with 294 more rows, and 5 more variables:
## #   spikiness <dbl>, linearity <dbl>, curvature <dbl>,
## #   seasonal_peak_year <dbl>, seasonal_trough_year <dbl>
```

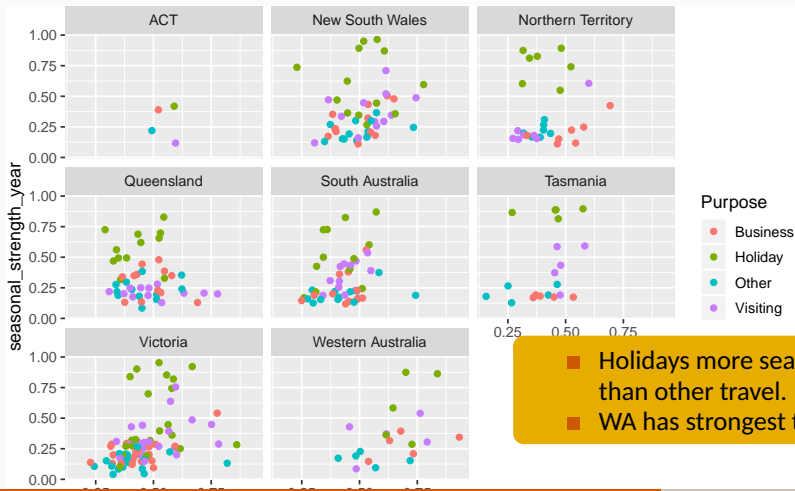

Feature extraction and statistics

```
tourism %>% features(Trips, feat_stl) %>%  
  ggplot(aes(x=trend_strength, y=seasonal_strength_year, col=Purpose)) +  
  geom_point() + facet_wrap(vars(State))
```



Feature extraction and statistics

```
tourism %>% features(Trips, feat_stl) %>%  
  ggplot(aes(x=trend_strength, y=seasonal_strength_year, col=Purpose)) +  
  geom_point() + facet_wrap(vars(State))
```



Feature extraction and statistics

Find the most seasonal time series:

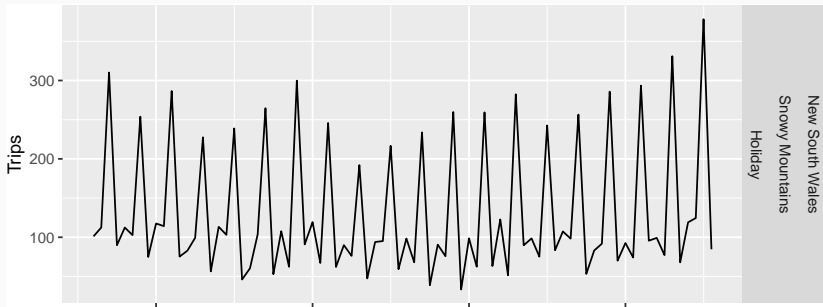
```
most_seasonal <- tourism %>%  
  features(Trips, feat_stl) %>%  
  filter(seasonal_strength_year == max(seasonal_strength_year))
```

Feature extraction and statistics

Find the most seasonal time series:

```
most_seasonal <- tourism %>%  
  features(Trips, feat_stl) %>%  
  filter(seasonal_strength_year == max(seasonal_strength_year))
```

```
tourism %>%  
  right_join(most_seasonal, by = c("State", "Region", "Purpose")) %>%  
  ggplot(aes(x = Quarter, y = Trips)) + geom_line() +  
  facet_grid(vars(State, Region, Purpose))
```



Feature extraction and statistics

```
tourism_features <- tourism %>%  
  features(Trips, feature_set(pkgs="feasts"))
```

All features from
the feasts
package

