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3547 Intelligent Agents & Reinforcement Learning

Module 7: Dynamic Programming & Monte Carlo Methods

Course Plan

Module Titles
Module 1 – Introduction to Intelligent Agents
Module 2 – Search
Module 3 – Logical Inference
Module 4 – Planning and Knowledge Representation
Module 5 – Probabilistic Reasoning
Module 6 – Intro to Reinforcement Learning and Finite Markov Decision Processes
Module 7 – **Current Focus: Dynamic Programming and Monte Carlo Methods**
Module 8 – Temporal Difference Learning
Module 9 – Function Approximation for RL
Module 10 – Deep Reinforcement Learning and Policy Gradient Methods
Module 11 – Introduction to Advanced DRL
Module 12 – Presentations (no content)

Learning Outcomes for this Module

- Discuss how an optimum policy can be computed when a model is known
- See how an optimum policy can be learned when the environment dynamics aren’t known in advance

Topics for this Module

- **7.1** Dynamic Programming
- **7.2** Monte Carlo Methods
- **7.3** Resources and Wrap-up

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Module 7 – Section 1

Dynamic Programming

Review of the Four Elements

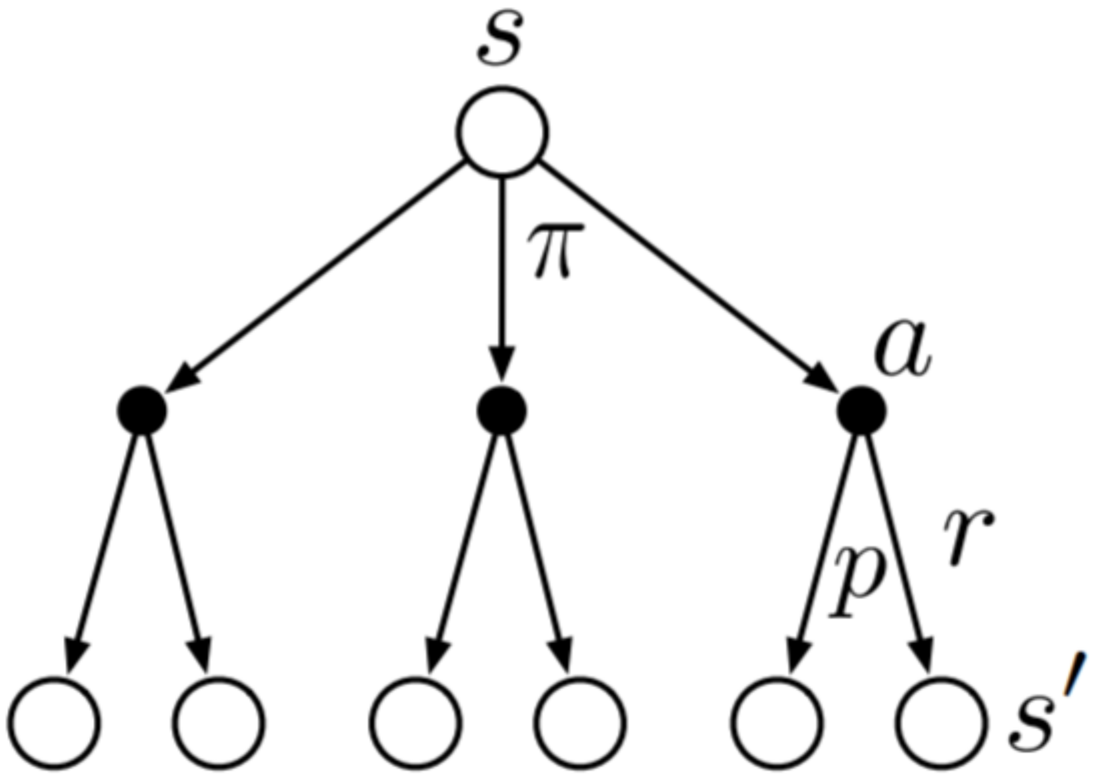
- **Policy**
  - $\pi(a|s)$
- **Reward Signal**
  - R
- **Value Function**
  - Value of a state: $v_\pi(s) = E_\pi(G_t|S_t = s)$
  - Value of a state-action: $q_\pi(s,a) = E_\pi(G_t|S_t = s, A_t = a)$
- **Model of the Environment**
  - $p(s', r|s, a) = \Pr(S_t = s', R_t = r|S_{t-1} = s, A_{t-1} = a)$

