Python

Variables & Data type

- Data is raw facts and figures
- Data type is the way in which we represent information in computer programming language.
  Depending upon the nature of information we want to represent, we have different data types in python.
- Variable is an entity that represents the info from RAM

Data type

① Numeric

→ int
→ float point
→ Complex. → c = 3+8j
print(c)
print(type(c))

To know the type of the variable → type(variable_name) or type(" ")

② Text type

→ String → represent " " / ' '
To calculate length → len(variable_name)

③ Sequence Type

→ List.

Array concept; mutable.
any data type object can be come in the list.
represent → [, ,]
len → no: of variable ~~to~~ return [eg:- ~~dig~~ len(digits)]

→ Tuple

represent → (.)

any datatype object can be come in tuple.

immutable.

* Tuples are similar to list but are immutable once you create a tuple, you can't change, add /remove items.

This immutability can help protect data from accidental modification and also makes tuple more m/y - efficient.

Indexing operation (do in list & tuple).

variable_name[0] → 1st element. (list) & (tuple).

→ Range.

function & datatype.

immutable.

range(start, stop, step)

④ Mapping Type.

→ Dictionary

unordered

mutable collection

data store → key-value pair

key → unique, values → duplicated.

Variabl.name = { Key : value }

Access the value → variable_name["key"]

add item → variable_name[key] = pair

Access key

dict-name.keys()

Access value

dict_name.values().

⑤ Set Type

→ Sets.

unordered
unique items, duplication can't possibl.
represent {;;;}
mix of various data type possible.
empty_set = set().
No indexing.

⑥ Boolean Type

→ Boolean.

True or False. ← First letter capital.
used conditional statements & expression to make decisions in code. (comparison / condition verification)
Control design for programs.

⑦ None Type

→ None

absence of ~~a~~ value / null value.
eg:- name = None.

Operators in python

① Arithmetic operators

Addition +
subtraction -
multiplication *
division /
modulus % (Remainder part give)
exponentiation **
floor division → fractional part is no
// only ~~Quot~~ Quotient.

② Assignment operators.

equals = (Assign)

③ comparison operators.

return Boolean ~~e~~value (True / False).

== (a==b)
!= Not equal
> Greater than
< less than
>= Greater or equal to
<= less than or equal to

④ logical operators

AND ~~(either or~~ both are true → it is true
OR (any one is true → it is true
NOT

⑤ Membership operators. (Test in lists, strings, etc)

IN
NOT IN

⑥ Identity operators.

- : 2 variables refers to the same object in mry.

IS
IS NOT

Conditional Statement

if

```
if True :
    print ("Inside if block")
```

4 space or tab.

if - else

```
if True:
    print (" Inside if block")
else:
    print (" Inside else block")
```

if - elif

```
if condition:
    pass
elif condition:
    pass
elif condition:
    pass.
```

if - ~~else~~ elif - else

```
if condition:
    pass
elif condition:
    pass
else:
    pass.
```

Nested - if

```
if condition:
    pass
    if condition:
        pass
    elif condition:
        pass
    else:
        pass.
```

1. write code to check whether a given string is palindrome or not.
2. write code to do check a given number is divisible by 9 if it is odd & divisible by 4 if it is even.
3. Grade checker: create a program that asks the user for a score (b/w 0 and 100) & prints the grade based on the score.

   90 & above: A
   80 to 89 : B
   70 to 79 : C
   60 to 69 : D
   Below 60 : F.

(4) Number sign: write a program that takes a number as input and prints whether the number is "positive", "Negative", or zero.

(5) Range→
print 100-0, odd numbers.

(6) create a list, then print
second item
last item
second last items.

(7) print age & year of birth accepting user i/p & print formatting.

operator overloading

means giving extended meaning beyond their predefined operational meaning.

eg:- operator '+' is used to add 2 integers as well as used to join two strings and merge two lists.

```
>> "I am" + "Lekshmi"
o/p -> 'I am Lekshmi'
```

*

```
>> "lekshmi" * 2
-> 'lekshmilekshmi'
```

→ Create a dictionary to store movies and its rating. Print all movies and its rating.

```
movie_rating = {"Drishyam": 9, "premam": 7,
                "The God Father": 9, "Bangalore Days": 5,
                "Angry Men": 4, "kumbalangi Nights": 10}
Print(movie_rating).
```