```
## # A tibble: 304 x 45  
##   Region State Purpose trend_strength seasonal_strength  
##   <chr> <chr> <chr>          <dbl>          <dbl>  
## 1 Adela~ Sout~ Busine~          0.451          0.380  
## 2 Adela~ Sout~ Holiday          0.541          0.601  
## 3 Adela~ Sout~ Other            0.743          0.189  
## 4 Adela~ Sout~ Visiti~          0.433          0.446  
## 5 Adela~ Sout~ Busine~          0.453          0.140  
## 6 Adela~ Sout~ Holiday          0.512          0.244  
## 7 Adela~ Sout~ Other            0.584          0.374  
## 8 Adela~ Sout~ Visiti~          0.481          0.228  
## 9 Alice~ Nort~ Busine~          0.526          0.224  
## 10 Alice~ Nort~ Holiday          0.377          0.827  
## # ... with 294 more rows, and 40 more variables:  
## #   spikiness <dbl>, linearity <dbl>, curvature <dbl>,  
## #   seasonal_peak_year <dbl>, seasonal_trough_year <dbl>,  
## #   acf1 <dbl>, acf10 <dbl>, diff1_acf1 <dbl>,  
## #   diff1_acf10 <dbl>, diff2_acf1 <dbl>, diff2_acf10 <dbl>,  
## #   ...
```

Feature extraction and statistics

