Tutorial 0A

Vectors

1. Let $\mathbf{u} = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 5 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ -3 \end{bmatrix}$. Find scalars a and b so that $a\mathbf{u} + b\mathbf{v} = \begin{bmatrix} 3 \\ -3 \\ -3 \\ 11 \end{bmatrix}$.

2. Find the initial point of a non-zero vector $\mathbf{u}$ with terminal point $Q(3, 0, -5)$ such that $\mathbf{u}$ is oppositely directed to $\mathbf{v} = \begin{bmatrix} 4 \\ -2 \\ -1 \end{bmatrix}$.

3. For $\mathbf{u} = \begin{bmatrix} -2 \\ -1 \\ 4 \\ 5 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ -5 \\ 7 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -6 \\ 2 \\ 1 \\ 1 \end{bmatrix}$, evaluate $\|3\mathbf{u} - 5\mathbf{v} + \mathbf{w}\|$ and $\|3\mathbf{u}\| - \|5\mathbf{v}\| + \|\mathbf{w}\|$.

4. Find the Euclidean distance and the cosine of the angle between the vectors $\mathbf{u} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$.

5. Given vector $\mathbf{u} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$, find a unit vector that (a) has the same direction as $\mathbf{u}$, (b) is oppositely directed to $\mathbf{u}$.

6. Determine if the points $A(1, 1, 1), B(-2, 0, 3)$ and $C(-3, -1, 1)$ form the vertices of a right angle triangle.

7. Find the work done by the force $\mathbf{F} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$ in moving a particle from a point $P(1, 4, -1)$ to a point $Q(-2, 3, 1)$. Use the relation Work $= \mathbf{F} \cdot \mathbf{r}$, where $\mathbf{r}$ is the displacement.

8. Find the point-normal form of the equation of a plane passing through $P(-1, 3, -2)$ and having normal $\mathbf{n} = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$.

9. Find the vector and parametric equations of a plane containing the point (-3,1,0) and the vectors $\mathbf{v}_1 = \begin{bmatrix} 0 \\ -3 \\ 6 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -5 \\ 1 \\ 2 \end{bmatrix}$.

10. The position vectors of the points A and B are $\begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}$, respectively.

    Find the vector equation of the line AB and find the points where the line intersects the $x - y$ plane.

Answers

1. $a = 1, b = -2$
2. **u**=(7,-2,-6) is one possible answer.
3. $\sqrt{2570}$ and $3\sqrt{46} - 10\sqrt{21} + \sqrt{42}$.
4. $\sqrt{14}$, $\cos\theta = \frac{15}{\sqrt{27}\sqrt{17}}$
5. (a) $(\frac{3}{5}, -\frac{4}{5})$

   (b) $(-\frac{3}{5}, \frac{4}{5})$
6. Yes
7. 3 units
8. $-2(x + 1) + (y - 3) - (z + 2) = 0$
9. Vector equation: $(x, y, z) = (-3, 1, 0) + t_1(0, -3, 6) + t_2(-5, 1, 2)$

   Parametric equation: $x = -3 - 5t_2, y = 1 - 3t_1 + t_2, z = 6t_1 + 2t_2$
10. $\mathbf{r} = (1, 4, 6) + t(2, 0, 1)$; point of intersection with $x - y$ plane is (-11,4,0).

End