In [1]:

```python
# Import Required Packages

import requests
import zipfile
import io
import pandas as pd
import numpy as np
import datetime as dt
import statsmodels.api as sm
import math as math
import matplotlib.pyplot as plt
```

Lejing Lu

1. House Price Dynamics

In [2]:

```python
#download and read a .csv file of price returns by census tract
url = "https://www.fhfa.gov/DataTools/Downloads/Documents/HPI/HPI_AT_BDL_tract.csv"
df = pd.read_csv(url)
df
```

Out[2]:

	tract	state_abbr	year	annual_change	hpi	hpi1990	hpi2000
0	1001020100	AL	1998	NaN	100.00	NaN	100.96
1	1001020100	AL	1999	-5.71	94.29	NaN	95.19
2	1001020100	AL	2000	5.05	99.05	NaN	100.00
3	1001020100	AL	2001	6.95	105.93	NaN	106.95
4	1001020100	AL	2002	7.60	113.98	NaN	115.07
...	...	...	...	...	...	...	...
1986388	56045951300	WY	2018	2.82	180.67	NaN	NaN
1986389	56045951300	WY	2019	7.49	194.21	NaN	NaN
1986390	56045951300	WY	2020	0.79	195.75	NaN	NaN
1986391	56045951300	WY	2021	14.07	223.29	NaN	NaN
1986392	56045951300	WY	2022	-2.01	218.80	NaN	NaN

1986393 rows × 7 columns

In [3]:

```
#Filter dataframe such that the first year is 1990. Then, convert the data type of the variable
#annual change to a float.
dfFiltered = df
dfFiltered = dfFiltered[(dfFiltered['year'] >= 1990) & (dfFiltered['year'] <= 2019)]
dfFiltered = dfFiltered[dfFiltered['annual_change'] != "."]
dfFiltered['annual_change'] = dfFiltered['annual_change'].astype(float)
dfFiltered
```

Out[3]:

	tract	state_abbr	year	annual_change	hpi	hpi1990	hpi2000
0	1001020100	AL	1998	NaN	100.00	NaN	100.96
1	1001020100	AL	1999	-5.71	94.29	NaN	95.19
2	1001020100	AL	2000	5.05	99.05	NaN	100.00
3	1001020100	AL	2001	6.95	105.93	NaN	106.95
4	1001020100	AL	2002	7.60	113.98	NaN	115.07
...	...	...	...	...	...	...	...
1986385	56045951300	WY	2015	-2.58	173.52	NaN	NaN
1986386	56045951300	WY	2016	2.40	177.67	NaN	NaN
1986387	56045951300	WY	2017	-1.10	175.71	NaN	NaN
1986388	56045951300	WY	2018	2.82	180.67	NaN	NaN
1986389	56045951300	WY	2019	7.49	194.21	NaN	NaN

1662188 rows × 7 columns

In [4]:

```
#1c  Read the crosswalk file tract-metro-crosswalk.csv and create a new dataframe consisting of a list
#of census tracts in San Diego. Note that San Diego’s metro code is 41740.
dfCrosswalk = pd.read_csv('tract-metro-crosswalk.csv')
dfSD = dfCrosswalk[dfCrosswalk['metro'] == 41740]
dfSD
```

Out[4]:

	tract	metro
8498	6059032023	41740
8533	6059042103	41740
8535	6059042106	41740
8537	6059042107	41740
8539	6059042108	41740
...	...	...
10968	6073021600	41740
10969	6073021800	41740
10970	6073021900	41740
10971	6073022000	41740
10972	6073022100	41740

654 rows × 2 columns

In [5]:

```
#Read the auxiliary dataset tract-centrality.csv and drop observations with missing data on distance to the city center
dfDistance = pd.read_csv('tract-centrality.csv')
dfDistance = dfDistance.dropna(subset=['distance_to_city_center'])
dfDistance
```

Out[5]:

	tract	distance_to_city_center
0	1001020100	14.486390
1	1001020200	13.577132
2	1001020300	12.906942
3	1001020400	12.091412
4	1001020500	10.527886
...	...	...
55311	56043000200	81.299417
55312	56043000301	98.385588
55313	56043000302	98.707554
55314	56045951100	68.830917
55315	56045951300	52.973136

