ORIGINAL

NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF ENGINEERING

EXAMINATION FOR

(Semester I: 2019/2020)

EE5103 / ME5403– COMPUTER CONTROL SYSTEMS

November 2019 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **SIX** (6) printed pages.
2. Answer all **FOUR** (4) questions.
3. All questions carry **EQUAL** marks. The **TOTAL** marks are 100.
4. This is a CLOSED BOOK examination. But the candidate is permitted to bring into the examination hall a single A4 size *help sheet*. The candidate may refer to this sheet during the examination.
5. Calculators can be used in the examination, but no programmable calculator is allowed.

Q.1 A system is described by

$$x(k+1) = \begin{bmatrix} 2 & 1 \\ \alpha & -1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \omega(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$

where α is a constant parameters, $x(k) = [x_1(k), x_2(k)]^T$ is the state vector, $y(k)$ is the output, $u(k)$ is the input, and $\omega(k)$ is the disturbance.

a) Find the range of α such that the system is both controllable and observable.

$\Phi = \begin{bmatrix} 2 & 1 \\ \alpha & -1 \end{bmatrix}$ $\Gamma = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C = [1\ 0]$

$W_c = [\Gamma\ \Phi\Gamma] = \begin{bmatrix} 1 & 2 \\ 0 & \alpha \end{bmatrix}$ $\det(W_c) = \alpha \neq 0$ $W_c^{-1} = \frac{1}{\alpha}\begin{bmatrix} \alpha & -2 \\ 0 & 1 \end{bmatrix}$ $\Phi^2 = \begin{bmatrix} 2+\alpha & 1 \\ \alpha & \alpha-1 \end{bmatrix}$ $W_o^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

(2 Marks)

$W_o = \begin{bmatrix} C \\ C\Phi \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $\det(W_o) \neq 0$ Thus $\alpha \neq 2$ so that the system is controllable and observable.

b) Assuming that there is no disturbance and the state variables are accessible, design a deadbeat state feedback controller.

(6 Marks)

For deadbeat controller, the desired poles should be all zero.
Close loop transfer function is $P(z) = z^2$
Assume the state feedback gain is L. The closeloop CP is $\det(zI - (\Phi - \Gamma L)) = z^2$

Let $L = [l_1\ l_2]$, according to Ackerman's fomula,
$L = [0\ 1] W_c^{-1} \Phi^2$
$= \frac{1}{\alpha}[0\ 1]\begin{bmatrix} 2+\alpha & 1 \\ \alpha & \alpha-1 \end{bmatrix} = \frac{1}{\alpha}[\alpha\ \alpha-1]$
$u(k) = -Lx(k)$

c) Assuming that there is no disturbance and only the output $y(k)$ is available, design a deadbeat observer to estimate the state variables, and use these estimates to design an output-feedback controller.

(6 Marks)

Assume the state variables $x(k)$ is avaliable.
$u(k) = -Lx(k)$ and desiderd CP is z^2

$L = [0\ 1] W_c^{-1} \Phi^2$
$= \frac{1}{\alpha}[\alpha\ \alpha-1]$

observer can be build as
$\hat{x}(k+1) = \Phi\hat{x}(k) + \Gamma u(k) + K(y(k) - \hat{y}(k))$
$\hat{y}(k) = C\hat{x}(k)$, $e(k+1) = (\Phi - KC)e(k)$

$A_o(z) = z^2$, $K = A_o(\Phi) W_o^{-1} [0\ 1]^T$
$= \frac{1}{2}\begin{bmatrix} 2+\alpha & 1 \\ \alpha & \alpha-1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ \alpha-1 \end{bmatrix}$

d) Assuming that the disturbance is an unknown constant, design a deadbeat observer to estimate both the state variables and the disturbance, and use these estimates to design an output-feedback controller such that the effect of the disturbance may be completely eliminated.

(6 Marks)

HW2 Q2.c and HW3 Q2.a

e) Assuming that there are time-delays in both the state variables and the input, the corresponding model of the system is given as

$$x(k+1) = \begin{bmatrix} 2 & 1 \\ \alpha & -1 \end{bmatrix} x(k-2) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k-1)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$

What is the state of the system at time k? What is the order of the system? Justify your answers.

(5 Marks)

Apply z-transformation

$z Y(z) = Y(z) + 2U(z) + \alpha z^{-1} U(z)$

$Y(z)(z-1) = U(z)(2+\alpha z^{-1})$

Q.2 $H(z) = \dfrac{Y(z)}{U(z)} = \dfrac{2+\alpha z^{-1}}{z-1} = \dfrac{\alpha + 2z}{z(z-1)}$

Poles: $z_1 = 0,\ z_2 = 1$

Zeros: $z_1 = -\frac{\alpha}{2}$

$H_m(z) = \frac{1}{z^2}$ $\quad B(z) = \alpha + 2z$

Thus $R(z)$ need to contain $\alpha + 2z$.

a) A system is described by

HW3 Q3.a and HW3 Q4

$$y(k+1) = y(k) + 2u(k) + \alpha u(k-1),$$

Also we need to have perfect tracking, $R(z)$ contains $z-1$

where $y(k)$ and $u(k)$ are the output and input signals of the system, and α is a constant parameter.

