ORIGINAL

NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF ENGINEERING

EXAMINATION FOR

(Semester I: 2018/2019)

EE5103 / ME5403– COMPUTER CONTROL SYSTEMS

December 2018 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **FIVE** (5) printed pages.

2. Answer all **FOUR** (4) questions.

3. All questions carry **EQUAL** marks. The **TOTAL** marks are 100.

4. This is a CLOSED BOOK examination. But the candidate is permitted to bring into the examination hall a single A4 size *help sheet*. The candidate may refer to this sheet during the examination.

5. Calculators can be used in the examination, but no programmable calculator is allowed.

Q.1 Consider the discrete-time process

$$x(k+1)=\begin{bmatrix}1 & 1\\ \alpha & 0\end{bmatrix}x(k)+\begin{bmatrix}1\\ 0\end{bmatrix}u(k)+\begin{bmatrix}1\\ 0\end{bmatrix}\omega(k)$$

$$y(k)=\begin{bmatrix}1 & 0\end{bmatrix}x(k)$$

where α is a constant, $x(k)=[x_1(k),x_2(k)]^T$ is the state vector, $y(k)$ is the output, $u(k)$ is the input, and $\omega(k)$ is the disturbance.

(a) Find the range of α such that the system is both controllable and observable.

(3 Marks)

(b) Assuming that there is no disturbance and the state variables are accessible, design a deadbeat state feedback controller.

(6 Marks)

(c) Assuming that there is no disturbance and only the output $y(k)$ is available, design a deadbeat observer to estimate the state variables, and use these estimates to design an output-feedback controller.

(6 Marks)

(d) Assuming that the disturbance is an unknown constant, design a deadbeat observer to estimate both the state variables and the disturbance, and use these estimates to design an output-feedback controller such that the effect of the disturbance may be completely eliminated.

(6 Marks)

(e) Consider the case when $\alpha=0$. Is the system controllable? Is it still possible to design a deadbeat state feedback controller? Justify your answers.

(4 Marks)

Q.2

(a) A process is described by the transfer function

$$H(z) = \frac{z+\alpha}{z^2 - z - 2}.$$

Design a two-degree-of-freedom controller in the form of

$$R(q)u(k) = T(q)u_c(k) - S(q)y(k)$$

such that the transfer function from the command signal, $u_c(k)$, to the system output, $y(k)$, follows the reference model, $\frac{1}{z^2}$. Discuss the condition on the parameter α such that perfect tracking is attainable.

(15 Marks)

(b) In some systems the process output y(k) and the command signals $u_c(k)$ are not available because only the error $e = u_c - y$ is measured. This case is called error feedback. Mathematically it means that the control law becomes

$$U(z) = C(z)\,\mathrm{E}(z),$$

where C(z) is the transfer function of the controller, and the error signal

$$\mathrm{E}(z) = U_c(z) - Y(z).$$

Assume that the process is described by the same transfer function in part (a) where $\alpha = 0$. Is it possible to design an error feedback controller C(z) such that the closed loop transfer function from the command signal, $u_c(k)$, to the system output, $y(k)$, follows the reference model, $\frac{1}{z^2}$? Justify your answers.

(10 Marks)

Q.3 Consider the first-order process model

$$x(k+1) = ax(k) + w(k)$$
$$y(k) = x(k) + v(k)$$

which is also the model used by the Kalman filter where $w(k)$ and $v(k)$ are independent Gaussian noises with variances R_1 and R_2 respectively.

(a) Find the batch least-squares estimates $\hat{x}(0)$ and $\hat{x}(1)$ in terms of a, $y(0)$, $y(1)$, R_1, and R_2 that minimises the objective function

$$J = \frac{1}{2}\left(\frac{w(0)^2}{R_1} + \frac{v(0)^2}{R_2} + \frac{v(1)^2}{R_2}\right)$$

(8 Marks)

(b) Given $a = 0.5$, $R_1 = 0$, $R_2 = 1$, $y(0) = 1$, $y(1) = 0$, compute the following: (i) the batch least-squares estimates $\hat{x}(0)$ and $\hat{x}(1)$, (ii) the Kalman filter estimates $\hat{x}(0|0)$ and $\hat{x}(1|1)$ with initial conditions $\hat{x}(0|-1) = 0$ and $P(0|-1) = \infty$.

(10 Marks)

(c) Compare $\hat{x}(0|0)$ with $\hat{x}(0)$ and $\hat{x}(1|1)$ with $\hat{x}(1)$. Explain your observation.

(7 Marks)

Q.4 A process can be modelled as

$$C\dot{y}(t) = u(t) - \frac{y(t)}{R}$$

where C and R are constants, $u(t)$ and $y(t)$ are the input and output respectively.

(a) Defining $y(t)$ as the state $x_p(t)$, obtain the discretized state-space model with sampling interval of 1.

(4 Marks)

(b) The process is put under closed-loop model predictive control with integration. Obtain the state-space model of the system.

(5 Marks)

(c) Given $C = 5$, $R = 1$, prediction horizon $N_p = 3$, control horizon $N_c = 2$, weight $r_w = 0.04$, find $u(k)$ and $y(k)$ of the model predictive control system for $k = 0$ and 1. The initial conditions are given as $x_p(k) = y(k) = 0$ for $k \leq 0$, $u(k) = 0$ for $k < 0$ and set-point

$$r(k) = \begin{cases} 0 & k < 0 \\ 1 & k \geq 0 \end{cases}$$

(8 Marks)

(d) Repeat Part (c) assuming that the states are not measurable and the Kalman filter (predicted estimate) is used as an estimate of the state. The steady-state Kalman filter gain $K_{ob} = [0.6059 \quad 1.5093]^T$ and the initial state estimate $\hat{x}(0) = [-0.1 \quad -0.1]^T$.

(8 Marks)

Appendix A - Table of Laplace Transform and Z Transform

The following table contains some frequently used time functions x(t), and their Laplace transforms X(s) and Z transforms X(z).

Entry #	Laplace Domain	Time Domain	Z Domain (t=kT)
1	1	$\delta(t)$ unit impulse	1
2	$\frac{1}{s}$	$u(t)$ unit step	$\frac{z}{z-1}$
3	$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
4	$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
5		$b^k \quad (b=e^{-aT})$	$\frac{z}{z-b}$
6	$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
7	$\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$	$\frac{z(1-e^{-aT})}{a(z-1)(z-e^{-aT})}$
8	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
9	$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}-\frac{e^{-at}}{a(b-a)}-\frac{e^{-bt}}{b(a-b)}$	
10	$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	
11	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	
12	$\frac{b}{s^2+b^2}$	$\sin(bt)$	$\frac{z\sin(bT)}{z^2-2z\cos(bT)+1}$
13	$\frac{s}{s^2+b^2}$	$\cos(bt)$	$\frac{z(z-\cos(bT))}{z^2-2z\cos(bT)+1}$
14	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin(bt)$	$\frac{ze^{-aT}\sin(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$
15	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos(bt)$	$\frac{z^2-ze^{-aT}\cos(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$

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