54727 rows × 2 columns

In [6]:
```
#Merge the dataframe on centrality with the dataframe containing the list of San Diego census
#tracts, using an inner join.
dfSD = dfDistance.merge(dfSD, how = 'inner', left_on = 'tract', right_on = 'tract')
```

In [7]:
```
dfSD = dfFiltered.merge(dfSD, how = 'inner', left_on = 'tract', right_on = 'tract')
```

In [8]:
```
#After performing the merge, sort census tracts into 5 quintiles based on their distance to San
#Diego’s city center.
dfSD['quintile'] = pd.qcut(dfSD['distance_to_city_center'], 5, labels = False) + 1
```

In lecture, we made an analogy to the stock market by calling census tracts “stocks”: if there is substantial house price variation within census tracts,is this analogy still appropriate? What would be the more appropriate analogue to a “stock” if there is substantial house price variation within census tracts?

Maybe bonds?

In [9]:
```
#calculate average annual house price and standard deviation of return for each quintile
meanSD = dfSD.groupby('quintile')['annual_change'].mean()
sdSD = dfSD.groupby('quintile')['annual_change'].std()
print(meanSD)
print(sdSD)
```

```
quintile
1    5.662363
2    4.873536
3    4.584100
4    4.415206
5    4.381964
Name: annual_change, dtype: float64
quintile
1    11.899407
2    10.267739
3    10.195707
4     9.195459
5    10.824235
Name: annual_change, dtype: float64
```

First quintile has the highest average return, fifth quintile has lowest return. That is suggesting there is a negative relationship between average return and distance to the city, closer the distance, the higher the return.
There does not seem to be a relationship between standard deviation of return and distance to the city center. Investors typically are risk-averse, it makes sence that closer to the downtown, the higher the return. Because closer to the center usually means the area is already established, there are more resources and more desireable by the population. Therefore the risk is lower compared to other undevelopped part (bigger distance to center).
The Sharpe ratio (risk-adjusted return) decreases as quintile increases(distance from center increases). The risk adjusted return has a negative relationship with the distance from center. This ratio use statistic principle of mean/sqrt(var) to adjust for the different volatility (similar to how in normal distribution, we would adjust Z-distribution by (x-mean)/sd to transfrom to Z(0,1)). It indicates how much excess return an investment gets for each unit of risk it rakes on, higher is better. Sharpe ratio and distance to the city center are negatively correlated. The closer to the city center, the higher Sharpe ratio. The further to the city center, Sharpe ratio is lower.

In [10]:
```
Sharpe_ratio = meanSD/sdSD
Sharpe_ratio
```

Out[10]:
```
quintile
1    0.475853
2    0.474645
3    0.449611
4    0.480151
5    0.404829
Name: annual_change, dtype: float64
```

    Although Price Growth one indication of return, Return = Capitalization rate+ Price Growth. Capitalization Rate, which is rent-expense. Rent is monthly income generated from renting out the property. Rent to Price ratio might not be consistant across different centrality. Expense could include HOA, Taxes, repair cost, management fee, etc. The Expense and rent ratio is not the same for all properties, therefore we cannot conclude total return is higher in city center with only Price Growth. If we have these information, we could use these many parameters to do a new analysis of total growth rate, or we could perform the same task we just did by switching the parameter to each of the other component to get individual relationship of these component to the annual change in return.

2.Mortgage Underwriting

Study the probability a mortgage loan application is denied based on (a) the applicant's loan-to-income ratio (LTI); and (b) the applicant's race. Perform this analysis using data from the Home Mortgage Disclosure Act (HMDA).

In [11]:
```
#Read the file hmda-2016.csv as a dataframe.
df2 = pd.read_csv('hmda-2016.csv')
```

Create a new dataframe consisting of a random 80% subsample of the full data. Sample without replacement.

