P. 解，由题意可知.

(a) 令 $B = A + cd^T \Rightarrow cd^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix}$，有 $c_3 d_2 = 2$，其它的值为 0.

故 $B^{-1} = (A + cd^T)^{-1} = A^{-1} - \frac{A^{-1}cd^TA^{-1}}{1 + d^TA^{-1}c}$

其中，$A^{-1}cd^TA^{-1} = \begin{pmatrix} 0 & 2 & -2 \\ 0 & -2 & 2 \\ 0 & 4 & -4 \end{pmatrix}$

可知，$d = \begin{pmatrix} 0 \\ d_2 \\ 0 \end{pmatrix}$，$c = \begin{pmatrix} 0 \\ 0 \\ c_3 \end{pmatrix}$，$c_3 d_2 = 2$

$\therefore d^TA^{-1}c = (0, d_2, -d_2)\begin{pmatrix} 0 \\ 0 \\ c_3 \end{pmatrix} = -d_2c_3 = -2$

$\therefore B^{-1} = A^{-1} + \begin{pmatrix} 0 & 2 & -2 \\ 0 & -2 & 2 \\ 0 & 4 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 1 & 4 & -2 \end{pmatrix}$

~~(b) 令 $C = A + cd^T \Rightarrow cd^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & 2 \end{pmatrix}$，同理，有~~

(b) 令 $C = A + mn^T$，$mn^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & 1 \end{pmatrix}$，有 $m_3n_2 = 2$，$m_3n_3 = 1$

可知，$m = \begin{pmatrix} 0 \\ 0 \\ m_3 \end{pmatrix}$，$n = \begin{pmatrix} 0 \\ n_2 \\ n_3 \end{pmatrix}$

$A^{-1}mn^TA^{-1} = \begin{pmatrix} 1 & 2 & 0 \\ -1 & -2 & 0 \\ 2 & 4 & 0 \end{pmatrix}$

$n^TA^{-1}m = (n_3, n_2, 2n_3 - n_2) \cdot \begin{pmatrix} 0 \\ 0 \\ m_3 \end{pmatrix} = (2n_3 - n_2) \cdot m_3 = 0$

$\therefore C^{-1} = A^{-1} - A^{-1}mn^TA^{-1} = \begin{pmatrix} 0 & -2 & 1 \\ 1 & 3 & -1 \\ -1 & -4 & 2 \end{pmatrix}$

11. 解，由题意可知.

(a) 利用相应方法，得到如下变换：

$$(A|b) \longrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 4 & 17 & 17 \\ 3 & 6 & -12 & 3 & 3 \\ 2 & 3 & -3 & 2 & 3 \\ 0 & 2 & -2 & 6 & 4 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 3 \\ 1 & 2 & 4 & 17 & 17 \\ 2 & 3 & -3 & 2 & 3 \\ 0 & 2 & -2 & 6 & 4 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 3 \\ \frac{1}{3} & 0 & 8 & 16 & 17 \\ \frac{2}{3} & -1 & 5 & 0 & 3 \\ 0 & 2 & -2 & 6 & 4 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 3 \\ 0 & 2 & -2 & 6 & 4 \\ \frac{2}{3} & -1 & 5 & 0 & 3 \\ \frac{1}{3} & 0 & 8 & 16 & 17 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 3 \\ 0 & 2 & -2 & 6 & 4 \\ \frac{2}{3} & -\frac{1}{2} & 4 & 3 & 3 \\ \frac{1}{3} & 0 & 8 & 16 & 17 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 3 \\ 0 & 2 & -2 & 6 & 4 \\ \frac{1}{3} & 0 & 8 & 16 & 17 \\ \frac{2}{3} & -\frac{1}{2} & 4 & 3 & 3 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 3 \\ 0 & 2 & -2 & 6 & 4 \\ \frac{1}{3} & 0 & 8 & 16 & 17 \\ \frac{2}{3} & -\frac{1}{2} & \frac{1}{2} & -5 & 3 \end{array}\right)$$

$$\therefore P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{3} & 0 & 1 & 0 \\ \frac{2}{3} & -\frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix} \quad U = \begin{pmatrix} 3 & 6 & -12 & 3 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 8 & 16 \\ 0 & 0 & 0 & -5 \end{pmatrix}$$

(b)： 可知，$PAx = Pb = \begin{pmatrix} 3 \\ 4 \\ 17 \\ 3 \end{pmatrix}$

$\because PA = LU$ 有 $PAx = LUx$，令 $Ux = y$

$\therefore Ly = Pb$

$\Rightarrow y = \begin{pmatrix} 3 \\ 4 \\ 16 \\ -5 \end{pmatrix}$

$\because Ux = y \Rightarrow x = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}$

$\therefore$ 求解得 $x = (2, -1, 0, 1)^T$.