Tutorial 0B

Matrices

1. Given matrices $A = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$,

   and $D = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$, find

   (i) -3 trace $(A - 3B)$
   (ii) trace $(C^T D^T + 2B^T)$.

2. Given matrices $A = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$, use the row
   method or column method (as appropriate) to find

   (i) the first row of BA
   (ii) the second column of AB.

3. For the matrices A and B in Q2, express each column vector of BA as a
   linear combination of the column vectors of B.

4. If a matrix A satisfies the equation $(7A)^{-1} = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}$, find A.

Answers

1. (i) 75   (ii) 28

2. (i) $\begin{bmatrix} 6 & -6 & 70 \end{bmatrix}$.

   (ii) $\begin{bmatrix} 41 \\ 21 \\ 67 \end{bmatrix}$.

3. First column of $BA = 3 \begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix} + 6 \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} + 0 \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 63 \end{bmatrix}$. Similarly for the
   other columns.

4. $\begin{bmatrix} 2/7 & 1 \\ 1/7 & 3/7 \end{bmatrix}$

<div align="center">End</div>