EECS 50 - Discrete Time Signals and Systems - Spring 2017

Midterm Exam I Solution

Student Name:

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Instructions: The table below is for grading purposes only.

Problem No.	Points
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Total Score	

1. (10 pts) The signal $x(t) = 2\cos\left(\frac{27}{5}\pi t + \frac{\pi}{3}\right)$ is sampled every T_s seconds to produce $x[n]$.

   (a) (2 pts) State the range of values of T_s that satisfy the Nyquist criterion.

   (b) (1 pt) Assume $T_s = 0.2$ seconds. Find $x[n]$.

   (c) (2 pts) Assume $T_s = 0.2$ seconds. Is $x[n]$ periodic? If so, then find its fundamental period.

   (d) (3 pts) Assume $T_s = 0.2$ seconds. Will aliasing occur in recovering $x(t)$ from $x[n]$? If so, then find the aliased frequency, i.e., the frequency of the recovered signal.

   (e) (2 pt) Find another value of sampling interval, $T_s' \neq 0.2$, such that sampling $x(t)$ every T_s' seconds will produce the same $x[n]$ as found in part (b).

Sol:

(a) Nyquist criterion: $f_s > 2|f|$. (1 point)
$f = 27/10$. So $f_s > 27/5$. $T_s = 1/f_s$, so the range of values of T_s that satisfies Nyquist criterion is $T_s < 5/27$ (1 point).

(b) $x[n] = 2\cos\left(\frac{27}{5}\pi n T_s + \frac{\pi}{3}\right) = 2\cos\left(\frac{27}{25}\pi n + \frac{\pi}{3}\right)$. (1 point)

(c) Yes, $x[n]$ is periodic. (1 point).
Fundamental period, $N_o = 50k/27 = 50$ samples. (1 point).

(d) $f_s = 1/T_s = 5 < 2f = 5.4$. Yes, aliasing will occur. (1 point).
Aliased frequency

$$\begin{aligned} f' &= f + kf_s \text{ (1 point)} && (1)\\ &= 2.7 + 5k && (2)\\ &= -2.3 \text{ (1 point)} && (3) \end{aligned}$$

for $k = -1$ is the only choice that satisfies $2|f'| < f_s$.

(e)

$$\begin{aligned} x[n] &= 2\cos\left(\frac{27}{25}\pi n + \frac{\pi}{3}\right) && (4)\\ &= 2\cos\left(\left(\frac{27}{25}\pi + 2\pi k\right) n + \frac{\pi}{3}\right) && (5)\\ &= 2\cos\left(2\pi\left(\frac{27}{50} + k\right) n + \frac{\pi}{3}\right) && (6)\\ \frac{f}{f_s'} &= \frac{27}{50} + k \text{ (1 point)} && (7)\\ T_s' = \frac{1}{f_s'} &= \frac{27/50 + k}{27/10} = 0.2 + 10k/27 \text{ seconds.} \quad \text{(1 point)} && (8) \end{aligned}$$

Any non-zero integer value of k that produces $T_s' > 0$ is acceptable. For example, $k = 27$ produces $T_s' = 10.2$ seconds.

2. (10 pts)

(a) (3 pts) Draw the block diagram for the system

$$T_o(x[n]) \quad = \quad T_2\Big(x[n] + T_1\big(x[n] + T_2(x[n])\big)\Big)$$

(b) (4 pts) With the systems T_1, T_2, defined as follows

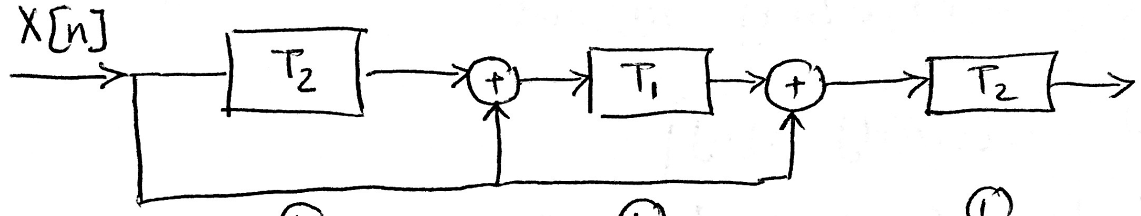
$$\begin{aligned} T_1(x[n]) &= n^2 x[n-1] \\ T_2(x[n]) &= x[-3-n] \end{aligned}$$

write the direct definition of the system $T(x[n]) = T_1(T_2(x[n]))$.

(c) (3 pts) For T_2 as defined in part (b), **sketch** the signal $g[n] = T_2\big(2\delta[4-2n]\big)$.

Question 2:

a)



① point ① point ① point

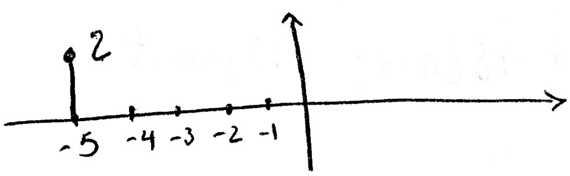
b) $T_1(T_2(X[n])) = T_1(X[-3-n]) = T_1(Y[n])$ ② points

$T_1(Y[n]) = Y[n-1] \times n^2 \Rightarrow T_1(X[-3-n]) = n^2 X[-3-(n-1)] =$

$n^2 X[-n-2]$ ② points

c) $T_2(2\delta[4-2n]) = T_2(X[n])$

$T_2(X[n]) = X[-3-n] = 2\delta[4-2(-3-n)] = 2\delta[10+2n]$ ① point



② points

3. (10 pts) For the discrete-time system defined below, state if it is (i) memoryless, (ii) causal, (iii) invertible, (iv) BIBO stable, (v) linear, and (vi) time-invariant. Provide a proof for (v) and (vi).

