Stability And Evolution Of Conway's Game Of Life

Leo BECHET
UFR Sciences et Techniques, Besancon, FRANCE
leo.bechet@edu.univ-fcomte.fr

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Abstract

Short introduction to subject of the paper ...

1 Introduction

Conway's Game of Life, conceived by mathematician John Conway in 1970, is a cellular automaton operating on a grid of cells. Governed by four simple rules, each cell's status is determined by its neighboring cells, leading to a rich tapestry of patterns ranging from static formations to dynamic entities. Beyond its recreational appeal, the Game of Life finds application in scientific domains such as computer science, mathematics, and biology. Its enduring significance lies in its ability to stimulate inquiry into emergent complexity in various systems. In this article we will explore the evolution of the automaton for different starting conditions.

2 Experiment context

The focus lies on exploring the dynamics of Conway's Game of Life cellular automaton within a computationally feasible framework. The study employs a 500 by 500 grid with wrapping around the edges to effectively simulate an infinite grid, thus accommodating the inherent spatial constraints of computational resources. While acknowledging the potential introduction of periodic errors due to wrapping, the experiment adopts the perspective that in an infinite grid, such errors would manifest as an infinite grid can be considered as a repetition of an infinitely large grid. This approach allows us to investigate emergent phenomena and general properties of the Game of Life, leveraging the convenience of a finite grid while approximating the behavior of an infinite system.

3 Initial analysis

The initial analysis phase of the experiment involves generating random grids with varying filling percentages, ranging from 0% to 100% filled, with each percentage representing a sweep across the spectrum of possible initial conditions. For each filling percentage, the simulation is run for 1000 cycles, allowing the grid to evolve according to the rules of Conway's Game of Life cellular automaton. This process is repeated 10 times for each percentage to account for stochastic variability in the initial conditions. To quantify the equilibrium filling of the grid at each percentage, 20 evenly distributed points are selected across the simulation timeline. The final filled percentage is computed as the average of the last points of each of the 10 simulations conducted for that particular percentage. These average values are then plotted to generate a graph illustrating the relationship between the initial filling percentage and the corresponding equilibrium filling state. The resulting graph is given in

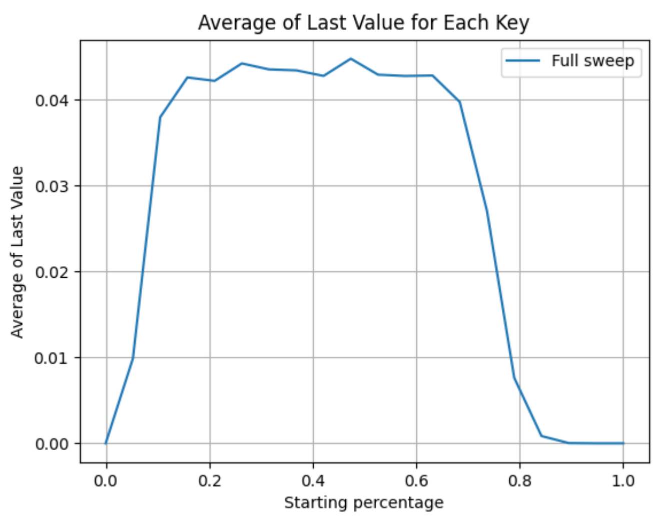

Figure 1: Final filling percentage for a sweep of 20 points accross 0% to 100% starting filling.