→ ~~pr~~

→ update a movie rating from 8 to 5 or 5 to 9.

```
movie_rating["Bangalore Days"] = 9.
print (movie_rating)
```

or

```
for x, y in movee_rating.items():
    if y == 5:
        movie-rating[x] = 9
Print(movie_rating).
```

→ add 2 new movies and their ratings to dictionary.

```
movie_rating.update({"ustad Hotel": 10, "Take off": 7,
                     "Traffic": 4})
Print (movie_rating).
```

→ Find the highest rated movies from the dictionary.

```
highest_rating = 0
top_list / highest rating movie = [ ].

for x,y in movie_rating.items():
    if y > highest_rating
        highest_rating = y
        top_list = [x]
    elif y == highest_rating
        top_list.append(x)
Print(f" Highest rating movie was {top_list} \n
  rating {highest_rating}).
```

→ removes from dictionary if rating below 5

```
for x,y in movies_rating.item():
    if y < 5:
        del movies_rating[y]
Print(movie_rating).
```

→ list = []

for x, y in movie_rating ~~[]~~ items():

    if y < 5:

    list.append(x)

for z in list ~~()~~:

    del ~~list~~ movie-rating [z]

Print (movie_rating).

} we must delete the list other wise it print the entire dictionary. (like we only list the deleted item not delete them that's why we should delete the list)

or

→ ~~fo~~ movie_rating = { k: v for k, v in movi_rating() if v > 5 }.

print (movie-rating)

Iterative statements - (used for Repetitions)

For loop

For loops can be applied on any sequential object (list, string, tuple, range, etc).

eg. for char in "Hello":
        Print(char)

eg:- for i in range(10):
        print(i)

while loop

i = 1
while i < 4:
    Print (i)
    i = i+1

eg.- msg = "welcome"
    i = 0
    while i < 7:
        Print(message[i])
        i = i+1

o/p → w
e
l
c
o
m
e

Break and continue statements in loops

- control the loop flow.

Break

If we want to out of the loop if a condition met. then we use "break".

eg:-
```
for i in range(1,6):
    if i==4:
        break
    Print(i)
print ("outside loop")
```
o/p →
```
1
2
3
outside loop.
```

Continue:

After continue then skip the statements after that.

eg:-
```
for i in range(1,6):
    if i==4:
        continue.
    print (i)
print ("outside loop")
```
o/p →
```
1
2
3
5
outside loop.
```

Functions

- In python, functions are usable blocks of code that perform a specific task.
- help make programs modular, readable & easier to maintain.
- built-in or user-defined blocks.
- Built-in functions (provided by python)
  eg:- print(), len(), type(), sum(), max() etc.

user-defined functions (created by the programmer)
using 'def' keyword., followed by function name.

Syntax → def function-name (parameters):
            # statement.
            return expression.

1. Simple function

eg:- def greet():
        print ("Hello, welcome to python!")
     greet()

o/p → Hello, welcome to python!

2. Function with parameters

```
def add-num (a,b):
    return a+b
result = add-num (5,3)
print ("sum", result)
```

o/p → sum: 8.

3. function with default parameters

```
def greet_user (name = "guest"):
        print (f"Hello, {name}!")
greet_user("Alice")
greet_user()
```

o/p → Hello, Alice!
Hello, guest!

4. Function with variable Arguments (*args and **kwargs)

*args - Allows passing a variable number of positional arguments.

**kwargs - Allows passing a variable number of keyword arguments.

eg:-
```
def display_info (*args, **kwargs)
      print ("positional argument:," args)
      print ("keyword arguments:," kwargs)
display_info (1,2,3, name = "Alice", age = 25)
```

o/p → positional arguments : (1, 2, 3)
keyword arguments : {'name': 'Alice', 'age': 25}

5. Returning multiple values.

```
def Calculate (a,b):
      return a+b, a-b, a*b, a/b
add, sub, mul, div = Calculate (10,5)
Print (f"Add : {add}, Sub: {sub}, Mul: {mul}, Div : {div}")
```

o/p → Add: 15 , ~~Mul~~ : sub: 5 , Mul : 50, Div : 2.0

key points

1. return → return a value from a function

2. scope → variables defined inside a function are local to that functions.
"global" keyword used to modify ~~loca~~ global variables inside a function

3. Docstrings → documentation string to describe their purpose

```
def ~~sto~~greet():
    """ This function prints a greeting msg."""
    print("Hello!")
```

4. Lambda functions → Anonymous, single expression function.

eg:-
```
square = lambda x : x**2
print (square (4))
```
o/p → 16

object oriented programming in python

- Is a programming paradigm that organizes code into objects and classes to model real-world entities and their behaviour.
- Python is a object oriented language
- supports ~~p~~ oop principles such as encapsulation, inheritance, polymorphism and abstraction.

