Regulation Computer Assignment

Group 4: Ran Ao, Gelin Gao, Lejing Xu, Xiaohe Yu

December 8, 2017

3 CVA/DVA Exercise

1.

Assume first that neither the collateral nor the downgrade protection clauses in Section 2 are in effect. Using Monte Carlo simulation with 50,000 paths, compute and graph the present value as seen from B of the present value of expected exposure $PVEE(T)$, for all T on a monthly schedule out to 10 years. Do this first assuming that B receives the fixed coupon (receiver swap), and then assuming that B pays the coupon (payer swap). The result graph

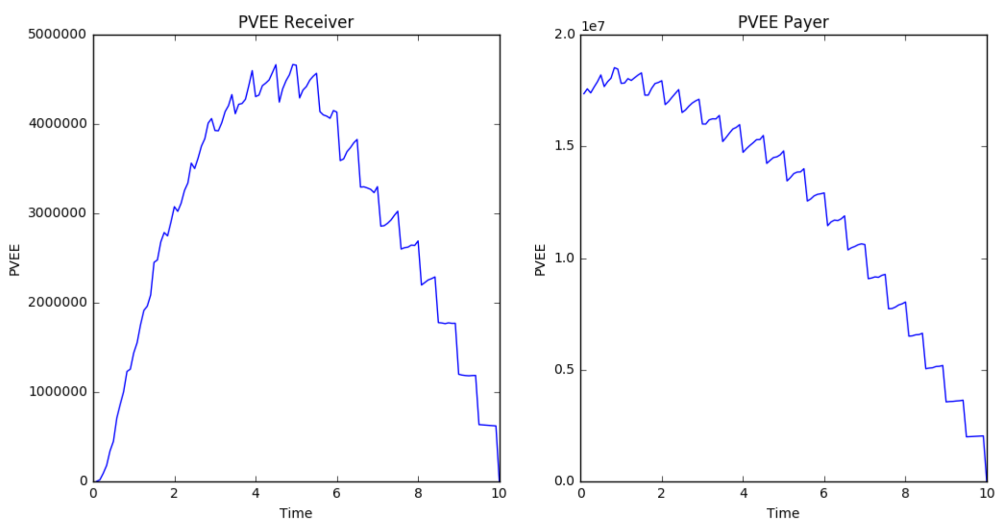

Figure 1: PVEE(T) for payer and receiver swap

2.

Use the results of Exercise 1 to compute the (unilateral) CVA from the perspective of B (for both payer and receiver swap). The result

```
Unilateral CVA as a payer for B is 23943435.3289
Unilateral CVA as a receiver for B is 5341804.71754
```

Figure 2: Unilateral CVA from the perspective of B

3.

Compute the unilateral DVA, for both payer and receiver swap. Also compute the net unilateral CVA. The result

```
Unilateral DVA as a payer for B is 8607414.6679
Unilateral DVA as a receiver for B is 1960446.38474
Net Unilateral CVA as a payer for B is 15336020.661
Net Unilateral CVA as a receiver for B is 3381358.33279
```

Figure 3: Unilateral DVA and net unilateral CVA from the persective of B

4.

For the receiver swap, graph the unilateral CVA, DVA, and net CVA against the interest rate model parameters σ_r and κ_2 (two separate graphs). Explain the results. The result

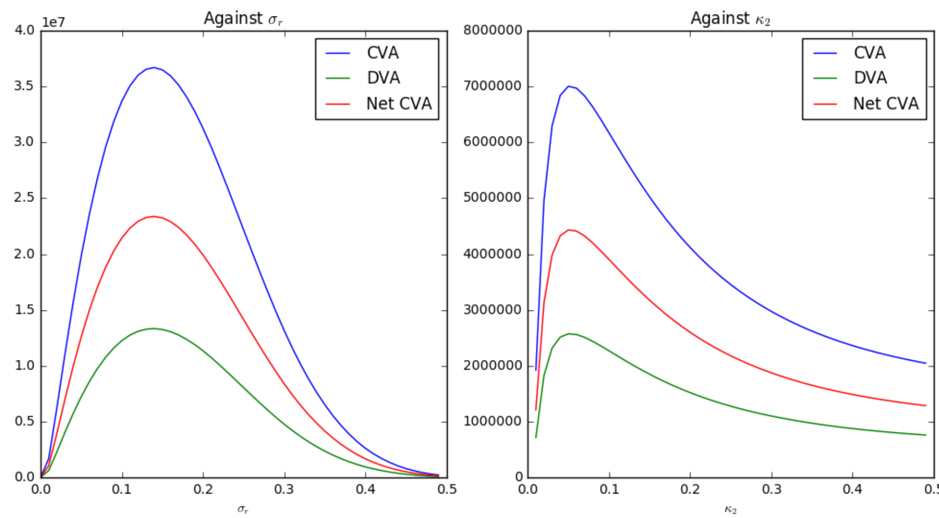

Figure 4: CVA, DVA, net CVA against σ_r and κ_2

Explanation:

5.