The Bellman Equation for v_π

- Tells us a recursive condition that must hold for any policy:

$$
\begin{aligned}
v_\pi(s) &= E_\pi(G_t|S_t = s) \\
&= E_\pi(R_{t+1} + \gamma G_{t+1}|S_t = s) \\
&= \sum_a \pi(a|s) \sum_{s\prime} \sum_r p(s', r|s, a)[r + \gamma E_\pi(G_{t+1}|S_{t+1} = s')] \\
&= \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)[r + \gamma v_\pi(s')], \text{ for all } s \in S
\end{aligned}
$$

The Backup Diagram for v_π



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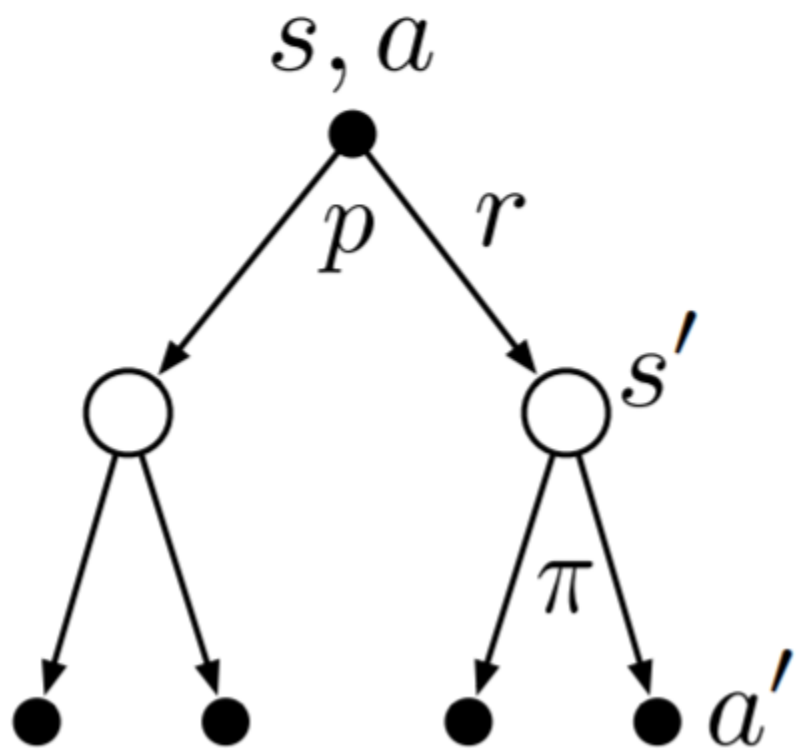
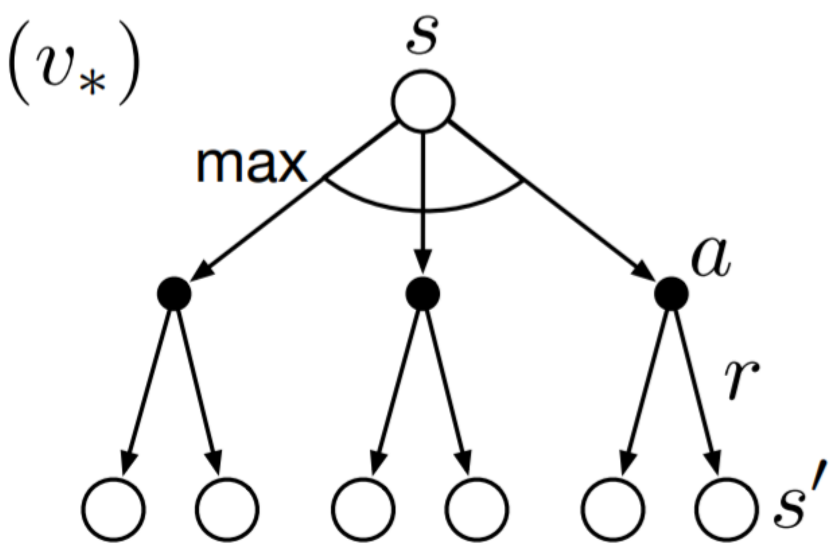
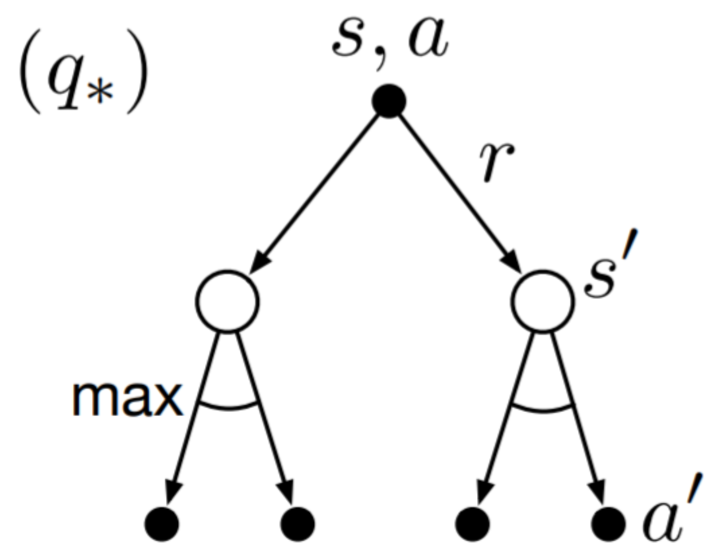
The Backup Diagram for q_π



