Title

Subtitle

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Outline

1. Objective
2. Method
   - Data Analysis
3. Result

Initial Research Question

Lagrangian Particle Tracking of Snowflakes

Find a general formula for the drag coefficient of blowing snow

$$c_D = c_D(Re, \text{parameters})$$

And implement it in PoliMIce with some general rule for the choice of the main parameters.

Literature Review

What is not available

- c_D formula tuned for snowflakes
- Experimental measurements of snowflakes in the blowing regime
- Shape of the typical snowflake (they are almost unique)

What is available

- c_D formulae tuned for arbitrary shaped bodies
- Experimental measurements of falling snowflakes terminal velocities
- Bulk parameters for the description of the shape and the orientation of a general particle

Updated Research Question

Drag coefficient formula

Find a suitable existing model to infer the main parameters in the falling regime and use them in the blowing one

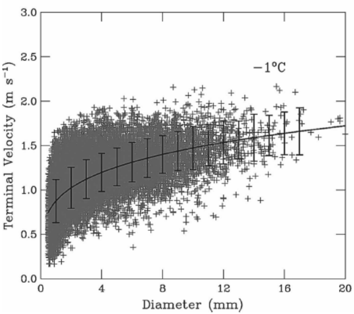
Snow Parameters

Develop a method to find the discrete set of properties that on average describe a certain *cloud*

Outline

Data Generation

Reproduce an artificial data set from the *velocity-diameter* relation of Brandes et al - 2008. The diameter distribution is not taken into account.



The Drag Model

Ganser - 1993

$$c_D = K_2 \left(\frac{24}{ReK_1K_2}(1 + 0.1118(ReK_1K_2)^{0.6567}) + \frac{0.4305}{1 + \frac{3305}{ReK_1K_2}} \right)$$

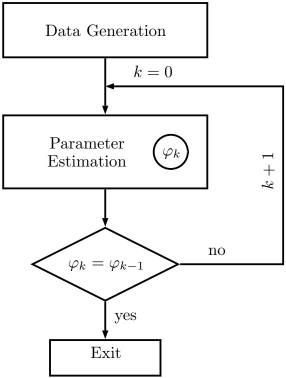
Stokes' Shape Factor

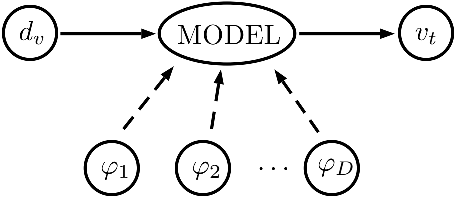
$$K_1 = \left(\frac{1}{3}\frac{d_n}{d_v} + \frac{2}{3}\Phi^{-\frac{1}{2}} \right)^{-1}$$

Newton's Shape Factor

$$K_2 = 10^{1.8148(-log(\Phi))^{0.5743}}$$

General Scheme





- (d_v, v_t): Experimental data
- $\varphi_1, \varphi_2 \dots \varphi_D = \underline{\varphi}$: Free parameters
- D: Number of parameters $\sim$ problem dimension

Bayesian Approach

Bayes' Theorem

$$\underbrace{prob(\delta HP|D,I)}_{Posterior\,Probability} \propto \underbrace{prob(D|\delta HP,I)}_{Likelihood\,Function} \cdot \underbrace{prob(\delta HP|I)}_{Prior\,Probability}$$

- *Prior:* State of knowledge (or ignorance) before analyzing the data
- *Likelihood:* Experimental measurements
- *Posterior:* State of knowledge in light of the data

Marginalization Equation and Sum Rule

$$prob(HP|D,I) = \int_{-\infty}^{+\infty} prob(\delta HP|D,I)\delta HP = 1$$

Problem formulation

Prior

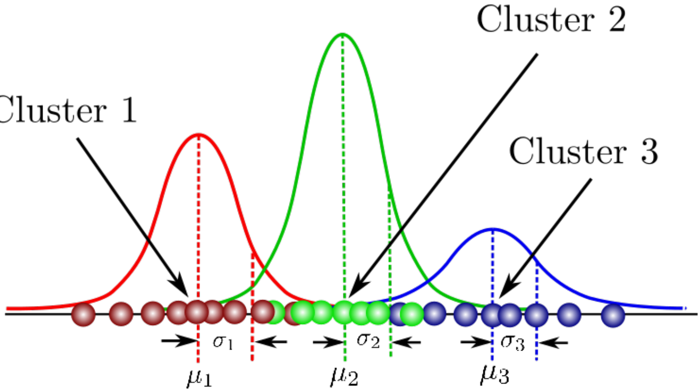
$prob(HP|I) = prob(\underline{\varphi})$: *Uniform* in the range of physical solution.

$$= \begin{cases} \dfrac{1}{(\varphi_{1,M} - \varphi_{1,m})(\varphi_{2,M} - \varphi_{2,m}) \dots (\varphi_{D,M} - \varphi_{D,m})} \\ 0 \end{cases}$$

