Regulation Computer Assignment

Group 4: Ran Ao, Gelin Gao, Lejing Xu, Xiaohe Yu

December 9, 2017

3 CVA/DVA Exercise

1.

Assume first that neither the collateral nor the downgrade protection clauses in Section 2 are in effect. Using Monte Carlo simulation with 50,000 paths, compute and graph the present value as seen from B of the present value of expected exposure $PVEE(T)$, for all T on a monthly schedule out to 10 years. Do this first assuming that B receives the fixed coupon (receiver swap), and then assuming that B pays the coupon (payer swap). The result graph

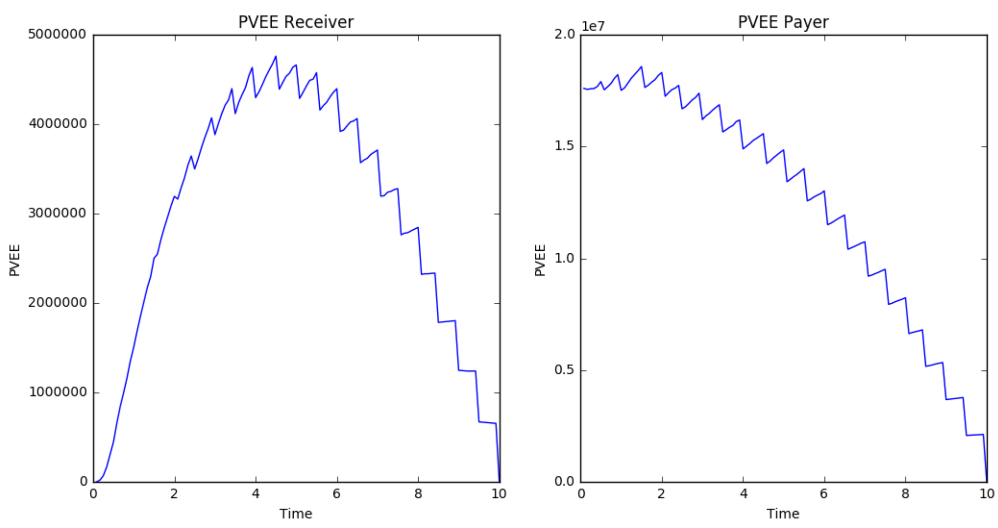


Figure 1: PVEE(T) for payer and receiver swap

2.

Use the results of Exercise 1 to compute the (unilateral) CVA from the perspective of B (for both payer and receiver swap). The result

```
Unilateral CVA as a payer for B is 24154053.9417
Unilateral CVA as a receiver for B is 5527176.44162
```

Figure 2: Unilateral CVA from the perspective of B

3.

Compute the unilateral DVA, for both payer and receiver swap. Also compute the net unilateral CVA. The result

```
Unilateral DVA as a payer for B is 8685378.04369
Unilateral DVA as a receiver for B is 2029735.41657
Net Unilateral CVA as a payer for B is 15468675.898
Net Unilateral CVA as a receiver for B is 3497441.02505
```

Figure 3: Unilateral DVA and net unilateral CVA from the persective of B

4.

For the receiver swap, graph the unilateral CVA, DVA, and net CVA against the interest rate model parameters σ_r and κ_2 (two separate graphs). Explain the results. The result

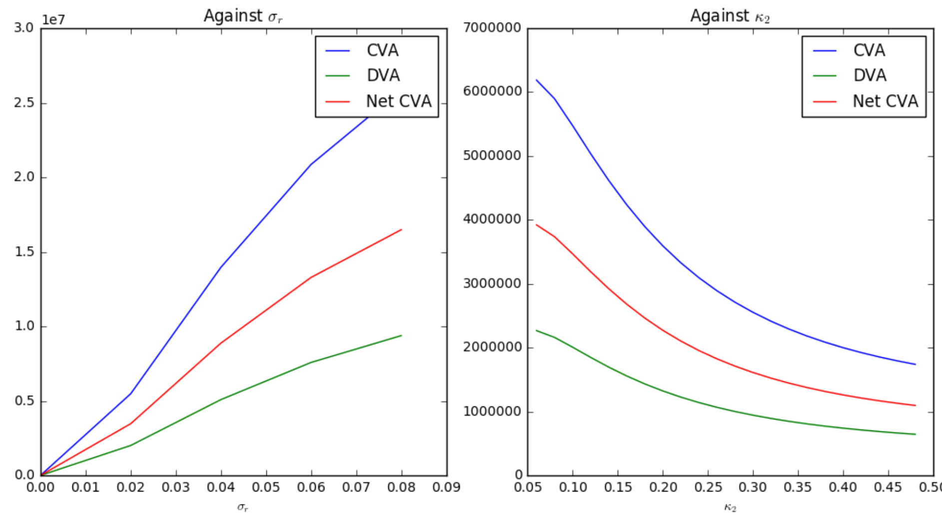


Figure 4: CVA, DVA, net CVA against σ_r and κ_2

The graph shows that as volatility increases, CVA, DVA and net CVA all increases. Explain: Receiver Swap pays floating rate and receives fixed rate, so the receiver will default when floating is higher than fixed. As volatility increases, the probability that floating goes above fixed rate increases, so receiver is more likely to default, indicating an increasing DVA. Similarly, CVA will increase with increasing volatility.

The κ graph shows as kappa increases, CVA and DVA increases. Explain: as κ increases, the rate the interest rate reverses to mean increases, which means the floating rate will less likely to be too small or too large. This makes CVA and DVA decreases using the similar argument in the explanation of volatility.

5.

Some of the correlations in Section 1.3 control wrong- and right-way risk in the unilateral CVA computation. Construct a test to demonstrate (e.g., via a graph) the effects of these correlations (receiver swap only). From the perspective of B owning a receiver swap, B receives the fixed coupon and pays the floating. Wrong-way risk would be when LIBOR rate goes down and the default intensity of C increases. In this situation, the expected profit of B increases, but B is also facing higher default probability from counterparty C. Thus, $corr(d\lambda_C, df_{OIS}) < 0$ control the wrong-way risk. The more negative the correlation, the higher wrong-way risk. On the other hand, when LIBOR rate goes down and the default intensity of C decreases, this is right-way risk. In this situation, the expected profit of B increases, and

C is less likely to default. Thus, $corr(d\lambda_C, df_{OIS}) > 0$ control the right-way risk. The more positive the correlation, the higher right-way risk.

We first experiment by changing only one correlation at a time, keeping the other four correlations as given. The result graph does not lead to meaningful conclusion.