1. Class

A blueprint for creating objects.
Defines attributes (data) and method (behaviour)

eg:-
```
class person():
    def __init__(self, name, age):
        self.name = name
        Self.age = age.
    def greet(self):
        print(f"Hello, my name is {self.name} and
               I am {self.age} year old.")
```

2. object

An instance of a class.

eg:-
```
person1 = person ("Alice", 30)
person1.greet()
```

o/p → Hello, my name is Alice and I am 30 year old.

3. Encapsulation

- Hides internal details of a class
- Achieves using private attributes (__attribute)

eg:-
```
class BankAccount:
    def __init__(self, Balance):
        self.__balance = balance.
    def deposite(self, amount):
        self.__balance += amount
    def get_balance(self):
        ~~account~~
        return self.__balance
account = BankAccount(1000)
account.deposite(500)
print(account.get_balance())
```

o/p → 1500

4. Inheritance

Allows one class (child) to acquire propeties and methods from another class (parent).

eg:-
```
class Animal:
    def speak(self):
        print("Animal speaks")
class Dog(Animal):
    def speak(self):
        print("Dog barks")
dog = Dog()
dog.speak()
```

o/p → Dog barks

5. polymorphism

The ability to use a single interface for different datatypes or classes.

eg:-
```
class Birde:
    def fly(self):
        Print ("Bird can fly")
class penguin (Bird):
    def fly(self):
        Print ("penguin cannot fly")
def flying-test(bird):
    bird.fly()
Bird = Bird()
penguin = Penguin()
flying-test(bird)
flying-test(penguin)
```

o/p → Bird can fly
penguin cannot fly..

6. Abstraction.

- Hides implementation details, showing only essential features
- Implemented using Abstract Base class (ABC).

eg:- from abc import ABC, abstractmethod.

```
class shape(ABC):
    @abstractmethod
    def area(self)
        pass

class Circle(shape):
    def __init__(self, radius):
        self.radius = radius

    def area(self):
        radius = 3.14 * self.radius * self.radius

circle = Circle(5)
print(circle.area())
```

o/p → 78.5.

Advantage of OOP

- modularity → code is organized into classes
- Reusability → classes and method can be reused
- Maintainability → Encapsulation helps manage code better
- Scalability → Inheritance & polymorphism make extending easy.

Constructors

To initalize the objects attributes and set up any necessary state for the object.
defined using the __init__() method

Complete oop program

```
class vehicle:
    def ~~b~~ __init__(self, brand, model):
        self.brand = brand
        self.model = model.
    def display-info(self):
        print(f"vehicle: {self.brand} {self.model}")
class car(vehicle):
    def __init__(self, brand, model, doors):
        super().__init__(brand, modi)
        self.door = doors.
~~class~~ def display-info(self):
        super().display-info()
        print(f"Doors: {self.doors}")
class Bike (vehicle):
    def __init__(self, brand, model, type-of-bike):
        super(). ~~init~~ __init__(brand, model)
        self.type-of-bike = type-of-bike
    def display-info (self):
        super().display-info()
        print(f"Type : {self.type-of-bike}")
car = car("Toyota", "Corolla", 4)
bike = Bike ("Yamaha", "MT-15", "sports")
car. display_info
Bike. display_info
```

o/p → Vehicle: Toyota Corolla
Doors: 4
Vehicle: Yamaha MT-15
Type: sports.

This eg combine inheritance, method overriding, encapsulation, and reuseability.

Math Notation, statistics, Measure central tendency

μ - population mean (Total number, divide n) → Average of entire group

$\bar{x}$ - sample mean (Total number, divide $n-1$) → Average of subset of the population.

σ - standard deviation (SD)

σ^2 - Variance

$$SD = \sqrt{Variance} = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$$

$$Variance = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

Sample S.D

$$\sqrt{\frac{1}{N-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

population S.D

$$\sqrt{\frac{1}{N} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Sample Variance

$$\frac{\Sigma (x_i - \bar{x})^2}{n-1}$$

population Variance

$$\frac{\Sigma (x_i - \bar{x})^2}{n}$$

~~Range~~ of class interval = upper limit - lower limit.

frequency → Count of occurrences (how often a value appears)

Range → difference b/w largest & smallest value = Max - Min.