In [12]:
```
dfs = df2.sample(frac = 0.80, replace = False, random_state = 1)
dfs
```

Out[12]:

	year	id	agency	loanType	propType	loanPurpose	occupancy	amount	action	msa	...	tract	appEth	appRace1	ap
138305	2016	22257	3	1	1	1	1	28	1	NA	...	9703.00	2	5	
62574	2016	0000451965	9	1	1	3	1	175	6	46060	...	40.44	2	5	
162126	2016	23-2470039	7	1	1	3	1	411	4	40140	...	406.13	2	2	
27292	2016	41-1795868	7	2	1	3	1	77	4	NA	...	9501.00	2	3	
139388	2016	76-0629353	7	1	1	1	1	200	6	26420	...	6729.0	2	2	
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
19685	2016	17117	5	1	1	1	3	113	2	NA	...	NA	2	5	
9057	2016	26-0021318	7	1	1	3	1	60	3	19340	...	0202.00	2	5	
150344	2016	0000066328	5	2	1	1	1	53	3	19380	...	301.0	3	5	
125892	2016	146672	9	1	1	3	1	179	3	47664	...	1360.0	2	2	
152371	2016	0000656733	9	2	1	1	1	74	1	19804	...	5779.0	1	5	

130664 rows × 22 columns

In [13]:

```
#Filter the dataset such that: the edit status is empty; the loan is for the purchase of a home;
#the property is single family (i.e. one-to-four family); the property is owner-occupied;
#and the borrower’s income is non empty.
dfs = dfs[(dfs['editStatus'] == " ") & (dfs['loanPurpose'] == 1) & (dfs['msa'] != "NA") &
          (dfs['occupancy'] == 1) & (dfs['propType'] == 1) & (dfs['income'] != "NA  ") &
          (dfs['income'] != "NA")]
```

Create the following new variables: an indicator for whether the loan application was denied; the borrower’s loan-to-income ratio (LTI); and an indicator for whether the borrower is African¬American or Hispanic. To construct the last variable, review the HMDA code sheet here, and note the value for Race which codes whether the borrower is African-American and the value for Ethnicity which codes whether the borrower is Hispanic.

In [14]:

```
#if loan application is denied
dfs['denied'] = (dfs["action"] == 3).astype(int)
#loan to income ratio
dfs['income'] = dfs['income'].astype(float)
dfs['amount'] = dfs['amount'].astype(float)
```

```python
dfs['lti'] = dfs['amount'] / dfs['income']
#minority
dfs['minority'] = ((dfs['appEth'] == 1) | (dfs['appRace1'] == 3)).astype(int)
dfs
```

```
C:\Users\vivia\AppData\Local\Temp\ipykernel_19148\2621871534.py:2: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a
-view-versus-a-copy
  dfs['denied'] = (dfs["action"] == 3).astype(int)
C:\Users\vivia\AppData\Local\Temp\ipykernel_19148\2621871534.py:4: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a
-view-versus-a-copy
  dfs['income'] = dfs['income'].astype(float)
C:\Users\vivia\AppData\Local\Temp\ipykernel_19148\2621871534.py:5: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a
-view-versus-a-copy
  dfs['amount'] = dfs['amount'].astype(float)
C:\Users\vivia\AppData\Local\Temp\ipykernel_19148\2621871534.py:6: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a
-view-versus-a-copy
  dfs['lti'] = dfs['amount'] / dfs['income']
C:\Users\vivia\AppData\Local\Temp\ipykernel_19148\2621871534.py:8: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a
-view-versus-a-copy
  dfs['minority'] = ((dfs['appEth'] == 1) | (dfs['appRace1'] == 3)).astype(int)
```

