
# Title

Subtitle

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# Outline

# Objective

# Initial Research Question



## Lagrangian Particle Tracking of Snowflakes

Find a general formula for the drag coefficient of blowing snow

$$c_D = c_D(Re, \text{parameters})$$

And implement it in PoliMIce with some general rule for the choice of the main parameters.

# Literature Review



## What is not available

- $c_D$ formula tuned for snowflakes
- Experimental measurements of blowing snowflakes velocities
- Shape of the typical snowflake (they are almost unique)

## What is available

- $c_D$ formulae tuned for arbitrary shaped bodies
- Experimental measurements of falling snowflakes terminal velocities
- General parameters that describe the shape (CAMBIARE)

# Updated Research Question



## Drag coefficient formula

Find a suitable existing model to infer the main parameters in the falling regime and use them in the blowing one

## Snow Parameters

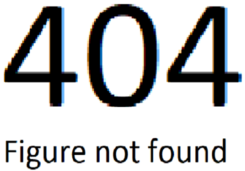
Develop a method to find the discrete set of properties that on average describe a certain *cloud*

## Outline

# Data Generation



Reproduce an artificial data set from the *velocity-diameter* relation of Brandes et al - 2008. The diameter distribution is not taken into account.



# The Drag Model



## Ganser - 1993

$$c_D = K_2 \left( \frac{24}{ReK_1K_2}(1 + 0.1118(ReK_1K_2)^{0.6567}) + \frac{0.4305}{1 + \frac{3305}{ReK_1K_2}} \right)$$

## Stokes' Shape Factor

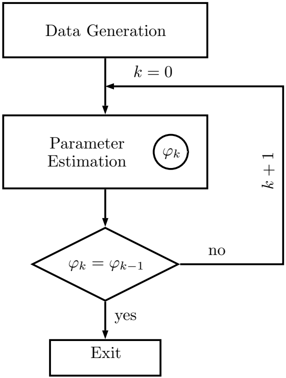
$$K_1 = \left( \frac{1}{3}\frac{d_n}{d_v} + \frac{2}{3}\Phi^{-\frac{1}{2}} \right)^{-1}$$

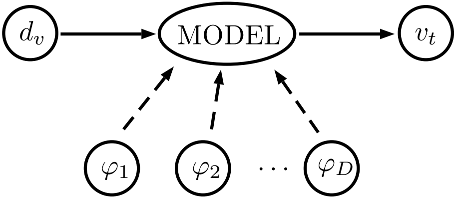
## Newton's Shape Factor

$$K_2 = 10^{1.8148(-log(\Phi))^{0.5743}}$$

# General Scheme






- $(d_v, v_t)$: Experimental data
- $\varphi_1, \varphi_2 \dots \varphi_D = \underline{\varphi}$: Free parameters
- $D$: Number of parameters $\sim$ problem dimension

## Bayes' Theorem

$$\underbrace{prob(\delta HP|D,I)}_{Posterior\,Probability} \propto \underbrace{prob(D|\delta HP,I)}_{Likelihood\,Function} \cdot \underbrace{prob(\delta HP|I)}_{Prior\,Probability}$$

- *Prior:* State of knowledge (or ignorance) before analyzing the data
- *Likelihood:* Experimental measurements
- *Posterior:* State of knowledge in light of the data

## Marginalization Equation and Sum Rule

$$prob(HP|D,I) = \int_{-\infty}^{+\infty} prob(\delta HP|D,I)\delta HP = 1$$

# Problem formulation



## Prior

$prob(HP|I) = prob(\underline{\varphi})$ : *Uniform* in the range of physical solution.

$$= \begin{cases} \dfrac{1}{(\varphi_{1,M} - \varphi_{1,m})(\varphi_{2,M} - \varphi_{2,m}) \dots (\varphi_{D,M} - \varphi_{D,m})} \\ 0 \end{cases}$$

## Likelihood

$$prob(D|HP,I) = \prod_{n=1}^{N_{data}} \sum_{k=1}^{N_{modes}} \pi_k \; e^{-\frac{1}{2}\left(\frac{v_{t,n} - v_t(d_{v,n}, \underline{\varphi}_k)}{\sigma_n}\right)^2}$$

## Gaussian Mixture Models