

Week 4 – Profiling and High-Performance NumPy

# 02613 Python and High-Performance Computing

# Last week on 02613

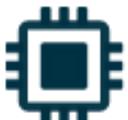
- Any questions about caches and memory hierarchy?

# Today

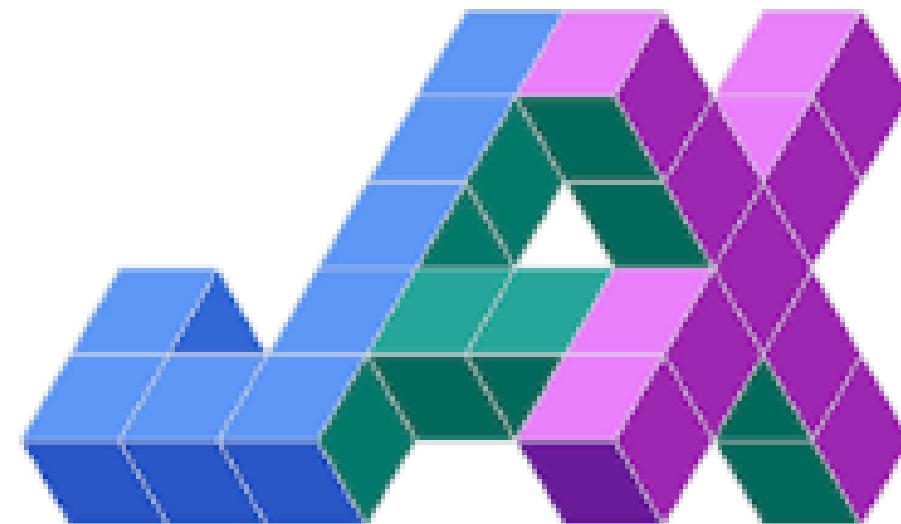
1. NumPy

2. Profiling



Quantum Computing	Statistical Computing	Signal Processing	Image Processing	Graphs and Networks	Astronomy	Cognitive Psychology
						
<a href="#">QuTiP</a>	<a href="#">Pandas</a>	<a href="#">SciPy</a>	<a href="#">Scikit-image</a>	<a href="#">NetworkX</a>	<a href="#">AstroPy</a>	<a href="#">PsychoPy</a>
<a href="#">PyQuil</a>	<a href="#">statsmodels</a>	<a href="#">PyWavelets</a>	<a href="#">OpenCV</a>	<a href="#">graph-tool</a>	<a href="#">SunPy</a>	
<a href="#">Qiskit</a>	<a href="#">Xarray</a>	<a href="#">python-control</a>	<a href="#">Mahotas</a>	<a href="#">igraph</a>	<a href="#">SpacePy</a>	
<a href="#">PennyLane</a>	<a href="#">Seaborn</a>			<a href="#">PyGSP</a>		
Bioinformatics	Bayesian Inference	Mathematical Analysis	Chemistry	Geoscience	Geographic Processing	Architecture & Engineering
						
<a href="#">BioPython</a>	<a href="#">PyStan</a>	<a href="#">SciPy</a>	<a href="#">Cantera</a>	<a href="#">Pangeo</a>	<a href="#">Shapely</a>	<a href="#">COMPAS</a>
<a href="#">Scikit-Bio</a>	<a href="#">PyMC3</a>	<a href="#">SymPy</a>	<a href="#">MDAnalysis</a>	<a href="#">Simpeg</a>	<a href="#">GeoPandas</a>	<a href="#">City Energy Analyst</a>
<a href="#">PyEnsembl</a>	<a href="#">ArviZ</a>	<a href="#">cvxpy</a>	<a href="#">RDKit</a>	<a href="#">ObsPy</a>	<a href="#">Folium</a>	<a href="#">Sverchok</a>
<a href="#">ETE</a>	<a href="#">emcee</a>	<a href="#">FEniCS</a>	<a href="#">PyBaMM</a>	<a href="#">Fatiando a Terra</a>		

numpy.org

The PyTorch logo consists of a red stylized flame or swirl icon followed by the word "PyTorch" in a large, bold, black sans-serif font.

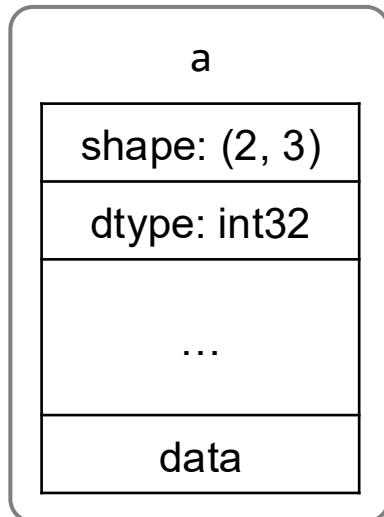
# What is a NumPy array

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])
```

1	2	3
4	5	6

# What is a NumPy array

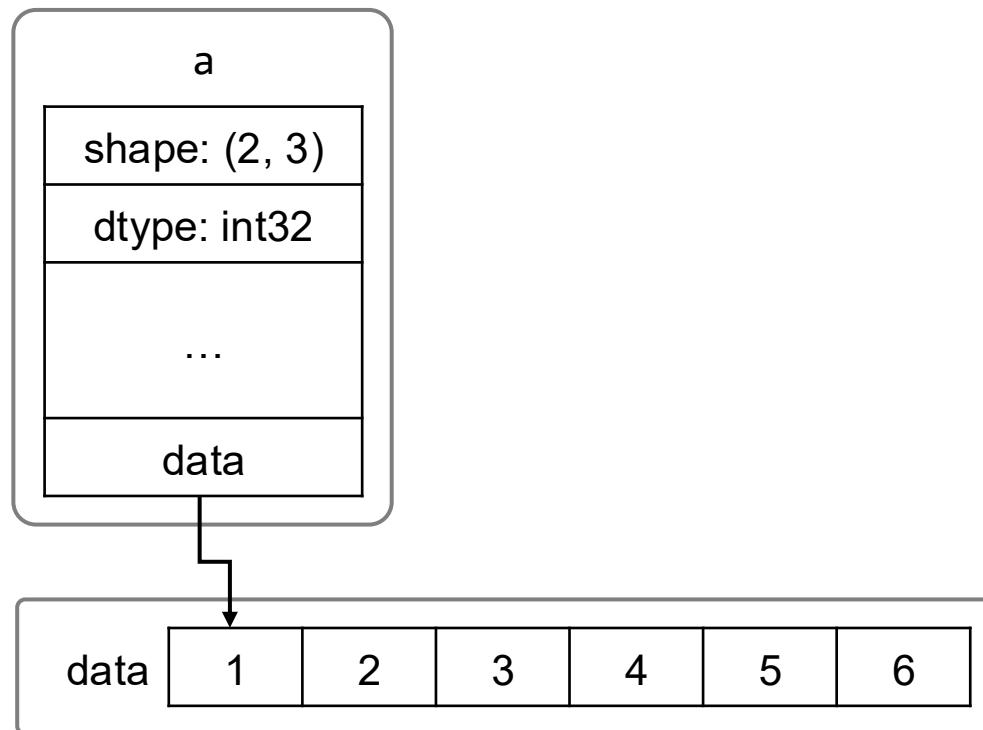
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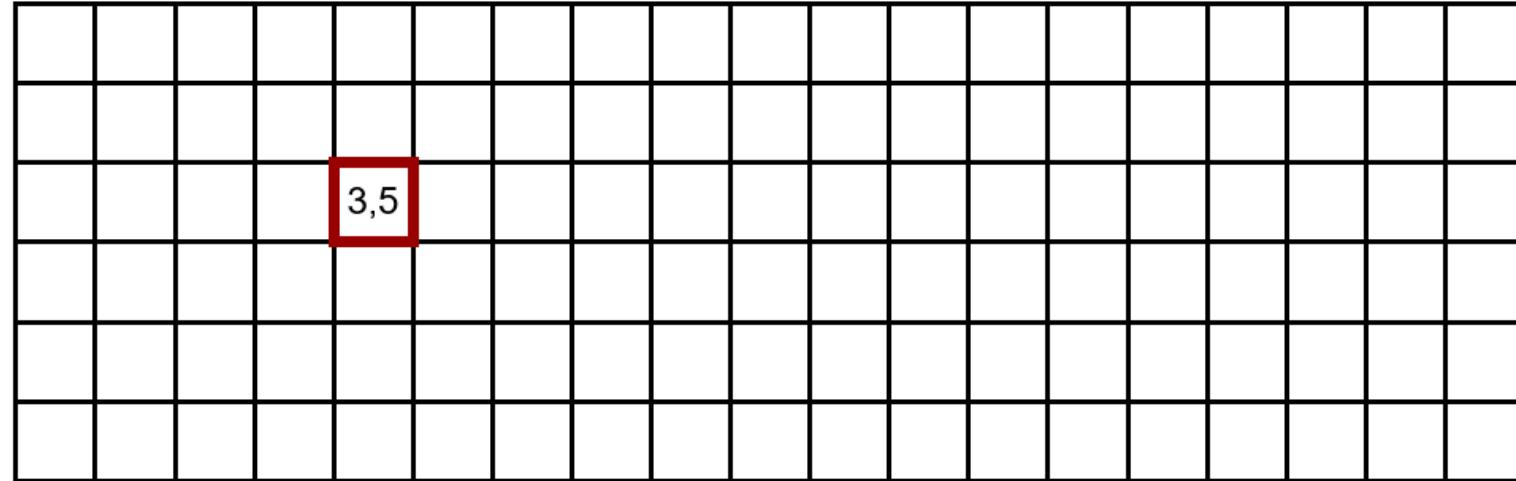
```
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```



1	2	3
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# Caches: multidimensional arrays

x =

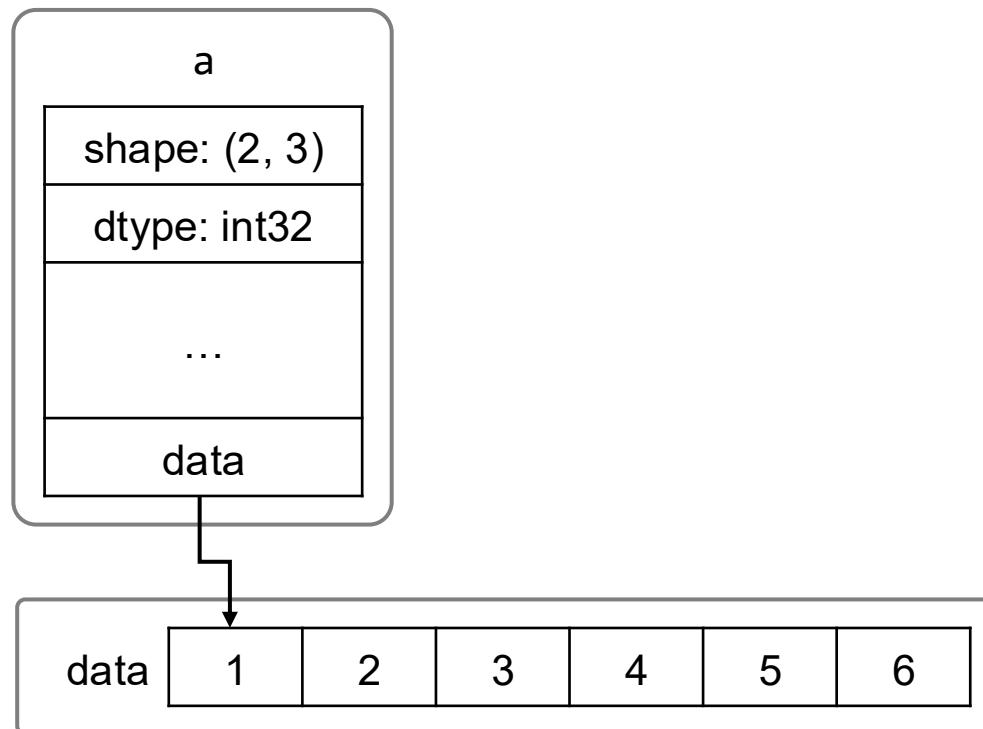


x =

Row 1	Row 2		Row 3	Row 3
-------	-------	--	-------	-------

# What is a NumPy array

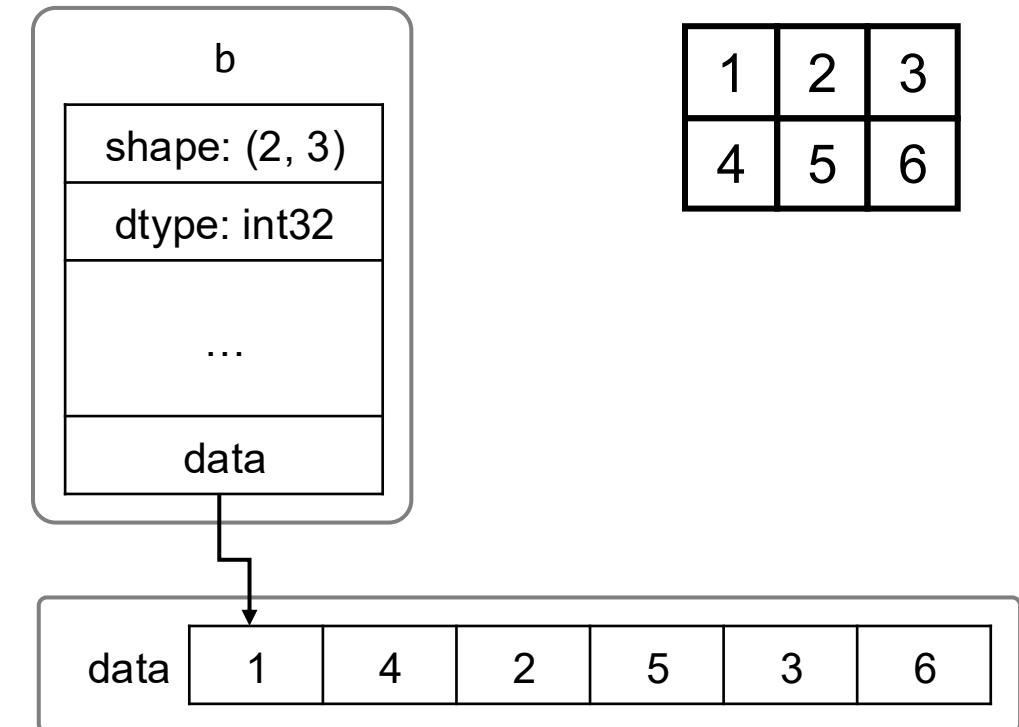
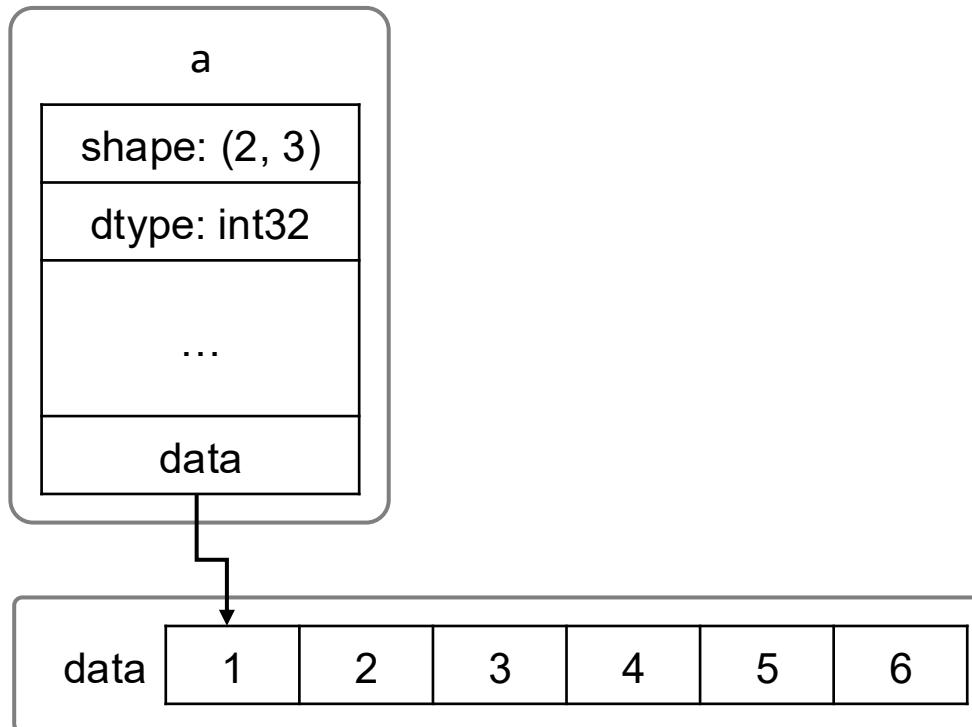
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```