Out[14]:

	year	id	agency	loanType	propType	loanPurpose	occupancy	amount	action	msa	...	appSex	income	purchaserTyp
138305	2016	22257	3	1	1	1	1	28.0	1	NA	...	2	33.0	
139388	2016	76-0629353	7	1	1	1	1	200.0	6	26420	...	1	112.0	
46254	2016	3827009995	7	4	1	1	1	158.0	5	NA	...	1	43.0	
161045	2016	74-2508160	7	3	1	1	1	255.0	1	41700	...	1	79.0	
143522	2016	31-1197926	9	1	1	1	1	311.0	6	39300	...	1	125.0	
...	...	...	...	...	...	...	...	...	...	...	...	...	...	
19652	2016	26-0595342	7	1	1	1	1	75.0	1	39580	...	2	103.0	
154178	2016	20-2749403	1	1	1	1	1	213.0	1	23104	...	1	83.0	
13491	2016	02-0640271	7	2	1	1	1	216.0	4	21780	...	1	92.0	
97140	2016	39-1767726	7	1	1	1	1	202.0	1	24580	...	1	131.0	
150344	2016	0000066328	5	2	1	1	1	53.0	3	19380	...	1	1.0	

40094 rows × 25 columns

Use a logistic regression to estimate the probability that a borrower is denied as a function of her loan-to-income ratio (LTI) and a constant. Note that this regression is similar to that estimated in lecture, except that the outcome variable differs (i.e. loan denial vs. loan sale). What are the estimated coefficients on the variables in your regression (i.e. β0, β1)? Using these estimated coefficients, calculate each loan application's probability of being denied.

β0 = -2.5857 β1 = 0.0559

probability of being denied = 1/(1+exp[-β0 -β1*LTI]) = 1/(1+exp[2.5857-0.0559*LTI)

In [33]:

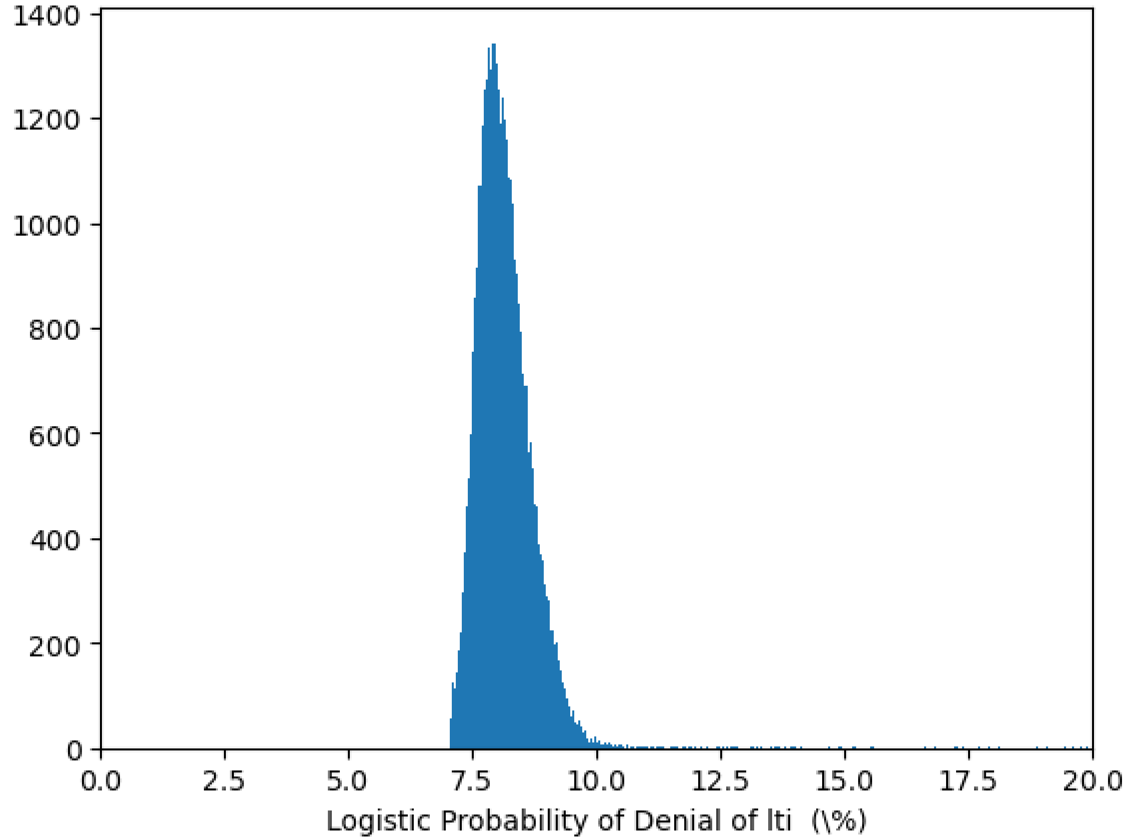
```
#2e: prepare data
df0 = dfs

x = sm.add_constant(np.array(df0['lti']))
y = np.array(df0['denied'])