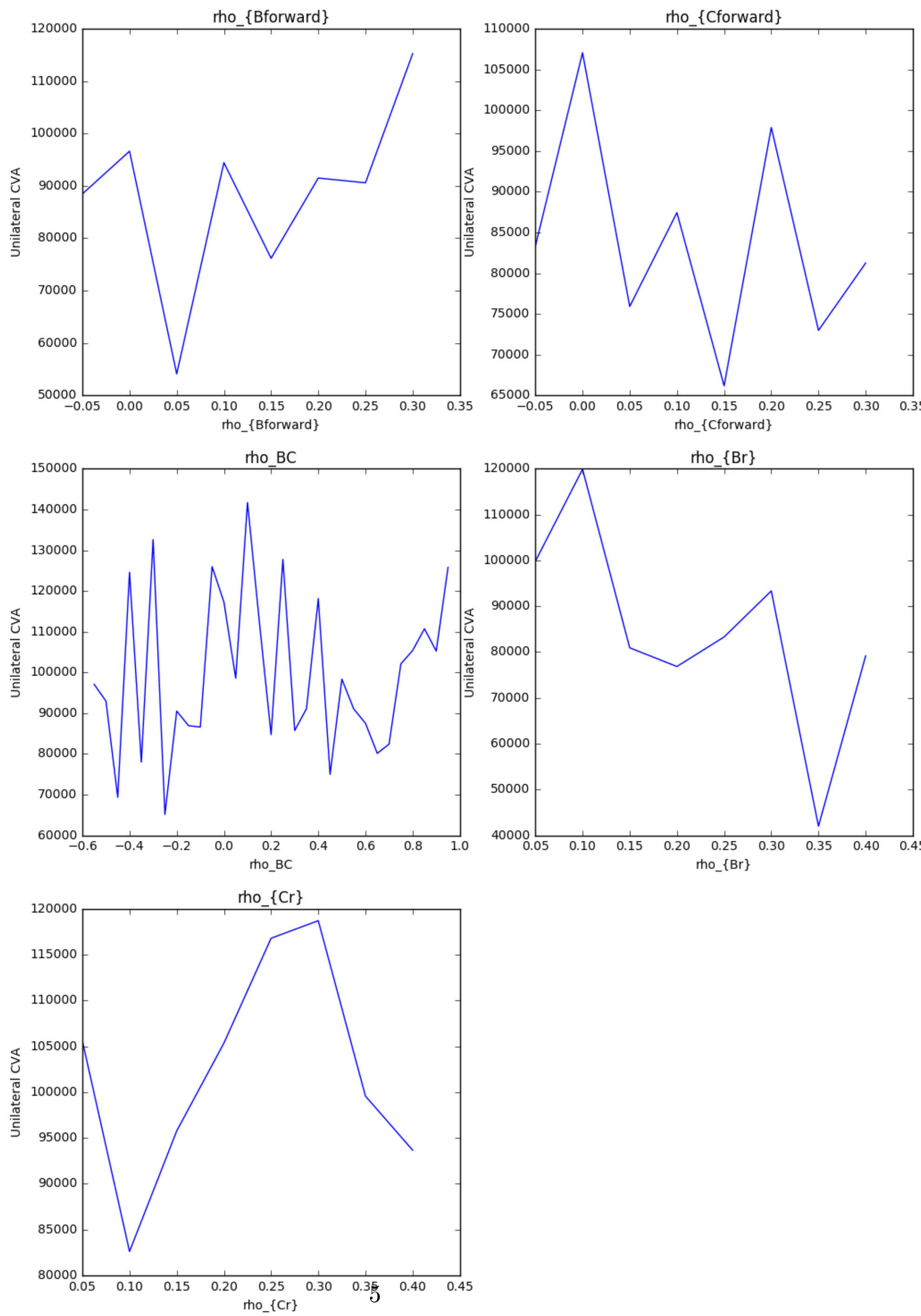


Figure 5: Exploring Wrong- and Right-way risk

The reason is that $corr(d\lambda_C, df_{OIS})$ is correlated with $corr(d\lambda_C, dr_{OIS})$ since the short rate $r(t) = f(t,t) = f(0,t) + x(t)$. We should not hold one constant while changing the other. So next we try changing $corr(d\lambda_C, df_{OIS})$ and $corr(d\lambda_C, dr_{OIS})$ at the same time, making sure the 4×4 correlation matrix is still semi-positive definite. We get the following result:

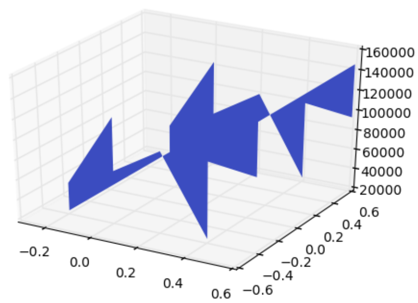


Figure 6: CVA charge with changing $corr(d\lambda_C, df_{OIS})$ and $corr(d\lambda_C, dr_{OIS})$

Now the result is clear: as $corr(d\lambda_C, df_{OIS})$ and $corr(d\lambda_C, dr_{OIS})$ become more positive, CVA is higher, meaning more right-way risk. On the other hand, as $corr(d\lambda_C, df_{OIS})$ and $corr(d\lambda_C, dr_{OIS})$ become more negative, CVA is lower, showing more wrong-way risk.

6.

Now consider the credit mitigants in Section 2. Repeat Exercise 1 with a) the collateral agreement in place; b) the termination agreement in place; c) both agreements in place. (Three separate graphs). Compare the results to those in Exercise 1. The result graph

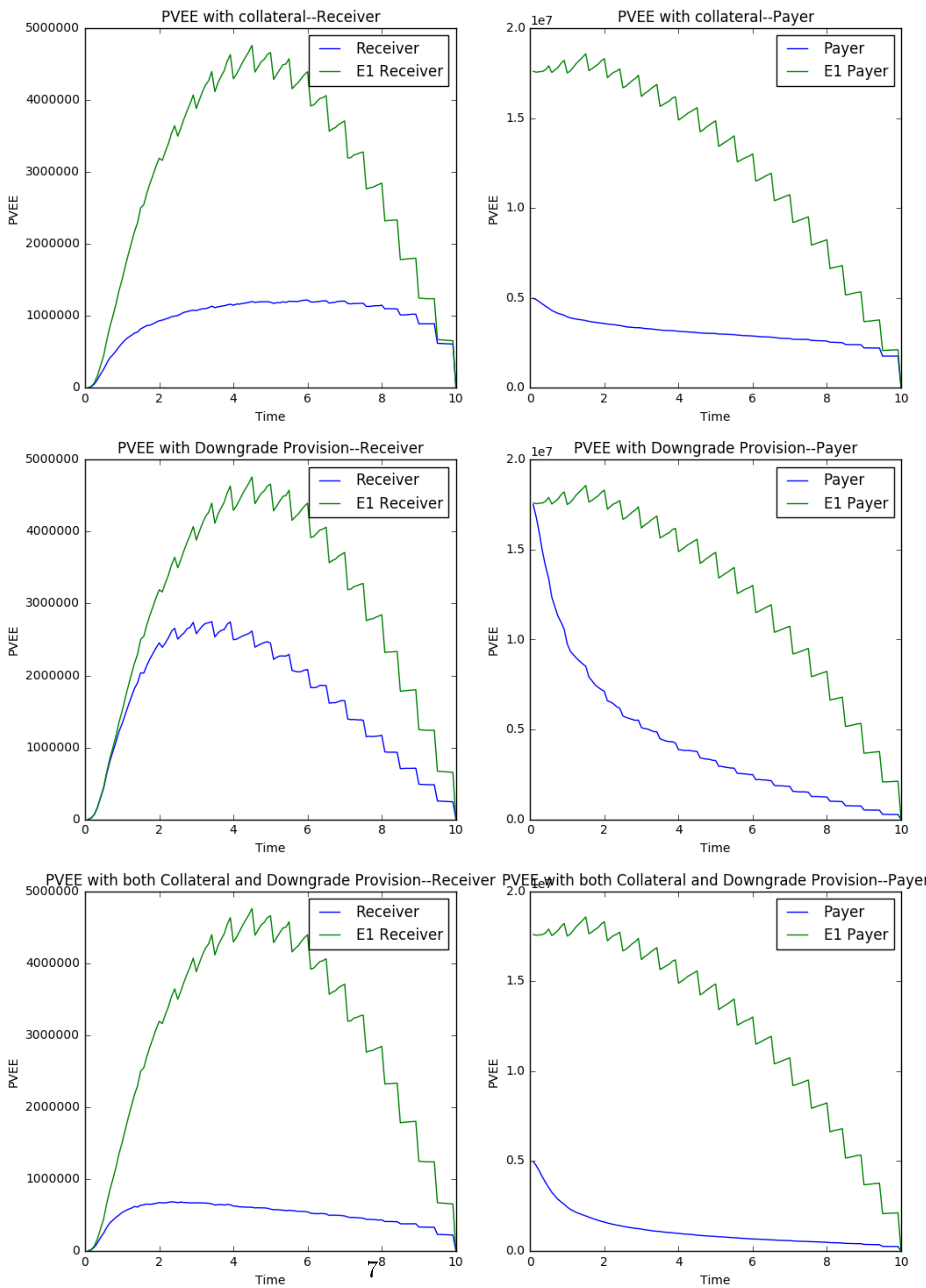


Figure 7: With Credit Mitigants

Both collateral and downgrade provision decreases the $PVEE$. Collateral decreases $PVEE$ more than downgrade provision does.

7.

Turn off the credit mitigants again, and now compute the bilateral CVA, DVA, and net CVA for the naked receiver swap position. Compare against the results in Exercises 2 and 3. Explain. The result

	CVA	DVA	Net CVA
Unilateral	5.527e+06	2.03e+06	3.497e+06
Bilateral	5.27e+06	1.757e+06	3.513e+06

Figure 8: Bilateral CVA, DVA, and net CVA

Explanation:

We can see the $CVA_{unilateral} > CVA_{bilateral}$ and $DVA_{unilateral} > DVA_{bilateral}$, this is because when calculating bilateral numbers, there is an extra discount factor of survival probability.

4 IMM Exercise

1.

