Universidad Politecnica De Valencia

Sistemas Complejos Bioinspirados

Inertial Spaceship in the Earth-Moon System:

Simulation of simplified 3-body problem

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Definitions:

Apoapsis	the point on the elliptical orbit with the greatest distance from the central gravitational body and lowest orbital speed, it is called Apogee when Earth is the central body.
Periapsis	the point on the elliptical orbit with the least distance from the central gravitational body and highest orbital speed, it is called Perigee when Earth is the central body.

1. Introduction

This essay describes a project designed and brought into operation according to the requirements of the "Sistemas Complejos Bioinspirados" classes. The author wished to create an mathematical model allowing to calculate the trajectory of a inertial body (e.g. a spaceship) in the 2 dimensional neighborhood of Earth – Moon system. Taking into account the distances and time periods in the considered system the computational requirements of obtaining the results with sufficient precision via a purely numerical approach is beyond the capabilities of a single PC computer. Thus a hybrid approach had to be devised and applied. The model presented here endevours to merge the simplicity of calculations of a Keplerian 2-body problem in the vicinity of a central body and the precision of numerical calculations in free space between the two celestial objects. There are, however, some disadvantages of this approach which will be explained in detail further on.

This paper will continue as follows: in the next chapter the reader will find a general explanation of the problem to be solved, the details of the assumed physical model and the astrodynamical equations ruling the motion of concerned bodies. In chapter 3 the author presents a detailed description of the applied solution presenting all stages of trajectory's calculations. All the limitations of the resolving algorithm are also contained here. In the next chapter, the reader will find some interesting results obtained from simulations which are followed by a short discussion of both advantages and shortcomings of the designed model. Lastly, the author presents some final conclusions as regards to the applied solution.

2. Description of the problem

The main objective of this project was to determine the flight path of the third inertial body, or „the spaceship" as it will be refered to in this paper, in the Earth-Moon vicinity. In this problem the center of the Earth is considered to be main reference point, the Moon follows its elliptical orbit with speed varying according to Kepler's 2nd law of planetary motion and finally the spaceship follows its path on the plane of Lunar orbit. The spaceship has no drive of its own so its path is determined only by the terrestrial and lunar forces of gravity, calculated from the expression (1) and the initial state of the system.

$$\vec{F_S} = -G \cdot \frac{M \cdot m_S}{\left(\left|r_S\right|\right)^3} \cdot \vec{r_S} \qquad (1)$$

where F_s is the gravitational force on the spaceship and r_s is the position vector from the center of the planet or moon.

Furthermore, since mass of the mentioned spaceship is negligible (when compared to planetary masses) the vehicle itself does not affect the positions of two other bodies in the system and thus it is considered to be a simplified 3-body problem of celestial mechanics.

The initial state of the system is stated by the position and velocity vectors of the spaceship, as well as by the starting position of the Moon on its orbit. Two other variables which are to be defined before commencing the simulation are: the time of flight and the minimum time increment applied. In an ideal case with system defined in such a way the path of the third body depends solely on time, however as the precision of numerical calculations is related to the time increment used some differences of results for the same initial state are expected according to the variations of the minimum time increment.

What's more, so as to ensure the efficient operation of the applied algorithm several simplifications have been introduced to the physical model ruling the motions of all bodies in the simulation. The following subsections shall discuss the details of the assumed physical model and the astrodynamical background ruling the motion of concerned bodies.

2.1 Assumed Physical Model

As it has already been mentioned it the model takes into account only 2 spatial dimensions corresponding to the plane of Moon's orbit. The plane of simulations has been divided into 3 zones: A, B and C according to the proportion of terrestrial and lunar gravitational influences respectively.

The **A zone** corresponds to the neighborhood of Earth, where only its gravitational influence is considered to affect the flight path of the spaceship and thus the lunar gravity force is completely disregarded. Its lower limit is somewhat contractual as it is the mean Earth radius plus about 200km, i.e the height at which the atmospheric drag is assumed to be negligible in the periods of time considered in the simulations, thus we have approximately: $r_{Eatm} = 6.6 \cdot 10^3$km.

By assuming that the Terrestrial gravity force comprises 99% of all forces acting on the spaceship using (1) we can find: $r_{E99} = r_{mn} \cdot (\sqrt{(0.01 \cdot M)})/(\sqrt{(0.01 \cdot M)}+\sqrt{(m_{mn})})$ which can be approximated to $r_{E99} = 1.723 \cdot 10^5$km, where: r_{mn} is height of lunar perigee, r_{E99} is the outer limit of the A zone, M and m_{mn} are the masses of Earth and Moon respectively.