Bellman Optimality





Solving for Optimality

- The objective is to optimize long-run total reward
- We've seen we can do this by always choosing the next action with the largest $q_t(a)$ (or an approximation $Q_t(a)$ if that's all we have)
- For Finite MDP's the Bellman equation is a system of n non-linear equations in n unknowns
- Is there another way we can solve it, given that it is defined recursively?

Policy Evaluation (Prediction)

- Yes! Dynamic Programming!
- We start by solving for any v_π, not just optimal ones
- We convert the Bellman equation into an update rule:

$$v_{k+1}(s) = \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)[r + \gamma v_k(s')]$$

  and (very) fortunately the sequence $\{v_k\}$ converges to v_π as $k \to \infty$
- Rather than using separate arrays for v_k and v_{k+1} it turns out that this usually converges faster if we use a single array
- We sweep through the entire array v updating all the values on each iteration

Policy Improvement

- If we have any old policy π and its state-value function v_π we can find better policies by being greedy and choosing the action in each state that takes us to a state with the biggest expected value of $v_\pi(s')$

$$\pi'(s) = \operatorname{argmax}_a \sum_{s',r} p(s', r|s, a)[r + \gamma v_\pi(s')]$$

- Remember, in general π is actually π(a|s), a probability distribution over actions for each state so if several actions are equally optimal, we can keep them all and select between them with any distribution we like