Turn off all credit mitigants, and compute and graph the expected exposure profile (as in Section 4.2 of [1]) for both the payer and receiver swap. Contrast the results with those of Exercise 1 in Section 3 above. The result graph:

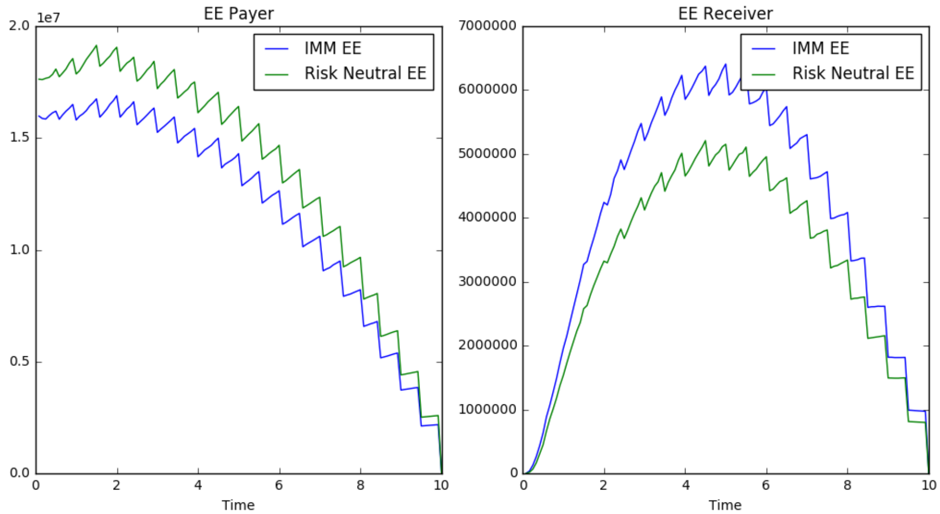


Figure 9: IMM Expected Exposure Profile

EE for payer using historical measure is lower than that using risk-neutral measure, while EE for receiver using historical measure is higher than that using risk-neutral measure.

2.

Using the IMM formulas in [3], compute the EEPE, the EAD and the weighted maturity (M) the payer and receiver swaps (as seen from B's perspective). The result

	EEPE	EAD(Basel 2)	Maturity
Payer	1.802e+07	2.523e+07	1.652
Receiver	6.357e+05	8.9e+05	5

Figure 10: EEPE, EAD, and Weighted Maturity

3.

Assuming that the 1-year (historical) default probability for rm C is PD = 1%; use the results of Exercise 3 to compute rm B's regulatory credit capital for the receiver and payer swap positions, respectively. The result

```
Regulatory Capital for payer is 1575406.2666
Regulatory Capital for Receiver is 84538.1772223
```

Figure 11: Regulatory Credit Capital

Appendix: Some derivations

Simulate OIS curve

From equation (18) and (19) from [1], we get

$$\sigma_{22} = \sigma_2$$

$$\sigma_{21} = \sigma_1 \times \rho_x$$

$$\sigma_{11} = \sqrt{\sigma_1^2 - \sigma_{21}^2} = \sqrt{\sigma_1^2 - \sigma_1^2 \times \rho_x^2}$$

For $y(t)$, we calculate the involved integral manually:

$$\begin{aligned}
\int_0^t a(u)^T a(u) du &= \int_0^t \begin{bmatrix} \sigma_{11}^2 e^{2\kappa_1 u} + \sigma_{21}^2 e^{2\kappa_1 u} & \sigma_{21}\sigma_{22} e^{(\kappa_1+\kappa_2)u} \\ \sigma_{21}\sigma_{22} e^{(\kappa_1+\kappa_2)u} & \sigma_{22}^2 e^{2\kappa_2 u} \end{bmatrix} du \\
&= \begin{bmatrix} \frac{\sigma_{11}^2+\sigma_{21}^2}{2\kappa_1}(e^{2\kappa_1 t} - 1) & \frac{\sigma_{21}\sigma_{22}}{\kappa_1+\kappa_2}(e^{(\kappa_1+\kappa_2)t} - 1) \\ \frac{\sigma_{21}\sigma_{22}}{\kappa_1+\kappa_2}(e^{(\kappa_1+\kappa_2)t} - 1) & \frac{\sigma_{22}^2}{2\kappa_2}(e^{2\kappa_2 t} - 1) \end{bmatrix}
\end{aligned}$$

For $P(t, x(t), T)$, we know

$$P(t, x(t), T) = e^{-\int_t^T f(t,x(t),u)du}$$

where

$$f(t, x(t), u) = f(0, u) + M(t, u)^T (x(t) + y(t)G(t, u))$$

Since $f(0, u)$ is constant, then

$$\begin{aligned}
P(t, x(t), T) &= e^{-f(0,T)+f(0,t)} e^{-x(t)\int_t^T M(t,u)du} e^{-y(t)\int_t^T M(t,u)^T G(t,u)du} \\
&= \frac{P(0, T)}{P(0, t)} exp\big(-G(t, T)^T x(t) - \frac{1}{2} G(t, T)^T y(t) G(t, T)\big)
\end{aligned}$$

where

$$M(t, T) = \begin{bmatrix} e^{-\kappa_1(T-t)} \\ e^{-\kappa_2(T-t)} \end{bmatrix}$$

and

$$G(t, T) = \int_t^T M(t, u) du = \int_t^T \begin{bmatrix} e^{-\kappa_1(u-t)} \\ e^{-\kappa_2(u-t)} \end{bmatrix} du = \begin{bmatrix} \frac{1-e^{-\kappa_1(T-t)}}{\kappa_1} \\ \frac{1-e^{-\kappa_2(T-t)}}{\kappa_2} \end{bmatrix}$$

Regulation_Project_Group_4-Copy1

December 9, 2017

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import fmt
        import pandas as pd

        from swap import Swap,priceSwap
        from marketSetup import simulateOIS,simulateSurvivalProb
        from valuationAdjustment import calculatePVEE,calculateUniCVA,calculateUniD
```

0.0.1 3 CVA/DVA Exercise

Simulate OIS curve

From equation (18) and (19) from [1], we get

$$\sigma_{22} = \sigma_2$$

$$\sigma_{21} = \sigma_1 \times \rho_x$$

$$\sigma_{11} = \sqrt{\sigma_1^2 - \sigma_{21}^2} = \sqrt{\sigma_1^2 - \sigma_1^2 \times \rho_x^2}$$

For $y(t)$, we calculate the involved integral manually:

$$\int_0^t a(u)^T a(u) du = \int_0^t \begin{bmatrix} \sigma_{11}^2 e^{2\kappa_1 u} + \sigma_{21}^2 e^{2\kappa_1 u} & \sigma_{21}\sigma_{22} e^{(\kappa_1+\kappa_2)u} \\ \sigma_{21}\sigma_{22} e^{(\kappa_1+\kappa_2)u} & \sigma_{22}^2 e^{2\kappa_2 u} \end{bmatrix} du = \begin{bmatrix} \frac{\sigma_{11}^2+\sigma_{21}^2}{2\kappa_1}(e^{2\kappa_1 t} - 1) & \frac{\sigma_{21}\sigma_{22}}{\kappa_1+\kappa_2}(e^{(\kappa_1+\kappa_2)t} - 1) \\ \frac{\sigma_{21}\sigma_{22}}{\kappa_1+\kappa_2}(e^{(\kappa_1+\kappa_2)t} - 1) & \frac{\sigma_{22}^2}{2\kappa_2}(e^{2\kappa_2 t} - 1) \end{bmatrix}$$

For $P(t, x(t), T)$, we know

$$P(t, x(t), T) = e^{-\int_t^T f(t,x(t),u)du}$$

where

$$f(t, x(t), u) = f(0, u) + M(t, u)^T (x(t) + y(t)G(t, u))$$

Since $f(0, u)$ is constant, then

$$P(t, x(t), T) = e^{-f(0,T)+f(0,t)} e^{-x(t)\int_t^T M(t,u)du} e^{-y(t)\int_t^T M(t,u)^T G(t,u)du} = \frac{P(0,T)}{P(0,t)} exp\big(-G(t,T)^T x(t) - \frac{1}{2} G(t,T)^T y(t) G$$

where

$$M(t, T) = \begin{bmatrix} e^{-\kappa_1(T-t)} \\ e^{-\kappa_2(T-t)} \end{bmatrix}$$

and

$$G(t,T) = \int_t^T M(t,u)du = \int_t^T \begin{bmatrix} e^{-\kappa_1(u-t)} \\ e^{-\kappa_2(u-t)} \end{bmatrix} du = \begin{bmatrix} \frac{1-e^{-\kappa_1(T-t)}}{\kappa_1} \\ \frac{1-e^{-\kappa_2(T-t)}}{\kappa_2} \end{bmatrix}$$

Inputs

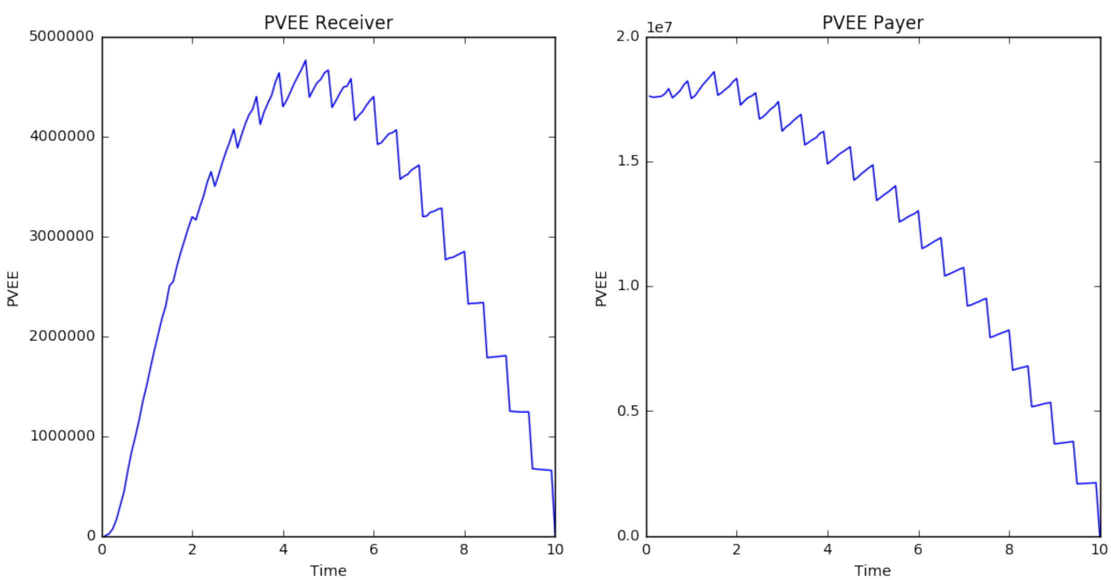
In [2]:
```
num_simulation = 1000
sim_freq = 12

