Quadrature detection methods for FM Demodulation

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***Abstract*— The Telecommand system enables the control of launch vehicle during the flight. Command system uses a FM/FSK modulation on a UHF-Band link. In this paper, different quadrature detection methods for FM demodulation like arctan method, CORDIC vector rotation method etc. are studied and simulated in MATLAB. The Algorithms are then compared based on SNR, frequency spectrum and complexity so as to find out the best algorithm for a telecommand receiver in a spacecraft. Same algorithm is implemented in VHDL also.**

Keywords—FM demodulation, telecomand receiver, ARCTAN, CORDIC, SNR, MATLAB .

I. Introduction

The purpose of the telecommand system is to serve as a primary control link between ground station and space vehicles. It plays a vital role in telecommunicating the information necessary to the success of a space vehicle's mission, which cannot be loaded into the space vehicle, prior to launch.

Many commands are necessary to the routine operation and control of spacecraft functions. Some are meant for changing mission emphasis if unusual or unexpected conditions are encountered. Some others are required to correct the erratic operations or partially salvage the mission if spacecraft failure modes occur. Usually, all of the commands will fall into two categories (1) the real time (RT) commands for switching scientific experiments and inhouse subsystems (ON/OFF), activating deployment mechanisms, terminating the flight in case the launch vehicle fail posing danger to the safety of the people and (2) the data commands for initiating orbital manoeuvres and remote programing of spacecraft computers. While the real time commands constitute definite word patterns, a data command can have any pattern.

Command systems are characterised mainly by the number of commands that they are capable of handling. Three types of command systems are in use in space vehicles. They are tone, tone-digital, and digital command systems. Early satellites used tone command system or tone digital command system. Both the systems are prone to transmission errors and have limited handling capacity of RT commands. Typical spacecraft of present day requires commands of the order of one to two thousand. Digital command systems have the capability of supplying large number of commands at a high command rate. This system is amenable to incorporation of error control techniques for guarding the commands against induced errors and scrambling techniques for maintaining privacy. Telecommanding is a digital communication process.

Modulation technique used for telecommanding is Frequency Modulation (FM).The design and operation of a command system is influenced by the need for high accuracy and reliability . Accuracy is measured by the system's ability to receive commands without error or at least within the limits of some prescribed error probability. It is a function of communication link capabilities as well and limitations, and channel noise.

In this paper, six FM demodulation algorithms are studied and implemented in MATLAB. The algorithms are then compared with each other based output SNR versus input SNR plot, harmonic distortion in the demodulated output and complexity. Output SNR is calculated using mean square error (MSE) method. FM modulation basics and FM Digital receivers are discussed in Section II. In section III and IV ARCTAN algorithm and baseband delay demodulator are studied respectively. Section V discusses CORDIC algorithm polar discriminator based FM demodulator. Section IV shows simulation results and comparison of the algorithms.

II. FM Modulation and Digital FM Receivers

A. FM Modulation

Let the message signal and carrier signal be,

$$x_m(t) = A_m \sin\omega_m t \quad (1)$$
$$x_c(t) = A_c \cos\omega_c t \quad (2)$$

then frequency modulated signal is given by,

$$y(t) = Ac \cos(2\ \ f_c t + 2\ \ k \int x_m(\tau))d\tau) \quad (3)$$

$$\int x_m(\tau)\, d\tau = A_m \cos(2\ \ f_m t) /2\ \ f_m \quad (4)$$

where k is the maximum frequency deviation

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$$y(t) = A_c \cos(2 \;\; f_c t + k\, A_m \cos(2 \;\; f_m t) /2 \;\; f_m)$$
$$= Ac \cos(2 \;\; fct + \beta\cos(2 \;\; f_m t) \qquad (5)$$

Where β is the modulation index given by,

$$\beta = k / f_m \qquad (6)$$

B. Digital Telecommand Receiver

Frequency modulated signal is received by the telecommand receiver. It is a double superheterodyne receiver. After passing through RF and IF stages, FM signal is down converted to an intermediate frequency of 10.7MHz. Conventional type of telecommand receivers used analog FM demodulators. Digital signal processing can offer more system flexibility, programmability & easy upgrading than fixed analog systems. Therefore analog IF signal is sampled and a complex baseband FM signal is generated out of it. Complex data is generated by mixing the FM signal with a cosine and sine local oscillator as shown in the Fig. 2.

