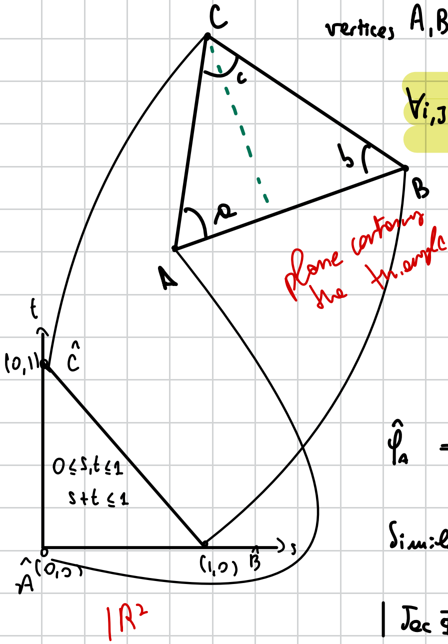
vertices $A, B, C \in \mathbb{R}^3$



Change of variable

$$\forall i,j \in \{A,B,C\} \text{ RMN: } \int_{ABC} \underbrace{\varphi_i}_{\in \mathbb{P}_1} \varphi_j \overset{\downarrow}{=} \int_{\hat{A}\hat{B}\hat{C}} \hat{\varphi}_i \hat{\varphi}_j \, |\text{Jacobian } \Phi| \, ds \, dt$$

$$\hat{\varphi}_i(s,t) = A_i + B_i s + C_i t$$

$$\hat{\varphi}_A(s,t) = 1 \quad \text{IF } (s,t) = (s_A, t_A) \quad \text{ELSE } = 0$$

$$\hat{\varphi}_B(s,t) = 1 \quad \text{IF } (s,t) = (s_B, t_B) \quad \text{ELSE } = 0$$

$$\hat{\varphi}_C(s,t) = 1 \quad \text{IF } (s,t) = (s_C, t_C) \quad \text{ELSE } = 0$$

$$\hat{\varphi}_A \implies \begin{cases} a_A + b_A \cdot s_A + c_A \cdot t_A = 1 \\ a_A + b_A \cdot s_B + c_A \cdot t_B = 0 \\ a_A + b_A \cdot s_C + c_A \cdot t_C = 0 \end{cases} \implies \hat{\varphi}_A(s,t) = 1 - s - t$$

Similarly solve the linear system to find $\hat{\varphi}_B(s,t) = s$, $\hat{\varphi}_C(s,t) = t$

$$|\text{Jac } \Phi| = \frac{\text{Area}(ABC)}{\text{Area}(\hat{A}\hat{B}\hat{C})} = \frac{\text{Area}(ABC)}{(1/2)} = 2 \,\text{Area}(ABC)$$

$$M_{ij} = \begin{cases} \frac{1}{\cancel{24}\,12} \cdot \cancel{2}\, \text{Area}(ABC) & \text{if } i \neq j \\ \frac{1}{\cancel{12}\,6} \cdot \cancel{2}\, \text{Area}(ABC) & \text{if } i == j \end{cases}$$

"mass matrix"

For the stiffness matrix:

$$G = J^T J$$

$$S_{ij} = \int_{ABC} \nabla\varphi_i \cdot \nabla\varphi_j \underset{\text{change of variables}}{=} \int_{\hat{A}\hat{B}\hat{C}} \underbrace{(\nabla\hat{\varphi}_i)^T G^{-1} (\nabla\hat{\varphi}_j) \, |\text{Jac } \Phi|}_{\text{CONSTANT } C_{ij}} \, ds \, dt =$$

$$C_{ij} \int_{\hat{A}\hat{B}\hat{C}} ds \, dt = C_{ij} \,\text{area}(\hat{A}\hat{B}\hat{C})$$

According to the constant C_{ij} I have the element S_{ij}