

091M4041H - Assignment 2

Algorithm Design and Analysis

November 1, 2019

Notice:

1. Please submit your answer in hard copy AND submit a digital version to UCAS website <http://sep.ucas.ac.cn>.
2. Hard copy should be submitted before 9 am. November 11 and digital version should be submitted before 11 pm. November 11.
3. You can choose **three** from problems 1-5, and you should do at least the following things:
 - (a) Describe the optimal substructure and DP equation;
 - (b) Describe your algorithm in daily language or pseudo-code;
 - (c) Prove the correctness of your algorithm;
 - (d) Analyse the complexity of your algorithm.
4. You should finish problems 6-7 on Universal Online Judge before 11 pm. November 2.

1 Money Robbing

A robber is planning to rob houses along a street. Each house has a certain amount of money stashed, the only constraint stopping you from robbing each of them is that adjacent houses have security system connected and it will automatically contact the police if two adjacent houses were broken into on the same night.

1. Given a list of non-negative integers representing the amount of money of each house, determine the maximum amount of money you can rob tonight without alerting the police.
2. What if all houses are arranged in a circle?

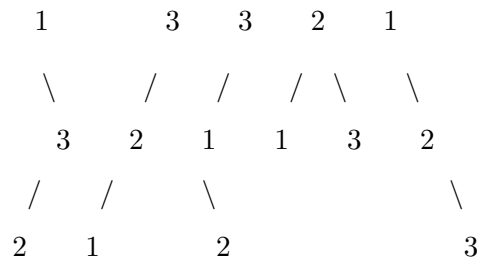
2 Node Selection

You are given a binary tree, and each node in the tree has a positive integer weight. If you select a node, then its children and parent nodes cannot be selected. Your task is to find a set of nodes, which has the maximum sum of weight.

3 Unique Binary Search Trees

Given n , how many structurally unique BST's (binary search trees) that store values $1...n$?

Note: Given $n = 3$, there are a total of 5 unique BST's:



4 Coin Change

You are given coins of different denominations and a total amount of money amount. Write a function to compute **the total number of ways** to make up that amount using some of those coins.

Note: You may assume that you have an infinite number of each kind of coin.

5 Maximum Profit of Transactions

You have an array for which the i -th element is the price of a given stock on day i .

Design an algorithm and implement it to find the maximum profit. You may complete at most two transactions.

Note: You may not engage in multiple transactions at the same time (ie, you must sell the stock before you buy again), and one transaction includes buying and selling.

6 Longest Common Subsequence

Given two sequences, find the length of the longest common subsequence(LCS)

Note: A subsequence is different from a substring, for the former need **NOT** be consecutive of the original sequence. For example, for “*ABCD*” and “*AEFC*”, the longest common subsequence is “*AC*”, and the length of it is 2.

7 Maximum Number of Users

There is a server which has the disk space of M and the memory of N . Given some tasks, the i -th task needs the disk space of X_i and the memory of Y_i , and it can serve U_i users. Design an algorithm and implement it to find out the maximum number of users that this server can serve simultaneously.