My Results *for* 50S17Quiz14

Done

Quiz still open - You cannot take this quiz again

- You can view your submission (all questions and your answers will be shown)
- Your incorrect answers are marked and feedback will be shown, if available
- Your correct answers are marked and feedback will be shown, if available

Submission (Tuesday, May 23, 2017 at 11:06am)

Key: C Correct/full credit P Partial credit I Incorrect/no credit Pending score Unscored/unreleased

1. Consider the causal LTI system whose impulse response has the Z-transform shown below.

   Find its Region of Convergence.

$$H(z) = \frac{(z-1)(z-2)}{(z-0.5)(z+0.9)}$$

A causal system has a ROC equal to
|z| > |p_far|
where p_far is the pole farthest from the origin.
In this problem, the poles are at 0.5 and -0.9.
The farthest pole is -0.9.
So the ROC is |z|>0.9.

z greater than - 0.9

- |z| greater than 0.5
- (C) (•) |z| greater than 0.9
- |z| greater than or equal to 0.5
- |z| greater than or equal to 0.9

2. Find the difference equation that describes the LTI system of Problem 1.

> Y(z) = H(z) X(z) = (z-1)(z-2)/((z-0.5)(z+0.9))X(z)
> So,
> (z-0.5)(z+0.9) Y(z) = (z-1)(z-2)X(z)
> (z^2 + 0.4z -0.45) Y(z) = (z^2-3z +2) X(z)
> Going to the time domain
> y[n+2] + 0.4 y[n+1] -0.45 y[n] = x[n+2] -3x[n+1] +2x[n]
>
> Replacing "n" with "n-2" everywhere
>
> y[n] +0.4y[n-1] - 0.45 y[n-2] = x[n] - 3x[n-1] +2 x[n-2]
>
> Equivalently,
>
> y[n] = x[n]-3x[n-1]+2x[n-2]-0.4y[n-1]+0.45y[n-2]

- y[n] = x[n] - 3x[n-1] + 2x[n-2] + 0.4 y[n-1] -0.45 y[n-2]
- y[n] = x[n+2] - 3x[n+1] + 2x[n] - 0.4 y[n+1] +0.45 y[n]
- (C) (•) y[n] = x[n] - 3x[n-1] + 2x[n-2] - 0.4 y[n-1] +0.45 y[n-2]
- none of the above

3. Is the causal LTI system of Problem 1 also a stable system?

> The region of convergence |z|>0.9
> includes the unit circle, |z|=1.
> So the system is stable.

- (C) (•) Yes
- No

4. For the LTI system of Problem 1, evaluate the following:

$$\sum_{n=-\infty}^{\infty} h[n]$$

Note that if we replace z=1 in the definition of z-transform,
then H(1) = sum_{-infinity to infinity} h[n].

Note that z=1 is in the ROC.
So we can plug in z=1 in the definition of H(z) that is given for Problem 1.

Since H(z) has a zero at z=1, the answer is H(1)=0.

- 1
- -1
- C 0
- none of the above

5. For the LTI system of Problem 1, evaluate the following:
(Hint: Similar to previous problem.)

$$\sum_{n=-\infty}^{\infty} \frac{h[n]}{2^n}$$

From the definition
H(z) = sum_{-infinity to infinity} h[n]z^(-n)
if we replace z=2, we have
sum_{-infinity to infinity}h[n]2^(-n)

Note that z=2 is in the ROC, so we
can compute H(2) from the expression
for H(z) provided in Problem 1.
Since the system has a zero at z=2,
H(2) = 0 is the answer.

-1

1

none of the above

6. Find the z-transform of the signal

$$h[n] \quad = \quad (3+4j)^n u[n]$$

Use the standard transform pair:

a^n u[n] -----> z/(z-a), with |z|>|a|

set a= 3+4j
and note that |a| = 5.

- H(z) = z/(z-3-4j), |z|>3
- (C) H(z) = z/(z-3-4j), |z|>5
- H(z) = z/(z-3-4j), z>3+4j
- none of the above

7. Find the z-transform of

$$h[n] \quad = \quad 2^{n+1} u[n-2]$$

h[n] = 2^(n+1)u[n-2] is a right sided exponential signal.
We want to use the known transform of 2^n u[n].

2^n u[n] has the transform z/(z-2)

Useful fact from lecture: Delay the signal by 2, the z-transform gets multiplied by z^(-2). So,

2^(n-2) u[n-2] has the transform (z^(-1))/(z-2)=1/(z(z-2))

Now scale the signal by 8 = 2^3, the z-transform gets scaled by the same factor. So, ROC is |z|>2, because it is a right sided signal.

