NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF ENGINEERING

EXAMINATION FOR

(Semester I: 2017/2018)

EE5103 / ME5403– COMPUTER CONTROL SYSTEMS

December 2017 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **SIX** (6) printed pages.

2. Answer all **FOUR** (4) questions.

3. All questions carry **EQUAL** marks. The **TOTAL** marks are 100.

4. This is a CLOSED BOOK examination. But the candidate is permitted to bring into the examination hall a single A4 size *help sheet*. The candidate may refer to this sheet during the examination.

6. Calculators can be used in the examination, but no programmable calculator is allowed.

Q.1 Consider the discrete-time process

$$x(k+1)=\begin{bmatrix}\alpha & \beta \\ 1 & 0\end{bmatrix}x(k)+\begin{bmatrix}1 \\ 0\end{bmatrix}u(k)$$
$$y(k)=\begin{bmatrix}1 & 0\end{bmatrix}x(k)$$

(a) Find the range of α and β such that the system is both controllable and observable.

(2 Marks)

(b) Assume that the state variables are accessible, design a deadbeat state feedback controller.

(7 Marks)

(c) Assume that the system is observable and only the output $y(k)$ is available, design a deadbeat observer to estimate the state variables, and use these estimates to design an output-feedback controller.

(7 Marks)

(d) If $\beta=0$, is it still possible to design a deadbeat observer to estimate the state variables? Justify your answer.

(3 Marks)

(e) Assume that there are time-delays in the model as shown below,

$$x(k+1)=\begin{bmatrix}\alpha & \beta \\ 1 & 0\end{bmatrix}x(k-1)+\begin{bmatrix}1 \\ 0\end{bmatrix}u(k-2)$$
$$y(k)=\begin{bmatrix}1 & 0\end{bmatrix}x(k)$$

What is the state of the system at time k? What is the order of the system? Is the system controllable? Justify your answers.

(6 Marks)

Q.2 (a) A process is described by the transfer function

$$H(z) = \frac{z+\alpha}{z^2+3z+2}$$

Design a controller in the form of

$$R(q)u(k) = T(q)u_c(k) - S(q)y(k)$$

such that the transfer function from the command signal, $u_c(k)$, to the system output, $y(k)$, follows the reference model, $\frac{1}{z^2}$. It is also desired that the controller can reject the effect of unknown constant disturbance. Discuss the condition on the parameter α such that perfect tracking is attainable.

(15 Marks)

(b) A nonlinear system is described by

$$y(k+1) = a \sin y(k) + u(k-1) + bu(k-2)$$

where a and b are constant parameters.

Design a one-step-ahead controller to make the output of the system, $y(k)$, follow an arbitrary desired output, $r(k)$. Discuss the condition on the parameters a and b such that perfect tracking is attainable.

(10 Marks)

Q.3 The true state position $x_1(k)$ and velocity $x_2(k)$ of a moving target are given by the following equations

$$x(k+1) = Ax(k) + \begin{bmatrix} \frac{1}{2}T^2 \\ T \end{bmatrix} w(k) \qquad (3.1)$$

$$y(k) = Cx(k) + v(k) \qquad (3.2)$$

where

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$w(k)$ and $v(k)$ are zero-mean independent Gaussian random variables with standard deviations σ_w and σ_v respectively. The sampling period is $T = 1$. A Kalman filter using the model of (3.1) and (3.2) with initial condition $P(0|-1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times 10^5$ was implemented.

(a) If $\sigma_w = \sigma_v = 1$, fill up the table below with the the Kalman filter gains and covariance matrices for $k = 0$ and 1.

k	$K_f(k)$	$K(k)$	$P(k\|k)$	$P(k+1\|k)$
0				
1				

(10 marks)

(b) If $\sigma_w = 0$ and $\sigma_v = 1$, find the steady-state Kalman filter gain K_f.

(7 marks)

(c) If $\sigma_w = 1$ and $\sigma_v = 0$, find the steady-state Kalman filter gain K_f.

(8 marks)

Q.4 Consider the process

$$x_p(k+1) = A_p x_p(k) + B_p u(k)$$
$$y(k) = C_p x_p(k)$$

where $A_p = 1$, $B_p = 1$, $C_p = 1$. The process input and output are given by $u(k)$ and $y(k)$ respectively. Consider the augmented process

$$x(k+1) = Ax(k) + B\Delta u(k)$$
$$y(k) = Cx(k)$$

where

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} \Delta x_p(k) \\ y(k) \end{bmatrix} = \begin{bmatrix} x_p(k) - x_p(k-1) \\ C_p x_p(k) \end{bmatrix}$$
$$\Delta u(k) = u(k) - u(k-1)$$
$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

The process is under closed-loop model predictive control with control weighting $r_w = 5$, control horizon $N_c = 1$, prediction horizon $N_p = 2$ and set-point $r(k) = 1$.

(a) Formulate a quadratic programming problem with constraints

$$u(k) \leq u^{max}$$
$$\begin{bmatrix} y(k+1) \\ y(k+2) \end{bmatrix} \leq \begin{bmatrix} y^{max} \\ y^{max} \end{bmatrix}$$

(12 marks)

(b) Given $u(-1) = 0$, $x(0) = [0 \quad 0]^T$, $y^{max} = 1.1$ and $u^{max} = 0.3$, find $\Delta u(k)$, $u(k)$ and $x(k)$ for $k = 0$, 1, and 2.

(13 marks)

Appendix A - **Table of Laplace Transform and Z Transform**

The following table contains some frequently used time functions x(t), and their Laplace transforms X(s) and Z transforms X(z).

Entry #	Laplace Domain	Time Domain	Z Domain (t=kT)
1	1	$\delta(t)$ unit impulse	1
2	$\frac{1}{s}$	$u(t)$ unit step	$\frac{z}{z-1}$
3	$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
4	$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
5		$b^k \quad (b=e^{-aT})$	$\frac{z}{z-b}$
6	$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{\left(z-e^{-aT}\right)^2}$
7	$\frac{1}{s(s+a)}$	$\frac{1}{a}\left(1-e^{-at}\right)$	$\frac{z\left(1-e^{-aT}\right)}{a(z-1)\left(z-e^{-aT}\right)}$
8	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$\frac{z\left(e^{-aT}-e^{-bT}\right)}{\left(z-e^{-aT}\right)\left(z-e^{-bT}\right)}$
9	$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}-\frac{e^{-at}}{a(b-a)}-\frac{e^{-bt}}{b(a-b)}$	
10	$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}\left(1-e^{-at}-ate^{-at}\right)$	
11	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	
12	$\frac{b}{s^2+b^2}$	$\sin(bt)$	$\frac{z\sin(bT)}{z^2-2z\cos(bT)+1}$
13	$\frac{s}{s^2+b^2}$	$\cos(bt)$	$\frac{z(z-\cos(bT))}{z^2-2z\cos(bT)+1}$
14	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin(bt)$	$\frac{ze^{-aT}\sin(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$
15	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos(bt)$	$\frac{z^2-ze^{-aT}\cos(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$

END OF PAPER