Some of the correlations in Section 1.3 control wrong- and right-way risk in the unilateral CVA computation. Construct a test to demonstrate (e.g., via a graph) the effects of these correlations (receiver swap only). From the perspective of B owning a receiver swap, B receives the fixed coupon and pays the floating. Wrong-way risk would be when LIBOR rate goes down and the default intensity of C increases. In this situation, the expected profit of B increases, but B is also facing higher default probability from counterparty C. Thus, $corr(d\lambda_C, df_{OIS}) < 0$ control the wrong-way risk. The more negative the correlation, the higher wrong-way risk. On the other hand, when LIBOR rate goes down and the default intensity of C decreases, this is right-way risk. In this situation, the expected profit of B increases, and C is less likely to default. Thus, $corr(d\lambda_C, df_{OIS}) > 0$ control the right-way risk. The more positive the correlation, the higher right-way risk.

We first experiment by changing only one correlation at a time, keeping the other four correlations as given. The result graph does not lead to meaningful conclusion.

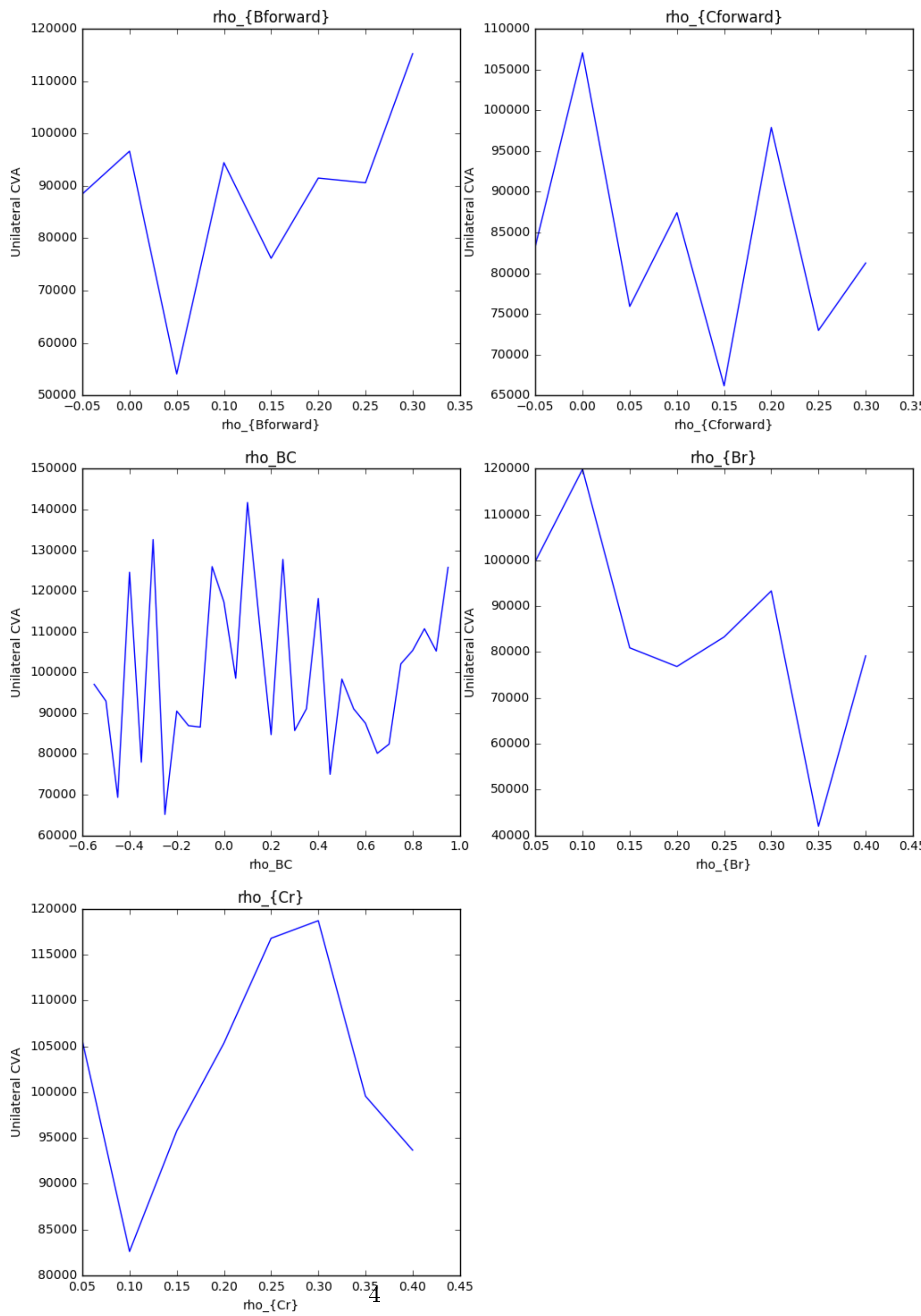

