

DTU





Model-based Machine Learning

Introduction

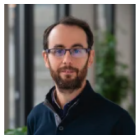
Motivation

A practical example

Probabilistic programming


Structure

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- Matteo Pozzar

Motivation - focus on the problem!

- Traditional machine learning
 - A very wide range of algorithms

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 - the number and types of variables in the problem domain
 - which variables affect each other
 - what the effect of changing one variable is on another variable
 - encode that knowledge in the form of a model

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 - Focus is on the problem
 - includes all the assumptions about the problem domain
 - highly tailored models for specific scenarios

MBML in a nutshell

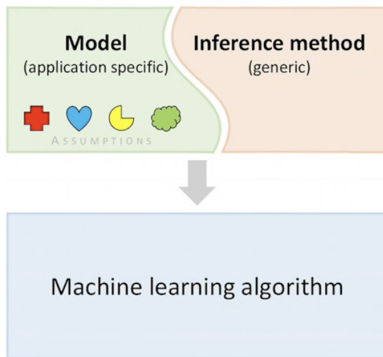


Figure: MBML big picture¹

¹In Christopher Bishop's Keynote, "Model-Based Machine Learning"
<https://www.microsoft.com/en-us/research/video/keynote-model-based-machine-learning>

Motivation - Why probabilistic models?

- **Uncertainty** in the real world
 - partial knowledge of state of the world
 - noisy observations
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- **Uncertainty** in the real world
 - partial knowledge of state of the world
 - noisy observations
 - inherent stochasticity
- It is essential to account for uncertainty when building models of reality!
- **Probability theory** provides us with a consistent framework for quantifying and manipulating uncertainty

Motivation - A recent example...



starting to cross



crossing



stopping



bending-in

Motivation - A recent example...

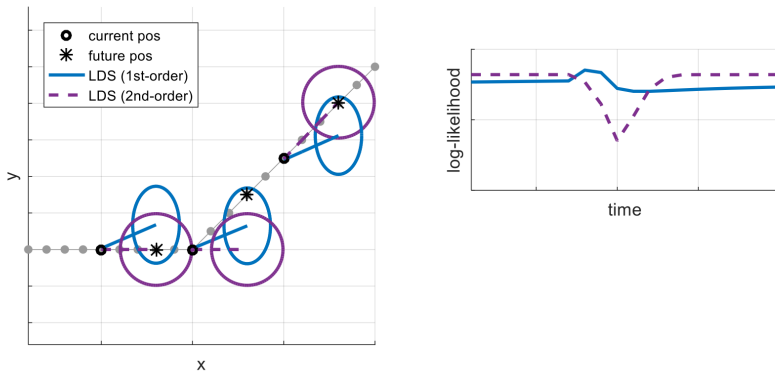
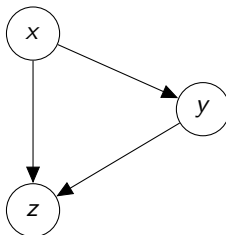


Figure: From Julian Kooij, TUDelft

Motivation - A recent example...

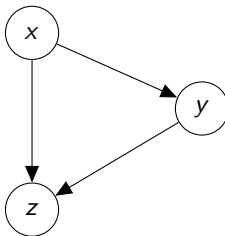
- Video

Motivation - Why probabilistic graphical models (PGMs)?



- **Nodes** - Represent random variables
- **Arrows** - Represent causal relationships

Motivation - Why probabilistic graphical models (PGMs)?



- Provide an **intuitive** and **compact** way of representing the structure of a probabilistic model
 - gives us insights about the properties of the model (e.g. relationships between variables, conditional independencies)
 - helps us communicate and design new models

A practical example

- A recruiter looking for the most intelligent students

A practical example

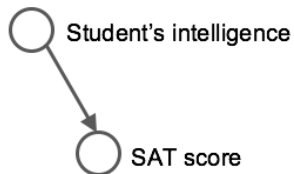
- A recruiter looking for the most intelligent students

