量子信息与量子密码 第二次作业

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6. 解，计算可得：

(1) $\rho = |\varphi\rangle\langle\varphi| = \begin{pmatrix} e^{i\phi}\cos\theta \\ \sin\theta \end{pmatrix} (e^{-i\phi}\cos\theta, \sin\theta)$

$$= \begin{pmatrix} \cos^2\theta & e^{i\phi}\sin\theta\cos\theta \\ e^{-i\phi}\sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$$

∴(2) 根据(1)的计算，$tr(\rho) = \cos^2\theta + \sin^2\theta = 1$

(3) $\langle\varphi|\varphi\rangle = (e^{-i\phi}\cos\theta \quad \sin\theta) \begin{pmatrix} e^{i\phi}\cos\theta \\ \sin\theta \end{pmatrix} = \cos^2\theta + \sin^2\theta = 1$

$\therefore \rho^2 = |\varphi\rangle\langle\varphi|\varphi\rangle\langle\varphi| = |\varphi\rangle\langle\varphi| = \rho = \begin{pmatrix} \cos^2\theta & e^{i\phi}\cos\theta\sin\theta \\ e^{-i\phi}\sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$

7. 受控非门的矩阵表示为：

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

设 $|c\rangle = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ $|t\rangle = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$

则 $|c\rangle|t\rangle = \begin{pmatrix} a_1a_2 \\ a_1b_2 \\ b_1a_2 \\ b_1b_2 \end{pmatrix}$ 而 $|c\rangle|t\oplus c\rangle = \begin{pmatrix} a_1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} + \begin{pmatrix} 0 \\ b_1 \end{pmatrix} \otimes \begin{pmatrix} b_2 \\ a_2 \end{pmatrix} = \begin{pmatrix} a_1a_2 \\ a_1b_2 \\ b_1b_2 \\ b_1a_2 \end{pmatrix}$

而 $CNOT \cdot |c\rangle|t\rangle$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a_1a_2 \\ a_1b_2 \\ b_1a_2 \\ b_1b_2 \end{pmatrix} = \begin{pmatrix} a_1a_2 \\ a_1b_2 \\ b_1b_2 \\ b_1a_2 \end{pmatrix} = |c\rangle|t\oplus c\rangle$$

故得证。

8、解，根据泡利矩阵和 Hadamard 变换定义标准，有：

$$X=\begin{pmatrix}0&1\\1&0\end{pmatrix}\quad Y=\begin{pmatrix}0&-i\\i&0\end{pmatrix}\quad Z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}\quad H=\begin{pmatrix}\frac{1}{\sqrt2}&\frac{1}{\sqrt2}\\\frac{1}{\sqrt2}&-\frac{1}{\sqrt2}\end{pmatrix}$$

∴ ① $HXH=\frac12\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$

$$=\frac12\begin{pmatrix}1&1\\-1&1\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}=\frac12\begin{pmatrix}2&0\\0&-2\end{pmatrix}=Z$$

② $HYH=\frac12\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$

$$=\frac12\begin{pmatrix}i&-i\\-i&-i\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}=\frac12\begin{pmatrix}0&2i\\-2i&0\end{pmatrix}=-Y$$

③ $HZH=HHXHH=X$

∵ H 为 Hadamard 变换为酉变换，∴ $HH^{\dagger}=I$，且 H 为厄米阵

∴ $HZH=X$

9. $(i[A,B])^{\dagger}=-i[A,B]^{\dagger}=-i[(AB)^{\dagger}-(BA)^{\dagger}]$

$$=-i[B^{\dagger}A^{\dagger}-A^{\dagger}B^{\dagger}]$$

$$=-i\cdot-[A,B]$$

$$=i[A,B]$$

∴ $i[A,B]$ 是厄米的.

Date

10. 根据单量子状态描述，通常可以将一个单量子比特混合态描述为题6的形式，即 $|\psi\rangle = e^{i\phi}\cos\theta|0\rangle + \sin\theta|1\rangle$

$$= \begin{pmatrix} e^{i\phi}\cos\theta \\ \sin\theta \end{pmatrix}$$

$$\therefore \rho = \begin{pmatrix} \cos^2\theta & e^{i\phi}\sin\theta\cos\theta \\ e^{-i\phi}\sin\theta\cos\theta & \sin^2\theta \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1+\cos2\theta & e^{i\phi}\sin2\theta \\ e^{-i\phi}\sin2\theta & 1-\cos2\theta \end{pmatrix}$$

假设 $\vec{r} = (\alpha, \beta, \gamma)$

那么，$\frac{I+\vec{r}\cdot\vec{\sigma}}{2} = \frac{1}{2}\cdot[I+\alpha\cdot\vec{\sigma}_x + \beta\vec{\sigma}_y + \gamma\cdot\vec{\sigma}_z]$

$$= \frac{1}{2}\begin{pmatrix} 1+\gamma & \alpha-\beta i \\ \alpha+\beta i & 1-\gamma \end{pmatrix}$$

故，对应的，令：$\begin{cases} \alpha = \sin2\theta\cos\phi \\ \beta = -\sin2\theta\sin\phi \\ \gamma = \cos2\theta \end{cases}$

因有 $||\vec{r}|| = (\alpha^2+\beta^2+\gamma^2) = 1 \leq 1$

∴ 得证