矩阵分析与应用 第四次作业

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12. 解，由题意可知，首先对矩阵A进行QR分解.

① $k=1$: $r_{11} \leftarrow \|a_1\| = \sqrt{3}$

∴有 $q_1 = \frac{a_1}{r_{11}} = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\\0\end{pmatrix}$

② $k=2$, $\Rightarrow r_{12} = q_1^T a_2 = \sqrt{3}$, $q_2 = a_2 - r_{12}q_1 = \begin{pmatrix}-1\\1\\0\\1\end{pmatrix}$

$r_{22} = \|q_2\| = \sqrt{3}$, 因标准化后 $q_2 = \frac{q_2}{r_{22}} = \frac{1}{\sqrt{3}}\begin{pmatrix}-1\\1\\0\\1\end{pmatrix}$

③ $k=3$. $r_{13} = q_1^T a_3 = -\sqrt{3}$, $r_{23} = q_2^T a_3 = \sqrt{3}$

$q_3 = a_3 - r_{13}q_1 - r_{23}q_2 = \begin{pmatrix}1\\1\\-2\\0\end{pmatrix}$

$r_{33} = \|q_3\| = \sqrt{6}$　　$q_3 = \frac{q_3}{r_{33}} = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\1\\-2\\0\end{pmatrix}$

∴得到：$Q = \frac{1}{\sqrt{6}}\begin{pmatrix}\sqrt{2} & -\sqrt{2} & \sqrt{2}\\ \sqrt{2} & \sqrt{2} & \sqrt{2}\\ \sqrt{2} & 0 & -2\sqrt{2}\\ 0 & \sqrt{2} & 0\end{pmatrix}$

$R = \begin{pmatrix}\sqrt{3} & \sqrt{3} & -\sqrt{3}\\ 0 & \sqrt{3} & \sqrt{3}\\ 0 & 0 & \sqrt{6}\end{pmatrix}$.

(2) 根据相应的定理，要求解 $Ax=b$，实际上只需要求解 $Rx = Q^Tb$

其中，$Q^Tb = \frac{1}{\sqrt{3}}\begin{pmatrix}3\\1\\0\end{pmatrix}$　　即有 $Rx = Q^Tb$

$\Rightarrow x = \begin{pmatrix}\frac{2}{3}\\ \frac{1}{3}\\ 0\end{pmatrix}$

金美莱

16. 解. 由题意可知, 有.

(a) the orthogonal projection of u onto span$\{v\}$ is:

$$vv^Tu = \begin{pmatrix} 1 \\ 4 \\ 0 \\ -1 \end{pmatrix} \begin{pmatrix} 1 & 4 & 0 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ 3 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & 0 & -1 \\ 4 & 16 & 0 & -4 \\ 0 & 0 & 0 & 0 \\ -1 & -4 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ 3 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 12 \\ 0 \\ -3 \end{pmatrix}$$

(b) the orthogonal projection of v onto span$\{u\}$ is:

$$uu^Tv = \begin{pmatrix} -2 \\ 1 \\ 3 \\ -1 \end{pmatrix} \begin{pmatrix} -2 & 1 & 3 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ 0 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -2 & -6 & 2 \\ -2 & 1 & 3 & -1 \\ -6 & 3 & 9 & -3 \\ 2 & -1 & -3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ 0 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} -6 \\ 3 \\ 9 \\ -3 \end{pmatrix}$$

(c) the orthogonal projection of u onto $v_\perp$ is:

$$(I - vv^T)u = Iu - vv^Tu = u - vv^Tu$$

$$= \begin{pmatrix} -2 \\ 1 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 12 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} -5 \\ -11 \\ 3 \\ 2 \end{pmatrix}$$

(d) the orthogonal projection of v onto $u_\perp$ is:

$$(I - uu^T)v = Iv - uu^Tv = v - uu^Tv = \begin{pmatrix} 1 \\ 4 \\ 0 \\ -1 \end{pmatrix} - \begin{pmatrix} -6 \\ 3 \\ 9 \\ -3 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ -9 \\ 2 \end{pmatrix}$$