Within the zone A motion of the Spaceship is simplified to a 2 body problem with Earth as the central body and is dealt with using equations resulting from Keplerian Laws of Planetary Motion (detailed description in 2.2.3 and 2.2.4).

The **C zone** corresponds to the lunar neighborhood, where the terrestrial gravity is completely disregarded, as it is assumed to have no influence (less than 1% in reality) on the flight path of the spaceship. Since the mass of the Moon is only 0.0123 of Earth's mass this zone is very small with outer limit restricted to $r_{M99} = 4 \cdot 10^3$km from Moon's center (to calculate this just swap masses in the expression for r_{E99}). The lower limit corresponds to Moon's equatorial radius giving approximately $r_{Msurf} = 1.74 \cdot 10^3$km.

Similarly as in zone A within zone C motion of the Spaceship is simplified to a 2 body problem with Moon as the central body and is dealt with using classical celestial mechanics. It is worth noting, however, that zone C is moving in the Earth frame of reference and thus on the entry to this zone spaceship position and velocity vectors must be transformed to Moon frame of reference. Although Moons orbital velocity vector is varying in value and direction, those variations are cause by Earth gravitational field so in this model it is assumed that both terrestrial and lunar frames of reference are inertial.

The **B zone** covers so called „Free space" that is all the regions of the movement plane not contained in A or C. Here both massive bodies have significant influence on the trajectory of the spaceship's flight so a numerical method is applied to determine this trajectory (more details in 2.2.4). Motion in this zone is described in the Earth frame of reference so if in the initial state of the system the spaceship does not start in this zone it may be entered either directly from zone A or from zone C through a transformation of coordinates from Moon's back to Earth's frame of reference.

The model presented in this paper does not take into account any perturbations in the orbits of the Moon nor of the spaceship that would be caused by the Sun, the Solar system planets nor any other massive objects like comets. What's more, both massive bodies are assumed to be perfectly spherical and thus there is no precession of the orbits confined within zones A or C. Orbits that pass through zone B are subjected to the influences of both massive bodies and thus will vary in time.

2.2 Astronomical Background

This sections is ment to inform the reader on the details of the astrodynamical calculations that are executed during the simulation. All pertinent equations of celestial mechanics as well as the details of numerical method used can be found below.

2.2.1 Frames of reference

The natural way to describe the position and velocity of a body following conic section curve in a gravitational field of one massive body is to use angular coordinates (r, θ) with the origin in the center of the main gravitational body and zero argument coresponding to the positive x axis. In the presented case the frame of reference is usually bound to Earth centre, except when the spaceship is in zone C, when its position and velocity are described in respect to Moon's center.

The velocity vector in the presented model is described as (v, ψ), where v states the value and, through its sign the orbit direction. Orbit of the body is direct (counterclockwise) if $v > 0$ and retrograde if $v < 0$. The velocity vector direction is described in relation to the body's position angle θ. The angle ψ is an angle between the v vector and a circle of radius r centred in the origin. ψ ranges from -90 (object in diving towards the central body) to 90 (object departing the central body).

2.2.2 Orbital Elements

Before we can describe the position of the body on the orbit first the orbit itself has to be defined. On a plane the orientation of an orbit (either elliptical, parabolic or hyperbolic) is described using 4 magnitudes (refered to as orbital elements):

- Eccentricity *e* defines the absolute shape of the orbit:
  - $e = 0$ orbit is a circle,
  - $0 < e < 1$ object follows an elliptical orbit
  - $e = 1$ corresponds to a parabolic or radial trajectory
  - $e > 1$ is a hyperbola
- Semi-major axis *a* is the longest diameter of an orbit:
  - $a > 0$ for all elliptic orbits (including circular ones),
  - $a \to \infty$ in case of parabolaes,
  - $a < 0$ in case of hyperbolaes.

  It is sometimes more convenient to express semi-major axis in terms of semi-latus rectum: $lts = a \cdot (1-e^2)$ as it has a finite positive value for all conical curves.
- Argument of periapsis *periapsisArg* an angle defining the orientation of the orbit in the orbital plane, in the presented model argument of periapsis of lunar orbit equal to zero so the periapsis of this orbit is exactly to the right of the origin of Earth's frame of reference.
- Mean Anomaly *MeanAnom* defines the position of the orbiting body along the ellipse at a specific time, it is a mathematically convenient "angle" which varies linearly with time, but which does not correspond to a real geometric angle. The real geometrical angle between the periapsis of the orbit and the present position of the body is called True Anomaly *TrueAnom* (which is not proportional to time) and it can be calculated from Mean Anomaly using Kepler's procedure.