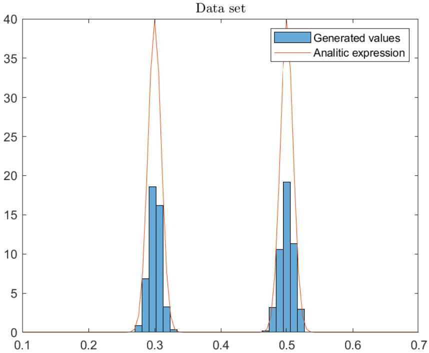
Likelihood

$$prob(D|HP, I) = \prod_{n=1}^{N_{data}} \sum_{k=1}^{N_{modes}} \pi_k \; e^{-\frac{1}{2}\left(\frac{v_{t,n} - v_t(d_{v,n}, \underline{\varphi}_k)}{\sigma_n}\right)^2}$$

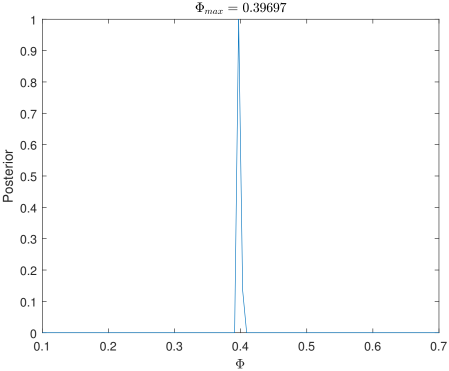
Gaussian Mixture Models



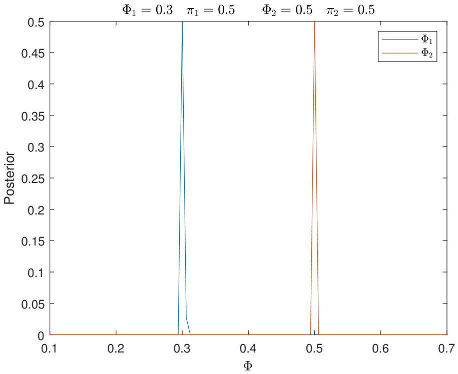
Validation


Data set
Generated values
Analitic expression
40
35
30
25
20
15
10
5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Validation

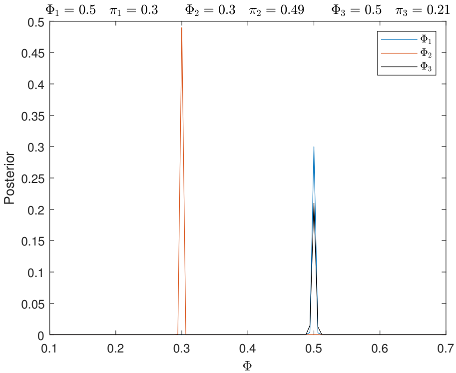

$\Phi_{max} = 0.39697$
Posterior
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Φ

Validation



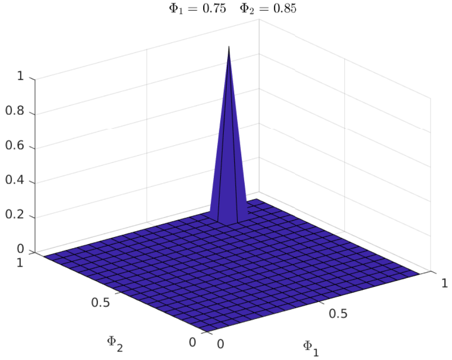
$\Phi_1 = 0.3 \quad \pi_1 = 0.5 \qquad \Phi_2 = 0.5 \quad \pi_2 = 0.5$
Posterior
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Φ
Φ_1
Φ_2

Validation

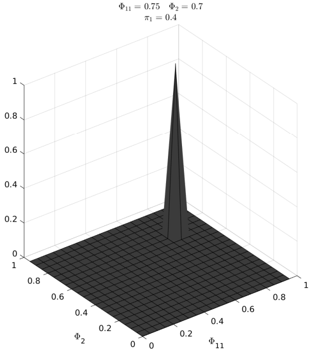


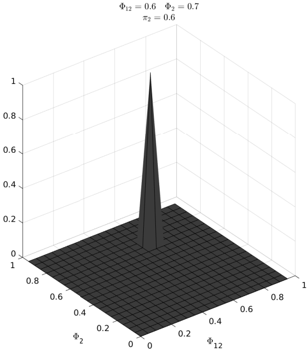
$\Phi_1 = 0.5 \quad \pi_1 = 0.3 \qquad \Phi_2 = 0.3 \quad \pi_2 = 0.49 \qquad \Phi_3 = 0.5 \quad \pi_3 = 0.21$
Posterior
Φ
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Φ_1
Φ_2
Φ_3

Brandes et al. - 2008

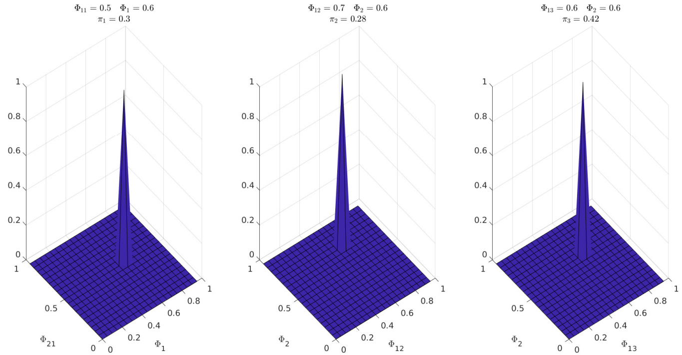


Brandes et al. - 2008





Brandes et al. - 2008



Brandes et al. - 2008