1	2	3
4	5	6

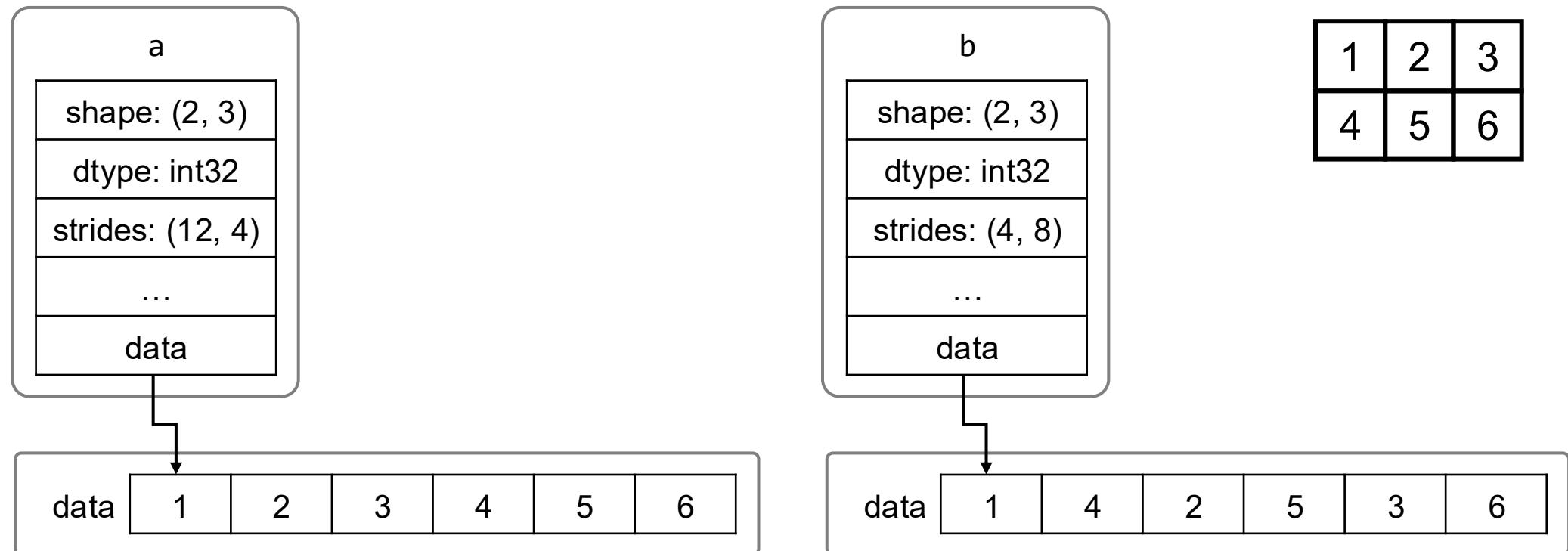
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```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> b = np.array([[1, 2, 3], [4, 5, 6]]),  
          order='F') # F=Fortran
```



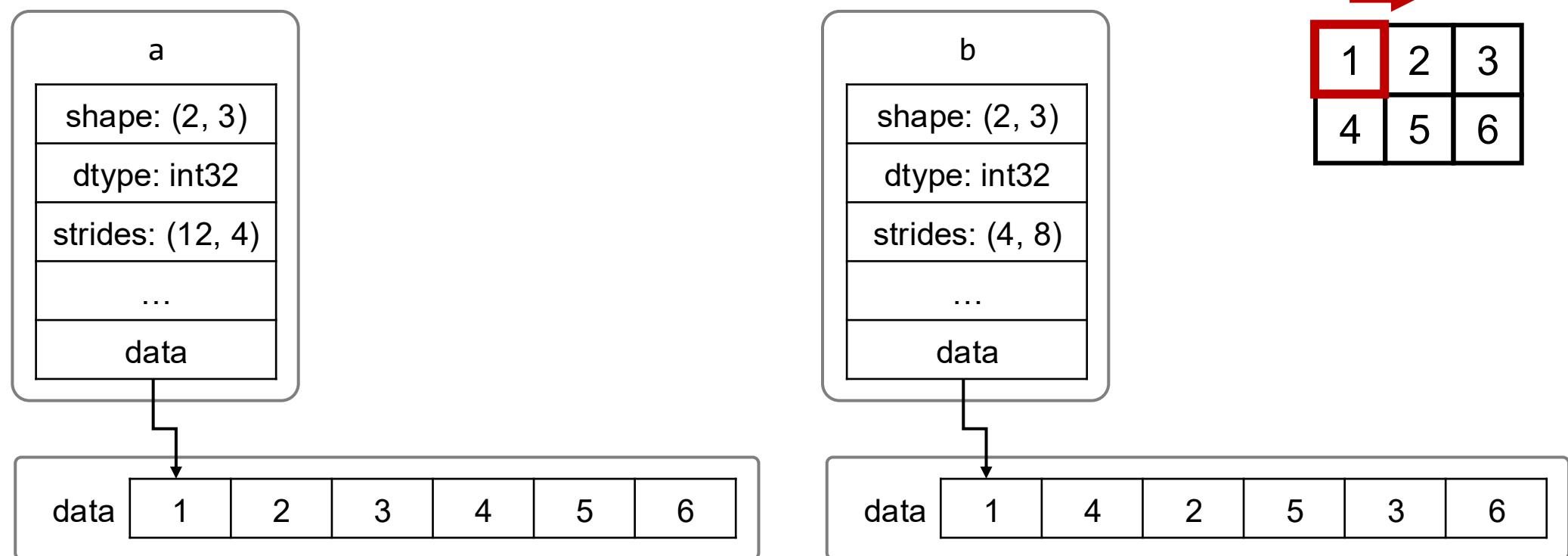
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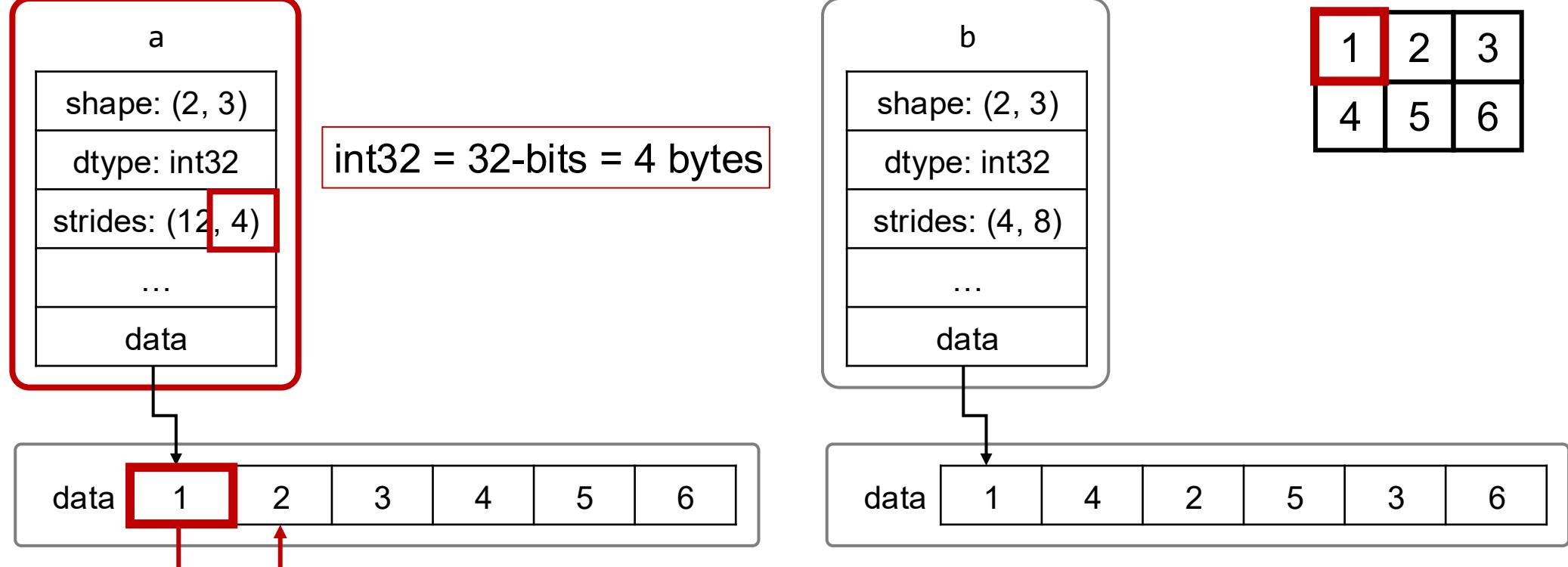
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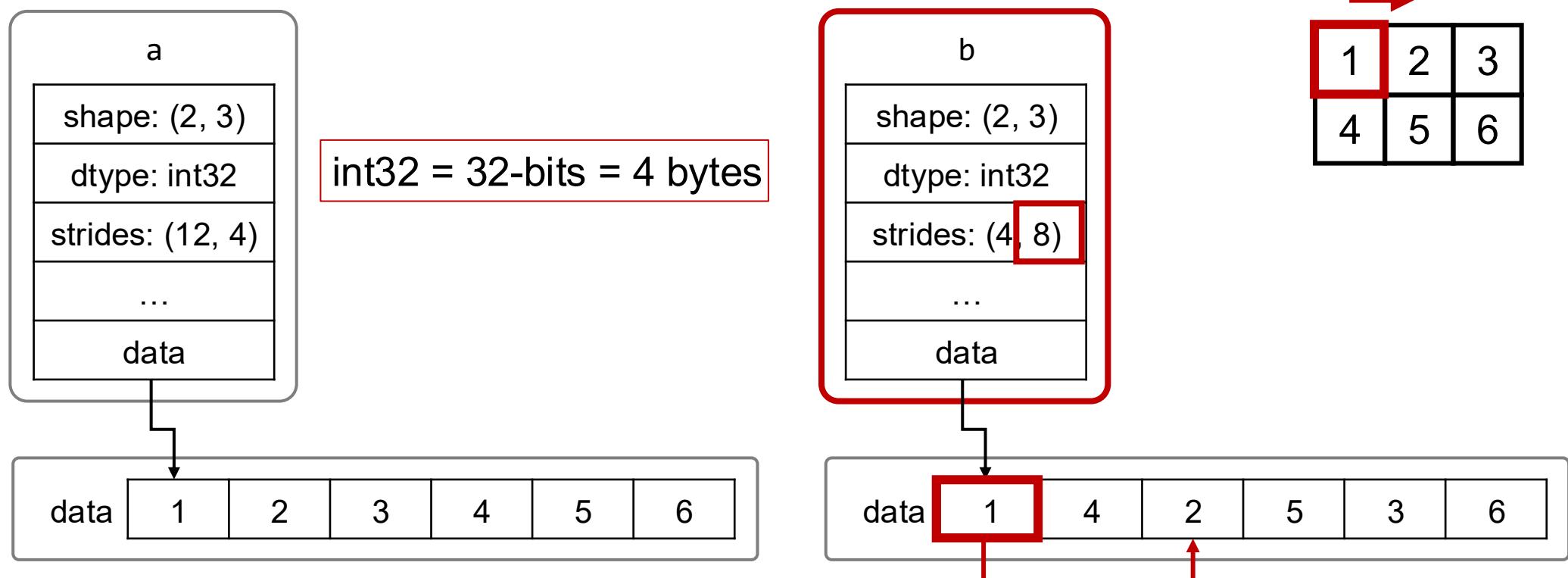
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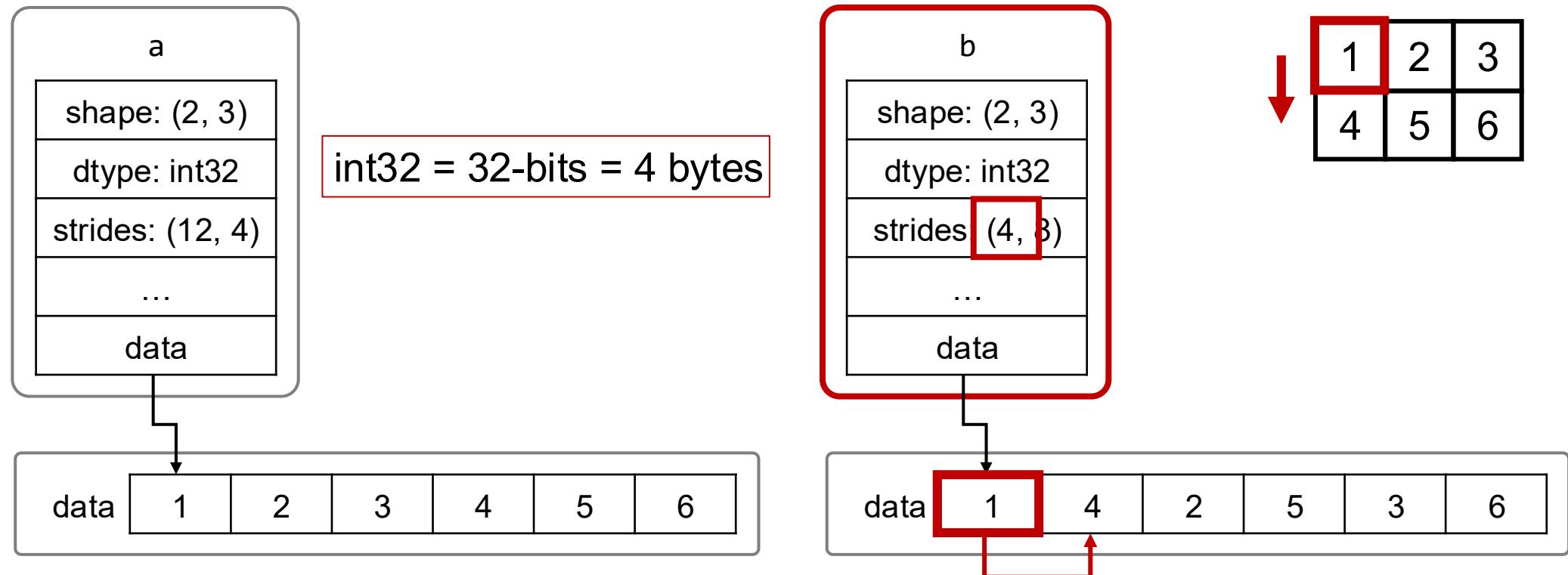
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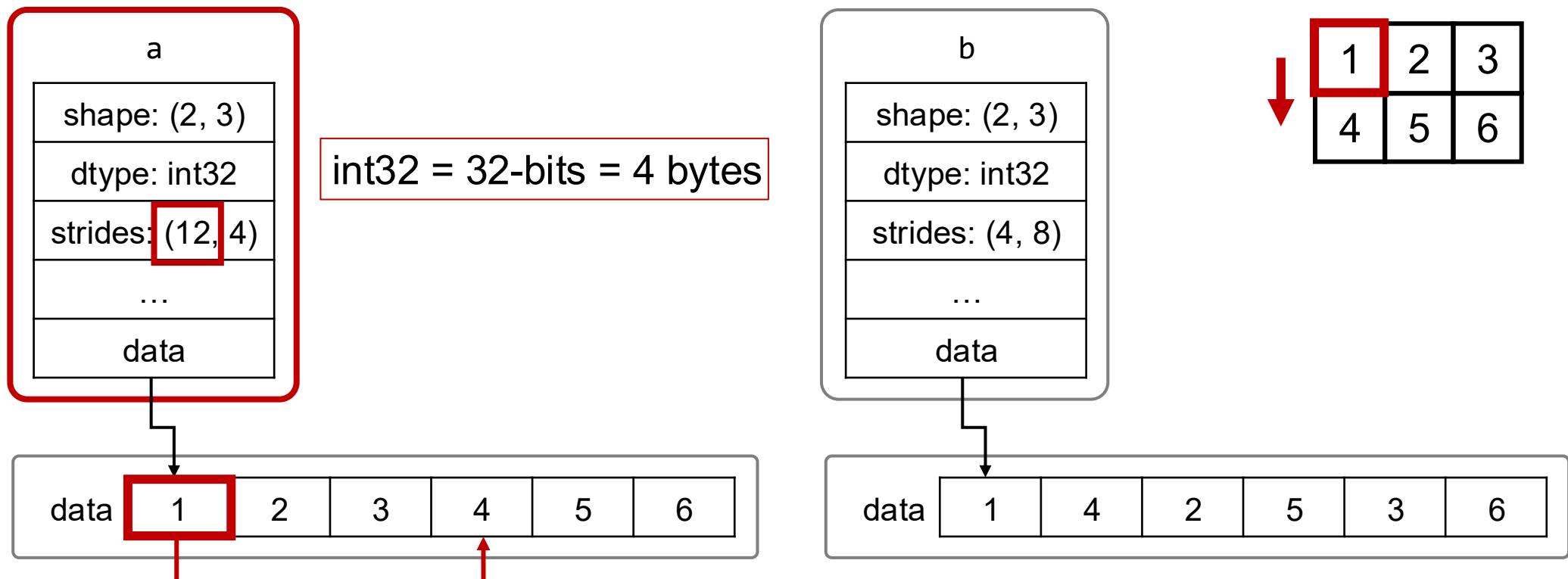
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```



# What is a NumPy array

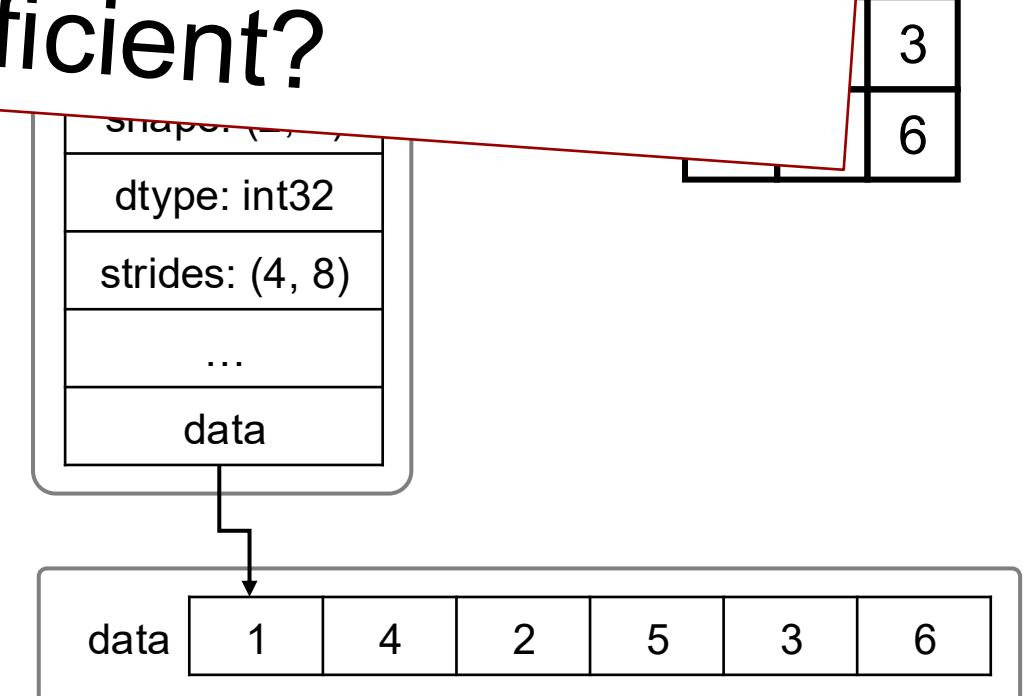
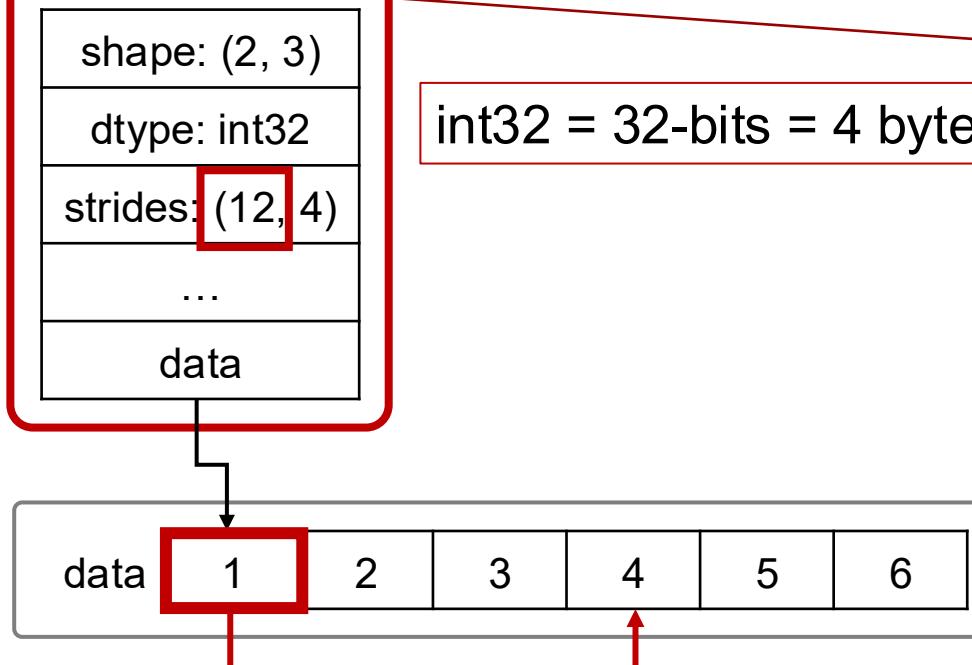
```
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# What is a NumPy array

