Literature review on non-spherical particles

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Chhabra, Agarwal, Sinha - 1998

Critical review of a selection of widely used correlation formulae for the estimation of c_D of non-spherical particles in incompressible viscous flow.

- **Data points:** 1900
- **Sphericity:** 0.09 - 1
- **Re:** $10^{-4} - 5 \cdot 10^{5}$
- **Methods analyzed:** Ganser [1993], Haider and Levenspiel [1989], Hartman [1994], Chien [1994] and Swamme and Ojha [1991] - Ordered by decreasing performance.

Chhabra, Agarwal, Sinha - 1998

Conclusions:

- **Reference length:** equal volume sphere diameter $d_{\mathsf{v}} = \sqrt{\frac{6V}{\pi}}$
- **Shape parameter:** Swamme and Ojha uses the *Corey shape factor* (β) while the other four uses the *Sphericity* Φ
- **Swamme and Ojha** Poor prediction (only one set of data was used), valid only for $Re > 1$.
- **Ganser:** Best overall method if Lasso and Weideman's results are not taken into account (hollow cylinder and agglomerates made of spherical particles). It's the only one that takes in consideration the **orientation of the particle**.
  Nb. After Chhabra's review, this will be a minimum requisite for a drag model.

Holzer and Sommerfeld - 2008

- Huge literature and data review (2061 values)
- Interpolation of different previous models
- Arbitrary shape (and Orientation!)
- Valid for all the Subcritical Regime

$$c_D = \frac{8}{Re}\frac{1}{\sqrt{\Phi_{//}}} + \frac{16}{Re}\frac{1}{\Phi} + \frac{3}{\sqrt{Re}}\frac{1}{\Phi^{\frac{3}{4}}} + 0.4210^{0.4(-\log\Phi)^{0.2}}\frac{1}{\Phi_{\perp}}$$

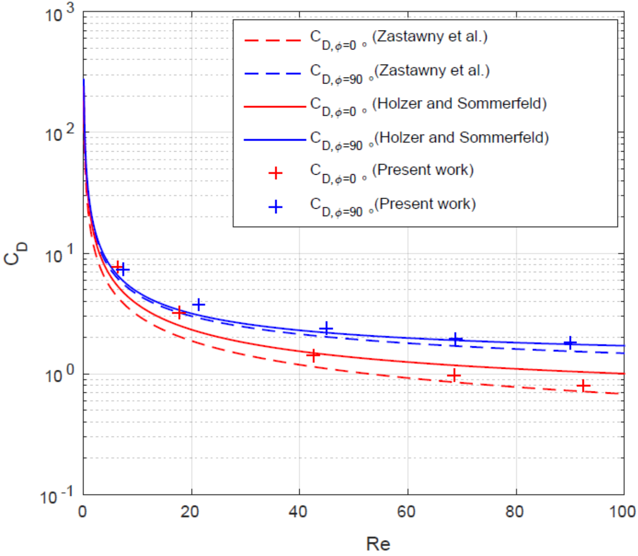
Holzer and Sommerfeld - 2008

	Haider and Levenspiel [1]	Ganser incl. Leith [2]	Present Eq. (9)	Present Eq. (10)
Mean relative deviation				
Sphere (683 values)	6.59%	10.9%	9.17%	9.17%
Isometric particles (655 values)	6.65%	6.46%	10.5%	10.9%
Cuboids and cylinders (337 values)	42.3%	38.4%	27.2%	29%
Disks and plates (386 values)	$2\ 10^3$%	$1.8\ 10^3$%	17.7%	16.8%
All Data (2061 values)	383%	348%	14.1%	14.4%
Maximum relative deviation				
Sphere (683 values)	44%	43%	45%	45%
Isometric particles (655 values)	50%	55%	68%	68%
Cuboids and cylinders (337 values)	$1.1\ 10^3$%	$1.1\ 10^3$%	88%	88%
Disks and plates (386 values)	$2.1\ 10^4$%	$2.4\ 10^4$%	75%	75%