2.2.3 Equations of 2 Body Motion

In case of a ideal 2 body problem, with one body having a negligible mass, its movement (with accordance to 1st Kepler law) follows a conic section can be described by the equation (2) as:

$$r(\text{TrueAnom}) = \frac{\text{lts}}{e\cdot\cos(\text{TrueAnom}) + 1} \quad (2)$$

assuming that for parabolic and hyperbolic orbits the TrueAnomaly has a limited range.

From the 2nd Kepler law we know that a specific angular momentum *h* of an orbit is conserved in a closed 2 body system (e.g. within the zones A and C). The expression for the specific angular momentum is as follows:

$$h = r\times v = r\_periapsis \cdot v\_periapsis = r\_apoapsis \cdot v\_apoapsis$$

$$h = r\times v = r\cdot v\cdot\cos(\psi) \quad (3)$$

in the last expression ψ is the path angle as defined in subpart 2.2.1 (i.e. not the angle between vectors r and v but its complement).

Additionaly, we will use the orbital energy conservation equations (also known as vis-viva equation) which is a variation of the law of total energy conservation.

$$v^2 = u\cdot\left(\frac{2}{r} - \frac{1}{a}\right) \quad (4)$$

where: v – the speed of orbiting body, r – its distance from the orbit's focus, u – grawitational parameter (*u = G·M*).

Using the geometrical relation: *r_periapsis = a·(1-e)* and the expressions (3), (4) we can obtain:

$$h^2 = u\cdot a\cdot\left(1 - e^2\right) = u\cdot\text{lts} \quad (5)$$

Equations (3), (4) and (5) together with above stated geometrical relations are sufficient to establish all characteristics of a curve that the satelite (or spaceship) will follow, when given the initial position and velocity vectors. Finaly, the true anomaly that can be calculated from (2) determines the present angular position of the satelite on that curve.

For elliptic orbits it is also possible to calculate the orbital period from 3rd Kepler law:

$$T\_orbit = 2\cdot\pi\cdot\sqrt{\frac{a^3}{u}} \quad (6)$$

2.2.4 Relations of Time and Position

As it has been pointed out in subpart 2.2.2 the relation between *TrueAnom* and time is not linear so in order to calculate the time of flight along an elliptic orbit the *MeanAnom* must be determined first.

$$\begin{pmatrix} \text{EccAnom} = \operatorname{atan}\left(\dfrac{\sin(\text{TrueAnom})\cdot\sqrt{1-e^2}}{e + \cos(\text{TrueAnom})}\right) \\ \text{MeanAnom} = \text{EccAnom} - e\cdot\sin(\text{EccAnom}) \\ t = \dfrac{\text{MeanAnom}}{2\cdot\pi}\cdot T\_orbit \end{pmatrix} \quad (7)$$

The set of equations (7) allows us to compute the time from the last periapsis for a given TrueAnom. If it is desirable to determine the time between two points on the orbit (when neither of those is the periapsis), the equations (7) should be used for both points and the results either added (when the points are on the oposite sides of periapsis) or substracted (when the points are on the same side).

As the 2nd equation from (7) does not have a closed form solution for eccentric anomaly *EccAnom*, the *TrueAnom* cannot be analiticaly determined when the time t is given. However it is possible to obtain the inverse version of the 2nd equation numericaly – for the purpose of the presented model it was done by the Newton-Raphson mehod. The rest of the equations from (7) has an inverse functions.

For hyperbolic trajectories the time of flight from last periapsis cannot be calculated from (7), but as the areal velocity h is known, integrating the total area swept by a line joining the central body and the satelite will enable us to calculate the time of flight from periapsis. The above mentioned integral take the following form:

$$A_{tot} = \frac{lts^2}{2\sqrt{e^2 - 1}} \cdot \frac{e \cdot \sin(TrueAnom)}{(e \cdot \cos(TrueAnom) + 1)} - 2 \cdot \operatorname{atanh}\left[\frac{(e-1) \cdot \tan\left(\frac{TrueAnom}{2}\right)}{\sqrt{e^2 - 1}}\right] \quad (8)$$

and the time of flight from periapsis is given by: $t = 2 \cdot A_{tot}/h$.

2.2.5 Numerical Approach

When the number of interacting bodies increases the newtonian differential equations of motion are becoming to complicated to allow for an analitical solution. This is the case in zone B of this model where an iterative approach is taken.

$$\overrightarrow{F_{tot}} = -G \cdot \sum_i \frac{M \cdot m_s}{\left(\left|r_i\right|\right)^3} \cdot \vec{r_i} \quad (8)$$

The summated gravitational force (8) is used to calculated in the initial point of an iteration and the resulting acceleration, along with a finite time increment are used to calculate the speed and new position at the end of the iteration and so on. The smaller the time increment the more accurate are the calculations however the computational requirements increase as more iterations are required.