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> b = np.array([[1, 2, 3], [4, 5, 6]]),  
an
```

**Question:** what direction is most cache efficient?

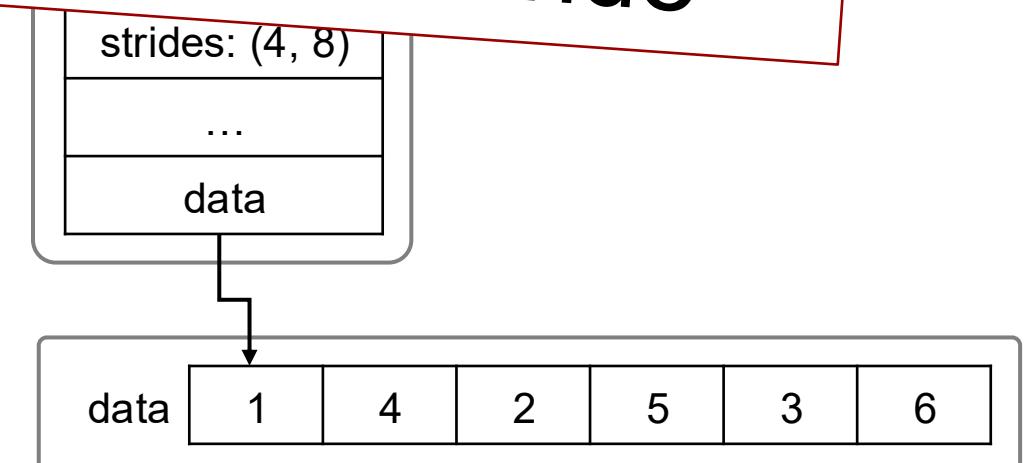
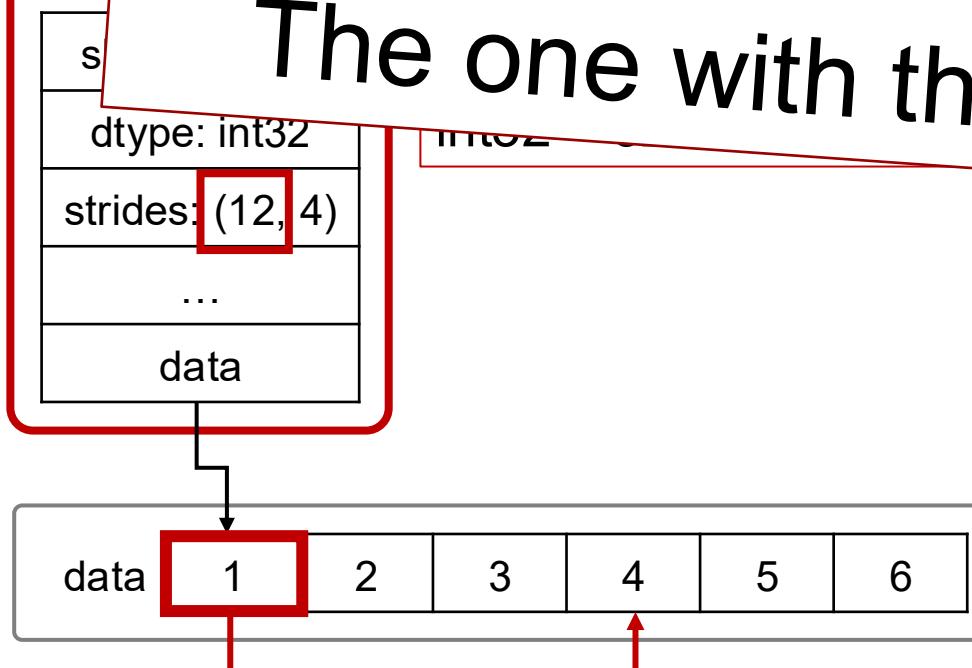


# What is a NumPy array

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
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```

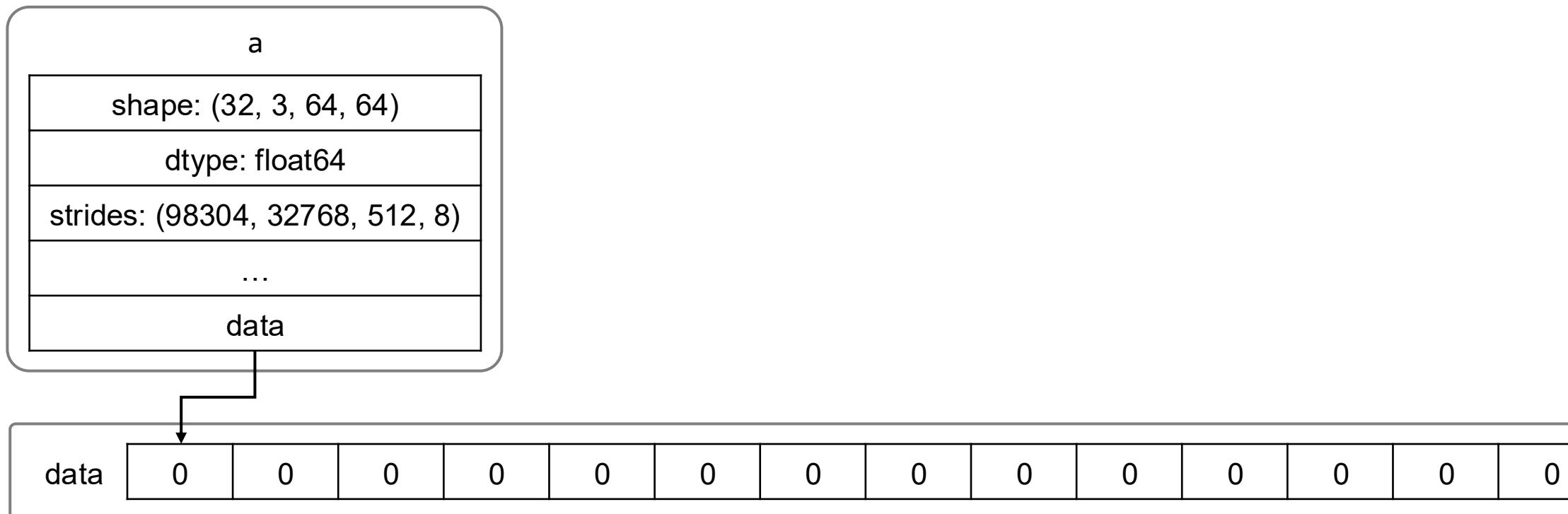
an

**Question:** what direction is most cache efficient?  
The one with the smallest stride

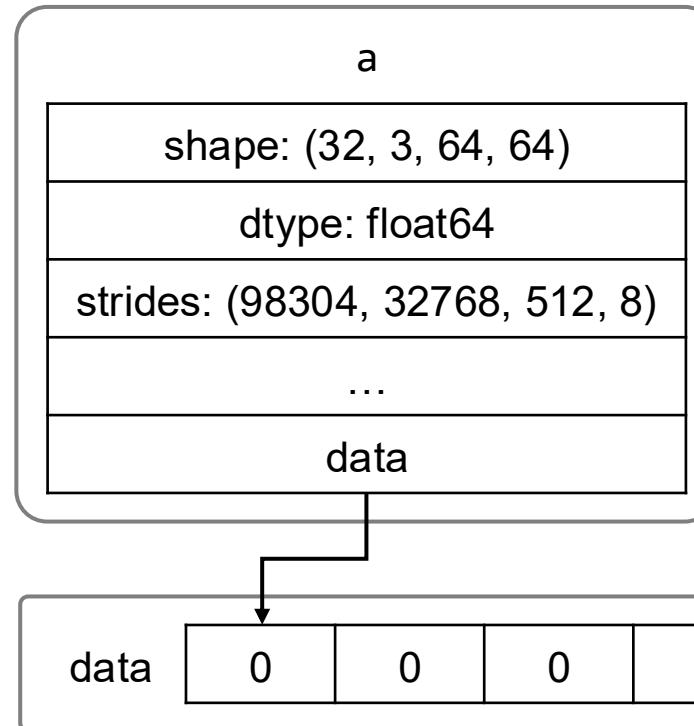


# What is a NumPy array

```
>>> a = np.zeros((32, 3, 64, 64))
```



# What is a NumPy array



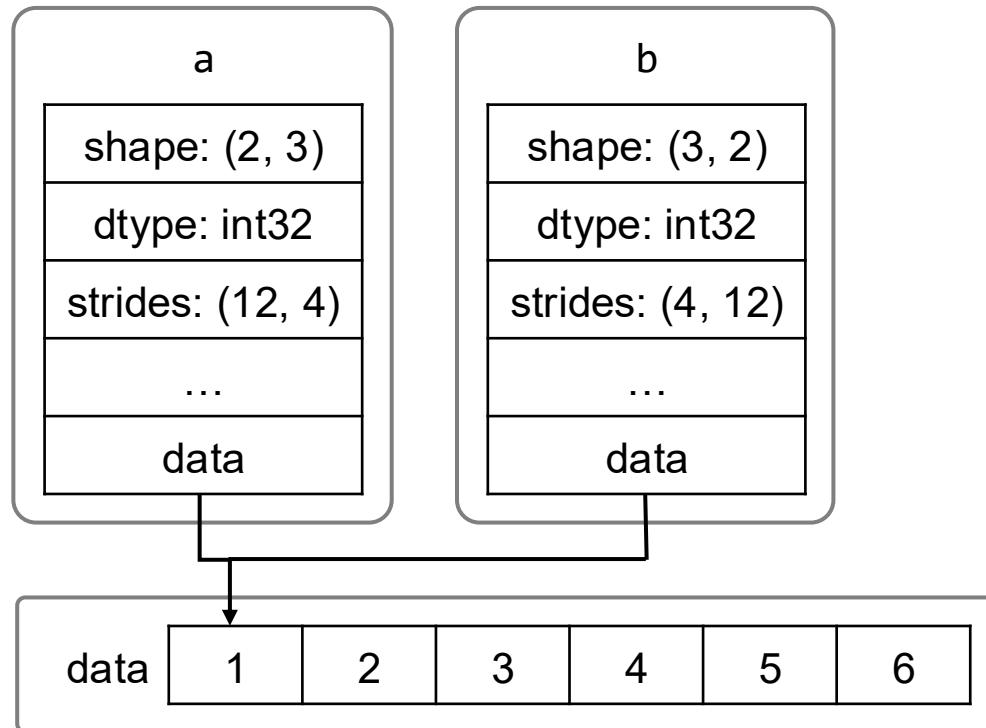
```
>>> a = np.zeros((32, 3, 64, 64))  
>>> a[2, 1, 4, 7]
```

```
index = 2 * strides[0]  
+ 1 * strides[1]  
+ 4 * strides[2]  
+ 7 * strides[3]
```

# NumPy: Views vs Copies

# Views vs Copies

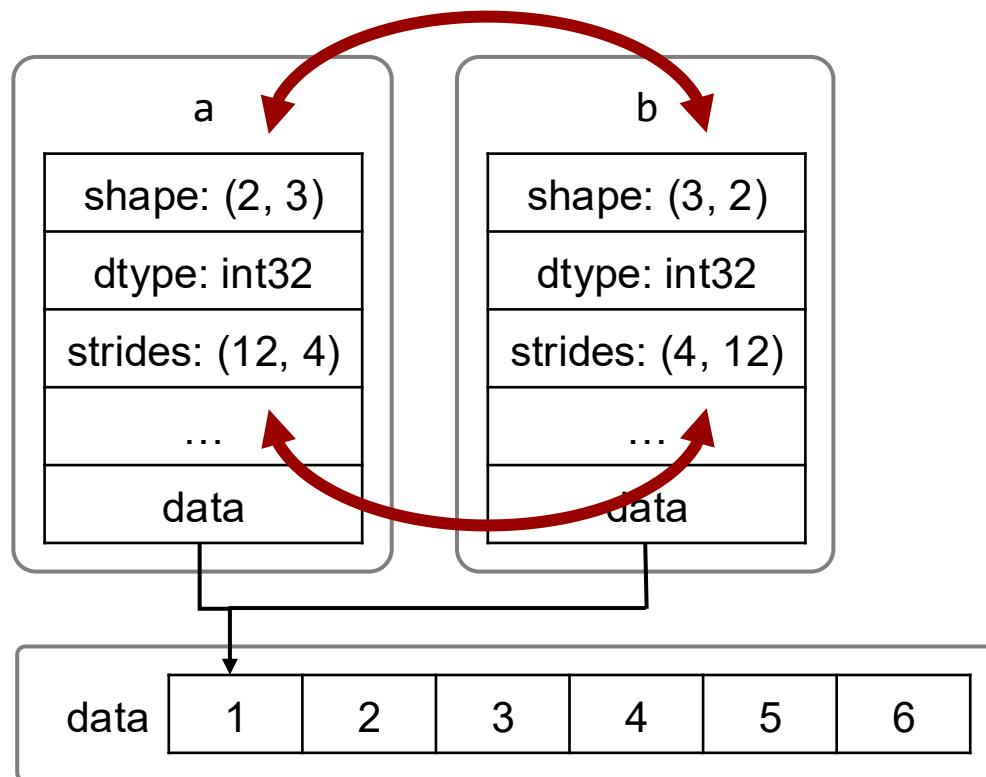
```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> b = a.T # Transpose
```



$$\begin{aligned} \mathbf{a} &= \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline \end{array} \\ \mathbf{b} &= \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 & 6 \\ \hline \end{array} \end{aligned}$$

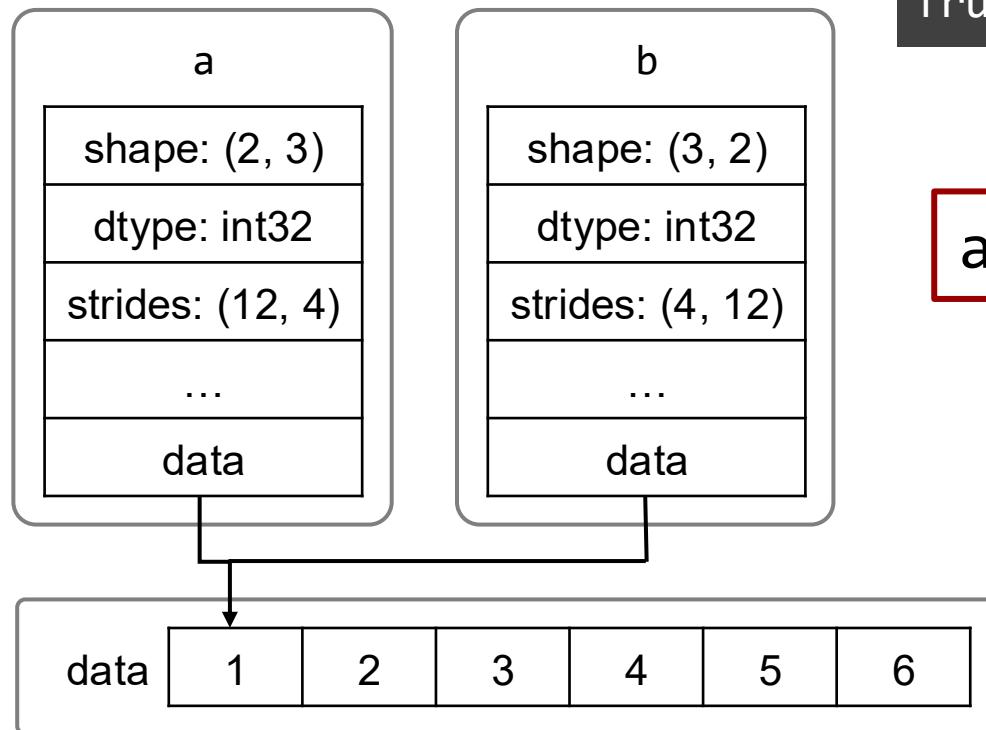
# Views vs Copies

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> b = a.T # Transpose
```



$$\begin{aligned} \mathbf{a} &= \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline \end{array} \\ \mathbf{b} &= \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 & 6 \\ \hline \end{array} \end{aligned}$$

