# 量子信息与量子密码 第七次作业

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41. 由于混合态中，是均匀分布制备

∴ 混合态密度矩阵为

$$\rho_{mix} = \sum_{i=1}^{3} p_i |\psi_i\rangle\langle\psi_i|$$

$$= \frac{1}{3}|\psi_1\rangle\langle\psi_1| + \frac{1}{3}|\psi_2\rangle\langle\psi_2| + \frac{1}{3}|\psi_3\rangle\langle\psi_3|$$

$$= \frac{1}{3}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} \frac{1}{4} & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{3}{4} \end{pmatrix} + \frac{1}{3}\begin{pmatrix} \frac{1}{4} & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{3}{4} \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 1+\frac{1}{4}+\frac{1}{4} & 0 \\ 0 & \frac{3}{4}+\frac{3}{4} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

43. 定义 $\rho = \frac{3}{4}|0\rangle\langle 0| + \frac{1}{4}|1\rangle\langle 1|$，$\sigma = \frac{2}{3}|+\rangle\langle +| + \frac{1}{3}|-\rangle\langle -|$

其中 $|+\rangle\langle +| = \frac{1}{2}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|)$

$|-\rangle\langle -| = \frac{1}{2}(|0\rangle\langle 0| - |0\rangle\langle 1| - |1\rangle\langle 0| + |1\rangle\langle 1|)$

$\therefore \rho - \sigma = (\frac{3}{4} - \frac{1}{2})|0\rangle\langle 0| - \frac{1}{6}(|0\rangle\langle 1| + |1\rangle\langle 0|) + (\frac{1}{4} - \frac{1}{2})|1\rangle\langle 1|$

$= \frac{1}{4}|0\rangle\langle 0| - \frac{1}{6}(|0\rangle\langle 1| + |1\rangle\langle 0|) - \frac{1}{4}|1\rangle\langle 1|$

而 $(\rho-\sigma)^{+}(\rho-\sigma) = \frac{1}{4^2}|0\rangle\langle 0| - \frac{1}{4\cdot 6}|0\rangle\langle 1| + \frac{1}{6^2}|0\rangle\langle 0| + \frac{1}{24}|0\rangle\langle 1| - \frac{1}{24}|1\rangle\langle 0|$

$+ \frac{1}{36}|1\rangle\langle 0| + \frac{1}{16}|1\rangle\langle 1|$

$= (\frac{1}{16} + \frac{1}{36})|0\rangle\langle 0| + |1\rangle\langle 1|$

$\therefore D(\rho,\sigma) = \frac{1}{2}Tr|\rho - \sigma|$

$= \sqrt{\frac{1}{16} + \frac{1}{36}}$

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44. $\rho = p|0\rangle\langle 0| + (1-p)|+\rangle\langle +|$

$\Rightarrow \rho = \begin{pmatrix} \frac{1+p}{2} & \frac{1-p}{2} \\ \frac{1-p}{2} & \frac{1-p}{2} \end{pmatrix}$

可以根据矩阵求得特征值为 $\lambda = \frac{1}{2}(1 \pm \sqrt{1-2p})$

有 $S(\rho) = -(p'\log p' + (1-p')\log(1-p'))$

其中 $p' = \frac{1}{2}(1+\sqrt{1-2p})$

$\Rightarrow S(\rho) = -\left[\frac{1}{2}(1+\sqrt{1-2p})\log\frac{1}{2}(1+\sqrt{1-2p}) + \frac{1}{2}(1-\sqrt{1-2p})\log\frac{1}{2}(1-\sqrt{1-2p}\right.$

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45. 根据相应的计算公式.

$F(\rho, \varepsilon) = \sum_i |tr(E_i\rho)|^2.$

$= |tr(E_1\rho)|^2 + |tr(E_2\rho)|^2.$

知 $\rho = \begin{pmatrix} \frac{1+p}{2} & \frac{1-p}{2} \\ \frac{1-p}{2} & \frac{1-p}{2} \end{pmatrix}$ 代入可得

$E_1\rho = \sqrt{p}\begin{pmatrix} \frac{1+p}{2} & \frac{1-p}{2} \\ \frac{1-p}{2} & \frac{1-p}{2} \end{pmatrix} \Rightarrow tr(E_1\rho) = \sqrt{p}\left(\frac{1+p}{2} + \frac{1-p}{2}\right) = \sqrt{p}$

$E_2\rho = \sqrt{1-p}\, Z\begin{pmatrix} \frac{1+p}{2} & \frac{1-p}{2} \\ \frac{1-p}{2} & \frac{1-p}{2} \end{pmatrix} = \sqrt{1-p}\begin{pmatrix} \frac{1+p}{2} & \frac{1-p}{2} \\ -\frac{1-p}{2} & -\frac{1-p}{2} \end{pmatrix}$

代入知 $tr(E_2\rho) = \sqrt{1-p}\left(\frac{1+p}{2} - \frac{1-p}{2}\right)$

$= p\sqrt{1-p}$

$\therefore F(\rho, E) = |\sqrt{p}|^2 + |p\sqrt{1-p}|^2$

$= p + p^2(1-p)$

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50. 证明如下，

令$|\phi_i\rangle$和$|\psi_i\rangle$为$\rho_i$和$\sigma_i$的纯化状态，使得有 $F(\rho_i, \sigma_i) = \langle\phi_i|\psi_i\rangle$

引入一个具有标准正交基状态$|i\rangle$的辅助状态，有

$$|\phi\rangle = \sum_i \sqrt{p_i}\,|\phi_i\rangle|i\rangle$$

$$|\psi\rangle = \sum_i \sqrt{q_i}\,|\psi_i\rangle|i\rangle,$$

注意到，$|\phi\rangle$为$\sum_i p_i\rho_i$的一个纯化状态，$|\psi\rangle$为$\sum_i q_i\sigma_i$的一个纯化状态，

则根据 Uhlmann 公式，得到：

$$F\Big(\sum_i p_i\rho_i, \sum_i q_i\sigma_i\Big) \geq |\langle\phi|\psi\rangle|$$

$$= \sum_i \sqrt{p_i q_i}\,|\langle\phi_i|\psi_i\rangle|$$

若令 $p_i = q_i$，代入得

$$F\Big(\sum_i p_i\rho_i, \sum_i p_i\sigma_i\Big) \geq \sum_i p_i\,|\langle\phi_i|\psi_i\rangle|.$$

$$= \sum_i p_i F(\rho_i, \sigma_i)$$