The main problem with a finite time increment is that the double integration that takes place (speed from acceleration, position from speed) results in an error that grows as the period of simulation increases. The value of this error depends on: the curvature of the trajectory (the more it deviates from a stright the bigger the error), time increment applied and the lenght of simulation period.

3. Applied solution

All calculations that are required to determine the trajectory are in the MATLAB workspace by use of scripts and functions explained below. In order to reach a compromise between the precision of trajectory calculations and the time used for computations a hybrid modular approach has been applied to the 3 body problem. This section presents the particular modules that correspond to zones A, B and C described in section 2.1, it also explains the details of visualsation algorithms applied as well as gives some guidelines to the proper usage of the created model.

3.1 Computational Modules

3.2.1 EarthMoonSystem

This is the main script of the model in which calls to all other parts are made during the process of trajectory calculation. It is also the only part of the m code to which the user ought to enter the initial conditions of the system. After doing so running this script in the MATLAB will produce outcomes and draw the trajectory.

EarthMoonSystem calls the function *KeplerianOrbit* when a spaceship passes through zone A or C, or the function *InBetween* when the ship goes through zone B. What's more, good part of the visualisation algoritms is realised in here.

3.2.2 KeplerianOrbit

The *KeplerianOrbit* function executes calculations of trajectories in both terrestrial and lunar neighborhoods i.e. when the spaceship is in zones A or B respectively. All computations within this function are carried out according to the laws ruling 2 body Kepler problems, and as it is so all limitations stated in section 2.1 are pertinent here.

First, using the initial state of the object (which is either defined by the user or passed from the B zone) orbital elements and a type of conical curve to be followed are determined. Afterwards, the algoritm distingushes between 5 general cases that may take place, according to which the spaceship will do one of the following:

- leave the massive body neighborhood via an elliptic trajectory,
- enter planet's atmosphere (crash into the Moon's surface) along an elliptic orbit,
- maintain a steady orbit around the massive body,
- leave the Earth's (Moon's) reagion via hyperbolic or parabolic trajectory,
- enter planet's atmosphere (crash into the Moon's surface) along a hyperbolic or parabolic curve.

In each of those cases the pertinent outcomes are calculated and ascribed to a variable argument vector *varargout1*. In all cases the *varargout1* contains the output position and velocity vectors expressed with respect to the central body, as well as the height and velocities at periapsis and (if applicable) apoapsis.

The trajectory points that will later be used in the visualisation outside of this function are calculated together after going through the branching part. If this function was called when the spaceship was in the Earth's neighborhood, an additional plot depicting this part of flight will be shown.

3.2.3 InBetween

When the inertial body travels through the zone B the *InBetween* function is called to execute numerical approximation of its path. The main calculations are made as described in 2.2.5 except the fact that the applied numerical algorithm varies the time increment between consecutive iterations according to the curvature of the trajectory. Since the error of numerical approximations increases with the curvature the time increment is decreased to compensate for this. On the other hand, when the flight path is straight or close to being straight the time increment increases up to 120 sec in order to lower the computational load.

The curvature of a 2 dimensional is given by:

$$\kappa = \left| \frac{d}{dt} r \times \frac{d^2}{dt^2} r \right| \cdot \left| \left(\frac{d}{dt} r \right)^{-3} \right|$$

which in our case can be simplified to:

$$\kappa = \frac{|v \times a|}{|v^3|} \quad (9)$$

and the time increment dt used in this model is obtained from: dt = 1/k, with the reservation for the maximum and minimum values of dt. The minimum value can be accessed by the user for modifications.

3.2.4 Auxiliary Functions

In the here described model there are 2 auxiliary functions: *TrueAnom* and *AngCoordConv*. The former calculates the true anomaly from a known mean anomaly and eccentricity of the pertinent orbit using the Newton-Raphson method. The latter is called when the spaceship crosses the boundary of the zones B and C, that is when there is a need for the coordinate transformation from the Earth bound frame to Moon based frame of reference. The *AngCoordConv* deals solely with the translation of the origin to a moving inertial system, there is no rotation involved as the lunar and terrestrial frames have the same orientation on the simulation plane.

3.3 Visualisation

The main plot depicts the whole trajectory calculated during the simulation in the Earth's frame of reference. The part of Moon's orbit covered during the simulation will also be shown on this plot unless the spaceship's trajectory was confined solely to zone A (i.e. terrestrial neighborhood) and the Moon's position did not affect it.

It is important to remember that the Earth is allways depicted in the origin of main plot, which sometimes impedes the comprehension of trajectory representation as the scale of th plot is very large. In that case the use of manual zoom is advised.