# Views vs Copies

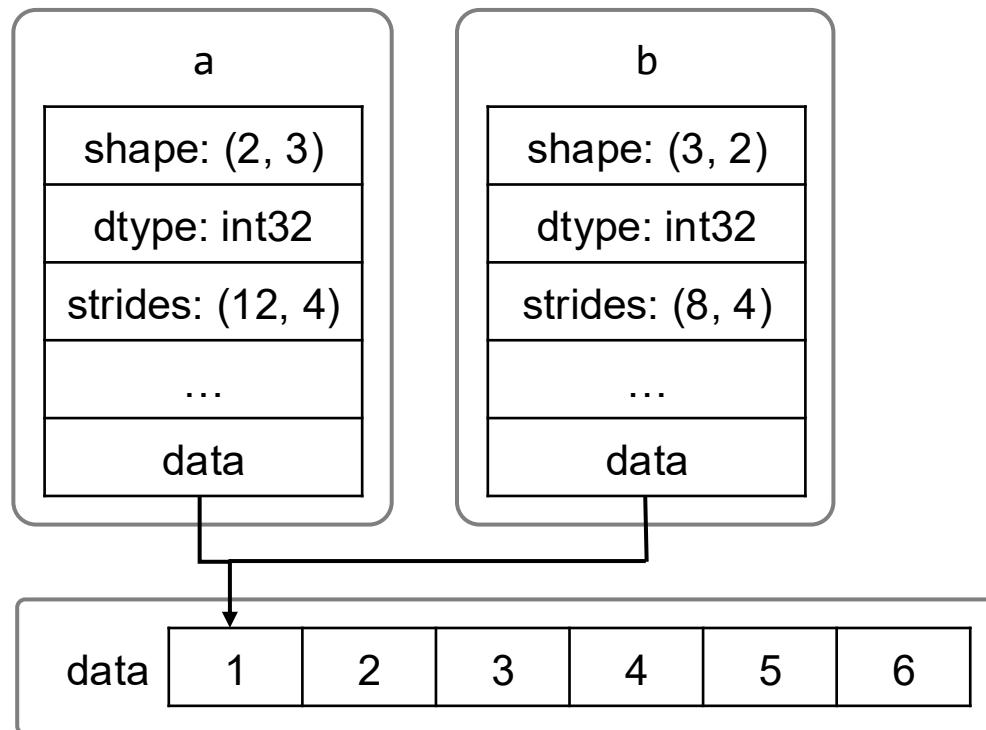


```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> b = a.T # Transpose  
>>> np.shares_memory(a, b)  
True
```

a and b are *views* to the same data

# Views vs Copies

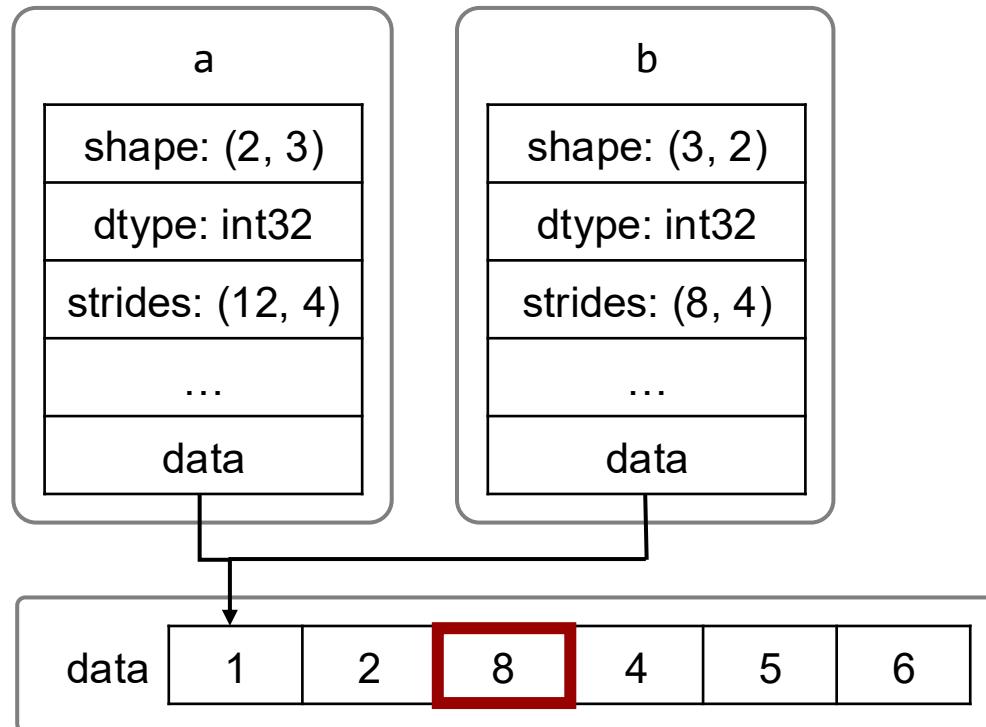
```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> b = a.reshape(3, 2)
```



$$\begin{aligned} \mathbf{a} &= \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline \end{array} \\ \mathbf{b} &= \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline 5 & 6 \\ \hline \end{array} \end{aligned}$$

# Views vs Copies

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> b = a.reshape(3, 2)  
>>> b[1, 0] = 8
```



**a** =

1	2	8
4	5	6

**b** =

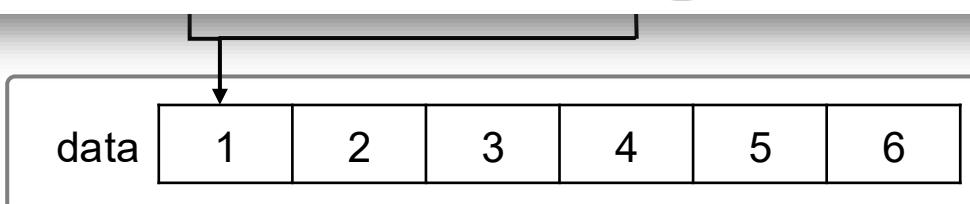
1	2
8	4
5	6

# Views vs Copies

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> b = a.reshape(3, 2)  
... b[1, 0] = 8
```

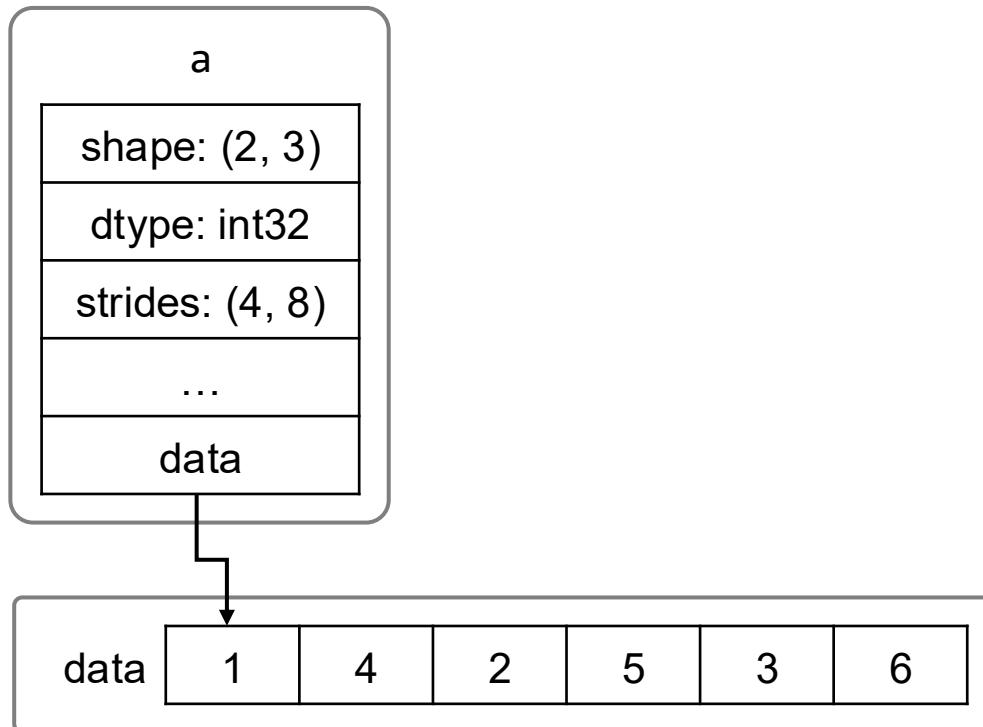
Returns: **reshaped\_array : ndarray**

This will be a new view object if possible; otherwise, it will be a copy. Note there is no guarantee of the *memory layout* (C- or Fortran-contiguous) of the returned array.



# Views vs Copies

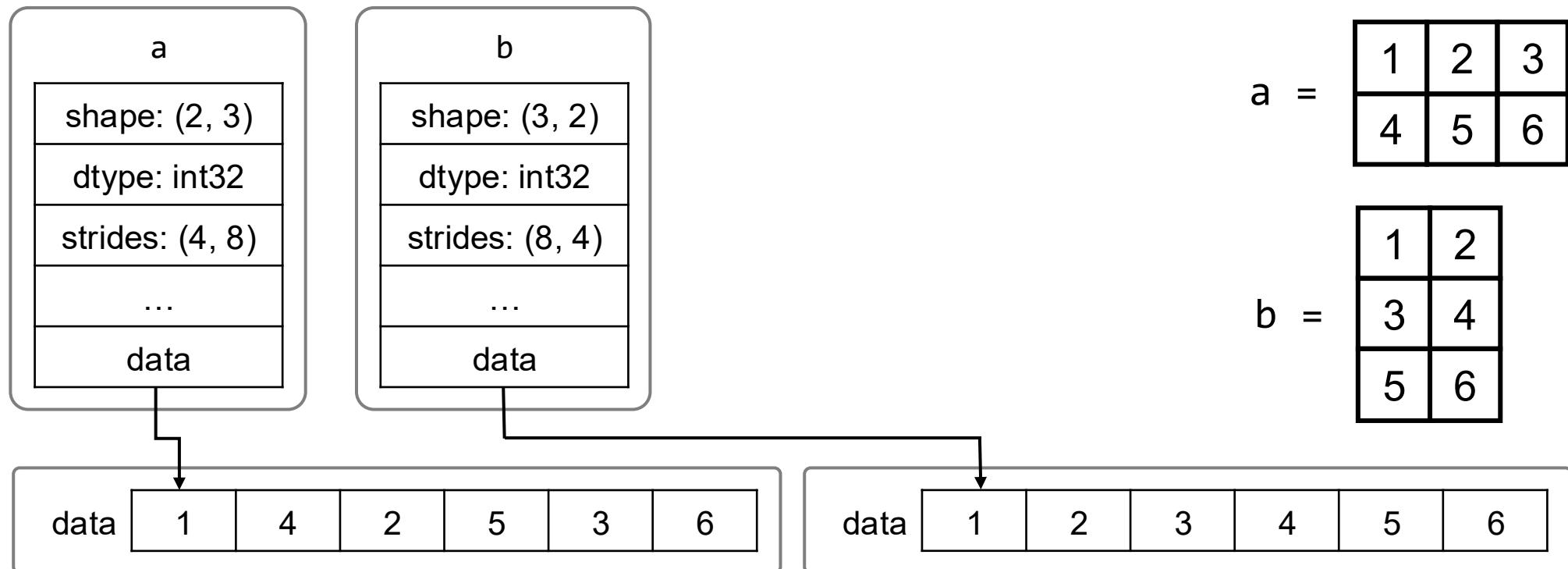
```
>>> a = np.array([[1, 2, 3], [4, 5, 6]],  
                 order='F') # Fortran  
>>> b = a.reshape(3, 2)
```



$$\begin{aligned} a &= \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline \end{array} \\ b &= \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline 5 & 6 \\ \hline \end{array} \end{aligned}$$

# Views vs Copies

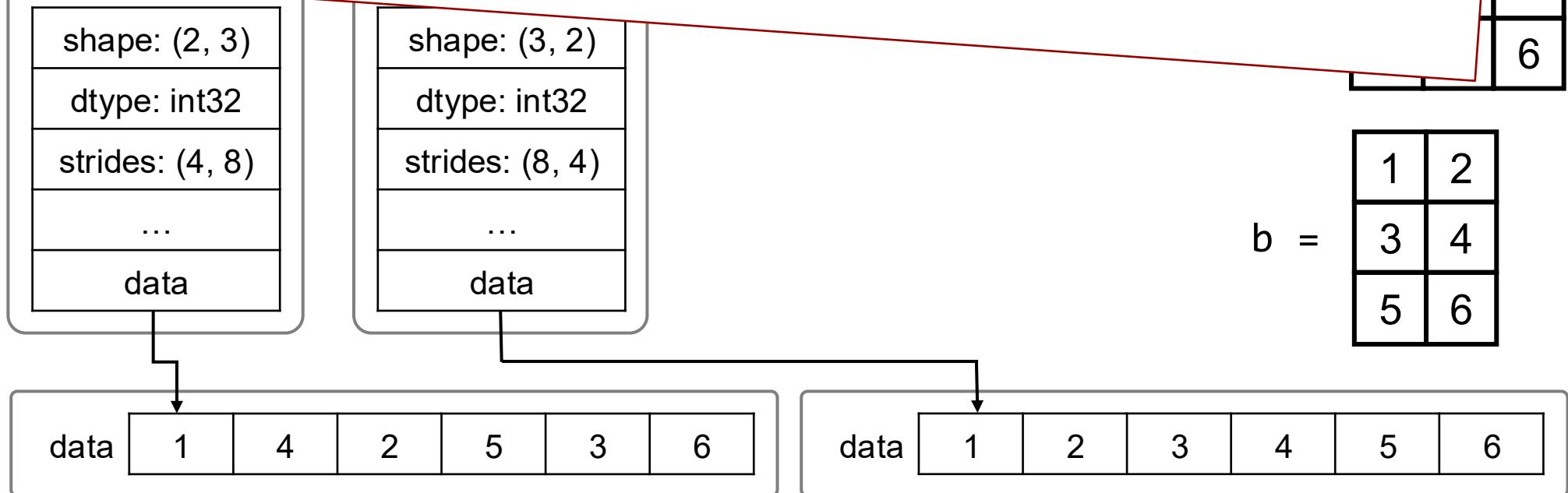
```
>>> a = np.array([[1, 2, 3], [4, 5, 6]],  
                 order='F') # Fortran  
>>> b = a.reshape(3, 2)
```



