CS 267 Applications of Parallel Computers

Hierarchical Methods for the N-Body problem

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Big Idea

- Suppose the answer at each point depends on data at all the other points
  - Electrostatic, gravitational force
  - Solution of elliptic PDEs
  - Graph partitioning
- Seems to require at least $O(n^2)$ work, communication
- If the dependence on "distant" data can be compressed
  - Because it gets smaller, smoother, simpler…
- Then by compressing data of groups of nearby points, can cut cost (work, communication) at distant points
  - Apply idea recursively: cost drops to O(n log n) or even O(n)
- Examples:
  - Barnes-Hut or Fast Multipole Method (FMM) for electrostatics/gravity/…
  - Multigrid for elliptic PDE
  - Multilevel graph partitioning (METIS, Chaco,…)

Outline

- Motivation
  - Obvious algorithm for computing gravitational or electrostatic force on N bodies takes $O(N^2)$ work
- How to reduce the number of particles in the force sum
  - We must settle for an approximate answer (say 2 decimal digits, or perhaps 16 …)
- Basic Data Structures: Quad Trees and Oct Trees
- The Barnes-Hut Algorithm (BH)
  - An O(N log N) approximate algorithm for the N-Body problem
- The Fast Multipole Method (FMM)
  - An O(N) approximate algorithm for the N-Body problem
- Parallelizing BH, FMM and related algorithms

Particle Simulation

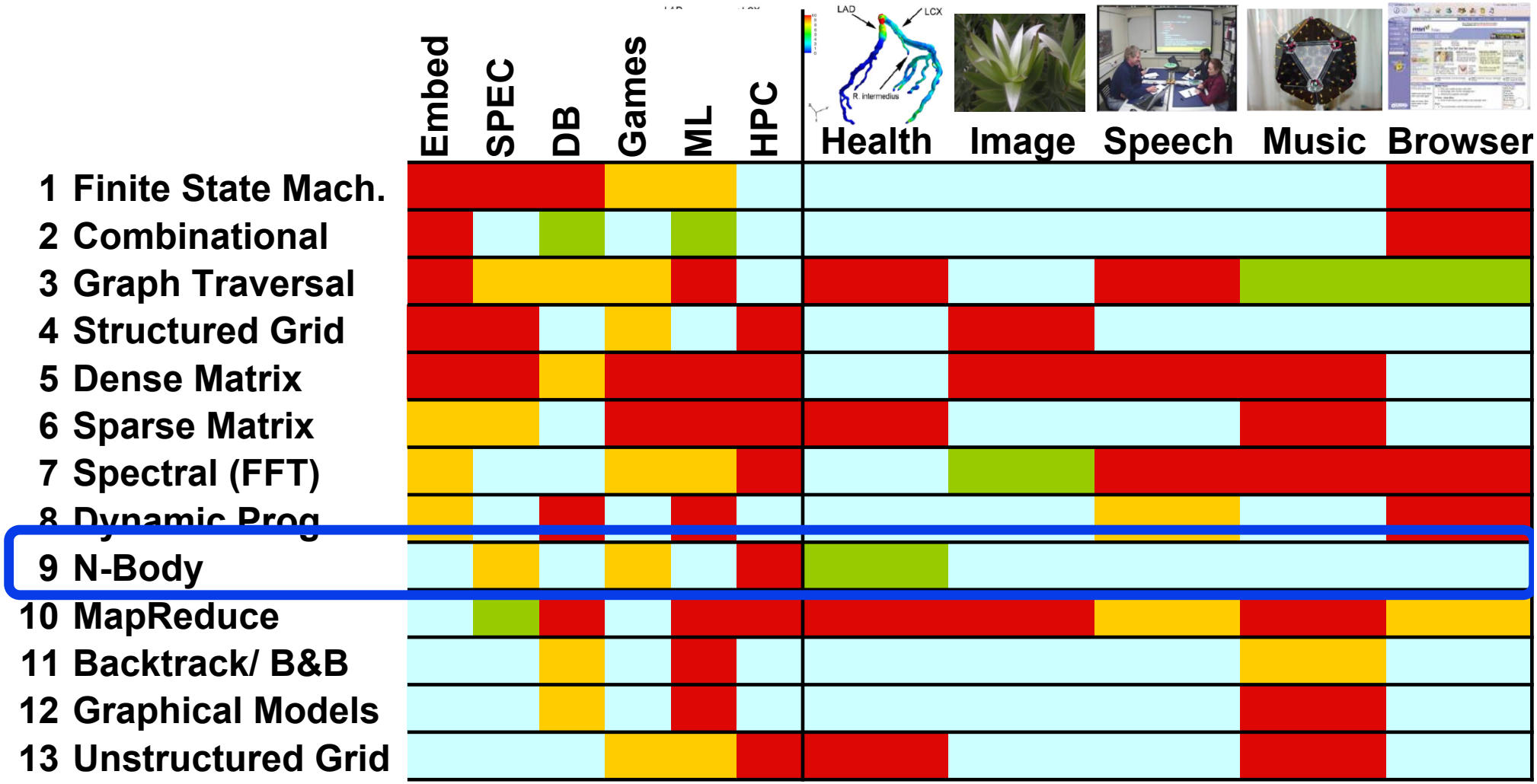
```
t = 0
while t < t_final
    for i = 1 to n              … n = number of particles
        compute f(i) = force on particle i
    for i = 1 to n
        move particle i under force f(i) for time dt    … using F=ma
    compute interesting properties of particles (energy, etc.)
    t = t + dt
end while
```

° **f(i) = external_force + nearest_neighbor_force + N-Body_force**

- External_force is usually embarrassingly parallel and costs O(N) for all particles
  - external current in Sharks and Fish
- Nearest_neighbor_force requires interacting with a few neighbors, so still O(N)
  - van der Waals, bouncing balls (HW2)
- N-Body_force (gravity or electrostatics) requires all-to-all interactions
  - $f(i) = \sum_{k \neq i} f(i,k)$ … f(i,k) = force on i from k
  - $f(i,k) = c*v/||v||^3$ in 3 dimensions or $f(i,k) = c*v/||v||^2$ in 2 dimensions
    - v = vector from particle i to particle k , c = product of masses or charges
    - ||v|| = length of v
  - Obvious algorithm costs $O(n^2)$, but we can do better...

What do commercial and CSE applications have in common?

Motif/Dwarf: Common Computational Methods (Red Hot → Blue Cool)

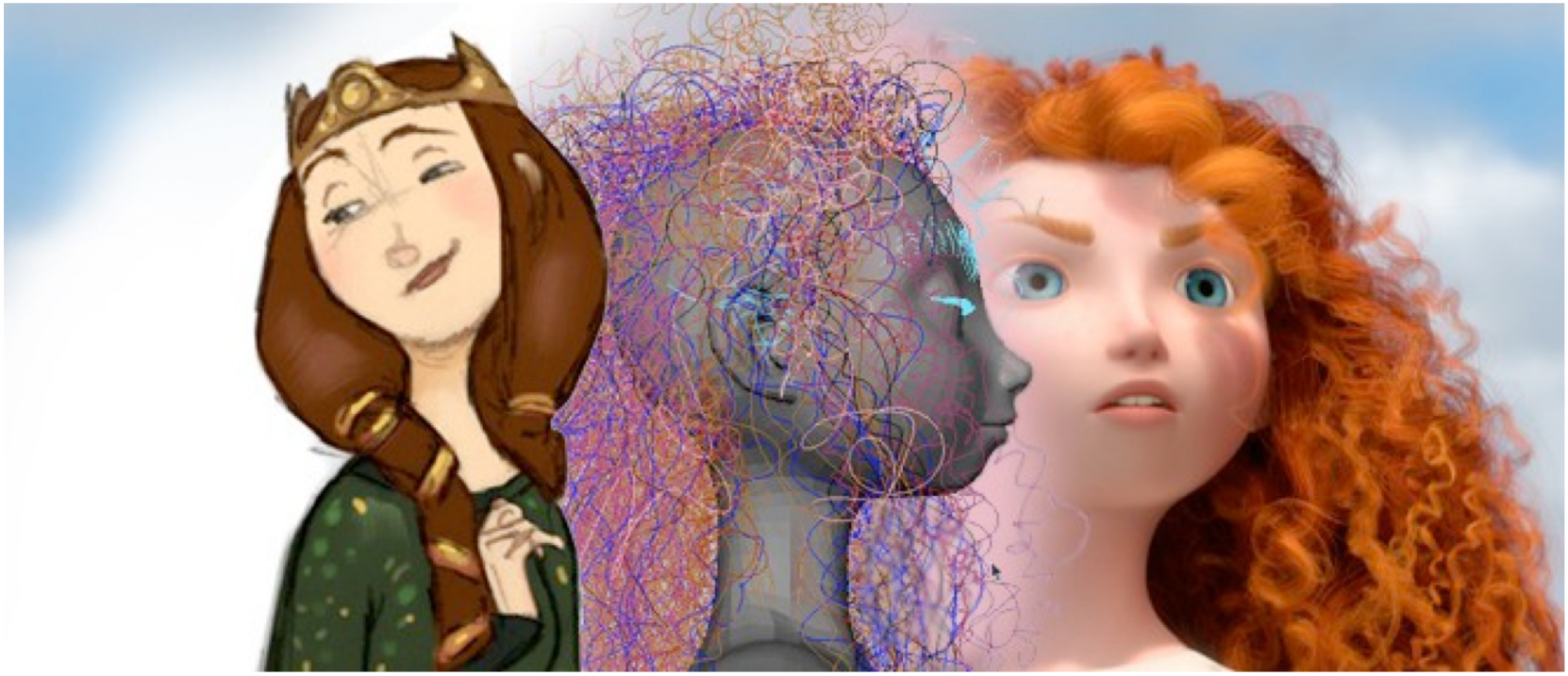


Applications (1/2)

