Multicriterio y Escalas Ordinales

Juan B Cabral

Universidad Gastón Dachary

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About Me

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About Me
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- Software engineer.
- Data scientist.
- Grad. Student In Machine Learning for Astrophysics.

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Disclaimer: Wikipedia powered slides!

Decision-making

Is the study of identifying and choosing alternatives based on the values and preferences of the decision maker.

Rational choice theory encompasses the notion that people try to maximize benefits while minimizing costs.

Perspectives:

- Psychological
- Cognitive
- Normative

Decision-making (Cont.)

- A major part of decision-making involves the analysis of a finite set of alternatives described in terms of evaluative criteria.
- These criteria may be benefit or cost in nature. (*Maximization* an *Minimization* problems).
- The problem might be to rank these alternatives in terms of how attractive they are to the decision-maker(s) when all the criteria are considered simultaneously.

Multiple-criteria decision analysis

- In our daily lives or in professional settings, there are typically multiple conflicting criteria that need to be evaluated in making decisions.
- We usually weigh multiple criteria implicitly and we may be comfortable with the consequences of such decisions that are made based on only intuition.
- When stakes are high, it is important to properly structure the problem and explicitly evaluate multiple criteria.

Multiple-criteria decision analysis (cont.)

MCDM or MCDA is concerned with structuring and **solving** decision and planning problems involving multiple criteria.

"Solving" can be:

- Best alternative, small set of best alternatives or grouping alternatives.
- An extreme interpretation could be to find all "efficient" or "nondominated" alternatives.

MCDA - Typologies

- **Multiple-criteria evaluation problems:** These problems consist of a finite number of alternatives, explicitly known in the beginning of the solution process. Each alternative is represented by its performance in multiple criteria. The problem may be defined as finding the best alternative for a decision maker (DM), or finding a set of good alternatives.
- **Multiple-criteria design problems (multiple objective mathematical programming problems):** In these problems, the alternatives are not explicitly known. An alternative (solution) can be found by solving a mathematical model. The number of alternatives is either infinite and not countable (when some variables are continuous) or typically very large if countable (when all variables are discrete).

MCDA - Typologies (cont.)

- There are methods that require the DM's preference information at the start of the process, transforming the problem into essentially a single criterion problem. (**prior articulation of preferences**).
- Some methods require preference information from the DM throughout the solution process. (**progressive articulation of preferences**).
- MC design problems typically require the solution of a series of mathematical programming models in order to reveal implicitly defined solutions. (**posterior articulation of preferences"**).

Representations and definitions

The MCDM problem can be represented in the criterion space or the decision space. Alternatively, if different criteria are combined by a weighted linear function, it is also possible to represent the problem in the weight space.

Criterion Space	Decision Space
$\text{"max" } \boldsymbol{q}$	$\text{"max" } \boldsymbol{q} = f(x) = (f_1(\mathbf{x}), \ldots, f_k(\mathbf{x}))$
subject to	subject to
$q \in Q$	$q \in Q = \{f(x) : x \in X, X \subseteq R^n\},$

where **q** is the vector of **k** criterion functions (objective functions) and **Q** is the feasible set, **Q ☐ R^k.**

If Q is defined explicitly or implicity (by a set of alternatives), the resulting problem is called a Multiple Criteria Evaluation or Design problem.

X is the feasible set and x is the decision variable vector of size n.

Te quotation mark indicate the maximization is not well-defined.

Representations and definitions (cont.)

Criterion Space

Max f_1

Max f_2

subject to

$f_1 + 2f_2 \le 12$

$2f_1 + f_2 \le 12$

$f_1 + f_2 \le 7$

$f_1 - f_2 \le 9$

$-f_1 + f_2 \le 9$

$f_1 + 2f_2 \ge 0$

$2f_1 + f_2 \ge 0$

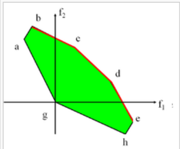

Figure 2. Demonstration of the solutions in the criterion space

Decision Space

Max $f_1(\boldsymbol{x}) = -x_1 + 2x_2$

Max $f_2(\boldsymbol{x}) = 2x_1 - x_2$

subject to

$x_1 \le 4$

$x_2 \le 4$

$x_1 + x_2 \le 7$

$-x_1 + x_2 \le 3$

$x_1 - x_2 \le 3$

$x_1, x_2 \ge 0$

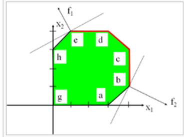

Figure 1. Demonstration of the decision space

Definitions:

Definition 1. $\boldsymbol{q}^* \in Q$ is nondominated if there does not exist another $\boldsymbol{q} \in Q$ such that $\boldsymbol{q} \ge \boldsymbol{q}^*$ and $\boldsymbol{q} \ne \boldsymbol{q}^*$.

Definition 2. $\boldsymbol{x}^* \in X$ is efficient if there does not exist another $\boldsymbol{x} \in X$ such that $f(\boldsymbol{x}) \ge f(\boldsymbol{x}^*)$ and $f(\boldsymbol{x}) \ne f(\boldsymbol{x}^*)$.

Definition 3. $\boldsymbol{q}^* \in Q$ is weakly nondominated if there does not exist another $\boldsymbol{q} \in Q$ such that $\boldsymbol{q} > \boldsymbol{q}^*$.