# Views vs Copies

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]],  
order='F') # Fortran
```

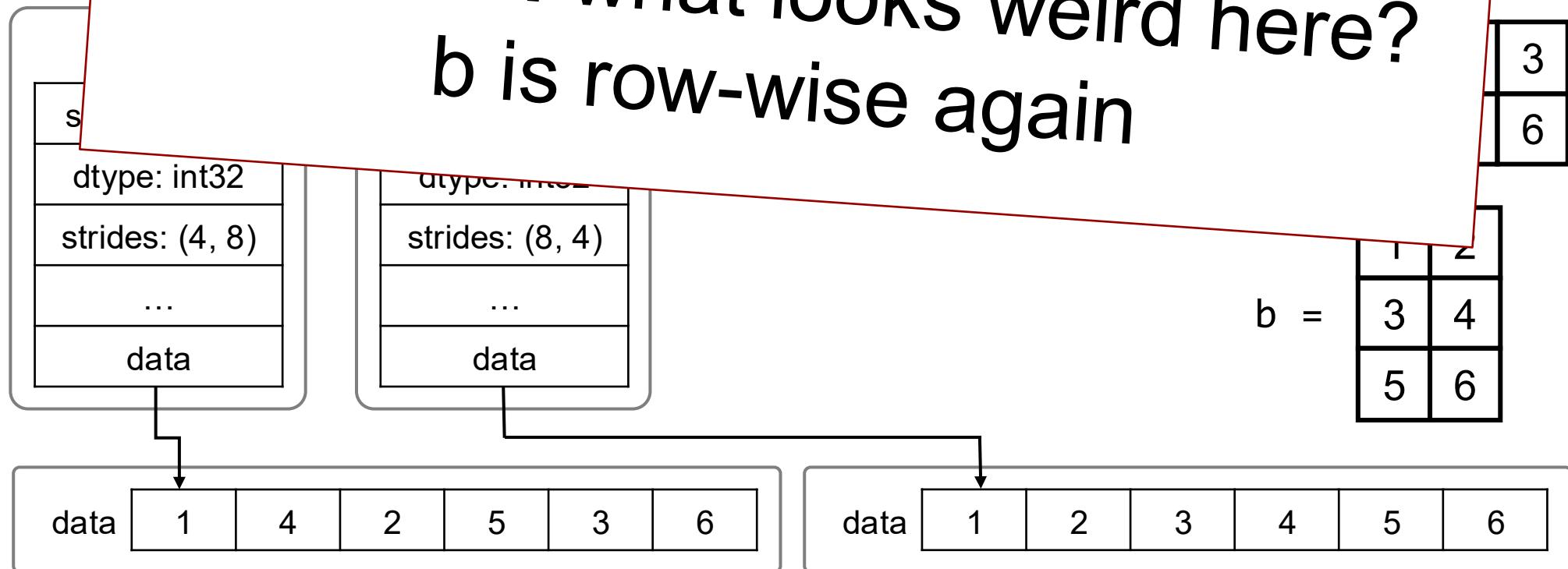
**Question:** what looks weird here?



# Views vs Copies

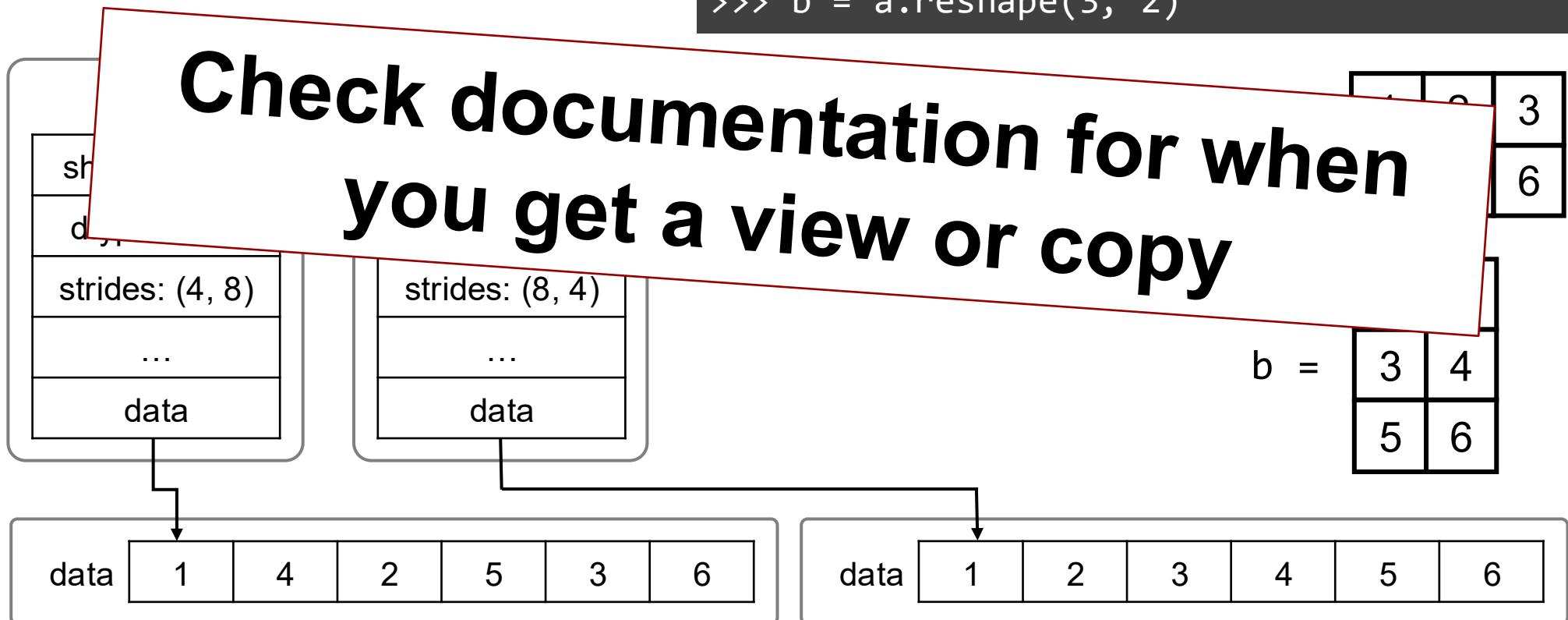
```
>>> a = np.array([[1, 2, 3], [4, 5, 6]],  
order='F') # Fortran
```

**Question:** what looks weird here?  
**b** is row-wise again



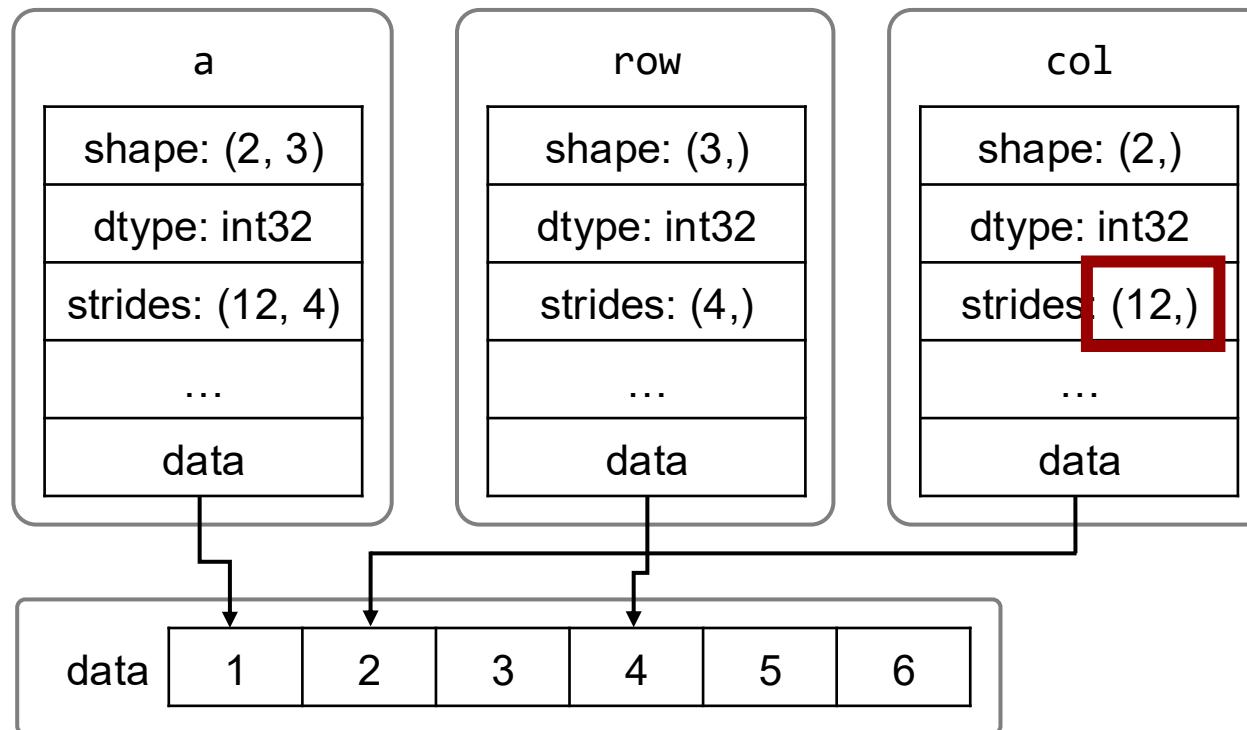
# Views vs Copies

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]],  
                 order='F') # Fortran  
>>> b = a.reshape(3, 2)
```



# Views vs Copies: Indexing

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> row, col = a[1, :], a[:, 1]
```

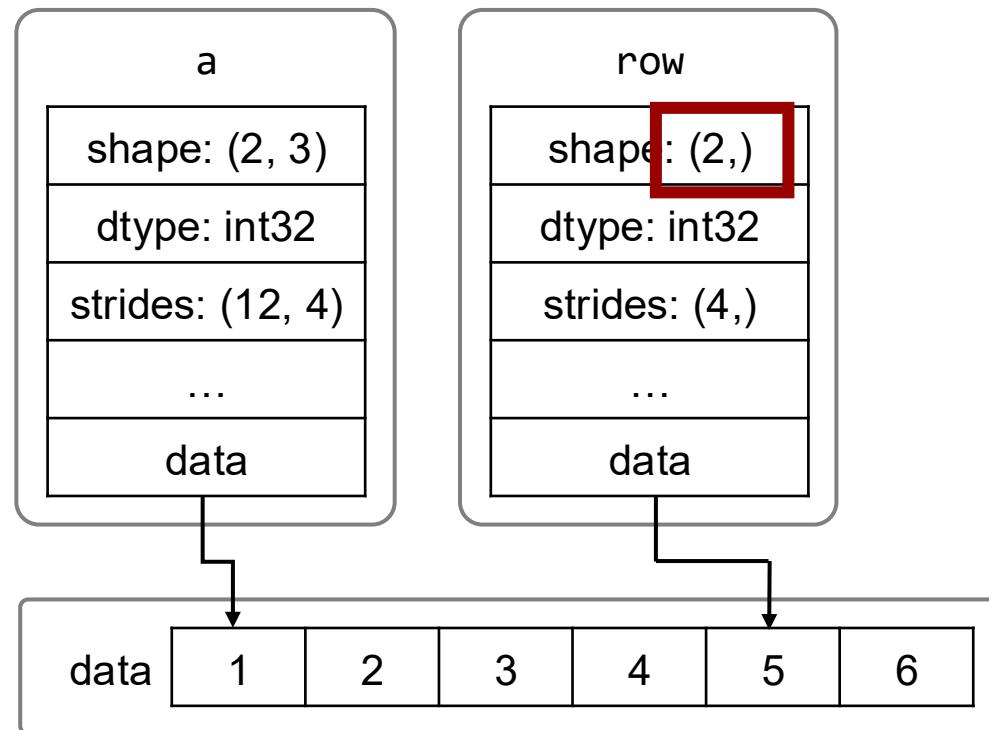


`a` =

1	2	3
4	5	6

# Views vs Copies: Indexing

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> row = a[1, 1:]; row  
array([5, 6])
```

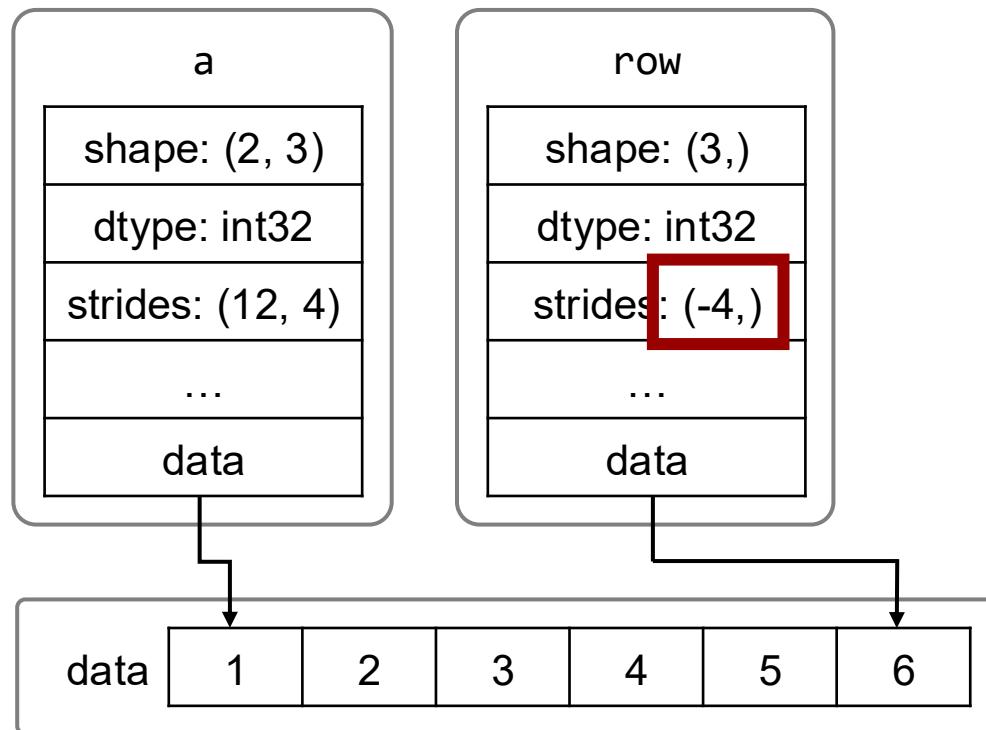


`a` =

1	2	3
4	5	6

# Views vs Copies: Indexing

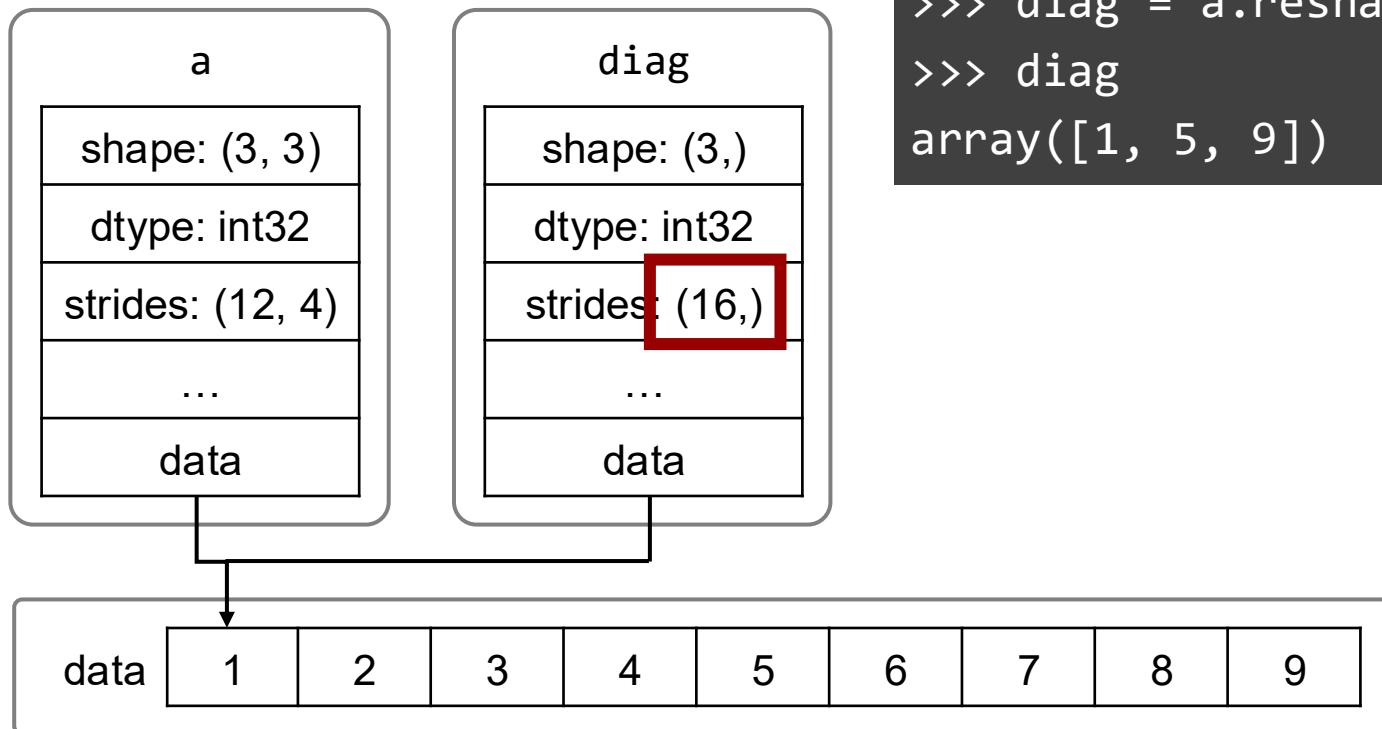
```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> row = a[1, ::-1]; row  
array([6, 5, 4])
```



`a` =

1	2	3
4	5	6

# Views vs Copies: Indexing



```
>>> a = np.array([[1, 2, 3],  
                 [4, 5, 6],  
                 [7, 8, 9]])  
>>> diag = a.reshape(-1)[::4]  
>>> diag  
array([1, 5, 9])
```

