1. A continuous time sinusoidal signal $x(t)$ is sampled at the rate $f_s = 10$ samples/second to produce the discrete-time signal $x[n] = 10\cos\left(\frac{22\pi n}{5} + \frac{\pi}{7}\right)$.

   (a) (3 pts) What are the possible signals $x(t)$ that could produce this $x[n]$.
   Sol: Sampling
   $$x(t) = 10\cos(2\pi f t + \pi/7)$$
   at $f_s = 10$ samples/sec produces
   $$x[n] = 10\cos(2\pi(f/10)n + \pi/7) = 10\cos(22\pi n/5 + \pi/7)$$
   $\Rightarrow f = 22$ Hz. But all values $f' = f + kf_s = 22 + 10k$ for integer k, produce the same $x[n]$ as well. So the possible signals are
   $$x(t) = 10\cos(2\pi(22 + 10k)t + \pi/7)$$

   (b) (3 pts) Is $x[n]$ a periodic signal? If so, then find its fundamental period.
   Sol: Yes, $x[n]$ is periodic because $\Omega = 22\pi/5$ is a rational multiple of π. For this signal
   $$F = \Omega/2\pi = 11/5$$
   is a rational number. The fundamental period
   $$N_o = k/F = 5k/11$$
   where k is the smallest positive integer value which makes $5k/11$ an integer. In this case, that would be $k = 11$ and therefore $N_o = 5$ samples.

   (c) (3 pts) Find $F \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ such that $10\cos\left(2\pi F n + \frac{\pi}{7}\right) = 10\cos\left(\frac{22\pi n}{5} + \frac{\pi}{7}\right)$.
   Sol: Adding any integer value to F does not change the discrete time sinusoid. So the equivalent values of F are $11/5 + k$. Choosing $k = -2$, we have $11/5 - 2 = 1/5$ which lies in the interval $(-1/2, 1/2)$.

   (d) (3 pts) Assuming the Nyquist criterion was satisfied find $x(t)$.
   Sol: From the possible values $f' = f + kf_s = 22 + 10k$ the only value, $\bar{f}$, that satisfies the Nyquist criterion, i.e., lies in the range $(-f_s/2, f_s/2) = (-5, 5)$ Hz is when $k = -2$, i.e., $\bar{f} = 22 - 20 = 2$. So the $x(t)$ that satisfies the Nyquist criterion is
   $$x(t) = 10\cos(4\pi t + \pi/7)$$

   (e) (2 pt) What is the Nyquist sampling rate for the $x(t)$ found in part (d)?
   Sol: The $x(t)$ found in part (d) is a sinusoid of frequency 2 Hz. So the Nyquist rate is 4 Hz, i.e., any sampling frequency higher than 4 samples/second will avoid aliasing.

   (f) (3 pts) If the same $x(t)$ was sampled at the rate of $f'_s = 1.5$ samples/sec, to produce the discrete time signal $x'[n]$, find $x'[n]$.
   Sol: Sampling $x(t) = 10\cos(4\pi t + \pi/7)$ at $f'_s = 1.5$ samples/sec, produces the discrete time signal
   $$x'[n] = 10\cos(2\pi(2/1.5)n + \pi/7) = 10\cos(8\pi n/3 + \pi/7)$$

   (g) (3 pts) Will aliasing occur in recovering $x(t)$ from $x'[n]$? If so, then find the aliased frequency.
   Sol: Yes, because f'_s does not satisfy the Nyquist criterion, there will be aliasing. All frequencies $f' = f + kf'_s = 2 + 1.5k$ produce equivalent discrete time signals, and the reconstructed signal will choose the value that lies in the interval $(-f'_s/2, f'_s/2) = (-0.75, 0.75)$. The only value of $2 + 1.5k$ that lies in this interval is obtained when $k = -1$, which gives us $2 - 1.5 = 0.5$ Hz. This is the frequency of the reconstructed signal, i.e., the aliased frequency.