Measure of central tendency

$$Mean (\mu) = \frac{\text{Sum of values}}{\text{No: of values}}$$

median → arrange descending / ascending order, for odd no: of data point - middle value (median) else even - average of 2 middle values.

mode - most frequently occurring value in the sample / population.

Measure of Dispersion

1. Range = max - min

2. Variance (σ^2) = $\frac{\Sigma(x-\mu)^2}{n}$

3. S.D (σ) = $\sqrt{(\sigma^2)}$

4. Interquartile Range (IQR)

$$IQR = Q_3 - Q_1$$

5. Quartiles (Q) :- divides the data set into 4 equal parts.

Q_1 - median of the lower 25% (25%) → percentile

Q_2 - median (50%) (50%) → percentile

Q_3 - median of the upper 25% (75%) → percentile.

Outliers - extreme values (far from the actual values).

6. Co-efficient of Variance (CV)

$$CV = \left(\frac{\sigma}{\mu}\right) * 100$$

σ - SD

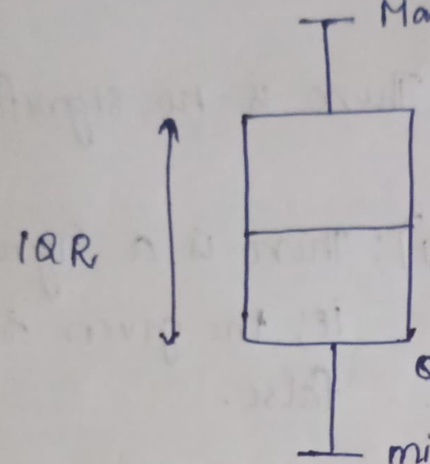
μ - mean.

7. Mean Absolute Deviation.

$$MAD = \frac{\sum_{i=1}^{n} |x-\mu|}{n}$$

Box plot

data visualization that summarizes a dataset's distribution



upper limit $= Q_3 + 1.5 * IQR$

lower limit $= Q_1 - 1.5 * IQR$.

condition for outliers.

outliers < lower limit
or
outliers > upper limit

Covariance

$$cov(x, y) = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n}.$$

Correlation

$$\rho(x, y) = \frac{cov(x, y)}{\sigma_x \sigma_y}$$

range → -1 to 1

0 → light

-1 → highly -ve. correlated

+1 → highly +ve correlated.

z-score

$$z\text{-score} = \frac{x_i - \mu}{\sigma}$$

x_i → data value.　σ → SD

μ → mean

~~Z-Test~~

Hypothesis Testing -

Null Hypothesis (H_0) : There is no significant difference or effect.

Alternative Hypothesis (H_1) : There is a significant effect ie; the given statement can be false.

level of significance (α) : It is a threshold used to determine statistical significance.
(values common - 0.05, 0.01 or 0.10)

medicine (0.01)

e-commerce (0.05)

Z - Test

used to compare means when sample size is large $n > 30$. and population variance - known.

Steps

1. State Null Hypothesis
2. State Alternate Hypothesis
3. choose your significance level (α)
4. Calculate your Z-test statistics

$$Z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

$\bar{x}$ - sample mean

μ - population mean

σ - population S.D

n - sample size.

5. Find p-value using z-table & z-test statistics computed

6. if p-value > α,

then we fail to reject null Hypothesis (Accept Alternate Hypothesis)

7. There are one-tailed & two-tailed tests.

~~One~~ Two tailed, Alternative hypothesis look like this

$$H_0 : \mu = \mu_0 \quad Vs \quad H_1 : \mu \neq \mu_0 .$$

~~two~~ One tailed, look like (A.H)

Right tailed, $H_0 : \mu \leq \mu_0$ Vs $H_1 : \mu > \mu_0$

Left tailed, $H_0 : \mu \geq \mu_0$ Vs $H_1 : \mu < \mu_0$

eg:- A particular companies chocolate bars are supposed to have an average weight of 50 grams according to the manufacturer. We want to test if a sample of chocolate bars deviates significantly from this weight. Data : 50.8, 49.5, 50.2, 51, 49.7, 50.3, 49.8, 50.5, 49.6, 50.1. population Standard deviation : 1.5 grams.