`a` =

1	2	3
4	5	6
7	8	9

# Quiz time!

# NumPy: Broadcasting

# Broadcasting

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> b = np.array([2, 3, 4])  
>>> a + b  
array([[ 3,  5,  7],  
       [ 6,  8, 10]])
```

$$\begin{aligned} \mathbf{a} &= \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline \end{array} \\ \mathbf{b} &= \begin{array}{|c|c|c|}\hline 2 & 3 & 4 \\ \hline \end{array} \end{aligned}$$

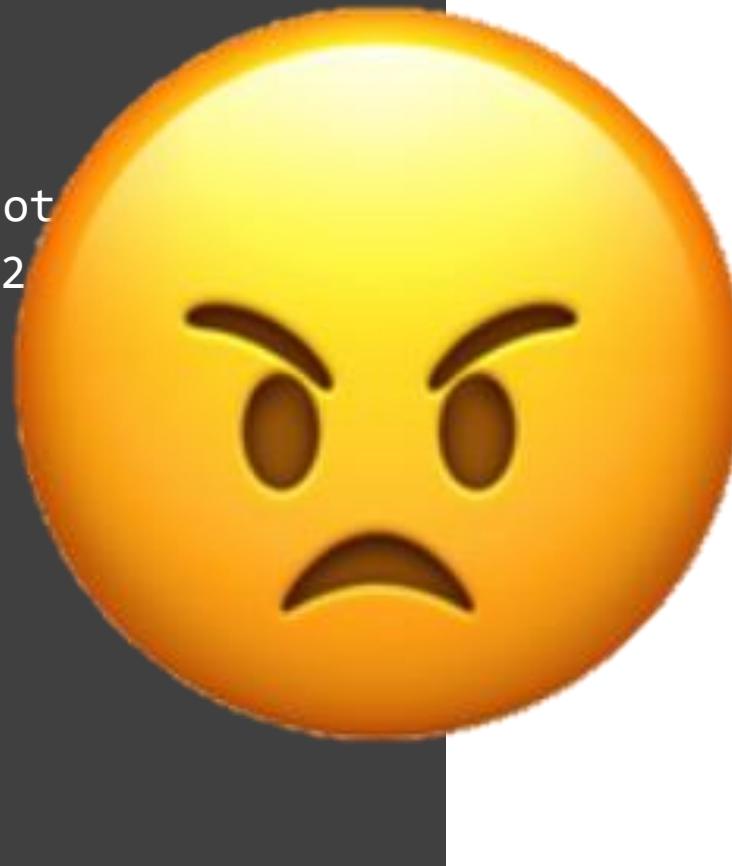
# Broadcasting

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
>>> b = np.array([2, 3])  
>>> a + b  
...  
ValueError: operands could not be broadcast  
together with shapes (2,3) (2,)
```

$$\begin{array}{l} a = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline \end{array} \\ b = \begin{array}{|c|c|}\hline 2 & 3 \\ \hline \end{array} \end{array}$$

# Broadcasting

```
>>> a = np.array([[1, 2, 3], [4, 5, 6]])  
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>>> a + b  
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ValueError: operands could not  
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```



$$\begin{array}{l} \text{a} = \\ \boxed{\begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline \end{array}} \\ \text{b} = \boxed{\begin{array}{|c|c|} \hline 2 & 3 \\ \hline \end{array}} \end{array}$$

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(3,)
```

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(3, )
```

1. Line up the shapes to the right

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(1, 3,)
```

1. Line up the shapes to the right
2. Left-pad the shortest with 1s

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(2, 3,)
```

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(2, 3,)
```

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(2,)
```

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(2, )
```

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(1, 2,)
```

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(2, 2,) Mismatch!
```

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(2,)  
>>> b = b[:, None] # Add dimension  
>>> b.shape  
(2, 1,)
```

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(2,)  
>>> b = b[:, None] # Add dimension  
>>> b.shape  
(2, 1,)
```

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(2,)  
>>> b = b[:, None] # Add dimension  
>>> b.shape  
(2, 3,)
```

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(128, 3, 64, 64,)  
N, C, W, H
```

Stack of 128 RGB images  
of shape 64 x 64

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(128, 3, 64, 64,)  
>>> m.shape  
(3,)
```

Stack of 128 RGB images  
of shape 64 x 64

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(128, 3, 64, 64,)  
>>> m.shape  
(3,)  
>>> a - m  
...  
ValueError: operands could not be broadcast  
together with shapes (128,3,64,64) (3,)
```

Stack of 128 RGB images  
of shape 64 x 64

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(128, 3, 64, 64,)  
>>> m.shape  
      (3,)  
>>> a - m  
...  
ValueError: operands could not be broadcast  
together with shapes (128,3,64,64) (3,)
```

Stack of 128 RGB images  
of shape 64 x 64

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(128, 3, 64, 64,)  
  
>>> m.shape  
( 1, 1, 1, 3,)  
  
>>> a - m  
...  
ValueError: operands could not be broadcast  
together with shapes (128,3,64,64) (3,)
```

Stack of 128 RGB images  
of shape 64 x 64

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(128, 3, 64, 64,)  
  
>>> m.shape  
(128, 3, 64, 3,)  
  
>>> a - m  
...  
ValueError: operands could not be broadcast  
together with shapes (128,3,64,64) (3,)
```

Stack of 128 RGB images  
of shape 64 x 64

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(128, 3, 64, 64,)  
>>> m.shape  
(3,)  
>>> m = m[:, None, None] # Add dimensions  
>>> m.shape  
(3, 1, 1,)  
>>> a - m  
>>>
```

Stack of 128 RGB images  
of shape 64 x 64

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(128, 3, 64, 64,)  
>>> m.shape  
(3,)  
>>> m = m[:, None, None] # Add dimensions  
>>> m.shape  
(3, 1, 1,)  
>>> a - m  
>>>
```

Stack of 128 RGB images  
of shape 64 x 64

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(128, 3, 64, 64,)  
>>> m.shape  
(3,)  
>>> m = m[:, None, None] # Add dimensions  
>>> m.shape  
(1, 3, 1, 1,)  
>>> a - m  
>>>
```

Stack of 128 RGB images  
of shape 64 x 64

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(128, 3, 64, 64,)  
  
>>> m.shape  
(3,)  
  
>>> m = m[:, None, None] # Add dimensions  
>>> m.shape  
(128, 3, 64, 64,)  
  
>>> a - m  
  
>>>
```

Stack of 128 RGB images  
of shape 64 x 64

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(128, 3, 64, 64,)  
  
>>> m.shape  
(3,)  
  
>>> m = m[:, None, None] # Add dimensions  
>>> m.shape  
(3, 1, 1,)  
  
>>> a - m  
  
>>> m[None].shape  
(1, 3, 1, 1,)
```

Stack of 128 RGB images  
of shape 64 x 64

1. Line up the shapes to the right
2. Left-pad the shortest with 1s
3. If an element is 1, replace with larger
4. If all elements match: broadcasted!

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(3,)
```

# Broadcasting

```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(3,)  
>>> a, b = np.broadcast_arrays(a, b)  
>>> print(f'a: {a.shape}, b: {b.shape}')  
a: (2, 3), b: (2, 3)
```

# Broadcasting

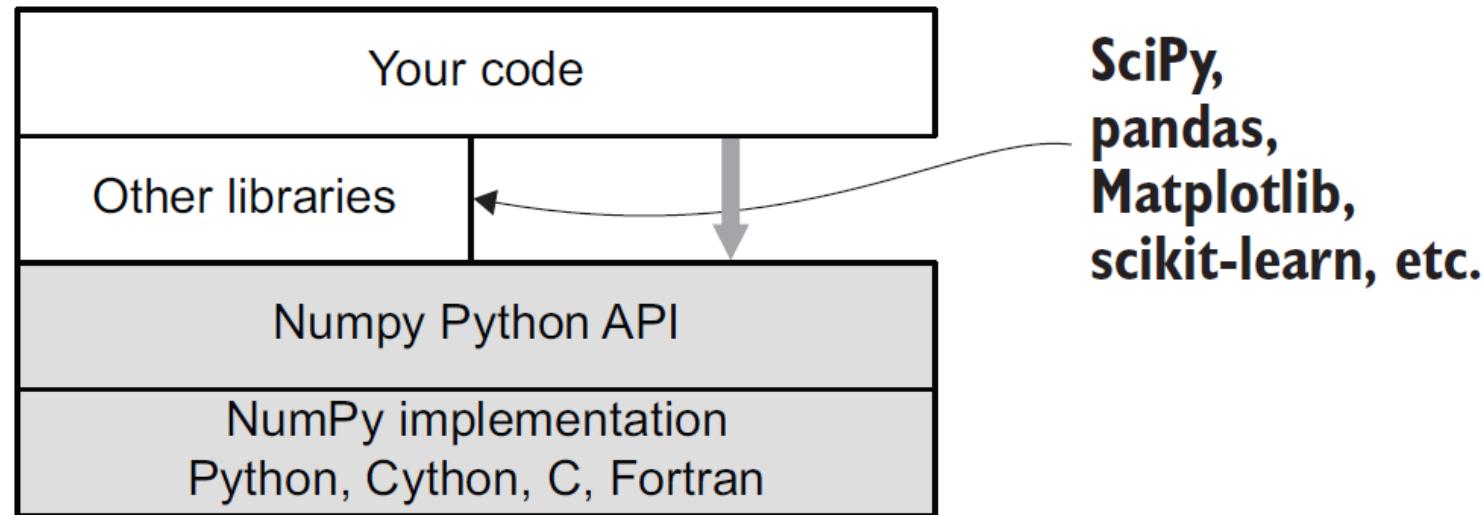
```
>>> a.shape  
(2, 3,)  
>>> b.shape  
(3,)  
>>> a, b = np.broadcast_arrays(a, b)  
>>> print(f'a: {a.shape}, b: {b.shape}')  
a: (2, 3), b: (2, 3)  
>>> print(f'a: {a.strides}, b: {b.strides}')  
a: (24, 8), b: (0, 8)
```



# Quiz time!

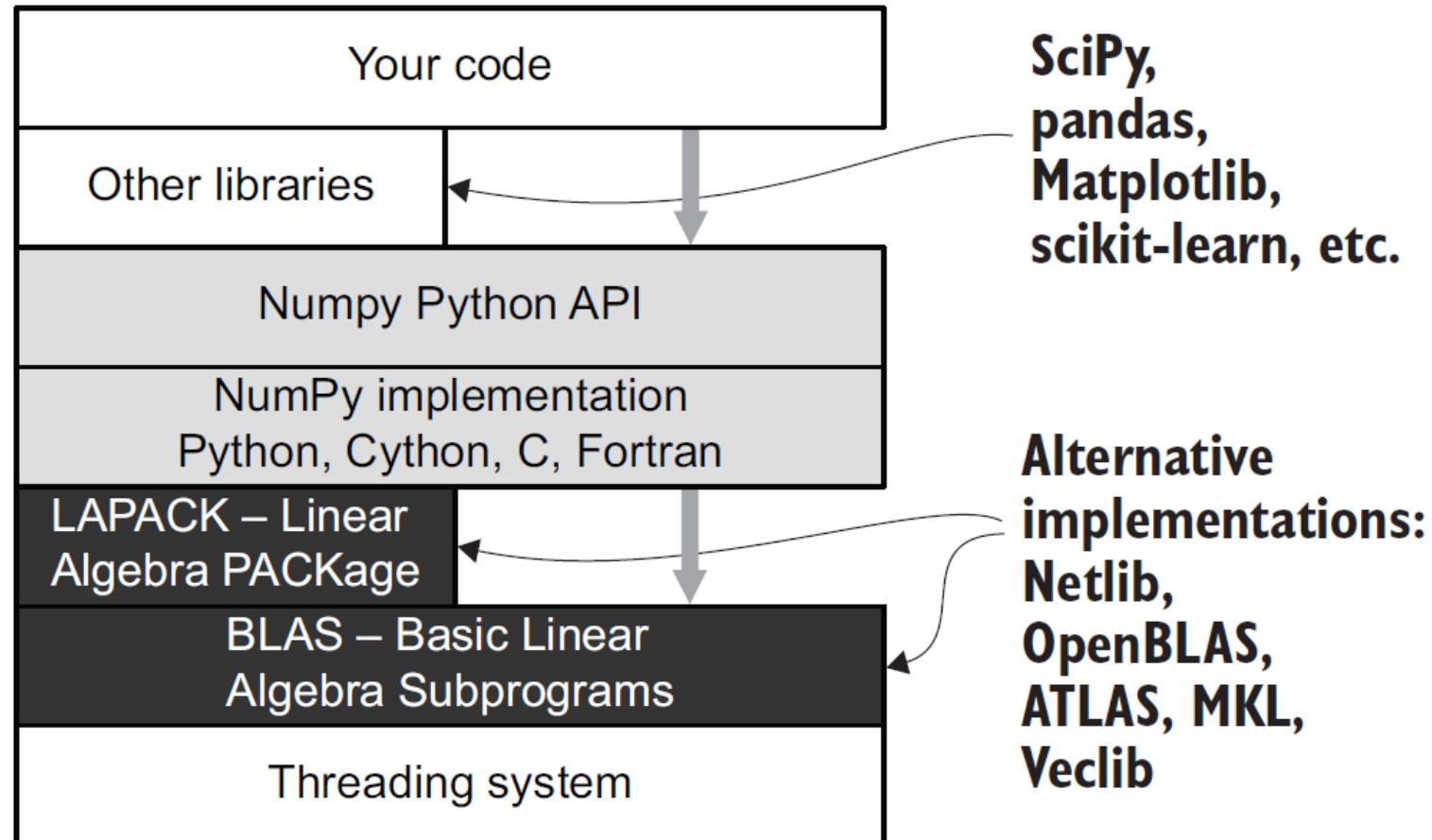
# NumPy Backends: The foundation of the foundation

# NumPy backends



Fast Python - Figure 4.11

# NumPy backends



Fast Python - Figure 4.11

# NumPy backends

```
$ conda install numpy
```

```
$ pip install numpy
```

```
$ conda install -c conda-forge numpy
```

# NumPy backends

```
$ conda install numpy  
$ python  
>>> import numpy as np  
>>> np.show_config()  
Build Dependencies:  
blas:  
    detection method: pkgconfig  
    found: true  
    include directory: ...  
    lib directory: ...  
    name: mkl-sdl  
    openblas configuration: unknown  
...
```

```
$ pip install numpy  
$ python  
>>> import numpy as np  
>>> np.show_config()  
...  
"Build Dependencies": {  
    "blas": {  
        "name": "openblas64",  
        "found": true,  
        "version": "0.3.23.dev",  
        "detection method": "pkgconfig",  
        "include directory": ...,  
        "lib directory": "/usr/local/lib",  
        "openblas configuration": ...  
    }  
}
```

```
$ conda install -c conda-forge numpy  
$ python  
>>> import numpy as np  
>>> np.show_config()  
...  
"Build Dependencies": {  
    "blas": {  
        "name": "blas",  
        "found": true,  
        "version": "3.9.0",  
        "detection method": "pkgconfig",  
        "include directory": ...,  
        "lib directory": ...,  
        "openblas configuration": ...  
    }  
}
```

# NumPy backends: A Laptop

```
$ conda install numpy
```

```
$ python numpy_bench.py
```

Dotted two 4096x4096 matrices in:  
1.76 s.

Dotted two vectors of length 524288 in:  
0.28 ms.

SVD of a 2048x1024 matrix in:  
0.72 s.

Cholesky decomposition of a  
2048x2048 matrix in:  
0.17 s.

Eigendecomposition of a 2048x2048  
matrix in:  
6.77 s.

```
$ pip install numpy
```

```
$ python numpy_bench.py
```

Dotted two 4096x4096 matrices in:  
1.49 s.

Dotted two vectors of length 524288 in:  
0.29 ms.

SVD of a 2048x1024 matrix in:  
1.48 s.

Cholesky decomposition of a  
2048x2048 matrix in:  
0.19 s.

Eigendecomposition of a 2048x2048  
matrix in:  
8.01 s.

```
$ conda install -c conda-forge numpy
```

```
$ python numpy_bench.py
```

Dotted two 4096x4096 matrices in:  
1.76 s.

Dotted two vectors of length 524288 in:  
0.30 ms.

SVD of a 2048x1024 matrix in:  
0.77 s.

Cholesky decomposition of a  
2048x2048 matrix in:  
0.18 s.

Eigendecomposition of a 2048x2048  
matrix in:  
6.92 s.

<https://web.archive.org/web/20230130105308/https://markus-beuckelmann.de/blog/boosting-numpy-blas.html>

# NumPy backends: HPC with XeonGold6226R

```
$ conda install numpy
```

```
$ python numpy_bench.py
```

Dotted two 4096x4096 matrices in:  
2.02 s.

Dotted two vectors of length 524288 in:  
0.35 ms.

SVD of a 2048x1024 matrix in:  
1.11 s.

Cholesky decomposition of a  
2048x2048 matrix in:  
0.30 s.

Eigendecomposition of a 2048x2048  
matrix in:  
9.23 s.

```
$ pip install numpy
```

```
$ python numpy_bench.py
```

Dotted two 4096x4096 matrices in:  
2.00 s.

Dotted two vectors of length 524288 in:  
0.37 ms.

SVD of a 2048x1024 matrix in:  
1.24 s.

Cholesky decomposition of a  
2048x2048 matrix in:  
0.35 s.

Eigendecomposition of a 2048x2048  
matrix in:  
8.70 s.

```
$ conda install -c conda-forge numpy
```

```
$ python numpy_bench.py
```

Dotted two 4096x4096 matrices in:  
1.97 s.

Dotted two vectors of length 524288 in:  
0.36 ms.

SVD of a 2048x1024 matrix in:  
1.12 s.

Cholesky decomposition of a  
2048x2048 matrix in:  
0.32 s.

Eigendecomposition of a 2048x2048  
matrix in:  
8.48 s.

<https://web.archive.org/web/20230130105308/https://markus-beuckelmann.de/blog/boosting-numpy-blas.html>

# NumPy backends: HPC with XeonE5\_2650v4

```
$ conda install numpy
```

```
$ python numpy_bench.py
```

Dotted two 4096x4096 matrices in:  
4.02 s.

Dotted two vectors of length 524288 in:  
0.27 ms.

SVD of a 2048x1024 matrix in:  
1.30 s.

Cholesky decomposition of a  
2048x2048 matrix in:  
0.22 s.

Eigendecomposition of a 2048x2048  
matrix in:  
10.32 s.

