$w = \sum_{i=1}^{n} a_i y^{(i)} x^{(i)}$

$\sum_{i=1}^{n} a_i y^{(i)} = 0$

$\theta_D(\alpha) = \min_{w,b} L(w,b,\alpha) = \min_{w,b} \frac{1}{2}\|w\|_2^2 - \sum_{i=1}^{n} a_i [y^{(i)}(w^T x^{(i)} + b) - 1]$

当 $\frac{\partial L}{\partial w} = 0$, $\frac{\partial L}{\partial b} = 0$ 时 即 $w = \sum_{i=1}^{n} a_i y^{(i)} x^{(i)}$, $\sum_{i=1}^{n} a_i y^{(i)} = 0$ 时, $L(w,b,\alpha)$

取最小值.

即 $\theta_D(\alpha) = \frac{1}{2}\| \sum_{i=1}^{n} a_i y^{(i)} x^{(i)} \|_2^2 - \sum_{i=1}^{n} a_i [y^{(i)}\{(\sum_{i=1}^{n} a_i y^{(i)} x^{(i)})^T x^{(i)} + b\} - 1]$

$= \frac{1}{2}\| \sum_{i=1}^{n} a_i y^{(i)} x^{(i)} \|_2^2 - \sum_{i=1}^{n} [a_i y^{(i)} (\sum_{i=1}^{n} a_i y^{(i)} x^{(i)})^T x^{(i)} + a_i y^{(i)} b] + \sum_{i=1}^{n} a_i$

$= \frac{1}{2}\| \sum_{i=1}^{n} a_i y^{(i)} x^{(i)} \|_2^2 - \sum_{i=1}^{n} a_i y^{(i)} (\sum_{j=1}^{n} a_j y^{(j)} (x^{(j)})^T x^{(i)}) + \sum_{i=1}^{n} a_i$

$= \frac{1}{2} (\sum_{i=1}^{n} a_i y^{(i)} x^{(i)})^T (\sum_{i=1}^{n} a_i y^{(i)} x^{(i)}) - \sum_{i=1}^{n} a_i y^{(i)} [\sum_{j=1}^{n} a_j y^{(j)} (x^{(j)})^T x^{(i)}] + \sum_{i=1}^{n} a_i$

$= \frac{1}{2} \sum_{i=1}^{n} a_i y^{(i)} (x^{(i)})^T (\sum_{j=1}^{n} a_j y^{(j)} x^{(j)}) - \sum_{i=1}^{n} a_i y^{(i)} [\sum_{j=1}^{n} a_j y^{(j)} (x^{(j)})^T x^{(i)}] + \sum_{i=1}^{n} a_i$

$(x^{(i)})^T x^{(j)} = (x^{(j)})^T x^{(i)}$

$= -\frac{1}{2} \sum_{i=1}^{n} a_i y^{(i)} (\sum_{j=1}^{n} a_j y^{(j)} (x^{(j)})^T x^{(i)}) + \sum_{i=1}^{n} a_i = \sum_{i=1}^{n} a_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} y^{(i)} y^{(j)} a_i a_j (x^{(i)})^T x^{(j)}$