Discussion Session no.: ___

EECS145 Fall 2015 First Midterm Exam Name:______________________

ID No.:__________________

You must write your answer within the space provided

Problem 1

$r(t) = (\alpha\cos t, \alpha\sin t, \beta t)$

(a) $r'(t) = (-\alpha\sin t, \alpha\cos t, \beta)$ 5

$s(t) = \int_0^t \sqrt{r' \cdot r'}\, dt$ 3

$= \int_0^t \sqrt{\alpha^2\sin^2 t + \alpha^2\cos^2 t + \beta^2}\, dt$

$= \int_0^t (\alpha^2+\beta^2)^{\frac{1}{2}} dt = (\alpha^2+\beta^2)^{\frac{1}{2}} t$

$k = (\alpha^2+\beta^2)^{\frac{1}{2}} \rightarrow s = kt \rightarrow t = \frac{s}{k}$ 1

$r(t) = (\alpha\cos\frac{s}{k}, \alpha\sin\frac{s}{k}, \beta\frac{s}{k})$ 1

(b) Curvature $\kappa(s) = |u'(s)| = |r''(s)|$ 2

$r'(s) = (\frac{-\alpha}{k}\sin\frac{s}{k}, \frac{\alpha}{k}\cos\frac{s}{k}, \frac{\beta}{k})$ 3

$r''(s) = (\frac{-\alpha}{k^2}\cos\frac{s}{k}, \frac{-\alpha}{k^2}\sin\frac{s}{k}, 0)$ 3

$|r''(s)| = \frac{\alpha}{k^2}$ 2

Problem 2

$$f(x,y,z) = z / [(x^2+y^2)]^{\frac{1}{2}}$$

(a) $\text{grad}\, f(x,y,z) = \nabla f = [\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}]$ ③

$$= \left(\frac{-zx}{(x^2+y^2)^{\frac{3}{2}}}, \frac{-zy}{(x^2+y^2)^{\frac{3}{2}}}, \frac{1}{(x^2+y^2)^{\frac{1}{2}}} \right)$$ ③

(b)

$$\nabla f(x,y,z)\Big|_{(3,-4,1)} = \left(\frac{-(1)(3)}{(3^2+4^2)^{\frac{3}{2}}}, \frac{-(1)(-4)}{(3^2+4^2)^{\frac{3}{2}}}, \frac{1}{(3^2+4^2)^{\frac{1}{2}}} \right)$$

$$= \left(\frac{-3}{125}, \frac{4}{125}, \frac{1}{5} \right)$$ ⑥

-0.024 0.032 0.2

(c) Directional derivative

$D_b f = \hat{b} \cdot \text{grad}\, f$ ④

$$\hat{b} = \frac{(1,-2,1)}{\sqrt{1^2+(-2)^2+(1)^2}} = \left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$$ ②

$$D_b f\Big|_{(3,-4,1)} = \left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right) \left(\frac{-3}{125}, \frac{4}{125}, \frac{1}{5} \right)$$

$$= \frac{(-3)+(-8)+25}{125\sqrt{6}} \text{①} = \frac{14}{125\sqrt{6}} \text{①}$$ ②

$$= \frac{7\sqrt{6}}{375}$$

$= 0.045723$

Problem 3

$F(x,y,z) = (2x - y^2, 3z + x^2, 4y - z^2)$

$$\text{Curl } F = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x-y^2 & 3z+x^2 & 4y-z^2 \end{vmatrix} \; 5 \; = \begin{array}{l} (4-3)\,\hat{i} \\ (0+0)\,\hat{j} \\ (2x+2y)\,\hat{k} \end{array}$$

$$= (1, 0, 2x+2y) \; 5$$

$$\text{div } F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} \; 5$$

$$= 2 + 0 + -2z$$

$$= 2 - 2z \; 5$$

Problem 4

$I = \int_S F \cdot dS$

$\vec{F} = (3y, 2x^2, z^3)$

$S: x^2 + y^2 = 1,\ x > 0,\ 0 < z < 1$

Let $x = \cos u,\ y = \sin u,\ z = v$ (3)

$\vec{r_u} = (-\sin u, \cos u, 0)$ (2)

$\vec{r_v} = (0, 0, 1)$ (2)

$$\vec{N} = \vec{r_u} \times \vec{r_v} = \begin{vmatrix} i & j & k \\ -\sin u & \cos u & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos u\, i + \sin u\, j + 0\, k$$ (3)

$I = \int_S F \cdot dS = \iint \vec{F} \cdot \vec{r_x} \cdot \vec{r_y}\, dx\, dy$

$= \iint (3\sin u, 2\cos^2 u, 0)(\cos u, \sin u, 0)\, du\, dv$ (3)

$= \int_0^1 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3\sin u \cos u + 2\sin u \cos^2 u\, du\, dv$ (4)

$= \int_0^1 \left(-\frac{3}{2}\cos^2 u - \frac{2}{3}\cos^3 u\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$= 0$

(3)

$m = \rho \cdot dV$ (3)

Problem 5 (continued)

$D^2 = x^2 + y^2$ (4)

$I = \iiint \mu D^2 \, dx\,dy\,dz$ (3)

$= \iiint \mu (x^2+y^2) \, dx\,dy\,dz$

$= \int_0^1 \int_0^1 \int_1^2 (x^2+y^2) \, dx\,dy\,dz$ (5)

$= \int_0^1 \int_0^1 \left(\frac{x^3}{3} + xy^2 \Big|_1^2 \right) dy\,dz$

$= \int_0^1 \int_0^1 \left[\frac{8}{3} + 2y^2 - \left(\frac{1}{3} + y^2\right) \right] dy\,dz$

$= \int_0^1 \int_0^1 \left(\frac{7}{3} + y^2 \right) dy\,dz$

$= \int_0^1 \left(\frac{7}{3}y + \frac{y^3}{3} \Big|_0^1 \right) dz$

$= \int_0^1 \left(\frac{7}{3} + \frac{1}{3} \right) dz$

$= \frac{8}{3}$ (5)