$H_0 : \mu = 50$

$H_1 : \mu \neq 50$. 2 tailed use (deviates or not)

$\alpha = 0.05$

$\bar{x} = 50.15$

$$S.D = \frac{\sigma}{\sqrt{n}} = \frac{1.5}{\sqrt{10}} = 0.47434$$

$$z = \frac{\bar{x} - \mu_0}{SE} = \frac{50.15 - 50}{0.47434}$$

$$z = \underline{\underline{0.3162}}$$

P value : 2 tailed.

$P = 0.7518$.

Here $\alpha = 0.05$, $P = 0.7518$, $z = 0.3162$

$\alpha \quad P$

$0.05 < 0.7518$

So fail to reject H_0.

$\therefore$ No significant deviation from 50 g.

T-Test

- we need T-Test because population standard deviation, σ is not always available.
- we will computing T-test statistic (based on problem statement & type of t-test.

Type of T-Test	Null Hypothesis	Alternative Hypothesis	Degree of freedom
• one sample T-test	sample mean = reference mean	sample mean $\neq$ reference mean	$df = n-1$
• Independent Sample T-test	mean values in both groups are same	mean values in both groups not same	$df = n_1 + n_2 - 2$

- paired sample t-test | mean value of the difference b/w the pairs is zero | mean value of difference b/w the pairs is not zero | df=n-1

one-sample T-Test - steps

Compares the sample mean to a known population mean.

Steps:

1. State the Null Hypothesis
2. State the Alternate Hypothesis
3. choose your significance level (α)
4. calculate your T-Test statistics.

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)}$$

$\bar{x}$ - sample mean
μ - population mean
s - sample SD.

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

5. critical t-value find from T-Table using α.
6. If t-statistic < critical t-value, we fail to reject Null Hypothesis

Q. Same eg. as we do in z-test.

$H_0 : \mu = 50$

$H_1 : \mu \neq 50$.

$\bar{x} = 50.15$; $s = 0.5104$, $n = 10$, $SE = \frac{s}{\sqrt{n}} = 0.1614$

T-Test statistic, $t = \frac{\bar{x} - \mu_0}{SE} = \frac{50.15 - 50}{0.1614} = \underline{0.929}$

$$df = 10 - 1$$
$$= \underline{9}$$

T ~~P~~ value (2-tailed)

~~P = 0.377~~ T-value = 2.262

t-test statistic T-value.

$$0.929 < 2.262$$

$\therefore$ we fail to reject null Hypothesis.

Independent Sample T-Test

compares means of 2 independent groups.

Steps:

1. State null Hypothesis
2. State Alternate Hypothesis.
3. choose your α.
4. Calculate your T-Test statistic

$$\cancel{t = \frac{\bar{x}_1 - \bar{x}_2}{\frac{s_1^2 + s_2^2}{n_1}}} \qquad t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}, \text{ for one tailed.}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \text{ for 2 tailed}$$

sample mean

$\rightarrow$ sample SD

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

pooled SD.

5. Find the critical t-value from T-Table using α.

6. if t-statistic < critical t-value, we fail to reject Null Hypothesis.

eg:- Brand A : 49.5, 50.1, 49.8, 50.3, 50

Brand B : 50.4, 49.9, 50.6, 50.2, 50.1

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A \neq \mu_B$.

$\bar{x}_A = 49.94 \qquad n_A = 5$

$\bar{x}_B = 50.24. \qquad n_B = 5$

$\bar{x}_A - \bar{x}_B = -0.30$

$S_A = 0.316$

$S_B = 0.265.$

$$S_p = \sqrt{\frac{(5-1)(0.316)^2 + (5-1)(0.265)^2}{5+5-2}} = 0.292$$

$$t = \frac{49.94 - 50.24}{0.292\sqrt{\frac{1}{5}+\frac{1}{5}}} = \frac{-0.3}{0.1847} = \underline{\underline{-1.624}}.$$

$df = n_1 + n_2 - 2 = \underline{\underline{8}}$

$\alpha = 0.05$

critical t_value = ~~8~~ 2.306

t_statistic　　critical t_value

$-1.624 < 2.306$.

Here we fail to reject the null Hypothesis.

Matrix

$r \times c \rightarrow m \times n$ [order]

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & \cdots & \cdots & a_{mn} \end{bmatrix}$$

Square matrix only: Transpose, Symmetric, Identity, Determinant

$a_{ij} = a_{ji} \rightarrow$ Symmetric

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \text{Identity matrix.}$$

$$D = \underset{3\times 2}{\begin{bmatrix} 6 & 3 \\ 9 & 4 \\ 0 & 7 \end{bmatrix}} \quad C = \underset{3\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 11 & 31 \end{bmatrix}} \rightarrow D+C = \underset{3\times 2}{\begin{bmatrix} 7 & 5 \\ 12 & 8 \\ 11 & 38 \end{bmatrix}}$$

Single value x matrix → scaler matrix.

multiplication with 2 matrix.