- Astrophysics and Celestial Mechanics - 1992
  - Intel Delta = 1992 supercomputer, 512 Intel i860s
  - 17 million particles, 600 time steps, 24 hours elapsed time
    - M. Warren and J. Salmon
    - Gordon Bell Prize at Supercomputing 1992
  - Sustained 5.2 Gigaflops = 44K Flops/particle/time step
  - 1% accuracy
  - Direct method (17 Flops/particle/time step) at 5.2 Gflops would have taken 18 years, 6570 times longer
- Vortex particle simulation of turbulence – 2009
  - Cluster of 256 NVIDIA GeForce 8800 GPUs
  - 16.8 million particles
    - T. Hamada, R. Yokota, K. Nitadori. T. Narumi, K. Yasoki et al
    - Gordon Bell Prize for Price/Performance at Supercomputing 2009
  - Sustained 20 Teraflops, or $8/Gigaflop

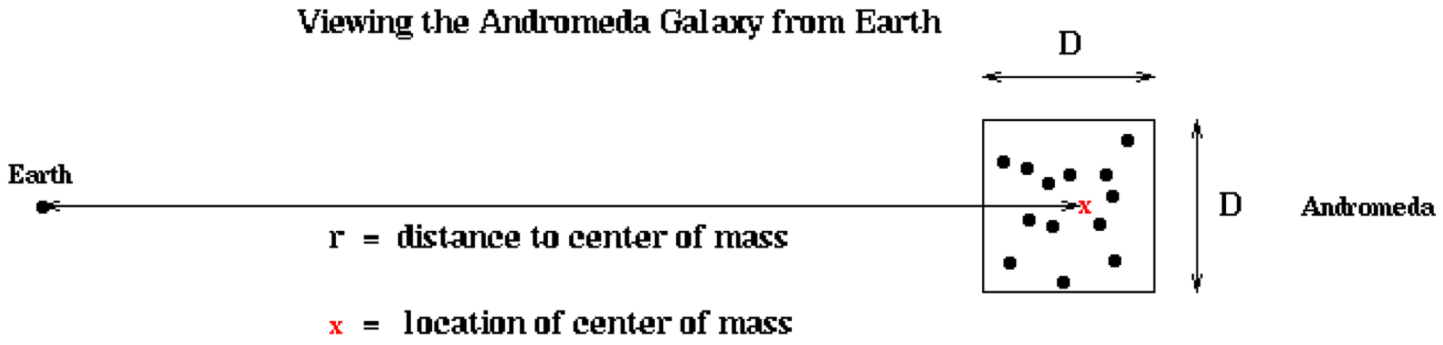
Applications (2/2)

- Molecular Dynamics
- Plasma Simulation
- Electron-Beam Lithography Device Simulation
- Hair ...
  - www.fxguide.com/featured/brave-new-hair/
  - graphics.pixar.com/library/CurlyHairA/paper.pdf



Reducing the number of particles in the force sum

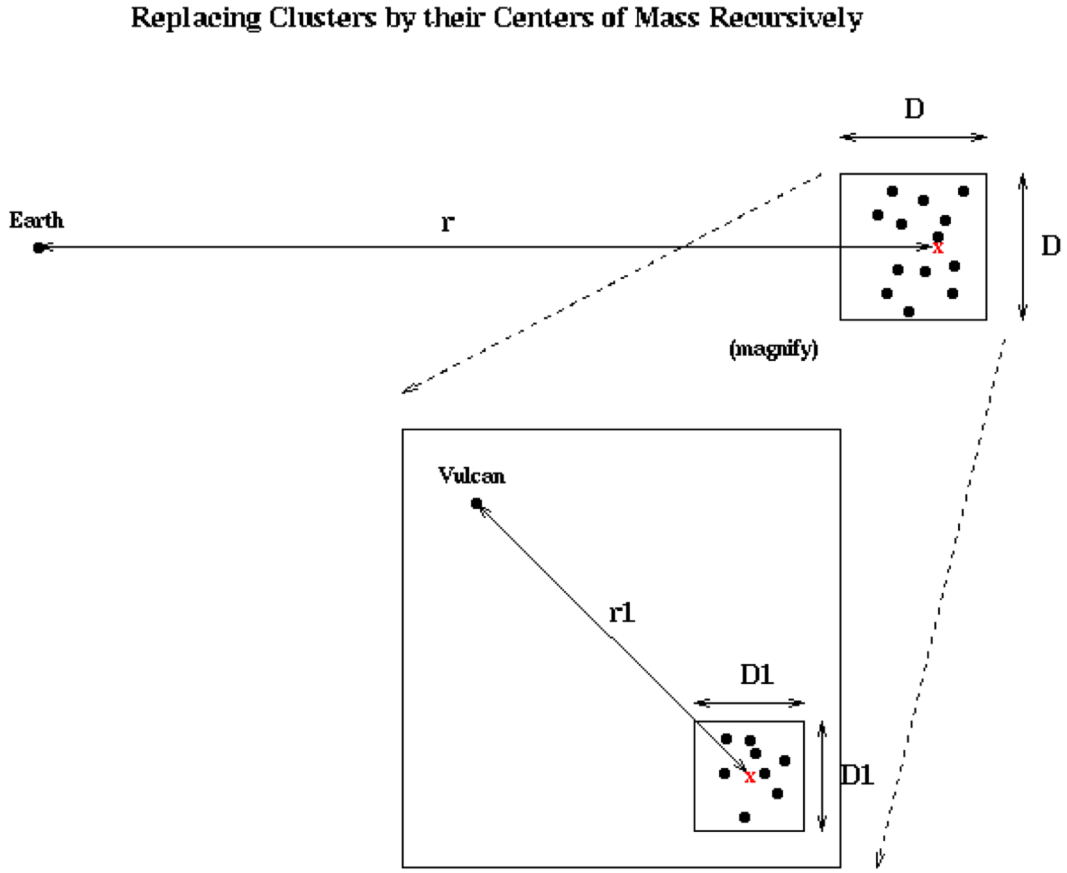
- All later divide and conquer algorithms use same intuition
- Consider computing force on earth due to all celestial bodies
  - Look at night sky, # terms in force sum ≥ number of visible stars
  - Oops! One "star" is really the Andromeda galaxy, which contains billions of real stars
    - Seems like a lot more work than we thought …
- Don't worry, ok to approximate all stars in Andromeda by a single point at its center of mass (CM) with same total mass (TM)
  - D = size of box containing Andromeda , r = distance of CM to Earth
  - Require that D/r be "small enough"



  - Idea not new: Newton approximated earth and falling apple by CMs

What is new: Using points at CM recursively

- From Andromeda's point of view, Milky Way is also a point mass
- Within Andromeda, picture repeats itself
  - As long as D1/r1 is small enough, stars inside smaller box can be replaced by their CM to compute the force on Vulcan
  - Boxes nest in boxes recursively