Figure 5: Exploring Wrong- and Right-way risk

The reason is that $corr(d\lambda_C, df_{OIS})$ is correlated with $corr(d\lambda_C, dr_{OIS})$ since the short rate $r(t) = f(t,t) = f(0,t) + x(t)$. We should not hold one constant while changing the other. So next we try changing $corr(d\lambda_C, df_{OIS})$ and $corr(d\lambda_C, dr_{OIS})$ at the same time, making sure the 4×4 correlation matrix is still semi-positive definite. We get the following result:

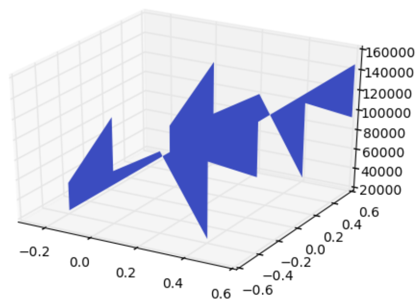

Figure 6: CVA charge with changing $corr(d\lambda_C, df_{OIS})$ and $corr(d\lambda_C, dr_{OIS})$

Now the result is clear: as $corr(d\lambda_C, df_{OIS})$ and $corr(d\lambda_C, dr_{OIS})$ become more positive, CVA is higher, meaning more right-way risk. On the other hand, as $corr(d\lambda_C, df_{OIS})$ and $corr(d\lambda_C, dr_{OIS})$ become more negative, CVA is lower, showing more wrong-way risk.

6.

Now consider the credit mitigants in Section 2. Repeat Exercise 1 with a) the collateral agreement in place; b) the termination agreement in place; c) both agreements in place. (Three separate graphs). Compare the results to those in Exercise 1. The result graph