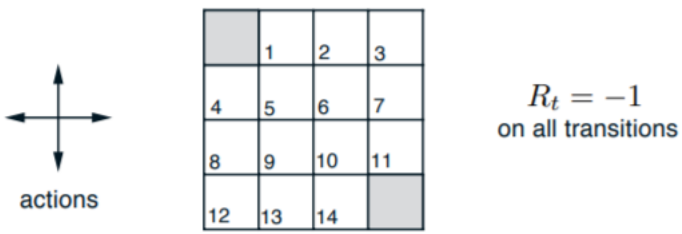
Policy Iteration

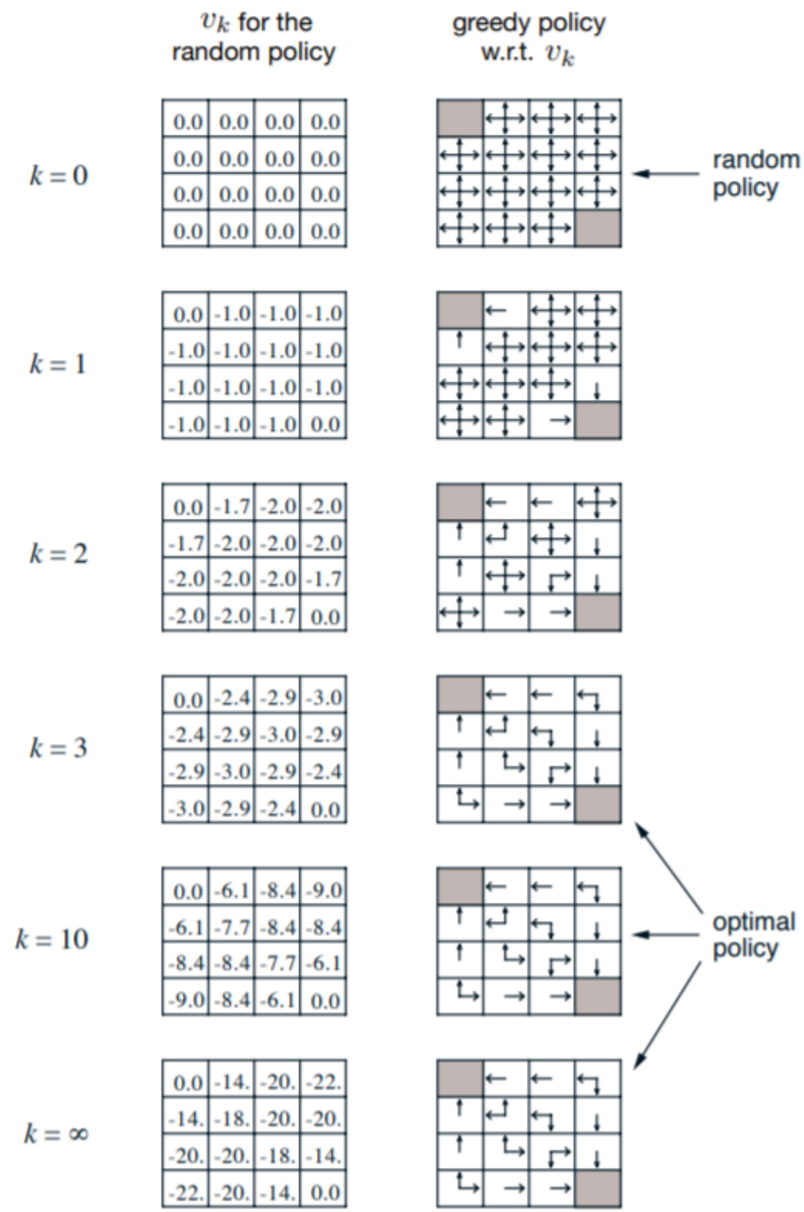
- Now we have a better policy π_{k+1}, but our value function v_k is not going to be still right for it, so we need to re-evaluate it to get v_{k+1}
- But then we can find a better π: π_{k+2}, and so on…
- We can interleave recalculations of v and π
- Repeatedly switching between policy evaluation and improvement will eventually converge to our optimal π_*

Value Iteration

- Each iteration of *policy iteration* requires a *policy evaluation* which means computing successive values of $v(s)$ over all of the states potentially many times before it converges to v_π
- What if we just make $v(s)$ a little closer to v_π (just one sweep through the states) then do a policy improvement?
- Turns out this converges too (sometimes even quicker) and is called **value iteration**