$R(z)$ should be in the form $(2z+\alpha)(z-1)$. $A_{cl}(z) = (2z+\alpha)A_m(z)A_o(z)$

$A(z)R(z)$ is in degree of 4. and $S(z) = s_0 z^2 + s_1 z + s_2$. We choose $A_o(z) = z$

$A(z)R(z) + B(z)S(z) = z(z-1)(z-1)(2z+\alpha) + (2z+\alpha)(s_0 z^2 + s_1 z + s_2)$

$A_{cl}(z) = (2z+\alpha) z^3$

Design a controller in the form of

$$R(q)u(k) = T(q)u_c(k) - S(q)y(k)$$

such that the transfer function from the command signal, $u_c(k)$, to the system output, $y(k)$ follows the reference model, $\dfrac{1}{z^2}$. Discuss the condition on the parameter, α, such that perfect tracking is attainable.

$a_1 = -1 \quad b_1 = 2$

$a_2 = 0 \quad b_2 = \alpha$

(12 Marks)

(b)

Assume $L = [l_1\ l_2]$

$u(k) = -L\,x(k)$

$H_m(z) = \dfrac{B_m(z)}{A_m(z)} = \dfrac{1}{z^2}$

b) Is it possible to use the state-space approach to design the controller to meet the same design specifications as those in part (a)? Justify your answers.

(8 Marks)

define $x_1(k) = y(k)$

$x_1(k+1) = x_1(k) + 2u(k) + \alpha u(k-1)$

define $x_2(k) = \alpha u(k-1)$

$x_1(k+1) = x_1(k) + 2u(k) + x_2(k)$

$x_2(k+1) = \alpha u(k)$

$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ \alpha \end{bmatrix} u(k)$

$y(k) = [1\ 0]\, x(k)$

The close loop sys becomes: $x(k+1) = (\Phi - \Gamma L)x(k)$

$\Gamma L = \begin{bmatrix} 2 \\ \alpha \end{bmatrix} [l_1\ l_2] = \begin{bmatrix} 2l_1 & 2l_2 \\ \alpha l_1 & \alpha l_2 \end{bmatrix}$

$\Phi - \Gamma L = \begin{bmatrix} 1-2l_1 & 1-2l_2 \\ -\alpha l_1 & -\alpha l_2 \end{bmatrix}$ $\quad W_c = [\Gamma\ \Phi\Gamma] = \begin{bmatrix} 2 & 2+\alpha \\ \alpha & 0 \end{bmatrix}$. $\alpha(2+\alpha) \neq 0$ $\quad \alpha \neq 0$ and $\alpha \neq -2$ $\quad L = [0\ \ 1]\, W_c^{-1} \Phi^2$

c) A nonlinear system is described by

$$y(k+1) = y(k) + y^3(k-1) + cu(k-1)$$

where c is a constant parameter.

Design a predictive controller to make the output of the system, $y(k)$, follow an arbitrary desired output, $y^*(k)$. Discuss the condition on the parameter c such that perfect tracking is attainable.

(5 Marks)

Forward shift by 1 $\Rightarrow$ $y(k+2) = y(k+1) + y^3(k) + cu(k)$

In one step-head controller, $\quad = y(k) + y^3(k-1) + cu(k-1) + y^3(k) + cu(k)$

just make $y(k+2) = r(k+2)$. gives $u(k) = \frac{1}{c}\left(r(k+2) - y(k) - y^3(k-1) - y^3(k)\right) - u(k-1)$

we have $y(k) = r(k)$.

$u(k) = \frac{1}{c}\left(r(k+2) - r(k+1) - r^3(k)\right)$

For any reference $r(k)$, as long as it is bounded, as long as c is not zero, RHS is also bounded.

Q.3 Consider the first-order process model

$$x(k+1) = ax(k) + w(k)$$
$$y(k) = x(k) + v(k)$$

which is also the model used by the Kalman filter $A=a$ $C=1$ let $P(k+1|k)=P$

$P_{k-1} = P(k|k-1)$

$$K_f(k) = P(k|k-1)C^T(CP(k|k-1)C^T + R_2)^{-1}$$
$$K(k) = (AP(k|k-1)C^T)(CP(k|k-1)C^T + R_2)^{-1}$$
$$\hat{x}(k|k) = \hat{x}(k|k-1) + K_f(k)(y(k) - C\hat{x}(k|k-1))$$
$$\hat{x}(k+1|k) = A\hat{x}(k|k-1) + Bu(k) + K(k)\,(y(k) - C\hat{x}(k|k-1))$$
$$P(k|k) = P(k|k-1) - P(k|k-1)C^T(CP(k|k-1)C^T + R_2)^{-1}CP(k|k-1)$$
$$P(k+1|k) = AP(k|k-1)A^T - K(k)(CP(k|k-1)C^T + R_2)K^T(k) + R_1$$

where $w(k)$ and $v(k)$ are independent Gaussian noises with variances $R_1 = 0$ and $R_2 = 1$ respectively. The initial estimate $\hat{x}(0|-1) = 0$, covariance $P(0|-1) = \infty$. The measurements $y(k)$ for $k = 0, 1, 2,$ and 3 are given.