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- **Basic Data Structures: Quad Trees and Oct Trees**
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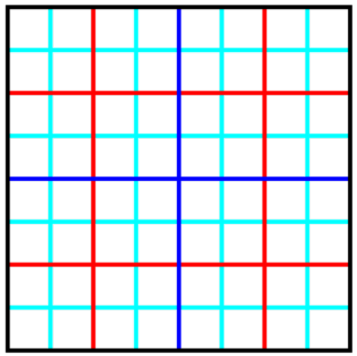
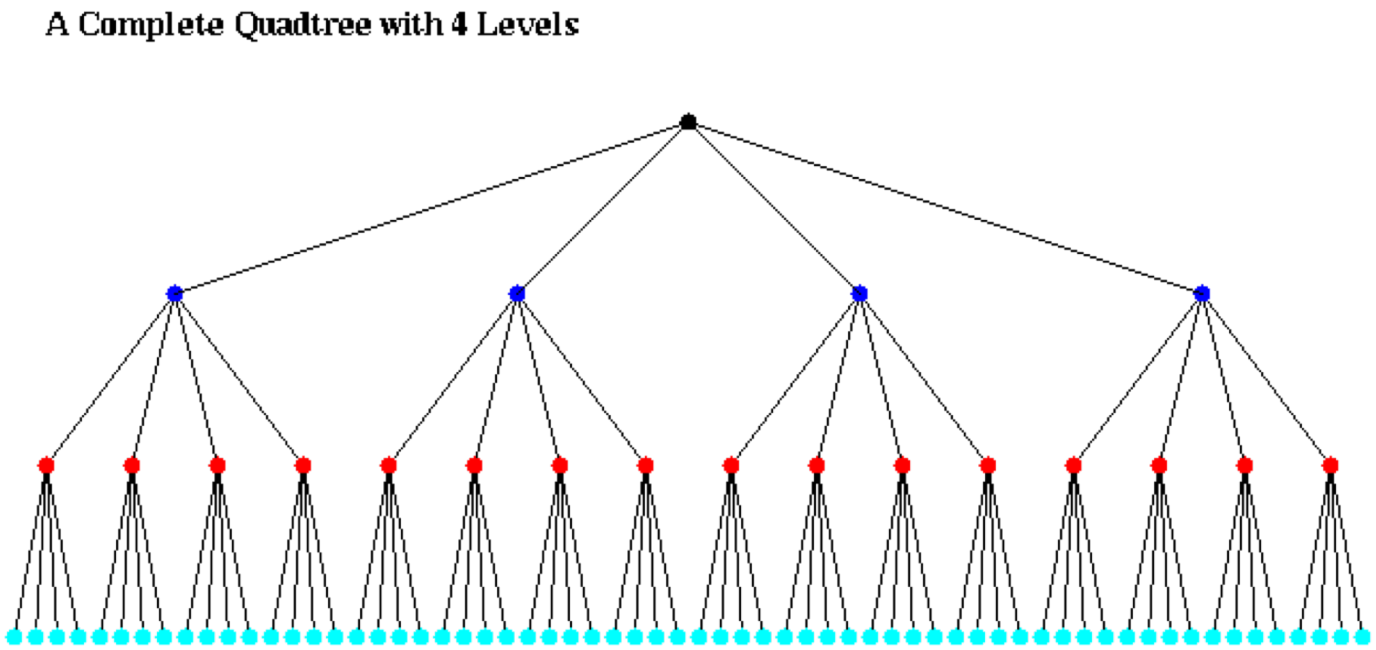
Quad Trees

- Data structure to subdivide the plane
  - Nodes can contain coordinates of center of box, side length
  - Eventually also coordinates of CM, total mass, etc.
- In a complete quad tree, each nonleaf node has 4 children

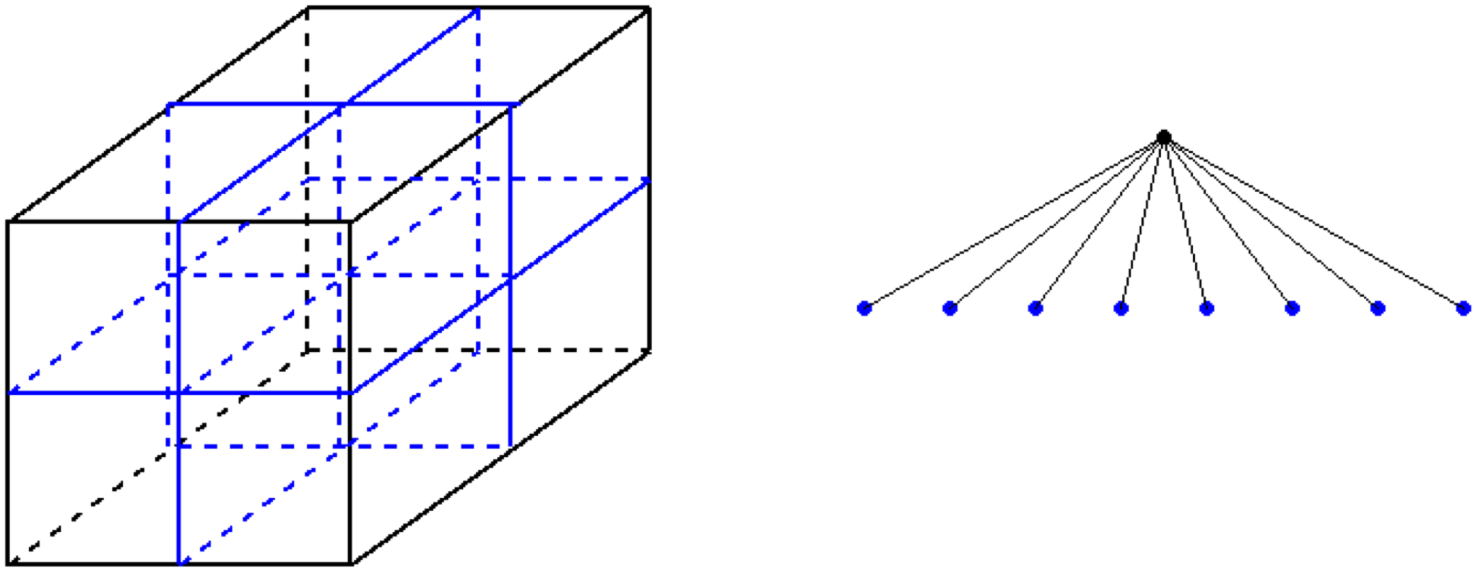




Oct Trees

- **Similar Data Structure to subdivide space**

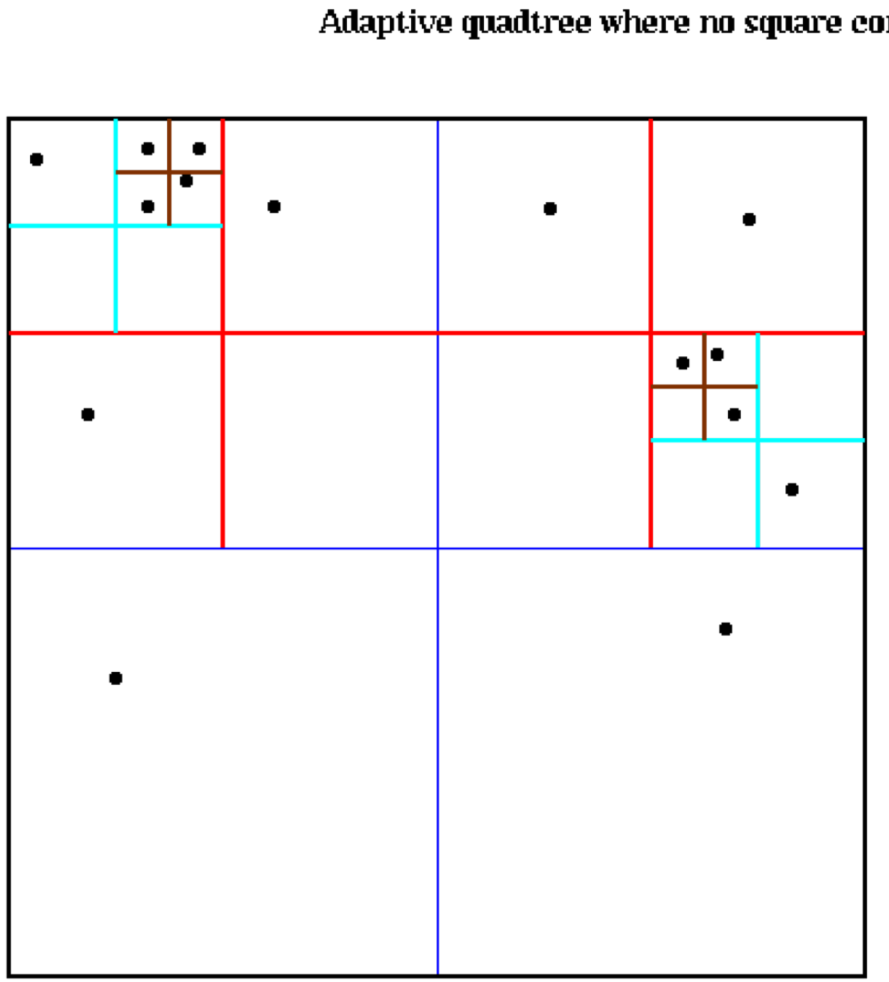
2 Levels of an Octree

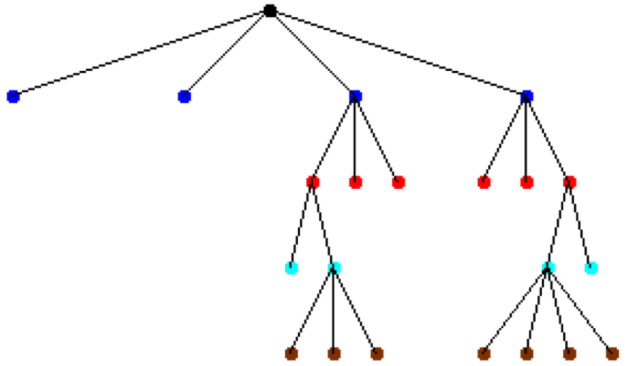


Using Quad Trees and Oct Trees

- All our algorithms begin by constructing a tree to hold all the particles
- Interesting cases have nonuniformly distributed particles
  - In a complete tree most nodes would be empty, a waste of space and time
- Adaptive Quad (Oct) Tree only subdivides space where particles are located