### Interest rate parameters
spread = 0.005 # spread i.e. f_LIBOR=f_OIS+spread
f0_OIS = 0.02
f0_LIBOR = f0_OIS+spread

### swap parameters
freq = 2
maturity = 10
coupon = 0.0265
notional = 150000000.

### Interest rate model parameters
sigma_r = 0.02
c = 0.35
kappa1,kappa2 = 0.02,0.1
rho_inf = 0.4
nu = np.sqrt(1./c/c - 1. - 2.*(rho_inf/c - 1.))
rho_x = (rho_inf/c - 1.)/nu
sigma_l = c * sigma_r
sigma1 = sigma_l
sigma2 = nu*sigma1

### Credit curve parameters
lbda0_B, lbda0_C = 0.01,0.03
sigmaB, sigmaC = 0.005,0.01
kappaB, kappaC = 0.1,0.1

rho_Bf, rho_Cf = 0.1,0.1
rho_BC = 0.75
rho_Br, rho_Cr = 0.25,0.25
rho_B1 = rho_Bf   # corr b/w lbdaB and x1
rho_C1 = rho_Cf   # corr b/w lbdaC and x1
rho_B2 = rho_Br*np.sqrt(nu*nu+1.+2*rho_x*nu)-nu*rho_B1   # corr b/w lbdaB a
rho_C2 = rho_Cr*np.sqrt(nu*nu+1.+2*rho_x*nu)-nu*rho_C1   # corr b/w lbdaC a

### correlation matrix among lbdaB,lbdaC,x1,x2 for simulation
corr = np.array([[1., rho_BC, rho_B1, rho_B2],\
                 [rho_BC, 1., rho_C1, rho_C2],\
                 [rho_B1, rho_C1, 1., rho_x],\
```

```
                 [rho_B2, rho_C2, rho_x, 1.]])
chol = np.linalg.cholesky(corr)

### Credit Mitigation
D = 0.0375   # intensity threshold for downgrade provision
collateral = 5000000.
rr = 0.4    # recovery rate
```

```
In [3]: swap = Swap(maturity, coupon, freq, notional)
        swap.__str__()


        Tis = np.arange(1./freq,maturity+1e-6,1./freq)
        ts = np.arange(1./sim_freq,maturity+1e-6,1./sim_freq)

        #num_simulation = 1000

        prices_payer=[]
        prices_receiver = []
        P_OISs = []
        P_LIBORs = []
        X_Bs = []
        X_Cs = []
        lbdaBs = []
        lbdaCs = []
        wts = []

        for num in range(num_simulation):
            # simulate correlated 4-D brownian motion
            wt = chol.dot(np.random.normal(0,1./np.sqrt(sim_freq),(4,sim_freq*matur
            wts.append(wt)
            P_OIS, P_LIBOR = simulateOIS(rho_x, sigma1, sigma2, kappa1, kappa2, sim
            X_B,X_C,lbdaB,lbdaC = simulateSurvivalProb(lbda0_B,lbda0_C,ts,sigmaB,ka
            price_one_path=[]
            price_one_path_payer = []
            for i in range(maturity*sim_freq):
                p = priceSwap(swap, 'payer', P_OIS, P_LIBOR, i, ts, Tis,sim_freq)
                price_one_path_payer.append(p)
                price_one_path.append(-p)

            prices_payer.append(price_one_path_payer)
            prices_receiver.append(price_one_path)
            P_OISs.append(P_OIS)
            P_LIBORs.append(P_LIBOR)
            X_Bs.append(X_B)
            X_Cs.append(X_C)
            lbdaBs.append(lbdaB)
            lbdaCs.append(lbdaC)
```

1 Plot $PVEE(T)$ as seen from B as payer and receiver respectively

```
In [5]: switch_collateral = False
        switch_downProv = False
        collateral = 0
        D = 0
        PVEE_payer,EE_payer = calculatePVEE(lbdaBs,lbdaCs,P_OISs,X_Cs,prices_payer,
        PVEE_receiver,EE_receiver = calculatePVEE(lbdaBs,lbdaCs,P_OISs,X_Cs,prices_
        #print "Payer",PVEE_payer
        #print "Receiver",PVEE_receiver

        plt.figure(figsize=[12,6])
        plt.subplot(1,2,1)
        plt.plot(ts,PVEE_receiver)
        plt.xlabel('Time')
        plt.ylabel('PVEE')
        plt.title('PVEE Receiver')

        plt.subplot(1,2,2)
        plt.plot(ts,PVEE_payer)
        plt.xlabel('Time')
        plt.ylabel('PVEE')
        plt.title('PVEE Payer')


        plt.savefig('3_1.png')
        plt.show()
```



2 The unilateral CVA from the perspective of B for both payer and receiver swap

```
In [6]: CVA_uni_payer = calculateUniCVA(EE_payer,P_OISs,X_Cs,lbdaCs,rr)
        CVA_uni_receiver = calculateUniCVA(EE_receiver,P_OISs,X_Cs,lbdaCs,rr)
        print "Unilateral CVA as a payer for B is", CVA_uni_payer
        print "Unilateral CVA as a receiver for B is",CVA_uni_receiver
```

```
Unilateral CVA as a payer for B is 24154053.9417
Unilateral CVA as a receiver for B is 5527176.44162
```

3 The unilateral DVA from the perspective of B for both payer and receiver swap, net unilateral CVA

