081203M04001H - Algorithm Design and Analysis

Assignment 4 - Linear Programming

November 24, 2021

Notice:

1. Please submit your answer in hard copy **AND** submit a digital version to UCAS website `http://sep.ucas.ac.cn`.
2. Hard copy should be submitted before 9 am. December 3 and digital version should be submitted before 11:30 pm. December 3.
3. You should finish **all the five problems**.
4. **Integer Linear Programming (ILP)** is different from the classic Linear Programming that some extra constraints such as

$$x_i \text{ is an integer, } \forall i = 1, 2, \cdots, n \tag{1}$$

   or

$$x_i \in 0, 1 \ \forall i = 1, 2, \cdots, n \tag{2}$$

   are added.
5. When you give the formulation of an LP or ILP, you should explain all mathematical symbols you are using if not appearing in the problem, and interpret the constraints if necessary.

1 Road Lighting Problem

Consider a road divided into n segments that is illuminated by m lamps. Let p_j be the power of the jth lamp and c is the cost to operate a single lamp per hour. The illumination I_i of the ith segment is assumed to be $\sum_{j=1}^{m} a_{ij} p_j$, where a_{ij} are known coefficients. Let I_i^* be the desired illumination of segment i. In order to prolong the life of the lamps, each lamp needs to be turned off for one hour within 24 hours. Assume that the start and end time for the lamp to remain off are integer numbers of hours.

We need to **choosing the lamp powers p_j and determining the turning off time for all lamps** so that the illuminations I_i meet the desired illuminations I_i^*. Formulate this problem as an ILP so as to minimize the cost in operating all lamps.

2 Profit Maximization

Your factory produces three kinds of product: A, B and C. All of them need two kinds of raw materials: nickel and aluminum. The profit and cost of each kind of product are shown in the following table.

Product	Profit($)	Nickel(kg)	Aluminum(kg)
A	10	3	4
B	8	3	3
C	16	2	7

You only have 200 kg of nickle and 300 kg of aluminum in stock. How to arrange production to maximize profits? Please formulate this problem as a LP and transform it into dual form. Then you may solve both primal and dual problems using GLPK or Gurobi or other similar tools.

3 Cutting Paper Minimization

Your factory has expanded its bussiness. Suppose you have an unlimited number of large rolls of paper, of width W meters per roll (W is a positive integer). However, different m customers demands are for smaller width of paper; in particular, customer i needs b_i rolls of paper of width w_i, $i = 1, 2, ..., m$. We assume that $w_i \leq W$ for each i, and each w_i is an integer. Smaller rolls are obtained by slicing a large roll in a certain way. You can slice one roll of paper for different customers only if their total width does not exceed W.

The goal of you is to minimize the number of large rolls used while satisfying customer demand. **Please formulate this problem as an ILP.** Assume that there is no cost for slicing.

4 Reformulation Problems with Absolute Values

Consider the problem:

$$\begin{aligned} \text{minimize} \quad & 2\,|x_1| + x_2 \\ \text{subject to} \quad & x_1 + x_2 \geq 4 \end{aligned} \tag{3}$$

Please reformulate this problem as a LP without absolute values.

5 Cook Recruitment for UCAS Canteen

Suppose that you are the canteen manager of UCAS and you are asked to recruit a group of cooks for improving the quality of meals. It is estimated that there are N stalls need to change cooks, and the $i(th)$ stall needs at least n_i cooks. The number of recruitment firms is F. Cooks from the $j(th)$ recruitment firm can cook different foods in several stalls S_j and the recruitment fee for one cook from the $j(th)$ recruitment firm is c_j. Note that S_j is a subset of $N = \{1, 2, \cdots, n\}$ and the union of S_j equals to N.

Your boss wants you to save money so your need to formulate this problem as an ILP and your goal is minimizing the recruitment fee of enough cooks.