Assignment 5

Algorithm Design and Analysis

December 26, 2021

Notice:

1. This homework is for exercise, without requirements of submissions.
2. When you're asked to give an algorithm, you should do at least the following things:
   - Describe the basic idea of your algorithm in natural language **AND** pseudo-code;
   - Prove the correctness of your algorithm.
   - Analyse the complexity of your algorithm.

1 Load balance

You have some different computers and jobs. For each job, it can only be done on one of two specified computers. The load of a computer is the number of jobs which have been done on the computer. Give the number of jobs and two computer ID for each job. You task is to minimize the max load.

(hint: binary search)

2 Matrix

For a matrix filled with 0 and 1, you know the sum of every row and column. You are asked to give such a matrix which satisfys the conditions.

3 Problem Reduction

There is a matrix with numbers which means the cost when you walk through this point. you are asked to walk through the matrix from the top left point to the right bottom point and then return to the top left point with the minimal cost. Note that when you walk from the top to the bottom you can

just walk to the right or bottom point and when you return, you can just walk to the top or left point. And each point CAN NOT be walked through more than once.

4 Network Cost

For a network, there is one source and one sink. Every edge is directed and has two value c and a. c means the maximum flow of the adge. a is a coefficient number which means that if the flow of the edge is x, the cost is ax^2.

Design an algorithm to get the Minimum Cost Maximum Flow.

5 Choose Numbers

Given a matrix $M = (M_{ij})^{n \times m}$ where $M_{ij} > 0$, for every two neighbor elements, **at least** one of them should be chosen. You are asked minimize the sum of chosen elements that meets the conditions.
(hint: neighbor means share common edge. That is, the neighbors of $M_{i,j}$ are $M_{i-1,j}, M_{i,j-1}, M_{i,j+1}, M_{i+1,j}$).

6 Maximum Weight Subgraph

Given an undirected graph with n weighted vertices and m weighted edges(weights are all positive). A subgraph of a graph is some set of the graph vertices and some set of the graph edges. The set of edges must meet the condition: both ends of each edge from the set must belong to the chosen set of vertices.
The weight of a subgraph is the sum of the weights of its edges, minus the sum of the weights of its vertices.
You need to find the maximum weight of subgraph of given graph.