```
pcs <- tourism_features %>% select(-State, -Region, -Purpose) %>%  
  prcomp(scale=TRUE) %>% augment(tourism_features)
```

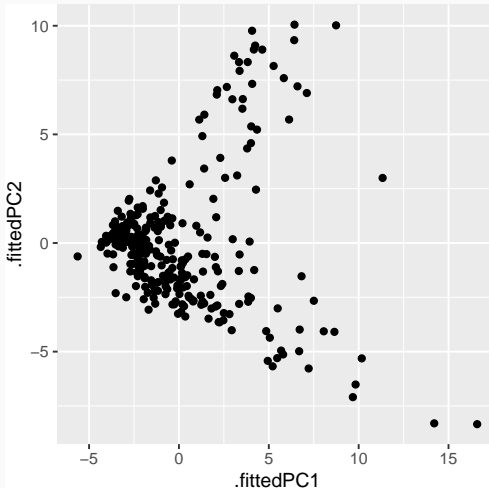
```
## # A tibble: 304 x 88
##   .rownames Region State Purpose trend_strength
##   <fct>      <chr> <chr> <chr>      <dbl>
## 1 1      Adela~ Sout~ Busine~      0.451
## 2 2      Adela~ Sout~ Holiday    0.541
## 3 3      Adela~ Sout~ Other      0.743
## 4 4      Adela~ Sout~ Visiti~    0.433
## 5 5      Adela~ Sout~ Busine~    0.453
## 6 6      Adela~ Sout~ Holiday    0.512
## 7 7      Adela~ Sout~ Other      0.584
## 8 8      Adela~ Sout~ Visiti~    0.481
## 9 9      Alice~ Nort~ Busine~    0.526
## 10 10     Alice~ Nort~ Holiday    0.377
## # ... with 294 more rows, and 83 more variables:
## #   seasonal_strength_year <dbl>, spikiness <dbl>,
## #   linearity <dbl>, curvature <dbl>,
## #   seasonal_peak_year <dbl>, seasonal_trough_year <dbl>,
## #   acf1 <dbl>, acf10 <dbl>, diff1_acf1 <dbl>,
## #   diff2_acf1 <dbl>, diff3_acf1 <dbl>, diff4_acf1 <dbl>,
## #   diff5_acf1 <dbl>, diff6_acf1 <dbl>, diff7_acf1 <dbl>,
## #   diff8_acf1 <dbl>, diff9_acf1 <dbl>, diff10_acf1 <dbl>,
## #   diff11_acf1 <dbl>, diff12_acf1 <dbl>, diff13_acf1 <dbl>,
## #   diff14_acf1 <dbl>, diff15_acf1 <dbl>, diff16_acf1 <dbl>,
## #   diff17_acf1 <dbl>, diff18_acf1 <dbl>, diff19_acf1 <dbl>,
## #   diff20_acf1 <dbl>, diff21_acf1 <dbl>, diff22_acf1 <dbl>,
## #   diff23_acf1 <dbl>, diff24_acf1 <dbl>, diff25_acf1 <dbl>,
## #   diff26_acf1 <dbl>, diff27_acf1 <dbl>, diff28_acf1 <dbl>,
## #   diff29_acf1 <dbl>, diff30_acf1 <dbl>, diff31_acf1 <dbl>,
## #   diff32_acf1 <dbl>, diff33_acf1 <dbl>, diff34_acf1 <dbl>,
## #   diff35_acf1 <dbl>, diff36_acf1 <dbl>, diff37_acf1 <dbl>,
## #   diff38_acf1 <dbl>, diff39_acf1 <dbl>, diff40_acf1 <dbl>,
## #   diff41_acf1 <dbl>, diff42_acf1 <dbl>, diff43_acf1 <dbl>,
## #   diff44_acf1 <dbl>, diff45_acf1 <dbl>, diff46_acf1 <dbl>,
## #   diff47_acf1 <dbl>, diff48_acf1 <dbl>, diff49_acf1 <dbl>,
## #   diff50_acf1 <dbl>, diff51_acf1 <dbl>, diff52_acf1 <dbl>,
## #   diff53_acf1 <dbl>, diff54_acf1 <dbl>, diff55_acf1 <dbl>,
## #   diff56_acf1 <dbl>, diff57_acf1 <dbl>, diff58_acf1 <dbl>,
## #   diff59_acf1 <dbl>, diff60_acf1 <dbl>, diff61_acf1 <dbl>,
## #   diff62_acf1 <dbl>, diff63_acf1 <dbl>, diff64_acf1 <dbl>,
## #   diff65_acf1 <dbl>, diff66_acf1 <dbl>, diff67_acf1 <dbl>,
## #   diff68_acf1 <dbl>, diff69_acf1 <dbl>, diff70_acf1 <dbl>,
## #   diff71_acf1 <dbl>, diff72_acf1 <dbl>, diff73_acf1 <dbl>,
## #   diff74_acf1 <dbl>, diff75_acf1 <dbl>, diff76_acf1 <dbl>,
## #   diff77_acf1 <dbl>, diff78_acf1 <dbl>, diff79_acf1 <dbl>,
## #   diff80_acf1 <dbl>, diff81_acf1 <dbl>, diff82_acf1 <dbl>
```

Principal components based on all features from the feasts package

Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2)) +  
  geom_point() + theme(aspect.ratio=1)
```

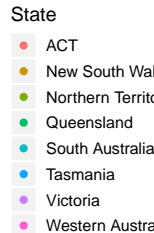
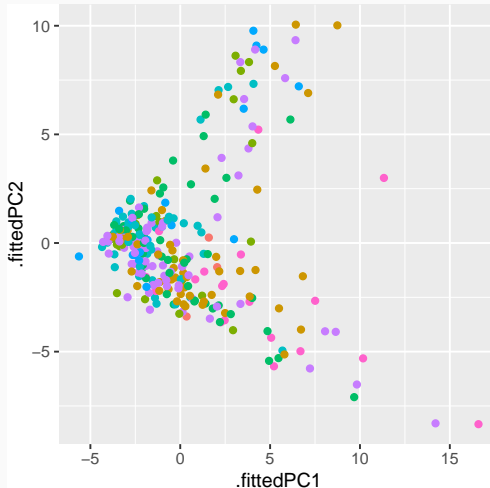
Principal components
based on all features
from the feasts
package



Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=State)) +  
  geom_point() + theme(aspect.ratio=1)
```

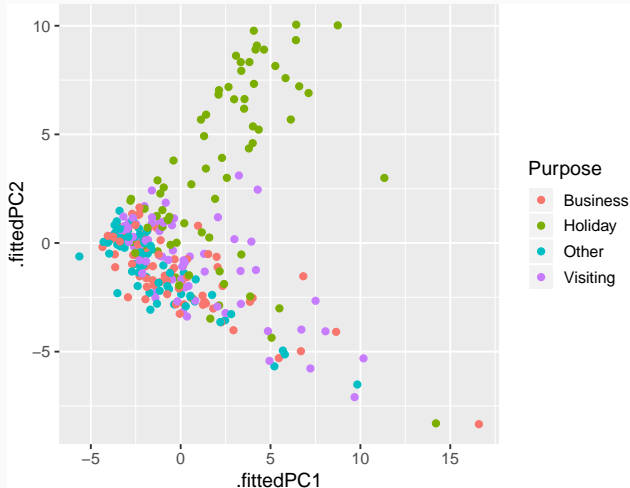
Principal components
based on all features
from the feasts
package



Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +  
  geom_point() + theme(aspect.ratio=1)
```

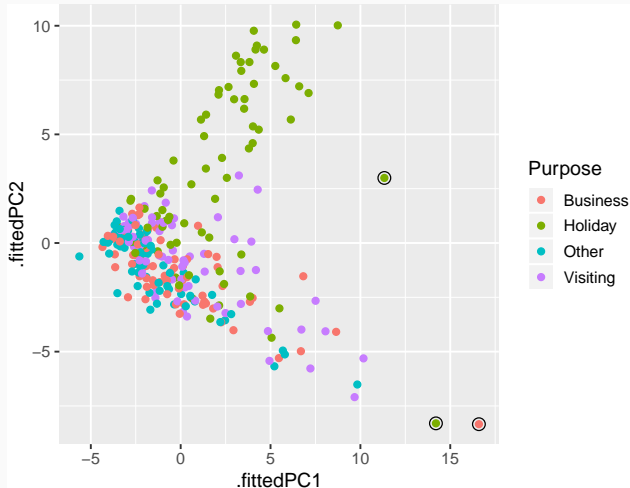
Principal components
based on all features
from the feasts
package



Feature extraction and statistics

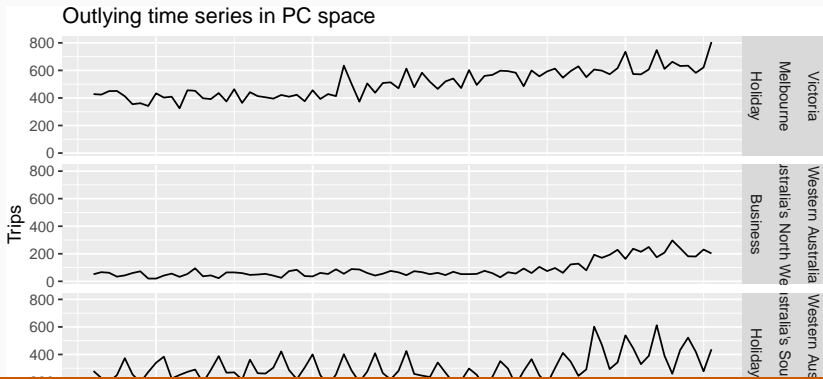
```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +  
  geom_point() + theme(aspect.ratio=1)
```

Principal components
based on all features
from the feasts
package



Feature extraction and statistics

```
outliers %>%  
  left_join(tourism, by = c("State", "Region", "Purpose")) %>%  
  ggplot(aes(x = Quarter, y = Trips)) +  
    geom_line() +  
    facet_grid(vars(State, Region, Purpose)) +  
    ggtitle("Outlying time series in PC space") +  
    theme(legend.position = "none")
```



Acknowledgements



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Earo Wang

feasts.tidyverts.org
robjhyndman.com