```
$ pip install numpy
```

```
$ python numpy_bench.py
```

Dotted two 4096x4096 matrices in:  
4.59 s.

Dotted two vectors of length 524288 in:  
0.27 ms.

SVD of a 2048x1024 matrix in:  
1.46 s.

Cholesky decomposition of a  
2048x2048 matrix in:  
0.22 s.

Eigendecomposition of a 2048x2048  
matrix in:  
10.81 s.

```
$ conda install -c conda-forge numpy
```

```
$ python numpy_bench.py
```

Dotted two 4096x4096 matrices in:  
4.69 s.

Dotted two vectors of length 524288 in:  
0.28 ms.

SVD of a 2048x1024 matrix in:  
1.62 s.

Cholesky decomposition of a  
2048x2048 matrix in:  
0.24 s.

Eigendecomposition of a 2048x2048  
matrix in:  
12.83 s.

<https://web.archive.org/web/20230130105308/https://markus-beuckelmann.de/blog/boosting-numpy-blas.html>

# Profiling

# Profiling

"Premature optimization is the root of all evil."  
- Donald E. Knuth

# Structured Programming with go to Statements

**DONALD E. KNUTH**

*Stanford University, Stanford, California 94305*

A consideration of several different examples sheds new light on the problem of creating reliable, well-structured programs that behave efficiently. This study focuses largely on two issues: (a) improved syntax for iterations and error exits, making it possible to write a larger class of programs clearly and efficiently without **go to** statements; (b) a methodology of program design, beginning with readable and correct, but possibly inefficient programs that are systematically transformed if necessary into efficient and correct, but possibly less readable code. The discussion brings out opposing points of view about whether or not **go to** statements should be abolished; some merit is found on both sides of this question. Finally, an attempt is made to define the true nature of structured programming, and to recommend fruitful directions for further study.

*Keywords and phrases:* structured programming, **go to** statements, language design, event indicators, recursion, Boolean variables, iteration, optimization of programs, program transformations, program manipulation systems searching, Quicksort, efficiency

*CR categories:* 4.0, 4.10, 4.20, 5.20, 5.5, 6.1 (5.23, 5.24, 5.25, 5.27)

There is no doubt that the grail of efficiency leads to abuse. Programmers waste enormous amounts of time thinking about, or worrying about, the speed of noncritical parts of their programs, and these attempts at efficiency actually have a strong negative impact when debugging and maintenance are considered. We *should* forget about small efficiencies, say about 97 % of the time: premature optimization is the root of all evil.

Yet we should not pass up our opportunities in that critical 3 %. A good programmer will not be lulled into complacency by such reasoning, he will be wise to look carefully at the critical code; but only *after* that code has been identified. It is often a mistake to make *a priori* judgments about what parts of a program are really critical, since the universal experience of programmers who have been using measurement tools has been that their intuitive guesses fail. After work-

There is no doubt that the grail of efficiency leads to abuse. Programmers waste enormous amounts of time thinking about, or worrying about, the speed of noncritical parts of their programs, and these attempts at efficiency actually have a strong negative impact when debugging and maintenance are considered. We *should* forget about small

— 97% of the time: pre-

## Measure first; then act

ties in that critical 3%. A good programmer will not be lulled into complacency by such reasoning, he will be wise to look carefully at the critical code; but only *after* that code has been identified. It is often a mistake to make *a priori* judgments about what parts of a program are really critical, since the universal experience of programmers who have been using measurement tools has been that their intuitive guesses fail. After work-

# Profiling

So far...

```
t = time()
for _ in range(num_repetitions):

    2 * mat[:, 2]
t = time() - t
avg = t / num_repetitions
```

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
    # Job security
    time.sleep(3)
    result = x * x
    return result

print(fast(10))
print(slow(10))
```

```
$ python script.py
12
100
```

script.py

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
    # Job security
    time.sleep(3)
    result = x * x
    return result

print(fast(10))
print(slow(10))
```

script.py

```
$ python -m cProfile -s cumulative script.py
```

Run cProfile module

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
    # Job security
    time.sleep(3)
    result = x * x
    return result

print(fast(10))
print(slow(10))
```

script.py

```
$ python -m cProfile -s cumulative script.py
```

Sort results by cumulative time

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
    # Job security
    time.sleep(3)
    result = x * x
    return result

print(fast(10))
print(slow(10))
```

script.py

```
$ python -m cProfile -s cumulative script.py
12
100
8 function calls in 3.000 seconds

Ordered by: cumulative time

      ncalls  tottime  percall  cumtime  percall filename:lineno(function)
                  1    0.000    0.000    3.000    3.000 {built-in method builtins.exec}
                  1    0.000    0.000    3.000    3.000 script.py:1(<module>)
                  1    0.000    0.000    3.000    3.000 script.py:8(slow)
                  1    3.000    3.000    3.000    3.000 {built-in method time.sleep}
                  2    0.000    0.000    0.000    0.000 {built-in method builtins.print}
                  1    0.000    0.000    0.000    0.000 {method 'disable' of...
                  1    0.000    0.000    0.000    0.000 script.py:3(fast)
```

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
    # Job security
    time.sleep(3)
    result = x * x
    return result

print(fast(10))
print(slow(10))
```

script.py

```
$ python -m cProfile -s cumulative script.py
```

```
12
```

```
100
```

```
8 function calls in 3.000 seconds
```

Ordered by: cumulative time

ncalls	tottime	percall	cumtime	percall	filename:lineno(function)
1	0.000	0.000	3.000	3.000	{built-in method builtins.exec}
1	0.000	0.000	3.000	3.000	script.py:1(<module>)
1	0.000	0.000	3.000	3.000	script.py:8(slow)
1	3.000	3.000	3.000	3.000	{built-in method time.sleep}
2	0.000	0.000	0.000	0.000	{built-in method builtins.print}
1	0.000	0.000	0.000	0.000	{method 'disable' of...
1	0.000	0.000	0.000	0.000	script.py:3(fast)

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
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    time.sleep(3)
    result = x * x
    return result

print(fast(10))
print(slow(10))
```

script.py

```
$ python -m cProfile -s cumulative script.py
```

```
12
```

```
100
```

```
8 function calls in 3.000 seconds
```

Ordered by: cumulative time

ncalls	tottime	percall	cumtime	percall	filename:lineno(function)
1	0.000	0.000	3.000	3.000	{built-in method builtins.exec}
1	0.000	0.000	3.000	3.000	script.py:1(<module>)
1	0.000	0.000	3.000	3.000	script.py:8(slow)
1	3.000	3.000	3.000	3.000	{built-in method time.sleep}
2	0.000	0.000	0.000	0.000	{built-in method builtins.print}
1	0.000	0.000	0.000	0.000	{method 'disable' of...
1	0.000	0.000	0.000	0.000	script.py:3(fast)

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
    # Job security
    time.sleep(3)
    result = x * x
    return result

print(fast(10))
print(slow(10))
```

script.py

```
$ python -m cProfile -s cumulative script.py
12
100
8 function calls in 3.000 seconds
```

Ordered by: cumulative time

ncalls	tottime	percall	cumtime	percall	filename:lineno(function)
1	0.000	0.000	3.000	3.000	{built-in method builtins.exec}
1	0.000	0.000	3.000	3.000	script.py:1(<module>)
1	0.000	0.000	3.000	3.000	script.py:8(slow)
1	3.000	3.000	3.000	3.000	{built-in method time.sleep}
2	0.000	0.000	0.000	0.000	{built-in method builtins.print}
1	0.000	0.000	0.000	0.000	{method 'disable' of...
1	0.000	0.000	0.000	0.000	script.py:3(fast)

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
    # Job security
    time.sleep(3)
    result = x * x
    return result

print(fast(10))
print(slow(10))
```

script.py

```
$ python -m cProfile -s cumulative script.py
12
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8 function calls in 3.000 seconds
```

Ordered by: cumulative time

ncalls	tottime	percall	cumtime	percall	filename:lineno(function)
1	0.000	0.000	3.000	3.000	{built-in method builtins.exec}
1	0.000	0.000	3.000	3.000	script.py:1(<module>)
1	0.000	0.000	3.000	3.000	script.py:8(slow)
1	3.000	3.000	3.000	3.000	{built-in method time.sleep}
2	0.000	0.000	0.000	0.000	{built-in method builtins.print}
1	0.000	0.000	0.000	0.000	{method 'disable' of...
1	0.000	0.000	0.000	0.000	script.py:3(fast)

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
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    time.sleep(3)
    result = x * x
    return result

print(fast(10))
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```

script.py

```
$ python -m cProfile -s cumulative script.py
12
100
8 function calls in 3.000 seconds

Ordered by: cumulative time

      ncalls  tottime  percall  cumtime  percall filename:lineno(function)
                  0.000   0.000   3.000   3.000 {built-in method builtins.exec}
                  0.000   0.000   3.000   3.000 script.py:1(<module>)
                  0.000   0.000   3.000   3.000 script.py:8(slow)
                  3.000   3.000   3.000   3.000 {built-in method time.sleep}
                  0.000   0.000   0.000   0.000 {built-in method builtins.print}
                  0.000   0.000   0.000   0.000 {method 'disable' of...
                  0.000   0.000   0.000   0.000 script.py:3(fast)
```

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
    # Job security
    time.sleep(3)
    result = x * x
    return result

print(fast(10))
print(slow(10))
```

script.py

```
$ python -m cProfile -s cumulative script.py
12
100
8 function calls in 3.000 seconds

Ordered by: cumulative time

      ncalls  tottime  percall  cumtime  percall filename:lineno(function)
                  1    0.000    0.000    3.000    3.000 {built-in method builtins.exec}
                  1    0.000    0.000    3.000    3.000 script.py:1(<module>)
                  1    0.000    0.000    3.000    3.000 script.py:8(slow)
                  1    3.000    3.000    3.000    3.000 {built-in method time.sleep}
                  2    0.000    0.000    0.000    0.000 {built-in method builtins.print}
                  1    0.000    0.000    0.000    0.000 {method 'disable' of...
                  1    0.000    0.000    0.000    0.000 script.py:3(fast)
```

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
    # Job security
    time.sleep(3)
    result = x * x
    return result

print(fast(10))
print(slow(10))
```

script.py

```
$ python -m cProfile -s cumulative script.py
12
100
8 function calls in 3.000 seconds

Ordered by: cumulative time
              tottime/ncalls      cumtime/ncalls
      ncalls  tottime  percall  cumtime  percall   filename:lineno(function)
                  0.000    0.000    3.000    3.000 {built-in method builtins.exec}
                  1.000    0.000    3.000    3.000 script.py:1(<module>)
                  1.000    0.000    3.000    3.000 script.py:8(slow)
                  1.000    3.000    3.000    3.000 {built-in method time.sleep}
                  2.000    0.000    0.000    0.000 {built-in method builtins.print}
                  1.000    0.000    0.000    0.000 {method 'disable' of...
                  1.000    0.000    0.000    0.000 script.py:3(fast)
```

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
    # Job security
    time.sleep(3)
    result = x * x
    return result

print(fast(10))
print(slow(10))
```

script.py

```
$ python -m cProfile -s cumulative script.py
12
100
```

# Profiling: function level

```
import time  
  
def fast(x):  
    result = x + 2  
    return result  
  
def slow(x):  
    # Job security  
    time.sleep(3)  
    result = x * x  
    return result  
  
print(fast(10))  
print(slow(10))
```

```
$ python -m cProfile -o script.prof script.py  
12  
100
```

script.py

# Profiling: function level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
    # Job security
    time.sleep(3)
    result = x * x
    return result

print(fast(10))
print(slow(10))
```

script.py

```
$ python -m cProfile -o script.prof script.py
12
100
$ snakeviz script.prof
```

# Profiling: function level

SnakeViz

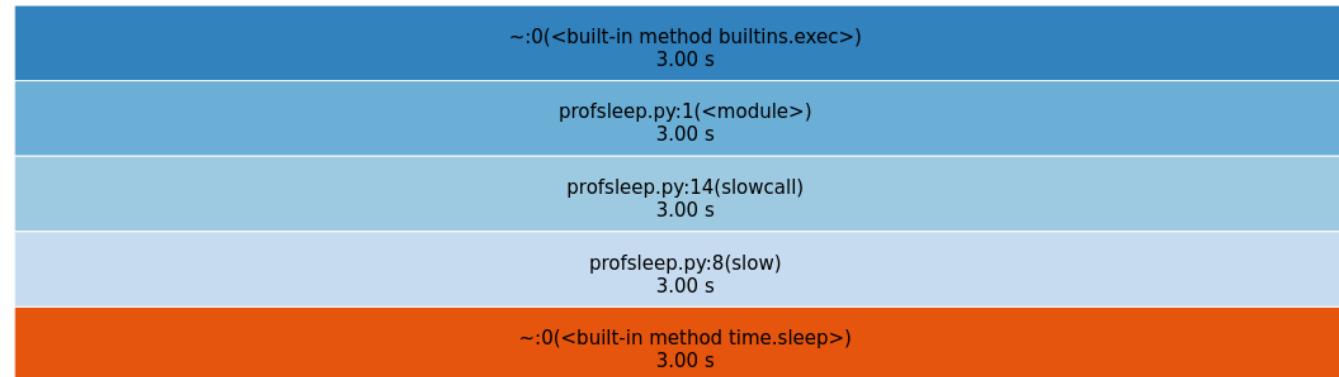
Reset Root

Reset Zoom

Style: Icicle

Depth: 10

Cutoff: 1 / 1000

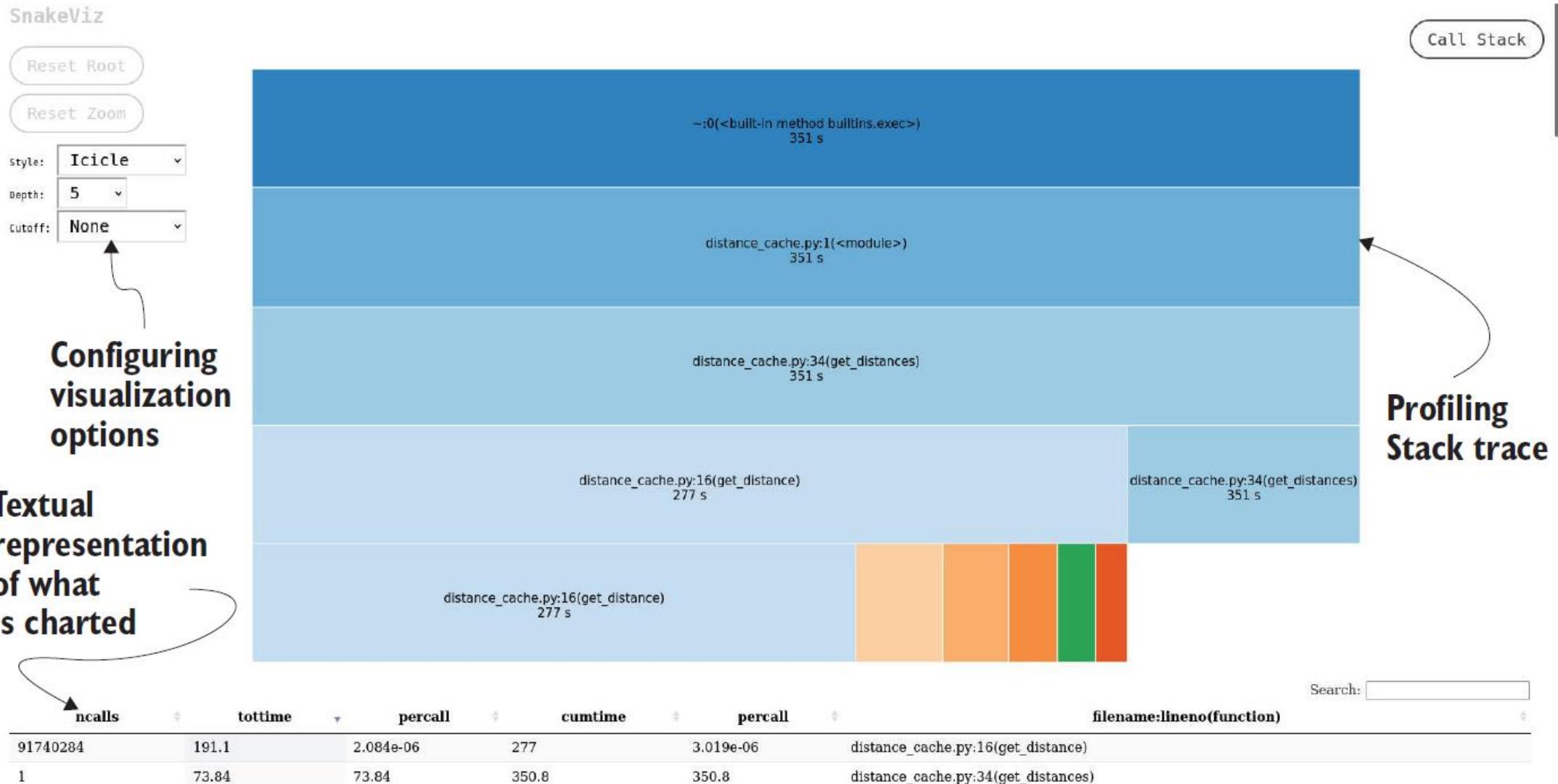


Call Stack

Search: 

ncalls	tottime	percall	cumtime	percall	filename:lineno(function)
1	3	3	3	3	~:0(<built-in method time.sleep>)
2	7.565e-05	3.782e-05	7.565e-05	3.782e-05	~:0(<built-in method builtins.print>)
1	1.473e-05	1.473e-05	3	3	profsleep.py:1(<module>)
1	8.189e-06	8.189e-06	3	3	~:0(<built-in method builtins.exec>)

# Profiling: function level



Fast Python – Figure 2.1

# Profiling: line level