#resultOLS = sm.OLS(y, x).fit()
resultLogit_lti = sm.Logit(y, x).fit()
```

```python
#print(resultOLS.summary())
print(resultLogit_lti.summary2())
plot = plt.hist(100/(1+np.exp(-np.sum(x*resultLogit_lti.params,axis=1))),bins='auto')
plt.xlim([0,20])
plt.xlabel("Logistic Probability of Denial of lti  (\%)")
plt.savefig("Log-prob-denial-lti.png")
```

```
Optimization terminated successfully.
         Current function value: 0.282519
         Iterations 7
                         Results: Logit
=================================================================
Model:              Logit            Method:           MLE
Dependent Variable: y                Pseudo R-squared: 0.003
Date:               2024-02-04 10:13 AIC:              22658.6202
No. Observations:   40094            BIC:              22675.8182
Df Model:           1                Log-Likelihood:   -11327.
Df Residuals:       40092            LL-Null:          -11365.
Converged:          1.0000           LLR p-value:      3.1634e-18
No. Iterations:     7.0000           Scale:            1.0000
-------------------------------------------------------------------
          Coef.    Std.Err.      z       P>|z|     [0.025    0.975]
-------------------------------------------------------------------
const    -2.5857     0.0309   -83.7544   0.0000   -2.6462   -2.5252
x1        0.0559     0.0082     6.8607   0.0000    0.0400    0.0719
=================================================================
```



