MIDTERM EXAM

NAME:

STUDENT ID:

- Please write down your NAME and Student ID#
- Please sign the attendee list when you return your answers
  Please, sign the honor code: **"In accordance with the honor code. I didn't receive or given any aid during this exam."**
- Duration of the exam is 80 minutes

ASSUMTIONS, CONSTANTS and FORMULAS

Assumptions:

- Unless opposite is stated, assume room temperature operation
- Effective mass ($m^*_{n,p}$) values of electron and holes are independent of temperature
- Between 200K and 500K electron and hole mobility for silicon can be calculated by using following formula
  $\mu_n = 3000 * e^{-(T-200)/125}$ cm²/V-Sec $\mu_p = 1000 * e^{-(T-200)/150}$ cm²/V-Sec

Constants

- $q = 1.6 \times 10^{-19}$ coul, $k = 8.617 \times 10^{-5}$ eV/K, $h = 6.63 \times 10^{-34}$ Joule-sec, $m_0 = 9.11 \times 10^{-34}$ kg, $m^*_n = 1.18 m_0$, $m^*_p = 0.81 m_0$, $E_G = 1.12$ eV for silicon.

Formulas

Carrier Drift: $\varepsilon = electric\ \ field\ (V/m)$

$$J_{n(p)\ drift} = q\mu_{n(p)} \cdot n(or\ p) \cdot \varepsilon$$

$$\rho = \frac{1}{q(\mu_n . n + \mu_p . p)}$$

$$\varepsilon = -\nabla V$$

$$\varepsilon = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{d}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

Diffusion

$$J_{n\ diff} = qD_n \cdot \nabla n \quad J_{p\ diff} = -qD_p \cdot \nabla p$$

Equilibrium Condition

$$J_{n\ drift} + J_{n\ diff} = 0 \rightarrow dn/dx = -(q/KT).n.\varepsilon$$

$$\frac{D_N}{\mu_n} = \frac{KT}{q}$$ Einsteins's relationship

(same applies to holes)

Continuity Equations

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla J_N + \left.\frac{\partial n}{\partial t}\right|_{Thermal\ R-G} + \left.\frac{\partial n}{\partial t}\right|_{other\ processes}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla J_p + \left.\frac{\partial p}{\partial t}\right|_{Thermal\ R-G} + \left.\frac{\partial p}{\partial t}\right|_{other\ processes}$$

Minority carrier diffusion equation in 1D

$$\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

$$\frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

General solutions to simple differential equations

$$a.\frac{d^2 y}{dx^2} - \frac{y}{\tau} = 0 \rightarrow y = A.e^{-x/L} + B.e^{x/L} \quad Where\ L = \sqrt{a.\tau}$$

$$\frac{dy}{dx} + \frac{y}{\tau} = 0 \rightarrow y = y(0).e^{-x/\tau}$$

$$a.\frac{d^2 y}{dx^2} = 0 \rightarrow y = A + B.x$$

$CH-2$

$$N_C = 2\left[\frac{m_n^* KT}{2\pi\hbar^2}\right]^{3/2} \quad N_V = 2\left[\frac{m_p^* KT}{2\pi\hbar^2}\right]^{3/2}$$

$$n = N_C.e^{(E_F - E_C)/KT} = n_i.e^{(E_F - E_i)/KT}$$

$$p = N_V.e^{(E_V - E_F)/KT} = n_i.e^{(E_i - E_F)/KT} \qquad n.p = n_i^2$$

Charge Neutrality and Fermi level

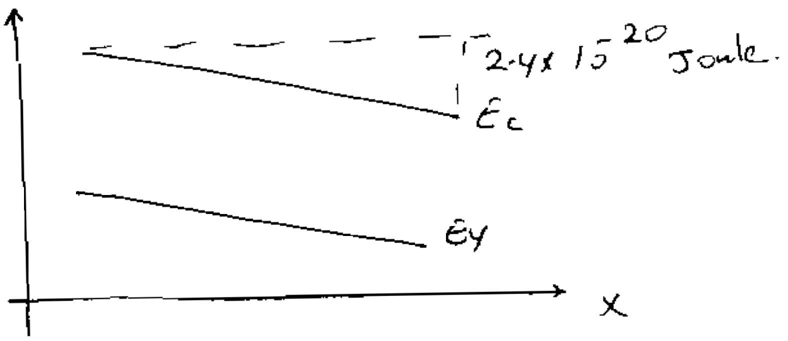
$$p - n + N_D - N_A = 0,$$

$$E_F = \frac{E_C + E_V}{2} + \frac{1}{2}KT\ln\left(\frac{N_V}{N_C}\right) + KT\ln\left(\frac{n}{n_i}\right)$$

MIDTERM SOLUTIONS:

10pt

(1)



$2.4\times10^{-20}\,\text{Joule} = 0.15\,\text{eV}$

$\frac{E(\text{Joule})}{q} = E(\text{eV})$

3pt a) $\mathcal{E} = \frac{1}{q}\frac{dE_c}{dx} = \frac{-0.15\,V}{3\times10^{-4}\,cm} = -500\,V/cm$

2pt b) $F = q\mathcal{E} \Rightarrow$ for electron $q = -1.6\times10^{-19}$

holes $q = 1.6\times10^{-19}$

$\mathcal{E} = -500V$

$\Rightarrow$ Force is (+) for electron towards +x direction.

and (−) for holes. towards +x direction

Electrons will accelerate towards +x direction.