Example of an Adaptive Quad Tree





Child nodes enumerated counterclockwise from SW corner, empty ones excluded

In practice, have q>1 particles/square; tuning parameter

(code to build data structure on hidden slide)

Cost of Adaptive Quad Tree Constrution

- Cost ≤ N * maximum cost of Quad_Tree_Insert
  = O(N * maximum depth of Quad_Tree)

- Uniform Distribution of particles
  - Depth of Quad_Tree = O(log N)
  - Cost ≤ O(N * log N)

- Arbitrary distribution of particles
  - Depth of Quad_Tree = O(# bits in particle coords) = O(b)
  - Cost ≤ O(b N)

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Barnes-Hut Algorithm

- "A Hierarchical O(n log n) force calculation algorithm", J. Barnes and P. Hut, Nature, v. 324 (1986), many later papers
- Good for low accuracy calculations:

$$\text{RMS error} = (\Sigma_k \,||\, \text{approx f(k)} - \text{true f(k)} \,||^2 \,/\, ||\, \text{true f(k)} \,||^2 \,/N)^{1/2}$$

$$\sim 1\%$$

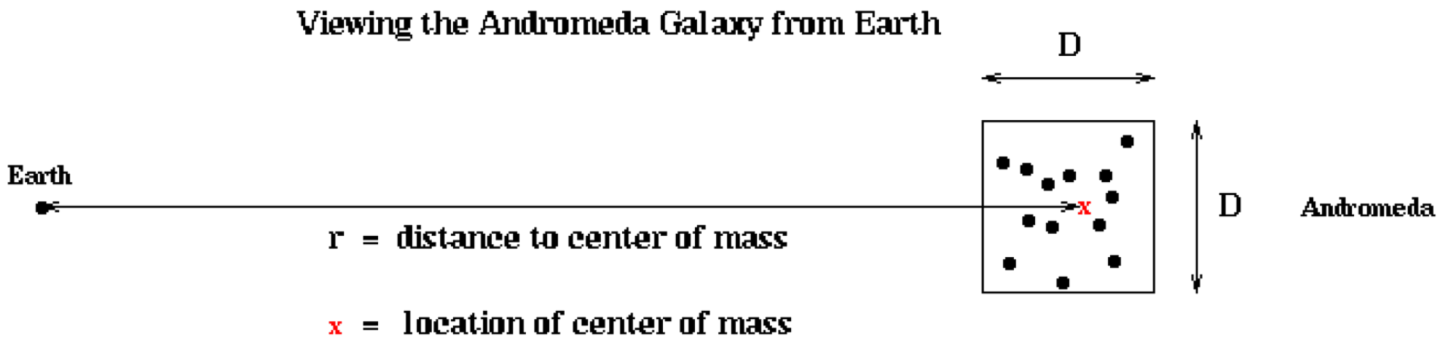
(other measures better if some true f(k) ~ 0)

- High Level Algorithm (in 2D, for simplicity)

1) Build the QuadTree using QuadTreeBuild
   … already described, cost = O(N log N) or O(b N)
2) For each node = subsquare in the QuadTree, compute the CM and total mass (TM) of all the particles it contains
   … "post order traversal" of QuadTree, cost = O(N log N) or O(b N)
   … (code on hidden slide)
3) For each particle, traverse the QuadTree to compute the force on it, using the CM and TM of "distant" subsquares
   … core of algorithm
   … cost depends on accuracy desired but still O(N log N) or O(bN)

Step 3 of BH: compute force on each particle

- For each node = square, can approximate force on particles outside the node due to particles inside node by using the node's CM and TM
- This will be accurate enough if the node if "far away enough" from the particle
- For each particle, use as few nodes as possible to compute force, subject to accuracy constraint
- Need criterion to decide if a node is far enough from a particle
  - D = side length of node
  - r = distance from particle to CM of node
  - θ = user supplied error tolerance < 1
  - Use CM and TM to approximate force of node on box if D/r < θ



Details of Step 3 of BH (analysis on hidden slide)

```
… for each particle, traverse the QuadTree to compute the force on it
for k = 1 to N
   f(k) = TreeForce( k, root )
                … compute force on particle k due to all particles inside root (except k)
endfor
```

```
function f = TreeForce( k, n )
   … compute force on particle k due to all particles inside node n (except k)
   f = 0
   if n contains one particle (not k)  … evaluate directly
      f = force computed using direct formula
   else
      r = distance from particle k to CM of particles in n
      D = size of n
      if  D/r  < θ    … ok to approximate by CM and TM
          compute f approximately using CM and TM
      else            … need to look inside node
          for all children c of n
               f = f + TreeForce ( k, c )
          end for
      end if
  end if
```

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Fast Multiple Method (FMM)

- "A fast algorithm for particle simulation", L. Greengard and V. Rokhlin, J. Comp. Phys. V. 73, 1987, many later papers
  - Many awards
- Differences from Barnes-Hut
  - FMM computes the *potential* at every point, not just the force
  - FMM uses more information in each box than the CM and TM, so it is both more accurate and more expensive
  - In compensation, FMM accesses a fixed set of boxes at every level, independent of D/r
  - BH uses fixed information (CM and TM) in every box, but # boxes increases with accuracy. FMM uses a fixed # boxes, but the amount of information per box increase with accuracy.
- FMM uses two kinds of expansions
  - Outer expansions represent potential outside node due to particles inside, analogous to (CM,TM)
  - Inner expansions represent potential inside node due to particles outside; *Computing this for every leaf node is the computational goal of FMM*
- First review potential, then return to FMM

Gravitational/Electrostatic Potential

- FMM will compute a compact expression for potential $\phi(x,y,z)$ which can be evaluated and/or differentiated at any point
- In 3D with x,y,z coordinates
  - Potential = $\phi(x,y,z) = -1/r = -1/(x^2 + y^2 + z^2)^{1/2}$
  - Force = -grad $\phi(x,y,z)$ = - $(d\phi/dx , d\phi/dy , d\phi/dz)$ = $-(x,y,z)/r^3$
- In 2D with x,y coordinates
  - Potential = $\phi(x,y) = \log r = \log (x^2 + y^2)^{1/2}$
  - Force = -grad $\phi(x,y)$ = - $(d\phi/dx , d\phi/dy)$ = $-(x,y)/r^2$
- In 2D with z = x+iy coordinates, i = sqrt(-1)
  - Potential = $\phi(z) = \log |z| = \text{Real}(\log z)$

    … because $\log z = \log |z|e^{i\theta} = \log |z| + i\theta$
  - Drop Real( ) from calculations, for simplicity
  - Force = $-(x,y)/r^2 = -z / |z|^2$
- Later: Kernel Independent FMM

2D Multipole Expansion (Taylor expansion in 1/z)

$\phi(z)$ = potential due to z_k, $k=1,\ldots,n$

$= \Sigma_k\, m_k * \log |z - z_k|$

$= \text{Real}(\Sigma_k\, m_k * \log (z - z_k))$

… since $\log z = \log |z| e^{i\theta} = \log |z| + i\theta$

… drop Real() from now on

$= \Sigma_k\, m_k * [\log(z) + \log (1 - z_k/z)]$

… how logarithms work

$= M * \log(z) + \Sigma_k\, m_k * \log (1 - z_k/z)$

… where $M = \Sigma_k\, m_k$

$= M * \log(z) - \Sigma_k\, m_k * \Sigma_{e\geq 1} (z_k/z)^e/e$

… Taylor expansion converges if $|z_k/z| < 1$

$= M * \log(z) - \Sigma_{e\geq 1}\, z^{-e}\, \Sigma_k\, m_k\, z_k^e/e$

… swap order of summation

$= M * \log(z) - \Sigma_{e\geq 1}\, z^{-e}\, \alpha_e$

… where $\alpha_e = \Sigma_k\, m_k\, z_k^e/e$ … called Multipole Expansion

Keep leading r terms of sum, error will decrease like $(\max |z_k|/|z|)^{r+1}$

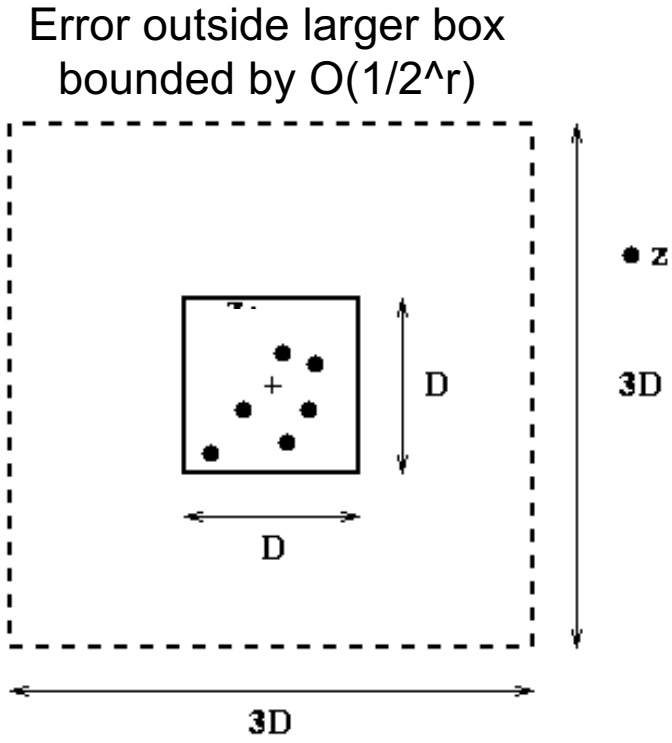
First two terms contain same info as BH (M and $\alpha_1 = \Sigma_k\, m_k\, z_k$)