The sign of the coefficient is positive. This makes sence because higher loan-to-income yields bigger risk, the borrower is less likely to pay back according to scheule, so more likely to get denied. One advantage to use OLS regression is that we bet B0 and B1, it is easy to interpret the relationship between two parameters, and easy to predict likelihood of denial when lti is given. Since we just need to plug into this one equation: probability of denial = $\beta_0 + \beta_1 * LTI$

Using a similar methodology, use a logistic regression to estimate the probability that a borrower is denied as a function of her loan-to-income ratio (LTI), a constant, and an indicator for whether the borrower is African-American or Hispanic.

$\beta_0 = -2.5547$ $\beta_1 = 0.0545$

probability of being denied = 1/(1+exp[-β_0 -β_1*LTI]) = 1/(1+exp[2.5547-0.0545*LTI)

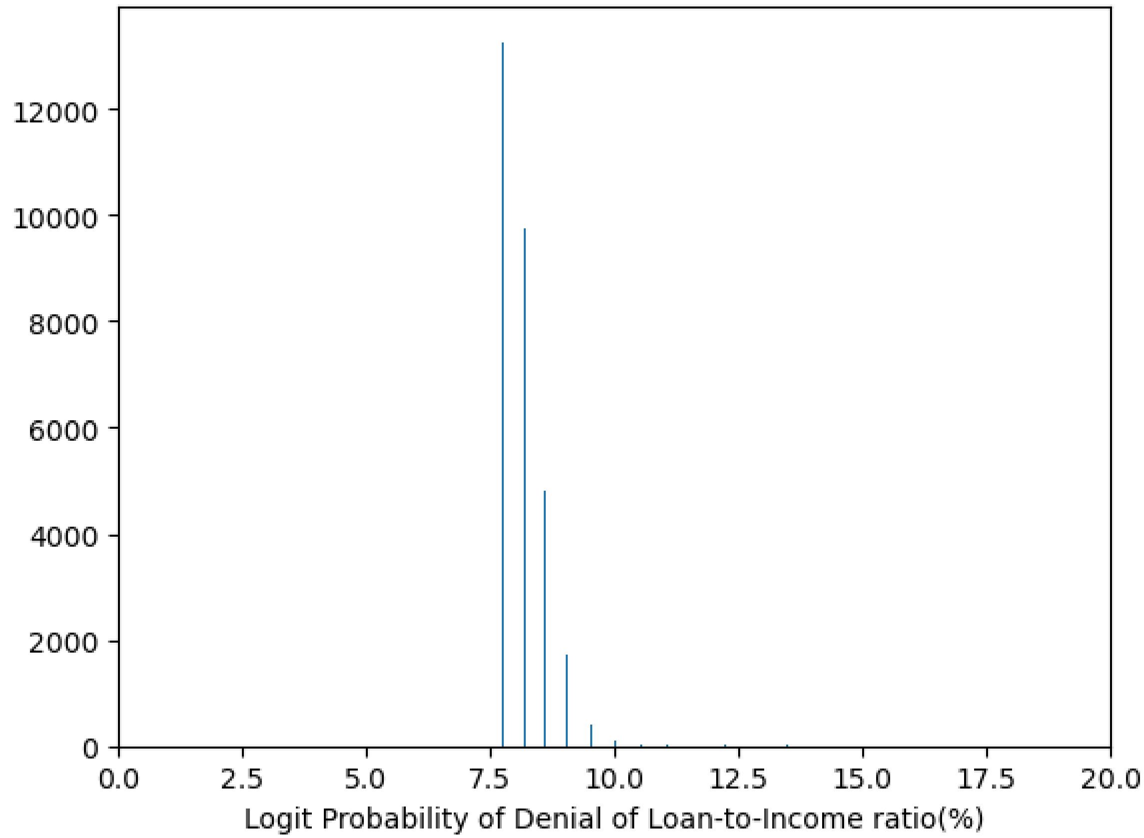
In [23]:
```
xg = sm.add_constant(np.array(dfO['lti'],dfO['minority']))
y = np.array(dfO['denied'])

#resultOLS = sm.OLS(y, x).fit()
resultLogit_minority = sm.Logit(y, xg).fit()
print(resultLogit_minority.summary2())
```

```
Optimization terminated successfully.
         Current function value: 0.282537
         Iterations 7
                         Results: Logit
=================================================================
Model:              Logit            Method:           MLE
Dependent Variable: y                Pseudo R-squared: 0.003
Date:               2024-02-04 10:08 AIC:              22660.0664
No. Observations:   40094            BIC:              22677.2644
Df Model:           1                Log-Likelihood:   -11328.
Df Residuals:       40092            LL-Null:          -11365.
Converged:          1.0000           LLR p-value:      6.5807e-18
No. Iterations:     7.0000           Scale:            1.0000
-------------------------------------------------------------------
          Coef.    Std.Err.      z       P>|z|     [0.025    0.975]
-------------------------------------------------------------------
const    -2.5547     0.0275   -92.8437   0.0000   -2.6086   -2.5007
x1        0.0545     0.0080     6.7873   0.0000    0.0387    0.0702
=================================================================
```

Two distribution looks different because they have different parameters, one is only with LTI data, one is with LTI and minority data. I think the part g with lti and minority data gives more accurate prediction because there are more parameters, so less confounding variables, maing the prediction more controlled and less error.

In [29]:
```
plot_g = plt.hist(100 / (1 + np.exp(- np.sum(xg * resultLogit_lti.params, axis = 1))), bins = 'auto')
plt.xlim([0,20])
plt.xlabel("Logit Probability of Denial of Loan-to-Income ratio(%)")
plt.savefig("logit-denied-lti-race.png")
```



We only analyz owner occupied homes because loan to income ratio can be different from owner-occupied properties and non-owner-occupied properties(usually rental/investment property). Because first, there is different purpose for the property, owner occupied is primary resident, there will not be rent income, whereas investment property earns rental income, so the cashflow differs, and return differs. Secondly, usually there would be a lower interest rate for owner-occupied property, and higher interest rate and requirements for investment properties, the underwritting procedure is different. Non-own-occupied property has higher risk since rental income and property damage could vary. Therefore, the loan-to-income ratio is less informative for non-owner-occupied properties.