$\underset{m\times 2}{A} \times \underset{2\times n}{B} \rightarrow \underset{m\times 2}{\text{Result}}$ [1st matrix column = 2nd matrix row]

$$A = \underset{2\times 3}{\begin{bmatrix} 15 & 6 & 4 \\ 1 & 2 & 3 \end{bmatrix}} \quad M = \underset{3\times 2}{\begin{bmatrix} -5 & -1 \\ -6 & 0 \\ 11 & 24 \end{bmatrix}} \quad A \times M = \begin{bmatrix} (-75-36+44) & (-15+0+96) \\ (-5-12+33) & (-1+0+72) \end{bmatrix}$$

$$= \underset{2\times 2}{\begin{bmatrix} -67 & 81 \\ 16 & 71 \end{bmatrix}}$$

$$M \times A = \begin{bmatrix} -76 & -32 & -23 \\ -90 & -36 & -24 \\ 189 & 114 & 116 \end{bmatrix}_{3 \times 3}$$

Transpose

$$A = \begin{bmatrix} 1 & 2 & 3 & 8 \\ 9 & 10 & 11 & 3 \end{bmatrix}_{2 \times 4} \qquad A^T = \begin{bmatrix} 1 & 9 \\ 2 & 10 \\ 3 & 11 \\ 8 & 3 \end{bmatrix}_{4 \times 2}$$

If $B = B^T$, symmetric matrix $\rightarrow$ (only in square matrix)

Identity matrix is also symmetric.

Determinant

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad |M| = (ad - bc)$$

$$M = \begin{bmatrix} 10 & 12 \\ 9 & 7 \end{bmatrix} \qquad |M| = (10 \times 7 - 12 \times 9)$$

$$= 70 - 108$$

$$= -38$$

$12 \times 9 = 108$

$108 - 70 = 38$

Inverse matrix

$M M^{-1} = I$

$M \downarrow$ matrix (M), $M^{-1} \downarrow$ Inverse of M, $I \downarrow$ Identity

* M^{-1} = inverse

M^{-1} doesn't exist when $|M| = 0$.

Eigen

$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}_{2\times2}$ $\quad V = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2\times1}$ $\rightarrow A \cdot V = \begin{bmatrix} 2 \\ 0 \end{bmatrix}_{2\times1}$

$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}_{2\times2}$ $\quad u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{2\times1}$ $\rightarrow A \cdot u = \begin{bmatrix} 0 \\ 3 \end{bmatrix}_{2\times1}$

$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}_{2\times2}$ $\quad w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times1}$ $\rightarrow A \cdot w = \begin{bmatrix} 2 \\ 3 \end{bmatrix}_{2\times1}$

2 x 2 matrix → 2 eigen value

3 x 3 matrix → 3 eigen value.

- - - . . . - - - - . .

direction doesn't change → Eigen vectors.

Here,

$Av = 2v$, 2 - eigen value

$Au = 3u$, 3 - eigen value

eigen value, λ

$[A V = \lambda V] \rightarrow$ eigen value condition.

λ ↓ eigen value

V ↓ eigen vector.

$A v = \lambda . v$

$A v - \lambda v = 0$

$(A - \lambda I) v = 0$

$|(A - \lambda I)| = 0$ but $v \rightarrow$ non-zero.

$\rightarrow B = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

$$\left| \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} (1-\lambda) & 4 \\ 3 & (2-\lambda) \end{vmatrix} = 0$$

$(1-\lambda)(2-\lambda) - 12 = 0$

$2 - \lambda - 2\lambda + \lambda^2 - 12 = 0$

$\lambda^2 - 3\lambda - 10 = 0.$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\lambda = 5, -2$

$$\lambda = \frac{-(-3) \pm \sqrt{(-3)^2 - (4 \times 1 \times -10)}}{2 \times 1}$$

$$= \frac{3 \pm \sqrt{9 + 40}}{2}$$

$$= \frac{3 \pm \sqrt{49}}{2}$$

$$= \frac{3 \pm 7}{2}$$

$$= \frac{3+7}{2} \text{ or } \frac{3-7}{2}$$

$= 5$ or -2

$(A - \lambda I) v = 0$

$\lambda = 5$

$$\left(\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\right)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\begin{bmatrix} -4 & 4 \\ 3 & -3 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$-4x_1 + 4x_2 = 0$ —— (1)