Two additional plots may also appear depending on the circumstances:

- if the zone A has been visited we will see a plot presenting only that part of the flight, which is the part of the flight nearest to Earth,
- if the C zone has been visited a similar plot depicting the close-up of lunar fly-by (or collision) will be displayed, this plot uses lunar frame of reference.

Both above mentioned plots show only the last fly-by so if the trajectory approached either of the bodies more than once previous fly-bies can be observed solely on the main plot.

One interesting feature provided by the main plot is that when the user defined time of flight has been reached and the object is within zones A or C a projected path will be automaticaly displayed (as a dashed line) until the boundary of B zone is reached. It is done so as this future path has been already determined as *KeplerianOrbit* function determines the whole conical section curve at once.

Further on, the dashed line projection is also very usefull when considering the closed elliptic orbits confined to A or C zones. In this case the central body could have been circumnavigated several times until the end of simulation so it is more convenient to depict the only the traveled part of the last orbit (thus pinpointing the exact final position of the spaceship) and the rest of the orbit is diplayed as a dashed line projection. The dashed line aslo connects the focus of the orbit with the final position of the body. Additionaly the plot's title of this sort of repetitive orbits has a counter stating the number of orbits already traveled.

The part of visualisation algorithm which is most complicated deals with proper transformation of trajectory points calculated by *KeplerianOrbit* function for the zone C as it moves along the orbit of the Moon. Some results of depicting the trajectories orbiting the Moon are presented in 4.1.

3.4 Handling Tips

The model is started from the MATLAB workspace by running the *EarthMoonSystem* m script, while the new initial state data may be entered by editing the top part of the same m script. The data that ought to be defined prior to starting the simulation are:

Variable	**Description**
r0, theta0	Radial and angular coordinates of position vector, as defined in 2.2.1
v0, psi0	Radial and angular coordinates of velocity vector, as defined in 2.2.1
T_sim_max	Maximal time period that is to be covered by the simulation
dt_min	Minimal time increment for numerical calculations
MnPos0	Angular coordinate of Moon position – corresponds to Moon's true anomaly

Caution should be taken not to define the initial state of the object within the Moon surface boundaries as this will yield undetermined resuls. Additionaly the trajectories that have eccentricity equal to unity or just slightly bigger might also result in singularities (especially for lunar orbits) and possibly distortions in the calculations of the flight time.

As the MATLAB workspace is the main user interface for this model, a table provided below explains some output variables that may be of interest when interpreting the trajectory plots.

Variable	**Description**
eOrbE, eOrbM	Eccentricity, terrestrial and lunar orbits respectively
ltsOrbE ltsOrbM	Semi-latus rectum, terrestrial and lunar orbits respectively
periapsisArgOrbE periapsisArgOrbM	Periapsis argument, terrestrial and lunar orbits respectively
r_periapsisOrbE r_periapsisOrbM	Height of periapsis, terrestrial and lunar orbits respectively
retroOrbE retroOrbM	Type of terrestrial and lunar orbits: 0-direct, 1-retrograde
v_periapsisOrbE v_periapsisOrbM	Velocity at periapsis, terrestrial and lunar orbits respectively
trayectory (array)	points of the trajectory in terrestrial FoR*
trayectoryVehMn (array)	points of the trajectory in lunar FoR* (only C zone)
rOut, thetaOut	Coordinates of position vector at the end of simulation
vOut, psiOut	Coordinates of velocity vector at the end of simulation
T_sim	Total time of fligh (diffrent than requested if collision or atmospheric entry occured)

*FoR – frame of reference

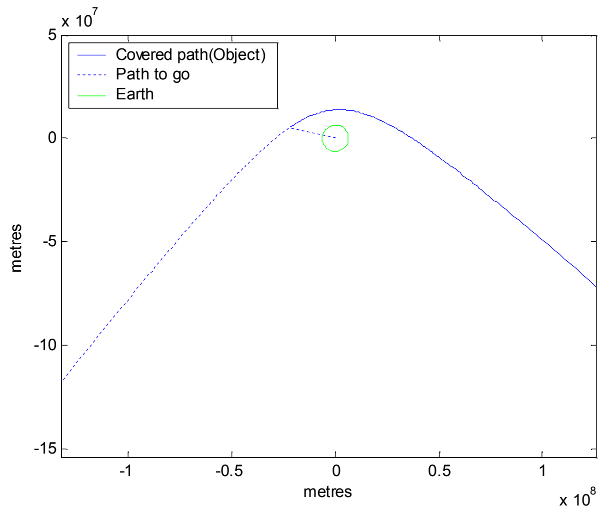
4. Outcomes

This chapter presents some results obtained from the simulations. It starts with the typical cases and passes to more interesting trajectories of Earth – Moon flights.

4.1 Basic Scenarios

Trajectories presented in this subpart are the most tipical cases and are shown here to confirm the correct operation of the created model.

