

# Vision and Perception

## First Exercises of second part of the course.

(The whole set of the second part of the course is delivered the 30th of April.)

### Instructions:

Only students already enrolled can do these exercises. You can consult the book of Hartley and Zisserman, we have essentially done the following parts, though essential paragraphs:

1. Part 0
  - (a) Chapter 2 - Sections 2.1-2.4, Section 2.7
  - (b) Chapter 3 - Section 3.1, 3.2.1
  - (c) Chapter 4 - Section 4.1

You can obviously look into the video channel.

### Exercise 1

1. Construct three collinear points  $x_1$ ,  $x_2$  and  $x_3$  and show they are on a line  $\ell$ .
2. Show that if  $H$  is a projective point transformation, then the transformation applied to the points  $x_1$ ,  $x_2$  and  $x_3$  and suitably to the line  $\ell$  maps the points into points  $x'_1$ ,  $x'_2$  and  $x'_3$  and the line into the line  $\ell'$  such that  $x'_1$ ,  $x'_2$  and  $x'_3$  are on  $\ell'$ .

### Exercise 2

Consider the following image (you can load from Google-Images any similar image you prefer.)



Figure 1: Perspectively distorted tiles

Compute four segments on the parallel sides of the tiles – you need two segments in one direction and two segments in the orthogonal direction, with respect to the scene. Using the extreme points of the two segments build the four lines and the corresponding pair of vanishing points.

The two vanishing points are the projections of the ideal points in the image, at the intersection of the parallel lines. Use the vanishing points to compute the vanishing line, that is, the projection of the line at infinity in the image.

You want to recover parallel lines on the image so that the tiles appear to be parallel as in the following image:



Figure 2: Tiles are parallel in the image

To obtain the above image you have to first define an homography  $H = H_A H_P$ , such that  $H^{-T} \ell^T = (0, 0, 1)^T$ .

1. The homography  $H_P$ , should map the vanishing line back to the line at infinity

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \ell_1 & \ell_2 & \ell_3 \end{bmatrix} \quad (1)$$

The last row is the vanishing line  $\ell$  that you have found.

2.  $H_A$  is any affine transformation.
3. You want that the last row of  $H$  is  $\ell$ . Verify that  $H^{-T} \ell^T$  results in the line at infinity, then apply  $H_P$  to the image, using the DLT algorithm to obtain the image in Figure 2. The DLT algorithm would be the same as we have seen for points, and we have 4 lines. Yet, if you are not able to use the DLT algorithm you can use `Skimage.transform.warp`. To those who are able to devise how to use the DLT algorithm also for lines, a big bonus.
4. After warping have we obtained a metric rectification? If not what would we need?

For the above exercise you can see Hartley and Zisserman book Section 2.4. Implement the solutions in Colab.

### Exercise 3

1. Show that the ratio of length of parallel line segments is preserved.
2. Show that a circle can be computed using 3 points plus the two circular points.