Policy Evaluation & Improvement

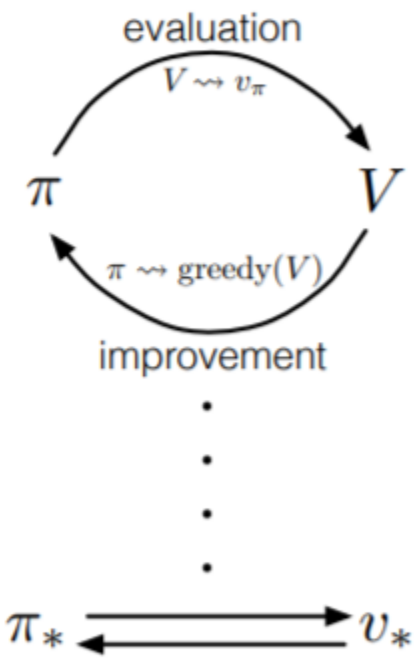


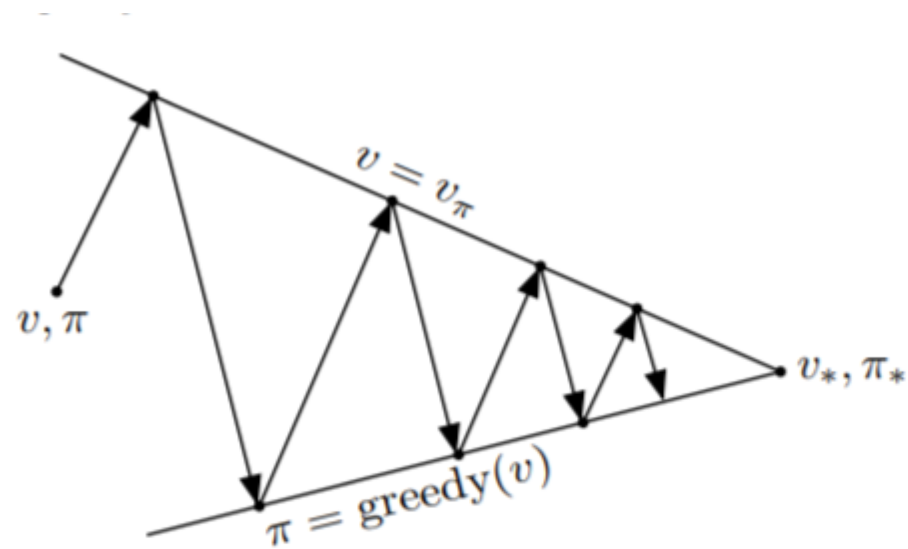


Source: Sutton & Barto

Generalized Policy Iteration

- Sutton and Barto call this notion of interacting policy evaluation and improvement **generalized policy iteration**





Source: Sutton & Barto

Asynchronous Dynamic Programming

- Turns out we don’t even need to do full state sweeps
- We can update states in any order and any number of times as long as all states are eventually updated
- Particularly useful if the agent is interacting with the environment so new information about some states is becoming available and we’d like to use it immediately
- Asynchronous DP algorithms are designed for these situations

Efficiency of Dynamic Programming

- DP is very efficient (polynomial in the number of states and actions)
- Can potentially be used with state spaces with millions of states
- But still not efficient enough for very large state spaces
- Here asynchronous DP is often better, or approximating methods we will look at next

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Module 7 – Section 2

Monte Carlo Methods

Monte Carlo Prediction

- What if we don't have a complete specification of the environment's dynamics?
- How can we learn from actual or simulated experience?
- Monte Carlo methods apply when we need to roll out to the end of an episode to determine the total reward
- We'll build on the methods we've already discussed

Monte Carlo Estimation of State Values

- When we can't determine $v_\pi(s)$ exactly we need to estimate it: $V(s)$
- The strategy: simulate many entire episodes and average them
- We can update $V(s)$ the first time we visit state s or every time we visit it: they both converge
- Consider how this would work in various games: Blackjack, chess, checkers, Go, WumpusWorld

Monte Carlo Estimation of Action Values

- We can roll out and update estimates $Q(s,a)$ of the expected return of state-action values $q_\pi(s,a)$ in the same way as we did for $V(s)$
- *But* we need to ensure that all valid actions can be tried, either by:
  - Intentionally starting in states chosen at random (**exploring starts**), or
  - Always having some chance of selecting any of the actions available in a state (for example, by being ε-greedy), or
  - By using a separate, exploratory policy for learning and adjusting the expected values of the rewards to more closely match what they should be under the current actual policy (called **off-policy estimation with importance sampling**)

Monte Carlo Control

- Sutton and Barto refer to the process of finding or approximating optimum policies as the **control problem**
- The issue is the huge number of episode rollouts required to fully learn $Q(s, a)$ so we need to compromise in some way:
  - Learn $Q(s, a)$ for many state-actions but not very accurately
  - Learn $Q(s, a)$ only for likely trajectories

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Module 7 – Section 3

Resources and Wrap-up

Resources

- Sutton & Barto. Reinforcement Learning, 2nd Ed. MIT Press. 2018.
- Richard Sutton's website: http://incompleteideas.net/
- https://gym.openai.com/
- Pumperla & Ferguson. Deep Learning and the Game of Go, Chap. 9. Manning. 2019.
- https://github.com/ShangtongZhang/reinforcement-learning-an-introduction/blob/master/chapter05/blackjack.py (note: you'll probably need to `pip install tqdm` if you want to try running it yourself)

Summary

- If we have a complete specification of the dynamics of the environment we can directly solve for an optimal set of actions
- If not, we can estimate the value functions probabilistically using Monte Carlo techniques

Next Week

- Temporal Difference Learning

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Any questions?

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Thank You

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