矩阵分析与应用 第五次作业

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1. 解，由题意可知：

① Householder reduction.

$$u_1 = A_{*1} - \|A_{*1}\| e_1 = \begin{pmatrix} -2 \\ -2 \\ 2 \end{pmatrix}$$

$$\therefore R_1 = I - 2\frac{u_1 u_1^T}{u_1^T u_1} = \frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$

$$\therefore R_1 A = \begin{pmatrix} 3 & 15 & 0 \\ 0 & -9 & 54 \\ 0 & 12 & 3 \end{pmatrix}, \quad \therefore A_2 = \begin{pmatrix} -9 & 54 \\ 12 & 3 \end{pmatrix}$$

$$\Rightarrow u_2 = \cancel{A_{*2} - \|A_{*2}} \; [A_2]_{*1} - \|[A_2]_{*1}\| e_1 = \begin{pmatrix} -24 \\ 12 \end{pmatrix}$$

$$\therefore \hat{R}_2 = I - 2\frac{u_2 u_2^T}{u_2^T u_2} = \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$$\hat{R}_2 A_2 = \begin{pmatrix} 25 & -30 \\ 0 & 45 \end{pmatrix}$$

$$\therefore R = R_2 R_1 A = \begin{pmatrix} 3 & 15 & 0 \\ 0 & 25 & -30 \\ 0 & 0 & 45 \end{pmatrix}$$

其QR分解中，$Q = (R_2 R_1)^T = \begin{pmatrix} \frac{1}{3} & \frac{14}{15} & -\frac{2}{15} \\ -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{2}{15} & \frac{11}{15} \end{pmatrix}$

② Givens Reduction

根据分析，有 $T_{12} = \begin{pmatrix} \frac{1}{\sqrt5} & -\frac{2}{\sqrt5} & 0 \\ \frac{2}{\sqrt5} & \frac{1}{\sqrt5} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad T_{12}A_{*1} = \begin{pmatrix} \sqrt5 \\ 0 \\ 2 \end{pmatrix}$

$T_{13} = \begin{pmatrix} \frac{\sqrt5}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 \\ -\frac{2}{3} & 0 & \frac{\sqrt5}{3} \end{pmatrix}$ $\quad \therefore T_{13}T_{12}A_{*1} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$

$$T_1 = T_{13}T_{12} = \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ \frac{2}{\sqrt5} & \frac{1}{\sqrt5} & 0 \\ -\frac{2}{3\sqrt5} & \frac{4}{3\sqrt5} & \frac{\sqrt5}{3} \end{pmatrix}$$

$T_1A = \begin{pmatrix} 3 & 15 & 0 \\ 0 & \frac{33}{\sqrt5} & -\frac{48}{\sqrt5} \\ 0 & -\frac{6}{\sqrt5} & \frac{111}{\sqrt5} \end{pmatrix} \Rightarrow$ 设 $A_2 = \begin{pmatrix} \frac{33}{\sqrt5} & -\frac{48}{\sqrt5} \\ -\frac{6}{\sqrt5} & \frac{111}{\sqrt5} \end{pmatrix}$

$\therefore T_{23} = \begin{pmatrix} \frac{11}{5\sqrt5} & \frac{-2}{5\sqrt5} \\ \frac{2}{5\sqrt5} & \frac{11}{5\sqrt5} \end{pmatrix}$ $\quad \therefore T_{23}A_2 = \begin{pmatrix} 15 & -30 \\ 0 & 45 \end{pmatrix}$

$$\therefore T = \begin{pmatrix} 1 & 0 \\ 0 & T_{23} \end{pmatrix} T_1 = \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ \frac{14}{15} & \frac{1}{3} & -\frac{2}{15} \\ -\frac{2}{15} & \frac{2}{3} & \frac{11}{15} \end{pmatrix}$$

$$Q. \therefore Q = T^T = \begin{pmatrix} \frac{1}{3} & \frac{14}{15} & -\frac{2}{15} \\ -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{2}{15} & \frac{11}{15} \end{pmatrix}$$

$$R = T_2T_1A = \begin{pmatrix} 3 & 15 & 0 \\ 0 & 15 & -30 \\ 0 & 0 & 45 \end{pmatrix}$$

2. 利用 Househoder reduction，分解为：

$$u = A_{*1} - \|A_{*1}\| e_1 = \begin{pmatrix} -1 \\ 2 \\ -2 \\ 1 \end{pmatrix}$$

$$R_1 = I - 2\frac{uu^T}{u^T u} = \frac{1}{5}\begin{pmatrix} 4 & 2 & -2 & 1 \\ 2 & 1 & 4 & -2 \\ -2 & 4 & 1 & 2 \\ 1 & -2 & 2 & 4 \end{pmatrix}$$

$$R_1 A = \begin{pmatrix} 5 & -15 & 5 \\ 0 & 10 & -5 \\ 0 & -10 & 2 \\ 0 & 5 & 14 \end{pmatrix} \qquad 令 A_1 = \begin{pmatrix} 10 & -5 \\ -10 & 2 \\ 5 & 14 \end{pmatrix}$$

则此时 $u = [A_1]_{*1} - \|[A_1]_{*1}\| e_1$

$$= \begin{pmatrix} -5 \\ -10 \\ 5 \end{pmatrix}$$

$$R_2' = I - 2\frac{uu^T}{u^T u} = \frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ -2 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

其中 $R_2' A_1 = \begin{pmatrix} 15 & 0 \\ 0 & 12 \\ 0 & 9 \end{pmatrix}$ 令 $A_2 = \begin{pmatrix} 12 \\ 9 \end{pmatrix}$

则此时 $u = [A_2]_{*1} - \|[A_2]_{*1}\| e_1 = \begin{pmatrix} -3 \\ 9 \end{pmatrix}$

$$\therefore R_3' = I - 2\frac{uu^T}{u^T u} = \frac{1}{5}\begin{pmatrix} 4 & 3 \\ 3 & -4 \end{pmatrix}$$

$\therefore Q = (R_3 R_2 R_1)^T$

则正交基为 $Q_{m\times n} = \frac{1}{15}\begin{pmatrix} 12 & 9 & 0 \\ 6 & -8 & -5 \\ -6 & 8 & 2 \\ 3 & -4 & 14 \end{pmatrix}$

即 $R(A)$ 的正交基为 $\frac{1}{15}\begin{pmatrix} 12 & 9 & 0 \\ 6 & -8 & -5 \\ -6 & 8 & 2 \\ 3 & -4 & 14 \end{pmatrix}$