Outer(n) and Outer Expansion

- Same idea for expansion around node n with center z_n :

  $\phi(z) \sim M * \log(z - z_n) - \Sigma_{r \geq e \geq 1} (z-z_n)^{-e} \alpha_e$

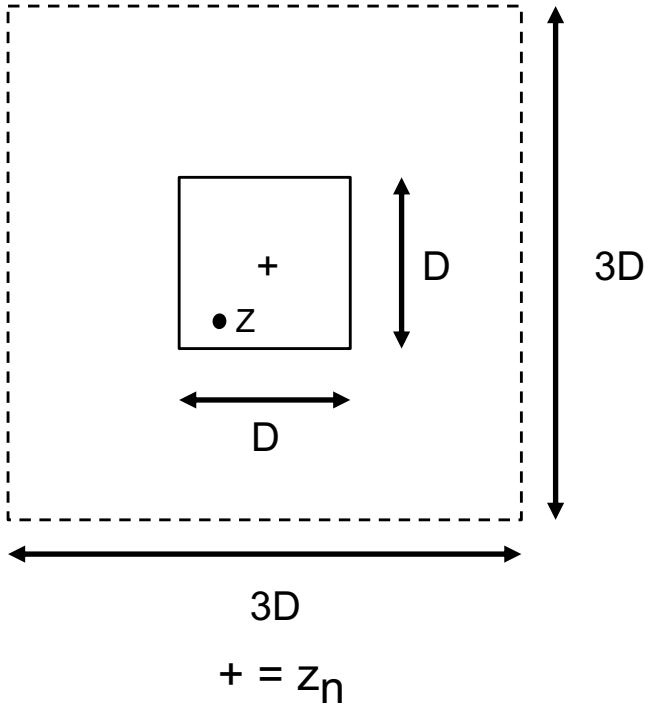
- Outer(n) = ($M, \alpha_1, \alpha_2, \ldots, \alpha_r, z_n$)
  - Stores data for evaluating $\phi(z)$ outside node n due to particles inside n
  - Error small for z outside dotted line
  - Cost of evaluating $\phi(z)$ is O(r), independent of #particles inside n
  - Cost grows linearly with desired number of bits of precision ~r
- Will be computed for each node in QuadTree
- Analogous to (TM,CM) in Barnes-Hut



Inner(n) and Inner Expansion

- Outer(n) used to evaluate potential outside node n due to particles inside n
- Inner(n) will be used to evaluate potential inside node n due to particles outside n

- $\Sigma_{0 \le e \le r} \beta_e * (z - z_n)^e$
- z_n = center of node n, a D-by-D box
- Inner(n) = (β_0 , β_1 , … , β_r , z_n)
- Particles outside n must lie outside 3D-by-3D box centered at z_n



Top Level Description of FMM

(1) Build the QuadTree

➡ *(2) Call Build_Outer(root), to compute outer expansions of each node n in the QuadTree*

… Traverse QuadTree from bottom to top,
… combining outer expansions of children
… to get out outer expansion of parent

(3) Call Build_ Inner(root), to compute inner expansions of each node n in the QuadTree

… Traverse QuadTree from top to bottom,
… converting outer to inner expansions
… and combining them

(4) For each leaf node n, add contributions of nearest particles directly to Inner(n)

… final Inner(n) is desired output: expansion for potential at each point due to all particles

Step 2 of FMM: compute Outer(n) for each node n in QuadTree

… Compute Outer(n) for each node of the QuadTree
outer = Build_Outer(root)

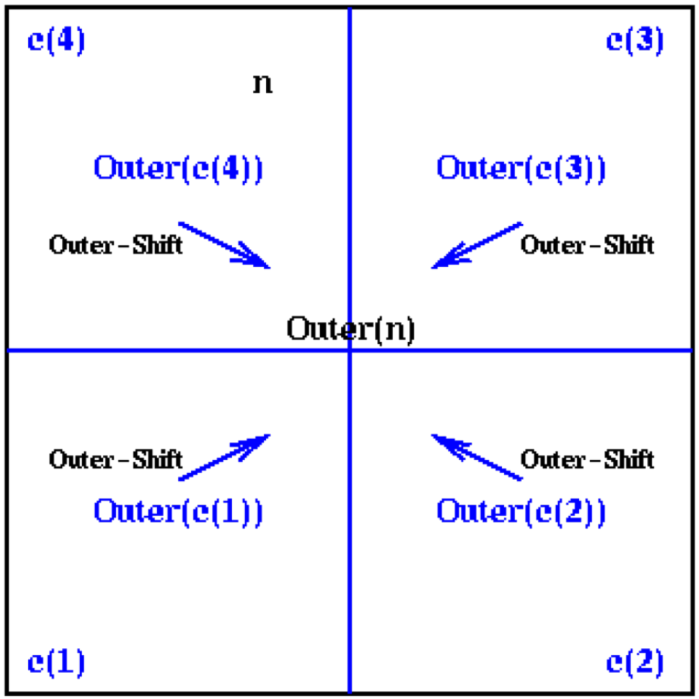
---

function (M, $\alpha_1,\ldots,\alpha_r$, z_n) = Build_Outer(n) … compute outer expansion of node n
  if n if a leaf … it contains 1 (or a few) particles
    compute and return Outer(n) = (M, $\alpha_1,\ldots,\alpha_r$, z_n) directly from
      its definition as a sum
  else … "post order traversal": process parent after all children
    Outer(n) = 0
    for all children c(k) of n … k = 1,2,3,4
      Outer(c(k)) = Build_Outer(c(k))
      Outer(n) = Outer(n) +
        Outer_shift(Outer(c(k)) , center(n))
        … convert expansion around c(k) to
        … expansion around center(n)
        … (details on hidden slides)
        … then add component by component
    endfor
    return Outer(n)
end if

Cost = O(# nodes in QuadTree) = O(N)

same as for Barnes-Hut

Inner Loop of Build_Outer



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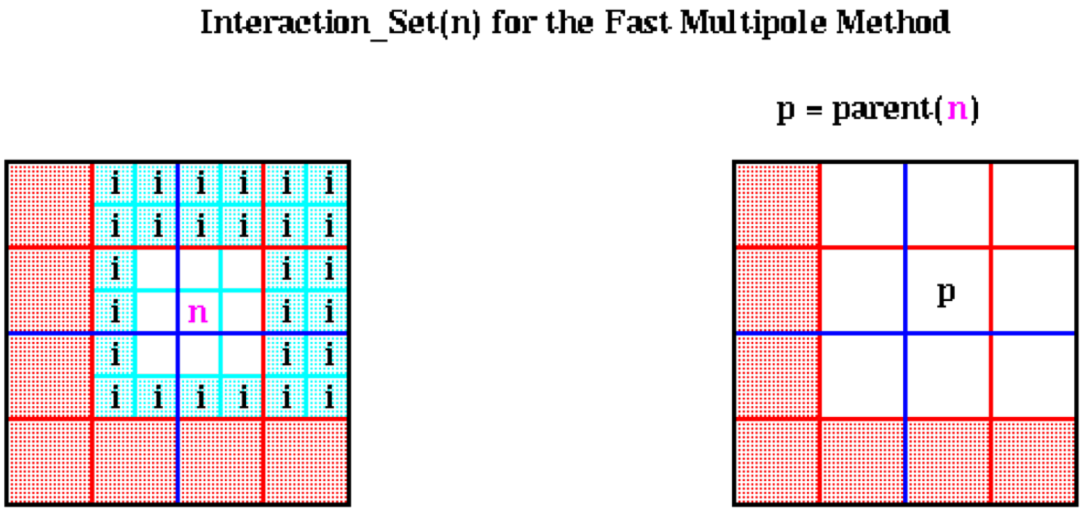
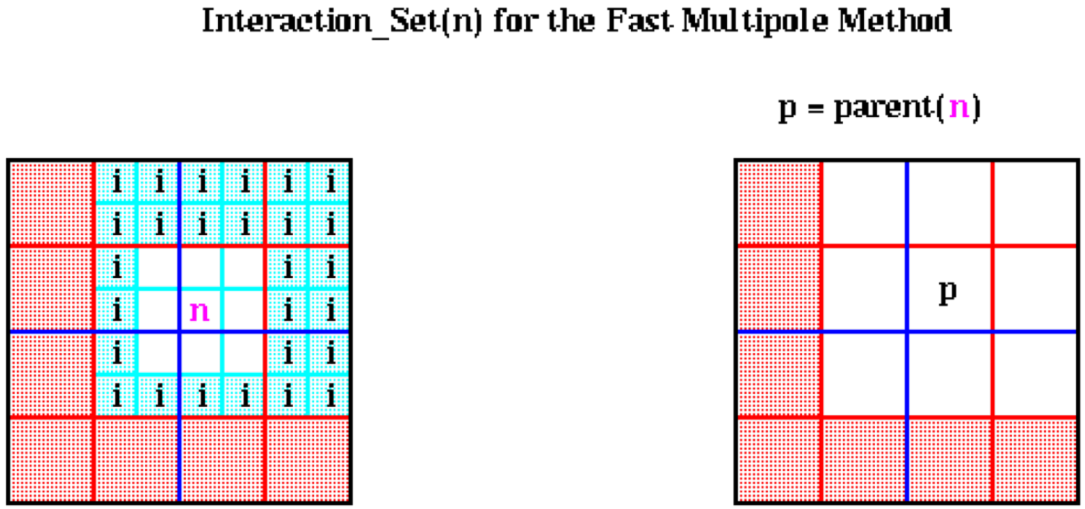
Step 3 of FMM: Computing Inner(n) from other expansions