- z/(z-2) + 2z, |z|>2
- 2 z^(-1)/(z-2), |z|>2
- C 8/(z(z-2)), |z|>2
- none of the above

8. Find the right-sided signal x[n] which has z-transform

$$X(z) = \frac{z^2+1}{z-1}$$

X(z) = z+1 +2/(z-1)
Now, let us find the inverse transform of each term
z has inverse transform delta[n+1]
1 has inverse transform delta[n]
All that is left is to find the inverse transform of 2/(z-1)

u[n] has transform z/(z-1)
So u[n-1] has transform z^(-1) z/(z-1) = 1/(z-1)
So 2 u[n-1] has transform 2/(z-1)

Putting it all together:
x[n] = delta[n+1]+delta[n]+2u[n-1]

- x[n] = delta[n]+2u[n-1]
- x[n] = 2u[n-1]
- C x[n] = delta[n+1]+delta[n]+2u[n-1]
- none of the above

9. Find the z-transform of the signal h[n] shown below.

$$h[n] = u[-n-2]$$

-a^n u[-n-1] has the transform z/(z-a), |z| less than 1.

Set a=1,

- u[-n-1] has the transform z/(z-1), |z| less than 1.

Scale by -1,

u[-n-1] has the transform z/(1-z), |z| less than 1.

Shift left by 1, i.e., n maps to n+1, which multiplies the transform by z (useful fact from lecture).

u[-n-2] has the transform z^2/(1-z), , |z| less than 1.

- H(z) = 1/(z-1), |z| is less than 1
- (C) H(z) = (z^2)/(1-z), |z| is less than 1
- H(z) = 1/(z(1-z)), |z| is less than 1
- none of the above

10. Using long division, find the first three non-zero terms of the causal signal h[n]

that has the z-transform H(z) shown below.

$$H(z) = \frac{z+2}{z^2 - 2z + 1}$$

Long division gives us
z^(-1) + 4z^(-2) + 7z^(-3) + ...

By the definition of the z-transform, the coefficient of z^(-n) is h[n].

Therefore, the coefficients of first 3 terms correspond to
h[1]=1, h[2]=4, and h[3]=7

- h[0]=1, h[1]=4, h[2]=7
- (C) h[1]=1, h[2]=4, h[3]=7
- h[-3]=7, h[-2]=4, h[-1]=1

11. A stable LTI system has impulse response h[n] which has the z-transform H(z) shown below.

Find the impulse response h[n].

$$H(z) = \frac{2z-5}{(z-2)(z-3)}$$

By partial fractions,
H(z) = 1/(z-2) + 1/(z-3)
Poles are at z=2, 3.

System is stable, so unit circle is included in the ROC.
Since the ROC is inside each of the two poles, each term represents a left-sided signal.

Now,
z/(z-2) is the transform of the left-sided signal -2^n u[-n-1].

Useful fact from lecture 12: Multiplying by z^(-1) produces a time shift (n --> n-1). So,

1/(z-2) is the transform of the left-sided signal -2^(n-1)u[-n].

Similarly, 1/(z-3) is the transform of the left-sided signal -3^(n-1)u[-n].

- h[n] = -2^n u[-n-1] -3^n u[-n-1]
- (C) h[n] = - 2^(n - 1)u[- n] - 3^(n - 1)u[- n]
- h[n] = - 2^(n - 1)u[- n - 2] - 3^(n - 1)u[- n - 2]
- none of the above

12. Consider the causal LTI system that has impulse response h[n] which has the z-transform H(z) shown below.

Find the output y[n] of this LTI system if the input is

x[n] = 5^n + 6^n, for all n.

$$H(z) = \frac{z-1}{z^2-5z+6}$$

5^n+6^n produces the output
H(5)5^n + H(6)6^n

The poles are at z=2, 3. So the ROC is |z|>3.

H(5) = 2/3, by plugging in z=5.
H(6) = 5/12, by plugging in z=6.

So the answer is

(2/3) 5^n + (5/12) 6^n

- y[n] = 5^n + 6^n
- y[n] = (2/3) 5^n + (5/12) 6^n
- y[n] = 5(2^n) u[n] +6(3^n) u[n]
- none of the above

13. Find the output of the same causal LTI system as in the previous problem.

to the input x[n] = (0.5)^n for all n.

2^(-n) is the same as 0.5^n.
The response to 0.5 should be H(0.5)(0.5^n)
However, H(0.5) is not defined
because the region of convergence of H(z)
is |z|>2. So the output is not defined.

- y[n] = - (2/15) 2^{ - n}
- output is not defined.
- y[n] = 0 for all n
- y[n] = 0.5^n

14. If $h[n]$ has z-transform $H(z) = e^{1/z}$, then find $h[3]$.
(Hint: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$)

By the definition of the z-transform, the coefficient of z^(-n)
is h[n].

So, the coefficient of z^(-3) is h[3].

From the expansion of the exponential function, this coefficient is
1/3! = 1/6.

- h[3] = 1/6
- h[3] = e^(1/3)

none of the above

15. If an LTI system is both causal and stable, then all its poles must lie inside the unit circle.

For a stable LTI system, the unit circle must be in the ROC.
For a causal LTI system, the ROC is outside the outermost pole.
For this ROC to contain the unit circle, clearly the outermost pole must be inside the unit circle.

Thus, all poles must be in the unit circle for a causal and stable LTI system.

C True

False

Done

Return to Top