2pt c) $n = n_i\, e^{(E_F - E_i)/kT}$

also from einstein's relationship.

$\frac{dn}{dx} = -\frac{q}{kT} n \mathcal{E}$ $\qquad \mathcal{E} =$ Constant

$\Rightarrow \frac{dn}{n} = -\frac{q}{kT}\mathcal{E}\,dx \Rightarrow \boxed{n(x) = n(0)\, e^{-\frac{q}{kT}\mathcal{E}x}}$

q. $P = \frac{n_i^2}{n}$

$\Rightarrow \rho = \frac{n_i^2}{n(0)} e^{\frac{q}{kT} \mathcal{E} x}$

3(d) $n = n_i e^{(E_F - E_i)/kT} = n(0) e^{\frac{-q}{kT} \mathcal{E} x}$

$\Rightarrow e^{(E_F - E_i)/kT} = \frac{n(0)}{n_i} e^{\frac{-q}{kT} \mathcal{E} x}$

take log of both sides.

$\frac{E_F - E_i}{kT} = \frac{-q}{kT} \mathcal{E} x + \ln\left(\frac{n(0)}{n_i}\right)$

$$\boxed{E_F - E_i = -q \mathcal{E} x + kT \ln\left(\frac{n(0)}{n_i}\right)}$$

$\frac{dE_F}{dx} = 0$ Always.

$E_i \simeq E_c - \frac{E_G}{2}$

(2) 20pt

(3)

5pt a) General form of diffusion equation.

$$\frac{\partial \Delta P_n}{\partial t} = D_p \frac{\partial^2 \Delta P_n}{\partial x^2} - \frac{\Delta P_n}{\tau_p} + G_L$$

However there is no electron-hole generation due to light inside the semiconductor

So $G_L = 0$ (everywhere except $\cancel{x}=0$, and $\cancel{x} = 50\mu m$)

5pt b) At steady state

$$\frac{\partial \Delta P_n}{\partial t} = 0.$$

$$\Rightarrow \boxed{D_p \frac{\partial^2 \Delta P_n}{\partial x^2} - \frac{\Delta P_n}{\tau_p} = 0}$$

General form of solution is

$$\frac{\Delta P_n(x)}{\cancel{\partial x}} = A\, e^{-x/L_p} + B e^{x/L_p}.$$

where $L_p = \sqrt{D_p \tau_p}$
$\approx$ ~~3μm~~ 34.6μm

For left side excitation

$$\Delta P_n'(x) = A\, e^{-x/L_p}$$

For right side excitation.

$$\Delta P_n''(x) = B\, e^{-(L-x)/L_p}$$

General solution.

$$\Delta P_n(x) = A e^{-x/L_p} + B e^{-(L-x)/L_p}$$

$L = 50\mu m$

Boundary values

$\Delta P_n(0) = 10^9\ cm^{-3}$

$\Delta P_n(\underbrace{50\mu m}_{L}) = 10^9\ cm^{-3}$

$\Rightarrow P_0 = \Delta P_n(0) = A + B e^{-L/L_p}$

$P_0 = \Delta P_n(0) = A e^{-L/L_p} + B$

$P_0(1 - e^{-L/L_p}) = B(1 - e^{-2L/L_p})$

$\Rightarrow \boxed{B = \frac{1 - e^{-L/L_p}}{1 - e^{-2L/L_p}} P_0}$

$P_0(1 - e^{-L/L_p}) = A(1 - e^{-2L/L_p})$

$\boxed{A = P_0 \frac{1 - e^{-L/L_p}}{1 - e^{-2L/L_p}}}$

6pt

c) $J_p = ?$ $\quad J_p = -q D_p \nabla P$

$\Rightarrow J_p = -q D_p \frac{\partial \Delta P}{\partial x} = -q D_p \left\{ \frac{-A}{L_p} e^{-x/L_p} + \frac{B}{L_p} e^{-(L-x)/L_p} \right\}$

at $x = 0$

$J_p = -q D_p \left(\frac{-A}{L_p} + \frac{B}{L_p} e^{-L/L_p} \right)$

4pt

d) $\Delta P_n(x) = A e^{-x/L_p} + B e^{-(L-x)/L_p}$

$10cm \gg L_p \Rightarrow \Delta P_n(5cm) \underline{\underline{\simeq 0}}$

everything will recombine before reaching mid point

③ ~~1pt~~ 25pt

$N_D = 10^{12}\ cm^{-3}$ $\quad N_A = 6\times10^{12}\ cm^{-3}$.