```
import time

def fast(x):
    result = x + 2
    return result

def slow(x):
    # Job security
    time.sleep(3)
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print(fast(10))
print(slow(10))
```

script.py

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```
$ kernprof -l script.py
12
100
Wrote profile results to script.py.lprof
Inspect results with:
python -m line_profiler -rmt "script.py.lprof"
```

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```
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Timer unit: 1e-06 s

Total time: 3.00014 s
File: script.py
Function: slow at line 7

Line #      Hits          Time  Per Hit   % Time  Line Contents
=====      ===      ======  ======   =====
                                         @profile
                                         def slow(x):
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10           1    3000132.9    3e+06   100.0
11           1            2.2       2.2     0.0
12           1            0.6       0.6     0.0

3.00 seconds - script.py:7 - slow
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script.py

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3.00 seconds - script.py:7 - slow

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           9
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3.00 seconds - script.py:7 - slow

script.py

# Profiling: when to use what

## Timing

Low overhead, very manual, no locations

Good for measuring performance

## Function profiling

Low-ish overhead, automatic, rough locations

Good for zooming in on problem areas

## Line profiling

High overhead, mostly automatic, precise locations

Good for pinning the exact problem, but should not be the start

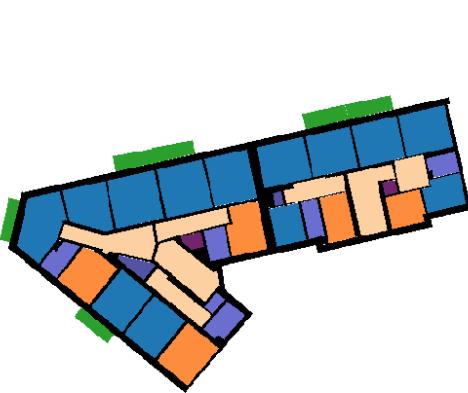
# Mini-project

# Mini-project now on Learn

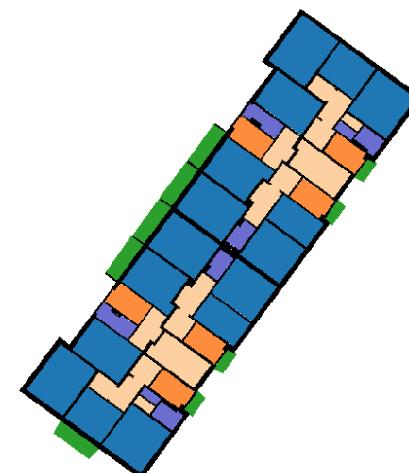
- Hand-in on Learn  
03.05.2026 (Sunday of semester week 12) 23:55
- Hand-in: PDF report and zip-file with code + job scripts in groups of 3-4. Make groups on Learn.
- Must *hand in and pass* project to attend the exam!

# Mini-project: Wall Heating

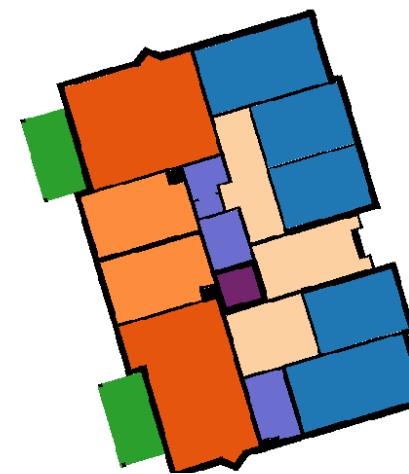
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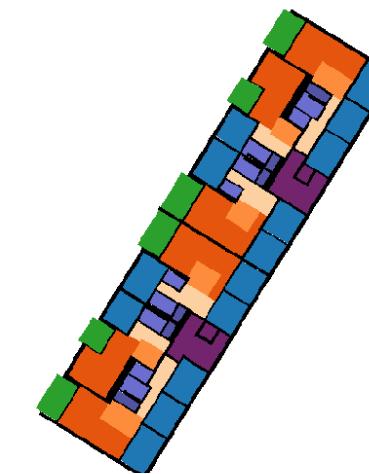
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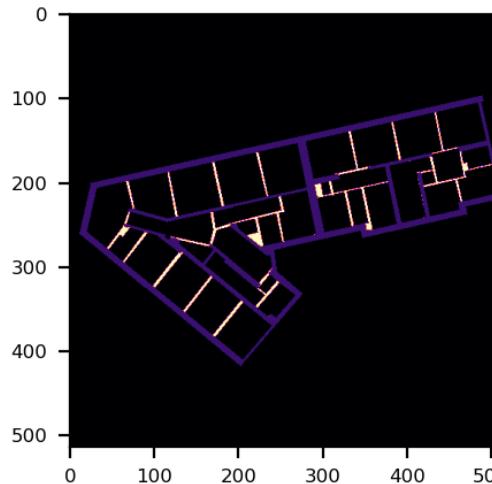
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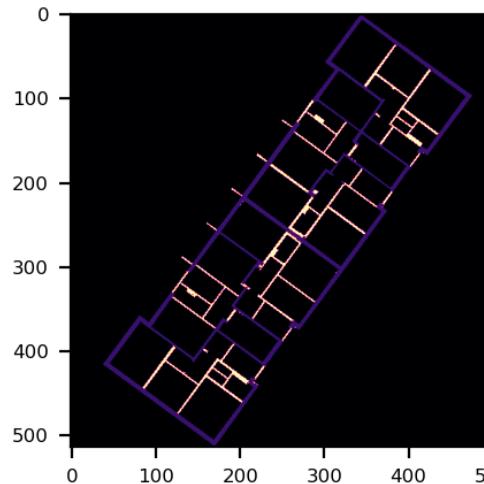
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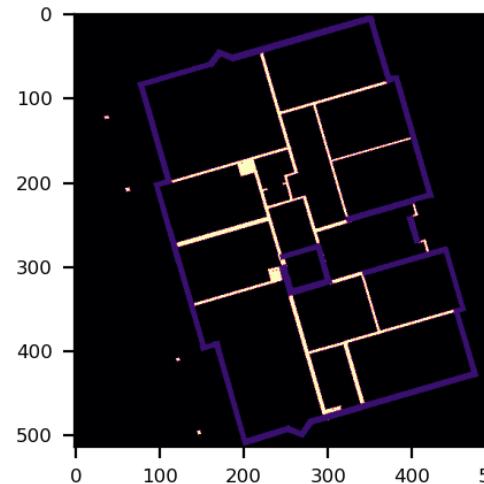
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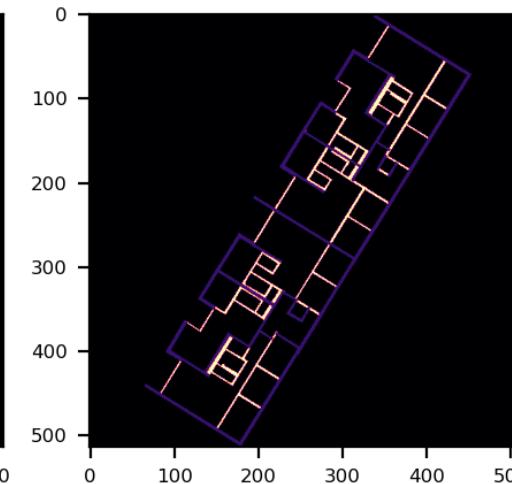
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ID: 10786

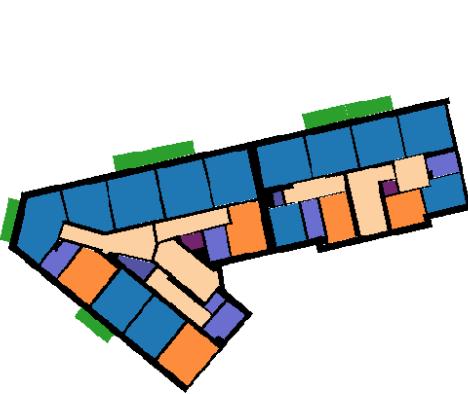


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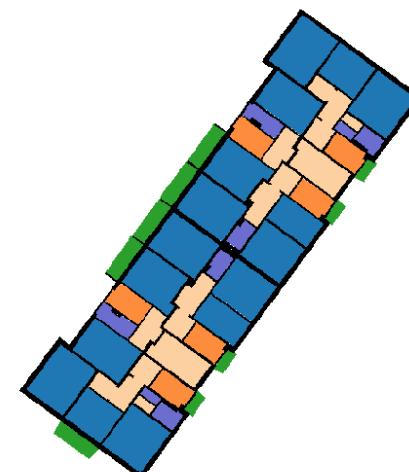


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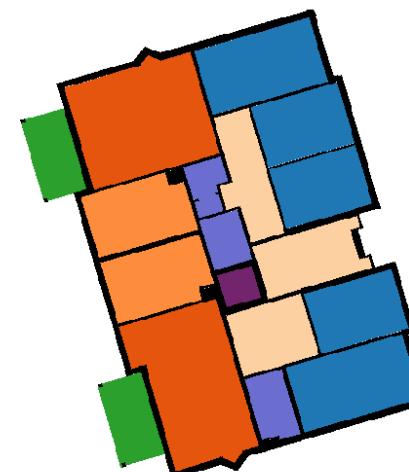
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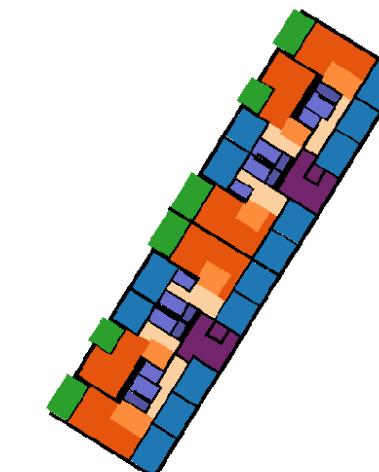
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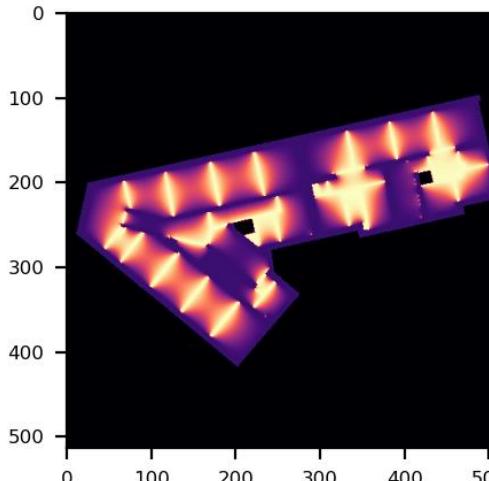
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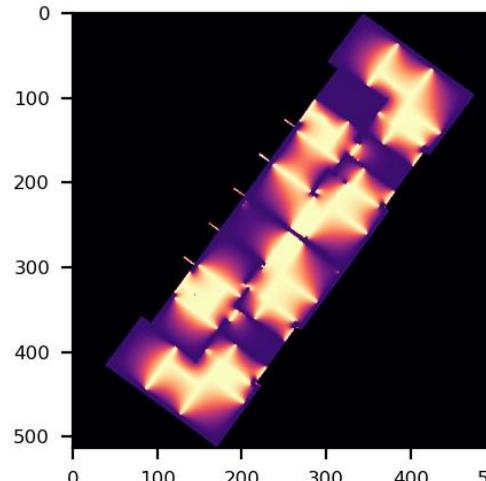
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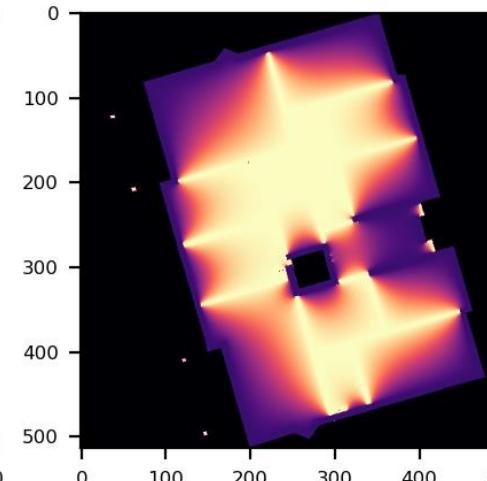
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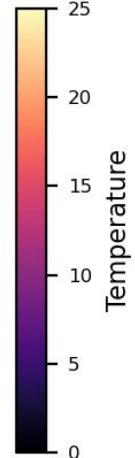
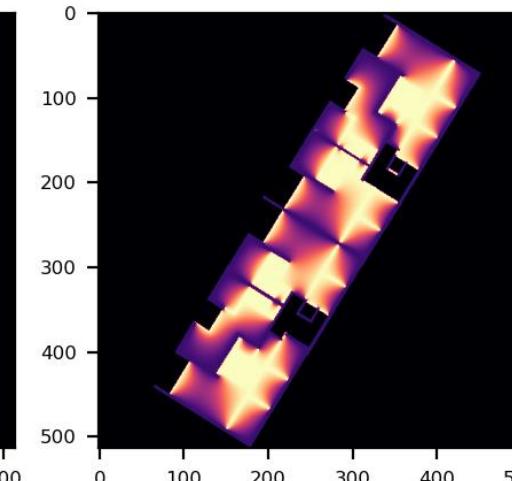
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# Mini-project: Wall Heating

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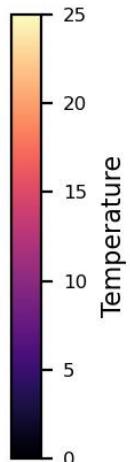
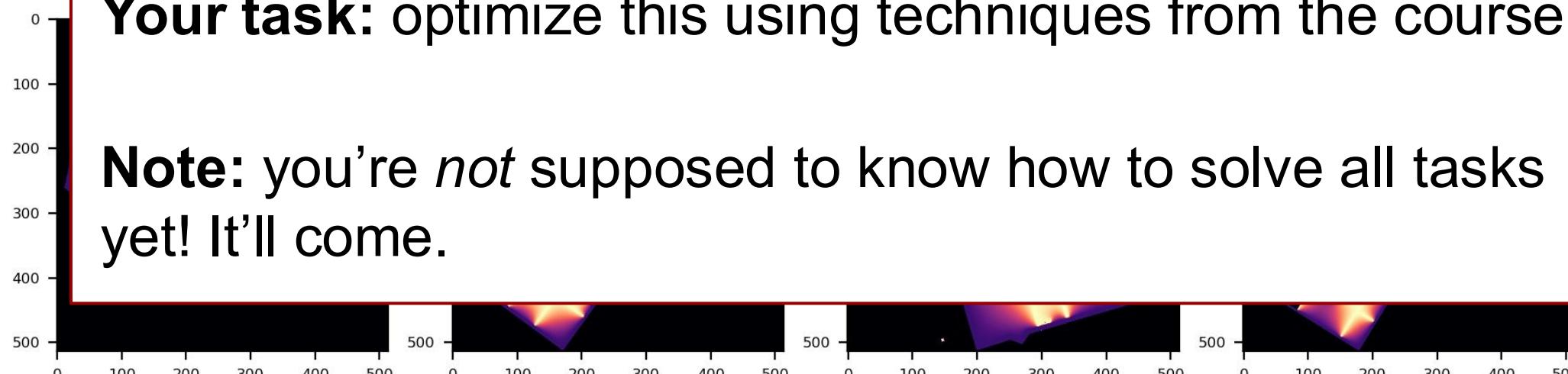
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Project has sample script to simulate heat and compute summary statistics.

*Please do **NOT** run sample script for all floorplans!*

**Your task:** optimize this using techniques from the course.

**Note:** you're *not* supposed to know how to solve all tasks yet! It'll come.



# Today's exercise

# Play with profiling and NumPy optimizations

- Use the different profilers to discover hotspots
- Fix them!
- Data is prepared on the cluster:  
`/dtu/projects/02613_2025/data/locations/`

# Useful concepts

## Add dimension to NumPy array

```
x[:, None]    # Add dimension at end  
x[None]       # Add dimension at begin  
x[:, None, :] # Add dimension in the middle
```

## cProfile for function profiling

```
python -m cProfile -s cumulative script.py # In terminal
```

```
python -m cProfile -o script.prof script.py # In snakeviz GUI  
snakeviz script.prof
```

## kernprof for line profiling

```
@profile # Decorator for profiled functions
```

```
kernprof -l script.py                      # 1. Run profiling  
python -m line_profiler -rmt "script.py.lprof" # 2. Show results
```

## Change to work node

```
linuxsh
```

## Submit job script

```
bsub < submit.sh
```

## Job status

```
bstat / bjobs
```

## Check job output

```
bpeek / bpeek <JOBID>
```

## Kill job

```
bkill <JOBID>
```