- **Which other expansions?**
  - As few as necessary to compute the potential accurately
  - Inner expansion of p = parent(n) will account for potential from particles far enough away from parent (red nodes below)
  - Outer expansions will account for potential from particles in boxes at same level in Interaction Set (nodes labeled i below)



Step 3 of FMM: Computing Inner(n) from other expansions

- We will use Inner_shift and ConvertO2I to build each Inner(n) by combining expansions from other nodes
- Which other nodes?
  - As few as necessary to compute the potential accurately
  - Inner_shift(Inner(parent(n)), center(n)) will account for potential from particles far enough away from parent (red nodes below)
  - ConvertO2I(Outer(i), center(n)) will account for potential from particles in boxes at same level in Interaction Set (nodes labeled i below)
  - (details on hidden slides)



Step 3 of FMM: Compute Inner(n) for each n in QuadTree

```
… Compute Inner(n) for each node of the QuadTree
outer = Build_ Inner( root )
```

```
function ( β1,…,βr , zn) = Build_ Inner( n )    … compute inner expansion of node n
    p = parent(n)    … p=nil if n = root
    Inner(n) = Inner_shift( Inner(p), center(n) )      … Inner(n) = 0 if n = root
    for all i in Interaction_Set(n)     …  Interaction_Set(root) is empty
        Inner(n) = Inner(n) + ConvertO2I( Outer(i), center(n) )
                    … add component by component
    end for
    for all children c of n   … complete preorder traversal of QuadTree
        Build_Inner( c )
    end for
```

Cost = O(# nodes in QuadTree)

= O(N)

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➡ *(4) For each leaf node n, add contributions of nearest particles directly into Inner(n)*
… if 1 node/leaf, then each particle accessed once,
… so cost = O(N)
… final Inner(n) is desired output: expansion for potential at each point due to all particles

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Parallelizing Hierachical N-Body codes

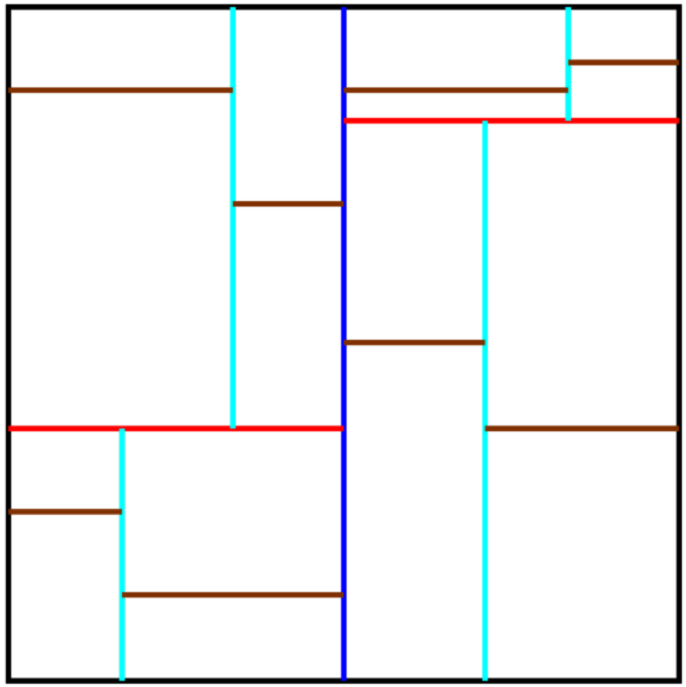
- Barnes-Hut, FMM and related algorithm have similar computational structure:
  1) Build the QuadTree
  2) Traverse QuadTree from leaves to root and build outer expansions (just (TM,CM) for Barnes-Hut)
  3) Traverse QuadTree from root to leaves and build any inner expansions (FMM only)
  4) Traverse QuadTree to accumulate forces for each particle
- One parallelization scheme will work for them all
  - Based on D. Blackston and T. Suel, Supercomputing 97
    - UCB PhD Thesis, David Blackston, "Pbody"
    - Autotuner for N-body codes
  - Assign regions of space to each processor
  - Regions may have different shapes, to get load balance
    - Each region will have about N/p particles
  - Each processor will store part of Quadtree containing all particles (=leaves) in its region, and their ancestors in Quadtree
    - Top of tree stored by all processors, lower nodes may also be shared
  - Each processor will also store adjoining parts of Quadtree needed to compute forces for particles it owns
    - Subset of Quadtree needed by a processor called the Locally Essential Tree (LET)
  - Given the LET, all force accumulations (step 4)) are done in parallel, without communication

Load Balancing Scheme 1: Orthogonal Recursive Bisection (ORB)

- Warren and Salmon, Supercomputing 92
- Recursively split region along axes into regions containing equal numbers of particles
- Works well for 2D, not 3D (available in Pbody)

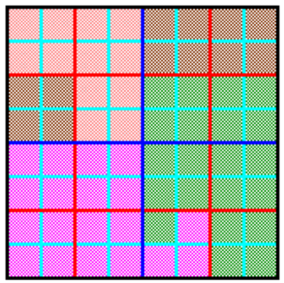
Orthogonal Recursive Bisection

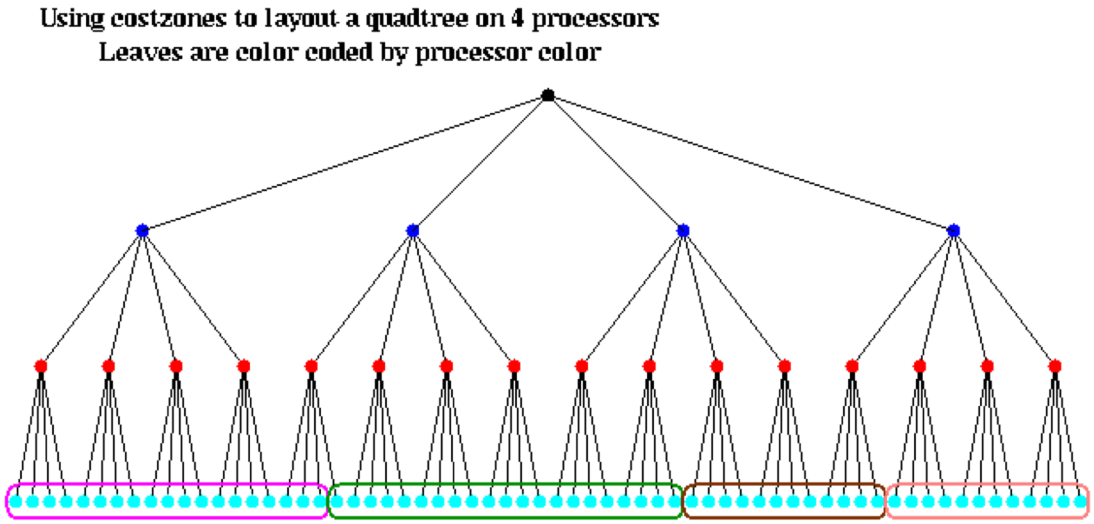
Partitioning for 16 procs:



