量子信息与量子密码 第四次作业

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1. 由于ρ为密度算子，可以对其进行谱分解，

$$\rho = \sum_i \lambda_i |i\rangle\langle i|$$

$$\therefore \quad \rho^2 = \sum_i \lambda_i^2 |i\rangle\langle i|$$

可以知，$tr(\rho) = \sum_i \lambda_i$， $tr(\rho^2) = \sum_i \lambda_i^2$

因知 $\sum_i \lambda_i = 1$，$\lambda_i \geq 0$，

∴当且仅当存在一个i时，$\lambda_i = 1$，其它$\lambda_i = 0$时，$tr(\rho^2) = 1$

∴当 $tr(\rho^2) = 1 \Rightarrow \rho$为纯态.

且若ρ为纯态，则 $\rho = |\varphi\rangle\langle\varphi|$，$\Rightarrow tr(\rho^2) = tr(\rho) = 1$.

∴得证.

2. Bell态 $|\Phi^+\rangle$的密度算子为：

$$\rho = \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)\left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)$$

$$= \frac{|00\rangle\langle 00| + |11\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 11|}{2}$$

求第一个量子比特的约化密度矩阵

$$\rho' = tr_2(\rho)$$

$$= \frac{tr(|00\rangle\langle 00| + |11\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 11|)}{2}$$

$$= \frac{|0\rangle\langle 0|\langle 0|0\rangle + |1\rangle\langle 0|\langle 0|1\rangle + |0\rangle\langle 1|\langle 1|0\rangle + |1\rangle\langle 1|\langle 1|1\rangle}{2}$$

$$= \frac{|0\rangle\langle 0| + |1\rangle\langle 1|}{2}$$

$$= \frac{I}{2}$$

Date

3. 假设 $|\varphi\rangle = \alpha|0\rangle+\beta|1\rangle$，下面分析该电路。

① 原系统为：$|P\rangle = |\varphi\rangle|\Phi^+\rangle$

$$= \alpha|0\rangle+\beta|1\rangle\left(\frac{|00\rangle+|11\rangle}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|000\rangle+\alpha|011\rangle+\beta|100\rangle+\beta|111\rangle\right)$$

~~② 经过H门之后：~~

$$\cancel{H|\varphi\rangle = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{1}{\sqrt{2}}(\alpha+\beta)|0\rangle+\frac{1}{\sqrt{2}}(\alpha-\beta)|1\rangle}$$

② 经过CNOT门之后。

$$CNOT_{12}|P\rangle = \alpha|0\rangle|\Phi^+\rangle + \beta|1\rangle\left(\frac{|10\rangle+|01\rangle}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|000\rangle+\alpha|011\rangle+\beta|110\rangle+\beta|101\rangle\right)$$

③ 经过H门之后

$$H_1|P\rangle = H\alpha|0\rangle|\Phi^+\rangle + H\beta|1\rangle\left(\frac{|10\rangle+|01\rangle}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\alpha(|0\rangle+|1\rangle)(|00\rangle+|11\rangle)+\frac{1}{2}\beta(|0\rangle-|1\rangle)(|10\rangle+|01\rangle)$$

$$= \frac{1}{2}\left[\alpha(|0\rangle+|1\rangle)(|00\rangle+|11\rangle)+\beta(|0\rangle-|1\rangle)(|10\rangle+|01\rangle)\right.$$

④ 经过受控X门之后：

$$X_{23}|P\rangle = \frac{1}{2}\left[\alpha(|0\rangle+|1\rangle)(|00\rangle+|10\rangle)+\beta(|0\rangle-|1\rangle)(|11\rangle+|01\rangle)\right.$$

⑤ 经过受控Z门之后：

$$Z_{13}|P\rangle = \frac{1}{2}\left[\alpha(|0\rangle+|1\rangle)(|00\rangle+|10\rangle)+\beta(|0\rangle+|1\rangle)(|11\rangle+|01\rangle)\right.$$

$$= \frac{1}{2}(|0\rangle+|1\rangle)(|0\rangle+|1\rangle)(\alpha|0\rangle+\beta|1\rangle)$$

∴ 可以证明得到，Bob端所得到的结果为 $\alpha|0\rangle+\beta|1\rangle = |\varphi\rangle$

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Date

4. 解，由题意知，

$$\rho_{AB} = \frac{1}{8} I + \frac{1}{2}|\psi^-\rangle\langle\psi^-|$$

$$= \frac{1}{8}\left(|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|\right) +$$

$$\frac{1}{4}\left(|01\rangle\langle 01| - |01\rangle\langle 10| - |10\rangle\langle 01| + |10\rangle\langle 10|\right)$$

$$= \frac{1}{8}|00\rangle\langle 00| + \frac{1}{8}|11\rangle\langle 11| + \frac{3}{8}|01\rangle\langle 01| + \frac{3}{8}|10\rangle\langle 10|$$

$$-\frac{1}{4}|01\rangle\langle 10| - \frac{1}{4}|10\rangle\langle 01|$$

$$= \frac{1}{8}|00\rangle\langle 00| + \frac{1}{8}|11\rangle\langle 11| + \frac{1}{16}(|01\rangle + |10\rangle)(|01\rangle + |10\rangle)$$

$$+ \frac{5}{16}(|01\rangle - |10\rangle)(|01\rangle - |10\rangle)$$

$\therefore$ ρ_{AB}的谱表示为

$$\rho_{AB} = \frac{1}{8}|00\rangle\langle 00| + \frac{1}{8}|11\rangle\langle 11| + \frac{1}{16}(|01\rangle + |10\rangle)(|01| + |10\rangle)$$

$$+ \frac{5}{16}(|01\rangle - |10\rangle)(|01\rangle - |10\rangle)$$

5. 根据基本知识知，$HW^T=0$，$W=AG$

$\Rightarrow$ $H(AG)^T=0$ 对任意 $A\in B^m$ 满足

则 $HG^T=0$，已知 H，则可分别求得生成矩阵 G.

(1) 知 $H_3=\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$

$\cancel{\Rightarrow G=\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}}$

$\Rightarrow G=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$

易知，该编码为 hamming 码，即为完全码，

$\therefore$ 能纠正一位错误.

(2) 类似可得到

$$G=\begin{pmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & & & & \vdots & 1 & 0 & [illegible] & 0 \\ 0 & & 1 & & & \vdots & 1 & 0 & 0 & 1 \\ 0 & & & \ddots & 0 & \vdots & 0 & 1 & 1 & 0 \\ 0 & & & & & \vdots & 0 & 1 & 0 & 1 \\ 0 & & & & & 0 & 0 & 0 & 1 & 1 \\ 0 & & 0 & & \ddots & \vdots & 1 & 1 & 1 & 0 \\ 0 & & & & & \vdots & 1 & 1 & 0 & 1 \\ 0 & & & & & \vdots & 1 & 0 & 1 & 1 \\ 0 & & & & \ddots & \vdots & 0 & 1 & 1 & 1 \\ 0 & \cdots & 0 & \cdots & & 1 & 1 & 1 & 1 & 1 \end{pmatrix}_{11\times 15}$$

且可知，该编码同样为 hamming 码，

故能纠正一位错误.