SSY130 - Hand-in 2
FIR Differentiator Design

Lucas Rath

December 10, 2018

student id: 19930425
secret key: Clamperl

Question 1)

The code 1 above, shows a FIR filter that performs approximate differentiation for low frequencies up to frequency 0:05 Hz, while blocking the frequency band above 0.1 Hz to reduce the influence of the noise:

Listing 1: *Differentiator FIR filter design*

```
dt = 1;          % s
f_s    = 1/dt;   % Hz

fcut  = 0.05;    % Hz
fstop = 0.1;     % Hz
N = 60;          % FIR filter order

H_f = [0, fcut, fstop, f_s/2] /(f_s/2);        % *pi [rad/sample]
H_a = [0,    1,     0,     0] .*H_f *pi*f_s;   % filter amplitude
h_diff = firpm(N,H_f,H_a,'differentiator');
```

As specified by the `firpm` Matlab command, the frequency vector `H_f` is given as normalized frequency points, specified in the range between 0 and 1, where 1 corresponds to the Nyquist frequency $f_s/2$. Moreover, in order to perform properly the differentiation in the pass-band region, the amplitude vector `H_a` must correspond to the value jw, where w is the non-normalized frequency in rad/s. Therefore, the amplitude vector makes use of the frequency vector multiplied by $2\pi \cdot (f_s/2) = \pi \cdot f_s$ which will result in the right unit. The ideal filter frequency response can be verified in figure 2. The `'differentiator'` argument will ensure constant phase of 90 °.

Question 2)

Figure 1 shows the designed filter impulse response and figure 2 the comparison between the desired ideal and the real FIR frequency response.

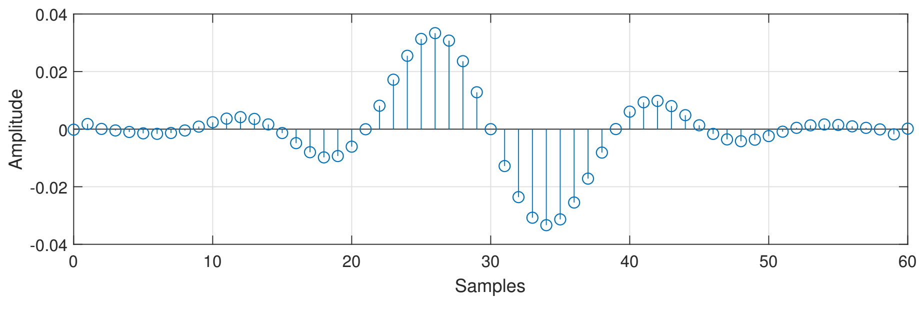


Figure 1: *FIR differentiator filter impulse response*

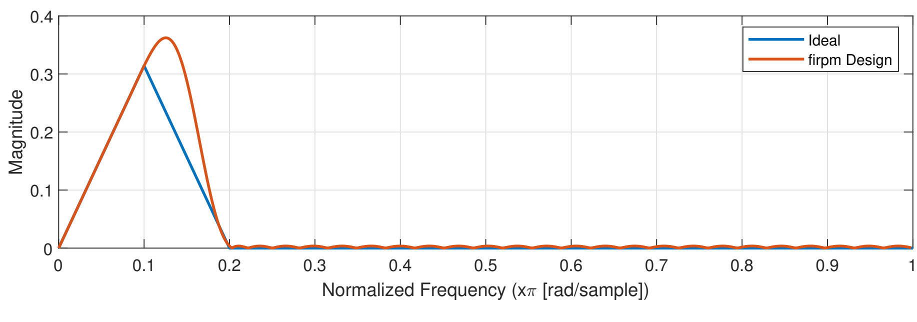


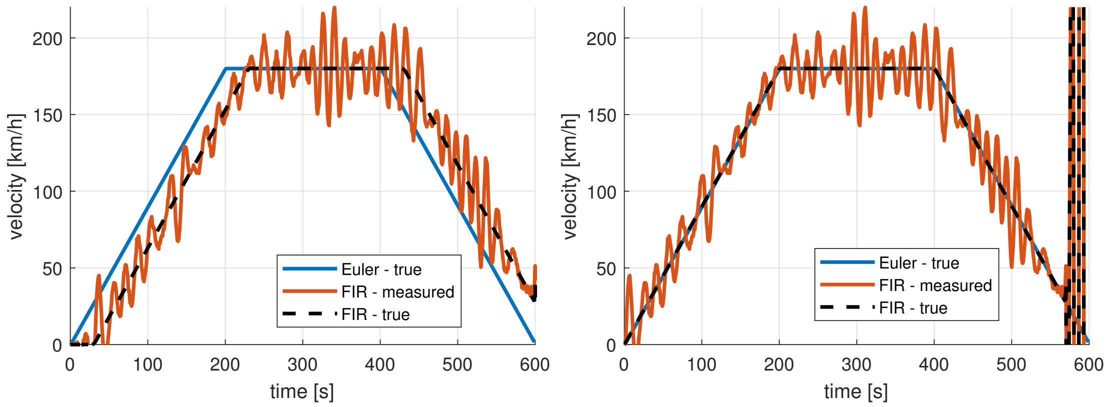
Figure 2: *FIR differentiator filter frequency response*

Question 3)

The Matlab function `firpm` designs a linear-phase FIR filter using the Parks-McClellan algorithm. In addition, we are designing an odd filter of order M-1, which has odd M coefficients. It follows then that since we are designing a linear-phase filter, its ideal impulse response is truncated to M=61 samples. The algorithm then introduces a delay of (M-1)/2=30 samples to make the filter causal. In time this is equivalent to $30 \cdot \Delta t = 30 \cdot 1s = 30s$.

