



POLITECNICO
MILANO 1863

Title

Subtitle

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- ① Objective
- ② Method
 - Data Analysis
- ③ Result

Lagrangian Particle Tracking of Snowflakes

Find a general formula for the drag coefficient of blowing snow

$$c_D = c_D(Re, \text{parameters})$$

And implement it in PoliMIce with some general rule for the choice of the main parameters.

What is not available

- c_D formula tuned for snowflakes
- Experimental measurements of snowflakes in the blowing regime
- Shape of the typical snowflake (they are almost unique)

What is available

- c_D formulae tuned for arbitrary shaped bodies
- Experimental measurements of falling snowflakes terminal velocities
- Bulk parameters for the description of the shape and the orientation of a general particle

Drag coefficient formula

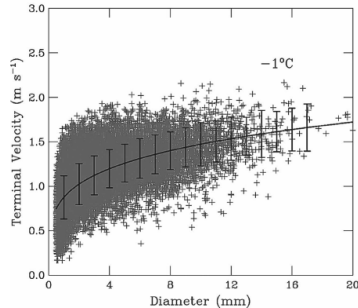
Find a suitable existing model to infer the main parameters in the falling regime and use them in the blowing one

Snow Parameters

Develop a method to find the discrete set of properties that on average describe a certain *cloud*

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Reproduce an artificial data set from the *velocity-diameter* relation of Brandes et al - 2008. The diameter distribution is not taken into account.



Ganser - 1993

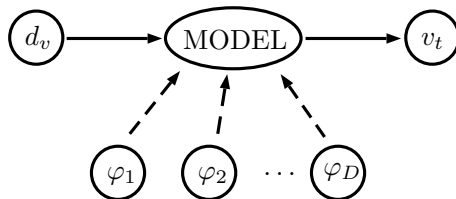
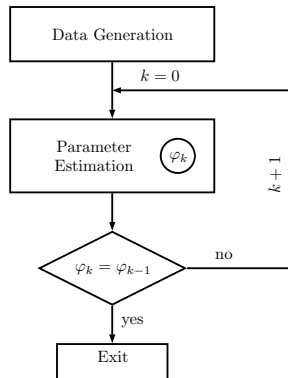
$$c_D = K_2 \left(\frac{24}{Re K_1 K_2} (1 + 0.1118 (Re K_1 K_2)^{0.6567}) + \frac{0.4305}{1 + \frac{3305}{Re K_1 K_2}} \right)$$

Stokes' Shape Factor

$$K_1 = \left(\frac{1}{3} \frac{d_n}{d_v} + \frac{2}{3} \Phi^{-\frac{1}{2}} \right)^{-1}$$

Newton's Shape Factor

$$K_2 = 10^{1.8148(-\log(\Phi))^{0.5743}}$$



- (d_v, v_t) : Experimental data
- $\varphi_1, \varphi_2 \dots \varphi_D = \underline{\varphi}$: Free parameters
- D : Number of parameters \sim problem dimension

Bayes' Theorem

$$\underbrace{prob(\delta HP|D, I)}_{\text{PosteriorProbability}} \propto \underbrace{prob(D|\delta HP, I)}_{\text{LikelihoodFunction}} \cdot \underbrace{prob(\delta HP|I)}_{\text{PriorProbability}}$$

- *Prior*: State of knowledge (or ignorance) before analyzing the data
- *Likelihood*: Experimental measurements
- *Posterior*: State of knowledge in light of the data

Marginalization Equation and Sum Rule

$$prob(HP|D, I) = \int_{-\infty}^{+\infty} prob(\delta HP|D, I) \delta HP = 1$$

Prior

$prob(HP|I) = prob(\underline{\varphi})$: *Uniform* in the range of physical solution.

$$= \begin{cases} \frac{1}{(\varphi_{1,M} - \varphi_{1,m})(\varphi_{2,M} - \varphi_{2,m}) \dots (\varphi_{D,M} - \varphi_{D,m})} \\ 0 \end{cases}$$

Likelihood

$$prob(D|HP, I) = \prod_{n=1}^{N_{data}} \sum_{k=1}^{N_{modes}} \pi_k e^{-\frac{1}{2} \left(\frac{v_{t,n} - v_t(d_{v,n}, \underline{\varphi}_k)}{\sigma_n} \right)^2}$$