After conducting the initial simulations, it was observed in Fig 1 that the final percentage of filled cells exhibits a non-linear trend with respect to the initial filling percentage. Initially, from 0% to 10% starting filling, the final percentage rises. Subsequently, between 10% and 70% initial filling, the final percentage stabilizes at around 4.5%. Beyond 70% initial filling, the final percentage begins to decline once again. This pattern suggests that the dynamics of Conway's Game of Life cellular automaton exhibit distinct behaviors at different ranges of initial filling percentages. The initial rise in final filling percentage may reflect the propagation and expansion of patterns from initially sparse configurations, while the subsequent stabilization could indicate a balance between birth and death rates of cells within moderately filled grids. The observed decline in final percentage at higher initial fillings may be attributed to increased competition and limited space for the growth and sustainability of patterns, leading to more frequent cell deaths. These findings highlight the intricate interplay between initial conditions and emergent behaviors in cellular automata systems. Further analysis and modeling efforts are warranted to fully elucidate the underlying mechanisms driving these observed phenomena. Moreover the plateau observed at 4.5% in the final percentage of filled cells suggests the presence of a stable equilibrium state within Conway's Game of Life, indicating a critical threshold where the interplay between cell birth and death rates leads to sustained patterns and structures within the system.

4 Investigating the first rise

Due to the resolution of our graph, we do not have much precision on the first rise. Thus we run 2 other simulations in order to increase the precision on that specific part.

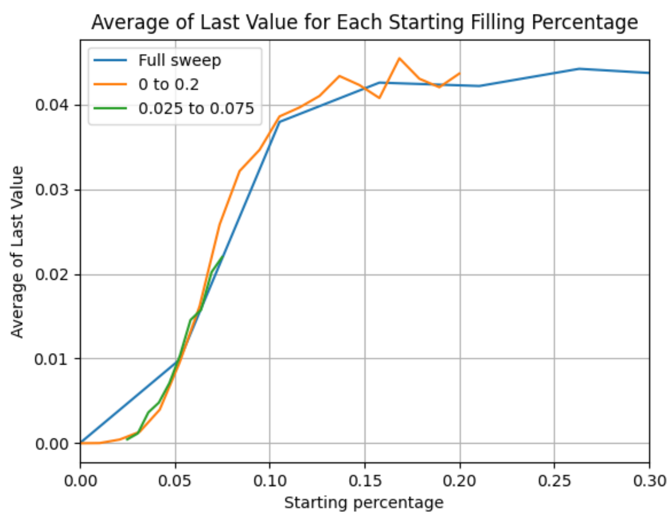

Figure 2: Increased resolution sweep on the first rise.

In Fig 2, the medium curve in orange gives us some insight on the evolution at low fillings. The highest resolution curve in green does not show much deviation from the previous curve. Though we would have expected a steeper curve, it appears to slowly go up in a quadratic fascion. We however speculate[1] that for

[1] Due to the complexity of the computations, and a loss of access to the CompuPhys server, we weren't able to investigate this supposition.

an infinite amount of cycle, the final sweep will be close to a gate function.

5 Evolution in time

We consider by evolution in time, the evolution of the automaton in relation to each step. In Fig 3 we plot the previous data by averaging each the sets of 10 runs for each percentages.

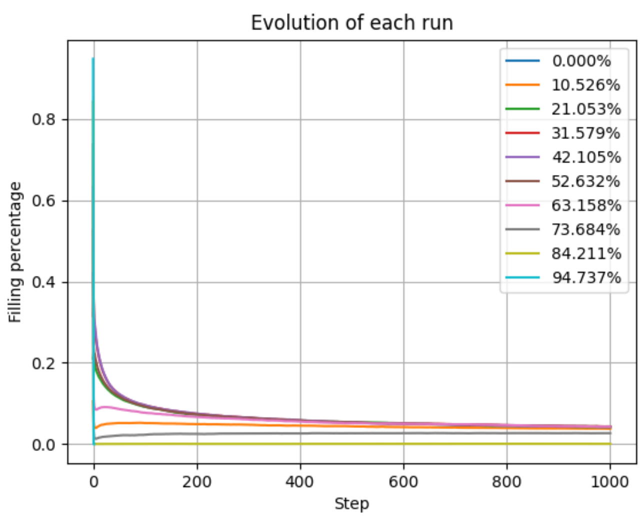

Figure 3: Evolution of the system for different starting fills.

