矩阵分析与应用 第六次作业

学号： 202128013229021 姓名： 刘炼

Date

7.解，由题意可知：

(1) B_x和B_y是线性无关的，这是X和Y的基。

有 $\text{rank}[X|Y] = \text{rank}\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{array}\right) = 3$，Y空间中的任意值不能在X中线性表出，$X+Y=0$

故 $B_x \cup B_y$是R^3空间的一个基，

有 X和Y是R^3空间中互补子空间

(2) 求 projector onto X along Y.

$$P = [X|0][X|Y]^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 1 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{pmatrix}$$

projector onto Y along X

$$Q = [Y|0][Y|X]^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 3 & 1 & 2 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 0 & -1 & 1 \\ 0 & -2 & 2 \\ 0 & -3 & 3 \end{pmatrix}$$

(3) 根据分析，求得为：$Qv = \begin{pmatrix} 0 & -1 & 1 \\ 0 & -2 & 2 \\ 0 & -3 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$

(4) $$P^2 = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{pmatrix} = P$$

$$Q^2 = \begin{pmatrix} 0 & -1 & 1 \\ 0 & -2 & 2 \\ 0 & -3 & 3 \end{pmatrix} \begin{pmatrix} 0 & -1 & 1 \\ 0 & -2 & 2 \\ 0 & -3 & 3 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 1 \\ 0 & -2 & 2 \\ 0 & -3 & 3 \end{pmatrix} = Q$$

∴ P和Q都是幂等的 (idempotent)

(5) 令 $x_1 = [B_x]_{*1} = \begin{pmatrix}1\\1\\1\end{pmatrix}$ $\quad x_2 = [B_x]_{*2} = \begin{pmatrix}1\\2\\2\end{pmatrix}$

$$y = B_y = \begin{pmatrix}1\\2\\3\end{pmatrix}$$

$$\begin{cases} Px_1 = \begin{pmatrix}1 & 1 & -1\\0 & 3 & -2\\0 & 3 & -2\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix} = x_1 \\ Qx_1 = \begin{pmatrix}0 & -1 & 1\\0 & -2 & 2\\0 & -3 & 3\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \end{cases}$$

$$\begin{cases} Px_2 = \begin{pmatrix}1 & 1 & -1\\0 & 3 & -2\\0 & 3 & -2\end{pmatrix}\begin{pmatrix}1\\2\\2\end{pmatrix} = \begin{pmatrix}1\\2\\2\end{pmatrix} = x_2 \\ Qx_2 = \begin{pmatrix}0 & -1 & 1\\0 & -2 & 2\\0 & -3 & 3\end{pmatrix}\begin{pmatrix}1\\2\\2\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \end{cases}$$

∴ $\cancel{P(X) = X \text{ 且 } N(P) = Y}$

∴ $R(P) = X = N(Q)$

$$\begin{cases} Py_1 = \begin{pmatrix}1 & 1 & -1\\0 & 3 & -2\\0 & 3 & -2\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \\ Qy_1 = \begin{pmatrix}0 & -1 & 1\\0 & -2 & 2\\0 & -3 & 3\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix} = y_1 \end{cases}$$

∴ $N(P) = Y = R(Q)$

∴ 验证如上.

15、解，由题意可知

(a) $R(A) = \text{span}\left\{\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$ $\quad R(A)^{\perp} = N(A^T) = \text{span}\left\{\begin{pmatrix}1\\0\\-1\end{pmatrix}\right\}$

$N(A) = \text{span}\left\{\begin{pmatrix}2\\-1\\0\end{pmatrix}\right\}$ $\quad N(A)^{\perp} = R(A^T) = \text{span}\left\{\begin{pmatrix}1\\2\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$

得到的4个基本子空间如上所示、

(b) 由 $N(A)^{\perp} = R(A^T) = \text{span}\left\{\begin{pmatrix}1\\2\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$

令 $M = \begin{pmatrix}1 & 0\\2 & 0\\0 & 1\end{pmatrix}$

$$\therefore P = M(M^TM)^{-1}M^T$$
$$= \begin{pmatrix}1 & 0\\2 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}\frac{1}{5} & 0\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 2 & 0\\0 & 0 & 1\end{pmatrix}$$
$$= \begin{pmatrix}\frac{1}{5} & \frac{2}{5} & 0\\\frac{2}{5} & \frac{4}{5} & 0\\0 & 0 & 1\end{pmatrix}$$

$\therefore$ 最近的点为 $Pb = \begin{pmatrix}\frac{1}{5} & \frac{2}{5} & 0\\\frac{2}{5} & \frac{4}{5} & 0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}\frac{3}{5}\\\frac{6}{5}\\1\end{pmatrix}$