

To prove:  $(\exists x)(I(x) \wedge \neg R(x))$

Given  $(\forall x) R(x) \supset L(x)$  whoever can read  $\Rightarrow \neg R(x) \vee L(x)$   
is literate

$(\forall x) D(x) \supset \neg L(x)$  Dolphins are not literate  $\Rightarrow \neg D(y) \vee \neg L(y)$

$(\exists x)(D(x) \wedge I(x))$  Some dolphins are intelligent  $\Rightarrow \left\{ \begin{array}{l} D(A) \\ I(A) \end{array} \right\}$

recall  
 $p \supset q$   
 $\neg p \vee q$

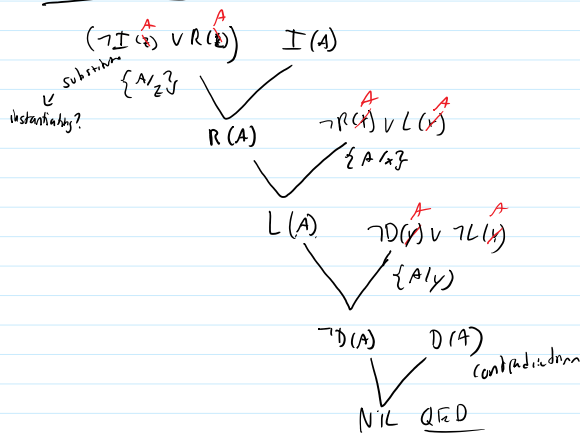
$\downarrow$   
 $D(A) \wedge I(A)$   
Not a rule of logic  
A is random variable

Negate

$$\neg(\exists x)(I(x) \wedge \neg R(x)) \equiv (\forall x) \neg(I(x) \wedge \neg R(x))$$

$$\equiv \forall x (\neg I(x) \vee R(x))$$

Resolving



$$\forall x \{ P(x) \supset \{ (\forall y) [P(y) \supset P(f(x,y))] \wedge \neg(\forall y)(Q(x,y) \supset P(y)) \} \}$$

Step 1: eliminate ' $\neg$ 's

$$(\forall x) \{ \neg P(x) \vee \{ (\forall y) [\neg P(y) \vee P(f(x,y))] \wedge \neg(\forall y)(\neg Q(x,y) \vee P(y)) \} \}$$

scope not valid

Step 2: Reduce the scope of negation so each negation symbol applies to at most one atomic formula or predicate

$$(\exists y) [Q(x,y) \wedge \neg P(y)]$$

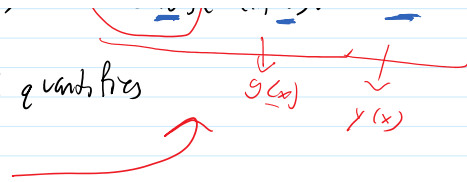
Step 3: standardize variables so that each quantifier has its own variable

Quantifier as in: Existential or universal

$$(\forall x) \{ \neg P(x) \vee \{ (\forall y) [\neg P(y) \vee P(f(x,y))] \wedge (\exists w) (Q(x,w) \wedge \neg P(w)) \} \}$$

Step 4: Skolemization  $\rightarrow$  gets rid of Existential quantifiers

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Step 5: Convert to Prenex form:

$$\underbrace{(\forall x)(\forall y)}_{\text{prefix}} \{ \neg P(x) \vee [ \neg P(y) \vee P(f(x,y)) ] \wedge [ Q(x, g(x)) \wedge \neg P(g(x)) ] \}$$

matrix

Step 6: Convert to matrix  $\rightarrow$  CNF

~~$(\forall x)(\forall y)$~~   $\{ [ \neg P(x) \vee \neg P(y) \vee P(f(x,y)) ] \wedge [ \neg P(x) \vee Q(x, g(x)) ] \wedge [ \neg P(x) \vee \neg P(g(x)) ] \}$

Step 7: Drop universal quantifiers

Step 8: Eliminate '!'s  $\rightarrow$

$$\{ [ \neg P(x_1) \vee \neg P(y_1) \vee P(f(x_1, y_1)) ] \wedge [ \neg P(x_2) \vee Q(x_1, g(x_2)) ] \wedge [ \neg P(x_3) \vee \neg P(g(x_3)) ] \}$$

Step 9: Rename variables so each clause has its own variables

Given  $(\forall x) (A \in (\text{John}, x) \supset A \in (\text{Fido}, x))$ . Fido is wherever John is  
 $A \in (\text{John}, \text{school}) \quad \hookrightarrow \neg(A \in (\text{John}, x) \vee A \in (\text{Fido}, x))$

Prove:  $(\exists x) A \in (\text{Fido}, x) \Rightarrow \neg A \in (\text{Fido}, x) \quad \neg A \in (\text{John}, x) \vee A \in (\text{Fido}, x)$

