量子信息与量子密码 第五次作业

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22. 令 $r_x = p_x - q_x$，令 U 为全集.

则 $\max\limits_S |p(S) - q(S)| = \max\limits_S \left|\sum\limits_{x\in S} p_x - \sum\limits_{x\in S} q_x\right|$

$= \max\limits_S \left|\sum\limits_{x\in S} (p_x - q_x)\right|$

$= \max\limits_S \left|\sum\limits_{x\in S} r_x\right|$

可将 r_x 中的值分为两部分，为 $r_x \geq 0$ 和 $r_x < 0$

即 $\sum\limits_{x\in S} r_x = \sum\limits_{\substack{x\in S \\ r_x \geq 0}} r_x + \sum\limits_{\substack{x\in S \\ r_x < 0}} r_x = 0$

$\Rightarrow \sum\limits_{x\in S_+} r_x = -\sum\limits_{x\in S_-} r_x = \max\limits_S \left|\sum\limits_{x\in S} r_x\right|$.

则 $D(p_x, q_x) = \frac{1}{2}\sum\limits_{x\in U} |p_x - q_x|$

$= \frac{1}{2}\sum\limits_{x\in U} |r_x|$

$= \frac{1}{2}\sum\limits_{x\in S^+} |r_x| + \frac{1}{2}\sum\limits_{x\in S_-} |r_x|$

$= \frac{1}{2}\sum\limits_{x\in S^+} r_x - \frac{1}{2}\sum\limits_{x\in S_-} r_x$

$= \sum\limits_{x\in S^+} r_x$

$= \max\limits_S \left|\sum\limits_{x\in S} r_x\right|$.

且 $\max\limits_S \left|\sum\limits_{x\in S} r_x\right| = \max\limits_S \sum\limits_{x\in S} r_x$

$\Rightarrow D(p_x, q_x) = \max\limits_S (p(S) - q(S)) = \max\limits_S \left(\sum\limits_{x\in S} p_x - \sum\limits_{x\in S} q_x\right.$

∴ 得证.

23. 证明:

根据保真度的基本定义，有 $F(|\varphi\rangle, \sigma) = \sqrt{\langle\varphi|\sigma|\varphi\rangle}$

$\therefore 1 - F^2(|\varphi\rangle, \sigma) = 1 - \langle\varphi|\sigma|\varphi\rangle$.

而根据迹距离的基本内容，可知:

$$\begin{aligned}
D(|\varphi\rangle, \sigma) &= \max_P Tr(P(\rho-\sigma)) \\
&\geq Tr(|\varphi\rangle\langle\varphi|(\rho-\sigma)) \\
&= \langle\varphi|(|\varphi\rangle\langle\varphi| - \sigma)|\varphi\rangle \\
&= 1 - \langle\varphi|\sigma|\varphi\rangle \\
&= 1 - F^2(|\varphi\rangle, \sigma)
\end{aligned}$$

$\therefore$ 得证.

Date

25. 令 $\rho = \frac{3}{4}|0\rangle\langle 0| + \frac{1}{4}|1\rangle\langle 1|$, $\sigma = \frac{2}{3}|0\rangle\langle 0| + \frac{1}{3}|1\rangle\langle 1|$

$$\begin{aligned}
D(\rho, \sigma) &= \frac{1}{2} Tr|\rho - \sigma| \\
&= D\left(\left(\frac{3}{4}, \frac{1}{4}\right), \left(\frac{2}{3}, \frac{1}{3}\right)\right) \\
&= \frac{1}{2}\left(\left|\frac{3}{4} - \frac{2}{3}\right| + \left|\frac{1}{4} - \frac{1}{3}\right|\right) \\
&= \frac{1}{2}\left(\frac{1}{12} + \frac{1}{12}\right) \\
&= \frac{1}{12}
\end{aligned}$$

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24. 证明如下:

$$\begin{aligned}
D(U\rho U^+, U\sigma U^+) &= \frac{1}{2} tr|U(\rho-\sigma)U^+| \\
&= \frac{1}{2} tr\sqrt{U(\rho-\sigma)^+U^+U(\rho-\sigma)U^+} \\
&= \frac{1}{2} tr\sqrt{U(\rho-\sigma)^+(\rho-\sigma)U^+} \\
&= \frac{1}{2} tr(U\sqrt{(\rho-\sigma)^+(\rho-\sigma)}U^+) \\
&= \frac{1}{2} tr(U|\rho-\sigma|U^+) \\
&= \frac{1}{2} tr|\rho-\sigma| \\
&= D(\rho, \sigma)
\end{aligned}$$

$\therefore$ 得证.

29、证明如下：

(1) 知 A 的特征值，则可分解为：

$$A = T^{-1}\begin{pmatrix}\lambda_1 & & \\ & \ddots & \\ & & \lambda_n\end{pmatrix} T$$

故 $\|A\| = \max\limits_{\langle u|u\rangle=1} |\langle u|A|u\rangle|$

$$= \max_{\langle u|u\rangle=1} \left|\langle u|T^{-1}\begin{pmatrix}\lambda_1 & & \\ & \ddots & \\ & & \lambda_n\end{pmatrix}T|u\rangle\right|$$

将 $|u\rangle$ 分解为：$|u\rangle = \sum_i a_i|x_i\rangle$，其中 $|x_i\rangle$ 为 T 的第 i 行；$\sum_i a_i^2=1$

将 $\langle u|$ 分解为：$\langle u| = \sum_i b_i\langle y_i|$，其中 $\langle y_i|$ 为 T^{-1} 的第 i 列；$\sum_i b_i^2=1$

则 $T|u\rangle = \begin{pmatrix}a_1\\ \vdots \\ a_n\end{pmatrix}$　　$\langle u|T^{-1} = (b_1, \dots, b_n)$

∴ $\|A\| = \max \sum_i |a_i\lambda_i b_i|$ ，$\sum_i a_i^2=1$，$\sum_i b_i^2=1$

故 $\|A\| \geq \max \sum_i a_i^2|\lambda_i| = \lambda$.

得证

(2) 知 $A^+=A$，则 A 为正规矩阵，$A = T^+\begin{pmatrix}\lambda_1 & & \\ & \ddots & \\ & & \lambda_n\end{pmatrix}T$.

∴ 上述式中 $a_i = b_i$

则 $\|A\| = \max \sum_i a_i^2|\lambda_i|$

$= \max|\lambda_i|$

$= \lambda$.

30. 由于 $|AB\rangle$ 是一个纯态，

则 $S(A,B)=0$

∴ $S(B|A) = -S(A) \leq 0$

且仅当 A 为纯态时，$S(A)=0$

故：

充分性 ① 若 $|AB\rangle$ 为纠缠态，则 $S(A)\neq 0 \Rightarrow S(B|A) = -S(A)<0$

必要性 ② 若 $S(B|A)<0$，$\Rightarrow S(A)<0 \Rightarrow S(A)\neq 0$，∴ $|AB\rangle$ 为纠缠态.