Applied Signal Processing
Computer Studio Session 3: Stochastic processes, Wiener filters and adaptive filters

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Histogram Use Matlab to generate $N = 1000$ random variables from the U(0,1) (`rand`) and N(0,1) (`randn`) distributions respectively. Plot the histograms using the `hist` command.

Correlation Generate two vectors x and y of $N = 1000$ independent N(0,1) random variables. Illustrate the joint distribution by plotting y versus x using `plot(x,y,'.')`. Note the shape of the "equiprobability" lines. We shall now form a modified variable z using the formula:

$$z = \alpha x + \sqrt{1-\alpha^2}\, y\,,\ -1 \le \alpha \le 1\,.$$

What is the distribution of the components of z (verify by plotting the histogram for some α)? What is the correlation

$$\phi_{xz} = \mathrm{E}[x_n z_n],$$

between the elements of x and z? Now, illustrate the effect of correlation by plotting x vs z for the correlation values $\phi_{xz} = \{0.5, -0.5, 0.9, -0.9\}$. As seen in the plots, we can intuitively interpret correlation as approximate linear dependence!

Sample correlation Assume now that x and z from the previous exercise are realizations of stationary stochastic processes. Thus, each of the entries x_n and z_n, $n = 1, 2, \ldots, N$ represents one observation. Note that x_n and x_m are uncorrelated for $n \ne m$ (and similarly for z) - what is such a random process called? Now, based on the available time samples, we wish to estimate the correlation between x and z. Since the processes are stationary, one can replace the ensemble average implied by the expectation operator, by a time average:

$$\hat{\phi}_{xz} = \frac{1}{N}\sum_{n=1}^{N} x_n z_n\,.$$

This is also called the *sample correlation*. What result did you get using $\alpha = -0.9$ and $N = 1000$? What would happen if $N \to \infty$? We shall also compute the sample auto-correlation function of x:

$$\hat{\phi}_{xx}(k) = \frac{1}{N} \sum_{n=|k|+1}^{N} x_n x_{n-|k|}$$

In Matlab's Signal Processing Toolbox, this is done using the `xcorr` command. Compute and plot the auto-correlation estimates for $|k| \leq 5$. Make sure you get the correct ("biased") scaling! How does the result agree with the true auto-correlation function $\phi_{xx}(k)$?

Simple Wiener filter Continuing the previous example, suppose we are now given observations of z only, and are asked to estimate x! A typical situation is that z is a noisy and distorted observation of some unmeasurable quantity x (which is in fact exacly how z was generated!). Since the signals are white in time, a natural estimate is simply

$$\hat{x}_n = a z_n \,.$$

The Mean-Square Error (MSE) $\mathrm{E}[(x_n - \hat{x}_n)^2]$ will then be a function of the "filter coefficient" a. Derive an explicit expression for the function

$$\xi(a) = \mathrm{E}[(x_n - \hat{x}_n)^2]$$

and plot $\xi(a)$ for the values `a=-5:0.01:5`. Derive the minimizing a by hand, and verify with the plot. Apply your Wiener filter to the available z samples, and compute the empirical MSE:

$$\hat{\xi} = \frac{1}{N} \sum_{n=1}^{N} (x_n - \hat{x}_n)^2$$

Does it agree with the theoretical minimum value?

LMS filter Open the `studio3_lms.m` m-file and use it as a MATLAB work book. Follow the directions in the file.

RLS filter Open the `studio3_rls.m` m-file and use it as a MATLAB work book. Follow the directions in the file.