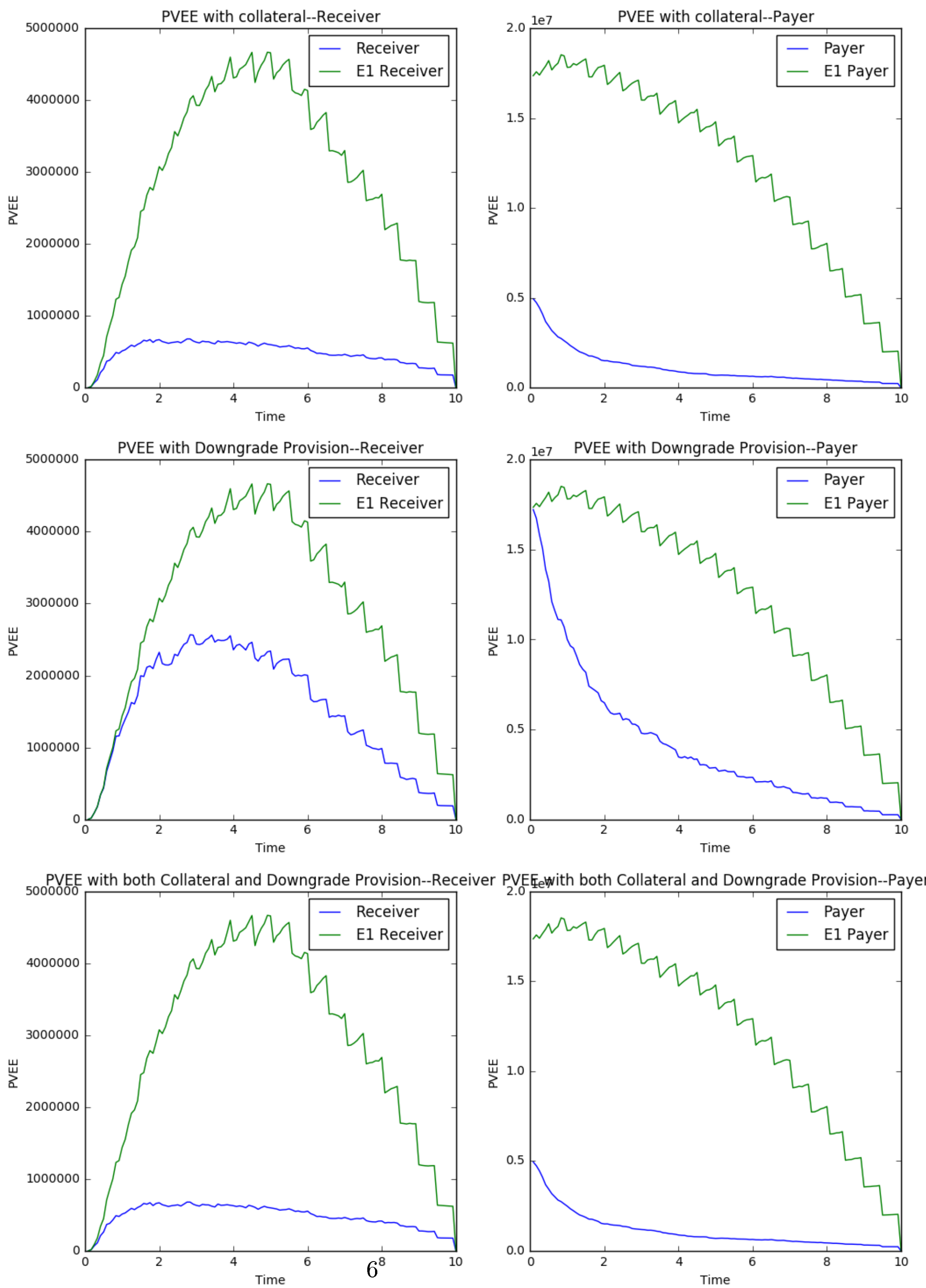

Figure 7: With Credit Mitigants

Both collateral and downgrade provision decreases the $PVEE$. Collateral decreases $PVEE$ more than downgrade provision does.

7.

Turn off the credit mitigants again, and now compute the bilateral CVA, DVA, and net CVA for the naked receiver swap position. Compare against the results in Exercises 2 and 3. Explain. The result

	CVA	DVA	Net CVA
Unilateral	5.342e+06	1.96e+06	3.381e+06
Bilateral	5.095e+06	1.698e+06	3.396e+06

Figure 8: Bilateral CVA, DVA, and net CVA

Explanation:

We can see the $CVA_{unilateral} > CVA_{bilateral}$ and $DVA_{unilateral} > DVA_{bilateral}$, this is because when calculating bilateral numbers, there is an extra discount factor of survival probability.

4 IMM Exercise

1.

Turn off all credit mitigants, and compute and graph the expected exposure profile (as in Section 4.2 of [1]) for both the payer and receiver swap. Contrast the results with those of Exercise 1 in Section 3 above. The result graph:

