第四章作业

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题目一

根据题目，可求得，三个不同类的均值向量分别为：

$$\begin{aligned} m_1 &= (\frac{4}{3}, \frac{1}{3})^T \\ m_2 &= (-\frac{2}{3}, \frac{2}{3})^T \\ m_3 &= (-\frac{1}{3}, -\frac{4}{3})^T \end{aligned} \tag{1}$$

且有： $P(w_1) = P(w_2) = P(w_3) = \frac{1}{3}$

所以，多类模式的总体均值向量为:

$$m_0 = E(x) = \sum_{i=1}^{3} P(w_i) m_i = (\frac{1}{9}, -\frac{1}{9})^T \tag{2}$$

对类内距离和类间距离分别求解得：

$$\begin{aligned} S_w &= \sum_{i=1}^{3} P(w_i) E\{(x - m_i)(x - m_i)^T\} \\ &= \frac{1}{3}\frac{1}{3}\sum_{x_i \in w_1}\{(x - m_1)(x - m_1)^T\} + \frac{1}{3}\frac{1}{3}\sum_{x_i \in w_2}\{(x - m_2)(x - m_2)^T\} \\ &+ \frac{1}{3}\frac{1}{3}\sum_{x_i \in w_3}\{(x - m_3)(x - m_3)^T\} \\ &= \begin{pmatrix} \frac{2}{9} & -\frac{1}{27} \\ -\frac{1}{27} & \frac{2}{9} \end{pmatrix} \end{aligned} \tag{3}$$

$$\begin{aligned} S_b &= \sum_{i=1}^{3} P(w_i)(m_i - m_0)(m_i - m_0)^T \\ &= \begin{pmatrix} \frac{97}{81} & \frac{13}{81} \\ \frac{13}{81} & \frac{1}{3} \end{pmatrix} \end{aligned} \tag{4}$$

题目2

根据题目，两类模式中两类的概率相等，应该要$P(w_1)=P(w_2)=0.5$，所以

$$m=0.5\times\frac{1}{4}(3,1,1)^T+0.5\times\frac{1}{4}(1,3,3)^T=(0.5,0.5,0.5)^T \tag{5}$$

这样的情况并不满足特征压缩的最佳条件，因此要进行变换

则，令变换后的向量为$t=x-m$

求解为；

$$R=\sum_{i=1}^{2}P(w_i)E(tt^T)=\begin{pmatrix}0.5 & 0.25 & 0.25\\ 0.25 & 0.5 & 0.25\\ 0.25 & 0.25 & 0.5\end{pmatrix} \tag{6}$$

解特征方程组$|R-\lambda I|=0$

得到特征值为：$\lambda_1=1,\lambda_2=0.25,\lambda_3=0.25$

其对应的特征向量可由$R\varphi_i=\lambda_i\varphi_i$求得：

$$\varphi_1=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\\ 0\end{pmatrix} \tag{7}$$

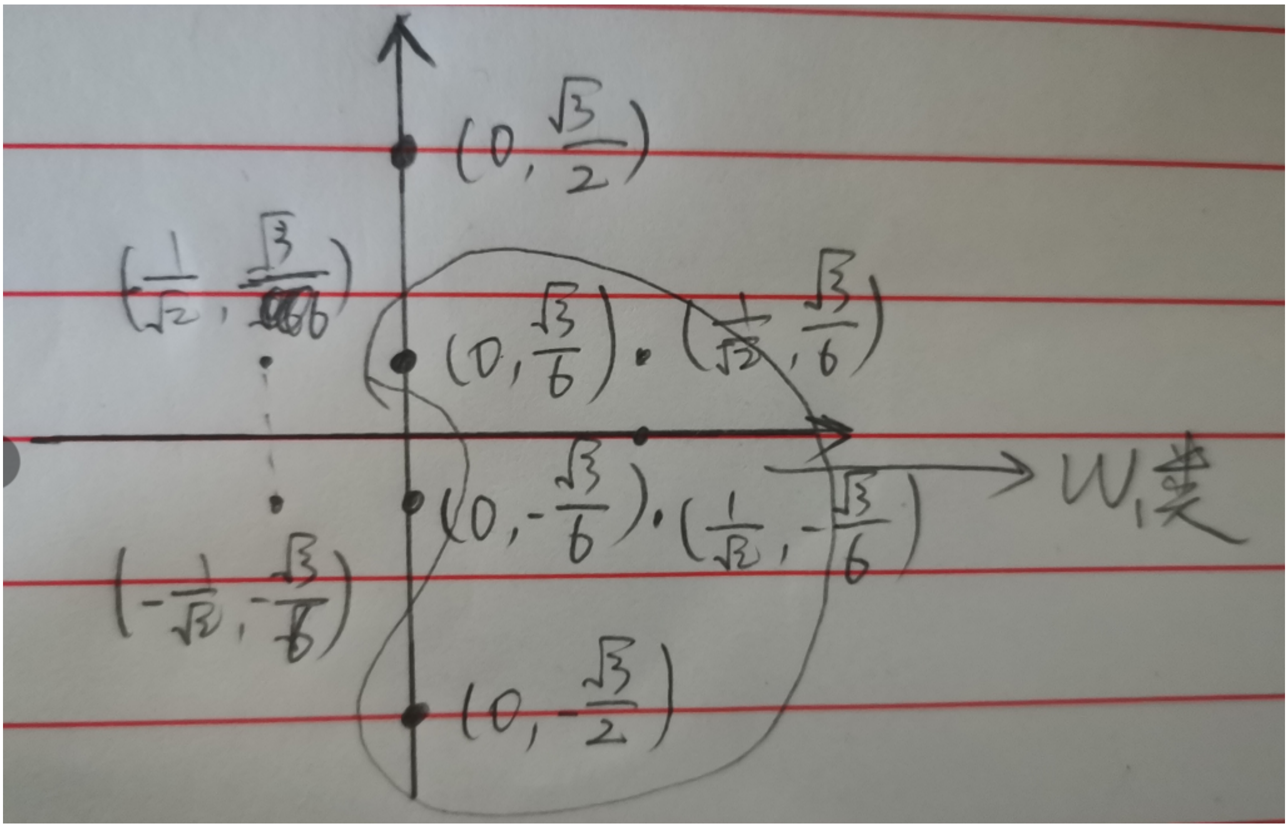
$$\varphi_2=\varphi_3=\frac{1}{\sqrt{3}}\begin{pmatrix}1\\ 1\\ 1\end{pmatrix} \tag{8}$$

故当需要降到二维时：

由$y=\phi^Tt=\phi^T(x-m)$求解，变换后的二维模式特征为：

$$\begin{aligned}w_1:\{(0\ -\frac{\sqrt{3}}{2})^T,(\frac{1}{\sqrt{2}}\ -\frac{\sqrt{3}}{6})^T,(\frac{1}{\sqrt{2}}\ \frac{\sqrt{3}}{6})^T,(0\ \frac{\sqrt{3}}{6})^T\}\\ w_2:\{(0\ -\frac{\sqrt{3}}{6})^T,(-\frac{1}{\sqrt{2}}\ -\frac{\sqrt{3}}{6})^T,(-\frac{1}{\sqrt{2}}\ \frac{\sqrt{3}}{6})^T,(0\ \frac{\sqrt{3}}{2})^T\}\end{aligned} \tag{9}$$

绘图为：



由$y=\phi^T t=\phi^T(x-m)$求解，变换后的一维模式特征为：

$$
\begin{aligned}
w_1 &: \{0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\} \\
w_2 &: \{0,-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0\}
\end{aligned} \tag{10}
$$

绘图为：