$$y[n] \quad = \quad x[n-1] + u[n-2]$$

3. not memoryless 1

causal 1

invertible 1

BIBO stable 1

not linear. 1

$$\text{not equal}\left\{\begin{array}{l} a_1 T[x_1(n)] + a_2 T[x_2(n)] \overset{1}{=} a_1 x_1(n-1) + a_1 u(n-2) \\ \qquad + a_2 x_2(n-1) + a_2 u(n-2) \\ T[a_1 x_1(n) + a_2 x_2(n)] \underset{1}{=} a_1 x_1(n-1) + a_2 x(n-2) + u(n-2) \end{array}\right.$$

other reasonable answers also work

not TI 1

$$\left.\begin{array}{l} y(n-n_0) = x(n-n_0-1) + u(n-n_0-2) \quad 1 \\ T[x(n-n_0)] = x(n-n_0-1) + u(n-2) \quad 1 \end{array}\right\rangle \text{not equal}$$

other reasonable answers also work

4. (10 pts) T_1 and T_2 are LTI systems, i.e., they are known to be **linear and time-invariant**.

(a) (3 pts) When the input to T_1 is chosen to be $x[n] = 2u[n-1]$, then the output of T_1 is

$$y[n] = 4\delta[n+1] + 2\delta[n-1].$$

Find the impulse response, $h_1[n]$, of the LTI system T_1.

(b) (4 pts) $T_2(\delta[n]) = 4\delta[n+1] + 2\delta[n-1]$. Find $T_2(T_2(\delta[n]))$.

(c) (3 pts) Is T_2 causal? Is T_2 stable?

Question 4

a) $\delta[n] = u[n] - u[n-1]$ ① point

$\Rightarrow \delta[n] = \frac{1}{2}[x[n+1] - x[n]]$

LTI system $\Rightarrow h_1[n] = \frac{1}{2}[y[n+1] - y[n]] = \frac{1}{2}[4\delta[n+2] + 2\delta[n] - 4\delta[n+1] - 2\delta[n-1]]$ ① point

$= 2\delta[n+2] + \delta[n] - 2\delta[n+1] - \delta[n-1]$ ① point

b) $T_2(\delta(n)) = 4\delta[n+1] + 2\delta[n-1]$

$\Rightarrow T_2(T_2(\delta(n))) = T_2(4\delta[n+1] + 2\delta[n-1])$ ① point

$= 4(4\delta[n+1+1] + 2\delta[n-1+1]) + 2(4\delta[n-1+1] + 2\delta[n-1-1])$ ② points

$= 16\,\delta[n+2] + 8\,\delta[n] + 8\,\delta[n] + 4\delta[n-2] =$

$16\,\delta[n+2] + 16\delta[n] + 4\delta[n-2]$ ① point

c.) Not causal (1.5) point

Stable (1.5) point

good explanation on part c with wrong answer (+0.5)

5. (10 pts) For each of the following statements, mark it as true or false.

True/False	Statement
F	The signal $x[n] = u[n] - u[n-5]$ is the same as the signal $x'[n] = u[n]u[5-n]$.
F	The signal $x[-3-n]$ can be obtained from $x[n]$ by first doing a time reversal, $n \to -n$, and then the time shift $n \to n-3$.
T	$x[n] = \cos(5n + \pi/6)$ is not a periodic signal.
F	For a general system T, if $T(u[n]) = u[n]$ then T must be a BIBO stable system.
T	For an LTI system T, if $T(u[n]) = u[n]$ then T must be a BIBO stable system.
F	If the input to a time-invariant system is zero for all n, then the output must be zero for all n.
T	The system T is defined as $T(x[n]) = x[n+1] * u[n-2]$. Then T must be causal.
T	$(2^n * \delta[n+2])\delta[n+1] = 2\delta[n+1]$
F	$(2^n \delta[n+2]) * \delta[n+1] = \frac{1}{8}\delta[n+3]$
T	If $T_1(x[n])$ and $T_2(x[n])$ are time-invariant systems, then $T_1(T_2(x[n]))$ must also be a time-invariant system.

Reasons (optional):

(a) False. $x[5] = 0$ but $x'[5] = 1$.

(b) False. The shift needed is $n \longrightarrow n+3$.

(c) True. F is not a rational multiple of π.

(d) False. Response to one particular bounded input cannot establish BIBO stability for a general system.

(e) True. This LTI system has impulse response $h[n] = \delta[n]$, so the system is $y[n] = x[n]$, which is stable.

(f) False. For example, $T(x[n]) = 1$ is a time-invariant system but its output is not zero even if its input is zero.

(g) $T(x[n] = x[n+1] * u[n-2] = x[n] * u[n-1]$. This is an LTI system with impulse response $h[n] = u[n-1]$. Since impulse response is zero for $n < 0$, it is causal.

(h) True. $2^n * \delta[n+2] = 2^{n+2}$ and then multiplying with $\delta[n+1]$ gives us $2\delta[n+1]$.

(i) False. $2^n\delta[n+2] * \delta[n+1] = 2^[n+1]\delta[n+3] = \frac{1}{4}\delta[n+3]$.

(j) True. Shifting $x[n]$ shifts $T_2(x[n])$ by the same amount, which shifts $T_1(T_2(x[n]))$ by the same amount as well.