```
In [7]: DVA_uni_payer = calculateUniDVA(EE_payer,P_OISs,X_Bs,lbdaBs,rr)
        DVA_uni_receiver = calculateUniDVA(EE_receiver,P_OISs,X_Bs,lbdaBs,rr)
        print "Unilateral DVA as a payer for B is", DVA_uni_payer
        print "Unilateral DVA as a receiver for B is",DVA_uni_receiver

        net_uni_CVA_payer = calculateNetUniCVA(CVA_uni_payer,DVA_uni_payer)
        net_uni_CVA_receiver = calculateNetUniCVA(CVA_uni_receiver,DVA_uni_receiver
        print "Net Unilateral CVA as a payer for B is", net_uni_CVA_payer
        print "Net Unilateral CVA as a receiver for B is",net_uni_CVA_receiver
```

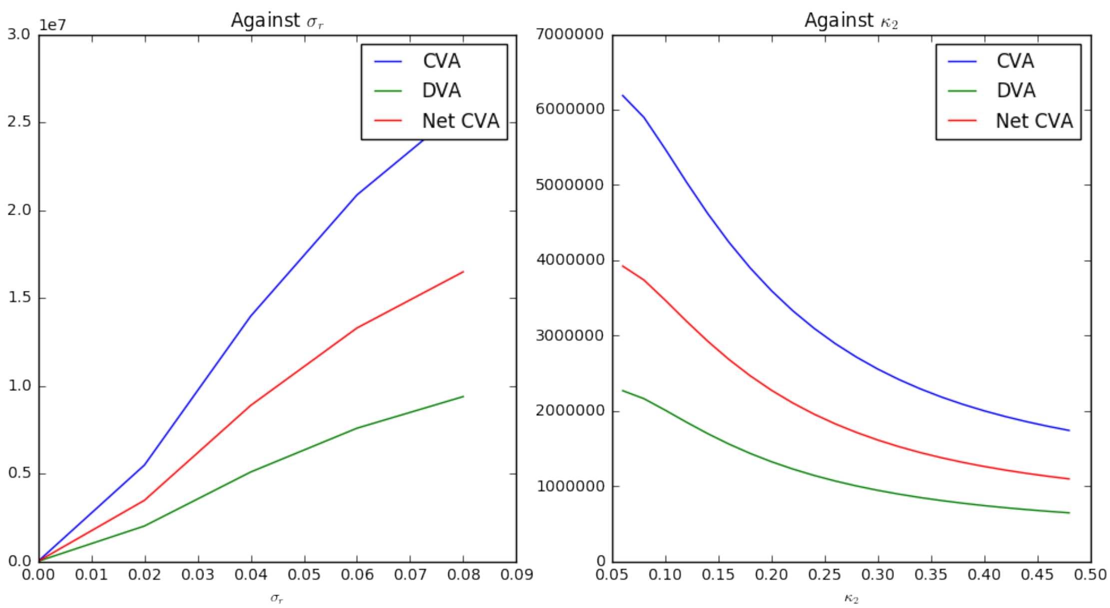
```
Unilateral DVA as a payer for B is 8685378.04369
Unilateral DVA as a receiver for B is 2029735.41657
Net Unilateral CVA as a payer for B is 15468675.898
Net Unilateral CVA as a receiver for B is 3497441.02505
```

4 For the receiver swap, graph the unilateral CVA, DVA, and net CVA against the interest rate model parameters σ_r and κ_2 (two separate graphs)

```
In [5]: switch_collateral = False
        switch_downProv = False
        collateral = 0
        D = 0
        ### sigma_r
        sigma_rs = np.arange(0,0.1,0.02)
        num_sim = 1000
        uniCVA_4s = []
        uniDVA_4s = []
        netCVA_4s = []
        for i in range(len(sigma_rs)):
            sigma_l_4 = c * sigma_rs[i]
            sigma1_4 = sigma_l_4
            sigma2_4 = nu*sigma1_4
            P_OISs_4 = []
            #P_LIBORs_4 = []
```

```
    prices_receiver_4 = []
    for j in range(num_sim):
        P_OIS, P_LIBOR = simulateOIS(rho_x, sigma1_4, sigma2_4, kappa1, kap
        #X_B,X_C,lbdaB,lbdaC = simulateSurvivalProb(lbda0_B,lbda0_C,ts,sigm
        price_one_path=[]
        for t in range(maturity*sim_freq):
            p =priceSwap(swap, 'receiver', P_OIS, P_LIBOR, t, ts, Tis,sim_f
            price_one_path.append(p)

        prices_receiver_4.append(price_one_path)
        P_OISs_4.append(P_OIS)
        #P_LIBORs_4.append(P_LIBOR)

    PVEE_4,EE_4 = calculatePVEE(lbdaBs,lbdaCs,P_OISs_4,X_Cs,prices_receiver
    uniCVA = calculateUniCVA(EE_4,P_OISs_4,X_Cs,lbdaCs,rr)
    uniDVA = calculateUniDVA(EE_4,P_OISs_4,X_Bs,lbdaBs,rr)
    netCVA = calculateNetUniCVA(uniCVA,uniDVA)
    uniCVA_4s.append(uniCVA)
    uniDVA_4s.append(uniDVA)
    netCVA_4s.append(netCVA)


### kappa_2
kappa2s = np.arange(0.06,0.5,0.02)
uniCVA_4k = []
uniDVA_4k = []
netCVA_4k = []
for i in range(len(kappa2s)):

    P_OISs_4k = []
    #P_LIBORs_4k = []
    prices_receiver_4k = []
    for j in range(num_sim):
        P_OIS, P_LIBOR = simulateOIS(rho_x, sigma1, sigma2, kappa1, kappa2s
        #X_B,X_C,lbdaB,lbdaC = simulateSurvivalProb(lbda0_B,lbda0_C,ts,sigm
        price_one_path=[]
        for t in range(maturity*sim_freq):
            p =priceSwap(swap, 'receiver', P_OIS, P_LIBOR, t, ts, Tis,sim_f
            price_one_path.append(p)

        prices_receiver_4k.append(price_one_path)
        P_OISs_4k.append(P_OIS)
        #P_LIBORs_4k.append(P_LIBOR)

    PVEE_4k,EE_4k = calculatePVEE(lbdaBs,lbdaCs,P_OISs_4k,X_Cs,prices_recei
    uniCVA = calculateUniCVA(EE_4k,P_OISs_4k,X_Cs,lbdaCs,rr)
    uniDVA = calculateUniDVA(EE_4k,P_OISs_4k,X_Bs,lbdaBs,rr)
    netCVA = calculateNetUniCVA(uniCVA,uniDVA)
```

```
        uniCVA_4k.append(uniCVA)
        uniDVA_4k.append(uniDVA)
        netCVA_4k.append(netCVA)


    ### plot
    plt.figure(figsize=[12,6])
    plt.subplot(1,2,1)
    plt.plot(sigma_rs,uniCVA_4s,sigma_rs,uniDVA_4s,sigma_rs,netCVA_4s)
    plt.title('Against $\sigma_r$')
    plt.xlabel('$\sigma_r$')
    plt.legend(['CVA','DVA','Net CVA'])

    plt.subplot(1,2,2)
    plt.plot(kappa2s,uniCVA_4k,kappa2s,uniDVA_4k,kappa2s,netCVA_4k)
    plt.title('Against $\kappa_2$')
    plt.xlabel('$\kappa_2$')
    plt.legend(['CVA','DVA','Net CVA'])
    plt.savefig('3_4.png')
    plt.show()
```

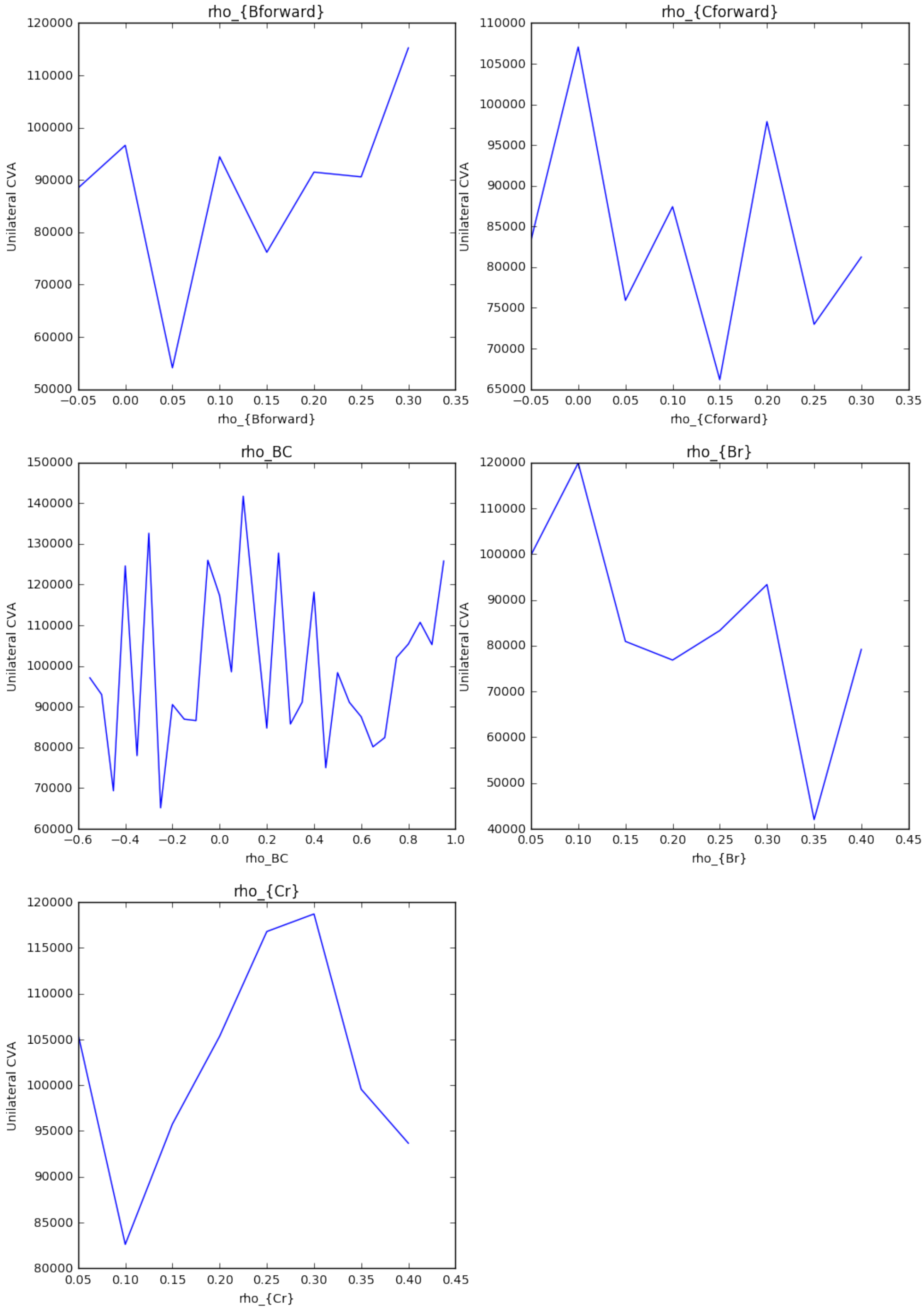


5 correlations that control wrong- and right-way risk