One aspect of the graph that can be spotted is that both 0% and 100% curves go straight to a 0% population. The simulation has checks to stop runs once if the population reaches 0%, thus we don't have any steps for those. The current graph only plots half of the test fills as to not overload the graph.

Second aspect, is that we can see a heavy decrease down under 4.5% for each percentages. However, zooming in on the lower part of the graph, it is obvious that the evolution does not always follow a exponential decrease. As shown on Fig 4, some rise up. This behavior is consistent for curves where the starting percentage isn't in the plateau identified of Fig 1.

Comparing with Fig 1, curves categorized inside the plateau appear to do follow the exponential decrease. However for the 63% curve, it appears that it starts with an important fall before rising up again. We do not know if that value is indeed inside of the plateau or slitghtly off.

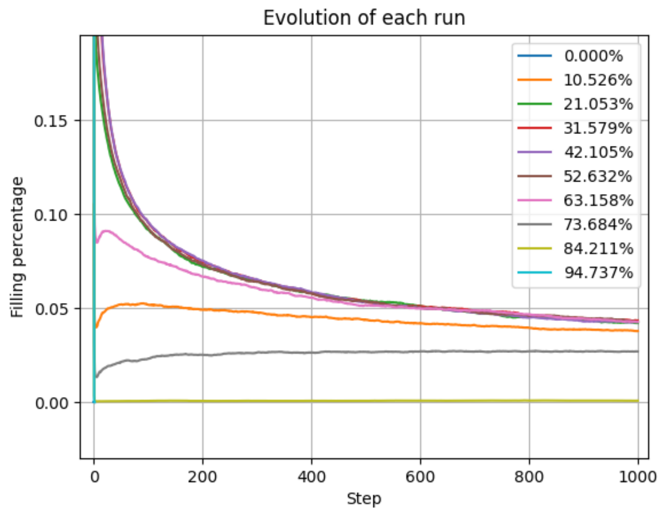

Figure 4: Evolution of the system for different starting fills zoomed on the lower part.

6 Fitting to an exponential decrease

In order to obtain a precise value for the plateau, we propose to fit one of the central curve to an exponential decay described in Eq. 1.

$$f(x) = e^{-ax+b} + c \tag{1}$$

The following fit resulted in values of $a = 0.00364699$, $b = -2.62888693$, $c = 0.04145781$. The comparison between the points and the fit is displayed in Fig 5. Fitting the whole data was not possible due to the first points. We believe that though the exponential decay looks to be a correct fit, it does not correspond to the right function. Fits using an inverse function were inconclusive. From the value of C, we expect that the plateau is around 4.1% of population. It is however necessary to state that we have no proof that it is flat, and it might aswell be bumped. The bump theory actually makes sense by allowing descrepancies in the final value for limited number of iterations, while still leaving the possibility of all percentage inside the plateau to converge to the same value for an infinite number of cycle.

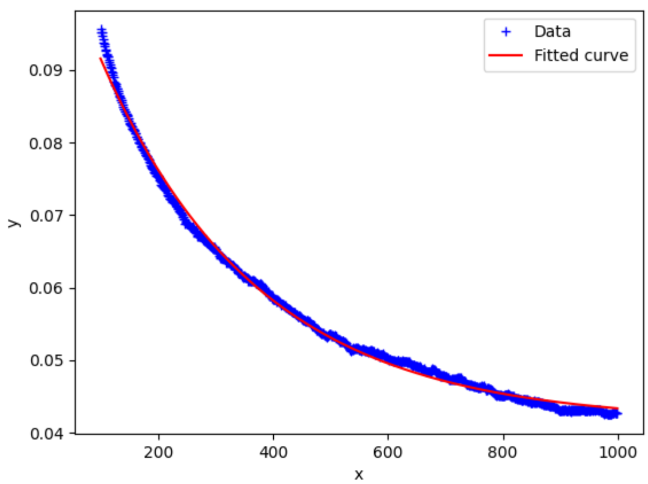

Figure 5: Fitting of the 42% curve to Eq. 1