$3x_1 - 3x_2 = 0$ —— (2)

(1) x -1 $\quad 4x_1 - 4x_2 = 0$ —— (3)

$3x_1 - 3x_2 = 0$ —— (2)

(3) - (2) $\quad x_1 - x_2 = 0$

$x_1 = x_2$

$u\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda = -2$

$$\left(\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}\right)\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = 0$$

$$\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = 0$$

$3y_1 + 4y_2 = 0$

$3y_1 + 4y_2 = 0$

$3y_1 = -4y_2$ $\quad -4+3$

$y_1 = -\frac{4}{3} y_2$

If $y_1 = 1$, $y_2 = \frac{-3}{4}$.

$y\begin{pmatrix} 1 \\ -3/4 \end{pmatrix}$

$\rightarrow O = \begin{bmatrix} 0 & 1 \\ +1 & 0 \end{bmatrix}_{2\times 2} \quad M = \begin{bmatrix} 2 \\ 0 \end{bmatrix}_{2\times 1} \Rightarrow OM = \begin{bmatrix} 0 \\ 2 \end{bmatrix}_{2\times 1}$

$\rightarrow O = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad M = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \Rightarrow OM = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$$\rightarrow \left| (O - \lambda I) \right| = 0$$

$$\left| \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$(-\lambda - \lambda) - 1 = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda = 1, -1$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{0 \pm \sqrt{0 - 4 \times 1 \times 1}}{2}$$

$$= \frac{\pm\sqrt{+4}}{2} = \frac{\pm 2}{2}$$

$$= \underline{-1 \text{ or } 1}$$

$\underline{\lambda = 1}$ $\quad (O - \lambda I)v = 0 \quad$ $\underline{\lambda = -1}$

$$\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$-x_1 + x_2 = 0$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

$$\cancel{x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}}$$

$$x_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} //$$

$$\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right) \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0$$

$$y_1 + y_2 = 0$$

$$y_1 + y_2 = 0$$

$$y_1 = -y_2$$

$$\cancel{y_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y_2 \begin{pmatrix} 0 \\ -1 \end{pmatrix}}$$

$$y \begin{pmatrix} 1 \\ -1 \end{pmatrix} //$$

$\rightarrow \quad M = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

$$|M - \lambda I| = 0$$

$$\left| \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} (2-\lambda) & 1 \\ 0 & (2-\lambda) \end{vmatrix} = 0$$

$$(2-\lambda)(2-\lambda) - 0 = 0$$

$$4 - 2\lambda - 2\lambda + \lambda^2 = 0$$

$$+\lambda^2 - 4\lambda + 4 = 0$$

$$\cancel{\lambda^2 + 4\lambda - 4 = 0}$$

$$\therefore \lambda = 2$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 4}}{2 \times 1}$$

$$= \frac{4 \pm \sqrt{16-16}}{2}$$

$$= \frac{4}{2} = 2.$$

$$(M - \lambda I)V = 0$$

$$\left(\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$x_2 = 0$$

$$x = \begin{pmatrix} x_1 \\ 0 \end{pmatrix}$$

$$x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$x = x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

153 391
~~39~~ +
135 345

Probability - chance of happen an event.

event - get an outcome from sample space

$$\text{Probability} = \frac{(\text{event}) \text{ count}}{(S) \text{ count}}$$

Event → Independent
→ dependent.

joint probability

· valid only for Independent event

$P(C, m) = P(C) \times P(m)$

$P(C, n) = P(C) \cdot P(n)$

$P(T, m) = P(T) \cdot P(m)$

$P(T, n) = P(T) \cdot P(n)$

→ $P(C) = \frac{15}{32}$ $P(T) = \frac{17}{32}$

$P(m) = \frac{2}{32}$ $P(n) = \frac{30}{32}$

	m	n	
C	$\frac{15 \times 2}{(32)^2}$	$\frac{15 \times 30}{(32)^2}$	→ $\frac{480}{(32)^2}$ $(P(C,m) + P(C,n))$
T	$\frac{17 \times 2}{(32)^2}$	$\frac{17 \times 30}{(32)^2}$	→ $\frac{544}{(32)^2}$ $(P(T,m) + P(T,n))$
	↓ $\frac{64}{(32)^2}$	↓ $\frac{960}{(32)^2}$	
	$P(C,m) + P(T,m)$	$P(C,n) + P(T,n)$	

$P(B, sp) = P(B) \times P(sp)$

$P(B, w.sp) = P(B) \times P(w.sp)$

$P(G, sp) = P(G) \times P(sp)$

$P(G, w.sp) = P(G) \times P(w.sp)$

32.

