Frequency Transformation for Digital Filters – Table 10.8

Prototype Lowpass
Filter Has Band Edge
Frequency ω_p

Type of transformation	Transformation	Parameters
Lowpass	$z^{-1} \longrightarrow \dfrac{z^{-1} - a}{1 - az^{-1}}$	ω'_p = band edge frequency new filter $$a = \frac{\sin[(\omega_p - \omega'_p)/2]}{\sin[(\omega_p + \omega'_p)/2]}$$
Highpass	$z^{-1} \longrightarrow -\dfrac{z^{-1} + a}{1 + az^{-1}}$	ω'_p = band edge frequency new filter $$a = -\frac{\cos[(\omega_p + \omega'_p)/2]}{\cos[(\omega_p - \omega'_p)/2]}$$
Bandpass	$z^{-1} \longrightarrow -\dfrac{z^{-2} - a_1 z^{-1} + a_2}{a_2 z^{-2} - a_1 z^{-1} + 1}$	ω_l = lower band edge frequency ω_u = upper band edge frequency $a_1 = 2\alpha K/(K+1)$ $a_2 = (K-1)/(K+1)$ $$\alpha = \frac{\cos[(\omega_u + \omega_l)/2]}{\cos[(\omega_u - \omega_l)/2]}$$ $K = \cot \dfrac{\omega_u - \omega_l}{2} \tan \dfrac{\omega_p}{2}$
Bandstop	$z^{-1} \longrightarrow \dfrac{z^{-2} - a_1 z^{-1} + a_2}{a_2 z^{-1} - a_1 z^{-1} + 1}$	ω_l = lower band edge frequency ω_u = upper band edge frequency $a_1 = 2\alpha/(K+1)$ $a_2 = (1-K)/(1+K)$ $$\alpha = \frac{\cos[(\omega_u + \omega_l)/2]}{\cos[(\omega_u - \omega_l)/2]}$$ $K = \tan \dfrac{\omega_u - \omega_l}{2} \tan \dfrac{\omega_p}{2}$

Frequency Transformation for Analog Filters – Table 10.7

TABLE 10.7 Frequency Transformations for Analog Filters (Prototype Lowpass Filter Has Band Edge Frequency Ω_p)

Type of transformation	Transformation	Band edge frequencies of new filter
Lowpass	$s \longrightarrow \dfrac{\Omega_p}{\Omega_p'} s$	Ω_p'
Highpass	$s \longrightarrow \dfrac{\Omega_p \Omega_p'}{s}$	Ω_p'
Bandpass	$s \longrightarrow \Omega_p \dfrac{s^2 + \Omega_l \Omega_u}{s(\Omega_u - \Omega_l)}$	Ω_l, Ω_u
Bandstop	$s \longrightarrow \Omega_p \dfrac{s(\Omega_u - \Omega_c)}{s^2 + \Omega_u \Omega_l}$	Ω_l, Ω_u