- Works better than the other for disks and plates
- Ganser works better for isometric particles
- Works well overall

Sanjeevi, Padding and Kuipers – 2015

DNS of a prolate Ellipsoid with $\Phi = 0.886$



Stokes Regime

$$c_D = \frac{8}{Re}\frac{1}{\sqrt{\Phi_\perp}} + \frac{16}{Re}\frac{1}{\Phi}$$

- Leith's (or Ganser's) model – 1993
- Valid for low Re (≤ 10)
- For sphere ($\Phi_\perp = \Phi_{//} = \Phi$) degenerates to Stokes' analytical solution
- Modification: $\Phi_\perp \rightarrow \Phi_{//}$ (for better approximation of the c_D as a function of the particle orientation)

Newton Regime

Blasius - 1908

Friction drag for plates and disks (small cross-sectional area)

$$c_D = 1.327 \cdot 2 \left(\frac{8}{9}\right)^{\frac{1}{4}} \pi^{\frac{1}{4}} \left(\frac{\text{depth}}{\text{length}}\right)^{\frac{1}{4}} \frac{1}{\Phi^{\frac{3}{4}}} \frac{1}{\sqrt{Re}}$$

for square plates: $c_D = 3.43/(\Phi^{\frac{3}{4}}\sqrt{Re})$

Tran-Cong (2004) and Ganser (1993)

Term for high Re proportional to the projected cross-sectional area (Tran-Cong) with the same factor of proportionality of Ganser.

$$c_D = 0.4210^{0.4(-\log\Phi)^{0.2}} \frac{1}{\Phi_\perp}$$

Symbols

Sphericity

$$\Phi = \frac{A_{\text{eq sphere}}}{A_{\text{particle}}}$$

Ratio between the surface area of the volume-equivalent sphere and the area of the actual particle

Crosswise Sphericity

$$\Phi_{\perp} = \frac{A_{\text{eq sphere, cross}}}{A_{\text{particle},\perp}}$$

Ratio between the cross-sectional area of the volume-equivalent sphere and the projected cross-sectional area of the actual particle

Symbols

Lengthwise Sphericity

$$\Phi_{//} = \frac{A_{\text{eq sphere, cross}}}{\Delta A}$$

$$\Delta A = \frac{A_{\text{particle}}}{2} - \bar{L}_{//}$$

Ratio between the cross-sectional area of the volume-equivalent sphere and the difference between half the surface area and the *average* mean longitudinal projected cross-sectional area of the actual particle.
Since $L_{//}$ depends on the angle of view, an arithmetic average over an entire revolution is used

Recent work – camera-based methods

China - 2017

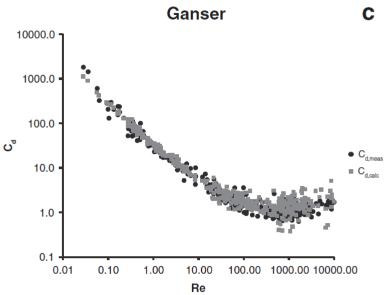
Correlation	Prediction error		
	r	s_1	s_2
Haider and Levenspiel [23]	13.84%	9.85	2.47
Swamee and Ojha [24]	21.27%	22.22	7.07
Ganser [25]	8.41%	3.46	0.73
Chien [1]	17.27%	13.21	3.23
Yow et al. [29]	47.77%	130.24	/
Holzer and Sommerfeld [36]	11.07%	5.75	1.32
Our equation	4.91%	1.45	0.30

Limitations:

- Only three major regular particle shapes investigated (sphere, cube, cylinder)

Recent work – camera-based methods

Italy (Bari) - 2015



Limitations:

- Small data sample
- No comparison with Holzer and Sommerfeld (although good agreement with Ganser)

To do list

- Read Loth-2008 (Snow??)
- Replicate the work of the workshop on SU2
- (Try the H&S formula on the Italian data)