```
In [8]: param = np.arange(-1.,1.,0.05)
        rhos = np.array([rho_Bf, rho_Cf,rho_BC,rho_Br, rho_Cr])
        all_rhos = []
        CVAs = []
```

```
        num_sim = 500
        for i in range(len(rhos)):
            rhos = np.array([rho_Bf, rho_Cf,rho_BC,rho_Br, rho_Cr])
            rho_this = []
            CVA_this = []
            for tester in param:
                rhos[i] = tester
                rho_B1_t = rhos[0]  # corr b/w lbdaB and x1
                rho_C1_t = rhos[1]   # corr b/w lbdaC and x1
                rho_B2_t = rhos[3]*np.sqrt(nu*nu+1.+2*rho_x*nu)-nu*rho_B1_t   # co
                rho_C2_t = rhos[4]*np.sqrt(nu*nu+1.+2*rho_x*nu)-nu*rho_C1_t   # co

                ### correlation matrix among lbdaB,lbdaC,x1,x2 for simulation
                corr = np.array([[1., rhos[2], rho_B1_t, rho_B2_t],\
                         [rhos[2], 1., rho_C1_t, rho_C2_t],\
                         [rho_B1_t, rho_C1_t, 1., rho_x],\
                         [rho_B2_t, rho_C2_t, rho_x, 1.]])
                if np.all(np.linalg.eigvals(corr) >= 0): # make sure corr is positi
                    chol = np.linalg.cholesky(corr)
                    rho_this.append(tester)
                    prices_receiver_t = []
                    P_OISs_t = []
                    X_Cs_t = []
                    lbdaBs_t = []
                    lbdaCs_t = []
                    for num in range(num_sim):
                        # simulate correlated 4-D brownian motion
                        wt = chol.dot(np.random.normal(0,1./sim_freq,(4,sim_freq*ma
                        P_OIS, P_LIBOR = simulateOIS(rho_x, sigma1, sigma2, kappa1,
                        X_B,X_C,lbdaB,lbdaC = simulateSurvivalProb(lbda0_B,lbda0_C,
                        price_one_path=[]
                        for j in range(maturity*sim_freq):
                            p =priceSwap(swap, 'receiver', P_OIS, P_LIBOR, j, ts, T
                            price_one_path.append(p)

                        prices_receiver_t.append(price_one_path)
                        P_OISs_t.append(P_OIS)
                        X_Cs_t.append(X_C)
                        lbdaCs_t.append(lbdaC)
                        lbdaBs_t.append(lbdaB)

                    PVEE_receiver_t,EE_receiver_t = calculatePVEE(lbdaBs_t,lbdaCs_t
                    CVA_uni_receiver_t = calculateUniCVA(EE_receiver_t,P_OISs_t,X_C
                    CVA_this.append(CVA_uni_receiver_t)
            all_rhos.append(rho_this)
            CVAs.append(CVA_this)

In [10]: ## plot
```

```
titles = ['rho_{Bforward}', 'rho_{Cforward}','rho_BC','rho_{Br}', 'rho_{Cr
plt.figure(figsize=[12,18])
for i in range(len(all_rhos)):

    plt.subplot(3,2,i+1)
    plt.plot(all_rhos[i],CVAs[i])
    plt.title(titles[i])
    plt.ylabel('Unilateral CVA')
    plt.xlabel(titles[i])

plt.savefig('3_5.png')
plt.show()
```

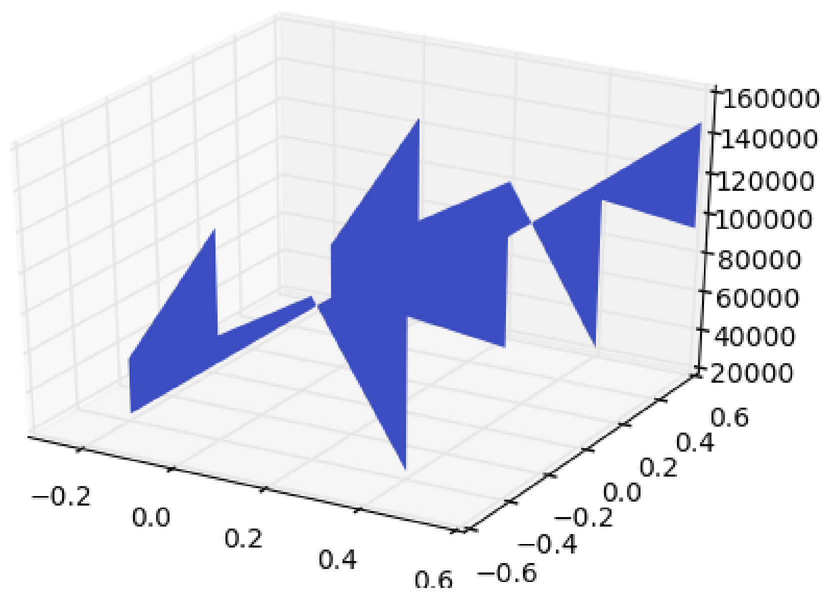



```
In [70]: # change cforward and cr only simultaneously
```

```
param = np.arange(-1.,1.,0.2)
rhos = np.array([rho_Bf, rho_Cf,rho_BC,rho_Br, rho_Cr])
#all_rhos = []
cfs = []
crs = []
CVAs = []
num_sim = 100
for cf in param:
    rhos = np.array([rho_Bf, cf,rho_BC,rho_Br, rho_Cr])
    #rho_this = []
    #CVA_this = []
    for cr in param:
        rhos[4] = cr
        rho_B1_t = rhos[0]  # corr b/w lbdaB and x1
        rho_C1_t = rhos[1]   # corr b/w lbdaC and x1
        rho_B2_t = rhos[3]*np.sqrt(nu*nu+1.+2*rho_x*nu)-nu*rho_B1_t   # co
        rho_C2_t = rhos[4]*np.sqrt(nu*nu+1.+2*rho_x*nu)-nu*rho_C1_t   # co

        ### correlation matrix among lbdaB,lbdaC,x1,x2 for simulation
        corr = np.array([[1., rhos[2], rho_B1_t, rho_B2_t],\
                 [rhos[2], 1., rho_C1_t, rho_C2_t],\
                 [rho_B1_t, rho_C1_t, 1., rho_x],\
                 [rho_B2_t, rho_C2_t, rho_x, 1.]])
        if np.all(np.linalg.eigvals(corr) >= 0): # make sure corr is posit
            chol = np.linalg.cholesky(corr)
            #rho_this.append(tester)
            prices_receiver_t = []
            P_OISs_t = []
            X_Cs_t = []
            lbdaBs_t = []
            lbdaCs_t = []
            for num in range(num_sim):
                # simulate correlated 4-D brownian motion
                wt = chol.dot(np.random.normal(0,1./sim_freq,(4,sim_freq*m
                P_OIS, P_LIBOR = simulateOIS(rho_x, sigma1, sigma2, kappa1
                X_B,X_C,lbdaB,lbdaC = simulateSurvivalProb(lbda0_B,lbda0_C
                price_one_path=[]
                for j in range(maturity*sim_freq):
                    p =priceSwap(swap, 'receiver', P_OIS, P_LIBOR, j, ts,
                    price_one_path.append(p)

                prices_receiver_t.append(price_one_path)
                P_OISs_t.append(P_OIS)
                X_Cs_t.append(X_C)
                lbdaCs_t.append(lbdaC)
                lbdaBs_t.append(lbdaB)
```