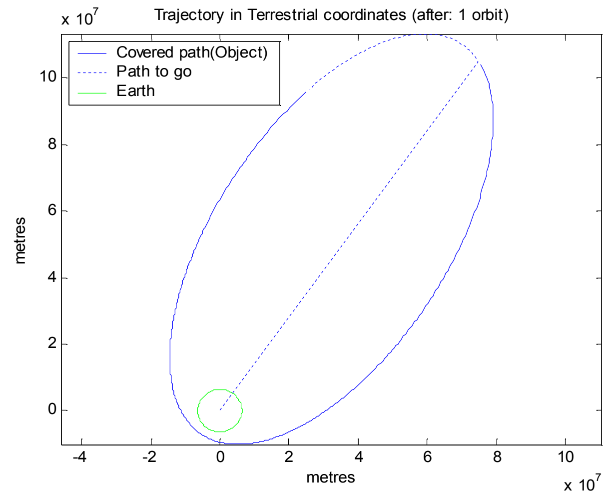
Earth fly-by along a hyperbolic trajectory – img.1
Initial state:
-position: r0 = 15*10^7m, theta0 = -pi/6rad;
-velocity: v0 = 4000m/s, psi0 = -1.38rad;
-others: dt_min = 10sec, T_sim_max = 35000sec, MnPos0 = 0rad,



Img.1 Hyperbolic trajectory

Final state:
-position: rOut = 2.27*10^7m, thetaOut = 2.907rad;
-velocity: vOut = 6770.2m/s, psiOut = 0.73639rad;
-Perigeum data: r_periapsis = 1.37*10^7m, v_periapsis = 8295.8m/s;
-Orbit eccentricity e = 1.3676

High eccentricity Earth orbit – img.2
Initial state:
-position: r0 = 10*10^7m, theta0 = 5/12*pi rad;
-velocity: v0 = 1500m/s, psi0 = -1rad;
-others: dt_min = 10sec, T_sim_max = 300000sec, MnPos0 = 0rad;

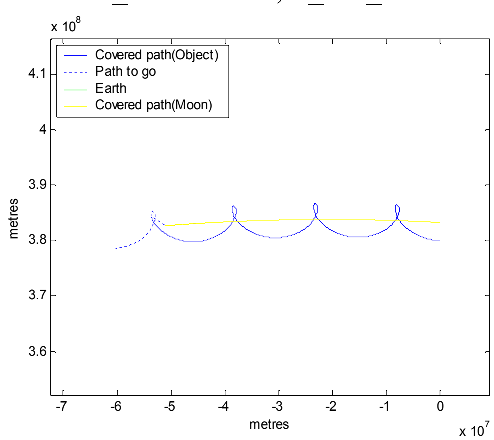


Img.2 Elliptic Earth orbit

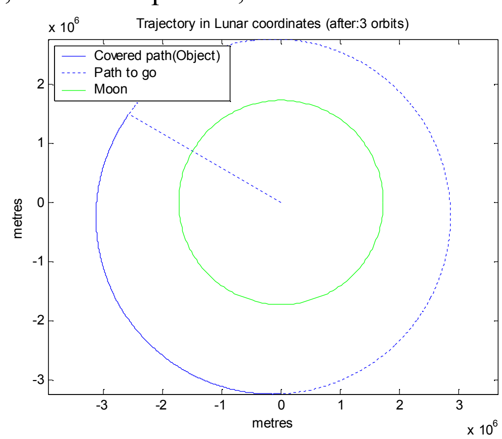
Final state:
-position: rOut = 1.29*10^8m, thetaOut = 0.951rad;
-velocity vOut = 679.7m/s, psiOut = 0.38968rad;
-perigeum: r_periapsis = 8.79*10^6m, v_periapsis = 9217.8m/s;
-apogeum: r_apoapsis = 1.3*10^8m, v_apoapsis = 620.96m/s;
-orbit eccentricity e = 0.8738

<u>Stationary lunar orbit – img.3 and img.4</u>

Initial state:
-position: r0 = 3.8*10^8m, theta0 = pi/2rad;
-velocity: v0 = 2200m/s, psi0 = 0rad;
-others: dt_min = 10sec, T_sim_max = 50000sec, MnPos0 = pi/2rad;



