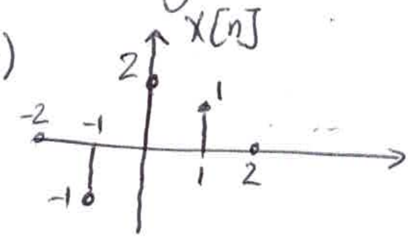
Summary

✓ Express any discrete signal using impulses.

Ex.)

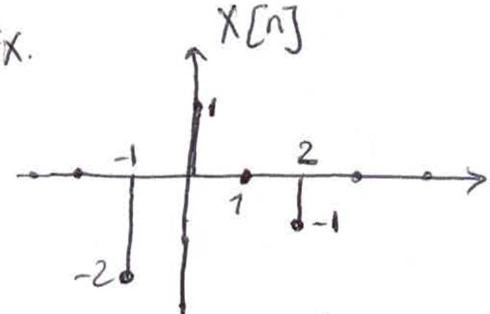


$x[n] = 2\delta[n] + \delta[n-1] - \delta[n+1]$

any signal : $x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-\delta]$

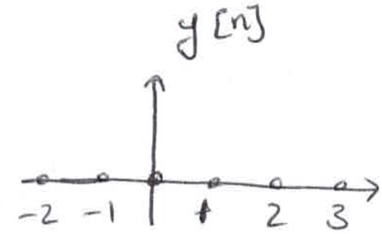
✓ Time Transformations (Shifting and Scaling)

Ex.



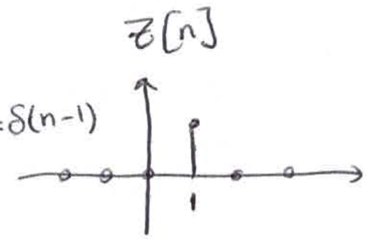
what is $y[n] \overset{?}{=} x[1-3n]$

$$\left.\begin{aligned} y[0] &= x[1-0] = x[1] = 0 \\ y[1] &= x[1-3\times 1] = x[-2] = 0 \\ y[-1] &= x(1-3(-1)] = x[4] = 0 \end{aligned}\right\} \Rightarrow y[n] = 0$$



what is $z[n] \overset{?}{=} x[(n-1)^2]$

$$\left.\begin{aligned} z[0] &= x[(0-1)^2] = x[1] = 0 \\ z[1] &= x[(1-1)^2] = x[0] = 1 \\ z[2] &= x[(2-1)^2] = x[1] = 0 \\ z[3] &= x[(3-1)^2] = x[4] = 0 \\ \cancel{x[2}\; z[-1] &= x[(-1-1)^2] = x[4] = 0 \end{aligned}\right\} \Rightarrow z[n] = 1[1] = \delta(n-1)$$



✓ Even and Odd signals.

For any arbitrary signal $X[n]$, we can find even and odd signal:

$X_e[n] = \frac{1}{2}[X[n] + X[-n]]$ $\Rightarrow X[n] = X_e[n] + X_o[n]$

$X_o[n] = \frac{1}{2}[X[n] - X[-n]]$

Ex.) $X[n] = 10\cos(10n + \theta)$ what is even part of $X[n]$?

$X_e[n] = \frac{1}{2}[X[n] + X[-n]] \Rightarrow X_e[n] = \frac{10}{2}[\cos(10n+\theta) + \cos(-10n+\theta)]$

$= 5[\cos(10n+\theta) + \cos(-10n+\theta)]$

we know that: $\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\Rightarrow X_e[n] = 5 \times 2 \times \cos\left(\frac{2\theta}{2}\right)\cos\left(\frac{20n}{2}\right) = 10\cos(\theta)\cos(10n)$

✓ Sampling of Sinsuids and Nyquist Rate

Ex.) $x(t)$ is sampled every $T_s = 0.5$ seconds to produce $x[n]$

$x(t) = 2\cos(3\pi t + \pi/3)$

a) Does this sampling frequency satisfy the Nyquist Criterien?

Answer: $2\pi f = 3\pi \Rightarrow f = 1.5$, $f_s = 1/T_s = 2$

Nyquist Criterion is not satisfied $\Rightarrow$ we have aliasing

b) Find $X[n]$?

$X[n] = X(t)\Big|_{t=nT_s} \Rightarrow X[n] = 2\cos(3\pi \times nT_s + \pi/3)\Big|_{T_s=0.5} \Rightarrow$

$X[n] = 2\cos(1.5\pi n + \pi/3)$

c) Is $X[n]$ periodic?

Yes, $N = \frac{2\pi k}{\frac{3\pi}{2}} = \frac{4}{3}k \Rightarrow N = 4$

d) Find all signals $x'[t]$ that will produce the same $x[n]$ when sampled every $T_s = 0.5$ seconds.

$x'(t) = 2\cos((1.5\pi + 2k\pi)t \times 2 + \pi/3)$ for all integer values of k

e) of the signals found in d) find the signal that satisfies the Nyquist Criterion?

when $k=-1 \Rightarrow x(t) = 2\cos((1.5\pi - 2\pi)2\pi t + \pi/3) =$

$2\cos(-\pi t + \pi/3) \xrightarrow{\cos(-\theta)=\cos(\theta)}$

$x(t) = 2\cos(\pi t - \pi/3)$ satisfied the Nyquist Criterien

f) Will aliasing happen? If yes, find the aliased signal.

Yes, Aliased signal is $x(t) = 2\cos(\pi t - \pi/3)$

✓Properties of discrete time systems

* Time Invariant
* Linear
* Memoryless
* Causal
* Invertible
* stable

Ex.) $y[n] = (n-1)\,x[n-1]$

i) Not memoryless, $y[0] = -x[-1]$ depends on $x[-1]$

ii) Causal, $y[n]$ doesn't depend on future $x[n]$

iii) Not invertible, $x_1[n] = \delta(n)$ and $x_2[n] = 3\delta(n)$ are two distinct inputs and produce $y_1(n) = y_2(n) = 0 \Rightarrow$ the same output

✓ LTI systems:

Ex.) A system $T(x[n])$ is LTI, we are given the information that

$$x_1(n) = \delta(n-1) \xrightarrow{T} y_1(n) = \delta(n) + 2\delta(n-1)$$

a) What is the impulse response?

$$x_2[n] = \delta(n) \xrightarrow[T]{} y_2(n) = h(n)$$

system is Time Invariant $\Rightarrow x_2[n] = x_1[n+1]$

$$\Rightarrow y_2[n] = y_1[n+1]$$

$$\Rightarrow y_2[n] = h[n] = \delta(n+1) + 2\delta(n)$$

b) $x_3[n] = \begin{cases} 1 & \text{if } 0 \leq n \leq 2 \\ 0 & \text{otherwise} \end{cases}$, what is the output?

$$x_3[n] = \delta(n) + \delta(n-1) + \delta(n-2)$$

$$\xRightarrow[\text{system}]{\text{LTI}} y_3[n] = h(n) + h(n-1) + h(n-2)$$

$$= \delta(n+1) + 2\delta(n) + \delta(n) + 2\delta(n-1) + \delta(n-1) + 2\delta(n-2)$$

$$= \delta(n+1) + 3\delta(n) + 3\delta(n-1) + 2\delta(n-2)$$