(a) For $a = 1$, find the Kalman filter $\hat{x}(0|0)$, $\hat{x}(1|1)$ and $\hat{x}(2|2)$.

Example 2 at Page 28

(7 Marks)

(b) For $a = 1$, find the least-squares estimate $\hat{x}(3)$.

(6 Marks)

(c) For $|a| < 1$ and after the Kalman filter gains have reached steady-state, find $K(k)$, $K_f(k)$, $P(k|k)$, $P(k+1|k)$ and the relationship between $\hat{x}(k|k)$ and $\hat{x}(k-1|k-1)$.

(12 Marks)

Q1 at Page 108

$P = a^2P + R_1 - \dfrac{a^2P^2}{P+R_2}$ $R_1=0$, $R_2=1$ $\Rightarrow$ $P = a^2P - \dfrac{a^2P^2}{P+1}$

$P^2 + ((1-a^2)R_2 - R_1)P - R_1R_2 = 0$ $P^2 + (1-a^2)R_2P = 0$ Since $|a|<1$ $\Rightarrow P=0$

$a^2P_{k-1} - K^2_{(k)}(P_{k-1}+1) = 0$ $\Rightarrow$ $P_{k-1} = 0$

$K(k) = \dfrac{aP_{k-1}}{P_{k-1}+1}$ $K(k) = 0.$

$K_f(k) = 0.$

$P(k|k) = 0$ $\hat{x}(k|k) = a\hat{x}(k-1|k-1)$

Q.4 Consider the first-order single-input and single-out process

$$x_p(k+1) = a_p x_p(k) + b_p u(k)$$
$$y(k) = x_p(k) \qquad C_p = 1$$

where u, y, x_p, and k are the input, output, state and sampling instance respectively. The model parameters are given as $a_p = 0.8$, and $b_p = 0.4$. The process is placed under model predictive control with prediction horizon $N_p = 3$, control horizon $N_c = 2$, and weight $r_w = 1$.

(a) Determine A, B, and C of the state-space model augmented with an integrator

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$$x(k+1) = Ax(k) + B\Delta u(k)$$
$$y(k) = Cx(k)$$

where $\Delta u(k) = u(k) - u(k-1)$, $x(k) = [\Delta x_p(k) \quad y(k)]^T$, $\Delta x_p(k) = x(k) - x(k-1)$.

(5 Marks)

$$A = \begin{bmatrix} A_p & 0_p^T \\ C_p A_p & 1 \end{bmatrix} = \begin{bmatrix} 0.8 & 0 \\ 0.8 & 1 \end{bmatrix} \quad B = \begin{bmatrix} B_p \\ C_p B_p \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.4 \end{bmatrix} \quad C = [0_p \ \ 1] = [0 \ \ 1]$$

(b) Assuming that the states are not measurable and an observer

$$\hat{x}(k+1) = A\hat{x}(k) + B\Delta u(k) + K_{ob}\big(y(k) - C\hat{x}(k)\big)$$

is used to obtain the estimate, $\hat{x}(k)$. Obtain the observer gain, K_{ob}, for observer poles specified at $z = 0.4, 0.4$.

Q5 Page 143

(8 marks)

(c) The initial conditions are given as $x_p(k) = 0$ for $k \le 0$, $\hat{x}(0) = [0.1 \quad 0.1]^T$, $u(k) = 0$ for $k < 0$ and set-point

$$r(k) = \begin{cases} 0 & k < 0 \\ 1 & k \ge 0 \end{cases}$$

Find $y(0)$, $y(1)$ and $y(2)$.

(12 marks)

Appendix A - Table of Laplace Transform and Z Transform

The following table contains some frequently used time functions x(t), and their Laplace transforms X(s) and Z transforms X(z).

Entry #	Laplace Domain	Time Domain	Z Domain (t=kT)
1	1	$\delta(t)$ unit impulse	1
2	$\frac{1}{s}$	$u(t)$ unit step	$\frac{z}{z-1}$
3	$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
4	$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
5		$b^k \quad (b = e^{-aT})$	$\frac{z}{z-b}$
6	$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
7	$\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$	$\frac{z(1-e^{-aT})}{a(z-1)(z-e^{-aT})}$
8	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
9	$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}-\frac{e^{-at}}{a(b-a)}-\frac{e^{-bt}}{b(a-b)}$	
10	$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	
11	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	
12	$\frac{b}{s^2+b^2}$	$\sin(bt)$	$\frac{z\sin(bT)}{z^2-2z\cos(bT)+1}$
13	$\frac{s}{s^2+b^2}$	$\cos(bt)$	$\frac{z(z-\cos(bT))}{z^2-2z\cos(bT)+1}$
14	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin(bt)$	$\frac{ze^{-aT}\sin(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$
15	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos(bt)$	$\frac{z^2-ze^{-aT}\cos(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$

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