Question 4)

Figure 3a shows the Euler and FIR filter for true and measured data. After offline delay compensation, the delay effect is totally removed, as shown in figure 3b:



(a) True and measured velocity using Euler and FIR filter

(b) True and measured velocity using Euler and FIR filter with off-line delay correction

At the end of the filter output very large oscillations occur due to the effect of convolution $v(n) = h(n) * s(n)$. If $h(n)$ has size N and $s(n)$ has size M, then the convolution will result in M+N-1 elements. It follows, that the last N-1 elements are calculated by zero-padding s(n) in N-1 elements. What happens is that the last element in s is very large and the zero-padding will artificially create a huge position displacement and consequently a huge velocity. That is why, we usually discard this part of the resulted convoluted signal.

Question 5)

Figure 4 reveals the result of applying a trivial Euler derivative filter in a signal corrupted by noise. Clearly, the obtained velocity turned to be dominated by the noise. This occurs because when taking the derivative of the signal, which is corrupted by the noise, components with higher frequencies will have its magnitudes largely amplified:

$$\frac{df(t)}{dt} \rightarrow jw \cdot F(w) \tag{1}$$

Derivative in time domain is equivalent to multiplying by jw in frequency domain. Therefore, high frequency noise components will be more amplified and will dominate the output signal.

If we are able to filter the noise and then take the derivative, we decrease this noise amplification effect and the signal is cleaner. Indeed, we still see the noise in figure 3a and we could decrease the pass-band frequency to filter this visible high frequency noises. However, we assume the risk of reducing also the gain of the true signal, that are close to this frequency. Each differentiator filter must be then adjusted according to the desired frequency operation range.

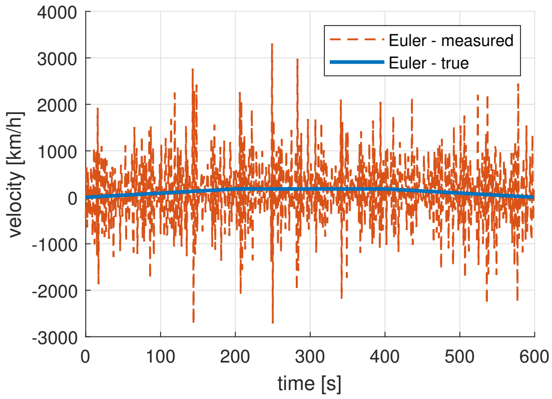


Figure 4: *True and filtered measured velocity*

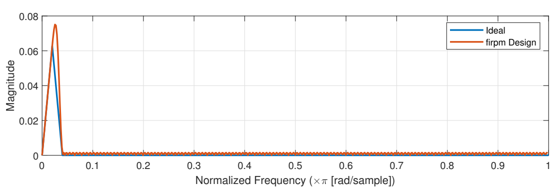
Question 6)

To determine the true maximum speed of the vehicle using the designed FIR filter, we should be able to eliminate completely the noise. As seen in figure 3a the noise is still very present, which means that the chosen pass-band frequency is still very high. Considering then that a vehicle usually achieves only low accelerations profiles, we can use the parameters above:

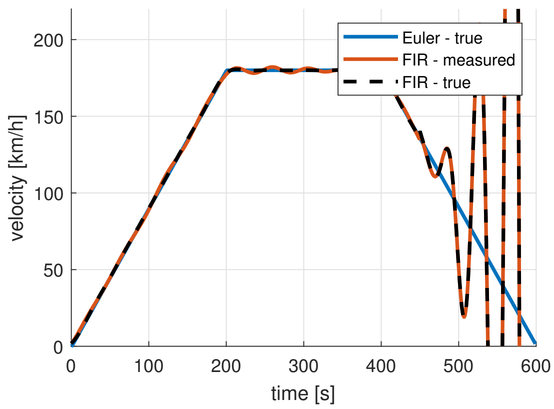
Listing 2: *Differentiator FIR filter design*

```
fcut  = 0.01;      % Hz
fstop = 0.02;      % Hz
N = 300;           % FIR filter order
```

Increasing the filter order will also ensure lower ripple magnitudes in the stop band region and therefore eliminate better the noise. On the other hand, we introduce more delay.



(a) FIR frequency response



(b) True and measured velocity using Euler and FIR filter with off-line delay correction

Evaluating the filter results until time equal 600-N/2 = 450s we get that the maximum speed of the vehicle around 180 km/h.