Img.3 Lunar orbit in Earth's frame of reference



Img.4 Lunar orbit in Moon's frame of reference

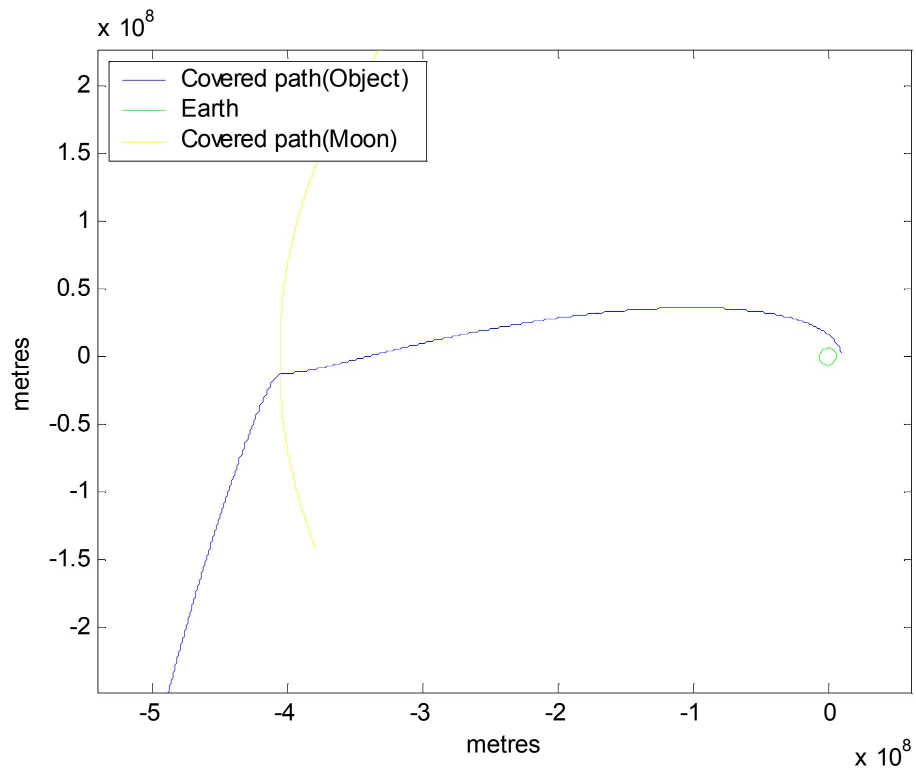
Final state:
position: rOut = 3.878*10^8m, thetaOut = 1.7095rad;
velocity: vOut = 2090.8m/s, psiOut = 0.705rad;
Lunar periapsis: r_periapsis = 2.73*10^6m, v_periapsis = 1398.5m/s;
Lunar apoapsis: r_apoapsis = 3.28*10^6m, v_apoapsis = 1165.7m/s;
orbit eccentricity e = 0.0908

4.2 Interesting Cases

4.2.1 Moon Gravity Assist

Gravity assist (also known as gravitational slingshot) is a manouver that makes use of the relative movement and gravity of a celestial body like the Moon in this exapmle to alter the path and velocity of a spacecraft, typically in order to save fuel and time. Here the Moon's gravitation increases velocity of our spaceship relative to Earth, thus increasing the total orbital energy of our spaceship and allowing it to leave the Earth's gravity field. This sort of manouver can also be used to decelerate the ship assuming that it were following an orbit with opposite angular momentum than the massive body (i.e retrograde in Earth – Moon system).

Initial state:
-position: r0 = 10^7m, theta0 = (12.5/180)*pi rad;
-velocity: v0 = 8850m/s, psi0 = 0rad;
-others: dt_min = 5sec, T_sim_max = 400000sec, MnPos0 = 2.5393rad



Img.5 Lunar gravity assist

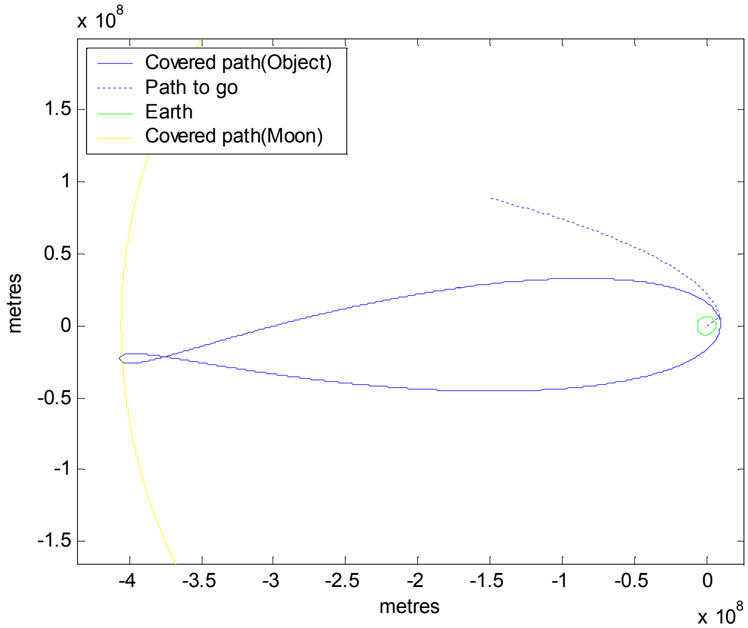
Final state:
position: rOut = 5.47*10^8, thetaOut = -2.6713rad;
velocity: vOut = 1730.7m/s, psiOut = 0.7263rad;
Lunar periapsis: r_periapsis = 1.9834*10^6m, v_periapsis = 2445.4m/s;
Lunar orbit eccentricity eOrbM = 1.4185
Earth orbit eccentricity eOrbE = 0.96448 (initial)