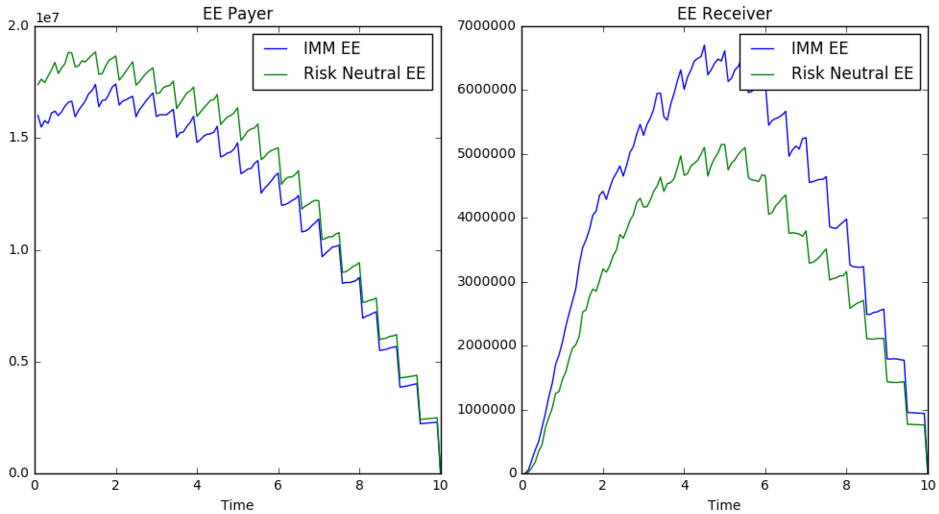

Figure 9: IMM Expected Exposure Profile

EE for payer using historical measure is lower than that using risk-neutral measure, while EE for receiver using historical measure is higher than that using risk-neutral measure.

2.

Using the IMM formulas in [3], compute the EEPE, the EAD and the weighted maturity (M) the payer and receiver swaps (as seen from B's perspective). The result

	EEPE	EAD(Basel 2)	Maturity
Payer	1.821e+07	2.549e+07	1.675
Receiver	6.413e+05	8.978e+05	5

Figure 10: EEPE, EAD, and Weighted Maturity

3.

Assuming that the 1-year (historical) default probability for rm C is PD = 1%; use the results of Exercise 3 to compute rm B's regulatory credit capital for the receiver and payer swap positions, respectively. The result

```
Regulatory Capital for payer is 1597453.32541
Regulatory Capital for Receiver is 85279.56136
```

Figure 11: Regulatory Credit Capital

Appendix: Some derivations

Simulate OIS curve

From equation (18) and (19) from [1], we get

$$\sigma_{22} = \sigma_2$$

$$\sigma_{21} = \sigma_1 \times \rho_x$$

$$\sigma_{11} = \sqrt{\sigma_1^2 - \sigma_{21}^2} = \sqrt{\sigma_1^2 - \sigma_1^2 \times \rho_x^2}$$

For $y(t)$, we calculate the involved integral manually:

$$\begin{aligned}\int_0^t a(u)^T a(u) du &= \int_0^t \begin{bmatrix} \sigma_{11}^2 e^{2\kappa_1 u} + \sigma_{21}^2 e^{2\kappa_1 u} & \sigma_{21}\sigma_{22} e^{(\kappa_1+\kappa_2)u} \\ \sigma_{21}\sigma_{22} e^{(\kappa_1+\kappa_2)u} & \sigma_{22}^2 e^{2\kappa_2 u} \end{bmatrix} du \\ &= \begin{bmatrix} \frac{\sigma_{11}^2+\sigma_{21}^2}{2\kappa_1}(e^{2\kappa_1 t} - 1) & \frac{\sigma_{21}\sigma_{22}}{\kappa_1+\kappa_2}(e^{(\kappa_1+\kappa_2)t} - 1) \\ \frac{\sigma_{21}\sigma_{22}}{\kappa_1+\kappa_2}(e^{(\kappa_1+\kappa_2)t} - 1) & \frac{\sigma_{22}^2}{2\kappa_2}(e^{2\kappa_2 t} - 1) \end{bmatrix}\end{aligned}$$

For $P(t, x(t), T)$, we know

$$P(t, x(t), T) = e^{-\int_t^T f(t,x(t),u)du}$$

where

$$f(t, x(t), u) = f(0, u) + M(t, u)^T (x(t) + y(t)G(t, u))$$

Since $f(0, u)$ is constant, then

$$\begin{aligned}P(t, x(t), T) &= e^{-f(0,T)+f(0,t)} e^{-x(t)\int_t^T M(t,u)du} e^{-y(t)\int_t^T M(t,u)^T G(t,u)du} \\ &= \frac{P(0,T)}{P(0,t)} exp\big(- G(t,T)^T x(t) - \frac{1}{2} G(t,T)^T y(t) G(t,T)\big)\end{aligned}$$

where

$$M(t, T) = \begin{bmatrix} e^{-\kappa_1(T-t)} \\ e^{-\kappa_2(T-t)} \end{bmatrix}$$

and

$$G(t, T) = \int_t^T M(t, u) du = \int_t^T \begin{bmatrix} e^{-\kappa_1(u-t)} \\ e^{-\kappa_2(u-t)} \end{bmatrix} du = \begin{bmatrix} \frac{1-e^{-\kappa_1(T-t)}}{\kappa_1} \\ \frac{1-e^{-\kappa_2(T-t)}}{\kappa_2} \end{bmatrix}$$