→ · sp - 9
w.sp - 23
girls - 15
boys - 17.

	sp	w.p	
B	$\frac{9 \times 17}{(32)^2}$	$\frac{23 \times 17}{(32)^2}$	→ $\frac{544}{(32)^2}$
G	$\frac{15 \times 9}{(32)^2}$	$\frac{15 \times 23}{(32)^2}$	→ $\frac{480}{(32)^2}$
	$\frac{288}{(32)^2}$	$\frac{\cancel{425}\ 736}{(32)^2}$	

Conditional probability - Baye's Theorem / Baye's Rule

P Cloudy → Raining $P(R|C) = \dfrac{P(R \cap C)}{P(C)}$

Raining → cloudy. $P(C/R) = \dfrac{P(C \cap R)}{P(R)}$

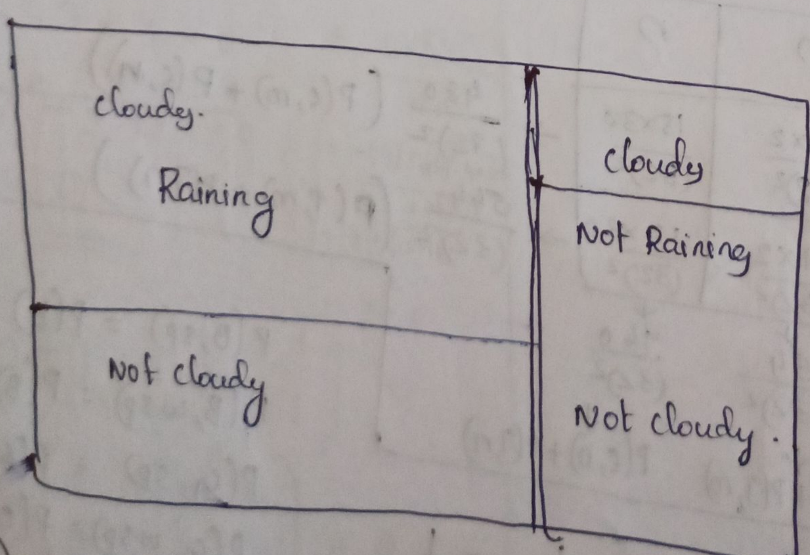
$\therefore P(R \cap C) = P(R|C) \times P(C)$

$P(C \cap R) = P(C/R) \times P(R)$

ie; $P(R/C) \times P(C) = P(C/R) \times P(R)$

$$P(R/C) = \frac{P(C/R) \times P(R)}{P(C)}.$$

→



$P(R) = 0.75$

$P(R') = 0.25$

Rain → $P(C) = 0.6$ $P(C/R)$

$P(C'/R)$ $P(C') = 0.4$

Not Rain → $P(C) = 0.1$ $P(C/R')$

$P(C'/R')$ $P(C') = 0.9$.

$$P(R/C) = \frac{P(C/R) \times P(R)}{P(C)}$$

~~$P(C)$~~

$P(C/R) = 0.6$

$P(C/R') = 0.1$

$\cancel{P(C) = P(C/R) \times P.}$

	R	R'	
C	$P(C\cap R)$	$P(C\cap R')$	$= P(C)$
C'	$P(C'\cap R)$	$P(C'\cap R')$	

$P(C\cap R) = ?$

$$P(C/R) = \frac{P(C\cap R)}{P(R)}$$

ie; $P(C\cap R) = P(C/R) \times P(R)$

$\cancel{= 0.6 \times 0.75}$

$= 0.6 \times 0.75$

$= 0.45$

$P(C\cap R') = ?$

$$P(C/R') = \frac{P(C\cap R')}{P(R')}$$

$P(C\cap R') = P(C/R') \times P(R')$

$= 0.1 \times 0.25$

$= 0.025$

$\therefore$ ~~$P(R$~~

$P(C) = P(C\cap R) + P(C\cap R')$

$P(C) = 0.45 + 0.025$

$= 0.475$

$$\therefore P(R/C) = \frac{0.6 \times 0.75}{0.475} = 0.947$$