Now using the output data and manualy determining (using equations (3), (4) and (5)) the resultant characteristics of the new orbit we get the eccetricity e = 2.4177 which clearly suggests a hyperbolic trajectory of much higher energy than the initial elipse.

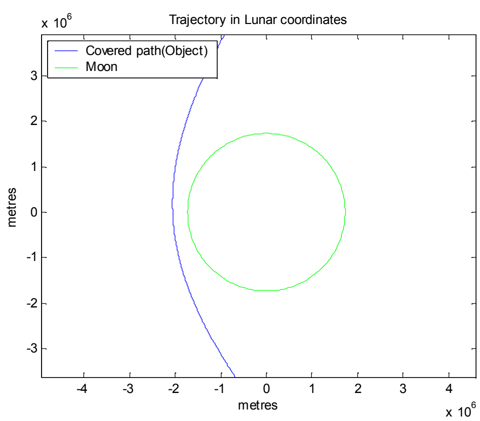
4.2.2 Earth to Moon – Safe Return Scenario

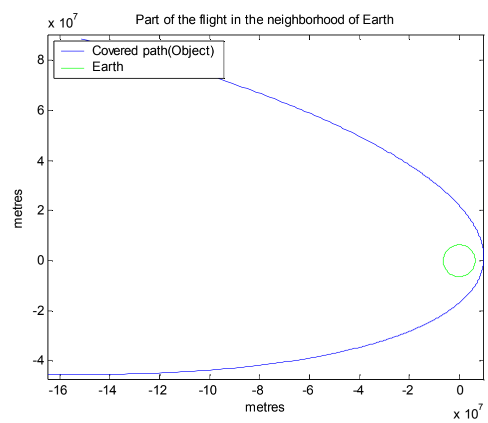
This case presents a safe return trajectory to Earth's natural satelite with the rendevous just after Moon's apoapsis. The total time of simulation 550000sec equals to 6 days 8 hours 46 mins and 40 sec which might be compared to the transition times of Apollo missions.

Initial state:
position: r0 = 10^7m, theta0 = (14.43/180)*pi rad; velocity: v0 = 8850m/s, psi0 = 0rad; dt_min = 5sec, T_sim_max = 550000sec, MnPos0 = 2.5393rad



Img.6 Safe return Earth-Moon transition





Img.7 Moon fly-by as seen from the satelite's frame of reference

Img.8 Earth fly-by on return from Moon

Final state:
-position: rOut = 1.152*10^7m, thetaOut = 0.71838rad,
-velocity: vOut = 8235, psiOut = 0.418,
-perigeum: r_periapsisOrbE = 9.58*10^6, v_periapsisOrbE = 9042.6,
-Moon's periapsis data: r_periapsisOrbM = 2.0541*10^6, v_periapsisOrbM = 2401,
-Moon's orbit eccentricity eOrbMn = 1.4146,
-Earth orbit eccentricity eOrbE = 0.9658

As we can see the initial periapsis characteristics (r0, v0) do not differ significantly from the same data at the return flight (r_periapsisOrbE, v_periapsisOrbE) which tells us that the total energy of the spaceship did not change during this flight. That is because the Moon's gravitation force influenced only the orientation of spaceships initial orbit, but not it's energy.

5. Conclusions

During the course of development of this project the author had a great opportunity to broaden his knowledge on the topic of celestial mechanics and to see how much more there is to learn. This assignment has posed a considerable challange to deal with and has been brought to the end with satisfactory compromise between the resolution of outcomes and the computational load required. Designed model deals equally well with all tested initial states of non-powered flight in free space, except following ideal parabolic orbits i.e. with eccentricity $e = 1$. However it is quite unlikely for those to occur in the simulated conditions as the eccentricity is almost never exactly equal to one.

As the chapter 4 has demonstrated, even complicated trajectories can be simulated using this mode and it might be considered a success as the designed algorithm correctly performs the tasks that were designated at the beginning of the project. The author encourages to try out the initial conditions for the model from the subchapter 4.2 (as well as any others) and enjoy the experiments.