```
            PVEE_receiver_t,EE_receiver_t = calculatePVEE(lbdaBs_t,lbdaCs_
            CVA_uni_receiver_t = calculateUniCVA(EE_receiver_t,P_OISs_t,X_
            CVAs.append(CVA_uni_receiver_t)
            cfs.append(cf)
            crs.append(cr)
```

In [72]:
```
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import cm
a,b = np.meshgrid(crs,cfs)
fig = plt.figure()
ax = fig.gca(projection='3d')
ax.plot_surface(a,b,CVAs,cmap=cm.coolwarm,linewidth=0, antialiased=False)
plt.savefig('3_5_2.png')
plt.show()
```



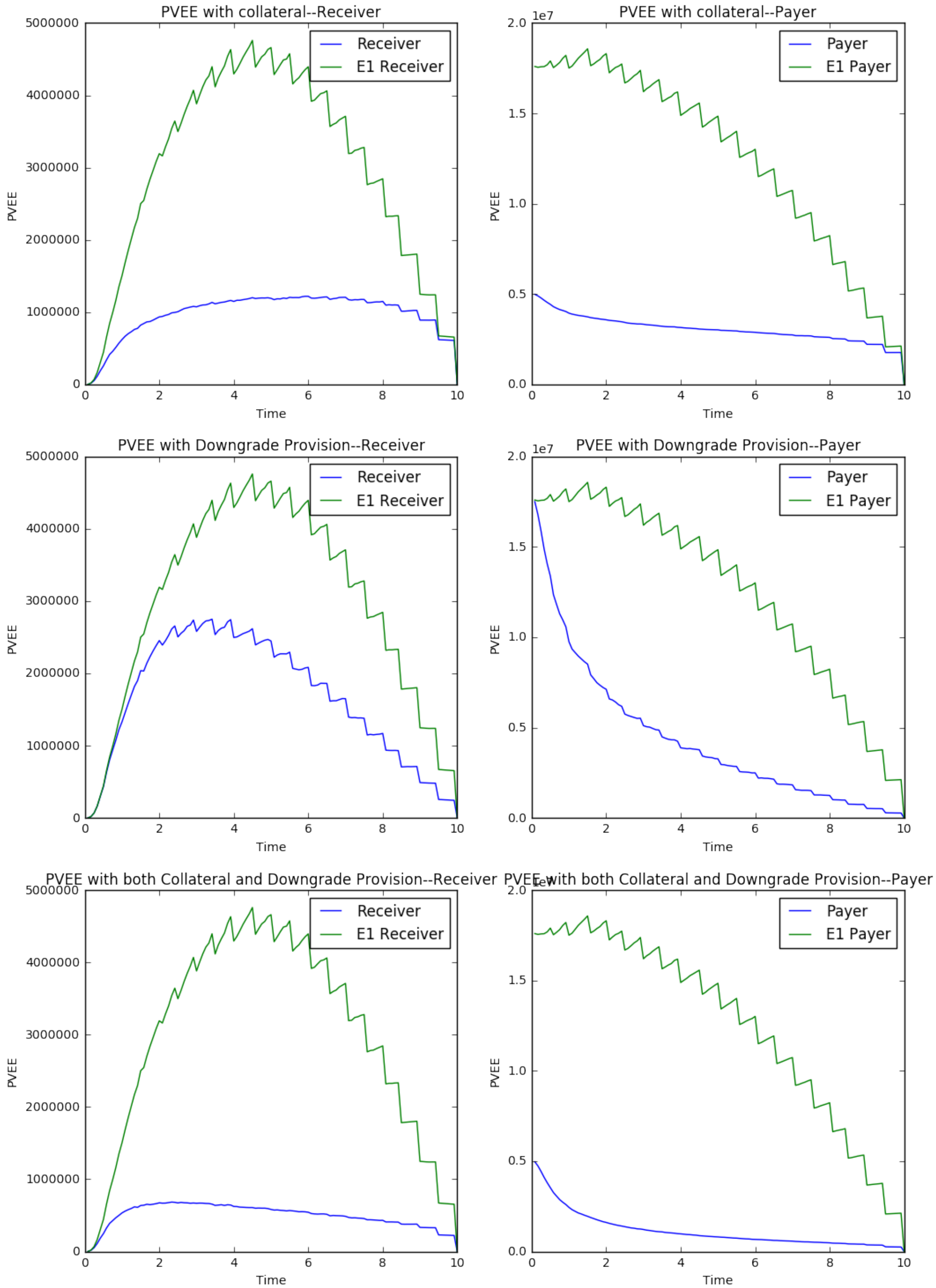
Explanation:
We can see from the graphs above,

6 with credit mitigants Repeat Exercise 1

In [8]:
```
## a) with collateral
switch_collateral = True
collateral = 5000000.
PVEE_payer_col,EE_payer_col = calculatePVEE(lbdaBs,lbdaCs,P_OISs,X_Cs,price
PVEE_receiver_col,EE_receiver_col = calculatePVEE(lbdaBs,lbdaCs,P_OISs,X_Cs
#print "Payer",PVEE_payer
```

```
#print "Receiver",PVEE_receiver

plt.figure(figsize=[12,18])
plt.subplot(3,2,1)
plt.plot(ts,PVEE_receiver_col,ts,PVEE_receiver)
plt.xlabel('Time')
plt.ylabel('PVEE')
plt.title('PVEE with collateral--Receiver')
plt.legend(['Receiver','E1 Receiver'],loc='best')

plt.subplot(3,2,2)
plt.plot(ts,PVEE_payer_col,ts,PVEE_payer)
plt.xlabel('Time')
plt.ylabel('PVEE')
plt.title('PVEE with collateral--Payer')
plt.legend(['Payer','E1 Payer'],loc='best')


## b) with termination
switch_collateral = False
collateral = 0
switch_downProv = True
D = 0.0375
PVEE_payer_down,EE_payer_down = calculatePVEE(lbdaBs,lbdaCs,P_OISs,X_Cs,pri
PVEE_receiver_down,EE_receiver_down = calculatePVEE(lbdaBs,lbdaCs,P_OISs,X_


plt.subplot(3,2,3)
plt.plot(ts,PVEE_receiver_down,ts,PVEE_receiver)
plt.xlabel('Time')
plt.ylabel('PVEE')
plt.title('PVEE with Downgrade Provision--Receiver')
plt.legend(['Receiver','E1 Receiver'],loc='best')

plt.subplot(3,2,4)
plt.plot(ts,PVEE_payer_down,ts,PVEE_payer)
plt.xlabel('Time')
plt.ylabel('PVEE')
plt.title('PVEE with Downgrade Provision--Payer')
plt.legend(['Payer','E1 Payer'],loc='best')

## c) with both collateral and termination
switch_collateral = True
collateral = 5000000.
PVEE_payer_both,EE_payer_both = calculatePVEE(lbdaBs,lbdaCs,P_OISs,X_Cs,pri
PVEE_receiver_both,EE_receiver_both = calculatePVEE(lbdaBs,lbdaCs,P_OISs,X_
#print "Payer",PVEE_payer
#print "Receiver",PVEE_receiver
```

```
plt.subplot(3,2,5)
plt.plot(ts,PVEE_receiver_both,ts,PVEE_receiver)
plt.xlabel('Time')
plt.ylabel('PVEE')
plt.title('PVEE with both Collateral and Downgrade Provision--Receiver')
plt.legend(['Receiver','E1 Receiver'],loc='best')

plt.subplot(3,2,6)
plt.plot(ts,PVEE_payer_both,ts,PVEE_payer)
plt.xlabel('Time')
plt.ylabel('PVEE')
plt.title('PVEE with both Collateral and Downgrade Provision--Payer')
plt.legend(['Payer','E1 Payer'],loc='best')
plt.savefig('3_6.png')
plt.show()
```



7 Compute the bilateral CVA,DVA,net CVA for the naked swap position

