Thursday, October 18, 2018 3:45 PM

To prove: $(\exists x)(I(x) \wedge \neg R(x))$

Given $(\forall x)\, R(x) \supset L(x)$ whoever can read is literate $\Rightarrow$ $\underline{\neg R(x) \vee L(x)}$

$(\forall x)\, D(x) \supset \neg L(x)$ Dolphins are not literate $\Rightarrow$ $\underline{\neg D(y) \vee \neg L(y)}$

$(\exists x)(D(x) \wedge I(x))$ Some dolphins are intelligent $\Rightarrow$ $\left\{ \begin{array}{l} D(A) \\ I(A) \end{array} \right.$

$\Downarrow$

$D(A) \wedge I(A)$

Gets rid of $(\exists x)$
A is random variable

Recall
$p \supset q$
$\equiv$
$\neg p \vee q$

Negate

$\neg(\exists x)(I(x) \wedge \neg R(x)) \equiv (\forall x)\, \neg I(x) \vee R(x))$

$\equiv$

$\cancel{\forall x}\; \underline{(\neg I(z) \vee R(z))}$

Resolving

$(\neg I(\overset{A}{\cancel{z}}) \vee R(\overset{A}{\cancel{z}}))$ $\quad I(A)$

substitution $\{A/z\}$
↙ instantiating?

$R(A)$ $\quad \neg R(\overset{A}{\cancel{x}}) \vee L(\overset{A}{\cancel{x}})$ $\quad \{A/x\}$

$L(A)$ $\quad \neg D(\overset{A}{\cancel{y}}) \vee \neg L(\overset{A}{\cancel{y}})$ $\quad \{A/y\}$

$\neg D(A)$ $\quad D(A)$ contradiction

NIL $\underline{\text{QED}}$

$$\forall x \; \{P(x) \supset \{(\forall y)\, [P(y) \supset P(f(x, y))] \wedge \neg(\forall y)(Q(x,y) \supset P(y))\}\}$$

Step 1: eliminate '$\supset$'s

$$(\forall x)\{\neg P(x) \vee \{(\forall y\, [\neg P(y) \vee P(f(x, y))] \wedge \neg(\forall y)(\neg Q(x,y) \vee P(y)]\}\}$$

scope not valid

Step 2: Reduce the scope of negation so each negation symbol applies to at most one atomic formula
↳ a predicate

$$(\exists y)\, [Q(x,y) \wedge \neg P(y)]$$

Step 3: standardize variables so that each quantifier has its own variable

Quantifier as in: Existential or universal

$$(\forall x)\{\neg P(x) \vee \{(\forall y)\, [\neg P(y) \vee P(f(x,y))] \wedge (\exists w)(Q(x, w) \wedge \neg P(w)\}\}$$

$g(x)$

Step 4: Skolemization $\rightarrow$ Gets rid of Existential quantifiers

Step 4: Skolemization → Gets rid of Existential quantifiers

$g(x)$ $y(x)$

Step 5: Convert to prenex form:

$$\underbrace{(\forall x)(\forall y)}_{\text{prefix}} \underbrace{\{\neg P(x) \vee \{[\neg P(y) \vee P(f(x,y))] \wedge [Q(x, g(x)) \wedge \neg P(g(x))]\}\}}_{\text{matrix}}$$

Step 6: Convert to matrix → CNF

$$\cancel{\forall x}\, \cancel{\forall y} \{ [\neg P(x) \vee \neg P(y) \vee P(f(x,y))] \wedge$$
$$[\neg P(x) \vee Q(x, g(x))] \wedge [\neg P(x) \vee \neg P(g(x))] \}$$

Step 7: Drop universal quantifiers

Step 8: Eliminate $\wedge$'s

$$\{ \boxed{[\neg P(x_1) \vee \neg P(y_1) \vee P(f(x_1,y_1))]} \wedge$$
$$\boxed{[\neg P(x_2) \vee Q(x_2, g(x_2))]} \wedge \boxed{[\neg P(x_3) \vee \neg P(g(x_3))]} \}$$

Step 9: Rename variables so each clause has its own variables.

Given $(\forall x)(At(John, x) \supset At(Fido, x))$. Fido is wherever John is

$At(John, school)$ ↳ $\neg At(John, x) \vee At(Fido, x)$

Prove: $(\exists x)\ At(Fido, x) \Rightarrow \neg At(Fido, x)$ $\quad \neg At(John, \cancel{y}^{x}) \vee At(Fido, \cancel{x}^{y})$

$\{x/y\}$

$\neg At(John, x)$ $\quad At(John, school)$

$\{school/x\}$

Ans(school) $\quad$ NIL