081203M04001H - Algorithm Design and Analysis

Assignment 1

September 17, 2021

Notice:

1. Please submit your answer in hard copy AND submit a digital version to UCAS website `http://sep.ucas.ac.cn`.
   Hard copy should be submitted before 9 am. October 8 and digital version should be submitted before 11 pm. October 8.
2. You can choose **three** from problems 1-6.
3. For problems 1-6, you should do at least the following things:
   (a) Describe your algorithm in natural language **AND** pseudo-code;
   (b) Draw a "subproblem reduction graph", where nodes represent subproblems, and edges describe the "reduction relationship" between them for every problem you choose in problems 1-6;
   (c) Prove the correctness of your algorithm;
   (d) Analyse the complexity of your algorithm.

1 Divide and Conquer

Given an integer array nums and an integer k, please return the k-th largest element in the array.

Your algorithm's runtime complexity must be in the order of O(logn), prove the correctnessand analyze the complexity.(k is much smaller than n, n is the length of the array.)

2 Divide and Conquer

Consider an n-node complete binary tree T, where $n = 2^d - 1$ for some d. Each node v of T is labeled with a real number x_v. You may assume that the real numbers labeling the nodes are all distinct. A node v of T is a local minimum if the label x_v is less than the label x_w for all nodes w that are joined to v by an edge.

You are given such a complete binary tree T, but the labeling is only specified in the following:

implicit way: for each node v, you can determine the value x_v by probing the node v.

Show how to find a local minimum of T using only $O(logn)$ probes to the nodes of T.

3 Divide and Conquer

Given an integer array, one or more consecutive integers in the array form a sub-array. Find the maximum value of the sum of all subarrays.

Please give an algorithm with O(nlogn) complexity

4 Divide and Conquer

Given an array of integers nums sorted in ascending order, find the starting and ending position of a given target value. If the target is not found in the array, return [-1, -1]. For example, if the array is $[5, 7, 7, 8, 8, 10]$ and the target is 8, then the output should be $[3, 4]$.

Your algorithm's runtime complexity must be in the order of $O(\log n)$, prove the correctness and analyze the complexity.

5 Divide and Conquer

Given a convex polygon with n vertices, we can divide it into several separated pieces, such that every piece is a triangle. When $n = 4$, there are two different ways to divide the polygon; When $n = 5$, there are five different ways.

Give an algorithm that decides how many ways we can divide a convex polygon with n vertices into triangles.

6 Divide and Conquer

Given an array of k linked-lists lists, each linked-list is sorted in ascending order. Given an O(knlogk) algorithm to merge all the linked-lists into one sorted linked-list. (Note that the length of a linked-lists is n)