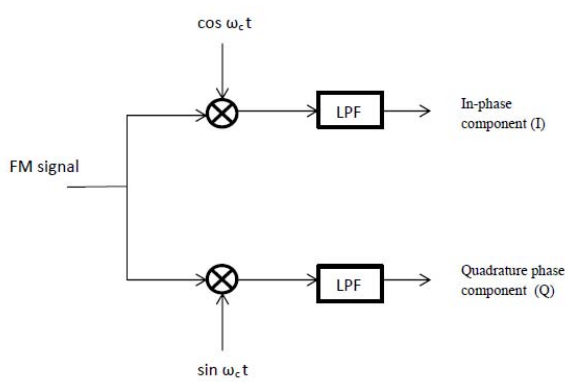

Figure 1. Generation of complex baseband FM signal

The cosine mixing term and sine mixing term are multiplied with the incoming FM signal. Both mixers oscillate at the FM carrier frequency w_c. The total mixing operation produces a real (in-phase) and imaginary (quadrature-phase) baseband component. Adding the in-phase and quadrature-phase baseband components results in the complex baseband FM signal.

$$I = A_C/2\{ \mathrm{Re} [e^{jy(t)}] \} \qquad (7)$$
$$Q = jA_C/2\{ \mathrm{Im} [e^{jy(t)}] \} \qquad (8)$$

Complex baseband FM signal is given by,

$$y(t)_{fm\text{-}baseband} = A_C/2\{ \mathrm{Re} [e^{jy(t)}] + j\, \mathrm{Im} [e^{jy(t)}] \} \qquad (9)$$

This complex baseband signal is used for FM demodulation. Basic idea behind all FM demodulators is to extract phase from the modulated signal and then differentiate for the recovery of original message signal.

III. ARCTAN ALGORITHM

Basic idea of ARCTAN method [1] is to take the ratio of quadrature signals I and Q signals. The phase is then extracted by using $\tan^{-1}$ operator. The phase angle is differentiated to obtain original message information. Since ratio of quadrature signals are taken, parasitic amplitude modulation can be eliminated. But when I signal is very small compared to Q, that may lead to saturation in the divider. Therefore a modified ARCTAN algorithm called differentiator demodulator has been proposed. This demodulator is possible since,

$$\frac{d}{dt} \tan^{-1} \alpha = \alpha' / (1+ \alpha^2) \qquad (9)$$

Differentiator demodulator [1] is capable of demodulating both wideband and narrowband signals. Look up table in a ROM that was used to get arctan values in case of ARCTAN demodulator is eliminated in case of differentiator demodulator. Since Q and I have almost same amplitudes, taking their ratio may lead to truncation errors. Inorder to avoid this, Q must be amplified before division and the corresponding gain must be compensated in the succeeding stages. A smoothening low pass filter is provided at the output to eliminate spikes and truncation errors.

IV. CORDIC ALGORITHM AND POLAR DISCRIMINATOR

CORDIC algorithm translates a point along a unit circle to implement various trigonometric and hyperbolic functions. These functions corresponds to mapping between rectangular and polar coordinate systems. It has two operating modes namely Vectoring mode and Rotation mode[2]. Vectoring mode converts a vector from Cartesian coordinate system to polar coordinate.

$$x=R\cos\theta \qquad (10)$$
$$y=R\sin\theta \qquad (11)$$
$$\theta=\tan^{-1}(y/x) \qquad (12)$$
$$R=(x^2+y^2)^{1/2} \qquad (13)$$

These are classic equations for translation between rectangular and polar system. For FM demodulation vectoring mode of CORDIC algorithm is used.

Consider a point (x_{in},y_{in}) offset from x-axis at an angle α. A new point (x_{final},y_{final}) can be created by rotating the initial point around unit circle by an angle θ.

$$x_{in} = R \cos\alpha \qquad (14)$$
$$y_{in} = R \sin\alpha \qquad (15)$$

$$x_{final} = R \cos(\theta+\alpha) \qquad (16)$$
$$y_{final} = R \sin(\theta+\alpha) \qquad (17)$$

using trigonometric identities,

$$x_{final} = R [\cos\alpha \cos\theta - \sin\alpha \sin \theta] \qquad (18)$$
$$y_{final} = R [\sin\alpha \cos\theta + \cos\alpha \sin \theta] \qquad (19)$$

from above equations,

$$x_{final} = x_{in} \cos\theta - y_{in} \sin\theta \quad (20)$$
$$y_{final} = y_{in} \cos\theta + x_{in} \sin\theta \quad (21)$$

if coordinates x_{in} and y_{in} are known and if $(x_{final}, y_{final}) = (R,0)$ angle swept will be equal to θ. This is rectangular to polar conversion[2]. In case of complex baseband FM signal, at each sample point let,

x_{in} = in-phase component (I)
y_{in} = quadrature phase component (Q)