Load Balancing Scheme 2: Costzones

- Called Costzones for Shared Memory
  - PhD thesis, J.P. Singh, Stanford, 1993
- Called "Hashed Oct Tree" for Distributed Memory
  - Warren and Salmon, Supercomputing 93
- We will use the name Costzones for both; also in Pbody
- Idea: partition QuadTree instead of space
  - Estimate work for each node, call total work W
  - Arrange nodes of QuadTree in some linear order (details on hidden slides)
  - Assign contiguous blocks of nodes with work W/p to processors: locality
  - Works well in 3D





Determining Costzones in Parallel

- Not practical to compute QuadTree, in order to compute Costzones, to then determine how to best build QuadTree
- Random Sampling:
  - All processors send small random sample of their particles to Proc 1
  - Proc 1 builds small Quadtree serially, determines its Costzones, and broadcasts them to all processors
  - Other processors build part of Quadtree they are assigned by these Costzones
- All processors know all Costzones; we need this later to compute LETs
- As particles move, may need to occasionally repeat construction, so should not be too slow

Computing Locally Essential Trees (LETs)

- Warren and Salmon, 1992; Liu and Bhatt, 1994
- Every processor needs a subset of the whole QuadTree, called the LET, to compute the force on all particles it owns
- Shared Memory
  - Receiver driven protocol
  - Each processor reads part of QuadTree it needs from shared memory on demand, keeps it in cache
  - Drawback: cache memory appears to need to grow proportionally to P to remain scalable
- Distributed Memory
  - Sender driven protocol
  - Each processor decides which other processors need parts of its local subset of the Quadtree, and sends these subsets

Locally Essential Trees in Distributed Memory

- How does each processor decide which other processors need parts of its local subset of the Quadtree?

- Barnes-Hut:
  - Let j and k be processors, n a node on processor j; Does k need n?
  - Let D(n) be the side length of n
  - Let r(n) be the shortest distance from n to any point owned by k
  - If either

    (1) $D(n)/r(n) < \theta$ and $D(parent(n))/r(parent(n)) \geq \theta$, or

    (2) $D(n)/r(n) \geq \theta$

    then node n is part of k's LET, and so proc j should send n to k
  - Condition (1) means (TM,CM) of n can be used on proc k, but this is not true of any ancestor
  - Condition (2) means that we need the ancestors of type (1) nodes too

- FMM
  - Simpler rules based just on relative positions in QuadTree (Interaction Set)

Performance Results - 1

- **512 Proc Intel Delta**
  - **Warren and Salmon, Supercomputing 92, Gordon Bell Prize**
  - **8.8 M particles, uniformly distributed**
  - **.1% to 1% RMS error, Barnes-Hut**
  - **114 seconds = 5.8 Gflops**
    - **Decomposing domain 7 secs**
    - **Building the OctTree 7 secs**
    - **Tree Traversal 33 secs**
    - **Communication during traversal 6 secs**
    - **Force evaluation 54 secs**
    - **Load imbalance 7 secs**
  - **Rises to 160 secs as distribution becomes nonuniform**

Performance Results - 2

° Cray T3E, running FMM

- Blackston, 1999
- 10^{-4} RMS error
- Generally 80% efficient on up to 32 processors
- Example: 50K particles, both uniform and nonuniform
  - preliminary results; lots of tuning parameters to set

	Uniform		Nonuniform	
	1 proc	4 procs	1 proc	4 procs
Tree size	2745	2745	5729	5729
MaxDepth	4	4	10	10
Time(secs)	172.4	38.9	14.7	2.4
Speedup		4.4		6.1
Speedup vs $O(n^2)$		>50		>500

° Ultimate goal - portable, tunable code including all useful variants

Performance Results - 3

Optimizing and Tuning the Fast Multipole Method for Multicore and Accelerator Systems

Georgia Tech
– *Aparna Chandramowlishwaran*, Aashay Shringarpure, Ilya Lashuk; George Biros, Richard Vuduc

Lawrence Berkeley National Laboratory
– Sam Williams, Lenny Oliker

- **Presented at IPDPS 2010**
- **Source: Richard Vuduc**

Summary

- **First cross-platform single-node multicore study of tuning the fast multipole method (FMM)**
  - **Explores data structures, SIMD, mixed-precision, multithreading, and tuning**
  - **Show**
    - **25x speedups on Intel Nehalem –**
      - 2-sockets x 4-cores/socket x **2-thr/core** = **16 threads**
    - **9.4x on AMD Barcelona**
      - 2-sockets x 4-cores/socket x **1-thr/core** = **8 threads**
    - **37.6x on Sun Victoria Falls**
      - 2-sockets x 8-cores/socket x **8-thr/core** = **128 threads**

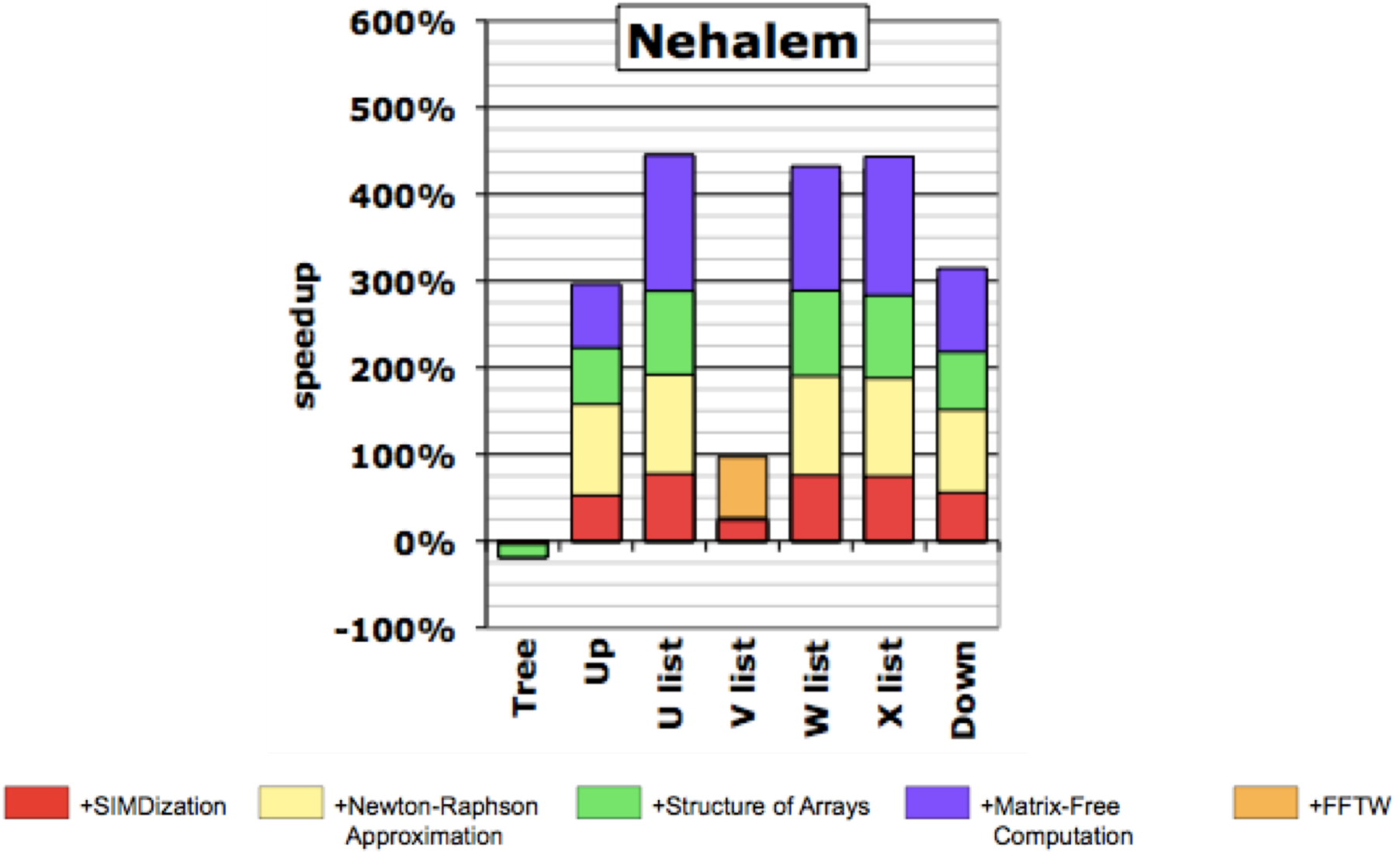
- **Surprise? Multicore ~ GPU in performance & energy efficiency for the FMM**

Optimizations tried (manual and autotuning)

- **Uses KIFMM = Kernel Independent FMM**
  - **Applies to "any" kernel, not just gravity/electrostatics**
  - **Requires subroutine to evaluate kernel, builds own expansions**
    - **Ex: (modified) Laplace, Stokes**
    - **Approximate particles inside square/box by evenly spaced particles on circle/sphere**
  - **FFT used to build expansions; tunable**
- **Single-core, manually coded & tuned**
  - ***Low-level*: SIMD vectorization (x86)**
  - ***Numerical*: `rsqrtps` + Newton-Raphson (x86)**
  - ***Data*: Structure reorg. (transpose or "SOA")**
  - ***Traffic*: Matrix-free via interprocedural loop fusion**
  - **FFTW plan optimization**
- **OpenMP parallelization**
- **Algorithmic tuning of max particles per box, *q***