Definition 4. $\boldsymbol{x}^* \in X$ is weakly efficient if there does not exist another $\boldsymbol{x} \in X$ such that $f(\boldsymbol{x}) > f(\boldsymbol{x}^*)$.

- **Ideal point:** (in criterion space) represents the best (the maximum for maximization problems and the minimum for minimization problems) of each objective function, and typically corresponds to an infeasible solution.
- **Nadir point:** (in criterion space) represents the worst (the minimum for maximization problems and the maximum for minimization problems) of each objective function among the points in the nondominated set, and is typically a dominated point.

Generating nondominated solutions

$$\max\ w^T.q = w^T.f(x),\ w > 0$$

subject to

$$x \in X$$

Only one dimension but... works!

Solving MCDM problems - Schools

- Multiple objective mathematical programming school
- Goal programming school
- Fuzzy-set theorists
- Multiattribute utility theorists
- French school
- Evolutionary multiobjective optimization school (EMO)
- Analytic hierarchy process (AHP)

How many methods?

MCDM methods (1/3)

- Aggregated Indices Randomization Method (AIRM)
- **Analytic hierarchy process (AHP)**
- Analytic network process (ANP)
- Best worst method (BWM)[30]
- Characteristic Objects METhod (COMET)
- Data envelopment analysis
- Decision EXpert (DEX)
- Disaggregation – Aggregation Approaches (UTA*, UTAII, UTADIS)
- Dominance-based rough set approach (DRSA)
- ELECTRE (Outranking)
- Evidential reasoning approach (ER)
- Goal programming
- Grey relational analysis (GRA)
- Inner product of vectors (IPV)

MCDM methods (2/3)

- Measuring Attractiveness by a categorical Based Evaluation Technique (MACBETH)
- Multi-Attribute Global Inference of Quality (MAGIQ)
- Multi-attribute utility theory (MAUT)
- Multi-attribute value theory (MAVT)
- New Approach to Appraisal (NATA)
- Nonstructural Fuzzy Decision Support System (NSFDSS)
- Potentially all pairwise rankings of all possible alternatives (PAPRIKA)
- PROMETHEE (Outranking)
- Superiority and inferiority ranking method (SIR method)
- Technique for the Order of Prioritisation by Similarity to Ideal Solution (TOPSIS)
- Value analysis (VA)
- Value engineering (VE)

MCDM methods (2/3)

- VIKOR method
- Fuzzy VIKOR method
- **Weighted product model (WPM)**
- **Weighted sum model (WSM)**
- **Multi-Objective Optimization by Ratio Analysis (MOORA)**

Decision-Makin Paradox

- Hay muchos metodos MCDA (normativos y descriptivos); y cada uno clama ser el mejos. Sin embargo muchos de estos metodos retornan diferentes resultados para los mismos problemas con exactamente los mismos datos.
- Encontrar un el mejor metodo es un problema de MCDA en si mismo.
- Naturalmente es necesario conocer el mejor metodo a-priori.

Decision-Makin Paradox (cont.)

- A traves de un estudio [Triantaphyllou1989]_ [Triantaphyllou,2000]_ Se realizo un experimento de selección de metodos utilizando 4 metodos. WSM, WPM y dos variantes de AHP. Cuando se usaba un metodo *X* (perteneciente a los anteriores, indicaba que *Y* era el mejor. Cuando se utilizaba *Y* el resultado decia que *Z* era mejor.
- Para enunciar el problema se utilizaron 2 criterios: #. Todo metodo debe ser tan precido en problemas multidimensionales como

  uni-dimensionales (se comparo resultados con WSUM) es lo llamaso ranking reversal tipo 5.

1. Resultados de Analisis de ranking reversals tipo 2.

Ranking Reversal

- Esencialmente son: Test Cases
- La idea es modificar las alternativas de tal forma que "suponemos" que las mejores alternativas no cambian.

Ranking Reversal (cont.)

Si tenemos tres Alternativas `A > B > C`

Tipos:

- **Tipo 1:** Agregamos una alternativa D igual o parecida a B o C y validamos que la mejor alternativa no cambie.
- **Tipo 2:** Reemplazamos B por D siendo D > B. Esperamos que A siga siendo la mejor.
- **Tipo 3:** Descomponemos el problema es problemas de 2 alternativas por ves, y verificamos que ninguno de ellos no se contradiga con el problema mayor.
- **Tipo 4:** Igual al tipo 3 pero solo comparamos entre ellos ignorando el general.
- **Tipo 5:** Comparaciones unidimensionales vs multidimensionales.

Ranking Reversal (cont.)

- Puede que una falla en un ranking reversal sea un resultado deseado.
- Se da en situaciones racionales.
- Conjetura: Intuyo que pasa en Machine Learning.

Ejemplo:

- Un comprador M1 que le gusta el lujo, Un comprador M2 que no tiene dinero.
- Un auto A1 lujoso y caro y un auto A2 barato y con poco confort.
- `M1 = A1 > A2` y `M2 = A2 > A1`

¿Preguntas?

- Charla:
- **Contactos:**
  - jbcabral.com
  - Juan B Cabral <jbc.develop@gmail.com>