The vector is then rotated until $y_{final} = 0$ in a series of angle steps θ_i that when summed gives θ (phase of the vector) as shown in Fig. 3.Then x_{final} gives the magnitude.

$$\theta = \sum \theta_i$$

θ_i is chosen so that

$$\tan \theta_i = \pm 2^{-i} \quad (22)$$

When $+2^{-i}$ is used, rotation will occur in a counter-clockwise direction. When -2^{-i} is used, rotation will occur in a clockwise direction.
Iterative rotation is expressed as,

$$d_i = -\,\mathrm{sgn}(y_i) \quad (23)$$

$$x_{i+1} = x_i - d_i\, y_i\, 2^{-i} \quad (24)$$
$$y_{i+1} = y_i + d_i\, x_i\, 2^{-i} \quad (25)$$
$$z_{i+1} = z_i - d_i\, \mathrm{atan}(2^{-i}) \quad (26)$$

Here $(x_0,y_0) = (I,Q)$ and $z_0 = 0$. After a number of iterations, number of subrotations (N) becomes large,
$y_N \rightarrow 0$, $z_N \approx \mathrm{atan}(y_0/x_0)$ and $x_N = (x^2+y^2)^{1/2}/K$.
where $K = \cos(\mathrm{atan}(2^{-i}))$.
z_N will give phase of the vector θ. Original message signal can be retrieved by differentiating phase of the signal.

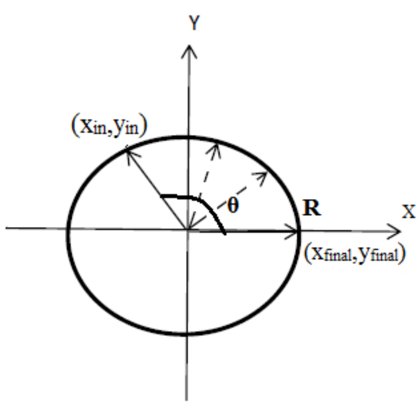

Figure 2. Rotation of a point about a circle

Polar discriminator [5] considers phase difference between successive samples of a complex valued signal. This phase difference corresponds to instantaneous frequency of sampled FM signal. It operates by taking successive complex-valued samples and multiplying the new sample by the conjugate of the old sample. In polar form it can be represented as,

$$y_1(n) = e^{j\theta 1} \quad (27)$$
$$y_2(n) = e^{j\theta 2} \quad (28)$$
$$y_2(n) * \mathrm{conj}(y_1(n)) = e^{j(\theta 2 - \theta 1)} \quad (29)$$

V. Baseband Delay Demodulator and Baseband Differentiator Demodulator

Baseband delay FM demodulator is similar to that of ARCTAN algorithm. Here the derivative of phase is obtained by first delaying the I and Q signals by one sample and then multiplying it with the un-delayed other followed by an arcsine function. Finally a scaling factor should be provided for obtaining the message signal.

$$I = \cos\Phi_{fm}(n)$$
$$Q = \sin\Phi_{fm}(n) \quad (30)$$

Where $\Phi_{fm}(n)$ represents phase of the FM modulated signal.

$$h_3(n) = \cos\Phi_{fm}(n-1)\sin\Phi_{fm}(n) - \sin\Phi_{fm}(n-1)\cos\Phi_{fm}(n)$$
$$h_3(n) = 1/2\sin[\Phi_{fm}(n) - \Phi_{fm}(n-1)] \quad (31)$$

$$h_4(n) = \sin^{-1}[h3(n)]$$
$$h_4(n) = \Phi_{fm}(n) - \Phi_{fm}(n-1) \quad (32)$$

Baseband differentiator involves differentiation instead of delay to determine the phase from I and Q signals. I and Q signals are differentiated and are multiplied other signal undifferentiated. The differentiated I-signal multiplied with the Q-signal is subtracted from the differentiated Q-signal multiplied with the I-signal.

$$\Phi'_{fm} = ((Q(n)-Q(n-1)).I(n)) - ((I(n)-I(n-1).Q(n))) \quad (33)$$

VI. Simulation Results

An FM modulated signal with f_m = 15KHz and f_C = 10.7MHz is generated in MATLAB software from Mathwork. FM signal is then sampled and Complex baseband signal is obtained. It is given as input to six different quadrature demodulators. Performance of demodulators[6] are compared based on frequency spectrum of output signal and input SNR versus output SNR plot. Output SNR is computed using mean square error (MSE) method[7].