☐ Student's intelligence

☐ SAT score

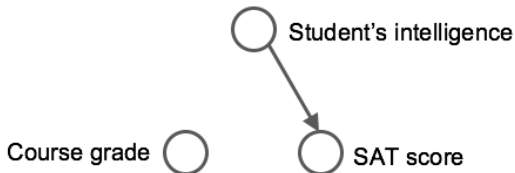
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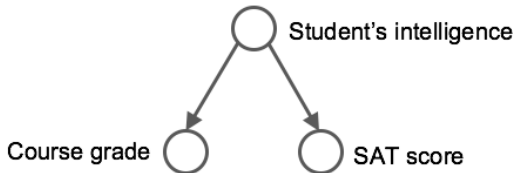
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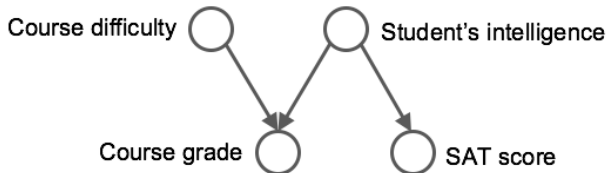
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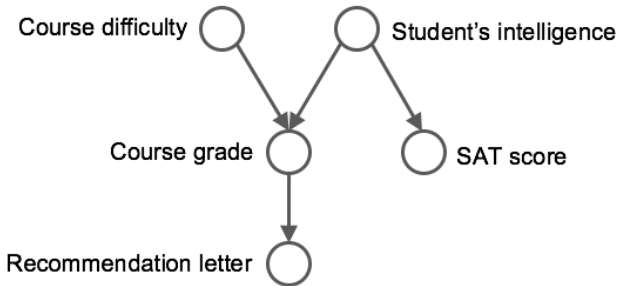
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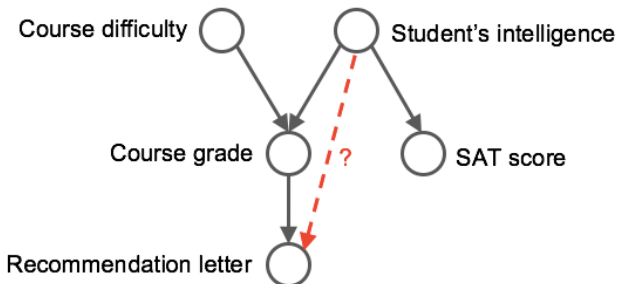
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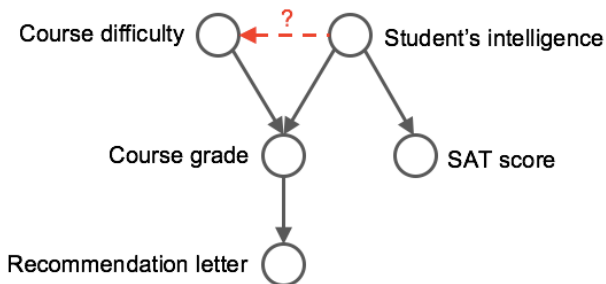
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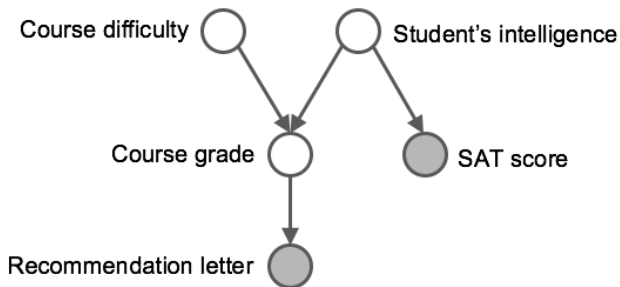
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A (familiar) example - Linear regression

- y - target (a.k.a. response or dependent variable)
- \mathbf{x} - vector of explanatory variables
(a.k.a. predictor or independent variables)
- Target y is assumed to be a function of \mathbf{x}

$$y = f(\mathbf{x})$$

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$$y = f(\mathbf{x})$$

- Linear regression assumes f to be a **linear function** of \mathbf{x}

$$y = \boldsymbol{\beta}^T \mathbf{x} + \epsilon$$

- $\boldsymbol{\beta}$ is a vector of coefficients
- $\epsilon \sim \mathcal{N}(0, \sigma^2)$ is an error term (accounts for observation noise)

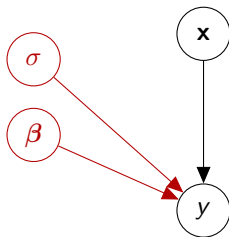
Linear regression as a graphical model

- We have a set of N observations of the targets y_n which depend on their corresponding explanatory variables \mathbf{x}_n



Linear regression as a graphical model

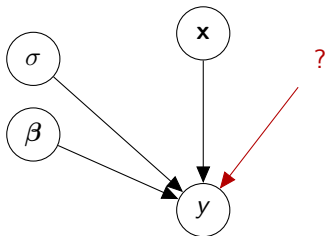
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- We need to include parameters...

