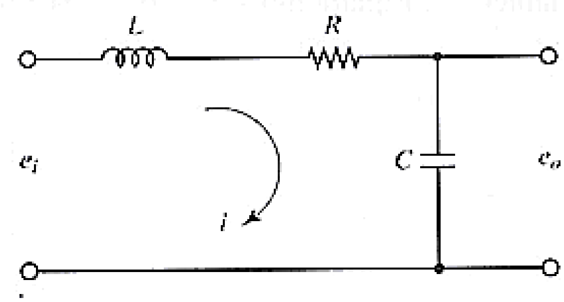
EE5103 Computer Control Systems: Homework #1
(Due date: 14/09/2025)

Q1. (10 Marks)
Consider the electrical circuit shown in the figure below. The circuit consists



of an inductance $L = 1$ henry, a resistance $R = 1$ ohm, and a capacitance $C = 1$ farad. Applying Kirchhoff's voltage law yields,

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = e_i$$
$$\frac{1}{C}\int idt = e_o$$

a) Assuming e_i is the input u, and e_o, the output y, derive the transfer function of the system from the input u to output y.

(2 Marks)

b) Define state variables by

$$x_1 = e_o$$
$$x_2 = \dot{e}_o$$

derive the state-space representation of the system.

(2 Marks)

c) Using zero-order-hold to sample the system, and assuming the sampling period $h = 1$, derive the state-space representation of the sampled system.

(2 Marks)

d) Apply z-transform to the state-space model derived in c), and obtain the input-output model of the system.

(2 Marks)

e) Assuming the initial conditions are $y(0) = 1$, and $\dot{y}(0) = 0$. Calculate the output sequence $y(k)$, under the unit step input, $u(k) = 1$ for $k \geq 0$.

(2 Marks)

Q2. (10 Marks)
Consider the system

$$G(s) = \frac{1}{s(s-1)}$$

a) Is the system stable? Does the system have a stable inverse? Justify your answers.

(3 Marks)

b) Is it possible to choose the sampling period h such that the sampled system is unstable? Justify your answer.

(3 Marks)

c) Is it possible to choose the sampling period h such that the sampled system has a stable inverse? Justify your answer.

(4 Marks)

Q3. (10 Marks)
Consider the system

$$x(k+1) = \begin{pmatrix} 0.1 & 0.2 \\ 0.1 & 0.2 \end{pmatrix} x(k) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(k)$$

a) Is the system stable? Is the system controllable? Is the system observable? Justify your answers.

(2 Marks)

b) Use z-transform to obtain the transfer function of the system. Write down the input-output difference equation.

(2 Marks)

c) Assume the system is controlled by a proportional controller

$$u(k) = K(u_c(k) - y(k))$$

Derive the transfer function from the command signal $u_c(k)$ to the output $y(k)$.

(2 Marks)

d) Apply Jury's stability criterion to determine the range of controller gain, K, such that the closed-loop system is stable.

(2 Marks)

e) Determine the steady-state error, $u_c - y$, when u_c is a unit step.

(2 Marks)

Q4. (10 Marks)
Consider the system described by the following difference equation

$$y(k+1) = -y(k) + y(k-1) + y(k-2) + u(k) + 2u(k-1) + u(k-2)$$

a) What is the transfer function? Is the system stable? Does it have a stable inverse?

(4 Marks)

b) Realize the system with a state space model. Check the controllability and observability of the realization.

(6 Marks)