```
In [9]: bi_CVA_receiver = calculateBiCVA(EE_receiver,P_OISs,X_Cs,lbdaCs,rr,X_Bs)
        bi_DVA_receiver = calculateBiDVA(EE_receiver,P_OISs,X_Cs,lbdaBs,rr,X_Bs)
        net_bi_CVA_receiver = calculateNetBiCVA(bi_CVA_receiver,bi_DVA_receiver)
        bilateral = [bi_CVA_receiver,bi_DVA_receiver,net_bi_CVA_receiver]
        unilateral = [CVA_uni_receiver,DVA_uni_receiver,net_uni_CVA_receiver]

        df = pd.DataFrame(np.asarray([unilateral,bilateral]),index = ['Unilateral',
        fmt.displayDF(df,'4g')
```

```
<IPython.core.display.HTML object>
```

Explain the results:

We can see the $CVA_{unilateral} > CVA_{bilateral}$ and $DVA_{unilateral}>DVA_{bilateral} $, this is because when calculating bilateral numbers, there is an extra discount factor of survival probability.

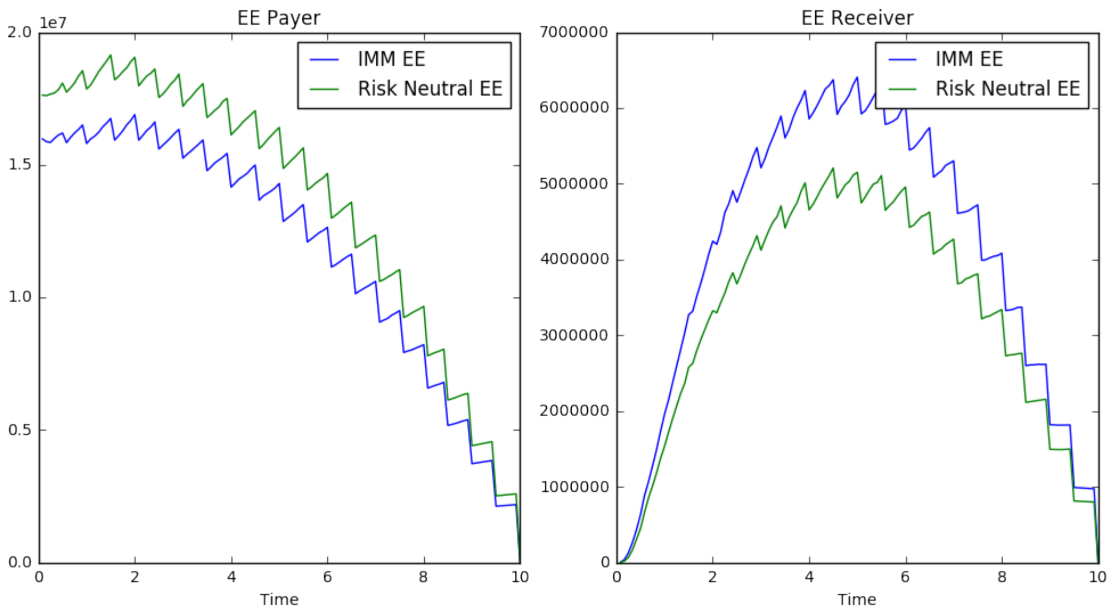
0.0.2 4 IMM Exercise

```
In [10]: from marketSetup import simulateOIS_IMM,simulateSurvivalProb_IMM
         prices_payer_IMM=[]
         prices_receiver_IMM = []
         P_OISs_IMM = []
         #P_LIBORs_IMM = []
         #X_Bs_IMM = []
         X_Cs_IMM = []
         #lbdaBs_IMM = []
         lbdaCs_IMM = []
         #wts_IMM = []

         for num in range(num_simulation):
             # simulate correlated 4-D brownian motion
             wt = chol.dot(np.random.normal(0,1./np.sqrt(sim_freq),(4,sim_freq*matu
             #wts.append(wt)
             P_OIS, P_LIBOR = simulateOIS_IMM(rho_x, sigma1, sigma2, kappa1, kappa2
             X_B,X_C,lbdaB,lbdaC = simulateSurvivalProb_IMM(lbda0_B,lbda0_C,ts,sigm
             price_one_path=[]
             price_one_path_payer = []
             for i in range(maturity*sim_freq):
                 p = priceSwap(swap, 'payer', P_OIS, P_LIBOR, i, ts, Tis,sim_freq)
                 price_one_path_payer.append(p)
                 price_one_path.append(-p)

             prices_payer_IMM.append(price_one_path_payer)
             prices_receiver_IMM.append(price_one_path)
             P_OISs_IMM.append(P_OIS)
             #P_LIBORs_IMM.append(P_LIBOR)
             #X_Bs_IMM.append(X_B)
```

```
            X_Cs_IMM.append(X_C)
            #lbdaBs_IMM.append(lbdaB)
            lbdaCs_IMM.append(lbdaC)

        #print "payer",np.average(prices_payer,axis=0)
        #print prices_payer
```

1 Expected Exposure Profile

```
In [11]: switch_collateral = False
         switch_downProv = False
         collateral = 0
         D = 0
         PVEE_payer_IMM,EE_payer_IMM = calculatePVEE(lbdaBs,lbdaCs_IMM,P_OISs_IMM,X
         PVEE_receiver_IMM,EE_receiver_IMM = calculatePVEE(lbdaBs,lbdaCs_IMM,P_OISs

         plt.figure(figsize=[12,6])
         plt.subplot(1,2,1)
         plt.plot(ts,EE_payer_IMM,ts,EE_payer)
         plt.xlabel('Time')
         plt.legend(['IMM EE','Risk Neutral EE'])
         plt.title('EE Payer')
         plt.subplot(1,2,2)
         plt.plot(ts,EE_receiver_IMM,ts,EE_receiver)
         plt.xlabel('Time')
         plt.legend(['IMM EE','Risk Neutral EE'])
         plt.title('EE Receiver')
         plt.savefig('4_1.png')
         plt.show()
```



2 Calculate EEPE, EAD, M

In [12]:
```
from IMM_fun import CalcEAD,RegulatoryCapital,EffectiveMaturity,CalcEEPE

alpha = 1.4
LGD = 0.4
PD = 0.01
# EEPE
EEPE_payer = CalcEEPE(EE_payer,sim_freq)
EEPE_receiver = CalcEEPE(EE_receiver,sim_freq)

# EAD
## Basel 2 EAD
EAD_payer_2 = CalcEAD('2',EEPE_payer,0,alpha,sim_freq)
EAD_receiver_2 = CalcEAD('2',EEPE_receiver,0,alpha,sim_freq)
'''## Basel 3 EAD
CVA_uni_payer_IMM = calculateUniCVA(EE_payer_IMM,P_OISs_IMM,X_Cs_IMM,lbda(
CVA_uni_receiver_IMM = calculateUniCVA(EE_receiver_IMM,P_OISs_IMM,X_Cs_IMM
EAD_payer_3 = CalcEAD('3',EEPE_payer,CVA_uni_payer_IMM,alpha,sim_freq)
EAD_receiver_3 = CalcEAD('3',EEPE_receiver,CVA_uni_receiver_IMM,alpha,sim_
print CVA_uni_receiver_IMM'''


# Effectvie Maturity
M_payer = EffectiveMaturity(EE_payer_IMM,sim_freq,P_OISs_IMM)
M_receiver = EffectiveMaturity(EE_receiver_IMM,sim_freq,P_OISs_IMM)

#df_payer = np.asarray([EEPE_payer,EAD_payer_2,EAD_payer_3,M_payer])
#df_receiver = np.asarray([EEPE_receiver,EAD_receiver_2,EAD_receiver_3,M_r
#inds = ['EEPE','EAD(Basel 2)','EAD(Basel 3)','Maturity']
df_payer = np.asarray([EEPE_payer,EAD_payer_2,M_payer])
df_receiver = np.asarray([EEPE_receiver,EAD_receiver_2,M_receiver])
inds = ['EEPE','EAD(Basel 2)','Maturity']

df2 = pd.DataFrame([df_payer, df_receiver],index = ['Payer','Receiver'],co
fmt.displayDF(df2,'4g')
```

```
<IPython.core.display.HTML object>
```

3 Regulatory Capital

In [13]:
```
RC_payer = RegulatoryCapital(EAD_payer_2,M_payer,LGD,PD)
RC_receiver = RegulatoryCapital(EAD_receiver_2,M_receiver,LGD,PD)
```

```python
    print "Regulatory Capital for payer is",RC_payer
    print "Regulatory Capital for Receiver is",RC_receiver
```

```
Regulatory Capital for payer is 1575406.2666
Regulatory Capital for Receiver is 84538.1772223
```