$n_i = 2\times10^{12}$

From charge neutrality.

$$p - n + N_D - N_A = 0$$

and

$$p \cdot n = n_i^2$$

8pt.

$$\Rightarrow \begin{cases} n = \dfrac{N_D - N_A}{2} + \left[\left(\dfrac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2} \\ p = \dfrac{n_i^2}{n} = \dfrac{N_A - N_D}{2} + \left[\left(\dfrac{N_A - N_D}{2}\right)^2 + n_i^2\right]^{1/2} \end{cases}$$

10pt

$$\Rightarrow n = \frac{-5\times10^{12}}{2} + \left[\left(\frac{5\times10^{12}}{2}\right)^2 + 4\times10^{24}\right]^{1/2}$$

$$= \frac{-5\times10^{12}}{2} + \left[12.5\times10^{24} + 4\times10^{24}\right]^{1/2}$$

7pt

[redacted] ~~5×10^{12}~~ cm^{-3}

$n = 7.0156\times10^{11}\ cm^{-3}$

$p = 5.702\times10^{12}\ cm^{-3}$

~~4~~ 25 pt

(6)

(4) $N_D = 10^{14}\, cm^{-3}$

3pt

$$E_F = \frac{E_C+E_V}{2} + \underbrace{\frac{1}{2} kT \ln\left(\frac{N_V}{N_C}\right)}_{\text{negligible at room temperature}} + kT \ln\left(\frac{n}{n_i}\right)$$

$n \simeq N_D = 10^{14}\, cm^{-3}$

$n_i = 10^{10}\, cm^{-3}$

$KT = 0.026\, eV.$

5pt

$$E_F = \frac{E_C+E_V}{2} + \underbrace{0.026 \times \ln\left(\frac{10^{14}}{10^{10}}\right)}_{\sim 0.24} = \frac{E_C+E_V}{2} + 0.24\, eV$$

5pt

$$N_C = 2\cdot\left[\frac{m_n^* kT}{2\pi\hbar^2}\right]^{3/2} \qquad N_V = 2\cdot\left[\frac{m_p^* kT}{2\pi\hbar^2}\right]^{3/2}$$

all values are given

5pt

matching intrinsic fermi level to $\frac{E_C+E_V}{2} + 0.24\, eV.$

$n = n_i$

$$E_i = \frac{E_C+E_V}{2} + \frac{1}{2} kT \ln\left(\frac{N_V}{N_C}\right) = \frac{E_C+E_V}{2} + 0.24\, eV.$$

$$\Rightarrow \quad \frac{1}{2} kT \ln\left(\frac{N_V}{N_C}\right) = 0.24\, eV.$$

only parameter tunable is $T \Rightarrow$

$$\Rightarrow \boxed{T = \frac{0.48}{K} \frac{1}{\ln\left(\frac{N_V}{N_C}\right)}}$$

2 pt

Density of states will increase with $T^{3/2}$

$\frac{A}{5}$ 25pt

This problem is very similar to the example solved in the class.

$$\left.\begin{aligned} n &= N_C e^{(E_F-E_C)/kT} \\ p &= N_V e^{(E_V-E_F)/kT} \end{aligned}\right\} 3pt$$

$$n \cdot p = n_i^2 = N_C N_V e^{-(E_G)/kT}$$

$$n_i = \sqrt{N_C N_V}\, e^{-E_G/2kT}$$

$$\left.\begin{aligned} n_{i,Si} &= \sqrt{N_C N_V}\, e^{-E_G/2kT} \\ n_{i,X} &= \sqrt{N_C N_V}\, e^{-E_{G,X}/2kT} \end{aligned}\right\} 6pt$$

Since N_C and N_V are the same both materials.

$$\left.\begin{aligned} \frac{n_{i,Si}}{n_{i,X}} &= \frac{e^{-E_{G,Si}/2kT}}{e^{-E_{G,X}/2kT}} = e^{-(E_{G,Si}-E_{G,X})/2kT} \\ &= e^{-(1.12-1.5)/2\times 0.026} \\ &= e^{+7.3} = (0.67\times 10^{-3})^{-1} \end{aligned}\right\} 5pt$$

$$\Rightarrow 6pt \left\{\begin{aligned} n_{i,X} &= 0.67\times 10^{-3} \times n_{i,Si} \\ &= 0.67\times 10^{-3} \times 10^{10} = 0.67\times 10^{7}\ cm^{-3} \end{aligned}\right.$$

intrinsic $n_X = p_X = 0.67\times 10^{7}\ cm^{-3}$

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

$R = \rho \cdot \frac{L}{A}$ $\quad L = 1\,cm$

$A = 50\,\mu m^2 = 50 \times 10^{-8}\,cm^2$

$V = I \cdot R \Rightarrow$ $\quad I = 0.1\,A$.

5pt