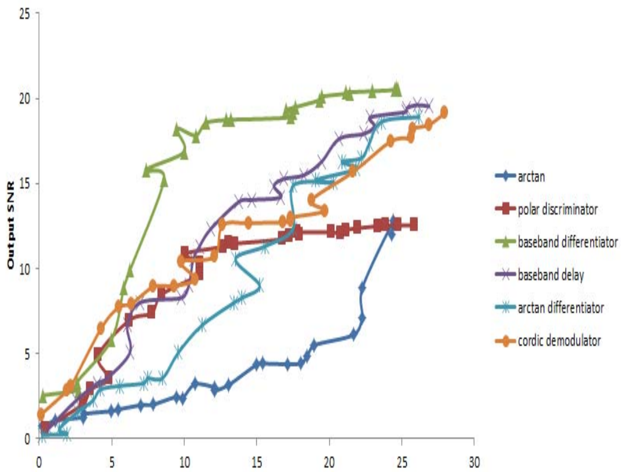

Figure 3. Output SNR versus Input SNR comparison

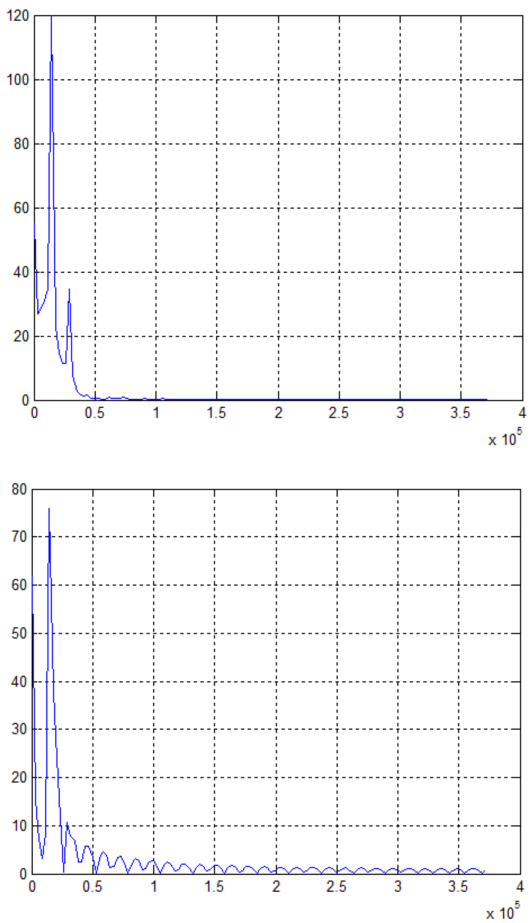

Figure 4. Frequency spectrum of arctan demodulator and baseband differentiator demodulator

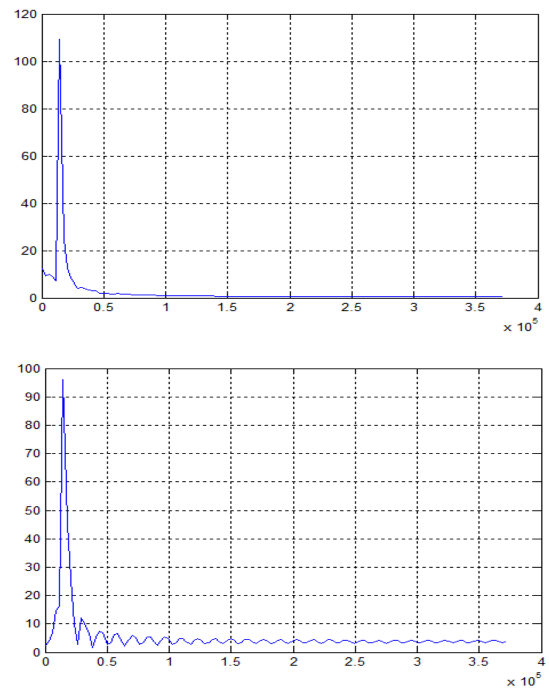

Figure 5. Frequency spectrum of CORDIC demodulator and Polar discriminator

VII. CONCLUSIONS

Six different quadrature detection methods for FM demodulation are studied and simulated in MATLAB. These methods are then compared using input SNR versus output SNR plot and frequency spectrum of output signal. It was found that baseband differentiator demodulator provides better SNR performance. But in terms of harmonic distortion, CORDIC demodulator output contains much less spurious signals compared to others. Since baseband differentiator offers acceptable SNR and due to the ease of implementation, it can be selected for telecommand receiver.

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