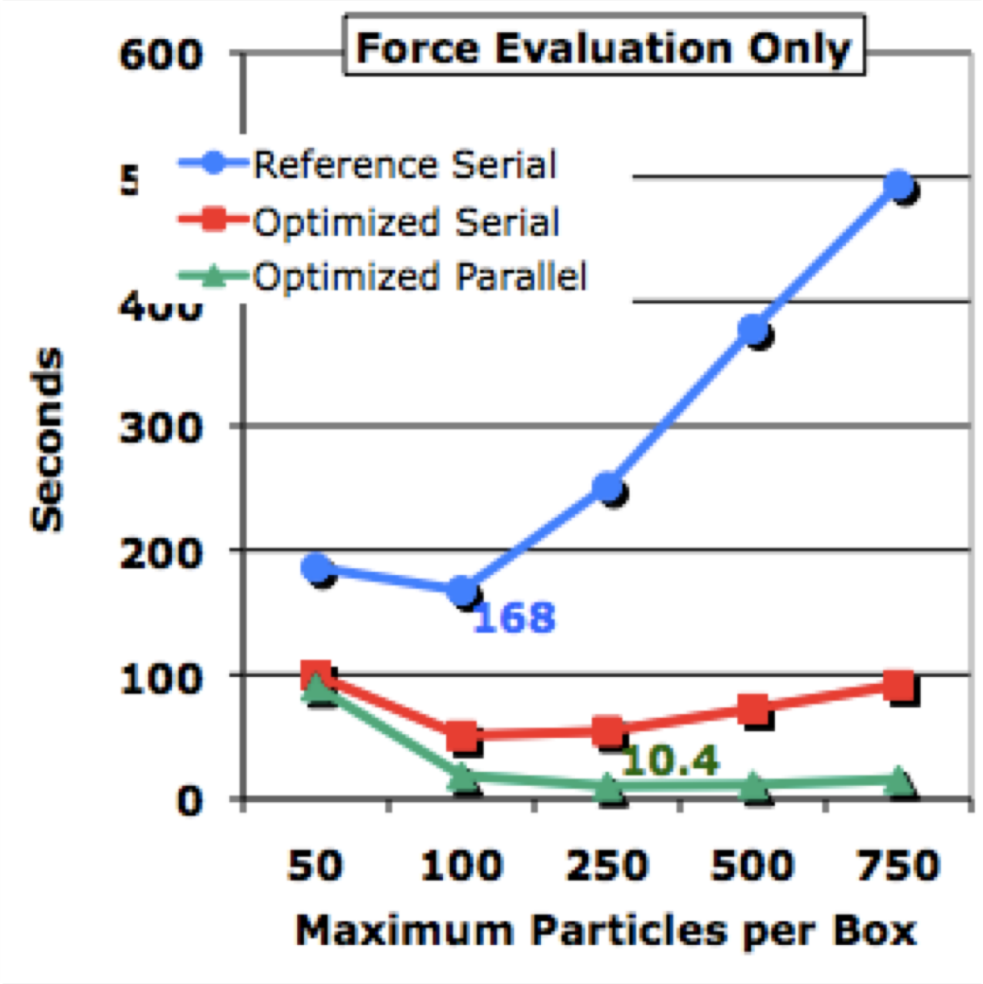
Single-core Optimizations

Double-Precision, Non-uniform (ellipsoidal)



***Reference*: kifmm3d [Ying, Langston, Zorin, Biros]**

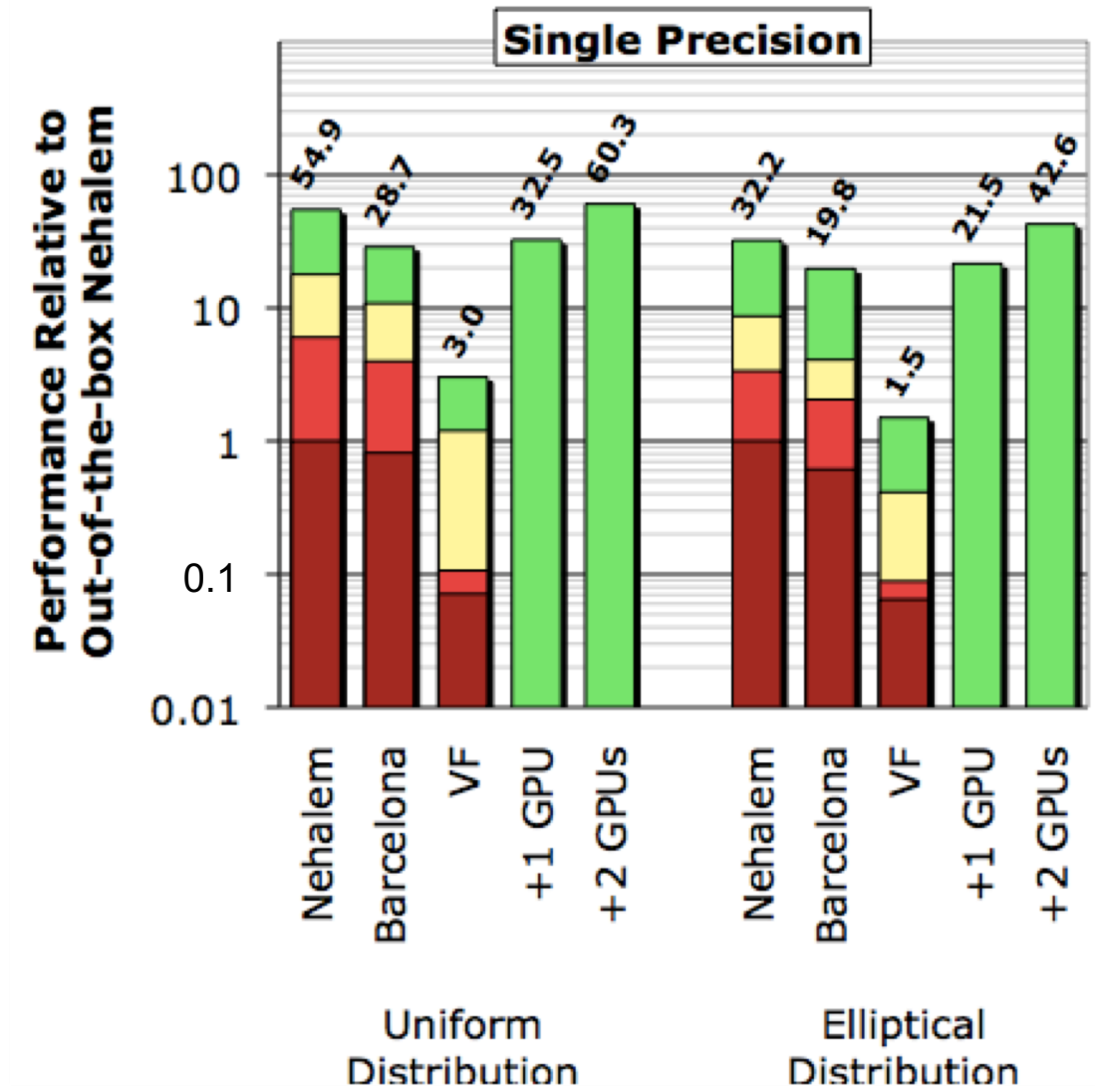
Algorithmic Tuning of *q* = Max pts / box - Nehalem



Shape of curve changes as we introduce optimizations.

Source: Richard Vuduc

Cross-Platform Performance Comparison (Summary)



GPU:
NCSA Lincoln Cluster
NVIDIA T10P +
dual socket Xeon

Nehalem outperforms 1-GPU case, a little slower than 2-GPU case.

Source: Richard Vuduc

Minimizing Communication in N-Body Problem

- **Hierarchical Methods**
  - Reducing arithmetic good for reducing communication too!
  - Deriving communication lower bounds is an open problem
    - Answer is approximate, so lower bound may depend on desired accuracy
    - Lower bound may also depend on particle distribution
    - Open problem (probably hard)
- **Direct methods**
  - Thm: Suppose p processors compute interactions among n particles, using local memories of size M. If each processor does an equal amount of work (n^2/p interactions) then the number of words that a processor must communicate is $\Omega((n^2/p)/M)$, and the number of messages is $\Omega((n^2/p)/M^2)$
  - If not computing all n^2 interactions (eg cutoff distance), replace n^2 by #interactions in Thm
  - Attainable when $n/p <= M <= n/p^{1/2}$
    - $M = n/p$: #words = n, #messages = p
    - $M = n/p^{1/2}$: #words = $n/p^{1/2}$, #messages = O(1) (12x speedups)