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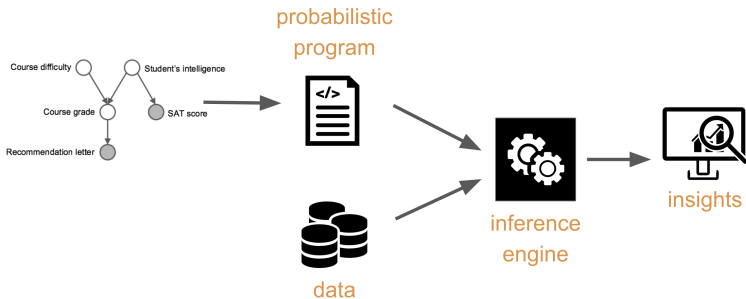
- We have a set of N observations of the targets y_n which depend on their corresponding explanatory variables \mathbf{x}_n



- We need to include parameters...
- In fact, we can even include other models (within our model)!

$$y = \beta_{lr}^T \mathbf{x} + \beta_k \text{kernel_model} + \beta_{dl} \text{deep_learning_model} \dots$$

Probabilistic programming



- Pyro is a probabilistic programming language written in Python and supported by PyTorch on the backend
 - Pyro enables flexible and expressive deep probabilistic modeling
 - Unifies the best of modern deep learning and Bayesian modeling
 - Check <https://pyro.ai>

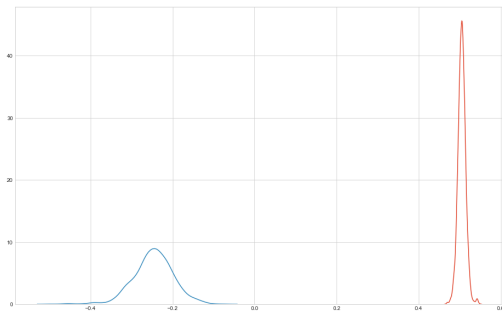
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- Example: Linear regression in Pyro

```
def model(x,y):  
    beta0 = pyro.sample("beta0", pyro.distributions.Normal(0, 1))  
    beta1 = pyro.sample("beta1", pyro.distributions.Normal(0, 1))  
    sigma = pyro.sample("sigma", pyro.distributions.HalfCauchy(1))  
    with pyro.plate("data", len(y)):  
        pyro.sample("y", pyro.distributions.Normal(beta0+beta1*x, sigma), obs=y)
```

Example with Linear Regression

- Running inference... (N=10)



```
mean
beta1 0.51
beta0 -0.25
sigma 0.1
```

Example with Linear Regression

- Running inference... (N=200)



```
mean
beta1  0.5
beta0 -0.19
sigma  0.1
```

Example with Linear Regression

- Running inference... (N=1000)



	mean
beta1	0.5
beta0	-0.2
sigma	0.1

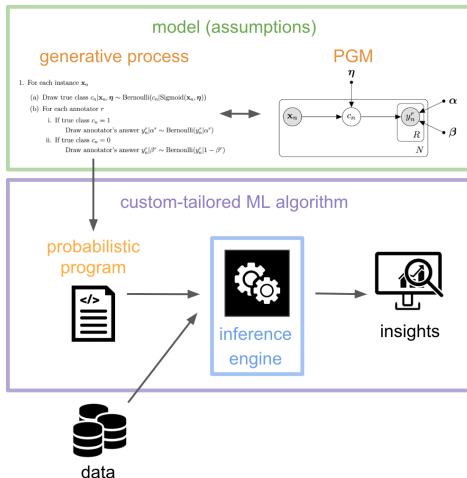
Foundations of PGMs and probabilistic programming

Probabilistic modelling in various contexts - your modelling toolbox

Bayesian inference (exact and approximate)

Week	Topic
1	Intro to the course + Prob. review
2	PGM foundations
3	PGM foundations II
4	Freq. vs Bayesian + Prob. Prog. + Mixture models
5	Regression models
6	Classification and Hierarchical models
7	Temporal models
8	Topic Models
9	Markov-chain Monte Carlo (MCMC)
10	Variational inference
11	Generative models
12	Gaussian processes
13	Project support

Model-based Machine Learning



- Interleaved blocks of theory and practice
- Python and Pyro(+PyTorch)
- Jupyter notebooks (in class)

Requirements

- Programming is important (preferably Python)
- Confident knowledge about basic statistics and probability
- Creativity, domain knowledge

- 2 in-class mini-tests, 25% each (dates: March 25, May 6)
 - Dates are final!
 - Multiple choice questions
 - Closed book - you can have 1 sheet of paper with you with notes
- Group project with individualized report, 50% (deadline: May 15)
 - Free topic
 - Groups of exactly 4 students (you will be randomly assigned if not)

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(we are not responsible for missed announcements!)

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- Some advice:
 - Don't skip class (keep the routine for you Wednesday afternoon)
 - Try to do all notebooks yourself before checking solutions
 - Don't be shy to ask for clarifications and help!